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ABSTRACT

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This paper provides a critique of the “unemployment invariance hypothesis,” according to which the behavior of the labor market ensures that the long-run unemployment rate is independent of the size of the capital stock, productivity, and the labor force. Using Solow growth and endogenous growth models, we show that the labor market need not contain all the equilibrating mechanisms to ensure unemployment invariance and that other markets may perform part of the equilibrating process as well. By implication, policies that stimulate investment and R&D and policies that affect the size of the labor force may influence the long-run unemployment rate. Layard-Nickell-Jackman “invariance condition” for labor market systems. This condition is meant to ensure that unemployment is not trended in response to growth in the capital stock, the labor force, or productivity.

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1 Introduction

Consider a policy that raises the growth path of capital, so that after the policy is implemented the capital stock is $x\%$ higher than it would otherwise be, and everything else (including the rate of capital accumulation) remained unchanged. Would the long-run unemployment rate be affected?

Along the same lines, would the long-run unemployment rate be affected by a proportional increase in the effective working-age population (induced, say, by early retirement measures or constraints on working time), or a proportional increase in productivity (generated, for example, by policies promoting R&D), all other things being equal? Questions of this sort have been central to the policy debate concerning unemployment over the past few decades.

A large and influential body of contributions (e.g. Layard, Nickell and Jackman (1991)) have addressed these questions with the view that, regardless of what the short-run impact of these policies might be, in the long run these policies are all ineffective. This view may be called the "unemployment invariance hypothesis." It asserts that the behavior of the labor market (labor demand, labor supply, and wage setting) ensures that the long-run unemployment rate is independent of the size of the capital stock, productivity, and the labor force. By implication, policies that stimulate capital accumulation or R&D lead to counterveiling changes in wages and labor supply so that there is no effect on the long-run unemployment rate. The hypothesis also implies that policies which reduce the size of the labor force will lead to a fall in the long-run employment rate while leaving the long-run unemployment rate unchanged.

On this account, it is often common practice to impose restrictions on macroeconomic labor market models, in both theoretical and empirical studies, to ensure unemployment invariance. These restrictions generally play a crucially important role in conditioning the behavior of these models.

Another equally influential body of contributions (e.g. Phelps (1994)) adopt a weaker form of this hypothesis, namely, that the long-run unemployment rate can influenced by the capital stock, productivity and the labor force only in trendless forms or combinations. For instance, given that the ratio of capital to labor (in efficiency units) is trendless, the long-run unemployment rate may depend on this ratio. Alternatively, if the growth rate of the capital stock and the working-age population are trendless, then they may also influence the long-run unemployment rate. By contrast, the unemployment invariance restrictions imply that unemployment equations (such as those of Phelps (1994, ch. 17) in which the unemployment rate depends on the capital stock, the labor force, or productivity must be misspecified.

Yet another approach is to deny the invariance hypothesis altogether. In particular, Rowthorn (1999) argues that the the capital stock does not affect the long-run unemployment rate in the model Layard, Nickell, and Jackman (henceforth LNJ) since it assumes a Cobb-Douglas production function, so that the elasticity of substitution between labor and capital is unity. If, more realistically, this elasticity is taken to be less than unity, then "weak invariance" follows.

At first sight, the empirical evidence appears to favor the unemployment invariance hypothesis, in its strong or weak forms. A striking feature of OECD labor markets over the past century is that unemployment rates have been essentially trendless, despite massive increases in productivity and population. Productivity growth stimulated labor demand; population growth stimulated labor supply. But these developments have not proceeded at the same rate. For most OECD countries, the rate of productivity growth over the past century significantly exceeded the rate of population growth, but nevertheless the unemployment rates have not followed a declining trend over the long run. The major swings in labor force growth, in response to baby booms and troughs, have not been closely related to the major swings in productivity growth. However the long-run unemployment rates appear to be largely independent of these developments.

This "unemployment invariance phenomenon" - the independence of long-run unemployment rates from the magnitude of the capital stock, productivity, and the labor force - may be illustrated in Fig. 1, which provides a conventional, stylized picture of a country's labor market.[1] In this context, increases in productivity - stemming, say, from capital accumulation and technological progress - shift the labor demand curve (LD) outwards, but over the long run the wage setting curve (WS) shifts inwards by the same amount, leaving the unemployment rate unchanged. Moreover, increases in population shift the labor supply curve (LS) outwards, but over the long run the wage setting shifts outwards by the same amount, once again leaving the unemployment rate unchanged.

The unemployment invariance phenomenon is one of the prominent styl-

[1]The labour demand curve LD specifies aggregate employment at any given real wage; the wage setting curve WS specifies the equilibrium real wage at any given aggregate employment level; and the labour supply curve LS gives the size of the labour force at any given real wage. If the labour market clears, the wage setting curve coincides with the labour supply curve; if the market does not clear - for efficiency wage, insider-outsider, labour union, or other reasons - then the wage setting curve lies to the left of the labour supply curve (as illustrated in the figure). The intersection of the labour demand curve and the wage setting curve yields the equilibrium employment level (E^*) and the equilibrium real wage (w^*). The difference between the labour supply (LS^*) and employment at the equilibrium real wage (w^*) is the equilibrium unemployment level (U^*).

ized facts of labor economics. The unemployment invariance hypothesis provides a possible explanation of this phenomenon. However, it is not the *only* possible explanation since, as noted, the hypothesis asserts that the behavior of the labor market, all by itself, is responsible for the unemployment invariance phenomenon. Yet, in practice, the behavior of other markets may be responsible as well.

This paper calls the unemployment invariance hypothesis into question. We will begin by examining the most widely used form of the invariance restrictions, namely those popularized by Layard, Nickell and Jackman (1991), among others - that we will call the "LNJ restrictions," for short. Many empirical models of the aggregate labor market[2] impose such restrictions, even though they are commonly rejected by the data (see Section 4 for an empirical illustration). These restrictions, in short, are an instance of theory overriding empirical considerations.

This paper will argue that the LNJ restrictions are completely unnecessary. Specifically, it is not necessary for the *labor market* to contain all the equilibrating mechanisms that guarantee unemployment invariance. Instead, what is required is merely that *all the markets* in the general equilibrium system contain such equilibrating mechanisms. In general, the labor market may be only one of possibly many markets doing the required equilibration.[3]

The paper makes this point in the context of the Solow growth model and a model of endogenous growth with unemployment.

2 The LNJ Invariance Conditions

The basic Layard-Nickell-Jackman (LNJ) model consists of a price mark-up equation and a wage mark-up equation. These equations are meant to describe equilibrium unemployment as the outcome of the "battle of the mark-ups."

First, firms set prices as a mark-up on expected wages:

$$P_t - W_t^e = \beta_0 - \beta_1 u_t, \tag{1a}$$

where P_t is the price level, W_t is the wage level, the superscript e stand for "expected," u_t is the unemployment rate, and the coefficients β_0 and β_1 are positive. Following the standard set-up, all variables (except the unemployment rate) are in logs.

[2] See, for example, Bean, Layard and Nickell (1987), Layard, Nickell, and Jackman (1991), Nickell (1995).

[3] Observe that this rationale for the absence of unemployment invariance is quite distinct from Rowthorn's critique of strong invariance.

Second, workers set wages as a mark-up on expected prices:

$$W_t - P_t^e = \gamma_0 - \gamma_1 u_t, \tag{1b}$$

where $\gamma_0, \gamma_1 > 0$.

In equilibrium, expectations are correct: $P_t^e = P_t$ and $W_t^e = W_t$. Thus, by (1a) and (1b), the equilibrium unemployment rate is

$$u^* = \frac{\beta_0 + \gamma_0}{\beta_1 + \gamma_1}. \tag{1c}$$

Next, let us extend this model to include the capital stock, the labor force, and productivity and specify the LNJ invariance conditions. The following is a simple way of doing so.

Let the price mark-up over the expected wage depend positively on employment E_t (due to diminishing returns to labor), negatively on the labor force L_t (since an increase in the labor force reduces the firms' search costs, *ceteris paribus*), negatively on the capital stock[4] K_{t-1} and a technological variable τ_t (since an increase in K_{t-1} or τ_t raises productivity and permits the firm to reduce its price relative to the wage):

$$P_t - W_t^e = \beta_0 + \beta_E E_t - \beta_L L_t - \beta_K K_{t-1} - \beta_\tau \tau_t, \tag{2a}$$

where the parameters $\beta_0, \beta_E, \beta_K, \beta_\tau$ are all positive.

Furthermore, let the wage mark-up over the expected price depend negatively on the unemployment rate u_t (as above), positively on the capital stock K_{t-1} and the technological variable (since productivity increases enable workers to claim higher wages):

$$W_t - P_t^e = \gamma_0 - \gamma_u u_t + \gamma_K K_{t-1} + \gamma_\tau \tau_t, \tag{2b}$$

where $\gamma_0, \gamma_u, \gamma_K, \gamma_\tau > 0$.

Since the labor force L_t and employment E_t are in logs, the unemployment rate may be approximated by

$$u_t = L_t - E_t. \tag{2c}$$

Recalling that $P_t^e = P_t$ and $W_t^e = W_t$ in equilibrium, the equilibrium unemployment rate becomes

$$u_t^* = \frac{(\beta_0 + \gamma_0) + (\beta_E - \beta_L)\, L_t + (\gamma_K - \beta_K)\, K_{t-1} + (\gamma_\tau - \beta_\tau)\, \tau_t}{\gamma_u + \beta_E} \tag{2d}$$

[4] In the analysis below, it will be important to distinguish between the capital stock in use at the beginning of the current period, denoted by K_{t-1}, and the capital stock in existence at the end of the period, denoted by K_t. The latter includes investment undertaken during the current period.

by (2a)-(2c).

In this context, the unemployment rate is independent of the capital stock, the labor force, and the productivity level in the long run if and only if the following invariance conditions hold:

$$\beta_E = \beta_L, \quad \beta_K = \gamma_K, \quad \beta_\tau = \gamma_\tau. \tag{3}$$

These are the "LNJ invariance restrictions."

3 The Role of Capital Accumulation

In this section we build a model containing both the labor market and the capital goods market, and show that when both of these markets are involved in the process of equilibration towards the steady state of economic activities, then the LNJ invariance conditions are unnecessarily restrictive and can be dropped altogether. The reason, of course, is that these conditions, being based solely on a model of the labor market, implicitly assume that the labor market does all the equilibration to establish unemployment invariance. We use the Solow growth model, however, to so how the capital goods market can be involved in the equilibration as well.

Let V_t be the nominal user cost of capital. Let the price mark-up over the expected user cost depend positively on the capital stock in use K_{t-1} (due to diminishing returns to capital), negatively on employment E_t (since a rise in employment is assumed to increase the productivity of capital, permitting the firm to reduce its price relative to the user cost), and negatively on the technological variable τ_t (since a rise in τ_t also raises the productivity of capital):

$$P_t - V_t^e = \alpha_0 + \alpha_K K_{t-1} - \alpha_E E_t - \alpha_\tau \tau_t, \tag{4a}$$

where the parameters $\alpha_0, \alpha_E, \alpha_K$, and α_τ are positive.

Furthermore, let the user cost mark-up over the expected price depend positively on investment I_t (viz., the supply of investment goods is positively related to the real expected user cost of capital), negatively on the capital stock in use K_{t-1} (since an increase in the inherited capital stock reduces the cost of producing new capital goods, i.e. investment), and positively on employment E_t and the technological variable τ_t (since an increase in E_t and τ_t raise productivity and enable firms to demand a higher user cost):

$$V_t - P_t^e = \delta_0 + \delta_I I_t - \delta_K K_{t-1} + \delta_E E_t + \delta_\tau \tau_t, \tag{4b}$$

where $\delta_0, \delta_I, \delta_K, \delta_E, \delta_\tau > 0$ and, for simplicity, we define $I_t \equiv \Delta K_t$.[5]

In equilibrium, $V_t^e = V_t$ and $P_t^e = P_t$. Then, by (4a) and (4b) and recalling that $u_t = L_t - E_t$, the equilibrium unemployment equation must satisfy the following condition:[6]

$$u_t^* = \frac{(\alpha_E - \delta_E) L_t - (\alpha_0 + \delta_0) - \delta_I I_t - (\alpha_K - \delta_K) K_{t-1} + (\alpha_\tau - \delta_\tau)\tau}{\alpha_E - \delta_E}. \quad (4c)$$

Let us now examine how the markets for labor and for capital goods can be jointly responsible for establishing unemployment invariance. Specifically, consider an economic system comprising a labor market (2a)-(2c) and a capital goods market (4a)-(4b). To keep our analysis as simple as possible, let us assume for the moment that the labor force is constant and there is no technological progress, i.e. $L_t = L$, and $\tau_t = \tau$. (These assumptions will be relaxed in the next section.) The resulting model will be a simple variant of the Solow growth model's explanation of capital accumulation. Along the lines of the Solow growth model, the rate of capital accumulation is such that both these markets are in equilibrium. Thus, in the long run of this variant of the Solow model, capital accumulation tends to zero.

For expositional simplicity, we rewrite the labor market equilibrium condition (2d) and the capital goods market equilibrium condition (4c) as follows:

$$u_t^* = a_0 + a_L L + a_K K_{t-1} + a_\tau \tau, \quad (5a)$$

$$u_t^* = b_0 + L + b_I I_t + b_\tau \tau + b_K K_{t-1}, \quad (5b)$$

where $a_0 = \frac{\beta_0 + \gamma_0}{\gamma_u + \beta_E}$, $a_L = \frac{\beta_E - \beta_L}{\gamma_u + \beta_E}$, $a_K = \frac{\gamma_K - \beta_K}{\gamma_u + \beta_E}$, $a_\tau = \frac{\gamma_\tau - \beta_\tau}{\gamma_u + \beta_E}$; and $b_0 = -\frac{\alpha_0 + \delta_0}{\alpha_E - \delta_E}$, $b_I = \frac{-\delta_I}{\alpha_E - \delta_E}$, $b_K = \frac{\delta_K - \alpha_K}{\alpha_E - \delta_E}$, $b_\tau = \frac{\alpha_\tau - \delta_\tau}{\alpha_E - \delta_E}$.

[5] Recall that these variables are in logs, so that I_t is the growth rate of capital stock (in levels), under the simplifying assumption that there is no depreciation.

Extending the model to positive depreciation would require only a minor amendment. Specifically, define $\widetilde{K}_t$ and $\widetilde{I}_t$ as the levels (rather than logs) of capital stock and investment, respectively. Then the capital stock (in levels) is $\widetilde{K}_t = \widetilde{K}_{t-1} + \widetilde{I}_t - \delta \widetilde{K}_{t-1}$, where δ denotes the depreciation rate. Thus the growth rate of capital stock is given by $\frac{\widetilde{K}_t - \widetilde{K}_{t-1}}{\widetilde{K}_{t-1}} = \frac{\widetilde{I}_t}{\widetilde{K}_{t-1}} - \delta = K_t - K_{t-1}$, since $\ln \widetilde{K}_t \equiv K_t$.

Consequently, when the depreciation rate is positive, the factor $-\delta_I \delta$ needs to be added to the constant terms of our equations.

[6] Just as unemployment in the LNJ framework equilibrates the labor market, so unemployment equilibrates the capital goods market here. If the capital goods market were to be solely responsible for establishing unemployment invariance, then (by (4c) and observing that I_t is constant in the long run) the invariance conditions would be $\alpha_E = \delta_E$, $\alpha_K = \delta_K$ and $\alpha_\tau = \delta_\tau$. These invariance conditions are the partial-equilibrium analogue in the capital goods market of the LNJ invariance conditions (3). We consider neither the LNJ model nor the model above (eq. (4c)) satisfactory for the determination of unemployment, since both adopt a partial-equilibrium approach.

Since the capital stock adjusts in Solow fashion so that both markets are in equilibrium, equations (5a) and (5b) may be combined to yield the following equation describing capital accumulation:[7]

$$I_t = \frac{1}{b_I}\left[(a_0 - b_0) + (a_L - 1)\, L + (a_\tau - b_\tau)\,\tau + (a_K - b_K)\, K_{t-1}\right]. \qquad (6)$$

Since the labor force is constant and there is no technological progress, in the long run $I_t = 0$ and the capital stock is constant: $K_t = K_{t-1} = K^{LR}$. Thus, by (6),

$$K^{LR} = \frac{1}{a_K - b_K}\left[(b_0 - a_0) + (1 - a_L)\, L + (b_\tau - a_\tau)\,\tau\right], \qquad (7)$$

where L and τ are constants.

In this context it is clearly unnecessary to impose the LNJ invariance restrictions, since the capital stock adjusts so as to ensure that the unemployment is constant in the long run. This preliminary result is of course obvious: the assumed constancy of the labor force and technology imply constancy of the long-run capital stock, and the constancy of all these variables ensures the long-run constancy of the unemployment rate. A real test of the LNJ invariance conditions, however, requires us to consider an economy in which the capital stock, the labor force, and productivity are all growing. We proceed to show that, in this context, the LNJ invariance conditions are also unnecessary to ensure that unemployment is constant in the long run.

4 The Role of Technological Change

We now relax the assumption that labor force growth and the rate of technological progress are both zero, assuming instead that the labor force grows at a constant rate $\Delta L_t \equiv g_L$ and the technological variable grows at a variable rate $\Delta\tau_t$. Our model here is an endogenous growth model in the spirit of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). In the Solow growth model, goods can be used for two purposes, consumption and investment. We now assume that they can serve a third purpose as well: R&D. For analytical simplicity (but without loss of generality),[8] assume that R&D is accomplished through labor alone. Accordingly,

[7] Setting the right-hand sides of (5a) and (5b) equal to one another, $a_0 + a_L L + a_K K_{-1} + a_\tau \tau = b_0 + b_I I + b_K K_{-1} + L + b_\tau \tau$, which implies equation (6).

[8] It is straightforward to extend the model to let R&D be generated by both labor and capital.

let E_t^τ be the amount of labor in the R&D sector, whose output is the rate of technological progress $\Delta\tau_t$. Let the production function of the R&D sector be

$$\Delta\tau_t = \frac{1}{\theta}E_t^\tau. \tag{8a}$$

The more labor is devoted to R&D, the less is available for the production of consumption and investment goods and the faster the rate of technological progress.

As in the previous section, let E_t be the amount of labor devoted to consumption and investment goods. Then the unemployment rate becomes

$$u_t = L_t - E_t - \theta\Delta\tau_t. \tag{8b}$$

By (2a), (2b), and (8b), the labor market equilibrium condition now becomes

$$u_t^* = \frac{(\beta_0 + \gamma_0) + (\beta_E - \beta_L) L_t + (\gamma_K - \beta_K) K_{t-1} + (\gamma_\tau - \beta_\tau) \tau_t - \beta_E \theta \Delta\tau_t}{\gamma_u + \beta_E}. \tag{8c}$$

Furthermore, by (4a), (4b), and (8b), the capital goods market equilibrium condition becomes

$$\begin{aligned} u_t^* &= \frac{(\alpha_E - \delta_E) L_t - (\alpha_0 + \delta_0) - \delta_I I_t - (\alpha_K - \delta_K) K_{t-1}}{\alpha_E - \delta_E} \\ &\quad + \frac{(\alpha_\tau - \delta_\tau) \tau_t - (\alpha_E - \delta_E) \theta \Delta\tau_t}{\alpha_E - \delta_E}. \end{aligned} \tag{8d}$$

Along the same lines as in the previous section, we may rewrite these two equilibrium conditions as

$$u_t^* = a_0 + a_L L_t + a_K K_{t-1} + a_\tau \tau_t + a_g \Delta\tau_t, \tag{9a}$$

$$u_t^* = b_0 + b_I I_t + b_K K_{t-1} + L_t + b_\tau \tau_t + b_g \Delta\tau_t, \tag{9b}$$

where $a_g = \frac{-\beta_E\theta}{\gamma_u + \beta_E}$ and $b_g = -\theta$.

As in a variety of endogenous growth models, capital accumulation and R&D adjust so that the economy approaches a steady state in which (a) the labor market and the capital goods market are both in equilibrium, (b) the capital stock grows at a constant rate, (c) the rate of technological progress is constant and (d) the unemployment rate is constant.

Furthermore, taking first differences in equations (9a) and (9b), and recalling that the unemployment rate is constant in the long run,[9] we obtain the long-run equilibrium growth rates of the capital stock (g_K) and technological progress (g_τ) as a function of the growth rate of labor force (g_L):[10]

$$g_K = \left(\frac{a_\tau - a_L b_\tau}{b_\tau a_K - a_\tau b_K}\right) g_L, \tag{10}$$

$$g_\tau = \left(\frac{-a_K + a_L b_K}{b_\tau a_K - a_\tau b_K}\right) g_L. \tag{11}$$

In the context of the model above, the LNJ invariance restrictions may be expressed as follows:

$$a_L = 0, \; a_K = 0, \text{ and } a_\tau = 0. \tag{12}$$

Now observe that these conditions are unnecessary to ensure unemployment invariance, since the long-run unemployment rate is constant even if

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[9] First differencing of (9a) and (9b) gives:

$$\Delta u^* = a_L g_L + a_K g_K - a_\tau g_\tau = 0,$$
$$\Delta u^* = g_L - b_K g_K - b_\tau g_\tau = 0,$$

respectively.

[10] Alternatively, we can follow the line of argument from the previous section and derive a capital accumulation equation as follows. When the labour market and the capital goods market are both in equilibrium, equations (9a) and (9b) both hold, implying that investment is: $I_t = \frac{1}{b_I}\left[(a_0 - b_0) + (a_g - b_g)\Delta\tau_t + (a_L - 1) L_t + (a_\tau - b_\tau)\tau_t + (a_K - b_K) K_{t-1}\right]$. (Setting the right-hand sides of (9a) and (9b) equal to one another, $a_0 + a_g \Delta\tau_t + a_L L_t + a_K K_{t-1} + a_\tau \tau_t = b_0 + b_g \Delta\tau_t + L_t + b_I I_t + b_\tau \tau_t + b_K K_{t-1}$, which implies the previous investment equation.)

Taking first differences, recalling that in the long-run $I_t \equiv \Delta K_t = \Delta K_{t-1} = g_K$, and thus $\Delta I_t = 0$, we obtain the long-run equilibrium growth rate of the capital stock (g_K) as a function of the growth rates of labour force (g_L) and technological progress (g_τ): $g_K = \frac{(1-a_L)g_L + (b_\tau - a_\tau)g_\tau}{a_K - b_K}$.

In a profound and important contribution, Rowthorn (1999) derives a similar condition for the growth rate of the capital stock that is required to offset the combined effects of labour supply growth and biased technological progress. He calls this the "natural" rate of growth of the capital stock, and shows that equilibrium unemployment (the NAIRU) remains constant if capital grows at this rate (p.421, eq. (20)). Rowthorn's results are based on a modified version of the LNJ model where the elasticity of substitution between labour and capital is below unity. He argues that the reason LNJ find the equilibrium unemployment rate to be unaffected by variations in aggregate capital stock, aggregate labour supply or technological progress is because they use the unrealistic assumption of a unitary elasticity of substitution.

conditions (12) are violated. For labor market models in which the LNJ invariance restrictions do not hold, it is sufficient to ensure that, in the long run, the capital stock, the labor force and productivity all grow at rates that make the labor demand, wage setting, and labor supply schedules shift outwards at the same rate, so that the unemployment rate remains constant.[11]

5 Empirical Illustration

Henry et al. (2000), using an autoregressive distributed lag (ARDL) approach to cointegration analysis,[12] estimated the following UK labor market model for the period 1964-1997:

$$\begin{array}{rcccc} \Delta E_t = & 3.16 & -0.31 E_{t-2} & -0.09 w_t & +0.14 K_t \\ & (1.05) & (0.05) & (0.05) & (0.03) \\ & +3.04 \Delta K_t & -1.98 \Delta K_{t-1} & -0.51 TR_t & -0.01 p_t^{oil}, \\ & (0.38) & (0.32) & (0.13) & (0.001) \end{array} \qquad (13)$$

[11] It is of course possible to derive a broader set of invariance restrictions, to ensure that the labor and capital market equilibrium conditions *together* imply unemployment invariance, regardless of the growth rates of the capital stock, the labor force, and productivity. These broader invariance restrictions however are also unnecessary.

Specifically, solve equation (9b) in terms of the labour force (L_t) and substitute the resulting expression into (9a), to get the following equilibrium unemployment rate that prevails in both markets: $u_t^* = \frac{(a_0 - a_L b_0) + (a_K - a_L b_K) K_{t-1} - a_L b_I I_t + (a_\tau - a_L b_\tau)\tau_t + (a_g - a_L b_g)\Delta\tau_t}{1 - a_L}$. In this context the invariance restrictions are given by:

$$\left\{ \begin{array}{c} a_K - a_L b_K = 0 \\ a_\tau - a_L b_\tau = 0 \end{array} \right\} \Rightarrow \frac{a_\tau}{b_\tau} = \frac{a_K}{b_K} \text{ or } b_\tau a_K - a_\tau b_K = 0.$$

However, these restrictions are unnecessary to ensure unemployment invariance. All that is required is that the growth rates of the explanatory variables of unemployment to evolve according to eq. (12a)-(12b). In other words, when labour supply (L_t) is exogenous and its growth rate (g_L) is given, the condition that guarantees the constancy the unemployment rate in the long run is that the ratio of capital stock growth and technological progress is a function of the parameters of the model:

$$\frac{g_K}{g_\tau} = \frac{a_\tau - a_L b_\tau}{a_L b_K - a_K}.$$

It is not difficult to see that the above condition implies that the long-run unemployment rate depends on $\left(\frac{g_\tau}{g_K} K_{t-1} - \tau_t\right)$. So if the growth rates of capital stock and technology are equal in the long-run, then u_t depends on the ratio of the capital stock and technological variables. (Note that $g_K = g_\tau \Leftrightarrow -(a_K - a_L b_K) = (a_\tau - a_L b_\tau)$).

[12] This approach has been developed by Pesaran and Shin (1995), Pesaran (1997), and Pesaran et al. (1996). We refer the reader to Henry, Karanassou, and Snower (2000) for the details of the estimation.

$$\Delta w_t = \underset{(0.11)}{-0.34} \quad \underset{(0.07)}{-0.31 w_{t-2}} \quad \underset{(0.04)}{+0.16 b_t} \quad \underset{(0.34)}{-1.18 \Delta TR_t} \quad \underset{(0.14)}{-0.50 u_t}, \tag{14}$$

$$\begin{aligned} \Delta L_t = & \underset{(0.02)}{-0.004} \quad \underset{(0.12)}{+0.41 \Delta L_{t-1}} \quad \underset{(0.07)}{-0.25 L_{t-2}} \quad \underset{(0.07)}{-0.16 \Delta u_t} \\ & \underset{(0.01)}{+0.02 w_t} \quad \underset{(*)}{+0.25 Z_t}, \end{aligned} \tag{15}$$

where Δ is the difference operator, standard errors are in parentheses[13], and the definitions of the variables are given below:

$$\begin{aligned} E_t &= \text{log of employment, } L_t = \text{log of labour force,} \\ u_t &= \text{unemployment rate } (u_t = L_t - E_t), \\ w_t &= \text{log of real compensation per person employed,} \\ K_t &= \text{log of real capital stock,} \\ p_t^{oil} &= \text{log of real oil price,} \\ b_t &= \text{log of real social security benefits per person,} \\ TR_t &= \text{indirect taxes as \% of GDP, } Z_t = \text{log of working age population.} \end{aligned}$$

For expositional purposes, we rewrite the labor market system (13)-(15) as

$$E_t = \beta_1 + E_{t-1} - \beta_2 E_{t-2} - \beta_3 w_t + \beta_4 K_t + \beta_5 \Delta K_t - \beta_6 \Delta K_{t-1} - \beta_7 TR_t - \beta_8 p_t^{oil}, \tag{16}$$

$$w_t = \beta_9 + w_{t-1} - \beta_{10} w_{t-2} + \beta_{11} b_t - \beta_{12} \Delta TR_t - \beta_{13} u_t, \tag{17}$$

$$L_t = \beta_{14} + (1 + \beta_{15}) L_{t-1} - (\beta_{15} + \beta_{16}) L_{t-2} - \beta_{17} \Delta u_t + \beta_{18} w_t + \beta_{16} Z_t, \tag{18}$$

where all the β's are positive. Using equations (16)-(18) together with the definition of the unemployment rate, $u_t = L_t - E_t$, Henry et al. (2000) derive the "reduced form" unemployment rate equation and its long-run solution[14] $\left(u_t^{LR}\right)$:

$$\begin{aligned} \lambda u_t^{LR} = & \beta_2 \beta_{10} (\beta_{14} + \beta_{16} Z_t) + (\beta_{16} \beta_3 + \beta_2 \beta_{18}) (\beta_9 + \beta_{11} b_t - \beta_{12} \Delta TR_t) \\ & - \beta_{16} \beta_{10} \left(\beta_1 + \beta_4 K_t + \beta_5 \Delta K_t - \beta_6 \Delta K_{t-1} - \beta_7 TR_t - \beta_8 p_t^{oil}\right), \end{aligned} \tag{19}$$

where $\lambda = \beta_2 \beta_{18} \beta_{13} + \beta_{16} \beta_3 \beta_{13} + \beta_2 \beta_{10} \beta_{16}$.

[13] The $(*)$ in equation (15) indicates that the restriction that the long-run elasticity of population is unity could not be rejected at the 5% size of the test.

[14] The existence of a long-run unemployment rate equation is guaranteed by the dynamic stability of the labour market system (13)-(15).

According to the LNJ restrictions, the long-run unemployment rate should not depend on any growing variables like capital stock (K_t) and working age population (Z_t). Therefore, in the context of eq. (19), we can express the LNJ restrictions as follows:

$$\begin{aligned} H_K &: \quad \beta_{16}\beta_{10}\beta_4 = 0, \\ H_Z &: \quad \beta_2\beta_{10}\beta_{16} = 0. \end{aligned}$$

Observe that for the null hypothesis H_K to be true we need $\beta_{16} = 0$, or $\beta_{10} = 0$, or $\beta_4 = 0$. (To see this express the hypothesis in terms of one of its parameters, for example, $H_K : \beta_{16} = \frac{0}{\beta_{10}\beta_4} = 0$.) Similarly, the null hypothesis H_Z will be valid when $\beta_2 = 0$, or $\beta_{10} = 0$, or $\beta_{16} = 0$.

In other words, given the above labor market model, the LNJ restrictions imply that either (i) the wage setting equation (17) is not dynamically stable, i.e. $\beta_{10} = 0$, or (ii) the labor supply equation (18) is not dynamically stable, i.e. $\beta_{16} = 0$, or (iii) the labor demand equation (16) does not depend on the level of capital stock $(\beta_4 = 0)$, and it does not satisfy the condition for dynamic stability $(\beta_2 = 0)$.

It is not difficult to see that our estimations reject the above hypotheses, since all the coefficients of the labor market system (13)-(15) are statistically different from zero at any conventional significance level.

Furthermore, to verify our conclusions from the above analysis, let us also test the LNJ restrictions in the standard integration - cointegration framework as opposed to the ARDL technique followed by Henry et al. (2000).

Consider the following error correction form of the autoregressive distributed lag equations (16)-(18):

$$\begin{aligned} \Delta E_t &= \beta_1 + \beta_2 \Delta E_{t-1} - \beta_3 \Delta w_t + (\beta_5 - \beta_4)\Delta K_t \\ &\quad - \beta_6 \Delta K_{t-1} - \beta_7 \Delta TR_t - \beta_8 \Delta p_t^{oil} \\ &\quad - \beta_2 \left(E_{t-1} + \frac{\beta_3}{\beta_2} w_{t-1} - \frac{\beta_4}{\beta_2} K_{t-1} + \frac{\beta_7}{\beta_2} TR_{t-1} + \frac{\beta_8}{\beta_2} p_t^{oil} \right), \quad (16') \\ \Delta w_t &= -\beta_9 + \beta_{10}\Delta w_{t-1} + \beta_{11}\Delta b_t - \beta_{12}\Delta TR_t - \beta_{13}\Delta u_t \\ &\quad - \beta_{10} \left(w_{t-1} - \frac{\beta_{11}}{\beta_{10}} b_{t-1} + \frac{\beta_{13}}{\beta_{10}} u_{t-1} \right), \quad (17') \\ \Delta L_t &= -\beta_{14} + (\beta_{15} + \beta_{16})\Delta L_{t-1} - \beta_{17}\Delta u_t + \beta_{18}\Delta w_t + \beta_{16}\Delta Z_t \\ &\quad - \beta_{16} \left(L_{t-1} - \frac{\beta_{18}}{\beta_{16}} w_{t-1} - Z_{t-1} \right), \quad (18') \end{aligned}$$

respectively. Therefore, the long-run solutions of (16)-(18),

$$\left[E_t - \frac{1}{\beta_2} \left(-\beta_3 w_t + \beta_4 K_t - \beta_7 TR_t - \beta_8 p_t^{oil} \right) \right], \quad (20)$$

$$\left[w_t - \frac{1}{\beta_{10}}\left(\beta_{11}b_t - \beta_{13}u_t\right)\right], \tag{21}$$

$$\left[L_t - \frac{1}{\beta_{16}}\left(\beta_{18}w_t + \beta_{16}Z_t\right)\right], \tag{22}$$

should represent cointegrating vectors.

Now recall that the LNJ restrictions require that either $\beta_{10} = 0$, or $\beta_{16} = 0$, or $\beta_2 = \beta_4 = 0$. Therefore, non-cointegration of the variables in any of the three vectors (20)-(22) would provide evidence for the validity of the LNJ restrictions. However, when we test for cointegration using the Johansen procedure, we cannot reject the hypotheses that the above linear combinations of variables are stationary.[15] In other words, the results obtained from the Johansen procedure validate our conclusion from the ARDL approach that the LNJ restrictions are rejected.

6 Conclusion

In sum, this paper has argued that the LNJ invariance restrictions are unnecessary to ensure that the long-run unemployment rate is independent of the capital stock, the labor force, and productivity. There is no reason to believe

[15] In particular, the evidence from the Johansen procedure is that the variables involved in each of the equations (20)-(22) are cointegrated. For example, using the max/trace statistics, we find that there is a cointegrating vector among the variables involved in the labor demand equation: E_t, w_t, K_t, TR_t, p_t^{oil}. We do not report these tests to save space.

Furthermore, using likelihood ratio statistics we test whether the coefficients of these cointegrating vectors conform with our ARDL estimations. The likelihood ratio tests for the restrictions imposed on the cointegrating vectors (20)-(22) are:

$$\begin{aligned} \chi^2(4) &= 8.27\ [0.08], \\ \chi^2(2) &= 7.24\ [0.03], \\ \chi^2(3) &= 8.27\ [0.34], \end{aligned}$$

repsectively. (Probabilities are given in square brackets.)

For example, the likelihood ratio statistic $\chi^2(4) = 8.27\,[0.08]$ tests whether the estimated linear combination (20) of the variables involved in our labor demand equation is stationary:

$$E_t - \frac{1}{\beta_2}\left(-\beta_3 w_t + \beta_4 K_t - \beta_7 TR_t - \beta_8 p_t^{oil}\right),$$

where the β's are the estimates of the ARDL approach.

At conventional significance levels, the above tests cannot reject the null that the long-run relationships estimated using the ARDL approach do indeed represent cointegrating vectors.

that *the labor market alone* is responsible for unemployment invariance. In general, equilibrating mechanisms in the labor market and other markets are jointly responsible for this phenomenon. Thus the LNJ restrictions need not be imposed on the specifications of labor market systems (such as the price mark-up and wage mark-up equations above), or on estimations of single-equation unemployment models. Restrictions on the relationships between the long-run growth rates of the capital stock, the labor force and technology are sufficient for this purpose.

By implication, policies that stimulate investment and promote R&D may have a long-run effect on the unemployment rate. The rates of capital accumulation and technological change may respond to these policies to ensure that unemployment stabilizes in the long run (so that the long-run unemployment rate is independent of the size of the capital stock and productivity in any given period of time), but these policies may nevertheless have a permanent effect on the unemployment rate. For instance, policies that stimulate productivity and thereby promote labor demand, may reduce the long-run unemployment rate. Along analogous lines, policies affecting the size of the labor force may affect the long-run unemployment rate as well.

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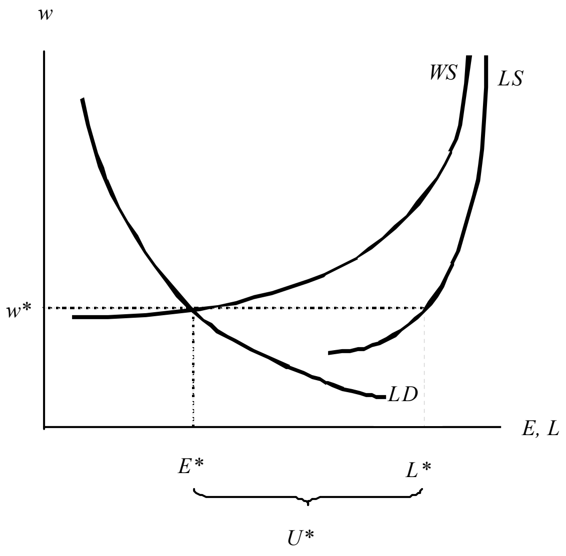

Figure 1:

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