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ABSTRACT

Polarization of Time and Income – A Multidimensional Approach with Well-Being Gap and Minimum 2DGAP: German Evidence

A growing polarization of society accompanied with an erosion of the middle class experiences more and more attention at least in the German recent economic and social policy discussion. Our study contributes to the polarization discussion with respect to multidimensional theoretical measurement and empirical application in two ways: First, we propose extended multidimensional polarization indices based on a CES-type well-being function and present a new measure to multidimensional polarization, the mean minimum polarization gap 2DGAP. This polarization intensity measure provides transparency with regard to each singular attributes – important for targeted policies – *and* ensures at the same time its interdependent relations. Second, the empirical application – in addition to the traditional income measure – incorporates time as a fundamental resource for any activity. In particular, genuine personal leisure time will take care of social participation in the spirit of social inclusion/exclusion and Amartya Sen's capability approach. Instead of arbitrarily choosing the attributes' parameters in the CES well-being function, the interdependent relations of time and income will be evaluated by German Society. With the German Socio-Economic Panel (SOEP) and detailed time use diary data of the available German Time Use Survey (GTUS) 1991/92 and 2001/02 we quantify available and extended multidimensional polarization measures as well as our new approach for the polarization development of the working poor and the working rich in Germany. Results: Genuine personal leisure time in addition to income is an important polarization attribute. Compensation is of economic and static significance. In particular supported by the new minimum 2DGAP approach, multidimensional polarization increased over that decade in Germany.

JEL Classification: I32, D31, J22

Keywords: multidimensional polarization, intensity of time and income poverty and affluence, interdependent multidimensional time and income poverty and affluence, minimum multidimensional polarization gap (2DGAP), extended economic well-being, satisfaction/happiness, working poor, CES well-being function, German Socio-Economic Panel, German Time Use Surveys 1991/92 and 2001/02

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Polarization of Time and Income – A Multidimensional Approach with Well-Being Gap and Minimum 2DGAP: German Evidence

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1 Introduction

A growing polarization in society accompanied with an erosion of the middle class experiences more and more attention, at least in Germany, in recent economic and social policy discussion. A drifting apart has many far reaching and multitude consequences for quality of life and requires engagement on many levels. If the complex topic is focused on economics the question is about a growing disperse of the "income scissors" which describes that "the poor are going to be poorer and the rich to be richer" (Grabka and Frick 2008). Though the case is important and there is a large literature about inequality and in particular income inequality with focus on the poor, there are only a few theoretical and empirical studies which explicitly encompass both poles of the income distribution (but see the approaches in section 2 of this study). Even less empirical evidence and theoretical approaches about polarization can be found when not only income but a multidimensional approach to economic well-being is considered, though multidimensional approaches have proven to be important for extended poverty analyses (Alkire and Foster 2011, Chakravarty 2009, Chakravarty and Silber 2008, Bourguignon and Chakravarty 2003, Atkinson 2003).

Our study contributes to the polarization discussion with respect to multidimensional theoretical measurement and empirical application in two ways:

First, methodical we propose extended multidimensional polarization indices based on a CES-type well-being function and present a new measure to multidimensional polarization, the mean minimum polarization gap 2DGAP. This unique polarization intensity measure provides transparency with regard to each singular attribute *and* ensures at the same time its interdependent relations.

Second, since polarization has many economic and social aspects, in the empirical application we respect both and argue, that – in addition to the traditional income measure – time as a fundamental resource for any activity should be incorporated into the multidimensional consideration. Genuine personal leisure time in particular will take care of the social participation aspect in the spirit of social inclusion/exclusion and Amartya Sen's capability approach (e.g. Sen 1999, 1985). The interdependent relations of time and income via the polarization attributes' parameters in the CES well-being function will be estimated and evaluated by the German Society instead of arbitrarily assigning values (as in Bourguignon and Chakravarty 2003, Lugo and Maasoumi 2009).

This empirical application is based on the German Socio-Economic Panel (SOEP) with additional detailed time use diary data from two available German Time Use Survey (GTUS) 1991/92 and 2001/02. We quantify available and extended polarization measures and our new approach for the polarization development over that decade in Germany according to time and income.

This study is the multidimensional polarization extension of our minimum 2DGAP approach and application recently proposed for multidimensional poverty analyses (Merz and Rathjen 2011a)

Overall, in addition to any one valued multidimensional polarization index quantified and

discussed in our study, the new transparent multidimensional polarization 2DGAP components with its empirical significance for Germany are important for any targeted polarization policies in particular.

The remainder of the study is organized as follows: Following the literature review on measuring polarization section 3 is about identification, aggregation and the multidimensional CES well-being function. This is used to capture and quantify the compensation of polarization attributes. The interdependent CES well-being approach then is the basis for our new multidimensional polarization index under compensation. Section 4 proposes the minimum multidimensional polarization gap (2DGAP) and is the main new contribution to multidimensional poverty measurement with transparent attribute contributions in the compensation context. Section 5 presents the empirical application and quantifies multidimensional polarization of genuine personal leisure time and income for different polarization regimes under compensation and for various socio-demographic groups in Germany for the decade 1991/92 to 2001/02.

Three prominent empirical results appear: First, genuine personal leisure time in addition to income is an important polarization attribute. Second, its compensation is evaluated by the German Society, and compensation is of economic and statistic significance. Third, in particular supported by the new minimum 2DGAP approach, multidimensional polarization increased over that decade in Germany.

2 Background: Measuring Polarization

2.1 Unidimensional Polarization Measures

First pioneering efforts of measuring polarization were done by Foster and Wolfson 2010 and Esteban and Ray 1994. They characterized polarization in two different ways. The first one, going back to Foster and Wolfson 2010, deals with the decline of the middle class, and measures how the center of the income distribution goes down, and the poles go up. The second consideration of polarization, following from Esteban and Ray, focuses on the rise of separated income groups. They define polarization by describing the homogeneity of the members of a group, and the differences that separate the groups from each other. Polarization increases the more equal the groups are in size.

The decline of the middle class, investigated by Foster and Wolfson (mimeo 1992, released 2010)

Foster and Wolfson 2010 characterized income polarization as a decrease of the middle class and an increase in the poles of the income distribution. These motions cause an increased income spread and an increased bipolarity. Both characteristics are modelled by two different polarization curves. The first polarization curve illustrates the income spread measured by the distance of income y to the median m normalized by m. A higher curve signals a larger spread. The second polarization curve is given by the area under the first polarisation curve. This curve represents the bipolarity. A higher curve signals a higher bipolarity. The polarization index then is given by twice the area beneath the second polarisation curve.

(1a) Income spread: $S(q) = \left| \frac{F^{-1}(q) - m}{m} \right|$

(1b) Bipolarity: $B(p) = \left| \int_{q}^{0.5} \frac{F^{-1}(p) - m}{m} \right| dp$

(1c) $P^{FW} = 2 \left| \int_{q}^{0.5} \frac{F^{-1}(q) - m}{m} \right| dp$

where F is the cumulative distribution function of income, q is the population fraction and m is the median income. This polarization index is closely related to the Gini coefficient

(1d) $P^{FW} = (G^{W} - G^{B}) \frac{\mu}{m}$.

Polarization is given by the Gini coefficient within the groups (G^W) minus the Gini coefficient between the groups (G^B) standardized by the mean income μ divided through the median income m (Foster and Wolfson 2010). Therefore the index reflects both the homogeneity of the individuals within the groups and the heterogeneity between the groups as well. If the distribution is left skewed μ will be smaller than m and P^{FW} will be between zero and unity. But if the distribution is right skewed, the index could be greater than one.

The advantage of using this index is the abandonment of income thresholds, since an arbitrary assignment of the population into subgroups is no longer necessary. The idea is based on the well- known Lorenz curve and Gini coefficient

The Foster and Wolfson polarization measure then was extended by Wang and Tsui 2000 and Scheicher 2010, 2009.

Wang and Tsui 2000

Wang and Tsui 2000 present a class of polarization indices which are based on the Foster and Wolfson index with relation to the median by:

(2) $P^{WT} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - m}{m} \right|^{\alpha}$

Where $\alpha \in [0,1]$, m is the median income and y_i is the income of individual i and n is the number of observations. This index measures the average relative distances to the median income. Thus, if the income has a large spread the index can be greater than one. If all individuals have the same income the index reaches its minimum zero.

Scheicher 2010

Scheicher 2010 defines polarization by aggregating measures of poverty and affluence. The focus thereby is on incomes outside the middle class interval. His index is an aggregation of the poverty index by Foster, Greer and Thorbecke 1984 and the affluence index by Peichl et al. 2010.

(3) $P^{S}_{univ} = \frac{1}{n_{poor}} \sum_{i \in poor} \left| \frac{z - y_i}{z} \right|^{\alpha} + \frac{1}{n_{rich}} \sum_{i \in rich} \left| \frac{y_i - r}{y_i} \right|^{\beta}$

where z is the poverty line and r is the affluence line, m is the median, and y_i describes individual income. The measure sums up the mean relative poverty gap and the mean relative affluence gap, respectively for the poor and the rich individuals.

The construction principle of this index – transferring well-known gap measures from poverty analysis to the analysis of the rich – reveals a general problem of measuring any gaps for the rich: whereas a poverty gap is restricted to the maximum interval z, the richness/affluence gap is unbounded. Thus a relative affluence gap then is related to the current y-value under investigation. Further implications are e.g. discussed in Peichl et al. 2010.

Nevertheless, the index by Scheicher has an intuitive appeal and provides the intensity of the poles by how far away from the poverty and affluence line income (y) is on average.

An advantage of the index is that polarization is understood as an income (y) aggregation of the poor and the rich. So the index with its both pole terms straightforwardly provides information about "the poor getting poorer and the rich getting richer" when two periods in time are compared.

Polarization with separate groups by Esteban and Ray 1994

Esteban and Ray 1994 have developed a group based polarization index in which the members of different groups are treated as being as dissimilar as possible. The index is based on the idea that members of the same group, who are homogenous, identify with each other strong, but members of different group feel alienated from each other. So the population is divided into g groups. Each group should be as similar as possible in terms of the members' attributes.

The degree of accordance is described by the population fraction of the group, the degree of alienation results from the absolute income distances. Thereby the group mean income is assigned to all group members.

Polarization of the population then is expressed as the sum of the accordance and alienation the individuals have relative to each other:

$$(4) \qquad P^{ER} = K \sum_{i=1}^{g} \sum_{j=1}^{g} \pi_i^{1+\alpha} \pi_j \left| \mu_i - \mu_j \right|$$

where π is the population fraction of group i and μ is the mean income of group i. Thus if all individuals have the same income, the index will be zero, so the minimum of the index is achieved at minimal inequality. The maximum of the measure is achieved when half of the population has minimum income and half of the population has the maximum income. Then the measure will be one normalized by a constant K.

The polarization sensitivity is measured by some α which weights the population frequency of each group. If alpha is equal to zero, the measure will be equal to the Gini coefficient. The larger α the greater is the difference to inequality measures. Esteban and Ray 1994 propose values for α in the interval between 1 and 1.6.

An advantage of the measure is that the plausible economic behaviour of accordance and alienation and the difference between inequality measures and polarization measures modelled by the polarization sensitivity. However, the disadvantage is that all income is reduced to its group mean. Any variation within the groups thus remains out of account.

This Esteban and Ray polarization approach with its grouping idea is extended by Esteban, Gradin and Ray 2007 and Duclos, Esteban and Ray 2004.

Esteban, Gradín and Ray 2007

One problem of the former Esteban and Ray index is the loss of the information about the dispersion of income within the group so the true polarization is overestimated by an underes-

timated inequality. Esteban, Gradín and Ray 2007 expanded the index by an approximation 2error ε which corrects this overestimation by an optimization process. This process classifies the given number of groups, so that the variance of the income within the groups is minimal.

$$(5) \qquad P^{EGR} = \sum_{i=1}^{g}\sum_{j=1}^{g} \pi_i^{1+\alpha}\pi_j \left|\mu_i - \mu_j\right| - \beta\varepsilon(G - G^{gruppiert})$$

β is a weight for the measurement error. An advantage of this extended polarization measure is that it is not necessary to classify the groups on the basis of arbitrary income constraints. Only the number of groups has to be chosen, and the group assignment is instead done by a classification algorithm.

Duclos, Esteban and Ray 2004

Duclos, Esteban and Ray 2004 developed a further extension of the polarization index by Esteban and Ray which is valid for continuous distributions. The measure does not require anymore the division into groups, which now are based on a non-parametric kernel density estimation. The polarization index then is obtained by describing the empirical distribution function by an estimated kernel density function.

$$(6) \qquad P_\alpha^{DER}(F) = \int_y f(y)^\alpha a(y)dF(y) \text{ with } a(y) \equiv \mu + y(2F(y)-1) - 2\int_{-\infty}^{y} xdF(x),$$

where F is the distribution function and f the associated density, μ is the income mean, $\alpha \in [0,1]$.

Therefore the measure is a translation from the Esteban and Ray discrete distribution case to a continuous distribution case. An increasing Duclos, Esteban and Ray index indicates increased polarization. The critique by Schmidt 2004 is about the empirical application: the measure would be not practical for an empirical analysis because the kernel density estimation requires population invariance, which might be not be given in the comparison of different time periods.

2.2 Multidimensional Polarization Measures

The extension of the traditional unidimensional income poverty measures, using polarization measurement by multidimensional approaches, has been of growing interest within the last years (see the overview of multidimensional poverty by Kakwani and Silber 2008). The European Union Laeken social inclusion indicator set is an example of multiple poverty dimensions with educational disadvantages, health inequalities, unemployment and worklessness as poverty dimensions (Atkinson 2003). Another example is the UNDP Human Development Index (HDI) which incorporates life expectancy, education and living standard (per capita GDP); see Alkire and Foster 2011 for some methodological background of the so called counting approach.

In the same spirit it is argued here, that polarization, connecting both poles of poverty and affluence should be measured in a multidimensional way. Though poverty and affluence might be measured in a similar fashion, obviously both are different in many aspects and require different economic and social policy approaches.

Gigliarano and Mosler 2009

Gigliarano and Mosler 2009 started with a multi-attribute measurement of polarization. They argue that the splitting of the population into groups should not be based only on income, but that other attributes like education, wealth or health have to be considered. Gigliarano and Mosler construct a class of multidimensional polarization measures by decomposing different inequality measures with measuring the relative group size. According to them polarization consists of inequality within groups, and inequality between the groups given a sufficient group size. Thus, this measure is a multidimensional extension of the group approach of Esteban and Ray 1994.

They decompose multivariate inequality measures in a within group inequality and a between group inequality. So the polarization increases if the between group inequality rises and decreases if the within group inequality rises. The more equal the different group sizes are, the greater is the polarization of the population. Based on this idea they construct three kinds of indices:

$$(7a) \qquad P_1^{GM} = \phi\left(\frac{B(X)}{W(X)+c}\right)\cdot S(X)$$

$$(7b) \qquad P_2^{GM} = \psi\left(B(X)-W(X)\right)\cdot S(X)$$

$$(7c) \qquad P_3^{GM} = \tau\left(\frac{B(X)}{B(X)+W(X)+c}\right)\cdot S(X)$$

where B(X) is the inequality between the groups and W(X) is the inequality within the groups φ, ψ, τ. Both are strictly increasing functions, and both parts results of the decomposition of multivariate inequality measures. S(X) is the relative group size which is measured by an inverse concentration measure like the Herfindahl index. X is a matrix which contains all considered individuals in the columns and their multiple attributes in the rows. Polarization is measured either via (8a), (8b) or (8c). An increasing inequality or an increasing group size raises polarization, an increasing inequality within the groups lowers polarization.

Scheicher 2010

Analogous to Scheicher's univariate polarization measure the multidimensional index is a combination of poverty and affluence measures. It is based on the distance of the income y of a middle class individual i to middle class thresholds.

$$(8a) \qquad d\left(y_{ij},\left[z_j,r_j\right]\right) = \begin{cases} \min\left\{\left|y_{ij}-z_j\right|,\left|y_{ij}-r_j\right|\right\} \; if \; y_{ij} \notin \left[z_j,r_j\right] \\ 0 \; if \; y_{ij} \in \left[z_j,r_j\right] \end{cases}$$

where z_j is the poverty line, r_j is the affluence line of attribute j, y_{ij} stands for the value of the jth attribute of individual i. For aggregation the single attribute specific distances are summed up over all attributes:

$$(8b) \qquad d\left(y_i,\left[z,r\right]\right) = \sum_j d(y_{ij},\left[z_j,r_j\right])$$

Finally, the mean of all aggregated distances of the poor and the rich individuals given the respective middle class thresholds builds the Scheicher 2010 multidimensional polarization index:

(8c) $$P^{S}_{mult} = \frac{1}{n}\sum_{i} d(y_i, [z, r]).$$

An advantage certainly is the incorporation of multiple attributes. However, since the attributes would be of different dimensions (say money, index, hour etc. dimensions) a convincing and comprehensive interpretation is at least hard to do.

3 Multidimensional Polarization: Identification, Aggregation and Multidimensional CES Well-Being Function

Multidimensionality in polarization measures as discussed so far respect multiple attributes via an attribute matrix in a grouping approach (like Gigliarano and Mosler 2009) or have respecting multiple attributes via distances to the pole thresholds (like Scheicher 2010). Further developments in the spirit of the counting approach are conceivable which are discussed in poverty analyses (Bossert, Chakravarty and D'Ambrosio 2013, Nolan and Whelan 2007, Atkinson 2003). Though the multidimensional approach obviously is the virtue, however, because of the different attribute dimensionalities, a combined "umbrella" which aggregates the single polarization attributes and respect its interdependence is still missing.

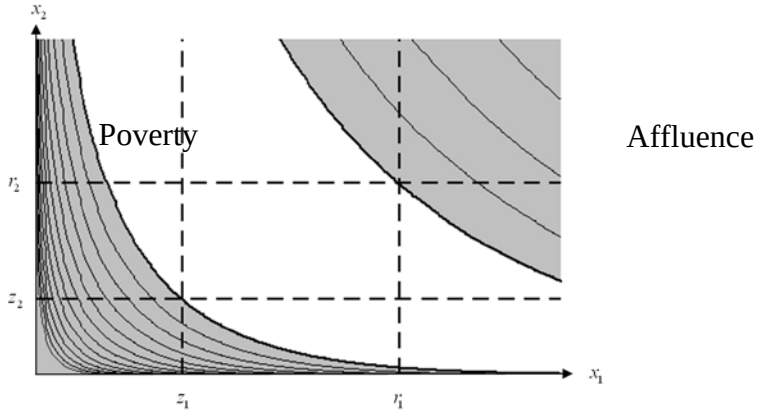
Such identification and aggregation is proposed in the following with a CES-type well-being function, which evaluates the relations between the polarization attributes and quantifies their possible respective compensation/substitution.

3.1 Multidimensional Polarization: Identification and Aggregation

To identify multidimensional polarization we refer mainly to the multidimensional poverty discussion. According to Bourguignon and Chakravarty 2003 multidimensional poverty identification is based on the strong and weak focus axiom. Instead of restricting compensation only to the intersection area of the two attributes (strong focus axiom) we follow the weak focus axiom (compensation approach), which allows possible compensation/substitution between poverty attributes over the whole attribute ranges. In the same spirit the affluent might be identified in general and in the following by the compensation approach (weak focus axiom).

Figure 1 illustrates the poverty and the affluence situation for the two-dimensional case. Under the unidimensional *perspective* the poor of the x_1 dimension, say income, are identified by

Figure 1: Multidimensional Isopolarization Contours - Compensation Approach (Weak Focus) in the Two-Dimensional Case



Source: own illustration

being left of z_1, the x_1 rich are identified by being right of r_1; z_1 and r_1 are the x_1 poverty and affluence thresholds to be defined. Similarly the x_2 poor, say time, are below z_2 (x_2 poverty threshold), the x_2 rich are above r_2 (x_2 affluence threshold).

The *multidimensional perspective* compensation (weak focus axiom) is described with the isoquants of the underlying still to be defined well-being function (see next section), which comprises all polarization attributes and their interdependent relations.

Polarization thresholds: Poverty and Affluence Lines

Two isoquants emerge: first, the isopoverty line which divides the poor and the non-poor crossing the unidimensional threshold at $z = (z_1, z_2)$, and second, the isoaffluence line which divides the affluent from the non-affluent at $r = (r_1, r_2)$.

3.2 Multidimensional Polarization: Multidimensional CES Well-Being Function

Two aggregation concepts are discussed with respect to multidimensionality poverty (Lugo and Maasoumi 2009, Bourguignon and Chakravarty 2003). These concepts are useful also for multidimensional affluence and its aggregation to the polarization lines of both distribution poles: one by "shortfall of well-being" (aggregate poverty line approach) and one by "well-being of the shortfalls" [1] (component poverty line approach). The first one relies on individual well-being compared to well-being at the threshold intersection. In the second one, the relative differences between the individual dimensional attributes and their thresholds are the respective input factors of the well-being function.

Based on the available data in our empirical application, we will evaluate the individual income and time situation in levels and accordingly concentrate on the "shortfall of well-being" approach, with levels rather than with relative deviations as arguments in the well-being function. However, with our proposed minimum multidimensional polarization gap indicator (2DGAP, see next section) we also carve out the single dimensional attributes when we disentangle them within the multidimensional context.

Within the multidimensional poverty discussion the interdependence of the (two) single poverty attributes is already expressed by a Constant Elasticity of Substitution (CES) type well-being function V_i^* (e.g. Lugo and Maasoumi 2009, pp. 12, 16, Bourguignon and Chakravarty 2003, p. 38) as

$$V_i^* = \left[w_1 \left(x_{1i} \right)^{\beta} + w_2 \left(x_{2i} \right)^{\beta} \right]^{\frac{1}{\beta}},$$

where β describes the level of substitutability with $\beta = 1$ for perfect substitution and $\beta = \infty$ for non-substitutes.

Similarly but with a slightly more flexible CES-type well-being function our individual well-being indicator V_i evaluates the interdependencies of both polarization dimensions by:

(9a) $$V_i = \gamma \left[w_1 \left(x_{1i} \right)^{-\rho} + w_2 \left(x_{2i} \right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}} \qquad \textit{weak focus axiom}$$

[1] Which corresponds to the Bourguignon and Chakravarty 2003 multidimensional poverty index.

with the substitution elasticity $\sigma = 1/(1+\rho)$, ρ as a curvature parameter of the isopolarization contours with $\rho \neq 0$, γ as a constant, υ as returns to scale, x_{1i} and x_{2i} as the input (polarization attribute) quantities, and the coefficients w_1 and $w_2 = 1 - w_1$ as distribution and weighting parameters describing the skewness of the isopolarization contours. This form of the CES well-being function among others already provided a better fit in the empirical poverty application (Merz and Rathjen 2011a, 2009).
Following the CES well-being approach the aggregated *multidimensional poverty line* will be defined by

$$(9b) \qquad V_z = \gamma \left[w_1 \left(z_1 \right)^{-\rho} + w_2 \left(z_2 \right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}} \qquad \textit{weak focus axiom}$$

resulting into the isopoverty contour, the isoquant, which is crossing the poverty threshold intersection at $z = (z_1, z_2)$. Once the CES well-being function is specified/estimated, all individuals with their calculated multidimensional well-being $V_x = V(x_{i1}, x_{i2})$ below the isopoverty line are assigned to be poor (see Figure 1).

Accordingly the aggregated multidimensional *affluence line is described by*

$$(9c) \qquad V_r = \gamma \left[w_1 \left(r_1 \right)^{-\rho} + w_2 \left(r_2 \right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}} \qquad \textit{weak focus axiom}$$

which delivers the isoaffluence contour, that isoquant which crosses the affluence intersection at $r = (r_1, r_2)$. All individuals with their calculated multidimensional well-being $V_x = V(x_{i1}, x_{i2})$ above the isoaffluence line are assigned to be affluent/rich.

3.3 Multidimensional Polarization: Measures Based on a Multidimensional (CES) Well-Being Function

We propose a straight forward measuring approach of multidimensional polarization which is based on the above compensation perspective by a multidimensional well-being function like our CES function.

Multidimensional Well-being Polarization (Median)

The first measure is an extension of the Wang and Tsui 2000 polarization measure (equation (2)) which now relates all well-being distances to the median well-being:

$$(10) \qquad P_{mult,m} = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{V(x_{i1}, x_{i2}) - V(m_1, m_2)}{V(m_1, m_2)} \right]^{\alpha} .$$

The greater the distance from the median well-being to the pole well-being the greater is this index. In contrast to Wang and Tsui 2000, who relates α to the interval [0,1], we follow the Foster Greer Thorbecke 1984 idea of α now describing a polarization aversion index, with α =1 as the relative well-being distance to the median and α=2 (or α ≥1) for greater weights of larger distances.

Multidimensional Well-Being Polarization (Poverty and Affluence Line)

Whereas this index comprises all below and above median values the next well-being meas-

ure, which extends Scheicher (2010), considers the *relative distance*s with respect to the poverty and the affluence lines: it is the sum of a mean relative poverty and a mean relative affluence well-being gap under the weak focus axiom:

$$(11a) \quad P_{mult,rel} = \frac{1}{n_{poor}} \sum_{i \in poor}^{n_{poor}} \left[\frac{V(z_1, z_2) - V(x_{i1}, x_{12})}{V(z_1, z_2)} \right]^{\alpha} + \frac{1}{n_{rich}} \sum_{i \in rich}^{n_{rich}} \left[\frac{V(x_{i1}, x_{i2}) - V(r_1, r_2)}{V(x_{i1}, x_{i2})} \right]^{\beta},$$

The first part of the polarization index $P_{mult,rel}$ expresses the relative poverty well-being gap of the well-being poverty line which in turn characterizes the maximum well-being poverty gap. The result is a mean percentage value (interval [0,1]). The second part relates the absolute affluence gap to the individual situation and might deliver values greater than one because of the unbounded (maximum) affluence gap. Though both parts have a different reference a larger value characterizes an increasing polarization as an increasing mean distance within the multidimensional distributional tale "ends".

The exponents α and β serve as polarization aversion coefficients with $\alpha = 0$ and $\beta = 0$ delivering the multidimensional polarization headcount number, $\alpha = 1$ and $\beta = 1$ an average relative polarization gap in well-being units is measured, and $\alpha > 1$ and $\beta > 1$, which reflects a higher aversion against strong polarization.

This proposed polarization index has its root in a multidimensional Foster-Greer-Thorbecke 1984 (FGT) poverty index under the weak focus axiom but according to well-being units (Lugo and Maasoumi 2009, Merz and Rathjen 2009, 2011a) which corresponds to the poverty part of equation (10). The affluence part refers to Peichl et al. and their discussion about the unavailable upper gap boundary. It extends Scheicher's 2010 unidimensional polarization measure as discussed with equation (3).

The building blocks of the multidimensional well-being polarization index beyond its compound description allows a characterization of a polarization asymmetry when the poverty part is compared to the affluence part.

The respective multidimensional polarization index in *absolute well-being deviations* expresses the pole well-being weights

$$(11b) \quad P_{mult,abs} = \frac{1}{n_{poor}} \sum_{i \in poor}^{n_{poor}} \left[V(z_1, z_2) - V(x_{i1}, x_{12}) \right]^{\alpha} + \frac{1}{n_{rich}} \sum_{i \in rich}^{n_{rich}} \left[V(x_{i1}, x_{i2}) - V(r_1, r_2) \right]^{\beta}.$$

Multidimensional Well-Being Polarization Asymmetry

Both well-being poles might be described by a *multidimensional well-being polarization gap ratio* between the poor and the rich gaps for the relative gaps

$$(12a) \quad P_{mult,rel,ratio} = \left\{ \frac{1}{n_{poor}} \sum_{i \in poor}^{n_{poor}} \left[\frac{V(z_1, z_2) - V(x_{i1}, x_{12})}{V(z_1, z_2)} \right]^{\alpha} \right\} / \left\{ \frac{1}{n_{rich}} \sum_{i \in rich}^{n_{rich}} \left[\frac{V(x_{i1}, x_{i2}) - V(r_1, r_2)}{V(x_{i1}, x_{i2})} \right]^{\beta} \right\}$$

and also absolute gaps

$$(12b) \quad P_{mult,abs,ratio} = \left\{ \frac{1}{n_{poor}} \sum_{i \in poor}^{n_{poor}} \left[V(z_1, z_2) - V(x_{i1}, x_{12}) \right]^{\alpha} \right\} / \left\{ \frac{1}{n_{rich}} \sum_{i \in rich}^{n_{rich}} \left[V(x_{i1}, x_{i2}) - V(r_1, r_2) \right]^{\beta} \right\},$$

where larger differences to one describe a greater asymmetry between the pole gaps.

4 Minimum Multidimensional Polarization Gap (2DGAP)

Multidimensional polarization by the compensation approach so far discussed is captured by a multidimensional well-being function which is at the heart of the respective polarization measures. The virtue of measuring multidimensional well-being and any well-being gap by a CES-type well-being function, is that it respects and quantifies the interdependence of multiple well-being attributes by a one value well-being index. However, such an aggregation of dimensions into an one well-being value is criticizable and questioned if it is still measuring "multidimensional" poverty, affluence or polarization. Transparency for the singular attributes within the multidimensional approach is desirable in a manner which allows a targeted attribute specific poverty/affluence/polarization policy.

The main motivation for the following multidimensional gap development is to unfold the singular attributes of a well-being gap to obtain a unique multidimensional intensity measure with its transparent singular attributes. This approach is based on our minimum poverty 2DGAP concept proposed in Merz and Rathjen 2011a.

As discussed, in the compensation (weak focus) approach all dimensions are combined and weighted via the respective CES well-being function by delivering a one value well-being level and index. In the polarization case both poles of a distribution, including the poor and the rich, are of interest and the respective poverty threshold and affluence threshold contours divide the well-being "mountain". Figure 2 (top) shows the CES well-being mountain and describes the two-dimensional poverty and affluence case: $V_z = V(z_1, z_2)$ is the well-being contour at the threshold isopoverty line at the singular poverty threshold $z = (z_1, z_2)$. $V_i = V(x_{1i}, x_{2i})$ is the individual well-being contour of the individual poverty attributes $x_i = (x_{i1}, x_{i2})$. The difference $V_z - V_i$ is the multidimensional poverty well-being gap. In an analogous way $V_i^r - V_r$ defines the multidimensional affluence well-being gap for a rich person with $V_i^r = V(x_{i1}^r, x_{i2}^r)$ and $V_r = V(r_1, r_2)$ at the affluence threshold $r = (r_1, r_2)$.

4.1 Minimum Multidimensional Poverty Gap (Minimum 2DGAP) – Concept, Condition and Properties

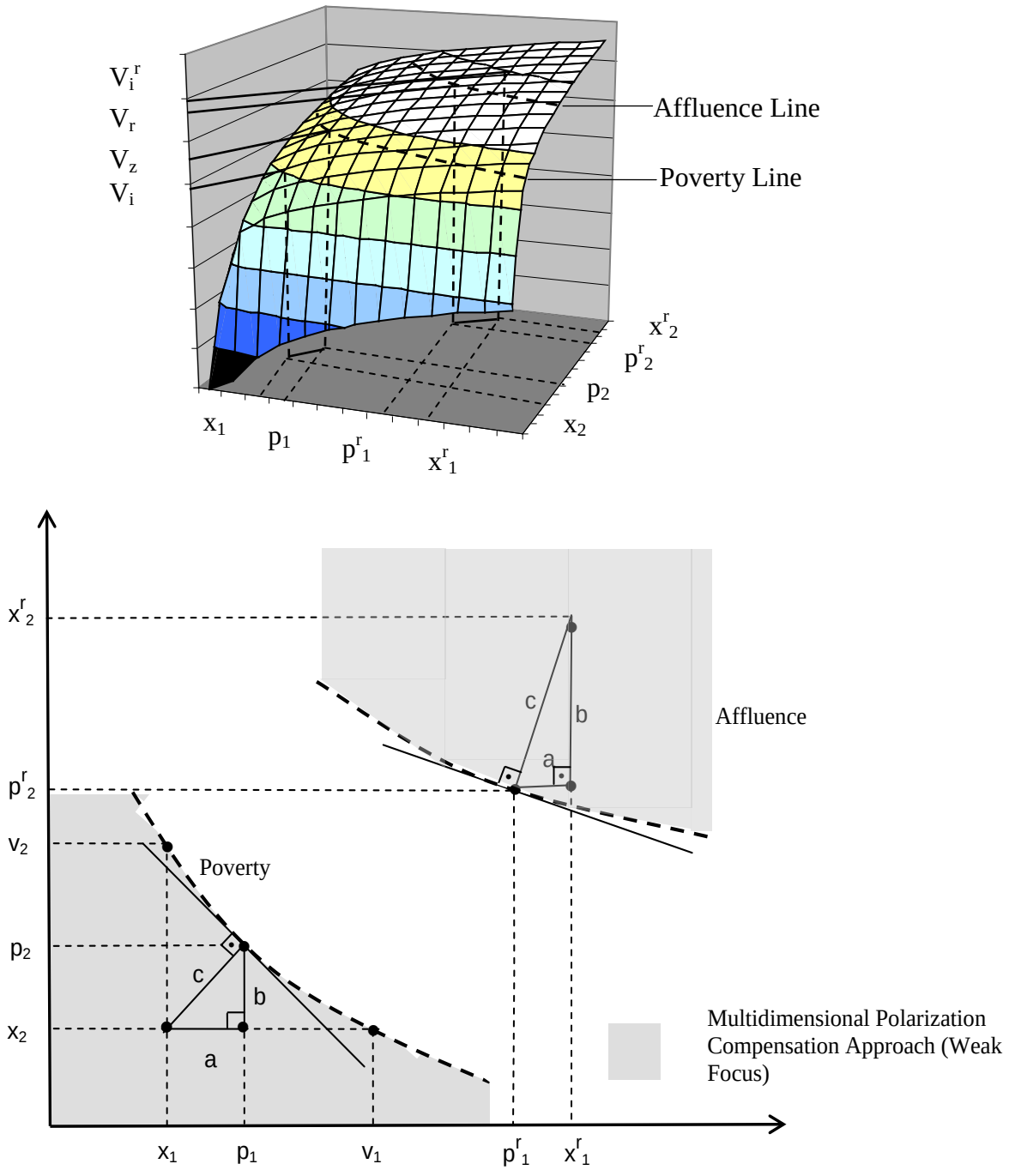
The mapping of the multidimensional well-being to its (two) singular dimensional space allows another appealing integrated approach for describing multidimensional polarization intensity. It consists of a unique distance between the individual situation and the poverty threshold respectively the affluence threshold which at the same time provides the contribution of the singular poverty attributes to the interdependent multiple poverty index.

As an illustration consider the two-dimensional case from the compensation approach and its attributes' space as in Figure 2 and regard again the poverty situation at $x = (x_1, x_2)$ for an individual. With respect to both dimensions there is a fan of distances from that point $x = (x_1, x_2)$ to the IMD isopoverty threshold. Each distance yields the same well-being difference $V_z - V_i$. However, each distance requires that different single attribute input intensities need to be overcome in order to escape multidimensional poverty.

The *shortest* path between $x = (x_1, x_2)$ and the corresponding point $p = (p_1, p_2)$ at the isopo-

verty threshold contour is prominent in Figure 2.[2] It requires the minimum input intensities in a sense of a minimum combined input "length" in order to escape multidimensional poverty.

Figure 2: Multidimensional Polarization: Well-Being Gap and Minimum 2DGAP



Source: own illustration

A natural measure for that length is the Euclidian distance of the singular attributes

[2] For the poverty case Lugo and Maasoumi (2008, p. 14, 2009, p. 12) already mention a distance from an individual point to the isopoverty line as the 'closest point' at the isopoverty line in the multidimensional case, however they do not determine any further characteristics and properties of that distance.

$c = \left[a^2 + b^2 \right]^{0,5}$ with the distances a and b as the singular poverty attribute gap intensities.

The solution for the shortest (or closest) length is characterized by the orthogonal path from the tangent at $p = (p_1, p_2)$ to $x = (x_1, x_2)$ (poverty case) respectively from the tangent at $p^r = (p_1^r, p_2^r)$ to $x_i^r = (x_{i1}^r, x_{i2}^r)$ (affluence case). We call the distance c the minimum multidimensional poverty respectively affluence 2DGAP (for two polarization dimensions), which is the measurable two-dimensional minimum mapping of the well-being distance between the individual well-being and the aggregate isopoverty respectively isoaffluence well-being threshold contour.

Minimum 2DGAP definition and property: For any individual point $x = (x_1, x_2)$ in the two-dimensional poverty space under the weak focus CES-type isopoverty respectively isoaffluence threshold the *minimum multidimensional poverty/affluence 2DGAP c* is defined as the shortest length (Euclidean norm) to the respective isothreshold line. The shortest length is the linear path orthogonal to the slope at the respective point $p = (p_1, p_2)$ on the CES-type isothreshold line:

$$\text{(13a)} \quad \begin{aligned} c = \| c \| &= \left[(p_1 - x_1)^2 + (p_2 - x_2)^2 \right]^{0,5} \\ &= \left[(p_1 - x_1)^2 + (f(p_1) - x_2)^2 \right]^{0,5} = \min! \end{aligned} .$$

where $f(p_1 | V_z)$ is the isothreshold contour with regard to ordinate values x_2 (time) of the CES multidimensional well-being function

$V_z = \gamma \left[w_1 \left(z_1 \right)^{-\rho} + w_2 \left(z_2 \right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}}$ as

$$\text{(13b)} \quad f(p_1 | V_z) = \left(\left(\left(\frac{V_z}{\gamma} \right)^{-\frac{\rho}{\upsilon}} - w_1 p_1^{-\rho} \right) / w_2 \right)^{-\frac{1}{\rho}} .$$

The solution p_1 (respectively p_1^r) of the minimizing problem then allows to calculate c by equation (1a) for a respective $x = (x_1, x_2)$. Note, because of the quadratic distances equation 13a is the solution for the poverty as well as for the affluence situation.

At point $(p_1, p_2 = f(p_1 | V_z))$ of the isothreshold contour the slope of c is orthogonal to the slope of the isothreshold contour. The orthogonal property allows another solution route via

$$c_\perp'(p_1) - f'(p_1 | V_z) = 0$$

$$\text{(14a)} \quad -\frac{p_1 - x_1}{f(p_1 | V_z) - x_2} + \left(\left(\left(\frac{V_z}{\gamma} \right)^{-\frac{\rho}{\upsilon}} - w_1 p_1^{-\rho} \right) / w_2 \right)^{-\frac{1}{\rho} - 1} \left(\frac{w_1}{w_2} \right) p_1^{-\rho - 1} = 0$$

with

$$\text{(14b)} \quad c_\perp' = -1/c' = -a/b$$

using

$$\text{(14c)} \quad c' = \frac{\partial c}{dp_1} = \tan(\alpha) = b/a \quad \textit{and} \quad \tan(\alpha + 90^\circ) = -1/\tan(\alpha) = -a/b .$$

The solution p_1 (respectively p_1^r) of the nonlinear equation 14 then allows to calculate c again by equation 13a for a respective $x = (x_1, x_2)$. The solution of equation 14 might also be found by an explicit iterative procedure in the interval $\left[x_1, v_1\right]$ [3] of changing c slopes until the slope of the isothreshold line is orthogonal to the slope of c through (x_1, x_2).[4]

Since the proposed CES well-being function is well behaved, there is always a unique solution (regardless the solution route) for the minimum 2DGAP (distance *c*).

4.2 Singular Attribute Gaps, Aggregation and Extensions

Singular poverty/affluence attribute gap intensities

Once $p = (p_1, p_2 = f(p_1 \mid V_z))$ is found the singular poverty repectively affluence attribute gap intensities can be calculated by

(15) $a = p_1 - x_1$ *und* $b = f(p_1) - x_2$

Relative 2DGAP

For the *poverty case* the 2DGAP might be defined relative to the maximum 2DGAP distance $c_{\max}$, which is the distance from the origin $(0,0)$ to the respective orthogonal slope of the IMD isopoverty threshold:

(16a) $c_{rel} = c / c_{\max}$ where $c_{\max} = \| c_{\max} \| = \left[(p_1)^2 + (f(p_1, V_z))^2 \right]^{0,5} = \min!$

with its corresponding **relative singular poverty attribute gap intensities**

(16b) $a_{rel} = \left[a / p_1 \mid a_{\max} \right]$ *and* $b_{rel} = \left[b / f(p_1, V_z) \mid b_{\max} \right]$.

For the *affluence case*, however and as discussed, there is no comparable genuine maximum distance since any affluence well-being or 2DGAP gap faces the problem of an open top interval.

A possible relative minimum affluence 2DGAP relating to the isoaffluence line precludes direct comparisons to poverty pole measures. A relating to the overall median for both pole minimum 2DGAPs would have a comparable reference but would not deliver transparent singular attribute components.

Aggregation and Mean Minimum Polarization 2DGAP

To retain the polarization singular attribute contributions in the multidimensional approach we propose a straight forward aggregation by the sum of the respective 2DGAP pole means, the mean minimum polarization 2DGAP:

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[3] v_1 is the abscissa value (here income) of the isopoverty contour of V_z equal x_2:

$$v_1(g(v_1 \mid V_z) \mid g(v_1 \mid V_z) = x_2) \quad \textit{with} \quad g(v_1 \mid V_z) = \left(\left(\left(\frac{V_z}{\gamma} \right)^{-\frac{\rho}{\upsilon}} - w_2 v_1^{-\rho} \right) / w_1 \right)^{-\frac{1}{\rho}}$$

where $g(v_1 \mid V_z)$ is the isopoverty contour with regard to abscissa values x_1 (here income).

[4] Stata minimum 2DGAP ado files for the minimum and slope solution are available from the authors by request.

(17a) $$C = \frac{1}{n_{poor}} \sum_{i \in poor}^{n} c_i + \frac{1}{n_{rich}} \sum_{i \in rich}^{n} c_i$$

with its singular aggregated components[5]

(17b) $$A = \frac{1}{n_{poor}} \sum_{i \in poor}^{n} a_i + \frac{1}{n_{rich}} \sum_{i \in rich}^{n} a_i, \quad B = \frac{1}{n_{poor}} \sum_{i \in poor}^{n} b_i + \frac{1}{n_{rich}} \sum_{i \in rich}^{n} b_i .$$

Minimum NDGAP

The minimum 2DGAP can be extended to the n-dimensional case, called minimum NDGAP, by a multivariate minimum search, where the slopes of the NDGAP linear distance are subject to the orthogonality of the n dimensional tangents to the isothreshold contours. A conceivable minimum 3DGAP for example would consider three dimensional isopolarization contours and a two-dimensional tangent plane resulting in a minimum 3DGAP which is right-angled to the tangent plane.

The Benefit: Visibility of Singular Attributes of Multidimensional Polarization

The minimum 2DGAP distance *c* itself measures the shortest multidimensional gap as the length of all dimensional gap intensities in combined attribute units but without direct interpretation in terms of the money or time-space. However, and this is the benefit of our proposed approach, both sides of the right-angled triangle (the distances a and b of Figure 2), are measurable and interpretable in a singular dimension, say income and time. Thus, beyond the compact interdependent multidimensional polarization description of the minimum 2DGAP, there is an additional singular dimension feature: each singular unidimensional attribute is transparent and visible from the two sides a and b, of the minimum 2DGAP triangles (Figure 2). In these triangles, with a as the amount of the first attribute, and *b* as the amount of the second attribute, the path to escape poverty or back to the affluence threshold is minimized while respecting its interdependence with its substitution/compensation.

In our application this would be income in money units (e.g. EURO) for *a*, and time in time units (e.g. minutes) for *b* as the singular attributes to escape multidimensional poverty. This information and transparency then allows singular dimension targeted anti polarization policies respecting its multidimensional interdependence. We discuss such policy possibilities in our concluding section.

5 Multidimensional Time and Income Polarization – The Case of Germany

The empirical application part of our study is about multidimensional polarization in Germany. Some polarization findings are already available (see Goebel, Goming and Häußermann 2010, Grabka and Frick 2008) however primarily based on unidimensional income polarization.[6] Scheicher 2009 provides empirical results based on his discussed multidimensional ap-

[5] The aggregation of the single poverty attributes a and b and of the 2DGAP c over all individuals might not result in the joint aggregate condition $c = (a^2 + b^2)^{0.5}$. With two degrees of freedom one remaining component (*a*, *b* or *c*) is computable from the other aggregates. In our application, alternative computations of the respective remaining component have shown close accordance to the orthogonal condition.

[6] Recent German empirical studies on unidimensional income inequality and income poverty results can be found e.g. in Grabka, Goebel and Schupp 2012, Groh-Samberg 2009 or Hauser 2008, Merz 2008, Becker and Hauser 2003.

proach with working hours and income as polarization attributes. For example, Gigliarano and Mosler 2009 analyze multidimensional polarization with regard to education and income according their group specific approach. Merz 2006 and Merz and Zwick 2005 analyse income polarization of self-employed as free-lancers and entrepreneurs.

Though available German results are showing – roughly speaking – some increasing polarization mainly in the first 2000 decade, the empirical results with our new polarization measures, polarization attributes, and type and periods of data are obviously new.

Note: In the tradition of inequality and polarization approaches we assign an individual as being poor or affluent according to defined poverty and affluence lines. This is naturally independent of any individual decision to live in such a situation, voluntarily or not.

Finally, a general remark to the empirical part: since our study is an extension of our multidimensional time and income poverty study (Merz and Rathjen 2009) further in-depth justifications and information is available there. It concerns the variables under investigation, time and income, and all further empirical definitions according to poverty and respectively built on parameters for the affluent and will not be referred to in here again.

5.1 Time and Income as Multidimensional Polarization Attributes

To understand poverty in a broader sense empirical multidimensional poverty studies incorporate various poverty attributes. An example is the European Union Laeken social inclusion/exclusion indicator set with educational disadvantages, health inequalities, unemployment and worklessness as poverty dimensions (Atkinson 2003). Whereas these and some others attributes are broadly accepted and available for economic and social policies in the poverty discussion, obviously a simple mirror image of affluence is misleading in many aspects.

Why income as a polarization dimension?

Income is the traditional and most-widely accepted poverty attribute and typically the focus of policy. The affluent are commonly defined by a large amount of material resources with focus on income and wealth. Thus, income is a natural first hand candidate as a polarization dimension for both poles.

Why time as a polarization dimension?

We argue that, in addition to income as a fundamental material resource, time is a similar fundamental immaterial resource and should be incorporated as a second attribute to better understand societal polarization. Time is a general requirement for daily living activity and is important for individual well-being simply by allowing or prohibiting desired activities for poor and rich alike. The importance of the time dimension for poverty analyses with different specific definitions is stressed meanwhile by other studies (see the discussion in Merz and Rathjen 2009, 2011a with Goodin et al. 2008, Burchardt 2008 Harvey and Mukhopadhyay 2007, Bittman 1999 or Vickery 1977).

In addition social participation with social inclusion/exclusion is an important aspect in the extended poverty discussion (Sen 1999, 1995), so we also think that social participation is of some similar importance for the affluent to have an integrated social life.

For this reason. Instead of a broad leisure time concept we propose g*enuine personal leisure time* as being essential to the multidimensional approach. Time poverty occurs when time, which is left after all paid and unpaid obligations, is below a certain level and does not allow or limit social participation with others of the society (see Merz and Rathjen 2009, 2011a) for

a further reasoning and discussion). Analogeously, time affluence occurs when genuine personal leisure time is above a certain level.

Why interdependent time and income polarization?

Time availability restricts market and non-market activities. Thus the more time is spent for income activities the less is available for leisure and vice versa. This trade-off is well-known and is central in the microeconomic optimal allocation and Becker's 1965 household production approach. Thus a certain trade-off between time and income is to be expected in any empirical analysis.

As discussed above, the trade-off will be quantified in our study by a CES well-being function with time and income as input factors to be weighted. Instead of arbitrarily chosen different trade-off weights and situations, and different compensation degrees, we let the data from German society identify the degree of interdependence and substitution between income and genuine personal leisure time.

5.2 Time, Income and Multidimensional Poverty and Affluence Threshold Lines

Singular poverty threshold lines z_i and singular affluence lines r_i define the respective poor and rich, the respective multidimensional well-being thresholds and finally the set of individuals in our polarization analysis. Yet, the empirical analysis requires concrete values.

Income: Poverty and Affluence Line

As an accepted measure in income poverty analysis income is measured as monthly household net equivalence income using equivalence scales like the OECD scale[7]. Conventional income-based poverty analyses from the European Union identify a person as income poor if net equivalized income is below 60% of the median income of all households (Bundesregierung 2005, XV). Hence, the 60% median line of the monthly household net equivalence income is adopted in the following as the *income poverty line*. For comparison, all subsequent income information for 1991/92 is adjusted to the 2001/02 price situation.

Whereas there is a common agreement about the income poverty line there is a longstanding and still open discussion about a respective affluence line. The Greek philosopher Plato (427-347 B.C.) already stated:"… *there should be* ***four different classes*** *appointed according to the amount of property.* ***The limit of affluence for the highest class****, which should not be passed over, should be the* ***fourfold value of the share in land (lot) of a citizen****; the poverty limit is the value itself which should not be diminished. …the* ***share in land (lot) of each citizen*** *should be large enough to satisfy a modest household,* ***and the total number of shares*** *should be large enough to enable its possessors to build an army great enough to protect against offences and to successfully help neighbours who are unfairly attacked*." (Platos laws, 5th book, pp.11-14, 39, 43)[8] Obviously a concrete empirical affluence line would be hard to find with respect to all the cited aspects.

The German Federal Administration for the first time explicitly focused on affluence in addition to poverty in their first "Poverty and Affluence Report" (Bundesregierung 2002) which was followed by three further Federal reports (Bundesregierung 2004, 2011, 2013). During

[7] With weight 1 for a household head, a weight of 0.5 to additional household members aged 15 years or older, and a weight of 0.3 to all others.

[8] Translation according to Ritter (1896, p. 43).

that period, top incomes gained increasing attention not only in Germany (Atkinson and Piketty 2007, Dell 2007 with German income tax micro data from 1891-1998, Merz, Hirschel und Zwick 2007 with German income tax microdata 1992-2003). Several affluence lines appeared: lines as a multiple of an income fraction like 200% (150%) of mean respective median income and as a top income percentile.

As a pragmatic approach, we are choosing a 150% median monthly household net equivalence income affluence line which is e.g. supported by the polarization analyses of Goebel et al. 2010 or Grabka and Frick 2008 from the German Economic Institute (DIW, Berlin).

Time: Poverty and Affluence Line

Compared to income the discussion about time poverty or even time affluence is still at its infancy (Bittman 1999 mentions a 50 % time poverty line). To be comparable to our income poverty and affluence line we chose 60% of the median genuine personal leisure time for poverty and 150% of the median as the time affluence line though such lines are certainly debatable.

5.3 Data: GSOEP and GTUS 1991/92 and 2001/02

The German Socio-Economic Panel (GSOEP)

The German Socio-Economic Panel (GSOEP) provides representative individual longitudinal data for all persons older than 16 years living in German households. The representative panel study started in 1984 and provides subjective as well as objective information about the individual living conditions in Germany (see the detailed presentation by Wagner, Frick and Schupp 2007). In particular, the SOEP is asking for satisfaction with regard to different topics, like income as well as general question about life satisfaction. The 11-point scale general satisfaction information is used for our well-being estimation and refers to the recent happiness/satisfaction literature (Clark et al. 2008, Frey and Stutzer 2005).

Since appropriate well-being data is only available within the German Socioeconomic Panel we use the GSOEP for the CES well-being estimation. Although in principle we could use the SOEP for our further analyses we prefer to use in addition time use diary data from both German Time Use Surveys (GTUS) from 1991/92 and 2001/02 (with no appropriate well-being information) since the time use diaries provide more additional in-depth information.

The German Time Use Surveys (GTUS) 1991/92 and 2001/02

The German Federal Statistical Office conducted two large representative time use surveys, the German Time Use Surveys 1991/92 and 2001/02 (Ehling, Holz and Kahle 2001, Ehling 2003). Therein all respondents older than 11 years in a household note their daily routines in diaries using their own words for two working days and a Saturday or Sunday. Person and household questionnaires also provide socio-economic background information. The final available data comprise 6,774 households with 15,366 persons and 30,732 diaries for 1991/92, and 5,144 households with 11,908 persons and 35,685 diaries for 2001/02.

5.4 Time, Income and Well-Being Multidimensional Poverty and Affluence Lines, Germany 1991/92 and 2001/02

The time and income singular poverty and affluence threshold lines based on both GTUS sur-

veys and are summarized in Table 1.[9]

In the GTUS surveys, the single *income poverty thresholds* are 665.78 € for 1991/92 and 793.55 € for 2001/02, the single *income affluence thresholds* are 1664.67€ for 1991/92 and 1983.97 € for 2001/02 (see Table 1). All income data are adjusted for price inflation by a 19.2% increase for the ten years between1991/92 to 2001/02.

The single *genuine personal leisure time poverty lines are* 159 minutes for 1991/92 and 186 minutes for 2001/02, while the single *genuine personal leisure time affluence lines* are 397.5 minutes for 1991/92 and 465 minutes for 2001/02.

The increase of personal leisure time median as well as the time poverty threshold and the affluence threshold over the ten years period is 17% and is somewhat lower as the increase in the respective median income by 19.2%.

Table 1: Income, Time and Well-Being Multidimensional Poverty and Affluence Lines, Germany 1991/92 and 2001/02

	1991/92	2001/02
Median Net Equivalence Income (in € per month and prices 2002)	1109.64	1322.58
Median Personal Leisure Time (in minutes per day)	265	310
Income Poverty Line (=60% Median Net Equivalence Income)	665.78	793.55
Time Poverty Line (=60% Median Personal Leisure Time)	159	186
Well-Being Poor $V^{poor} = f(I^{poor}, L^{poor})$	6.704	6.827
Income Affluence Line (=150% Median)	1664.46	1983.97
Time Affluence Line (=150% Median)	397.50	465.00
Well-Being Rich $V^{rich} = f(I^{rich}, L^{rich})$	7.402	7.538

Source: own calculations with GTUS 1991/92 and 2001/02, The time and income poverty lines and affluence lines by GTUS data are calculated for the total population for the median income; for the median genuine personal leisure time the population available older 11 years are respected.

With the SOEP reported general life satisfaction on an 11-point scale[10] an estimation of individual well-being requires rather a type of ordered response modelling. Yet, the Kmenta 1967 Taylor series approach allows a simple OLS estimator of the log transformed non-linear CES well-being function of equation (9a) as

$$(18) \qquad \ln V = \ln \gamma + \upsilon\delta \ln I + \upsilon(1-\delta)\ln L - \frac{1}{2}\rho\upsilon\delta(1-\delta)\left[\ln I - \ln L\right]^2 + \varepsilon$$

[9] Income is monthly net equivalized income. Time is personal genuine leisure time which is detailed in the individual time use diaries and includes one of the main categories "Contact, Conversations, Sociality" or "Media Use, Free-time Activities" in GTUS 1991/92 and the categories "Social Life and Entertainment", "Participation in athletic activities e.g. outdoor activities", "Hobbies and Games" and "Mass Media" in GTUS 2001/02.

[10] SOEP 2002 question 11 in the personal questionnaire.

with $I = x_{i1}$ for income and $L = x_{i2}$ for genuine personal leisure time providing efficient estimates. Some further conditions are fulfilled with the estimation as discussed in Merz and Rathjen 2009 and result in the CES well-being function

$$(19) \qquad V = f(I, L) = 3.550 \cdot \left(0.519 \cdot I^{0.297} + 0,481 \cdot L^{0.297}\right)^{\frac{0.108}{0.297}}$$

Significantly estimated coefficients together with the fulfilment of further consistency rules, quantify the relevance of the substitution/compensation between time and income. The population[11] based evaluation of the substitution/compensation between genuine time and income yields a substitution elasticity of $\sigma = 1.422$, which is a bit less distinct than in the Cobb-Douglas type ($\sigma = 1$) situation[12]. For comparison reasons the 1991/92 well-being function is specified by the same estimated parameters as in 2001/02.

The evaluated *well-being poverty line (compensation, weak focus axiom)* at the intersection of the singular time and income thresholds put into equation (10) is about a well-being level of 6.704 in 1991/92 and 6.827 in 2001/02.

In contrast, the evaluated *well-being affluence line (compensation, weak focus axiom)* at the intersection of the singular time and income thresholds yields a well-being level of 7.402 in 1991/92 and 7.538 in 2001/02.

Thus, the CES results suggest a slight increase in overall well-being within the ten years period. The estimated input coefficients, the weight w for income and $(1-w)$ for personal leisure, indicate a certain dominance of income. However, the evaluated time contribution is not that far away from a balanced 50% situation, and reflects the importance of time.

5.5 Polarization Overall: Uni- and Multidimensional Time and Income Poverty and Affluence

Our analysis concentrates on the active population. With regard to the working poor we shed light on the situation where despite particular governmental efforts poverty still exists. Thus we focus (for both distributional poles) on the more than part-time active population with more than five daily working hours.

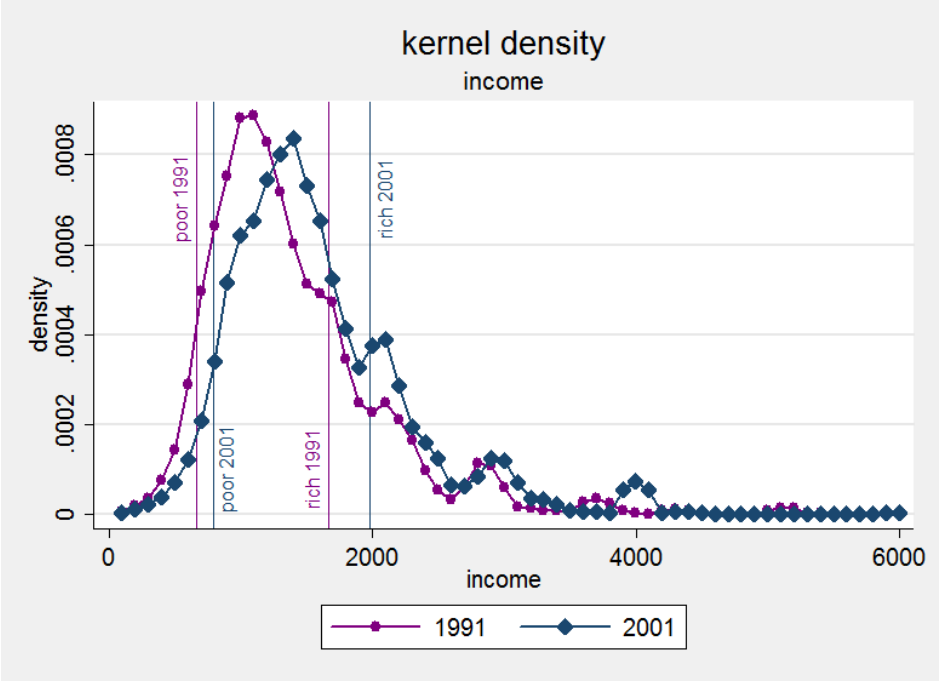
Graphical Illustration

Income and time Kernel density distributions for 1991/92 and 2001/02 are compared in Figures 3a and 3b. It is well to see, that the income and genuine personal leisure distributions have switched to higher income and time levels. The headcount ratios of the poor and rich active individuals are illustrated through the areas under the kernel density on the right and left sides of the poverty and affluence thresholds. The Kernel densities show some general shifts. However, since the respective thresholds as well as the distributions are changing an acceptable polarization description is hard to do; polarization measures have to help.

[11] Our CES well-being function estimates are based on the working population because the active population actually experiences work and leisure and therefore judges the trade-off between the two dimensions probably more appropriate.

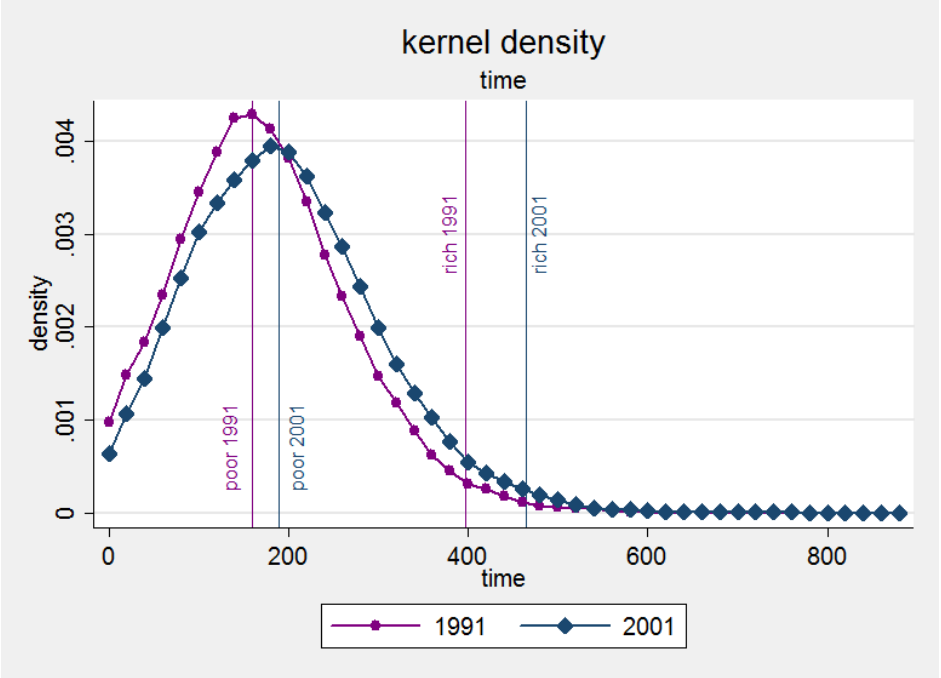
[12] Perfect substitution: ($\rho = -1, \sigma = \infty$), Cobb-Douglas case with ($\rho = 0,\ \sigma = 1$), no substitution at all (complementary input factors, $\rho = \infty, \sigma = 0$).

Figure 3a: Income Kernel Density for Germany 1991/92 and 2001/02



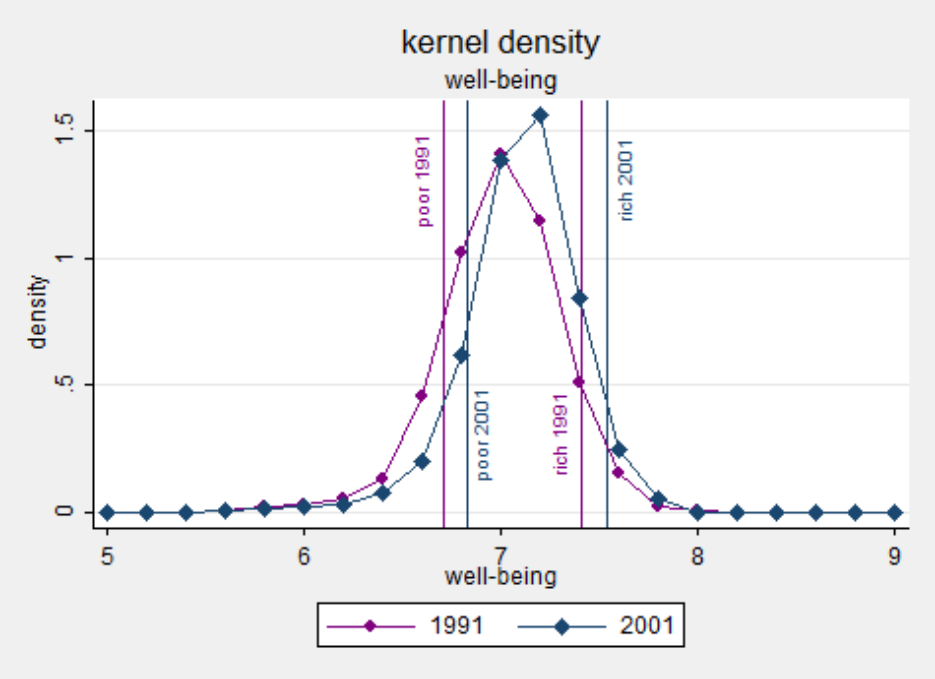
Source: own calculations with GTUS 1991/92 and 2001/02, active population

Figure 3b: Time Kernel Density for Germany 1991/92 and 2001/02



Source: own calculations with GTUS 1991/92 and 2001/02, active population

Figure 3c: Well-being Kernel Density for Germany 1991/92 and 2001/02



Source: own calculations with GTUS 1991/92 and 2001/02, active population

Figure 3c depicts the Kernel density of estimated CES well-being. Analogous to income and leisure time multidimensional well-being increased over the ten years. It seems to be that the area in the middle of the distribution has increased. However, a definite answer has to be given by the following with polarization measurement approaches.

Polarization Analysis by Measurement

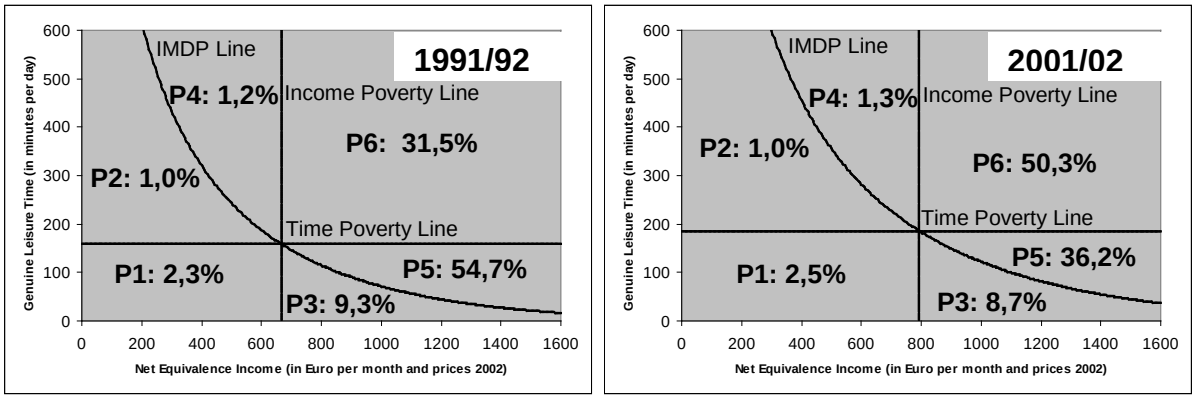
We identified the poor and the affluent by their headcount ratios further divided in respective six multidimensional regimes for both available years in Figure 4. The regimes identify unidimensional time and income polarization as well as multidimensional interdependent time and income polarization with their compensation regimes. Table 2 additionally summarizes all discussed unidimensional and multidimensional polarization measurement results including the poverty, affluence and polarization gaps with their 95% confidence intervals in both years.

Unidimensional Polarization Results

Unidimensional Income Poverty: Between 1991/92 to 2001/02 the percentage of income poor active individuals (headcount ratio) experienced marginal but significantly increased (α=0.043) from 4.2% to 4.8% (see Table 2 and Figure 4a, regimes P1, P2, P4). The first term of the polarisation index by Scheicher, which is the FGT index with α=1 for the poor, suggest an increasing poverty gap within the ten year period. The corresponding poverty index – measuring the average (relative) poverty gap – increases significantly (α=0.005) from 19.05% to 22.15%

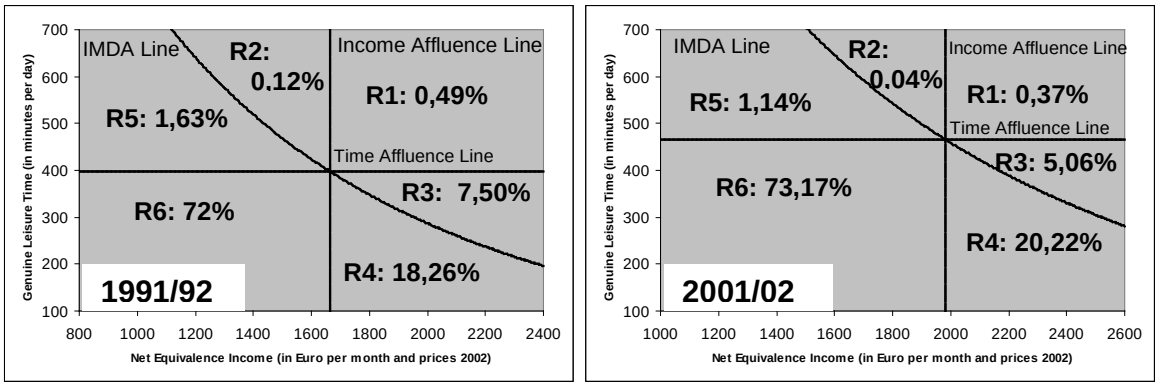
Unidimensional Income Affluence: From 1991/92 to 2001/02 the percentage of income rich individuals (headcount ratio) decreased from 26.3% to 25.7% (see Table 2 and Figure 4b, regimes R1, R2, R4). The 2001/02 headcount ratio of 25.7% is lying within the 95% confidence interval of the 1991/92 percentage. Accordingly, the decrease in the fraction of rich active individuals is not significant (α=0.361). The second term of the polarisation index by

Figure 4a: Interdependent Multidimensional and Unidimensional Poverty Thresholds and Headcount Ratios in Different Poverty Regimes for Germany 1991/92 and 2001/02



IMDP Line is the multidimensional time and income isopoverty threshold based on the CES estimates.
Source: own calculations with GTUS 1991/92 and 2001/02, active population;

Figure 4b: Interdependent Multidimensional and Unidimensional Affluence Thresholds and Headcount Ratios in Different Affluence Regimes for Germany 1991/92 and 2001/02



IMDA Line is the multidimensional time and income isoaffluence threshold based on the CES estimates.
Source: own calculations with GTUS 1991/92 and 2001/02, active population

Scheicher, which is measuring the average (relative) affluence gap, shows a declining average affluence gap within the ten years period. The affluence index decreases significantly (α=0.000) from 21.99 % to 18.20%.

Unidimensional Income Polarisation: The increased income poverty gap but decreased income affluence gap results in a slight (but not significantly) decreased combined polarization index by Scheicher from 0.4104 to 0.4034. The polarization index by Wang and Tsui, which is measuring the average (relative) gap to the median income decreases significantly (α=0.000) from 0.4356 to 0.3894. The polarization indices which are based on the Gini-coefficient, like the index by Foster and Wolfson and Esteban, Gradín and Ray show a slightly decrease of income polarization, too. Both of these declines are statistically significant (α=0.000).

Table 2: Interdependent Multidimensional and Unidimensional Time and Income Polarization 1991/92 and 2001/02, Germany

			1991/92			2001/02					
			Index	95% Confidence Interval		Index	95% Confidence Interval		Ratio 1991/92 =100	Diff. Test[1] p-values	
Headcount Ratio	Unidimensional										
	Income	poor	4.19	3.79	4.58	4.82	4.35	5.28	115	0.043	*
		rich	26.25	25.39	27.12	25.65	24.71	26.6	98	0.361	
	Time	poor	43.06	42.09	44.03	47.34	46.26	48.43	110	0.000	***
		rich	2.24	1.95	2.53	1.55	1.28	1.82	69	0.000	***
	Multidimensional										
	IMD[2]	poor	12.55	11.9	13.2	12.16	11.45	12.87	97	0.425	
		rich	8.11	7.57	8.65	5.47	4.97	5.96	67	0.000	***
		poor & ……rich	20.66	19.81	21.50	17.63	16.85	18.41	85	0.000	***
Polarization	Unidimensional										
	Income										
	Foster & Wolfson		0.0996	0.0969	0.1023	0.0908	0.0878	0.9386	91	0.000	***
	Esteban, Gradin & Ray		0.0506	0.0498	0.0516	0.0458	0.0445	0.047	91	0.000	***
	Gap Wang & Tsui		0.4356	0.4257	0.4455	0.3894	0.3801	0.3988	89	0.000	***
	Gap Scheicher		0.4104	0.4017	0.419	0.4034	0.3941	0.4127	98	0.286	
		poor	0.1905	0.1749	0.2061	0.2215	0.2063	0.2366	116	0.005	**
		rich	0.2199	0.2134	0.2263	0.182	0.1745	0.1895	83	0.000	***
	Time										
	Foster & Wolfson		0.1239	0.1201	0.1276	0.1214	0.1172	0.1255	98	0.379	
	Esteban, Gradin & Ray		0.0608	0.0596	0.0619	0.06	0.0586	0.0614	99	0.357	
	Gap Wang & Tsui		0.4074	0.4022	0.4125	0.4205	0.4149	0.426	103	0.000	***
	Gap Scheicher		0.5115	0.5033	0.5197	0.5073	0.4987	0.5158	99	0.484	
		poor	0.4037	0.3956	0.4118	0.3899	0.3818	0.3980	97	0.018	*
		rich	0.1078	0.0964	0.1192	0.1174	0.0989	0.1359	109	0.388	
	Multidimensional										
	Scheicher (Gap)		223.64	214.91	232.38	224.02	214.83	233.22	100	0.953	
	Multidimensional										
	IMD										
	$P_{mult, m}$		0.0328	0.0323	0.0334	0.0309	0.0303	0.0315	94	0.000	***
	$P_{mult,rel}$		0.0520	0.0502	0.0539	0.0487	0.0465	0.0508	94	0.020	*
		poor	0.0349	0.0328	0.037	0.0335	0.031	0.0359	96	0.394	
		rich	0.0172	0.0161	0.0183	0.0152	0.014	0.0164	88	0.018	*
	$P_{mult, rel, ratio}$		2.03			2.20					

[1] Two sample difference in means test with variance inhomogeneity and unequal variances; *** = significant on the 0.1% level; ** = significant on the 1% level; * = significant on the 5% level.

[2] IMD: Interdependent Multidimensional (IMD) compensation approach ; Poverty: CES well-being at 60% of income respective time median (CES well-being (1991/92) = 6.704, CES well-being (2001/02 = 6.827)
IMD: Interdependent Multidimensional (IMD) compensation approach; Affluence: CES well-being at 150% of income respective time median (CES well-being (1991/92) = 7.402, CES well-being (2001/02 = 7.538)

Source: GTUS 1991/92 and 2001/02, own calculations, weighted data

To summarize: the opposing trend of an increased income poverty but a decreased affluent headcount ratio and gap results in a diminishing overall income polarization over the decade for which Germany was analyzed.

Unidimensional Time Poverty: From 1991/92 to 2001/02 the percentage of time poor active individuals (headcount ratio) significantly increased from 43.1% to 47.4% (see Table 2 and Figure 4a, regime P1, P3, P5). The FGT-Index with $\alpha = 1$ slightly decreased significantly from 0.4037 to 0.3899.

Unidimensional Time Affluence: The percentage of time rich active individuals (headcount ratio) decreases significantly (α=0.000) from 2.2% to 1.6% in the considered decade. The average (relative) affluence gap slightly increases from 0.1078 to 0.1174 (Scheicher gap) but the change is insignificant.

Unidimensional Time Polarisation: The divergent development of time poverty (increasing) and time affluence (decreasing) combined yields for an insignificant decrease of the combined time polarization index by Scheicher from 0.5115 to 0.5073. Only the Wang and Tsui index which considered that the median gaps significantly increased in that decade. All other polarization indices are indifferent in their evaluation.

To summarize the unidimensional results: The combined time polarization picture is less selective than the income polarization picture; the still divergent poverty and affluence developments result only in an significant increased median specific polarization (Wang and Tsui). All other measured (Foster and Wolfson, Esteban, Gradin and Ray and Scheicher) polarization developments are insignificant with respect to time. However with respect to income all these measures (despite Scheicher) describe a significant income polarization decrease.

Multidimensional Polarization: Without any compensation the Scheicher multidimensional polarization gap (equation 8) additively combines time and income and shows no significant polarization differences between 1991/92 and 2001/02. However, an additive aggregation of minutes and income seems to be inappropriate for our application.

Multidimensional Well-Being Results (Compensation Approach, Weak Focus): The measures and results so far do not respect any compensation/substitution of time and income both polarization attributes are independent. The discussed *compensation approach (weak focus axiom),* however, allows a substitution between time and income and respects the interdependence of the polarization dimensions. This compensation is quantified by our CES approach and evaluated for the German population. The new results of our *well-being multidimensional polarization measures* and components over that decade in Germany are discussed now (IMD results in Figure 4 and Table 2) with $P_{mult,m}$ (equations 10), which is related to the median, $P_{mult,rel}$ (equation 11a), which measures the relative gaps, and $P_{mult,rel,ratio}$ (equation 12), which measures the asymmetry of both pole gaps.

Interdependent Multidimensional Poverty (Compensation Approach, Weak Focus): The headcount ratios of the multidimensional poor slightly declined from 12.6% in 1991/92 to 12.2% in 2001/02. Though the change is not significant, the absolute level of the working poor in both years yet is remarkable. Regime P3 is of particular importance: even an above income poverty threshold income is assigned not to compensate time poverty for 9.3% respectively 8.7% of the active population. Regime P3 is the prominent poverty regime under the

multidimensional perspective. For a further detailed discussion of the time and income multidimensional poverty development (compensation approach, weak focus axiom) with the same data is provided by Merz and Rathjen 2009 and 2011a.

Like the headcount ratios, the relative average multidimensional poverty gaps ($P_{mult,rel}$) slightly decrease, though not in a statistically significant fashion 0.0349 to 0.0335.

Interdependent Multidimensional Affluence (Compensation Approach, Weak Focus): Again, the compensation approach (weak focus axiom) also allows substitution/compensation between time and income when affluence is assigned. Multidimensional affluence by headcount ratios reduces significantly from 1991/92 to 2001/02 from 8.1% to 5.5% (Table 2 and Figure 4a, regime R1, R2, R3). Although the methodology of the regimes is similar for the poor and the rich, obviously the meaning is different and the focus is on the region above the isoaffluence line. The prominent regime in both years is regime R2: though being time poor there is enough income for compensation to be assigned as interdependent multidimensional affluent.

Furthermore, whereas this group and regime is diminished from 7.5% to 5.06% in 2001/02, regime R4 developed in the opposite direction: from 18.26% to 20.22%. Thus the headcount ratio of the income rich but not time affluent is increasing in Germany over that decade.

Remarkably there is only less than a half percent of the active population which is affluent in both attributes and both years (regime R1).

Concerning the distance of the affluent from the isoaffluence line, the relative average multidimensional affluence gaps as well as the headcount ratios $P_{mult,rel}$ are slightly but significantly decreasing from 1991/92 to 2001/02.

Multidimensional Polarization (Compensation Approach, WF) The overall multidimensional polarization index with compensation and regard to the median $R_{mult,m}$ decreases significantly over that decade (see above). However, if the well-being gaps refer to the respective isopoverty and isoaffluence lines $P_{mult,rel}$ the decrease is only of minor significance.

The multidimensional polarization gap ratio $P_{mult,rel,ratio}$ (equation 12) indicates the spread of the gaps, and as a result, the poverty gap is more than twice as large as the affluence gap. This spread grew, the divergence of the pole gap contributions expanded between 1991/92 to 2001/02.

Thus, based on the compensation evaluation of the German population with a CES well-being function, time and income polarization only declined in the 1990s when both distributional poles were measured from the median time and income values. Though time polarization increased over that period, the more rapidly decreasing income polarization with their compensation evaluation weaken the distributional poles in favour of some strengthening the remaining middle class. However, the median reference point neglects the different asymmetric definition of poverty and affluence. If this is accounted for by our $P_{mult,rel}$ measure then the well-being polarization decline is of minor importance.

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To summarize the overall picture: the unidimensional consideration of income and time suggest a decrease in income polarization and a slightly increase in time polarization in Germany within the considered decade. Concerning the single poles time as well income poverty in-

creases whereas income affluence decreases. Therefore there is no final evidence of an increasing bipolarity of the income and time distribution of the unidimensional measures are regarded together. The proposed interdependent multidimensional polarization well-being approach however provides such a combined analysis. The result up to now: only with respect to the median – but not to the poverty and affluence lines – a overall significant decreased interdependent multidimensional time and income polarization is visible.

5.6 Multidimensional Polarization by the Minimum Multidimensional Polarization Gap (2DGAP)

The interdependent multidimensional polarization gap so far discussed here [13] embraces the interdependence of time and income under the well-being shield. However, and in particular for targeted policy analysis, one might argue that the transparency for the singular polarization attributes is missing when only a one value indicator is given. Our proposed multidimensional polarization 2DGAP measure, developed in section 4 above, in fact provides such a singular time and income polarization contribution which respects compensation of the attributes.

Table 3 shows polarization results as mean minimum multidimensional polarization gaps (2DGAP) (equation 13) with the disentangled income and genuine personal leisure time components (equation 14) for 1991/92 and 2001/02 overall and divided for the respective poverty and affluence regimes. The mean absolute polarization well-being gaps $P_{mult,abs}$ (the sum of the poverty and affluence mean well-being gaps, equation 11b) in addition describe the underlying one valued well-being indicator, the already discussed polarization headcount ratios complete the overall multidimensional polarization picture given by Table 3.

Altogether (last line of Table 3) the mean multidimensional polarization 2DGAP c significantly (C, equation 17a) increased significantly by 18% starting with 141.30 in 1991/92. Thus, respecting the evaluated compensation between genuine personal leisure time and income evaluated by the German Society polarization increased in the last decade of the past century. Of particular interest are its mean components for income and time (A and B of equation 16b).

Income component: The summarized mean minimum *income 2DGAP* increases from 29.27 € to 38.58€ (2DGAP a). Though the mean income gap is small. The relative money increase of 32% is remarkable and highly significant. There is an asymmetry with a stronger poverty than affluence intensity: the poverty income component (17.72€) is greater than the affluence income component (11.55€). This might be an indication rather for an antipoverty policy than a reduction for higher income to diminish polarization with success.

Time component: The summarized mean minimum *time 2DGAP* of the poverty and affluence respective gaps (2DGAP b) increased significantly from 136 minutes per day to two and a half hours genuine personal leisure time by 18%. The asymmetry between the poor and the rich mean minimum time is remarkable: the affluence time gap in 1991/92 is twice as much as the poor time gap in 1991/92. It reduces to a 1.6 multiple in 2001/02. Though genuine personal leisure time is an important well-being polarization contribution, the significant all over polarization growth is mainly due to the significant growth of the income gap.

Polarization regimes: The minimum polarization gaps (2DGAP) measure the poverty and

[13] Except the Scheicher 2010 multivariate polarization index (equation 3)

Table 3: Multidimensional Polarization: Mean Minimum Multidimensional Polarization Gap (2DGAP) of Interdependent Multidimensional Time and Income, and Mean Well-Being Gap 1991/92 and 2001/02, Germany*

		Headcount Ratio				**Well-Being Gap**				**2DGAP: Mean Minimum 2DGAP c**				**2DGAP: Mean Minimum Income 2DGAP a (in €)**				**2DGAP: Mean Minimum Time 2DGAP b (in minutes per day)**			
	Year	1991	2001	Index 1991 =100	Diff test[4]	1991	2001	Index 1991 =100.	Diff test	1991	2001	Index 1991 =100	Diff test	1991	2001	Index 1991 =100	Diff test	1991	2001	Index 1991 =100	Diff . test
Poor	P1[1]	2.28	2.46	107		0.2593	0.3435	131	***	106.48	152.21	143	***	50.52	72.09	143	***	92.85	133.11	144	***
	P2[2]	1.04	1.02	98		0.0932	0.1080	18		56.32	74.75	133	*	35.51	46.67	131		43.19	57.82	134	*
	P3[3]	9.27	8.69	94		0.2186	0.1864	86	**	34.54	44.10	128	***	7.64	10.71	140	***	33.58	42.66	127	***
	IMD Poor	12.9	12.17	97		0.2160	0.2116	95		49.38	68.50	139	***	17.72	26.11	147	***	45.11	62.20	138	***
Rich	R1	0.49	0.37	76		0.1831	0.1639	88		188.66	204.65	108		36.26	40.30	111		183.98	199.59	108	
	R2	0.12	0.04	33	*	0.0291	0.0557	21		39.79	95.74	241	*	16.30	46.04	282	*	36.27	83.92	231	*
	R3	7.50	5.06	67	***	0.1296	0.1146	85	*	86.42	90.96	105		9.85	9.76	99		85.63	90.30	105	
	IMD Rich	8.11	5.47	67	***	0.1314	0.11752	92	*	91.92	98.73	107		11.55	12.09	107		90.87	97.69	108	
Rich & Poor	P1+R1	2.77	2.83	102		0.4424	0.5075	114	**	295.14	356.86	121	***	86.78	112.39	130	***	276.56	332.7	120	***
	P2+R2	1.16	1.06	91		0.1223	0.1638	141	**	96.11	170.49	177	***	51.81	92.71	179	***	79.46	141.74	178	***
	P3+R3	16.77	13.75	82	***	0.3483	0.3010	86	***	120.96	135.06	117	***	17.49	20.47	117	***	119.21	132.96	112	***
	IMD P + IMD R	**20.70**	**17.64**	**85**	***	**0.3473**	**0.3291**	**94**	*	**141.30**	**167.23**	**118**	***	**29.27**	**38.56**	**132**	***	**135.98**	**159.89**	**118**	***

* Poor describes multidimensional poverty, Rich multidimensional affluence and Rich & Poor multidimensional polarization in respective regimes; Headcount Ratio as $P_{mult,abs}$ (equation 11) with $\alpha=\beta=0$; Mean well-being gap as $P_{mult,abs}$ of equation 11 with $\alpha=\beta=1$

[1] P1/R1: regime of income *and* time poor/rich individuals [2] P2: regime of income poor but time not poor individuals [3] P3: regime time poor but not income poor individuals; R2 regime of time rich but not income rich individuals; R3 income rich but time not rich individuals

[4] Two sample difference in means test with variance inhomogeneity and unequal variances; *** = significant on the 1% level; ** = significant on the 5% level; * = significant on the 10% level. [5] IMD: Interdependent Multidimensional (IMD) compensation approach ; Poverty: CES well-being at 60% of income respective time median (CES well-being (1991/92) = 6.704, CES well-being (2001/02 = 6.827)

IMD: Interdependent Multidimensional (IMD) compensation approach; Affluence: CES well-being at 150% of income respective time median (CES well-being (1991/92) = 7.402, CES well-being (2001/02 = 7.538)

Source: own calculations with GTUS 1991/92 and 2001/02, active population

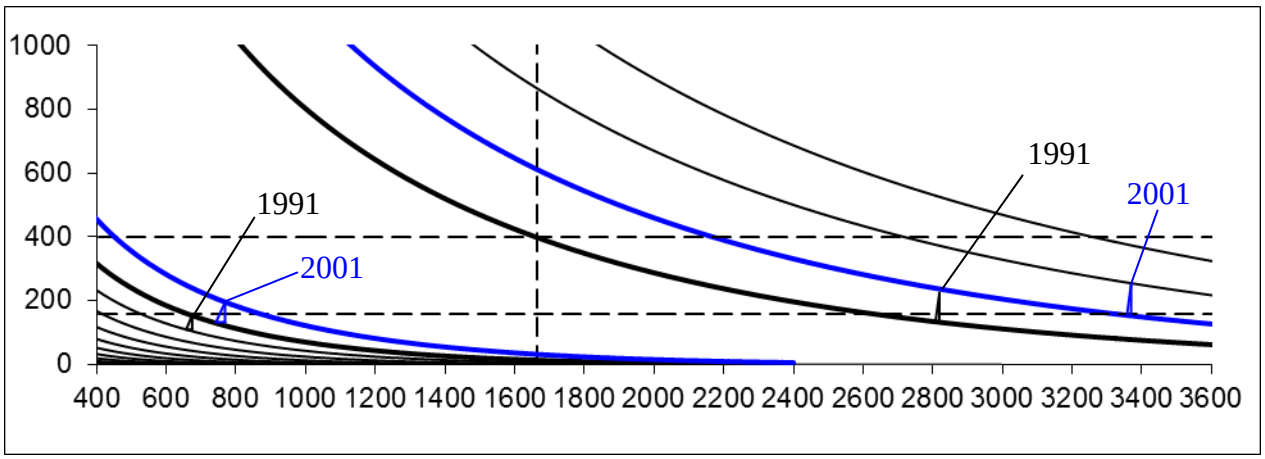
affluence intensities. The strongest polarization intensity is given in the intersection of time as well as income poverty overall and for each respective pole (Table 3, regimes P1, R1 and IMD poor and IMD rich). Those individuals face poverty and affluence beyond any compensation. This holds for the combined 2DGAP c as well as for the singular income (2DGAP a) and time (2DGAP b) components.

The strongest gap growth rate yet is seen in the R2 regime for the affluent and in the sequel for the overall regime polarization: there individuals are multidimensional affluent though income is below the affluence income threshold, less income is assigned to be compensated by time affluence.

Of specific interest and empirical importance are regimes P3 and R3 about time poverty which is assigned not to be compensated; in regime P3 even by above poverty threshold income, in R3 even by above affluence threshold income. These regimes show the highest headcount ratios for both years and emphasize the importance of genuine personal leisure time for the German population. And, polarization by headcount ratios for these regimes even increased significantly.

Figure 5a illustrates the positions of the mean minimum multidimensional polarization pole gaps (2DGAP) for 1991/92 (black) and 2001/02 (blue). Three results illustrate the numerical findings: first, the mean gaps are relative small, thus the poverty and affluence positions are relative near the respective interdependent multidimensional polarization thresholds. Second, there is a particular move of the mean affluent gap to higher income over the regarded decade. Third, relative steep ascending mean gaps pinpoints the importance of the time component.

Figure 5a: Multidimensional Polarization: Mean Minimum Multidimensional Polarization Pole Gaps (2DGAP c) 1991/92 (black) and 2001/02 (blue), Germany

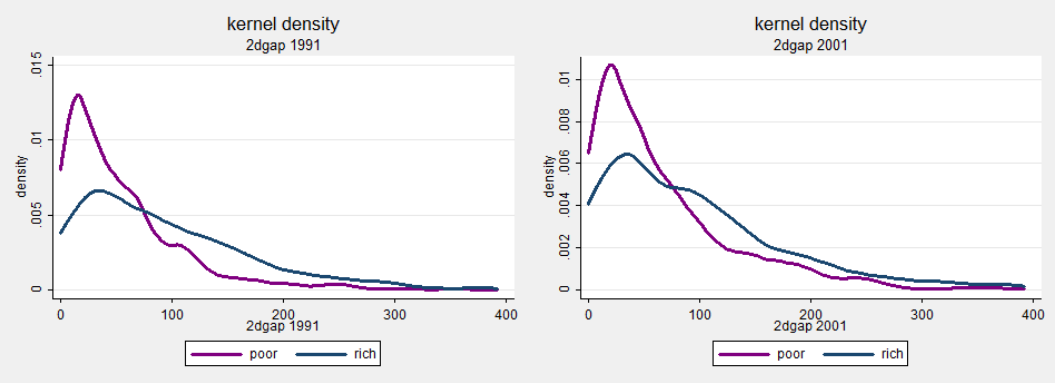


Source: own calculations

The Kernel densities of the poverty and affluence gap distributions for 1991/92 and 2001/02 of Figure 5b illustrates the different pole distributions of the minimum 2DGAP c measure: the affluent pole distributions are more right-skewed than the poor pole distribution and characterizes situations farer away from the polarization threshold.

Further interesting numerical results concerning the singular income and time 2DGAP components are available with Table 3.

Figure 5b: Kernel Densities of Minimum Multidimensional Polarization Pole Gaps (2DGAP c) 1991/92 and 2001/02, Germany



Source: own calculations

A last remark about differences between well-being and minimum 2DGAP results. The mean minimum polarization gaps (2DGAP) indicate a significant increase in multidimensional time and income polarization whereas the well-being polarization gaps indicate some decreasing polarization, but significant only for the median reference and of minor importance for the other measures ($P_{mult,rel}$, $P_{mult,abs}$). One explanation is the following: Any well-being difference is described by two respective contours as isoquants in the 2D attribute space. There, a fan of multitude gaps describe the difference from one point at an isoquant to the isopoverty respective isoaffluence line. Though there is only a one valued well-being gap, however, the indefinite possibilities from all income and time combinations result in a fuzzy allover well-being picture. The minimum 2DGAP approach however builds on a unique well-defined multidimensional distance with interpretable components.

Combined with high significant results we see a strong case for an increased multidimensional time and income polarization for Germany between 1991/92 and 2001/02.

5.7 Multidimensional Polarization in Socio-Economic Groups

It is to be expected that different individual resources and limitations will result in a different polarization picture for different socio-economic and socio-demographic groups. For various groups which experience attention in the public discussion Table 4a presents polarization headcount ratios, mean multidimensional polarization well-being gaps (IMD, compensation approach, weak focus) and mean minimum multidimensional 2DGAPs with its income and time components respecting compensation. The 2001/02 information also provides indices which describe the development since 1991/92 in Germany (the respective detailed results are given in the Appendix Tables 4b,c including information for the single poles).

There is a multitude of interesting single results given the compensation evaluation by the German Society. To be brief we will focus on selected results in particular with regard to our new multidimensional 2DGAP polarization measure and its components which describe the polarization intensity concerning income (in EURO) and genuine personal leisure time (in minutes).

Gender: Females are more often than males affected by poverty or affluence referring to unidimensional income and time polarization. In contrast males face a deeper multidimensional polarization gap (2DGAP c) allover *and* with respect to income and time. And, the 2DGAP polarization increased the most (2DGAP a, b, c) between 1991/92 to 2001/02 for males.

Age: Individuals who are over 65 years old and who are still working more than 5 daily hours a day are the group with the highest unidimensional polarization headcount ratios in the poles as well as under the multidimensional IMD polarization regimes. The older the individuals are the deeper are the polarization gaps (2DGAP c), too. The importance of age for both distributional poles is remarkable and underlines a particular erosion of the middle class for the elderly.

Education: Individuals with an A level ("Abitur") – in contrast to all other educational levels – show the most intense polarization. The higher the education level, the higher is the headount ratio of the affluent (Table 4b). Secondary schooling is connected with the fastest polarization growth.

Occupation: Self-employed are remarkable more often affected by income (52.88%), time (59.44%) and multidimensional IMD polarization (33.95%) than any other occupational group. Dividing further the Self-employed into the Liberal Professions ("Freie Berufe") and Entrepreneurs, reveals that the high percentages should be traced back to the high percentages of the Entrepreneurs with regard to time and IMD polarization, however not for the reverse income polarization. Furthermore, polarization intensity measured by multidimensional gaps show the highest spread for the self-employed (2DGAPc) and in particular for genuine personal leisure time (2DGAP b). And, multidimensional polarization for the self-employed grew the most followed by blue-collar workers.

This is a remarkable result since common sense tells that (liberal) professions (Freie Berufe) and entrepreneurs (tradesmen) as self-employed are rich by money and, because of their independence and time sovereignty, are rich by time, too. Since two thirds of the individuals in the two poles under IMD polarization are found in the poverty pole the deprived situation is of particular importance for the self-employed beyond the relatively dominant affluent gap contribution (see Appendix Tables 4b,c). This underlines self-employed results multidimensional time and income poverty results for the self-employed by Merz and Rathjen 2011b.

Working Hours: The highest polarization headcount ratios and the largest multidimensional polarization intensity (2DGAP) c are found for those with the most working hours, which, as to be expected, strengthen the affluent individuals.

Household/Family Structure: Whereas the IMD polarization headcount ratio for couples with two and more kids is the highest among the family groups the polarization is strongest for single parents with kids (2DGAP c). Single parents with kids also show the relative highest time gap. According to further results, this is mainly due to the poverty pole. They face a strong polarization increase by 33% (2DGAP c) over the decade. The increase is even stronger for single parents with more children (51%) and pinpoints growing tension for single parents.

Region: Though unidimensional income and time poverty headcount ratios are higher in West Germany the multidimensional picture is different. The relative number of individuals in the distributional poles are higher in East Germany (23.65% vs. 16.26% in West Germany) and the polarization intensity overall and with regard to income and time is greater in East Germany than in West Germany showing the influences of opposite economies.

Table 4a: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 2001/02, Germany

	Polarization Headcount Ratio						Well-Being Gap		Multidimensional Polarization Minimum 2DGAP					
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Index 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Gender														
Male	29.84	99	47.29	115	17.76	92	0.3525	103	182.97	134	40.51	152	175.34	133
Female	31.55	99	51.65	98	17.40	76	0.2845	80	135.34	92	33.88	105	128.46	91
Age														
12-17	17.05	76	40.57	103	17.90	135	0.2846	114	158.86	89	69.38	105	140.47	86
18-24	20.32	79	38.45	95	14.65	81	0.2776	84	158.23	111	45.83	126	148.90	111
25-44	28.79	101	50.92	109	17.97	84	0.3171	95	157.61	117	37.47	136	150.20	116
45-65	36.16	97	49.75	109	18.16	86	0.3520	95	174.20	120	36.08	130	167.79	120
>65	54.33	139	59.91	161	20.65	84	0.2662	45	189.54	110	55.76	283	175.89	103
Education														
A-Level	45.86	107	52.52	114	19.07	76	0.3579	97	172.75	113	36.81	113	166.35	115
Vocational Dipl.	33.50	71	48.46	104	18.46	70	0.2919	74	163.61	115	37.92	134	156.19	114
Second. School II	24.92	93	49.54	107	18.33	85	0.3212	96	167.12	117	42.47	130	158.45	116
Second. School I	22.90	87	45.16	103	15.08	91	0.3217	100	155.57	127	32.79	135	149.65	126
No certificate	28.69	219	48.84	158	17.64	145	0.2595	54	131.82	63	32.41	95	126.62	63
Occupation														
Self-employed	52.88	100	59.44	114	33.95	96	0.4563	95	240.22	129	59.59	168	227.72	128
Liberal. Prof.	59.84	-	49.64	-	28.29	-	0.4278	-	230.50	-	50.67	-	220.54	-
Entrepreneur	48.16	-	66.11	-	37.79	-	0.4510	-	231.47	-	62.20	-	218.11	-
Civil Servant	53.22	97	47.83	124	18.97	82	0.3220	80	135.19	104	16.37	84	133.84	104
White-Collar	33.88	102	50.14	103	14.34	69	0.2886	88	130.23	103	24.43	72	126.69	104
Blue-Collar	15.41	78	46.35	110	14.95	85	0.2561	88	155.62	127	37.06	124	149.97	128
Working Hours														
<20	21.70	75	47.74	116	22.24	105	0.2597	88	170.52	112	44.65	94	162.31	116
21-38	24.74	71	44.03	107	13.62	78	0.2745	82	151.23	112	31.58	115	145.84	112
39-40	27.98	115	47.70	103	14.23	71	0.3001	102	151.38	130	31.14	118	146.37	131
41-44	32.57	149	50.91	113	16.84	90	0.3508	78	187.05	100	36.04	106	182.01	100
…>45	46.75	106	60.99	115	27.9	92	0.4024	91	187.40	115	45.74	163	177.82	113
HH-Size														
Single-HH	30.13	132	50.40	122	15.88	81	0.3067	88	178.73	109	36.52	77	172.49	123
Couple 0 Kids	56.07	110	46.82	108	18.17	69	0.3294	90	150.37	106	26.76	112	145.84	106
Couple 1 Kid	23.95	136	47.47	102	14.04	95	0.2780	94	127.73	98	31.69	106	122.67	98
Couple 2 Kids	14.61	97	49.36	97	16.34	78	0.2833	85	146.62	97	42.85	148	138.69	94
Couple >2 Kids	20.94	114	57.65	100	31.68	118	0.2973	96	191.76	147	84.28	294	170.43	135
Single par. 1 Kid	31.18	121	40.46	78	22.23	105	0.3337	90	232.64	133	65.03	145	218.50	132
Single par. >1Kid	22.96	81	51.77	82	17.43	40	0.2491	119	146.09	151	47.44	114	135.88	158
Other structure	25.96	73	50.99	120	17.46	96	0.3693	102	160.14	129	52.63	212	148.68	124
Region														
West Germany	32.98	90	46.43	110	16.26	87	0.3193	85	161.59	121	36.16	141	154.57	120
East-Germany	19.38	114	59.77	114	23.65	93	0.3536	113	191.00	126	46.06	128	182.51	126

* IMD: Interdependent Multidimensional polarization compensation approach Source: own calculations with GTUS 1991/92 and 2001/02, active population

To summarize: As expected various socio-demographic groups show different uni- and multidimensional polarization and different growth for gender, age, education, the family structure and West vs. East German. Remarkably multidimensional polarization of time and income of self-employed as well single parents attract specific attention.

Our quantification of multidimensional time and income polarization for various socio-demographic groups Germany is important to detected groups of specific concern. Many further factors are expected to be included to explain and to formulate targeted policies. This discussion has to be postponed to further research.

6 Concluding Remarks

This study contributes to multidimensional polarization using new methodological approaches and empirical results. In particular, we propose a CES well-being function to capture the interdependence /compensation/substitution between the polarization attributes. This is the basis for new well-being polarization measures and for the new minimum multidimensional polarization (2DGAP) approach. In particular, the 2DGAP approach disentangles the singular polarization attributes, *and* ensures at the same time the compensation between the polarization attributes which is important for targeted economic and social policies.

The empirical application, beyond income, focuses on genuine personal leisure time in order to incorporate social participation aspects and income as polarization attributes. This is done by using the German time use diary data for 1991/92 up to 2001/02. Beyond unidimensional and multidimensional polarization results of measures found in the literature we develop new findings about polarization considering compensation which is estimated and evaluated by the German population.

The main finding is that a growing multidimensional interdependent time and income polarization has occurred over the analyzed decade in Germany for the working poor and working affluent. Though the pole gaps with reference to the *median* show a decreasing development, however when discussed *poverty and affluence thresholds* are respected, then the polarization intensity increased significantly. The new minimum 2DGAP polarization measure disentangles and quantifies income and time components in their own dimensions, that is in EUROS and minutes, and thus provides information for targeted polarization policies in Germany.

The largest poverty and affluent gaps (2DGAP), and thus the strongest poverty and affluence intensities, is found in the intersection regime of time as well as income poverty or affluence. This holds for the entire mean minimum polarization 2DGAP and for its time and income components. Yet there is a remarkable and significant impact of compensation between genuine personal leisure time and income in general and in the polarization regimes outside the intersections. The evaluated compensation also detects compensation and no compensation regimes. In particular, regimes of time poverty which are assigned not to be compensated even by above threshold income are important: time poverty even not compensated by above threshold poverty income, and time not compensated even by above affluence income. Those regimes are the most frequent ones. Further results for socio-demographic groups show remarkable polarization beyond gender, age or the East-West results for the self-employed and increasingly for single parents.

This and all other findings stress the relevance of genuine personal leisure time with its social participation aspect as an important polarization dimension. Economic and social policy will probably deal differently according to the poverty and affluence pole when a decline in polar-

ization is aspired. The more targeted pole information within the polarization picture therefore is needed; this is what our contribution is serving for.

Obviously available data enables and restricts the explanatory power of any analysis. In our case with survey data it is discussed if survey data would describe in a sufficient extent the situation for the affluent and in particular for high income. Compulsory income tax data, which arguably provides the most meaningful information about high income individuals, showed an increased unidimensional income polarization in Germany during the same period of analysis (Merz 2006). This is an indication that probably our measured increase of the multidimensional time and income polarization gap would be even higher if more informative data would be available.

Appendix

Table 4b: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

		Pole Headcount Ratio						Well-Being Gap			Pole Mean Minimum 2DGAP				
		Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Ind. 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Gender															
Male															
	poor	4.56	111	45.59	119	11.9	109	0.2212	105	71.13	164	27.24	184	64.57	162
	rich	25.28	97	1.71	66	5.86	69	0.1313	100	111.84	120	13.27	112	110.77	121
	poor+rich	29.84	99	47.29	115	17.76	92	0.3525	103	182.97	134	40.51	152	175.34	133
Female															
	poor	5.25	100	50.36	99	12.61	81	0.1959	87	64.24	113	24.28	114	58.34	113
	rich	26.30	99	1.29	79	4.79	64	0.0886	67	71.10	79	9.60	88	70.12	79
	poor+rich	31.55	99	51.65	98	17.40	76	0.2845	80	135.34	92	33.88	105	128.46	91
Age															
12-17															
	poor	13.68	146	33.08	104	16.50	140	0.2339	139	76.98	112	32.83	116	67.54	108
	rich	3.37	76	7.49	102	1.40	92	0.0507	62	81.88	75	36.55	96	72.93	72
	poor+rich	17.05	76	40.57	103	17.90	135	0.2846	114	158.86	89	69.38	105	140.47	86
18-24															
	poor	5.42	104	33.66	95	10.92	91	0.2022	87	64.62	124	25.57	122	57.99	125
	rich	14.82	73	4.79	95	3.73	62	0.0754	76	93.61	104	21.26	131	90.91	103
	poor+rich	20.32	79	38.45	95	14.65	81	0.2776	84	158.23	111	45.83	126	148.90	111
25-44															
	poor	5.24	117	49.81	110	13.89	98	0.2103	101	69.96	142	26.50	153	63.58	141
	rich	23.55	98	1.11	72	4.07	56	0.1068	86	87.65	103	10.97	106	86.62	103
	poor+rich	28.79	101	50.92	109	17.97	84	0.3171	95	157.61	117	37.47	136	150.20	116
45-65															
	poor	3.58	87	48.90	111	9.78	92	0.2194	98	66.55	140	24.82	150	60.87	139
	rich	33.16	100	0.86	49	8.38	80	0.1326	92	107.65	110	11.26	100	106.92	111
	poor+rich	36.16	97	49.75	109	18.16	86	0.3520	95	174.20	120	36.08	130	167.79	120

Appendix

Table 4b cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio						Well-Being Gap				Pole Mean Minimum 2DGAP			
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Ind. 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
>65														
poor	11.37	382	58.83	179	18.51	426	0.1426	36	69.53	206	37.61	768	57.25	172
rich	42.96	119	1.09	25	2.15	11	0.1236	61	120.01	86	18.15	123	118.64	86
poor+rich	54.33	139	59.91	161	20.65	84	0.2662	45	189.54	110	55.76	283	175.89	103
Education														
A-Level														
poor	3.76	126	51.48	116	8.89	86	0.2317	107	70.60	133	25.67	114	64.99	140
rich	42.11	106	1.05	56	10.18	69	0.1262	82	102.15	103	11.14	11	101.36	103
poor+rich	45.86	107	52.52	114	19.07	76	0.3579	97	172.75	113	36.81	113	166.35	115
Vocational Dipl.														
poor	3.64	226	45.70	102	11.88	113	0.1857	69	63.70	139	25.59	167	57.18	136
rich	29.86	65	2.76	143	6.57	41	0.1062	85	99.91	104	12.33	95	99.01	104
poor+rich	33.50	71	48.46	104	18.46	70	0.2919	74	163.61	115	37.92	134	156.19	114
Second. School II														
poor	5.60	95	47.60	108	14.32	94	0.2126	98	71.12	132	28.43	147	63.94	130
rich	19.33	92	1.93	84	4.01	65	0.1086	91	96.00	107	14.04	106	94.51	108
poor+rich	24.92	93	49.54	107	18.33	85	0.3212	96	167.12	117	42.47	130	158.45	116
Second. School I														
poor	4.70	110	44.25	107	12.22	109	0.2047	98	62.43	151	21.99	167	57.41	149
rich	18.2	83	0.91	38	2.86	53	0.1170	104	93.14	115	10.80	97	92.24	115
poor+rich	22.90	87	45.16	103	15.08	91	0.3217	100	155.57	127	32.79	135	149.65	126
No certificate														
poor	8.01	126	47.63	173	15.79	219	0.1810	142	57.12	91	20.26	71	52.91	96
rich	20.68	305	1.21	36	1.85	37	0.0785	22	74.70	51	12.15	215	73.71	51
poor+rich	28.69	219	48.84	158	17.64	145	0.2595	54	131.82	63	32.41	95	126.62	63

Appendix

Table 4b cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio						Well-Being Gap		Pole Mean Minimum 2DGAP					
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Ind. 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Occupation														
Self-employed														
poor	12.02	205	58.29	117	22.11	157	0.2964	109	108.23	174	46.24	182	96.60	175
rich	40.87	86	1.16	47	11.84	55	0.1599	77	131.99	107	13.35	133	131.12	107
poor+rich	52.88	100	59.44	114	33.95	96	0.4563	95	240.22	129	59.59	168	227.72	128
Liberal. Prof.														
poor	7.24	-	48.47	-	11.33	-	0.2566	-	88.39	-	37.32	-	79.11	-
rich	52.61	-	1.17	-	16.96	-	0.1712	-	142.11	-	13.35	-	141.43	-
poor+rich	59.84	-	49.64	-	28.29	-	0.4278	-	230.50	-	50.67	-	220.54	-
Entrepreneur														
poor	15.27	-	64.96	-	29.43	-	0.3068	-	113.43	-	48.57	-	101.18	-
rich	32.89	-	1.15	-	8.36	-	0.1442	-	118.04	-	13.63	-	116.93	-
poor+rich	48.16	-	66.11	-	37.79	-	0.4510	-	231.47	-	62.20	-	218.11	-
Civil Servant														
poor	0.32	49	45.29	125	4.27	88	0.1873	69	23.94	74	4.61	65	23.38	74
rich	52.90	97	2.55	118	14.69	81	0.1347	102	111.25	113	11.76	95	110.46	114
poor+rich	53.22	97	47.83	124	18.97	82	0.3220	80	135.19	104	16.37	84	133.84	104
White-Collar														
poor	1.59	63	48.97	104	7.91	68	0.1874	88	48.33	108	124.21	870	45.71	110
rich	32.28	105	1.17	84	6.43	70	0.1012	89	81.90	101	10.22	94	80.98	101
poor+rich	33.88	102	50.14	103	14.34	69	0.2886	88	130.23	103	134.43	536	126.69	104
Blue-Collar														
poor	5.31	87	44.96	113	13.99	97	0.1679	84	53.77	113	17.64	103	50.21	115
rich	10.10	74	1.39	65	0.96	31	0.0882	97	101.85	136	19.42	151	99.76	136
poor+rich	15.41	78	46.35	110	14.95	85	0.2561	88	155.62	127	37.06	124	149.97	128

Appendix

Table 4b cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio						Well-Being Gap			Pole Mean Minimum 2DGAP				
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Ind. 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Working Hours														
<20														
poor	7.06	64	45.41	119	18.13	111	0.1686	86	62.30	91	23.57	72	56.69	98
rich	14.64	82	2.33	82	4.11	85	0.0911	92	108.22	129	21.08	141	105.62	128
poor+rich	21.70	75	47.74	116	22.24	105	0.2597	88	170.52	112	44.65	94	162.31	116
21-38														
poor	3.51	142	42.03	110	9.95	122	0.1674	79	51.85	123	17.86	125	47.78	123
rich	21.23	66	2.00	70	3.67	40	0.1071	88	99.38	108	13.72	104	98.06	108
poor+rich	24.74	71	44.03	107	13.62	78	0.2745	82	151.23	112	31.58	115	145.84	112
39-40														
poor	2.96	59	46.56	105	9.74	67	0.1894	94	55.32	116	18.43	113	51.47	117
rich	25.02	129	1.14	58	45.00	84	0.1107	121	96.06	140	12.71	127	94.90	140
poor+rich	27.98	115	47.70	103	14.23	71	0.3001	102	151.38	130	31.14	118	146.37	131
41-44														
poor	3.94	78	49.79	116	12.40	117	0.2221	90	80.82	141	25.66	138	76.33	144
rich	28.63	170	1.12	50	4.44	55	0.1287	63	106.23	82	10.38	69	105.68	82
poor+rich	32.57	149	50.91	113	16.84	90	0.3508	78	187.05	100	36.04	106	182.01	100
…>45														
poor	6.83	149	59.99	116	16.23	106	0.2711	104	87.043	167	35.74	196	78.11	164
rich	39.91	101	1.00	70	11.67	77	0.1313	72	100.36	91	10.00	102	99.71	91
poor+rich	46.75	106	60.99	115	27.9	92	0.4024	91	187.40	115	45.74	163	177.82	113
HH-Size														
Single-HH														
poor	4.54	122	48.21	121	10.27	88	0.1697	80	62.19	87	23.48	64	56.94	96
rich	25.60	134	2.20	132	5.61	71	0.1370	102	116.54	126	13.04	127	115.55	142
poor+rich	30.13	132	50.40	122	15.88	81	0.3067	88	178.73	109	36.52	77	172.49	123

Appendix

Table 4b cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio						Well-Being Gap			Pole Mean Minimum 2DGAP				
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Ind. 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Couple 0 Kids														
poor	1.41	23	45.97	111	4.25	42	0.2001	95	49.88	122	16.65	123	46.04	121
rich	54.66	122	0.86	51	13.93	86	0.1293	84	100.49	100	10.11	96	99.80	100
poor+rich	56.07	110	46.82	108	18.17	69	0.3294	90	150.37	106	26.76	112	145.84	106
Couple 1 Kid														
poor	3.92	173	45.88	103	9.93	93	0.2108	115	58.66	131	18.69	122	55.09	134
rich	20.04	130	1.59	88	4.11	102	0.0672	60	69.07	81	13.00	90	67.58	81
poor+rich	23.95	136	47.47	102	14.04	95	0.2780	94	127.73	98	31.69	106	122.67	98
Couple 2 Kids														
poor	3.17	74	^45.88	97	9.93	81	0.2108	110	58.66	139	18.69	164	55.09	135
rich	11.44	107	1.28	83	1.24	52	0.0674	49	82.52	79	19.73	133	80.06	77
poor+rich	14.61	97	49.36	97	16.34	78	0.2833	85	146.62	97	42.85	148	138.69	94
Couple >2 Kids														
poor	15.00	133	55.36	99	31.55	122	0.2319	107	83.22	163	34.43	219	74.40	154
rich	5.94	84	2.30	124	0.13	13	0.0654	70	108.54	138	49.85	386	96.03	124
poor+rich	20.94	114	57.65	100	31.68	118	0.2973	96	191.76	147	84.28	294	170.43	135
Single par. 1 Kid														
poor	14.48	78	39.07	76	19.19	96	0.2025	87	83.83	99	38.42	107	72.65	96
rich	16.69	226	1.38	690	3.04	281	0.1312	97	148.81	166	26.61	301	145.85	163
poor+rich	31.18	121	40.46	78	22.23	105	0.3337	90	232.64	133	65.03	145	218.50	132
Single par. >1Kid														
poor	11.19	45	48.89	78	16.60	39	0.3124	99	96.42	119	39.51	132	68.42	115
rich	11.77	342	3.90	211	0.63	115	0.0636	301	69.18	218	14.88	87	67.46	252
poor+rich	22.96	81	51.77	82	17.43	40	0.2491	119	146.09	151	47.44	114	135.88	158

Appendix

Table 4b cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio						Well-Being Gap			Pole Mean Minimum 2DGAP				
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Ind. 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Other structure														
poor	5.75	219	49.89	127	16.60	180	0.3124	123	96.42	233	39.51	317	86.43	222
rich	20.21	61	1.10	32	086	10	0.0569	52	63.72	77	13.12	107	62.25	77
poor+rich	25.96	73	50.99	120	17.46	96	0.3693	102	160.14	129	52.63	212	148.68	124
Region														
West Germany														
poor	4.02	181	44.83	113	10.05	125	0.2022	84	64.00	152	24.41	169	57.98	151
rich	28.96	85	1.61	60	6.21	58	0.1171	88	97.59	107	11.75	104	96.59	107
poor+rich	32.98	90	46.43	110	16.26	87	0.3193	85	161.59	121	36.16	141	154.57	120
East-Germany														
poor	8.31	82	58.45	114	21.47	91	0.2310	118	77.81	141	29.63	145	70.92	140
rich	11.07	160	1.32	114	2.18	111	0.1226	105	113.19	118	16.43	106	111.59	118
poor+rich	19.38	114	59.77	114	23.65	93	0.3536	113	191.00	126	46.06	128	182.51	126

IMD: Interdependent Multidimensional polarization compensation approach

Source: own calculations with GTUS 1991/92 and 2001/02, active population ,

Table 4c: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio			Well-Being Gap	Pole Mean Minimum 2DGAP		
	Income	Time	IMD*	Mean	Mean c	Mean a Income (€)	Mean b (min.)
Gender							
Male							
poor	4.11	38.47	10.87	0.2100	43.30	14.84	39.80
rich	26.05	2.59	8.50	0.1310	92.97	11.85	91.87
poor+rich	30.15	41.05	19.37	0.3410	136.26	26.69	131.67
Female							
poor	5.24	51.01	15.56	0.2242	56.79	21.23	51.59
rich	26.61	1.64	7.44	0.1321	89.84	10.95	88.86
poor+rich	31.85	52.65	23.00	0.3563	146.63	32.18	140.45
Age							
12-17							
poor	9.34	31.91	11.78	0.1682	68.94	28.19	62.37
rich	13.10	7.31	1.52	0.0814	0.08	108.69	101.58
poor+rich	22.44	39.21	13.30	0.2496	177.62	66.07	163.94
18-24							
poor	5.22	35.6	12.06	0.2324	52.13	20.94	46.35
rich	20.26	5.02	6.06	0.0997	90.11	16.24	88.25
poor+rich	25.48	40.62	18.11	0.3321	142.24	37.18	134.59
25-44							
poor	4.47	45.16	14.11	0.2085	49.21	17.29	45.15
rich	24.10	1.55	7.26	0.1245	85.09	10.32	84.16
poor+rich	28.56	46.71	21.37	0.333	134.30	27.62	129.31
45-65							
poor	4.12	44.03	10.60	0.2242	47.54	16.56	43.75
rich	33.05	1.74	10.44	0.1446	97.44	11.27	96.50
poor+rich	37.17	45.77	21.05	0.3688	144.98	27.84	140.25
>65							
poor	2.98	32.84	4.35	0.3946	33.68	4.90	33.32
rich	36.00	4.32	20.30	0.2018	139.19	14.80	137.95
poor+rich	38.98	37.17	24.65	0.5964	172.88	19.70	171.27
Education							
A-Level							
poor	2.99	44.20	10.33	0.2166	53.03	22.48	46.40
rich	39.69	1.88	14.84	0.1540	99.47	1.02	98.68
poor+rich	42.68	46.08	25.17	0.3706	152.50	32.49	145.08
Vocational Dipl.							
poor	1.61	44.87	10.51	0.2684	45.99	15.35	42.18
rich	45.67	1.93	15.92	0.1244	96.01	13.00	94.93
poor+rich	47.28	46.80	26.43	0.3928	141.99	28.35	137.11
Second. School II							
poor	5.91	44.14	15.31	0.2165	53.85	19.31	49.37
rich	20.96	2.29	6.18	0.1187	89.11	13.29	87.76
poor+rich	26.87	46.42	21.49	0.3352	142.97	32.60	137.12
Second. School I							
poor	4.26	41.26	11.26	0.2093	41.25	13.18	38.47
rich	22.04	2.41	5.38	0.1126	81.28	11.15	80.18
poor+rich	26.30	43.67	16.65	0.3219	122.53	24.33	118.66
No certificate							
poor	6.34	27.46	7.21	0.1273	62.53	28.65	55.05
rich	6.77	3.37	4.98	0.3509	145.09	5.65	144.75
poor+rich	13.11	30.83	12.19	0.4782	207.62	34.30	199.80

Table 4c cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio			Well-Being Gap	Pole Mean Minimum 2DGAP		
	Income	Time	IMD*	Mean	Mean c	Mean a Income (€)	Mean b (min.)
Occupation							
Self-employed							
poor	5.86	49.82	14.11	0.2727	62.25	25.47	55.17
rich	47.28	2.46	21.36	0.2087	123.68	10.04	122.98
poor+rich	53.14	52.28	35.47	0.4814	185.93	35.51	178.15
Liberal. Prof.	-	-	-	-	-	-	-
Entrepreneur	-	-	-	-	-	-	-
Civil Servant							
poor	0.65	36.37	4.87	0.2711	32.45	7.05	31.50
rich	54.40	2.17	18.18	0.1325	98.17	12.32	97.11
poor+rich	55.06	38.54	23.05	0.4036	130.61	19.38	128.61
White-Collar							
poor	2.54	47.07	11.61	0.2125	44.59	14.28	41.50
rich	30.70	1.40	9.18	0.1139	81.35	10.82	80.38
poor+rich	33.23	48.47	20.79	0.3264	125.94	25.10	121.88
Blue-Collar							
poor	6.08	39.85	14.45	0.1991	47.69	17.08	43.67
rich	13.63	2.13	3.07	0.0911	74.78	12.88	73.32
poor+rich	19.70	41.98	17.51	0.2902	122.47	29.97	116.99
Working Hours							
<20							
poor	11.03	38.31	16.32	0.1951	68.27	32.84	57.86
rich	17.80	2.84	4.81	0.0990	83.89	14.90	82.24
poor+rich	28.83	41.15	21.13	0.2941	152.16	47.74	140.10
21-38							
poor	2.47	38.34	8.18	0.2130	42.25	14.32	38.86
rich	32.29	2.87	9.21	0.1214	92.38	13.17	91.09
poor+rich	34.76	41.22	17.40	0.3344	134.63	27.49	129.95
39-40							
poor	5.04	44.37	14.62	0.2021	47.51	16.37	43.83
rich	19.37	1.95	5.35	0.0914	68.68	10.04	67.69
poor+rich	24.41	46.32	19.97	0.2935	116.19	26.41	111.52
41-44							
poor	5.06	42.76	10.61	0.2459	57.14	18.84	53.16
rich	16.83	2.25	8.13	0.2037	130.23	15.14	128.87
poor+rich	21.89	45.02	18.74	0.4496	187.37	33.98	182.03
…>45							
poor	4.59	51.63	15.30	0.2602	51.98	18.23	47.63
rich	39.62	1.42	15.15	0.1815	110.78	9.80	110.10
poor+rich	44.21	53.06	30.45	0.4417	162.76	28.03	157.73
Household-Structure							
Single-HH							
poor	3.71	39.78	11.62	0.2125	71.57	36.93	59.30
rich	19.11	1.67	7.92	0.1345	92.16	10.23	81.29
poor+rich	22.82	41.45	19.54	0.3470	163.73	47.16	140.59
Couple 0 Kids							
poor	6.12	41.52	10.18	0.2113	10.78	13.49	37.92
rich	44.86	1.67	16.13	0.1544	100.68	10.49	99.89
poor+rich	50.98	43.19	26.31	0.3657	141.47	23.98	137.81
Couple 1 Kid							
poor	2.27	44.65	10.69	0.1827	44.94	15.33	41.25
rich	15.37	1.80	4.01	0.1124	85.36	14.49	83.60
poor+rich	17.65	46.44	14.71	0.2951	130.30	29.81	124.86

Table 4c cont.: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 1991/92, Germany

	Pole Headcount Ratio			Well-Being Gap	Pole Mean Minimum 2DGAP		
	Income	Time	IMD*	Mean	Mean c	Mean a Income (€)	Mean b (min.)
Couple 2 Kids							
poor	4.31	49.46	18.60	0.1959	46.15	14.06	43.45
rich	10.70	1.55	2.40	0.1388	104.82	14.83	103.34
poor+rich	15.01	51.02	21.00	0.3347	150.97	28.89	146.79
Couple >2 Kids							
poor	11.32	56.05	25.77	0.2173	51.11	15.72	48.25
rich	7.10	1.86	1.02	0.0935	78.93	12.90	77.55
poor+rich	18.42	57.91	26.79	0.3108	130.05	28.62	125.80
Single par. 1 Kid							
poor	18.45	51.70	20.07	0.2341	84.71	35.84	76.03
rich	7.37	0.20	1.08	0.1359	89.91	8.85	89.46
poor+rich	25.81	51.90	21.15	0.3700	174.62	44.69	165.49
Single par. >1Kid							
poor	24.84	61.34	42.64	0.1878	64.78	24.64	59.44
rich	3.44	1.85	0.55	0.0211	31.77	17.14	26.72
poor+rich	28.27	63.19	43.19	0.2089	96.55	41.78	86.15
Other structure							
poor	2.62	39.26	9.20	0.2541	41.45	12.47	38.96
rich	33.10	3.39	8.94	0.1092	82.55	12.30	81.30
poor+rich	35.72	42.65	18.14	0.3633	124.00	24.78	120.25
Region							
West Germany							
poor	2.22	39.60	8.05	0.2420	42.12	14.45	38.47
rich	34.27	2.69	10.66	0.1325	91.62	11.25	90.56
poor+rich	36.49	42.29	18.70	0.3745	133.74	25.71	129.07
East-Germany							
poor	10.08	51.41	23.55	0.1951	55.36	20.42	50.59
rich	6.91	1.16	1.96	0.1166	95.86	15.45	94.24
poor+rich	16.99	52.57	25.52	0.3117	151.22	35.86	144.83

IMD: Interdependent Multidimensional polarization compensation approach

Source: own calculations with GTUS 1991/92 and 2001/02, active population

References

Alkire, S. and J. Foster (2011), Counting and multidimensional poverty measurement, in: Journal of Public Economics, 95/7-8, 476-487.

Atkinson, A.B. (2003), Multidimensional Depriviation: Contrasting Social Welfare and Counting Approaches, in: Journal of Economic Inequality, 1, 51-65.

Atkinson, A.B. and T. Piketty (eds.), 2007, Top Incomes Over the 20th Century A Contrast Between Continental European and English Speaking Countries, Oxford University Press, Oxford.

Becker, G.S. (1965), A Theory of the Allocation of Time, in: Economic Journal 75, 493-517.

Becker, I. und R. Hauser (2003), Anatomie der Einkommensverteilung – Ergebnisse der Einkommens- und Verbrauchsstichproben 1969 – 1998, edition sigma, Berlin.

Bittman, M. (1999), Social Participation and Family Welfare: The Money and Time Cost of Leisure, SPRC Discussion Paper No. 95, Sydney.

Bossert, W., Chakravarty, S.R. and C. D'Ambrosio (2013), Multidimensional Poverty and Material Deprivation with Discrete Data, in: The Review of Income and Wealth, 59/1, 29-43.

Bourguignon, F. and S.R. Chakravarty (2003), The Measurement of Multidimensional Poverty, in: Journal of Economic Inequality 1, 1, 25-49.

Bundesregierung (2013, 2011, 2004, 2002), Lebenslagen in Deutschland, Der erste (zweite, dritte, vierte) Armuts- und Reichtumsbericht der Bundesregierung, Bundesministerium für Gesundheit und Soziale Sicherung, Berlin.

Burchardt, Tania (2008), Time and Income Poverty, Centre for Analysis of Social Exclusion, London School of Economics, London (UK).

Burkhardt et al. (2012), Mittelschicht unter Druck?, Bertelsmann Stiftung, 1. Aufl., Gütersloh.

Chakravarty, S. R. (2009): Inequality, Polarization and Poverty. Advances in Distributional Analysis. New York, NY: Springer-Verlag New York.

Chakravarty, S.R. and Silber, J. (2008), Measuring Multidimensional Poverty: The Axiomatic Approach, in: Kakwani, N. and J. Silber (Eds.), Quantitative Approaches to Multidimensional Poverty Measurement, Palgrave Macmillan, New York, 192-209.

Clark, A.E., Frijters, P. and M. Shields (2008), Relative Income, Happiness and Utility: An Explanation for the Easterlin Paradox and Other Puzzles, in: Journal of Economic Literature, 46, 95-144.

Dell, F., (2007), Top Incomes in Germany Throughout the Twentieth Century: 1891-1998, in: Atkinson and Piketty (2007), 365-425.

Duclos, J.-Y., Esteban, J.-M. and D. Ray (2004), Polarization: Concepts, Measurement, Estimation. in: Econometrica 72 (6), 1737–1772.

Ehling, M. (1999), The German Time Use Survey - Methods and Results, in: Merz, Joachim; Ehling, Manfred (Hg.): Time Use. Research, Data, and Policy, Baden-Baden: Nomos Verlagsgesellschaft, 89–105.

Ehling, M. (2003), Erhebungen zur Zeitverwendung 1991-92 und 2001-02. Skizzierung des methodischen Vorgehens, in: Wirtschaft und Statistik - Sonderausgabe anlässlich des 54. Weltkongresses des Internationalen Statistischen Instituts vom 13. bis 20. August 2003 in Berlin, 85-87.

Ehling, M., Holz, E. and I. Kahle (2001), Erhebungsdesign der Zeitbudgeterhebung 2001/2002, in: Wirtschaft und Statistik, 427–436.

Esteban, J.-M. and D. Ray (1994), On the Measurement of Polarization, in: Econometrica, 62 (4), 819–851.

Esteban, J.-M., Gradín, C. and D. Ray (2007), An Extension of a Measure of Polarization, with an Application to theIncome Distribution of Five OECD Countries, in: Journal of Economic Inequality 5 (1), 1–19.

Foster, J. and M.C. Wolfson (2010), Polarization and the Decline of the Middle Class. Canada and the U.S, in: Journal of Economic Inequality 8 (2), 247–273.

Foster, J., Greer, J. and E. Thorbecke (1984), A Class of Decomposable Poverty Measures, in: Econometrica, Vol. 52, No. 3, 761-766.

Frey, B.S. and A. Stutzer (2005), Happiness Research: State and Prospects, in: Review of Social Economy, Vol. LXII, No. 2, 207-228.

Gigliarano, C. and K. Mosler (2009), Constructing Indices of Multivariate Polarization, in: Journal of Economic Inequality, 7 (4), 435–460.

Goebel, J., Gorning, M. und H.Häußermann (2010), Polarisierung der Einkommen: Die Mittelschicht verliert, Wochenbericht des DIW Berlin, Nr. 24/2010, Berlin.

Goodin, R., Rice, J., Parpo, A. and Eriksson, L. (2008), Discretionary Time: A New Measure of Freedom, Cambridge University Press, Cambridge (UK).

Grabka, M. und J.R Frick (2008), Schrumpfende Mittelschicht – Anzeichen einer dauerhaften Polarisierung der verfügbaren Einkommen, Wochenbericht des DIW Berlin, Nr. 10/2008, Berlin.

Harvey, A.S. and A.K. Mukhopadhyay (2007), When Twenty-four Hours is Not Enough: Time Poverty of Working Parents, in: Social Indicators Research, 82, 1, 57-77.

Kakwani, N. and J. Silber (2008): Quantitative Approaches to Multidimensional Poverty Measurement, Houndmills, Basingstoke, Hampshire: Palgrave Macmillan.

Kmenta, J. (1967), On Estimation of the CES Production Function, in: International Economic Review, Vol. 8, No. 2.

Lugo, Maria Ana and Esfandiar Maasoumi (2008), Multidimensional poverty measures from an information theory perspective, Paper prepared for the 30th General Conference of The International Association for Research in Income and Wealth, Portoroz, Slovenia, August 24-30.

Lugo, Maria Ana and Esfandiar Maasoumi (2009), Multidimensional poverty measures from an information theory perspective, OPHI Working paper No. 10, Oxford Poverty & Human Development Initiative,10).

Merz, J. (2006), Polarisierung der Einkommen von Selbständigen? – Zur Dynamik der Einkommensverteilung von Freiberuflern und Unternehmern, in: Merz, J. (Ed.), Fortschritte der MittelstandsForschung, CREPS-Schriftenreihe Vol. 3, Lit Verlag, Münster, 395-415.

Merz, J. and M. Zwick (2005), Hohe Einkommen: Eine Verteilungsanalyse für Freie Berufe, Unternehmer und abhängig Beschäftigte mit Mikrodaten der Einkommensteuerstatistik, in: Journal of Applied Social Science Studies, Jg. 125/2, 313-328.

Merz, J. and T. Rathjen (2009), Time and income poverty - An interdependent multidimensional poverty approach with German time use diary data, Forschungsinstitut Freie Berufe (FFB), Leuphana Universität Lüneburg, FFB-Discussion Paper No.79, Lüneburg.

Merz, J. and T. Rathjen (2011a), Intensity of Time and Income Interdependent Multidimensional Poverty: Well-Being and Minimum 2DGAP – German Evidence, Forschungsinstitut Freie Berufe (FFB), Leuphana Universität Lüneburg, FFB-Discussion Paper No.92, Lüneburg.

Merz, J. and T. Rathjen (2011b), Sind Selbständige zeit- und einkommensarm? – Zur Dynamik interdependenter multidimensionaler Armut mit den deutschen Zeitbudgeterhebungen, in: Bekmeier-Feuerhahn, S., Martin, A., Merz, J. and U. Weisenfeld (Eds.), Die Dynamik tiefgreifenden Wandels in Gesellschaft, Wirtschaft und Unternehmen, LIT-Verlag, 219-239.

Merz, J., Hirschel, D. and M. Zwick (2005), Struktur und Verteilung hoher Einkommen - Mikroanalysen auf der Basis der Einkommensteuerstatistik, Gutachten zum zweiten Armuts- und Reichtumsbericht der Bundesregierung, Lebenslagen in Deutschland, Bundesministerium für Gesundheit und Soziale Sicherung, Berlin.

Nolan, B. and V.T. Whelan (2007), On the Multidimensionality of Poverty and Social Exclusion, in: Jenkins, S.P. and J. Micklewright (2007), Inequality and Poverty Re-examined, Oxford University Press, Oxford, 146-165.

Peichl, A. Schaefer, T. and C. Scheicher (2010), Measuring Richness and Poverty – A Micro Data Application to Europe and Germany, in: The Review of Income and Wealth, 56 (3), 597–619.

Ritter, C. (1896), Platos Gesetze, Neudruck der Ausgabe Leipzig 1896, Scientia Verlag Aalen.

Scheicher, C. (2009), Armut, Reichtum, Umverteilung: Begriff und statistische Messung, Eul Verlag, Lohmar.

Scheicher, C. (2010), Measuring Polarization via Poverty and Affluence, Köln Discussion Papers in Statistics and Econometrics, Köln.

Schmidt, A. (2004), Statistische Messung von Einkommenspolarisierung, Eul Verlag, Erste Auflage, Lohmar.

Sen, A.K. (1985), Commodities and Capabilities, North-Holland, Amsterdam.

Sen, A.K. (1999), Development as Freedom, Knopf Publishers, New York.

Sen, A.K. (2008), The Economics of Happiness and Capability, in: Bruni, L., Comim, F. and M. Pugno, Capabilities & Happiness, Oxford University Press, Oxford (UK), 16-27.

Vickery, C. (1977), The Time-Poor: A New Look at Poverty, in: Journal of Human Resources, 12, 1, 27-48.

Wagner, G.G., Frick. J.R. and J. Schupp (2007), The German Socio-Economic Panel Study (SOEP): Scope, Evolution and Enhancements, in: Journal of Applied Social Science Studies, 127, 1, 139-179.

Wang, Y.-Q. and K.-Y. Tsui (2000), Polarization Orderings and New Classes of Polarization Indices, in: Journal of Public Economic Theory, 2 (3), 349–363.