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ABSTRACT

Risk and Saving in Two-Person Households: More Scope for Precautionary Saving*

The existing literature suggests that when the saving decision of two-earner households under risk is analysed, standard results on the existence of precautionary saving no longer apply: precautionary saving is obtained if and only if very stringent conditions hold. This paper shows that when the two-earner household's saving decision is formulated more generally, standard assumptions suffice for precautionary saving to exist under increases in risk of the first and second orders, but not for higher orders.

JEL Classification: D10, D13, D14, D81, D91, E21

Keywords: two-earner households, risk order, precautionary saving

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1 Introduction

Almost without exception, the papers in the large literature on saving decisions under risk[1] take the decision unit to be a single individual and base the analysis on a model of individual preferences, most usually that of expected utility theory. This therefore ignores the fact that most saving is done by households in which typically there are two adults, and so actually or potentially two income earners. When making predictions or analysing data relating to this class of households, the implicit assumption must be that in some sense the two-person characteristics of the household do not matter. We do not have any precise idea however of the conditions under which such an assumption would be justified. Again almost without exception, the models in the large literature on the economics of family-based households[2], though dealing extensively with the two-earner case, ignore the existence of risk. There is a need to bring these two literatures together, in an analysis of two-person household decision taking under risk.

Mazzocco (2004) takes an important step in this direction. He proves a proposition that is not encouraging to those who assume that existing models of saving under risk can be applied regardless of the real nature of the household. A central topic in the theory of saving under risk is that of precautionary saving,[3] defined as saving that varies positively with the future income risk an individual faces. Mazocco shows that when a couple pool their individual incomes and share risk efficiently, thus, intuitively speaking, reducing risk relative to when they take their saving decisions independently, the reduction in saving that would follow from the precautionary motive cannot be guaranteed to happen. This is the case even if their probability beliefs and utility time-discount rates are identical, and their preferences are assumed to take the same special functional form, that of harmonic absolute risk aversion (HARA), which is sufficient for precautionary saving to characterise their *individual* decisions. This assumption has to be strengthened by the requirement that the curvature parameter of their utility functions must be equal - they must have almost identical preferences.

The result is that couples have to be assumed to behave essentially as single-person households,[4] with the advantage that they may be able to pool two

[1] For good surveys see Browning and Lusardi (1996), Eeckhoudt, Gollier and Schlesinger (2005) and Gollier (2001).

[2] For a recent survey of this literature see Apps and Rees (2009).

[3] See for example Browning and Lusardi (1996), Carroll and Kimball (2008) and Parker and Preston (2005).

[4] In this respect, the "collective model" of the household, with which Mazzocco works,

stochastic incomes.[5] This appears to deal quite a serious blow to the intuition supporting the prevalence of precautionary saving motives, which is based upon the argument that for risk averse decision takers, decreasing absolute risk aversion (DARA) is a plausible feature of individual risk preferences and, since this implies a positive third derivative of the utility function, makes precautionary saving equally plausible. The condition given by Mazzocco is much more stringent than that. It also suggests that we should not generally expect to find empirical evidence of precautionary saving in data generated by two-earner households.

The first step in this paper is to argue that these pessimistic conclusions are not warranted. As rigorous and insightful as Mazzocco's analysis is, it does not provide the answer to the question:

Given a cooperative two-person household taking its saving decision in the face of risky future incomes, how does it react to an increase in the riskiness of its future income distribution?

Rather, Mazzocco's analysis addresses the different question:

What happens to the saving of a couple who decide to pool their incomes and take their saving decision jointly rather than individually?

When we tackle the former problem, we can show that the very stringent conditions in Mazzocco's proposition are sufficient, but not necessary, to ensure that precautionary saving will always characterise the joint saving decision when it does so for individual decisions. We provide the complete necessary as well as sufficient conditions and show that for risk increases of the first and second order,[6] with which the literature on precautionary saving is primarily concerned, precautionary saving will take place whenever the individual preferences would exhibit it. Furthermore, we generalise the sense in which the income distribution becomes "more risky" to cases of risk increases of order higher than the second, to clarify when and under what conditions the assumption that the utility functions satisfy the condition for individual saving to increase with risk is sufficient to ensure that joint saving will also do so.

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becomes virtually identical to the "unitary model", which it was meant to replace, as Mazzocco points out. This is problematic, since there is now a great deal of empirical evidence rejecting the unitary model. See Apps and Rees (2009).

[5] At least in the majority of households. However, in most OECD countries roughly one-third of married or cohabiting couples have only a single earner. We should therefore be able to answer the question of whether this matters for saving decisions - would we expect the saving behaviour of these households to be the same as those of single-person households? This is further discussed below.

[6] See below for formal definitions of these terms.

The difference in results arises from the fact that we are carrying out a comparative statics analysis on a given household equilibrium, rather than comparing two different types of equilibrium. As with any comparative statics analysis, determinate predictions generally require restrictions on the functions we work with, but the analysis shows that it is possible to find interesting and less restrictive conditions than Mazocco's, under which precautionary saving also takes place in two-earner households.

2 Increases in risk and household saving

We begin by presenting Mazzocco's model, which is an extension of the "collective model" (CM) of the household[7] to a two-period economy with income uncertainty in the second period. Let $c_i, \tilde{c}_i$ denote the consumption of individual $i = 1, 2$ in the first and second periods respectively, with the latter a random variable. Their exogenously given (labour) incomes are likewise $y_i, \tilde{y}_i$ with their sums given by $y, \tilde{y}$. No insurance or asset markets exist that allow trade in state-contingent incomes, there is only a bond market with certain interest rate $r \geq 0$ which allows trade in incomes between periods. Individual utility functions $u_i(c_i)$ are neither time- nor state-dependent, though future utilities may be discounted by a "felicity discount factor" $\rho \in (0, 1]$. These utility functions are assumed to be continuously differentiable to any required order with $u_i'(.) > 0$, $u_i''(.) < 0$, $i = 1, 2$.

In this extension of the CM, the couple finds its optimal saving by solving the problem

$$\max_{c_i, s, \tilde{c}_i} \sum_{i=1}^{2} \mu_i \{u_i(c_i) + \rho E[u_i(\tilde{c}_i)]\} \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{2} c_i \leq y - s \tag{2}$$

$$\sum_{i=1}^{2} \tilde{c}_i \leq \tilde{y} + (1 + r)s \tag{3}$$

The parameters $\mu_i \in [0, 1]$, which we are free to normalise by setting $\sum_i \mu_i = 1$, are weights reflecting in some sense the "bargaining power" of individual i, or more generally, the weight the household gives to her wellbeing in its collective

[7] See Apps and Rees (1988), Chiappori (1988) and Browning and Chiappori (1998).

decision process.[8] The expectation $E[u_i(\tilde{c}_i)]$ embodies the assumption that the individuals have identical probability beliefs, which greatly simplifies the analysis and is also not unreasonable. We would imagine that the couple shares information and discusses future possibilities, and so might be expected to agree on the probabilities of future states.

The unique solution to this problem, denoted by $\{s^*, c_i^*, \tilde{c}_i^*\}$, $i = 1, 2$, is clearly (ex ante) Pareto efficient. We are also required to assume that the household at the initial decision point can commit to the future allocations of consumption $\tilde{c}_i^*$ in whatever state of the world is realised.[9] Furthermore, changes in risk are assumed not to change the weights μ_i.[10]

To this problem Mazzocco contrasts that of independent decision taking:

$$\max_{c_i, s_i, \tilde{c}_i} u_i(c_i) + \rho E[u_i(\tilde{c}_i)] \tag{4}$$

$$\text{s.t. } c_i \leq y_i - s_i \tag{5}$$

$$\tilde{c}_i \leq \tilde{y}_i + (1+r)s_i \tag{6}$$

for $i = 1, 2$. Denoting the solutions to these problems by s_i^*, the central proposition of Mazzocco's paper can be stated in the present notation as follows

Proposition 1: *Given the problems in (1)-(3) and (4)-(6) and the assumptions made so far, we have that* $s^* \leq \sum_i s_i^*$ *for any value of* $\mu_i \in (0, 1)$ *and strictly positive income vectors* $[y_i, \tilde{y}_i]$ *if and only if the household belongs to the class of households in which the individual utility functions* $u_i(.)$ *belong to the HARA class with identical curvature parameters.*

Proof: Mazzocco (2004).

To see the intuition underlying this proposition, note that when we move from independent to joint decision taking (i.e. the household is formed) there are two effects as far as risk is concerned. First, the incomes of the individuals

[8] Basu (2006) refers to these as measures of the "say" the individual has in household decisions. In general they are functions of variables exogenous to the household, but since in this paper these are assumed not to change, we follow Mazzocco in simply regarding them as given constants.

[9] This commitment issue, which of course differs from the standard "time consistency" requirement as discussed in the literature, does not appear to have been explicitly recognised by Mazzocco. See Fahn and Rees (2011) for an extensive analysis of the basis for this assumption.

[10] This assumption is not innocuous. For example, in a bargaining model, an increase in riskiness of total household income arising out of an increase in only one individual's income risk would worsen that individual's threat point and therefore reduce her bargaining power. We will however continue to exclude this possibility in the following discussion.

are pooled, and for well-known reasons each partner would perceive this as a reduction in risk. Therefore, given their prudent preferences, each would want to reduce saving. However, if they behave efficiently, they will wish to exchange state-contingent incomes,[11] i.e. insure each other against idiosyncratic risk, the optimal pattern of income exchanges across states being determined by the solution to the problem in (1)-(3). Mazzocco shows that even within the special class of cases in which the utility functions are in the HARA class, it is possible to construct cases in which efficient exchange of risky incomes leads to an increase in saving, in the sense that $s^* > \sum_i s_i^*$, when the curvature parameters differ. As long as an increment of income in a given state is allowed to have asymmetric effects on the demand for saving of each individual, it will be possible to find a set of risk-sharing transfers between the individuals, and the weights μ_i and preference parameters that support them, such that aggregate household demand for saving increases, and this could more than offset the reduction resulting from pooling. If and only if these effects of the income exchanges on the individuals' demands for saving are exactly offsetting are such possibilities ruled out. In that case, in each state the effect on the demand for saving of the individual receiving the transfer of income is exactly offset by the effect on the individual making the transfer and so efficient risk sharing will have no effect on aggregate household saving.[12] In this case only risk pooling is relevant to the demand for saving and this causes it to fall, in line with the theory of precautionary demand. The underlying point is that under joint decision taking the way in which the household efficiently shares its income - the properties of its *sharing rule* - must play a role, and this can cause results to deviate from those that might be expected from the analysis of single individuals' decision taking.[13]

However, as already suggested, although this result tells us something interesting about the effects of the formation of a household, it does not necessarily characterise the effects of a change in the riskiness of income endowments on the saving of an existing household which is initially in a risk-sharing equilibrium.

[11] Except in the uninteresting special case in which probability beliefs and endowments are such that there are no gains from trade.

[12] This suggests the close analogy with the Gorman conditions for exact aggregation in the theory of consumer demand. The linear sharing rules with the same coefficients on household income that result from the assumption are analogous to the Gorman polar form of expenditure function, which yields individual demand functions such that income redistributions have no effect on aggregate demand for a good.

[13] See also Mazzocco and Saini (2012), where this point is shown to have important implications for the design of empirical tests of the efficiency of risk sharing.

In fact we find that for first and second order changes in risk,[14] Mazzocco's conclusions do not apply, and the standard assumptions are also sufficient for precautionary saving to characterise the behaviour of couples. Only for higher orders of risk change is this no longer true. However, though sufficient, Mazzocco's conditions are not necessary in these cases. In the following section we carry out the analysis for the usual situation in which precautionary saving is analysed, that of a second order risk increase, also pointing out the results for a first order increase. We then go on to generalise the results to any higher order risk increase, and provide some numerical examples.

3 Household precautionary saving under first and second order risk increases

In the precautionary saving literature, the problem is usually taken to be[15] that of deriving the conditions under which saving increases when there is a mean preserving spread in the distribution of income, for example when a certain income y is replaced by the distribution of incomes $y + \tilde{\varepsilon} = \tilde{y}$, where $\tilde{\varepsilon}$ is a random variable with $E\tilde{\varepsilon} = 0$. More generally, $\tilde{y}$ corresponds to an initial uncertain income with cumulative distribution function $F(\tilde{y})$, which is replaced by a new uncertain income $\tilde{x}$ with cumulative distribution function $G(\tilde{x})$, each defined on a support in the interval (y^0, y^1). Then $\tilde{y}$ is taken to be less risky than $\tilde{x}$ in the sense of second order stochastic dominance.[16] Following Ekern (1980), replacing $\tilde{y}$ by $\tilde{x}$ is then a *second order increase in risk*. The underlying idea stems from the proposition that every risk averse decision taker strictly prefers $\tilde{y}$ to $\tilde{x}$.

Turning now to the household's saving decision, as modeled in (1)-(3) above, it is well-known[17] that the optimal allocations within any one state are independent of probabilities and the discount factor and can be found for any given

[14] As defined in the next section.
[15] Following Rothschild and Stiglitz (1970), (1971).
[16] Implying that $E\tilde{y} = E\tilde{x}$ and $F(z) = \int_{y^0}^{z} F(t)dt \leq G(z) = \int_{y^0}^{z} G(t)dt$ for all $z \in (y^0, y^1)$, with strict inequality for some z.
[17] See for example Eeckhoudt, Gollier and Schlesinger (2005) or Gollier (2001).

total income z available in that state[18] by solving the problem[19]

$$\max_{c_i} \sum_i \mu_i u_i(c_i) \text{ s.t. } \sum_i c_i \leq z \tag{7}$$

yielding as solution the two functions $c_1(z)$ and $c_2(z) \equiv z - c_1(z)$. Following Samuelson (1956), we call this solution the "sharing rule", and $c_i(z)$ the "share functions". Obviously the properties of these functions are determined by those of the $u_i(.)$, but are more complex than those of any one of these functions because of the effects of exchange between the individuals. Note in particular that, given the first order condition

$$\mu_1 u_1'(c_1(z)) = \mu_2 u_2'(z - c_1(z)) \tag{8}$$

we have from the Implicit Function Theorem:

$$c_1'(z) = \frac{\mu_2 u_2''}{\mu_1 u_1'' + \mu_2 u_2''} > 0 \tag{9}$$

with of course $\sum_i c_i'(z) = 1$. It is also worth noting at this point that $\sum_i c_i''(z) = \sum_i c_i'''(z) = 0$.

Define the *indirect household welfare function* (HWF):

$$H(z) = \sum_{i=1}^{2} \mu_i u_i[c_i(z)] \tag{10}$$

as the value function of the problem in (7), noting that in the initial period we have

$$z = z_0(s) \equiv y - s \tag{11}$$

and in each state in the second period

$$z = \tilde{z}(s) \equiv \tilde{y} + (1+r)s \tag{12}$$

Clearly, choice of s is equivalent to choice of z.

Then the household chooses its optimal saving and at the same time allocates total incomes to individual consumptions by solving the problem:

[18] That is, we allow z to denote either $y - s$ or $\tilde{y} + (1+r)s$, as the case may be.

[19] In a slight abuse of notation we allow c_i now to denote consumption in any period or state.

$$\max_s H[z_0(s)] + \rho E\{H[\tilde{z}(s)]\} \quad (13)$$

Given the individual's choice problem as set out in (4)-(6), it is well known[20] that strict concavity of the utility function is sufficient to satisfy the second order condition for the problem to yield a unique global saving optimum, and also for a first order increase in risk, under which $\tilde{x}$ is more risky than $\tilde{y}$ in the sense of first order stochastic dominance,[21] to result in an increase in saving. Furthermore, as already noted, prudence, or $u_i''' > 0$, is sufficient for a second order risk increase to increase saving. By comparing (4) with (13), it is clear that the function $H(.)$ in the two-earner saving problem plays exactly the same role as $u_i(.)$ in the individual problem, and so extending the results on precautionary saving in the former case simply requires us to examine the derivatives of $H(.)$. The important difference is that changes in saving affect individual utilities via the sharing rule, and this is the source of the additional complexity created by a two-person household.

We can characterise the optimal saving s^* by the first order condition, which, given the distribution $F(\tilde{y})$, is

$$H'[z_0(s^*)] = \rho(1+r)E_F\{H'[\tilde{z}(s^*)]\} \quad (14)$$

We establish the result on a first order increase in risk simply by showing that H is strictly concave in s. Although risk aversion would guarantee this in the case of single individuals, the presence of the sharing rule must now be taken into account. This is however straightforward.

Proposition 2: *A first order increase in risk at the household equilibrium will cause an increase in saving.*

Proof: Since $z_0(.)$ and $\tilde{z}(.)$ are linear in s, it suffices to show that

$$H''(z) = \sum_{i=1}^{2} \mu_i \{u_i''(c_i')^2 + u_i' c_i''\} < 0 \quad (15)$$

This follows immediately by noting that from the first order condition (8) and

[20] Again see Eeckhoudt, Gollier and Schlesinger (2005) or Gollier (2001).
[21] $F(z) \leq G(z)$ for all $z \in (y^0, y^1)$, with strict inequality for some z.

the fact that $\sum_i c_i''(z) = 0$ we have

$$H''(z) = \sum_{i=1}^{2} \mu_i u_i''(c_i')^2 < 0 \tag{16}$$

as a result of risk aversion.

This simple proposition has interesting economic applications. Consider for example a young couple planning to start a family. Since this will very likely be associated with a fall in income of at least one individual as time is diverted from market work to child care,[22] the couple will anticipate a first order increase in risk in future income - in every future state of the world household income will be lower, the cumulative distribution function shifts to the left. Therefore they will increase their current saving. However their saving on average after the arrival of the child will fall, since their average income will be lower. This "humped" shape of saving in younger households is strongly confirmed by the data.[23]

Turning now to the second order risk increase, we can establish our main result:

Proposition 3: *For a second order risk increase, the condition* $u_i''' \geq 0$ with strict inequality for at least one i *is sufficient for joint precautionary saving.*

Proof: Using standard arguments[24] we can show that for precautionary saving to result from a second order risk increase it is necessary and sufficient that

$$H''' = \sum_{i=1}^{2} \mu_i [u_i'''(c_i')^3 + 3u_i'' c_i' c_i'' + u_i' c_i'''] > 0 \tag{17}$$

We know that $c_i' > 0$, and so the first term in (17) is positive under the condition of the proposition. Substituting for c_1' from (9) into the second term in (17) and rearranging gives

$$\mu_1 u_1'' \frac{\mu_2 u_2''}{\mu_1 u_1'' + \mu_2 u_2''} = \mu_2 u_2'' \frac{\mu_1 u_1''}{\mu_1 u_1'' + \mu_2 u_2''} \tag{18}$$

which allows us to eliminate the second term. Finally using the first order condition (8) the third term becomes $\mu_1 u_1'(c_1''' + c_2''') = 0$ and this term also vanishes. This gives the result.

[22] Or there is an increase in expenditure required to provide non-parental child care.

[23] See for example Apps and Rees (2009), Ch. 5.

[24] Again see Eeckhoudt, Gollier and Schlesinger (2005), Gollier (2001) and Eeckhoudt and Schlesinger (2008).

Note that this result does not depend on the values of the μ_i. Note also that the condition (17) could be satisfied if $u_i''' > 0$ for only one of the individuals, but then if the other were strictly negative the relative weights and the precise values of the share functions would matter. However, the result would go through if say $u_1''' > 0$ and $u_2''' = 0$, so the utility functions do not have to be identical.

Although this result is relatively easy to establish it is certainly not trivial. We should expect in general that the existence of precautionary saving in the two-person household must depend on some conditions on the household sharing rule, since this is the element that the household model adds to the individual model. The key point about the second order risk increase case is that the comparative statics depend only on the first order derivatives of the share functions $c_i(z)$. The positivity of these derivatives, and the fact that they must sum to 1, suffices for the result. Higher order derivatives involve the curvature properties of the share functions and this is where problems arise.The simple results do not extend to higher orders of risk increase, since then the counterparts of condition (17) involve higher order derivatives of the share functions and conditions only on the signs of the derivatives of the utility functions are no longer sufficient. We now turn to the general case of n'th order risk increases, for $n \geq 3$.

4 Higher risk orders and saving

In general terms, a change in household income risk is a change in an initial cumulative distribution function of the random variable $\tilde{y}$, $F_1(\tilde{y})$, to a new distribution, $G_1(\tilde{y})$, where each is defined on a support in the interval (y^0, y^1). For the purpose of comparative statics analysis it is useful to put some structure on this change, and this is provided by the theory of stochastic dominance and the associated idea of the *order* of a risk increase.[25] If the distribution F_1 dominates G_1 by N'th order stochastic dominance[26] for $N = 1, 2, 3....$, and the first $N-1$ moments of the two distributions are equal, then G_1 is said to represent an N'th order risk increase over F_1.

The usefulness of the idea of risk order follows from the well-known relationship between the preferences of a risk averse decision taker over distributions that can be ordered by stochastic dominance and the signs of the derivatives of

[25] The discussion here is based on Ekern (1980).

[26] That is, defining $F_{n+1}(z) = \int_{y^0}^{z} F_n(y)dy$ for $n \geq 1$ and $G_{n+1}(z)$ similarly, F_1 dominates G_1 by N'th order stochastic dominance iff for all z $F_N(z) \leq G_N(z)$, with strict inequality for some z, and $F_n(y^1) \leq G_n(y^1)$ for $n = 1, .., N-1$.

her utility function, which is a powerful tool in comparative statics analysis of decisions under risk. In general terms, if F_1 dominates G_1 by N'th order stochastic dominance, then every expected utility maximising decision taker with utility function $u(y)$ will prefer F_1 to G_1 if $sgn[u^{(n)}] = (-1)^{n+1}$ for $n = 1, .., N$, where $u^{(n)}$ is the n'th derivative of $u(.)$.[27] An ordering of a given set of distributions by risk then gives the ordering of these distributions by preference. In most existing analyses of decision taking under risk, comparative statics analysis typically throws up expressions involving second and third order derivatives of the utility function, and so, in providing economic meaning to the conditions under which the comparative statics effects take a particular sign, it is useful to have this association of risk preferences and signs of these derivatives.

Eeckhoudt and Schlesinger (2008) give a thorough and illuminating analysis of the general relationship between risk orders and saving decisions in the case of a single individual decision taker. Here we give the straightforward extension of their result to the case of a two-person household. Given the decision problem of a single individual, as presented in (4)-(6), by substituting from the constraints we can write the problem as

$$\max_{s_i} U_i(s_i) = u_i(y_i - s_i) + \rho E[u_i(\tilde{y}_i + (1+r)s_i)] \tag{19}$$

Eeckhoudt and Schlesinger (2008) then generalise the standard proof of the proposition that prudence is necessary and sufficient for an increase in saving when the distribution of $\tilde{y}_i$ is subject to a second order increase in risk, to all orders of risk increase. Specifically, they prove:

Proposition 4: *For a risk increase of order $N = 1, 2, 3...$ to increase saving, it is necessary and sufficient that $sgn(u_i^{(n+1)}) = (-1)^n$ for $n = 1, ...N$*

Proof: Eeckhoudt and Schlesinger (2008)

Here, as in the foregoing section, we simply replace the individual utility function $u_i(.)$ by the indirect household welfare function $H(.)$.We can then rewrite Proposition 4 as

Proposition 4$'$: *For a risk increase of order $N = 1, 2, 3...$ to increase saving in the two-earner household, it is necessary and sufficient that $sgn(H^{(n+1)}) = (-1)^n$ for $n = 1, ...N$*

We have already seen two applications of this Proposition in the previous section. Note that in that analysis, the fact that incomes were pooled and that the distribution functions are defined on household income imply that the

[27] In the remainder of this paper we adopt this notation.

values of individual incomes do not influence the results.[28] Thus we conclude that even if there is only one earner, as long as the non-earner has some positive weight in the household decision process the saving behaviour of the two-person household will in general differ from that of a single person household.

The derivatives $H^{(n)}$ are clearly more complicated objects than the $u^{(n)}$ in the theory of individual saving, since they depend on the properties of the individual utility functions and the sharing rule functions, as well as being weighted sums of two possibly different functions. However, we can establish one general result which shows that the conditions in Mazzocco's theorem, Proposition 1 above, are sufficient for precautionary saving to exist for higher orders of risk increase.

Proposition 5: *For orders of risk increase $n \geq 3$, if the share functions are linear and both individuals would exhibit precautionary saving when taking their decisions independently, then the household will have positive precautionary saving.*

Proof: We are interested in the signs of the derivatives $d^n H(z))/dz^n \equiv H^{(n)}(z)$. For $n \geq 3, ...$ we have

$$H^{(n)}(z) = \sum_{i=1}^{2} \{\mu_i u_i^{(n)} (c_i^{(1)}(z))^n + S_i(c_i^{(1)}, c_i^{(2)}.., c_i^{(n)})\} \tag{20}$$

where $S_i(.)$ denotes a sum of terms each of which includes multiplicatively a derivative of $c_i(z)$ of higher order than 1. For a linear sharing rule and for all $n \geq 3$, $S_i(.) = 0$, $i = 1, 2$. Since $c_i^{(1)} > 0$, the signs of the derivatives in the first term in (20) are determined by the signs of the $u_i^{(n)}$. If these are such as to lead each individual to want precautionary saving in her independent decision, then the conditions for the household to want precautionary saving are also satisfied.

Where linearity of the share functions does not hold, we cannot guarantee precautionary saving in the case of orders of risk increase higher than 2, even when individual utility functions satisfy the conditions for individual precautionary saving, simply because the sums $S_i(.)$ defined above involve terms in derivatives of the utility functions of order 2 and higher that take opposite

[28] This would of course change if we included individual incomes among the determinants of the weights μ_i. However, although in this model they are exogenous, in a more general and realistic model with endogenous labour supplies they would be endogenous and it is preferable to take wage rates as the relevant determinants of the μ_i. In that case these would not be unaffected by changes in the distributions of wage rates.

signs. This is obvious for example from (9), which gives

$$c_1''(z) = \frac{\mu_2 u_2''' \sum \mu_i u_i'' - \mu_2 u_2'' \sum \mu_i u_i'''}{(\sum \mu_i u_i'')^2} \gtreqless 0 \tag{21}$$

with $c_1''(z) = -c_2''(z)$. We confirm this by constructing Example 2 in the following section.

5 Examples

Example 1: Here each individual is assumed to have a HARA utility function but the curvature parameters differ. We derive the functions that give saving as a function of first-period household income and show that saving at every income increases when we introduce risk increases of the first, second and third orders. The utility functions take the CRRA form

$$u_i(c_i) = \frac{c_i^{1-\gamma_i}}{1-\gamma_i} \quad i = 1, 2 \tag{22}$$

with $\gamma_1 = 0.3$ and $\gamma_2 = 0.7$. Expected income in the second period is equal to income y in the first period and $(1+r)^{-1} = 0.95$. There are 3 equiprobable states in the second period with a zero mean risk $\varepsilon \in \{-10, 0, 10\}$. The first order risk increase reduces income in every state by 1. The second order risk increase is a mean preserving spread which increases the variance from 10 to 11, but leaves skewness unchanged. The third order increase in risk is a shift to the distribution with equiprobable values $\varepsilon \in \{-13.6, 0.1, 3.5, 10\}$, which introduces negative skewness with the lower order moments held (approximately) constant. Solving for optimal saving as a function of income with the distributional weights $\mu_i = 0.5$ gives the results shown in Figure 1. In each case, saving increases with the increase in risk at every income level.

Figure 1 about here

Example 2: We retain the data of Example 1 but change the utility functions to

$$u_1(c_1) = -\exp(-0.1c_1) \tag{23}$$

$$u_2(c_2) = c_2 - 0.01c_2^2 \tag{24}$$

which are also both HARA. The saving functions are shown in Figure 2. Again the risk increases of the first 2 orders increase saving,[29] but now the third order risk increase leads to a fall in saving over a non-trivial range of household incomes. This example establishes that the counterpart of Proposition 3 for the case of a third order risk increase does not hold, since $u_i'''' \leq 0$ ("temperance") with strict inequality for at least one i does not guarantee the existence of precautionary saving.

Figure 2 about here

6 Conclusion

Mazzocco (2004) showed that only under very stringent conditions will the intuition hold, that total saving falls when two individuals pool their incomes and efficiently share risk, even when their utilities satisfy the necessary and sufficient conditions for individual precautionary saving. This paper poses a somewhat different question. Given an already existing two-person household saving in the face of an uncertain joint income, under what conditions will its saving increase when it experiences a risk increase in this joint income of any given order? This is the standard question of comparative statics that until now has been considered only for households consisting of single individuals.

For a first order risk increase the necessary and sufficient condition is very mild and corresponds to that for a single individual: we simply require strict concavity of the joint maximand in total income, i.e. that the first order necessary condition for optimum saving also be globally sufficient. This is guaranteed by risk aversion. For second order risk increases, the case typically considered in the literature, the conditions now appear to be more complex, depending as they do on the derivatives of the household share functions. However, because only the first order derivatives of these functions turn out to matter, the condition of a positive third derivative of individual utility - prudence - is still sufficient for the existence of precautionary saving. For higher order risk increases, unless the share functions are linear, the condition on the signs of the corresponding derivatives of the utility function is no longer sufficient, because the higher order derivatives of the share functions do in general come into play.

This should not come as a surprise, since efficient income sharing within a social group, such as a household or whole economy, has long been known to have a more complex structure than that attributed to individual decisions.

[29] Although in this example the changes in saving at lower income levels are almost imperceptible.

Nevertheless, precautionary saving for first and second orders of risk increase can hold under much more general conditions than in the problem studied by Mazzocco, in particular curvature parameters of HARA utilities do not have to be identical, and indeed utilities do not even have to be of the HARA type. In other words, nonlinear share functions, or linear share functions with differing slopes, are admissible.

One important restriction in the present analysis stems from the assumption that the HWF was of the weighted utilitarian type, implying the absence of aversion to inequality in ex ante expected utilities. Introducing strict concavity into the function would not only be a reasonably realistic step, but could, we conjecture, actually expand the set of cases in which precautionary saving holds, if conditions on the higher order derivatives of the HWF are placed which correspond to those placed on individual utility functions - that the higher order derivatives alternate in sign in a way that reflects prudence, temperance and so on. This suggests a fruitful intersection of the theories of risk taking and income distribution which in any case share a common formal structure.

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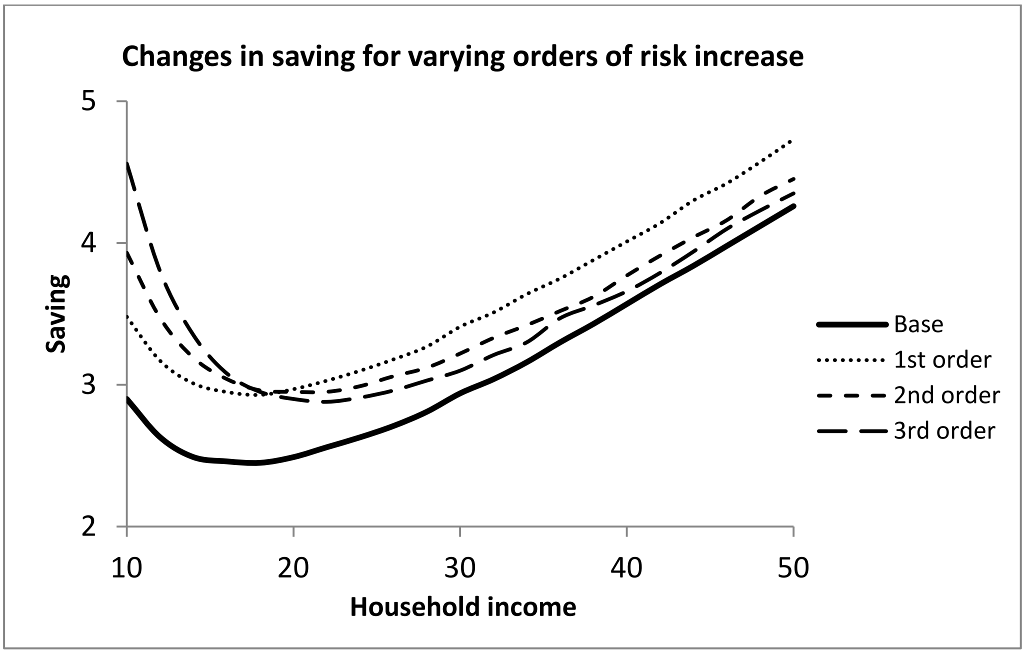

Figure 1: Example 1

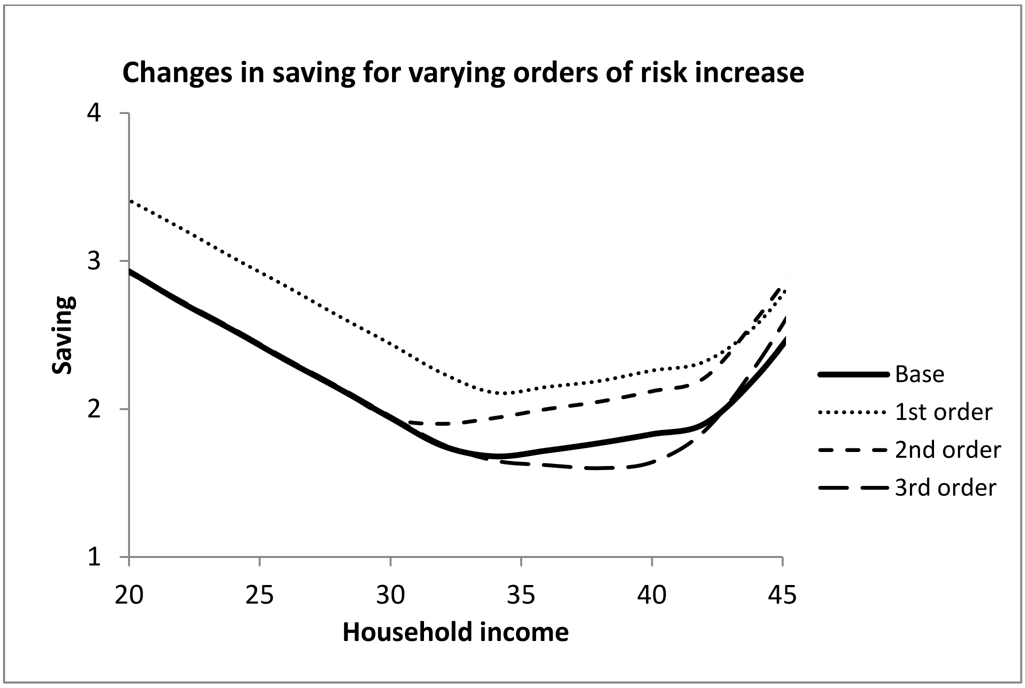

Figure 2: Example 2