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ABSTRACT

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This paper shows the effects on a pay-as-you-go pension system of the demographic change in the standard overlapping generations model. Firstly, we consider a setting with exogenous fertility and then a model with endogenous fertility. In both cases, population ageing due to increased longevity implies a reduction in pensions payouts.

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1 Introduction

A number of papers have studied the effects of population ageing on the pension system. In fact, the decline of the fertility rate and the increase in life expectancy have seriously undermined the solvency of the pay-as-you-go pension system in many developed countries that, because of ageing, have had to substantially change their pension schemes.

Theoretical models in the overlapping generations literature have studied the three main effects of a falling fertility: the intergenerational transfer effect, the capital dilution effect and the child quality effect.

The first effect is the classical externality arising from the fact that a fall in the fertility rate decreases the future number of workers who will support all the pensioners, leading to a decentralized equilibrium with too few children. Examples include Cigno (1996, 2010), Ehrlich and Lui (1998), Nishimura and Zhang (1992), Rosati (1996), Wigger (1999) and, more recently, Cremer et al. (2006) and Alders and Broer (2005).

The second effect works in the opposite direction: a fall in the fertility rate increases the capital labor ratio, increasing output per capita and pension payouts. Among the relevant papers here is Michel and Pestieau (1993) and Cigno (1993).

The third effect is due to the parental decision on investing in child education in a quality-quantity model: shifting child investment from quantity to quality, i.e. investing in children's human capital, increases future wages and tax revenues for the pension system. A very recent paper in this literature is Cremer et al. (2011). Another recent paper with education, focusing on agents investment in their own human capital in a model with pay-as-you-go pensions is Cipriani and Makris (2012).

The central issue in the various strands of the literature has been the study of the sustainability of a PAYG pension policy. However, a recent paper by Fanti and Gori (2012) shows that a falling birth rate does not necessarily cause a fall of pensions in the steady state of a standard OLG model with log utility and Cobb-Douglas production function. Their main policy implication is that no pension reforms may be necessary to face the fertility drop. In this paper we extend their textbook model to include life expectancy. We show that, as a result of an increasing longevity, pension transfers diminish in the steady state. Therefore, contrary to their results, we show that an ageing population (caused by falling fertility and increasing longevity) may adversely affect the pension system in the long run. Then we show that, in a model with endogenous fertility, an increase in life expectancy leads to a fall in fertility and adversely affects pensions.

The organization of the paper is as follows: Section 1 looks at a model with exogenous fertility, Section 2 considers an endogenous fertility setting, Section 3 concludes.

2 A Simple Model with Exogenous Fertility

The model consists of an economy with two overlapping generations in each period. We assume Cobb-Douglas functional forms in order to obtain definite results.[1] The superscript t, with $t = 1, ..., \infty$, denotes the time an agent was born. We model longevity as in Blackburn and Cipriani (2002): an individual lives for one period with certainty and faces a probability of surviving to old age. The expected utility function of an adult agent born in period t is described by:

$$U^t = \ln c_a^t + \beta\pi \ln c_o^t \tag{1}$$

where c_a^t is adult consumption of an individual belonging to generation t, π^t is the probability of surviving to the second period, c_o^t is consumption of an old agent born in period t and $\beta \in (0, 1)$ denotes the utility discount factor. The first period budget constraint is the following:

$$c_a^t = w_t(1 - \theta) - s_t - qw_t n \tag{2}$$

where $\theta < 1$ is the contribution rate to the PAYG pension system, $q > 0$ is the child rearing cost, assumed to be a fixed proportion of adult income, n is the fertility rate, s_t is the amount of resources saved in period t and w_t is the wage income. When financial markets intermediaries lend to or borrow from individuals, the corresponding rate must incorporate the risk involved due to the agents' uncertain lifetimes. Assuming, then, that intermediaries operate under conditions of perfect competition, and that entry is costless, we have that this rate of return is equal to R/π, where R is the risk-free rate of interest. If p_{t+1} is the pension paid to an old individual in period $t + 1$, the second period budget constraint is therefore:

$$c_o^t = R_{t+1}s_t/\pi + p_{t+1} \tag{3}$$

[1]Although this is an obvious limitation of our theory, it allows us to compare the results with those obtained by the large literature working with the "canonical" overlapping generations model.

Maximization of utility subject to the budget constraints gives the following savings function:

$$s^t = \frac{\beta w_t \left(1-\theta-qn\right)}{\left(1/\pi+\beta\right)} - \frac{p^e_{t+1}}{\left(1/\pi+\beta\right) R^e_{t+1}} \tag{4}$$

Assuming a perfectly competitive output market and a Cobb-Douglas production function $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$ where $A > 0$ and $0 < \alpha < 1$, with full capital depreciation after one period, profit maximization implies that $R_t = \alpha A k_t^{\alpha-1}$ and $w_t = (1-\alpha) A k_t^{\alpha}$, where k is per capita capital. Given the government budget constraint, $p_t \pi = \theta w_t n$, and the market clearing condition for the capital market, $nk_{t+1} = s$, the following dynamic equation for capital can easily be derived:

$$k_{t+1} = \frac{\beta w_t (1-\theta-qn)}{n(\frac{1}{\pi}+\beta)} - \frac{\theta w^e_{t+1}}{(\frac{1}{\pi}+\beta)\pi R^e_{t+1}} \tag{5}$$

Assuming perfect foresight, the steady state is defined as:

$$k^* = \left(\frac{A\beta\alpha\pi(1-\alpha)(1-\theta-qn)}{n\left(\alpha\pi(\frac{1}{\pi}+\beta\right) + \theta(1-\alpha))} \right)^{\frac{1}{1-\alpha}} \tag{6}$$

Hence, population ageing, either because of a fall in n or because of an increase in π, increases the per capita capital stock. Given the government budget constraint, and the steady state level of capital, k^*, the long-run pension benefits can be written as a function of the demographic variables $p^* = p\{n, \pi\}$:

$$p^* = \frac{\theta(1-\alpha)A}{\pi} \left(\frac{A\beta\alpha\pi(1-\alpha)(1-\theta-qn)}{\alpha\pi(\frac{1}{\pi}+\beta) + \theta(1-\alpha)} \right)^{\frac{\alpha}{1-\alpha}} n^{\frac{1-2\alpha}{1-\alpha}} \tag{7}$$

It is easy to show that, like in Fanti and Gori (2012), if $\alpha < 1/2$, this function is decreasing in n when $n > \frac{(1-\theta)(1-2\alpha)}{q(1-\alpha)}$. So, as in their paper, if the pension tax rate and the child rearing cost are sufficiently high, a fall in fertility leads to an increase in pensions. However, an increase in longevity, when $\alpha < 1/2$[2] negatively affects public pensions always. Therefore, population ageing resulting from increasing longevity and falling fertility has in general an ambiguous effect on pensions in this model. However, the effect of an increase in longevity is always unambiguously negative.[3]

[2] A well known empirical regularity of growth is that the capital share in added value, α, is about 1/3.

[3] Similar results have been obtained by Fanti and Gori (2008). However, in that paper the child rearing cost was not considered, while here it affects negatively both k^* and p^*.

3 Endogenous Fertility

We now turn to the case of endogenous fertility. Assume that parents choose the number of children to maximize the utility function

$$U = \ln c_a^t + \beta\pi \ln c_o^t + \gamma \ln n_t \tag{8}$$

This function, often used in the literature, assumes that parents are non-altruistic in the sense that they do not derive utility from the welfare of offspring, but merely from the production of children, who are treated as consumption goods yielding utility to their parents only during the period in which they are born. Also, both consumption and children are substituted intertemporally with unit elasticity. From the first order conditions, assuming that parents do not take into account the marginal effects of their own fertility choice on pensions we get:

$$n_t = \frac{\gamma\pi p_{t+1}^e + \gamma(1-\theta)w_t R_{t+1}^e}{q w_t R_{t+1}^e (1+\beta\pi+\gamma)} \tag{9}$$

$$s_t = \frac{\beta\pi w_t R_{t+1}^e (1-\theta) - (1+\gamma)\pi p_{t+1}^e}{R_{t+1}^e (1+\beta\pi+\gamma)} \tag{10}$$

Given the government budget contraint, optimal savings and fertility are:

$$n_t = \frac{\gamma(1-\theta)w_t R_{t+1}^e}{q w_t R_{t+1}^e (1+\beta\pi+\gamma) - \gamma\theta w_{t+1}^e} \tag{11}$$

$$s_t = \frac{w_t(1-\theta)}{(1+\beta\pi+\gamma)}\left[\beta\pi - \frac{\gamma\theta w_{t+1}^e}{q w_t R_{t+1}^e (1+\beta\pi+\gamma) - \gamma\theta w_{t+1}^e}\right] \tag{12}$$

With capital market clearing, and given the public pensions budget constraint, one can easily obtain the dynamic equation for capital as:

$$k_{t+1} = \frac{\beta\pi q w_t R_{t+1}^e - \gamma\theta w_{t+1}^e}{\gamma R_{t+1}^e} \tag{13}$$

Assuming perfect foresight, substituting for the real interest rate and the wage rate and solving for the steady state gives the following level of capital:

$$k^* = \left(\frac{A\alpha\beta\pi q(1-\alpha)}{\gamma(\alpha+\theta(1-\alpha))}\right)^{1/(1-\alpha)} \tag{14}$$

Hence an increase in life expectancy has, again, a positive effect on k^*. The steady-state level of fertility and pensions are:

$$n^* = \frac{\gamma(1-\theta)R^*}{qR^*(1+\beta\pi+\gamma)-\gamma\theta} \quad (15)$$

$$p^* = \frac{\gamma\theta(1-\theta)w^*R^*}{\pi\left[qR^*(1+\beta\pi+\gamma)-\gamma\theta\right]} \quad (16)$$

Note that the child rearing cost parameter, q, has a positive effect on k^* and a negative effect on n^* and p^*.[4] From here, substituting for w^* and R^* and deriving with respect to π, it is easy to show that an increase in life expectancy has always a negative effect on n^* and also a negative effect on p^* when $\alpha < 1/2$. In other words, if life expectancy increases there is a fall in fertility, which reinforces the ageing of population, and there is a consequent fall in PAYG pensions.

4 Conclusions

This paper presents a simple OLG model with PAYG pensions. When life expectancy increases, in a model with constant fertility, PAYG pension benefits decrease in the steady state. When the model includes endogenous fertility, an increase in longevity has a negative effect on fertility and on pension benefits. Thus, complementing the results of Fanti and Gori (2012), we find that population ageing may ideeed be a problem for the PAYG pension system as soon as one introduces longevity in the baseline model.

[4] In a similar setting, a discussion paper by Fanti and Gori (2009) assumes a fixed cost of child rearing. Here, instead, the time cost of child rearing introduces an additional negative substitution effect on fertility following an increase in longevity.

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