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IZA DP No. 5143

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August 2010

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

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IZA Discussion Paper No. 5143
August 2010

ABSTRACT

Sorting on Skills and Preferences: Tinbergen Meets Sattinger[*]

This paper proposes an assignment model where sorting occurs on attributes including both skills (Sattinger, 1979) and preferences (Tinbergen, 1956). The key feature of this model is that the wage function admits both jobs' and workers' attributes as arguments. Since this function is generically nonlinear (Ekeland et al., 2004), even under positive assortative matching, the correlation between the contribution of workers' attributes to wages and that of jobs' attributes can vary from -1 to 1 depending on the parameters of the model, i.e. preference, technology and the distribution of both sets of attributes. The paper discusses a closed form solution of the model, presents conditions under which nonadditive marginal utility and production function are nonparametrically identified using observations from a single hedonic market and proposes a nonparametric estimator.

JEL Classification: D3, J21, J23, J31

Keywords: hedonic models, personality traits, firms and workers fixed effects, nonparametric identification

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[*] Fourth draft. Part of this paper was written while on leave at Yale in spring 2009, I want to thank the department of economics for its hospitality. This paper benefited from discussions with Joe Altonji, Pierre Cahuc, Jeremy Fox, Alfred Galichon, Jim Heckman, Fabian Lange, Michael Sattinger and Ed Vytlacil and valuable comments from participants at the Winter meetings of the Econometric Society, 2010 and the 2010 workshop on Matching and Sorting: Theory and Applications, and seminars at CPB, CREST, SUNY at Albany, YALE and in particular, Michael Jerison, Guy Laroque, Adrian Masters, George Monokroussos and Coen Teulings.

1 Introduction

Recent emerging empirical literature (e.g. Borghans et al., 2008) has shown the importance of personality traits in economics and in particular for earnings (Bowles et al., 2001 and Mueller and Plug, 2006). This literature shows that earnings are related to personality traits like risk aversion or conscientiousness. One possible explanation for these wage differentials would be that personality traits are linked to preferences for certain jobs' attributes so that the correlation between personality and earnings reflects compensating wage differentials for jobs disamenities. Yet, another explanation would be that personality traits are linked to skills that enhance productivity on the job and hence lead to higher wages. For instance, the documented positive effect of conscientiousness on earnings could come about because conscientiousness enhances workers' productivity or because in equilibrium, more conscientious workers are mapped onto jobs whose attributes are associated with negative intrinsic utility (tax controller) and hence require a wage compensation.

The model presented in this paper is the first to allow sorting to occur simultaneously on skills and preferences. This assignment model is concerned with the process by which heterogenous workers, characterized by a vector of attributes t including both skills and preferences, are assigned to heterogenous jobs, characterized by a vector of attributes z including both required skills and disamenities.

This paper shows that in this type of assignment models, wages reflect two compensations that arise simultaneously, namely a compensation for the skills supplied and a compensation for jobs' disamenities. This implies that the wage function takes both workers' and jobs' attributes as arguments, i.e. $w(t, z)$. In this model, an equilibrium is defined by a mapping of workers' attributes t onto jobs' attributes z, a function say $t(z)$ or $z(t)$, together with a wage function $w(z, t)$ that depends on both workers' attributes and jobs' attributes.

The fact that the wage function admits both workers' and jobs' attributes as arguments has important implications for empirical applications and in particular for two noteworthy segments. First, the model has implications for the estimation of preference (technology respectively) parameters in hedonic models. Recently, Ekeland et al. (2002 and 2004) and Heckman et al. (2009) have shown conditions under which nonparametric identification of additive and nonadditive marginal utility models of the Tinbergen class, where sorting occurs on preferences only, is possible in a single hedonic market. Their identification strategy relies on the first order condition to utility maximization and builds on results from Matzkin (2003) on nonparametric estimation of nonadditive random functions. Crucial in this setting is the assumption that the slope of the wage function $\partial w/\partial z$, the left hand side of the first order condition, is identified from data on wages, workers' attributes and jobs' attributes.

In the Tinbergen class of models assumed in Ekeland et al. (2004) and Heckman et al. (2009), wages depend only on z so that data on wages and z, i.e. dw/dz, identify $\partial w/\partial z$. However, in the unified economy, where workers' attributes include both preferences and skills, wages are given by the unknown function $w(z,t)$ and hence $dw/dz = \partial w/\partial z + \partial w/\partial t \times t'(z)$. Without further assumptions, $\partial w/\partial z$ is not identified nonparametrically since for any value of t, the value of z is uniquely determined by the mapping function $z = z(t)$.

This paper shows conditions under which $\partial w/\partial z$ is identified nonparametrically and proposes a nonparametric estimator. First, it is shown that the mapping function $z(t)$ is identified nonparametrically using results from Matzkin (2003). This is a generalization to the unified economy of Heckman et al.'s (2009) result obtained for the Tinbergen class of models. Following the identification of $z(t)$, a method to nonparametrically identify $\partial w/\partial z$ is proposed. This method relies on imposing shape restrictions on the utility and production functions. In general, the method requires that i) the production function is additive separable in (z, t_{-i}) and t_i, with $t =< t_{-i}, t_i >$, where the contribution of t_i is a known differentiable function $r(t_i)$ and ii) t_i is a scalar attribute influencing job satisfaction. Condition i) insures that $\partial w/\partial t_i (= r'(t_i))$ is known. Condition ii) insures that z and hence $\partial w/\partial z$ vary with t_i. When Conditions i) and ii) are met, $\partial w/\partial z$ is identified from data in a single hedonic market as $\partial w/\partial z = 1/z'(t)(dw/dt_i - r'(t_i))$. A special case is met when t_i is a pure preference attribute, that is, t_i affects utility but not productivity, i.e. $r'(t_i) = 0$ for all z and t_i. Attribute t_i is an exclusion restriction in the equilibrium wage function $w(z,t)$ since it does not affect productivity but not in the equilibrium assignment $z(t)$ since it matters for job satisfaction.

Second, the model contributes to the literature on earnings regressions using matched employer-employee data, e.g. Abowd et al. (1999). This literature shows that in an earnings regression on matched employee-employer panel data, while both workers' and firms' fixed-effects correlate positively with measures of firms' productivity, their correlation is very low or even negative. The unified hedonic model presented in this paper offers a natural explanation for this puzzle. Since sorting occurs on both skills and preferences, wages are function of both workers' attributes and jobs' attributes. However, even when sorting exhibits positive assortative matching, the model does not imply that the contribution of workers' attributes to wages correlates positively with that of jobs' attributes. Both the sign and magnitude of the correlation between workers' and jobs' fixed-effects will depend on the preference parameters, the technology parameters and the distribution of workers' and jobs' attributes.

For instance, suppose that a worker's utility decreases with the distance between her own skills and the level of complexity of her job, as in Tinbergen (1956), but workers' skills and jobs' complexity are complement in production, as in Sattinger (1979). From both Tinbergen (1956) and Sattinger (1979) we know that in this economy more skilled workers will be assigned to more complex jobs in equilibrium, i.e. positive assortative matching arises. From Sattinger (1979) we know that the contribution of skills to wages is increasing in skills everywhere on the support of skills.[1] However, from Tinbergen (1956) we also know that the contribution of job complexity to wages is only increasing in job complexity (and hence in skills since there is positive assortative matching) in intervals of job complexity where job complexity exceeds the skills of the worker matched to that job. This means that if the distribution of skills dominates the distribution of jobs stochastically at the first order, then the contribution of job complexity to wages is decreasing in job complexity (skills) while the contribution of skills to wages is increasing in skills. In this case, the correlation between both contributions is negative even though equilibrium exhibits positive assortative matching.

Using the unidimensional quadratic-normal example, it is shown that one can calibrate the unified hedonic model so as to generate data where the contribution of workers' attributes to wages and that of job attributes both correlate positively with firms' productivity but not with each other. Changing the distribution of attributes over time induces the type of mobility of workers across firms that is necessary to identify workers' and firms' fixed-effects in matched employer-

[1] It is implicitly assumed that almost all workers have positive skills and almost all jobs have positive complexity. With normal distributions this is the case when the mean of each distribution is positive and large enough relative to the variance.

employee panel data set. It is shown that using this strategy for the unidimensional quadratic-normal example, one can generate a panel data and estimate workers' and firms' fixed-effects that both correlate with firms' productivity but not with each other.

The remaining structure of the paper is as follows. The next section reviews the related literature. Section 3 presents the unified model for the hedonic *endowment* economy. Section 4 presents a closed form solution for the wage function in the quadratic-normal setup. Section 5 discusses the implication of the model for the identification of preference parameters in a single hedonic market as well as for the literature on firms' and workers' fixed-effects in earnings regressions using matched employer-employee data. Section 6 summarizes and concludes.

2 Related literature

The model presented in this paper nests existing assignment models in the literature. This literature is divided into two distinct classes of models depending on the nature of the process governing assignment. One class of models, led by Tinbergen (1956), focuses on the assignment of workers to jobs based on preferences. Within this class of models, jobs' attributes z are seen as disamenity and workers derive intrinsic disutility from z. Although jobs with different attributes are unequally productive, output at a job with attribute z does not depend on workers' attributes t. Hence productivity is merely determined by jobs' attributes and all workers are equally productive at all jobs. In this class of models, workers select jobs' attributes to maximize their utility. The pricing function $w(z,t)$ does not depend on workers' attributes but merely on jobs' attributes, i.e. $w(z,t) = w(z)$, and is therefore interpreted as a compensating wage differential. As an example, the preference class of models indicates that risk loving workers will tend to become firemen as they command lower compensations for the risks taken on the job, but yet, assumes that risk loving workers would just make as good firemen as any other (risk averse) worker. While this class of models explains wage formation due to risk compensation, the model fails to explain wage formation due to productivity differentials across workers.

In contrast, the second class of models, led by Sattinger (1979), focuses on the assignment of workers to jobs based on skills. Jobs' attributes are seen as productive capacities and workers derive no intrinsic (dis-)utility from z. Both workers' and jobs' attributes matter for productivity. Workers with certain attributes are more productive at certain jobs than others. In this class of models, workers select jobs' attributes to maximize their wage and the wage function $w(z,t)$ does not depend on jobs' attributes but merely on workers' attributes, i.e. $w(z,t) = w(t)$. For instance, the skills class of models indicates that conscientious workers will tend to become tax controllers as conscientiousness is an important factor of productivity on the job, but yet, assumes that conscientious workers derive the same disutility from being a tax controller as less conscientious workers. While this class of models explains wage formation due to differential productivity across workers, these models fail to explain wage formation due to preference compensation.

The model presented in this paper nests both Tinbergen's and Sattinger's models.[2] Under the assumption that all jobs' attributes lead to intrinsic disutility and workers' attributes do not affect productivity the model collapses to Tinbergen's model. Under the assumption that all workers' attributes contribute to productivity and no job attributes lead to intrinsic disutility the model collapses to Sattinger's differential rents model.

There exist only few examples of closed form solutions for the hedonic price as a function of attributes, i.e. $w(z, t)$. The first was proposed by Tinbergen (1956). Assuming that i) workers derive intrinsic disutility from all their attributes, ii) price enters log linearly in the utility function, iii) intrinsic (dis-)utility is quadratic in jobs' attributes z, iv) the supply of products is exogenous (the hedonic *endowment* economy model) and v) both workers and jobs' attributes are normally distributed, Tinbergen showed that the *log* of the equilibrium price is a function quadratic in

[2]Sattinger (1977) developed a compensating wage differential model where workers differ in terms of productivity and jobs in terms of the satisfaction workers receive from working at it, both unidimensional. Workers and jobs attributes are encompassed in the definition of job satisfaction and cannot be distinguished from each other. Moreover, all jobs have similar productivity. There is no complementarity between workers skills and jobs requirements.

attributes z.[3] Assuming i), ii'), iii) and v) and relaxing iv) by allowing firms to produce job attributes and introducing production costs (the hedonic *production* economy model), Epple (1984) provided a closed form solution for the hedonic price function that is quadratic in z when production costs are also quadratic in z. Sattinger (1979 and 1980) provided closed for solutions when jobs and workers are differentiated along a single attribute (skills demanded and supplied) assuming that workers derive no intrinsic disutility from the level of skills demanded by their job, i.e. maximize their wage. This skills attribute affects productivity but does not provide intrinsic utility. The pricing function of interest in this model is the wage as a function of workers' skills. Since skills provide no intrinsic disutility and merely affect production, sorting in this model occurs on productive attributes rather than preference attributes as in Tinbergen and Epple. Sattinger's (1980) closed form solutions for the wage function are obtained when the distribution of jobs and workers are Pareto, production is multiplicative in attributes (i.e. Cobb-Douglas) and utility depends on wages only. This last assumption is characteristic of the differential rents model that precludes compensating wage differential for intrinsic disutility derived from the type of jobs.

A closed form solution for the unified model is proposed in section 4 of this paper. This solution is derived when workers' and jobs' attributes are normally

[3] In fact, replacing ii) by ii') price enters linearly in the utility function, the *level* of the equilibrium price would be quadratic in z.

distributed and intrinsic disutility is quadratic in jobs' attributes and productivity is quadratic in workers' attributes.

Finally, this paper relates to the general literature on hedonic models and not only on that segment focussing on the labor market. For instance, Epple's extension of Tinbergen's *endowment* economy to a *production* economy was originally written in a consumer/producer context, not a worker/firm context. In the consumer/producer model, the restriction that consumers' attributes do not affect the production of goods does not at first sight seem to be too strong. However, the generalization proposed in Appendix 3 of this paper is also relevant in that case. Think for instance of an economy where firms are endowed with a vector of attributes y. In this economy, to produce good z, firms need to hire a fixed number of workers, one and only one worker for simplicity. Suppose further that the attributes of that worker, say t', matter in the production process so that the costs (profits) of producing good z depend on t'. Firms need now to optimize not only on z but also on t'.

3 The unified hedonic *endowment* economy model

3.1 Setup

Consider a static labor market where workers match one-to-one with firms. Let each firm be endowed with a single machine. The supply of machines is therefore assumed exogenous to the model,[4] and the assumption that workers and firms match one-to-one therefore means that to produce output each machine must be operated by one and only one worker. Let a machine be characterized by a vector of attributes denoted by $z \in \mathbb{R}^{n_z}$. To fix ideas, machines attributes could be the level of physical strength involved in operating the machine, the level of intellectual complexity involved, the level of noise generated by the machine, the degree of risks taken while operating the machine, etc. Let $f_z(z)$ and $F_z(z)$ be the PDF and CDF of z respectively and let F_z be absolutely continuous with respect to Lebesgue measure.

Similarly, suppose that workers are endowed with a vector of attributes $t \in \mathbb{R}^{n_t}$. These attributes could refer to cognitive ability such as physical strength, intellectual ability but also personality traits such as conscientiousness, risk aversion etc.. Let the distribution of t be exogenous and let $f_t(t)$ and $F_t(t)$ be its PDF and

[4] The assumption that firms are endowed with a machine z can be released by supposing that firms are endowed with a vector of attributes y (investments capacity, managers' attributes etc.) and "produce" their machine z. The distribution of machines is then endogenous to the model. This case corresponds to the hedonic *production* economy and is dealt with in Appendix 2. The main results of the paper remain unchanged but the mechanic of the model simplifies significantly by assuming machines are endowed.

CDF respectively and let F_t be absolutely continuous with respect to Lebesgue measure.[5]

In contrast to Tinbergen (1956), Epple (1984), Ekeland et al. (2002 and 2004) and Heckman et al. (2009), the model does not require workers' attributes to be non productive. Let the output of each machine depend on its own attributes but also on the attributes of the worker operating this machine. Let $p(z,t)$ be a twice differentiable continuous function indicating the units of output produced by the pair (z,t). An attribute i is not a productive attribute if and only if $\frac{\partial p(z,t)}{\partial t_i} = 0$ for all z and t. Note that some attributes may be productive at some jobs but not at others. While skills of different types will clearly affect productivity, some preferences may also affect productivity, for instance, a risk averse person might also tend to operate a machine slower, conscientious workers may take better care of their machine, etc...

Let $w(z,t)$ be the wage of a worker with attributes t when assigned to a machine with attributes z and let $r(z,t)$ be the rents of a firm owning machine with attributes z when employing a worker with attributes t. Note that, by definition, product is exhausted so that $p(z,t) - w(z,t) = r(z,t)$.

[5] It should be noted here that the mass of workers is assumed to be equal to the mass of firms. The model could be accommodated to allow for different masses and would inevitably lead to unemployed workers or vacancies in equilibrium depending on whether the mass of workers exceeds that of firms. Although assignment models offer an interesting structure to analyze which agents are kept out of the market by the equilibrium pricing, the primary aim of this paper is to analyze wage formation when workers' attributes are both skills and personality traits. The assumption of equal mass does not seem to be restrictive with respect to this aim.

In contrast to Sattinger (1979), the model does not require that jobs' attributes do not affect intrinsic disutility. Assume a quasilinear utility function $u(z,t) = w(z,t) - j(z,t)$ where consumption equals wages $w(z,t)$ by assuming no unearned income. Let $j(z,t)$ be a continuous twice differentiable function capturing job dissatisfaction. The function j could take the specific form proposed by Tinbergen (1956), $j(z,t;A) = \frac{1}{2}(z-t)' A (z-t)$ where A is a positive definite matrix of parameters. A job attribute i does not provide intrinsic utility if and only if $\frac{\partial j(z,t)}{\partial z_i} = 0$ for all z and t.

3.2 Equilibrium

Definition 1 *An equilibrium is a wage function $w(z,t)$ and a mapping function $t(z)$ so that i) firms' supply of machines with attributes z equals workers' demand for machines with attributes z everywhere on the support of z, ii) workers maximize utility and iii) firms maximize rents.*[6]

Utility maximizing workers seek for a machine with attributes z so that:

[6] *See subsection 3.3 for a discussion of the existence, uniqueness and purity of the unified hedonic model.*

$$\frac{\partial u(z,t)}{\partial z} \equiv \frac{\partial w(z,t)}{\partial z} - \frac{\partial j(z,t)}{\partial z} = 0$$

$$\Leftrightarrow$$

$$\frac{\partial w(z,t)}{\partial z} = \frac{\partial j(z,t)}{\partial z} \tag{1}$$

Let $z(t)$ denote the implicit function that solves Equation 1 for z given $j(.,.)$ and $w(.,.)$. This function indicates the optimal machine a worker with attributes t chooses given job dissatisfaction $j(.,.)$ and the shape of the wage function and in particular the wage differential at z.

The second order condition for utility maximization reads as:

$$\frac{\partial^2 u(w(z,t), j(z,t))}{\partial z^2} < 0$$

$$\Leftrightarrow$$

$$\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t)}{\partial z^2} < 0$$

Rents maximizing firms will look for a worker with attributes t so that:

$$
\begin{aligned}
\frac{\partial r(z,t)}{\partial t} &= \frac{\partial p(z,t)}{\partial t} - \frac{\partial w(z,t)}{\partial t} = 0 \\
&\Leftrightarrow \\
\frac{\partial w(z,t)}{\partial t} &= \frac{\partial p(z,t)}{\partial t} \qquad (2)
\end{aligned}
$$

The second order condition for rents maximization reads as:

$$
\frac{\partial^2 p(z,t)}{\partial t^2} - \frac{\partial^2 w(t)}{\partial t^2} < 0
$$

Let $t(z)$ denote the implicit function that solves Equation 2 for t given $j(.,)$ and $w(.,.)$. This function indicates the optimal choice of a worker for a firm with machine z given productivity $p(.,.)$ and the shape of the wage function and in particular the differential at t.

It is important to note that the first order conditions determine the slopes of the equilibrium wage function while the second order conditions restrict the curvature. However, nothing is known about the cross-partial derivative. This suggests that if there exists a solution for the wage function, this solution will not be unique. All functions satisfying the first and second order conditions but with different cross-partial derivatives will also be solutions.[7]

Note that if the equilibrium is pure,[8] the two mapping functions $z(t)$ and $t(z)$ are invertible. We therefore have the restriction $t^{-1}(t) = z(t)$.

For an equilibrium allocation to be reached, the supply of machines with attributes z should be equal to workers' demand for machines with attributes z for all z. This means that:

$$f_z(z)dz = f_t(t(z)) \left| \frac{\partial t(z)}{\partial z} \right| dz$$

Equilibrium will be reached by choosing the right shape for the function $w(z,t)$ and in particular the right differentials at t and z. Workers and firms will participate if their wage and rents are larger than their reservation levels. To close the model, the usual assumption is to fix a reservation value for the utility, say $\underline{u}$ and rent $\underline{r}$ so that u and r must be larger than their respective thresholds.

3.3 Existence, Uniqueness and Purity

The results on the existence, uniqueness and purity of equilibrium in the unified hedonic model presented below build on Chiappori et al. (2009). Chiappori et al.

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[7]See Section 3.3.

[8]Conditions for purity are given in Section 3.3.

(2009) show equivalence results between hedonic models with quasi linear utility, stable matching models with transferable utilities and optimal transportation linear programming problem. These equivalence results allow Chiappori et al. (2009) to draw from the optimal transportation linear programming literature and prove the existence, uniqueness and purity in a great generality of hedonic models of the Tinbergen class.

Following this line of thought, write the optimal transport problem associated to the unified hedonic model above as follows. Let an assignment be defined as a measure $\gamma(z,t)$ on $\mathbb{R}^{n_z} \times \mathbb{R}^{n_t}$ whose marginals are F_z and F_t. Note that the measure γ corresponds to the mapping function introduced earlier $t = t(z)$. Also, let $s(z,t) \equiv p(z,t) - w(z,t) + w(z,t) - j(z,t) = p(z,t) - j(z,t)$ be the surplus of the pair worker t and firm z.

The primal program reads as:

$$
\begin{aligned}
& \max_{\gamma} \int_{\mathbb{R}^{n_z} \times \mathbb{R}^{n_t}} s(z,t) d\gamma(z,t) \\
& s.t. \\
\int_{\mathbb{R}^{n_z}} d\gamma(z,t) &= F_t(t) \\
\int_{\mathbb{R}^{n_t}} d\gamma(z,t) &= F_z(z)
\end{aligned}
$$

The dual program reads as:

$$\begin{aligned} & \min_{(U,V)} \int_{\mathbb{R}^{n_z}} V(z) dF_z(z) + \int_{\mathbb{R}^{n_t}} U(t) dF_t(t) \\ & s.t. \\ V(z) + U(t) \geq \; & s(z,t) \; \forall (z,t) \in \mathbb{R}^{n_z} \times \mathbb{R}^{n_t} \end{aligned}$$

where $U(t)$ and $V(z)$ can be seen as the payoff of worker t and firm z respectively.

It is important to note that all that matters in the optimal transportation linear programming problem is the surplus function $s(z,t)$ and the distributions of attributes $F_t(t)$ and $F_z(z)$. How this surplus is formed ($s(z,t) = p(z) - j(z,t)$ in the Tinbergen class studied by Chiappori et al., 2009, and $s(z,t) = p(z,t) - j(z,t)$ in the unified class) and whether the transfer is $w(z)$ or $w(z,t)$ respectively does not matter. This means that from the perspective of the primal and dual program, whether one considers the Tinbergen class or the unified class of models is irrelevant. However, the distinction is fundamental in the identification of how the surplus is built up, i.e. productivity and preferences.

There are two important results available from the optimal transport literature for the existence, uniqueness and purity of an equilibrium in the unified hedonic model. The first important result is that:

Solution 2 *If the surplus function $s(.,.)$ is upper semicontinuous, then a maximum in the primal program is attained so that an equilibrium allocation γ exists.*

In the unified hedonic model outlined in the previous section, the assumptions of continuity and twice differentiability of the production function and job dissatisfaction function carry over to $s(.,.)$. Since these properties satisfy −are in fact stronger than− the upper semicontinuous hypothesis required in the theorem, we conclude that an equilibrium assignment $\gamma(z,t)$ exists in the unified hedonic economy.

In addition, if the surplus function $s(.,.)$ satisfies the twisted-buyers/sellers condition (generalized Spence-Mirrlees conditions) given as:

$$\begin{aligned} \frac{\partial s(z,t^a)}{\partial z} &= \frac{\partial s(z,t^b)}{\partial z} \Longrightarrow t^b = t^a \\ \frac{\partial s(z^a,t)}{\partial t} &= \frac{\partial s(z^b,t)}{\partial t} \Longrightarrow z^b = z^a \end{aligned}$$

then the equilibrium assignment $\gamma(z,t)$ is unique and pure, i.e. the mapping function $t(z)$ is strictly monotonic −see Theorem 4.11 in Chiappori et al. (2009)−.

The second important result from the optimal transport literature reads as:

Solution 3 *A feasible triple* (γ, U, V) *produces* $\int_{\mathbb{R}^{n_z}\times\mathbb{R}^{n_t}} s(z,t)d\gamma(z,t) = \int_{\mathbb{R}^{n_z}} V(z)dF_z(z) + \int_{\mathbb{R}^{n_t}} U(t)dF_t(t)$ *if and only if* γ *solves the primal program and* (U, V) *solves the dual program.*

This result can be used to prove the existence of an equilibrium price function in the unified hedonic model. Since Chiappori et al. (2009) focussed on the Tinbergen class of hedonic models with $p(z,t) = p(z)$ and $w(z,t) = w(z)$, a formal proof for the existence of an equilibrium price function $w(z,t)$ in the unified hedonic model is proposed below.

Proof. Let $w : \mathbb{R}^{n_z} \times \mathbb{R}^{n_t} \to \mathbb{R}$ satisfy:

$$U(t) + j(z,t) \geq w(z,t) \geq p(z,t) - V(z) \tag{3}$$

The left hand side of this inequality is the minimum willingness of worker t to accept job z while the right hand side is the maximum willingness of firm z to pay for worker t.

Take a feasible triple (γ, U, V) and suppose that (U, V) solves the dual program so that $V(z) + U(t) \geq s(z,t) = p(z,t) - j(z,t)$. Rearranging obtains:

$$U(t) + j(z,t) \geq p(z,t) - V(z) \text{ for } (z,t) \in \mathbb{R}^{n_z} \times \mathbb{R}^{n_t} \tag{4}$$

Suppose also that γ solves the primal program so that $\int_{\mathbb{R}^{n_z}\times\mathbb{R}^{n_t}} s(z,t)d\gamma(z,t) =$

$\int_{\mathbb{R}^{n_z}} V(z)dF_z(z) + \int_{\mathbb{R}^{n_t}} U(t)dF_t(t)$. It follows that:

$$\int_{\mathbb{R}^{n_z}\times\mathbb{R}^{n_t}} s(z,t)d\gamma(z,t) = \int_{\mathbb{R}^{n_z}\times\mathbb{R}^{n_t}} (V(z)+U(t))d\gamma(z,t)$$

since $\int_{\mathbb{R}^{n_z}} d\gamma(z,t) = F_t(t)$ and $\int_{\mathbb{R}^{n_t}} d\gamma(z,t) = F_z(z)$. This yields $s(z,t) = V(z) + U(t)$ for $\gamma-$almost every (z,t), that is for $z = z(t)$. Consider a worker t^* that is matched with a firm z^*, i.e. $z^* = z(t^*)$. We have $s(z^*,t^*) = V(z^*) + U(t^*)$ and hence:

$$U(t^*) + j(z^*,t^*) = w(z^*,t^*) = p(z^*,t^*) - V(z^*) \tag{5}$$

From our choice of $w(z,t)$ in Inequality 3 we have$U(t^*) + j(z,t^*) \geq w(z,t^*)$ for worker t^* and $z \in \mathbb{R}^{n_z}$ and $w(z^*,t) \geq p(z^*,t) - V(z^*)$ for firm z^* and all $t \in \mathbb{R}^{n_t}$. Rearranging and using Equation 5 obtains:

$$p(z^*,t) - w(z^*,t) \quad \leq \quad V(z^*) = p(z^*,t^*) - w(z^*,t^*)\ \forall t \in \mathbb{R}^{n_t} \tag{6}$$

$$w(z,t^*) - j(z,t^*) \quad \leq \quad U(t^*) = w(z^*,t^*) - j(z^*,t^*)\ \forall z \in \mathbb{R}^{n_z} \tag{7}$$

It follows that t^* maximizes $p(z^*,t) - w(z^*,t)$ and z^* maximizes $w(z,t^*) - j(z,t^*)$. Since the equalities in 6 and 7 hold for $\gamma-$almost every (z,t) and γ exists, there exists a solution (γ, w) to the unified hedonic model. ■

Note however that the solution for $w(.,.)$ is not unique. Suppose that the surplus function $s(.,.)$ satisfies the twisted-buyers/sellers condition (generalized Spence-Mirrlees conditions). There exists a unique solution for the mapping function (a unique and pure solution that solves the primal program), say $t = \widehat{t}(z)$ where $\widehat{t}$ is a monotonic function. Let $\widehat{w}(z,t)$ be a solution for $w(z,t)$. The FOCs yield:

$$\begin{aligned}\frac{\partial \widehat{w}(z,\widehat{t}(z))}{\partial t} &= \frac{\partial p(z,\widehat{t}(z))}{\partial t}\\ \frac{\partial \widehat{w}(z,\widehat{t}(z))}{\partial z} &= \frac{\partial j(z,\widehat{t}(z))}{\partial z}\end{aligned}$$

Totally differentiating the FOCs with respect to z and rearranging yields (dropping the arguments of all functions for notational convenience):

$$\widehat{t}' = \frac{\frac{\partial^2 p}{\partial z \partial t} - \frac{\partial^2 \widehat{w}}{\partial z \partial t}}{\frac{\partial^2 \widehat{w}}{\partial t^2} - \frac{\partial^2 p}{\partial t^2}} \tag{8}$$

$$\frac{1}{\widehat{t}'} = \frac{\frac{\partial^2 j}{\partial z \partial t} - \frac{\partial^2 \widehat{w}}{\partial z \partial t}}{\frac{\partial^2 \widehat{w}}{\partial z^2} - \frac{\partial^2 j}{\partial z^2}} \tag{9}$$

since $\frac{\partial^2 \widehat{w}}{\partial t^2} - \frac{\partial^2 p}{\partial t^2} < 0$ and $\frac{\partial^2 \widehat{w}}{\partial z^2} - \frac{\partial^2 j}{\partial z^2} < 0$ from the SOCs.

It is easy to show that for all $z \in \mathbb{R}^{n_z}$, one could change the value of $\frac{\partial^2 \widehat{w}}{\partial z \partial t}$, $\frac{\partial^2 \widehat{w}}{\partial t^2}$ and $\frac{\partial^2 \widehat{w}}{\partial z^2}$ in such a way that the right hand sides of Equation 8 and 9 remain unchanged and the SOCs are still satisfied. This means that even though $\widehat{t}$ is unique (and pure), the solution for the wage function is not unique.

4 Quadratic-normal example

4.1 The model

Let $n_z = n_t = n$ and let workers' attributes be normally distributed with mean vector μ_t and variance-covariance matrix Σ_t and let z be normally distributed with mean vector μ_z and variance-covariance matrix Σ_z. Suppose that, as in Tinbergen (1956) job dissatisfaction is defined as $j(z,t;A) = \frac{1}{2}(z-t)' A (z-t)$, where A is a positive definite matrix of preference parameters. Suppose further that productivity is given by $p(z,t;E) = b_0 + b'z + c't + \frac{1}{2}z'Bz + \frac{1}{2}t'Ct + t'Dz$ and where b_0 is a constant, b and c are vectors and B, C and D are matrices of parameters. The parameters contained in D indicate the extent to which the attributes of machines complement or substitute workers' attributes, i.e. $\frac{\partial p(z,t;E)}{\partial z \partial t} = D$.

The generalized Spence-Mirrlees condition will be satisfied as long as $|D+A| \neq 0$. As long as $|D+A| \neq 0$, equilibrium is pure for any distributions F_t and F_z so that the mapping function $t(z)$ is invertible with inverse $z(t) \equiv t^{-1}(t)$. The mapping function $t(z)$ is linear when the distributions of t and z are normal.

The first order conditions read now as:[9]

$$
\begin{aligned}
\frac{\partial w(z,t)}{\partial z} &= A(z-t) \\
\frac{\partial w(z,t)}{\partial t} &= c + Ct + Dz
\end{aligned}
$$

It is now easy to see that the first order conditions yield linear mapping of jobs' attributes on workers' attributes if and only if $\frac{\partial w(z,t)}{\partial t}$ is linear in t and $\frac{\partial w(z,t)}{\partial z}$ is linear in z. This, in turns, implies that the equilibrium wage function is quadratic and reads as:[10]

$$
w(z,t) = \delta_0 + \delta' t + \frac{1}{2} t' \Delta t + \lambda' z + \frac{1}{2} z' \Lambda z \tag{10}
$$

[9] The second order conditions are trivial and given by:

$$
\begin{aligned}
\frac{\partial^2 w(z,t)}{\partial z^2} - A &< 0 \\
C - \frac{\partial^2 w(z,t)}{\partial t^2} &< 0
\end{aligned}
$$

[10] We implicitly assume that the wage function is additive separable in z and t. Although including the cross term $z'\Omega t$ in the wage equation would still produce linear mapping functions, the parameters of the wage function would not be identified unless we impose $\Omega = \boxed{0}$ where $\boxed{0}$ is a matrix filled with 0.

Using equation 10 in equations 1 and 2 respectively and rearranging yields:

$$\lambda + (\Lambda - A)\, z \;=\; -At \tag{11}$$

$$\delta - c + (\Delta - C)\, t \;=\; Dz \tag{12}$$

These are linear functions and the reduced form solution will be of the form $t = \pi_0 + \Pi_1 z$ or $z = -\Pi_1^{-1}\pi_0 + \Pi_1^{-1}t$. Plugging $t = \pi_0 + \Pi_1 z$ into 11 yields $\lambda = -A\pi_0$ and $\Lambda = A(I - \Pi_1)$. Plugging $z = -\Pi_1^{-1}\pi_0 + \Pi_1^{-1}t$ into Equation 12 yields $\delta = c - D\Pi_1^{-1}\pi_0$ and $\Delta = C + D\Pi_1^{-1}$.

As noted earlier by Tinbergen (1956) and Epple (1984), when attributes on both sides of the labor market are normally distributed, linear mapping functions of the form $t = \pi_0 + \Pi_1 z$ equilibrate supply and demand. Indeed, the equilibrium condition $f_t(t)dt_1dt_2...dt_N = f_z(z)dz_1dz_2...dz_N$ given normally distributed attributes, is equivalent to equating the means, i.e. $\mu_t = \pi_0 + \Pi_1\mu_z$ and equating the variances, i.e. $\Sigma_t = \Pi_1'\Sigma_z\Pi_1$.

To find the solution for π_0 and Π_1, first note that $\Sigma_t = \Pi_1'\Sigma_z\Pi_1 = (-\Pi_1)'\,\Sigma_z\,(-\Pi_1)$. There are therefore two solutions to this equilibrium condition, one with positive assortative matching $\Pi_1 > 0$ and one with negative assortative matching $\Pi_1 < 0$. These two solutions however will give rise to different total surplus. The one that maximizes total surplus will prevail. If workers' and jobs' attributes are globally complements (substitutes) in surplus, i.e. $D + A > 0$, then total surplus will be maximized by mapping higher t with higher (lower) z, i.e. $\Pi_1 > 0$ (< 0). The solution for π_0 and Π_1 is:[11]

$$\left.\begin{aligned} \pi_0 &= \mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z \\ \Pi_1 &= \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1} \end{aligned}\right\} \text{ if } D + A > 0$$

$$\left.\begin{aligned} \pi_0 &= \mu_t + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z \\ \Pi_1 &= -\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1} \end{aligned}\right\} \text{ if } D + A < 0$$

To see this, note first that $\Pi_1\left(\Pi_1^{-1}\right)' = \Pi_1^{-1}\Pi_1' = I$ where I is the identity matrix. Post-multiply both sides of the equation $\Sigma_t = \Pi_1'\Sigma_z\Pi_1$ by $\left(\Pi_1^{-1}\right)' = \Sigma_t^{-1/2}\Sigma_z^{1/2}$. This yields $\Sigma_t\Sigma_t^{-1/2}\Sigma_z^{1/2} = \Pi_1'\Sigma_z$. Pre-multiply both sides of this equation by Π_1^{-1}. This yields the identity $\Sigma_z^{1/2}\Sigma_t^{-1/2}\Sigma_t\Sigma_t^{-1/2}\Sigma_z^{1/2} = \Sigma_z$.

If $D + A > 0$, the equilibrium wage function has for parameters:

[11] Note that the power p, $p \in R$, $p \neq 0$, of a square matrix A of size $n \times n$ is obtained as $A^p X = X diag(\lambda)$ where X is a matrix of size $n \times n$ formed of the n eigenvectors of A and λ is the vector containing the corresponding eigenvalues. If in addition A is symmetric, then X is orthogonal so that $X'X = XX' = I$ and, post-multiplying both sides by X', the result simplifies to $A^p = X diag(\lambda)^p X'$. The matrix A^p will be real if and only if all eigenvalues λ are real and strictly positive that is if and only if A is positive definite. Since Σ_t and Σ_z are symmetric, the above result applies to $\Sigma_t^{-1/2}$ and $\Sigma_z^{1/2}$. (See Bosch, 1987)

$$\lambda = -A\left(\mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) \tag{13}$$

$$\Lambda = A\left(I - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\right) \tag{14}$$

$$\delta = c - D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)\left(\mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) \tag{15}$$

$$\Delta = C + D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right) \tag{16}$$

If $D + A < 0$, the equilibrium wage function has for parameters:

$$\lambda = -A\left(\mu_t + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) \tag{17}$$

$$\Lambda = A\left(I + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\right) \tag{18}$$

$$\delta = c + D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)\left(\mu_t + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) \tag{19}$$

$$\Delta = C - D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right) \tag{20}$$

The constant δ_0 is not identified. To close the model, the usual assumption (see Ekeland et al. (2002 and 2004), Heckman et al. (2009) and Sattinger (1979) among others) is to fix a reservation value for the utility, say $\underline{u}$ and rent $\underline{r}$ so that u and r must be larger than their respective thresholds.

To summarize, we have shown that the equilibrium parameters of the wage function λ, Λ, δ and Δ are retrieved from the distribution parameters of workers' and jobs' attributes using the mapping function $t(z) = \pi_0 + \Pi_1 z$ where Π_1 is identified by the variances and covariances of workers' and jobs' attributes only and π_0 is identified from the means of workers and jobs' attributes and from Π_1. The second order coefficients of the wage function Λ and Δ depend only on the matrix of variances and covariances of workers' and jobs' attributes. The first order coefficients λ and δ depend on the matrix variances and covariances and the means of workers' and jobs' attributes.

To illustrate the model, we programmed the closed form solution of the unidimensional quadratic-normal model in Mathematica. For a given set of parameters, this program illustrates the equilibrium with a panel of three graphics. These graphics represent 1) the equilibrium mapping function in the (z, t) plan, 2) the equilibrium compensation for skills (i.e. the contribution of workers' attributes to wages) and the equilibrium compensation for jobs disamenities and 3) the distribution of workers' and jobs' attributes. We used the command Manipulate to enable the user to visualize instantaneously the impact of changing structural parameters of the model on the equilibrium through these three graphics.[12] As an example, Figure 6 was generated for $\Sigma_z = 1$, $\Sigma_t = 4$, $\mu_z = 1$, $\mu_t = 0$, and $c = 10$, $C = -1$, $D = 3$ and $A = 3$.

[12] This program is available from the author upon request.

4.2 Relations to Tinbergen (1956) and Sattinger (1979)

Tinbergen (1956), Epple (1984) and Ekeland et al. (2002 and 2004) consider the case where workers' attributes do not contribute to production, i.e. $\frac{\partial p(z,t;E)}{\partial t} = 0$ for all t and z, so that the first order condition to rents maximization in equation 2 indicate no wage differentials across workers' attributes. In the quadratic-normal example, this condition is met when $D = C = \boxed{0}$ where $\boxed{0}$ is a matrix filled with zeros, $c' = \underline{0}$, so that equation 12 yields $\Delta = C = \boxed{0}$ and $\delta = c = \underline{0}$. Since $D = \boxed{0}$, the generalized Spence-Mirrlees condition for pure equilibrium is now satisfied for $|D + A| \neq 0$, the equilibrium will be pure in Tinbergen's model only if $|A| \neq 0$.

The model proposed above admits Sattinger's differential rents model as a special case. This is the case when t and z are unidimensional and z carries no intrinsic disutility so that $\frac{\partial j(z,t;A)}{\partial z} = 0$ for all z and t or $A = \boxed{0}$ in the quadratic-normal example. From equation 2 (equation 11 respectively in the quadratic-normal example) we then have $\Lambda = (A =)\boxed{0}$ and $\lambda = \underline{0}$ so that the wage function depends merely on t. As soon as t is loaded with intrinsic disutility, $A \neq \boxed{0}$, the slope of the rents function increases and the increase is more pronounced for higher z. Again, since $A = \boxed{0}$, the generalized Spence-Mirrlees condition is satisfied if $|D + A| \neq 0$.

5 Implications for empirical applications

5.1 Identification and estimation in a single hedonic market

5.1.1 Identification

Since the seminal work by Rosen (1974), the traditional approach to estimate preference parameters, the function $j(z, t)$,[13] has consisted of two steps. In the first step, using market data on wages and jobs' attributes, one estimates the wage function applying the functional form that fits best the data. In the second step, one uses the first order condition in Equation 1 together with the marginal wage derived from the first step, i.e. $\frac{dw(z)}{dz}$, to recover preference estimates of $j(z, t)$.

Early literature by Brown and Rosen (1982), Epple (1987), Bartik (1987) and Kahn and Lang (1988) has argued that $j(z, t)$ cannot be identified in a single hedonic market unless an arbitrary nonlinear marginal utility is assumed. Recently, Ekeland et al. (2002 and 2004) have shown that nonlinearity is a generic feature of the hedonic model, not an arbitrary choice, and Ekeland et al. (2002 and 2004) and Heckman et al. (2009) have provided conditions under which nonparametric identification of additive and nonadditive hedonic models of the Tinbergen class is possible in a single hedonic market.

[13] All techniques below apply also to the estimation of productivity parameters by symmetry.

These conditions build on results from Matzkin (2003) on nonparametric estimation of additive and nonadditive random functions. All the results from Ekeland et al. and Heckman et al. crucially depend on the assumption that $\frac{dw}{dz}$ identifies $\frac{\partial w}{\partial z}$. In the Tinbergen class of models, where wages depend only on z, identification of $\frac{\partial w}{\partial z}$ follows by assumption, $\frac{dw}{dz} = \frac{\partial w}{\partial z}$. However, in the unified economy, wages are given by the unknown function $w(z, t)$ so that $\frac{dw}{dz} = \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} t'$. Without further assumptions, $\frac{\partial w}{\partial z}$ is not identified nonparametrically since for any value of t, the value of z is uniquely determined through the mapping function $z = z(t)$.

Since all the identification results presented in Heckman et al. follow once $\frac{\partial w}{\partial z}$ is identified, this paper focuses on the identification of $\frac{\partial w}{\partial z}$ and refers the reader to Heckman et al. for identification results of $j(z, t)$ once $\frac{\partial w}{\partial z}$ is identified. The method proposed to identify $\frac{\partial w}{\partial z}$ relies on shape restrictions on the production function $p(z, t)$.

Following Ekeland et al. and Heckman at al., assume that z is a scalar and $t =< t^o, t^u >$ where t^o is a vector of observed attributes and t^u is a scalar unobserved (to the econometrician) attribute. Assume further that t^u is independent of t^o. The identification method requires first an identification of the mapping function $z = z(t^o, t^u)$. Lemma 4 is a generalization of the identification proof provided in Heckman et al. (2009) to the unified model.

Lemma 4 *If* $\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^u} - \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t^u} < 0$ *(or* > 0*), the mapping function* $z =$

$z(t^o, t^u)$ is identified in the unified hedonic model.

Proof. See Appendix 1. ■

Once the mapping function $z(t^o, t^u)$ is identified, we can proceed to the identification of $\frac{\partial w}{\partial z}$. The next theorem shows that imposing some shape restrictions on the production function $p(z, t)$ allows us to identify $\frac{\partial w}{\partial z}$.

Theorem 5 *Let there be at least one attribute t_i^o so that i) $\frac{\partial j(z,t)}{\partial z \partial t_i^o} \neq \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t_i^o}$ and ii) $p(z, t) = q(z, t_{-i}^o, t^u) + r(t_i^o)$ where $r(.)$ is a known differentiable function. Then for any (z, t^o, t^u), the function $\frac{\partial w}{\partial z}$ is identified.*

Proof. From the first order condition to rents maximization we have:

$$\frac{dw}{dt_i^o} \equiv \frac{\partial w}{\partial z} z'_{t_i^o} + \frac{\partial w}{\partial t_i^o} = \frac{\partial w}{\partial z} z'_{t_i^o} + r'$$

Moreover, from lemma 4, $z'_{t_i^o}$ is identified. Is $z'_{t_i^o} \neq 0$? Consider the first order condition to utility maximization. We have $\frac{\partial w(z(t),t)}{\partial z} - \frac{\partial j(z(t),t;A)}{\partial z} = 0$. Totally differentiating with respect to t_i^o and rearranging obtains:

$$z'_{t_i^o} = \frac{\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t_i^o} - \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t_i^o}}{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t)}{\partial z^2}}$$

From $\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t_i^o} - \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t_i^o} \neq 0$ we have $z'_{t_i^o} \neq 0$ and it follows that, $\frac{\partial w}{\partial z}$ is identified as:

$$\frac{\partial w}{\partial z} = \frac{1}{z'_{t_i^o}} \left(\frac{dw}{dt_i^o} - r' \right)$$

■

An important special case is met when $r(t_i^o)$ is a constant. This occurs when t_i^o is a pure preference attribute, $\frac{\partial p(z,t)}{\partial t_i^o} = r'(t_i^o) = 0$ for all z and t_i^o. This means that t_i^o plays the role of an exclusion restriction in the wage equation. Attribute t_i^o is an argument of the mapping function $z(.)$ but not of $w(.,.)$.

5.1.2 Estimation

Since the estimation results presented in Heckman et al. (2009) follow once we have estimated $\frac{\partial w}{\partial z}$, this paper focuses on the estimation of $\frac{\partial w}{\partial z}$ and refers the reader to Heckman at al. (2009) for the estimation of $j(z,t)$. To present the problem in terms of random functions, let W, T^o, Z be the observable variables of our model and let T^u be the unobservable variable. All variables are of dimension 1 except T^o that has dimension of at least 1. Let our model be $W = w(Z,(T^o,T^u))$ where w is an unknown function continuous in Z and (T^o,T^u) respectively. The function w is assumed to belong to the set of functions derived from the unified economy outlined above. Let $F_{T^u}(.)$ be the distribution of T^u and let $F_{W,Z,T^o}(.; w', F'_{T^u})$ be the joint distribution of the observable variables when $w = w'$ and $F'_{T^u} = F_{T^u}$. Assume that T^u is independent of T^o. Our data consist of a sample of N draws of W, Z and T^o from a single hedonic market.

To estimate $\frac{\partial w}{\partial z}$, we need first to estimate the mapping function. From the proof of Lemma 4 we know that $z(t^o, t^u)$ is strictly increasing in its last argument and since by assumption T^u is independent of T^o, we have:

$$\begin{aligned}
F_{T^u}(t^u) &= \Pr(T^u < t^u) \qquad (21) \\
&= \Pr(T^u < t^u | T^o = t^o) \\
&= \Pr(z(T^o, T^u) < z(t^o, t^u) | T^o = t^o) \\
&= F_{Z|T^o=t^o}(z(t^o, t^u))
\end{aligned}$$

The first equality follows by the definition of F_{T^u}, the second by the independence of T^o and T^u, the third by the monotonicity of $z(t^o, .)$ and the fourth by the definition of $F_{Z|T^o=t^o}$.

Suppose we know F_{T^u}, since $F^{-1}_{Z|,T^o}$ exists from the monotonicity of $z(t^o, .)$, we recover $z(t^o, t^u)$ as:

$$z(t^o, t^u) = F^{-1}_{Z|,T^o}(F_{T^u}(t^u))$$

Suppose instead that we normalize the mapping function so that for some $\bar{t}^o$ and all t^u we have $z(\bar{t}^o, t^u) = t^u$. From Equation 21, we have $F_{T^u}(t^u) = F_{Z|T^o=\bar{t}^o}(t^u)$ and we therefore identify the distribution of T^u from the distribution of Z conditional on $T^o = \bar{t}^o$. The expression of the function z is now given by noting that $F_{Z|T^o=t^o}(z(t^o, t^u)) = F_{T^u}(t^u) = F_{Z|T^o=\bar{t}^o}(z(\bar{t}^o, t^u))$. The first equality follows from the monotonicity of $z(t^o, .)$ in its last argument and the second holds from the previous normalization. This means that we have:

$$z(t^o, t^u) = F^{-1}_{Z|T^o=t^o}\left(F_{Z|T^o=\bar{t}^o}(t^u)\right)$$

Estimates of z are obtained by replacing the true distributions F by their kernel estimators $\widehat{F}$ following the definitions provided in Matzkin (2003) or in Heckman et al. (2009), in the above equalities. Denote $\widehat{z}(t^o, t^u)$ the estimated mapping function. Theorem 5 suggests the following estimator $\widehat{\frac{\partial w}{\partial z}}$ of $\frac{\partial w}{\partial z}$ for any $\bar{t}^o_{-i}, t^o_i, \bar{t}^u$:

$$\widehat{\frac{\partial w}{\partial z}} = \frac{1}{\widetilde{z}'_{t_i^o}}\left(\frac{dw}{dt_i^o} - r'\right)$$

where $\frac{dw}{dt_i^o}$ is the observed wage differential as t_i^o changes and using the known function r to calculate r'.

5.2 Literature on matched employer-employee data

This model makes interesting predictions with respect to the empirical literature estimating earnings functions using matched employer-employee panel data, e.g. Abowd et al. (1999). This literature typically finds that while both workers' and firms' fixed-effects positively correlate with measures of firms' productivity, the correlation between the two components of wages is low or even negative. While Shimer's (2005) unidimensional assignment model with coordinative frictions could generate low or even negative correlation if frictions are large enough,[14] the unified model presented above predicts that a frictionless economy could also be characterized by a low or negative correlation between the contribution of workers' attributes to wages and that of jobs' attributes even though both contributions positively correlate with measures of firms' productivity and sorting exhibits pos-

[14] Recent empirical literature, e.g. De Melo (2009) and Lise, Meghir and Robin (2009), argues that in equilibrium search models, correlations between the estimated employer and worker fixed effects may be misleading.

itive assortative matching. The key features of the unified hedonic model that makes these predictions possible are: 1) that sorting occurs on both skills and preferences and 2) the generic nonlinearity of the wage function (see Ekeland et al., 2004).

To illustrate this result, consider the unidimensional quadratic-normal unified hedonic economy. Let t be the skill level of workers and let z be the level of jobs' complexity. Let $A > 0$ so that workers have a preference for jobs of complexity corresponding to their skills level. Let $D > 0$ so that workers' skills complement jobs' complexity in production. It is further assumed that b, c, B, C and D are so that production increases in t and in z for almost all pairs (z, t).

Since $A + D > 0$, we have $\Pi_1 \left(= \sqrt{\frac{\Sigma_t}{\Sigma_z}}\right) > 0$, so that this economy is characterized by positive assortative matching.

The production of a firm holding job z when matched in equilibrium with worker $t(z) = \pi_0 + \Pi_1 z$ is given by $p(z, t(z)) = const_p + (b + \Pi_1 (c + \pi_0 C) + \pi_0 D) z + \left(\frac{B}{2} + \Pi_1 \left(\frac{C}{2}\Pi_1 + D\right)\right) z^2$ where $const_p = b_0 + c\pi_0 + \frac{C}{2}\pi_0^2$. Using the solution $\lambda = -A\pi_0$, $\Lambda = A(I - \Pi_1)$, $\delta = c - D\Pi_1^{-1}\pi_0$ and $\Delta = C + D\Pi_1^{-1}$ into Equation 10 and after some simplifications, obtains:

$$w(z,t) = \delta_0 + \vartheta(t) + \kappa(z) \tag{22}$$

where $\vartheta(t) \equiv \left(c - D\Pi_1^{-1}\pi_0\right) t + \frac{1}{2}\left(C + D\Pi_1^{-1}\right) t^2$ is the wage contribution of workers' attributes and $\kappa(z) \equiv -\pi_0 A z + \frac{1}{2}A(1-\Pi_1)z^2$ is the wage contribution of jobs' attributes.

Replacing t by $\pi_0 + \Pi_1 z$, the contribution of workers' attributes to wages is given by $\vartheta(t(z)) = const_\vartheta + \Pi_1 (c + \pi_0 C) z + \frac{\Pi_1}{2}(C\Pi_1 + D) z^2$ where $const_\vartheta = \pi_0 \left(c + \frac{\pi_0}{2}\left(C + D\Pi_1^{-1}\right)\right)$.

The three measures of interest are:

$$
\begin{aligned}
p(z, t(z)) &= const_p + \Omega_p z + \Upsilon_p z^2 \\
\vartheta(t(z)) &= const_\vartheta + \Omega_\vartheta z + \Upsilon_\vartheta z^2 \\
\kappa(z) &= const_\kappa + \Omega_\kappa z + \Upsilon_\kappa z^2
\end{aligned}
$$

where $\Omega_p = b + \Pi_1 (c + \pi_0 C) + \pi_0 D$, $\Upsilon_p = \frac{B}{2} + \Pi_1 \left(\frac{C}{2}\Pi_1 + D\right)$, $\Omega_\vartheta = \Pi_1 (c + \pi_0 C)$, $\Upsilon_\vartheta = \frac{\Pi_1}{2}(C\Pi_1 + D)$, $const_\kappa = 0$, $\Omega_\kappa = -\pi_0 A$ and $\Upsilon_\kappa = \frac{A}{2}(1 - \Pi_1)$.

The question arises whether we can calibrate the parameters of the model so that $COV(\vartheta, p) \gg 0$, $COV(\kappa, p) \gg 0$ and $COV(\vartheta, \kappa) = 0$ with the constraints that $A + D > 0$ and hence $\Pi_1 > 0$. These three conditions read as:

$$
\begin{aligned}
COV(\vartheta,p) &\gg 0 \Leftrightarrow \Omega_p\Omega_\vartheta\frac{\Sigma_z}{V(z^2)}+(\Upsilon_p\Omega_\vartheta+\Upsilon_\vartheta\Omega_p)\frac{COV(z,z^2)}{V(z^2)}+\Upsilon_p\Upsilon_\vartheta \gg 0\\
COV(\kappa,p) &\gg 0 \Leftrightarrow \Omega_p\Omega_\kappa\frac{\Sigma_z}{V(z^2)}+(\Upsilon_p\Omega_\kappa+\Upsilon_\kappa\Omega_p)\frac{COV(z,z^2)}{V(z^2)}+\Upsilon_p\Upsilon_\kappa \gg 0\\
COV(\vartheta,\kappa) &= 0 \Leftrightarrow \Omega_\kappa\Omega_\vartheta\frac{\Sigma_z}{V(z^2)}+(\Upsilon_\kappa\Omega_\vartheta+\Upsilon_\vartheta\Omega_\kappa)\frac{COV(z,z^2)}{V(z^2)}+\Upsilon_\kappa\Upsilon_\vartheta = 0
\end{aligned}
$$

Note first that one would not be able to satisfy these three conditions if $\vartheta(t(z))$ and $\kappa(z)$ were linear functions of z. For these conditions to be satisfied we need $\vartheta(t(z))$ and $\kappa(z)$ to be nonlinear functions of z.

Note also that $COV(z,z^2)$ and $V(z^2)$ merely depend on μ_z and Σ_z so that these three conditions are governed by 10 free parameters, i.e. μ_z, Σ_z, b, B, c, C, D, A, Π_1 (or Σ_t once Σ_z is given) and π_0 (or μ_t once μ_z and Π_1 are given) suggesting an infinity of solutions to the problem. However, it seems appropriate to restrict the domain of the parameters to ensure absolute advantage of workers and firms that is 1) more skilled workers are more productive in all jobs, 2) more complex jobs are more productive independently of the type of the worker. Formally these conditions read as:

$$
\begin{aligned}
\frac{\partial p(z,t;E)}{\partial t} &= c + Ct + Dz > 0 \text{ for almost all } t \text{ and } z \\
&\Leftrightarrow c > \min_{z,t}(-Ct - Dz) \\
& \quad \textit{and} \\
\frac{\partial p(z,t;E)}{\partial z} &= b + Bz + Dt > 0 \text{ for almost all } t \text{ and } z \\
&\Leftrightarrow b > \min_{z,t}(-Bz - Dt)
\end{aligned}
$$

Since z and t are normally distributed their support is the real line. For the conditions above to be met for almost all t and z would require c and b to be infinitely large. Instead, the conditions are imposed for all t within 2 standard deviations from the mean and all z within 2 standard deviations from the mean. Formally we impose:

$$
\begin{aligned}
c &> -C\left(\mu_t - l \times 2\sqrt{\Sigma_t}\right) - D\left(\mu_z - 2\sqrt{\Sigma_z}\right) \text{ where } l = \begin{cases} -1 \text{ if } C < 0 \\ 1 \; else \end{cases} \\
b &> -B\left(\mu_z - m \times 2\sqrt{\Sigma_z}\right) - D\left(\mu_t - 2\sqrt{\Sigma_t}\right) \text{ where } m = \begin{cases} -1 \text{ if } B < 0 \\ 1 \; else \end{cases}
\end{aligned}
$$

As it turns out, even with these additional restrictions imposed on the parameters, one can easily calibrate the model so as to generate $COV(\vartheta, p) \gg 0$,

$COV(\kappa, p) \gg 0$ and $COV(\vartheta, \kappa) = 0$. For instance, the calibration reported in Table 1, shows, for a sample of 5000 firms and workers, that $COV(\vartheta, p) = 0.93$, $COV(\kappa, p) = 0.40$ and $COV(\vartheta, \kappa) = 0.04$.

One can even go further and generate not just one cross section of wage data but several successive cross-sections. Provided there is enough mobility of workers across firms in the data, one would then be able to estimate firms' and workers' fixed-effects using firms' and workers' identity and wages as in Abowd et al. (1999).

The problem in hedonic models is to generate the kind of mobility necessary to identify these fixed-effects. The solution is to either let the distribution of jobs or the distribution of workers change over time. As either distribution changes over time, the mapping function changes (remember that $\Pi_1 = \sqrt{\frac{\Sigma_t}{\Sigma_z}}$ and $\pi_0 = \mu_t - \Pi_1 \mu_z$) which as the effect of shuffling the identity (defined by t) of the worker assigned to each firm (identity defined by z) over time.

In the following example, five successive years of data are generated for an economy of 5000 firms and 5000 workers and the parameters are calibrated as in Table 1. The required mobility is generated by successive increments of magnitude 0.05 in μ_t, i.e. μ_t increases from -0.04 to 0.16, keeping the variance of skills and the distribution of jobs constant over time. In each cross section the assignment of workers to firms is defined by $t = \pi_0 + \Pi_1 z$ and wages are determined as $\delta_0 + \vartheta(t(z)) + \kappa(z) + e$ where e a random error that follows a normal distribution.

As is well-known in this literature, identification of workers' and firms' fixed-effects is only possible within connected groups of workers and firms (see Abowd et al., 2002). Such a group contains all the workers who ever worked for any firm in the group and all the firms at which any worker in the group were ever employed. Given the generated panel data, that contains information on wages, firms' identity and workers' identity in five successive years, workers' and firms' fixed-effects are estimated using the $a2reg$ Stata command (see Ouazad, 2008) on the largest connected group (selected using the $a2group$ stata command). This group contains 4706 different firms and 4706 different workers observed on average in 3.3 years.

Denote $\widehat{\vartheta}$ and $\widehat{\kappa}$ the estimated fixed-effects of workers and firms respectively. Estimation results show first that the estimated firms' and workers' fixed-effects are highly correlated with their respective true effects ($COV\left(\vartheta, \widehat{\vartheta}\right) = 0.98$ and $COV(\kappa, \widehat{\kappa}) = 0.68$). Second, we find a negative correlation between the estimated firms' and workers' fixed-effects, i.e. $COV\left(\widehat{\vartheta}, \widehat{\kappa}\right) = -0.27$ (true correlation is $COV\left(\vartheta, \kappa\right) = 0.01$), whereas both sets of fixed-effects correlate positively with firms' productivity, i.e. $COV\left(\widehat{\vartheta}, p\right) = 0.87$ (where $COV\left(\vartheta, p\right) = 0.94$) and $COV\left(\widehat{\kappa}, p\right) = 0.21$ (where $COV\left(\kappa, p\right) = 0.23$).

6 Conclusion

This paper unifies the two classes of models within the sorting literature. The model nests both Tinbergen's model of sorting on job preferences and Sattinger's model of sorting on productivity. Under the assumption that all jobs' attributes lead to intrinsic disutility but workers' attributes do not affect productivity the model collapses to Tinbergen's model. Workers care about their job satisfaction but are equally productive at all jobs. This means that the wage function does not depend on workers' attributes but merely on jobs' attributes. Opposite to this, under the assumption that all workers' attributes contribute to productivity but no jobs' attributes lead to intrinsic disutility the model collapses to Sattinger's differential rents model. Workers do not care about job satisfaction, only about their wage, but workers with different attributes are unequally productive. This means that the wage function does not depend on jobs' attributes but merely on workers' attributes. In the more general case depicted in the unifying model, workers do care about job's satisfaction and productivity does depend on workers' attributes. As a result, the wage function has both workers' and jobs' attributes as arguments. An example of closed form solution is provided when productivity and job satisfaction are quadratic and attributes on both sides are normally distributed.

The model has implications for the estimation of preference (technology respectively) parameters in hedonic models. Recently, Ekeland et al. (2002 and

2004) and Heckman et al. (2009) have shown conditions under which nonparametric identification of additive and nonadditive marginal utility models of the Tinbergen class is possible in a single hedonic market. These conditions depend crucially on the assumption that $\frac{\partial w}{\partial z}$ is known (or estimated from data on wages and z). While this is true by definition in the Tinbergen class of models studied by Ekeland et al. and Heckman et al., in the unified economy, wages are given by the unknown function $w(z, t)$. Without further assumption, $\frac{\partial w}{\partial z}$ is not identified nonparametrically since for any value of t, the value of z is uniquely determined by the mapping function $z = z(t)$. This paper first shows in Lemma 4 that the mapping function $z(t)$ is identified nonparametrically using results from Matzkin (2003). Lemma 4 generalizes Heckman et al.'s (2009) results to the unified hedonic model. Using the identification result for $z(t)$, this paper shows conditions under which $\frac{\partial w}{\partial z}$ is identified nonparametrically in Theorem 5. These conditions impose shape restrictions on the production function $p(z, t)$. In particular, the method assumes that $p(z, t) = q(z, t_{-i}) + r(t_i)$ where $r(.)$ is a known differentiable function and with $t =< t_{-i}, t_i >$ and where t_i is a preference attribute, i.e. so that $\frac{\partial j(z,t)}{\partial z \partial t_i} \neq \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t_i^o}$. A special case is met when t_i is a pure preference attribute, that is, t_i affects utility but not productivity, i.e. $\frac{\partial p(z,t)}{\partial t_i} = 0$ for all z and t_i. Attribute t_i is an exclusion restriction in $w(.,.)$ since it does not affect productivity but not in $z(t)$ since it matters for job satisfaction.

The model is flexible enough to allow the correlation between the contribution of workers' attributes to wages and that of jobs' attributes to vary between -1 to 1. This correlation depends on preference parameters, technology parameters and the distribution of workers and jobs' attributes. The model therefore provides an explanation for Abowd et al.'s (1999) puzzling finding of a low or even negative correlation between workers' and firms' fixed-effects in wage regressions using matched employer-employee data that does not require (large) frictions (Shimer, 2005). The key features of the unified hedonic model that makes this prediction possible are: 1) that sorting occurs on both skills and preferences and 2) the generic nonlinearity of the wage function (see Ekeland et al., 2004).

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Appendix 1:

Proof of Lemma 4. To see this, consider the first order condition to utility maximization, $\frac{\partial w(z(t),t)}{\partial z} - \frac{\partial j(z(t),t;A)}{\partial z} = 0$. Totally differentiating with respect to t^u and rearranging yields:

$$\frac{\partial z(t^o,t^u)}{\partial t^u} = \frac{\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^u} - \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t^u}}{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t)}{\partial z^2}}$$

Using the second order condition, we have $\frac{\partial z(t^o,t^u)}{\partial t^u} > 0$ if $\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^u} - \frac{\partial^2 w(z(t^o,t^u),t)}{\partial z \partial t^u} < 0$. The mapping function is strictly increasing in t^u. Therefore, $z(t^o,t^u)$ is identified using normalization results from Matzkin (2003) and assuming that t^o and t^u are independently distributed. Normalization is required since there exist monotonic transformations g so that $(g \circ z, F_{t^u} \circ g^{-1})$ and (z, F_{t^u}), where F_{t^u} is the CDF of t^u, generate the same data. However, one can show that $z(t^o,t^u)$ is identified non-parametrically using a normalization (choosing one function g). One could either normalize the distribution of t^u (uniform for instance) or normalize the shape of the function $z(t^o,t^u)$ by imposing $z(\bar{t}^o,t^u) = t^u$ for some $\bar{t}^o$ for instance.

Suppose that we assume a certain distribution on t^u, so that F_{t^u} is known, then $F_{z|t^o=x}(z) = F_{t^u}(y)$ tells us that $z(x,y)$ is the same quantile of the distribution of z given t^o as the quantile that y is of the distribution of t^u. We recover $z(t^o,t^u)$ from $F^{-1}_{z|t^o}(\ F_{t^u}(t^u))$.

Suppose instead that we normalize the mapping function so that for some $\bar{t}^o$ and all t^u we have $z(\bar{t}^o, t^u) = t^u$. We can then show that $F_{t^u}(t^u) = F_{z|t^o=\bar{t}^o}(z(\bar{t}^o, t^u))$. We therefore identify the distribution of t^u from the distribution of z conditional on $t^o = \bar{t}^o$. The expression of the function z is now given by noting that $F_{z|t^o}(z(t^o, t^u)) = F_{t^u}(t^u) = F_{z|t^o=\bar{t}^o}(z(\bar{t}^o, t^u))$. The first equality holds since $z(t^o, t^u)$ is strictly increasing over t^u and the second holds from the previous normalization. This means that $z(t^o, t^u) = F^{-1}_{z|t^o}\left(F_{z|t^o=\bar{t}^o}(t^u)\right)$. ■

Appendix 2:

The unified hedonic production economy model

Suppose that instead of being endowed with a machine, firms can produce their own machine. For instance, firms could invest in less noisy machines, safer machines, machines requiring less physical strength to operate, high-tech machines etc. Suppose further that firms are endowed with a vector of attributes y, $y \in \mathbb{R}^{n_y}$. To fix ideas, these attributes could be related to investments capacities but also to the managers' attributes, again, either skills or preferences. Let $f_y(y)$ and $F_y(y)$ be the PDF and CDF of y respectively and let F_y be absolutely continuous with respect to Lebesgue measure.

Let the costs of producing a machine with attributes z for a firm with attributes y be given by the twice differentiable continuous function $c(y, z)$. It is still assumed that to produce output each machine needs to be operated by one and only one worker so that workers and firms match one-to-one. The profits of a firm with attributes y producing output with machine z and employing worker t are now given by:

$$r(z, t, y) = p(z, t) - w(z, t) - c(y, z)$$

The first order condition to utility maximization is unchanged and given by equation 1. We therefore have the mapping function $z(t)$ indicating the optimal machine demanded by a worker with attributes t when wage differential at z is given by w. However, firms are now maximizing profits by selecting the optimal combination of worker t and machine z. First order conditions for profit maximization read as:

$$\frac{\partial w(z,t)}{\partial z} = \frac{\partial p(z,t)}{\partial z} - \frac{\partial c(y,z)}{\partial z} \tag{23}$$

$$\frac{\partial w(z,t)}{\partial t} = \frac{\partial p(z,t)}{\partial t} \tag{24}$$

Let $t(z)$ denote the implicit function that solves Equation 24 for t given $p(.,.)$ and $w(.,.)$. This function indicates the optimal worker t to select for a firm supplying machine with attributes z. Let $z(y,t)$ denote the implicit function that solves Equation 23 for z given $p(.,.)$ and $c(.,.)$ and $w(.,.)$. This function indicates the optimal machine z to supply for a firm with attributes y employing worker with attributes t. Substituting $t(z)$ for t in $z(y,t)$ we obtain an implicit function $z(y)$ indicating the optimal machine $z^* = z(y)$ to supply for a firm with attributes y given productivity $p(.,.)$, costs $c(.,.)$ and wage function $w(.,.)$.

Assume further that the total surplus function $s(z,t) \equiv p(z,t) - c(y,z) - j(z,t)$

satisfies the generalized Spence-Mirrlees condition so that equilibrium is pure and the mapping functions $z(y)$ and $t(z)$ are invertible. Define these inverse functions as $y(z)$ and $z(t)$ respectively. Workers' demand for machines with attributes z is then given by $f_z^d(z)dz = f_t(t(z))\left|\frac{\partial t(z)}{\partial z}\right| dz$ while firms' supply is given by $f_z^s(z)dz = f_y(y(z))\left|\frac{\partial y(z)}{\partial z}\right| dz$. For an equilibrium to be reached, the supply of machines with attributes z should be equal to workers' demand for machines with attributes z for all z. This means that:

$$f_t(t(z))\left|\frac{\partial t(z)}{\partial z}\right| dz = f_y(y(z))\left|\frac{\partial y(z)}{\partial z}\right| dz$$

Equilibrium will be reached by choosing the right shape for the function w and in particular the right differentials at t and z. The equilibrium in this economy is therefore characterized by a wage function $w(z,t)$ and a mapping of workers' attributes onto jobs' attributes $t(z)$ and a mapping function of firms attributes onto jobs' attributes and workers' attributes $y(z)$ so that i) supply equals demand everywhere on the support of z −provided all workers and firms receive more than their reservation levels− , ii) workers maximize utility and iii) firms maximize profits (rents minus costs of producing z).

Appendix 3:

Generalization to other markets

This paper relates to the general literature on hedonic models and not only on that segment focussing on the labor market. For instance, Epple's (1984) extension of Tinbergen's endowed economy to a production economy was originally written in a consumer/producer context, not a worker/firm context. The classical consumer/producer setting reads as follow.

There is a market for a good of attributes z and let $p(z)$ be the hedonic equilibrium price. Producers are endowed attributes y, $y \in \mathbb{R}^{n_y}$. To fix ideas, these attributes could be related to investments capacities but also to the managers' attributes, again, either skills or preferences. Let $f_y(y)$ and $F_y(y)$ be the PDF and CDF of y respectively and let F_y be absolutely continuous with respect to Lebesgue measure. Firms profits are given by $p(z) - c(z, y)$. Consumers are endowed with attributes t, $t \in \mathbb{R}^{n_t}$, that reflect their preferences for the product. Let $f_t(t)$ and $F_t(t)$ be the PDF and CDF of t respectively and let F_t be absolutely continuous with respect to Lebesgue measure. Utility is given by $k(z, t) - p(z)$ where $k(z, t)$ is the indirect utility derived from consumption of z. In this classical setting, output is either produced without labor input or, labor input belongs to y and is fixed at the time firms decide what z to produce. Firms choose to produce the z that maximizes their profits and consumers/workers choose to consume the

z that maximizes their utility.

An extension of the classical hedonic model related to the unified model outline above would be to allow firms to choose the type of worker to hire simultaneously with their choice of z. Suppose that to produce a unit of good of quality z, firms need to hire one and only one worker, $-$say y is a machine that needs to be operated by fixed quantity of workers, one and only one worker$-$. Suppose that the costs of producing z for a firm y when employing worker with attributes t are $c(z, y, t)$. Profits for firm y employing t to produce z are $p(z) - c(z, y, t) - w(z, t)$ where $w(z, t)$ is the wage of worker t at job z. Suppose workers t derive indirect disutility $j(z', t)$ while working at producing z'. Workers utility is then given by $k(z, t) - j(z', t) + w(z', t) - p(z)$.

In this economy, firms/producers optimize on z and t' and workers/consumers optimize on z and z'. The first order conditions for profits maximization read as:

$$p'(z) = \frac{\partial c(z, y, t)}{\partial z} + \frac{\partial w(z, t')}{\partial z} \quad (25)$$

$$\frac{\partial c(z, y, t)}{\partial t} = \frac{\partial w(z, t')}{\partial t'} \quad (26)$$

Let $z(y, t')$ be the implicit function that solves Equation 25 for z the optimal good to produce for firm y with worker t' and let $t'(z, y)$ be the implicit function solving Equation 26 for t' the optimal worker to hire for firm y to produce z. Substituting $t'(z, y)$ for t' in $z(y, t')$ yields $z(y)$ the optimal quality of good to produce for a firm with attributes y given production technology $p(.,.)$, the equilibrium wage function $w(z, t)$ –the slopes– and the equilibrium price function $p(z)$.

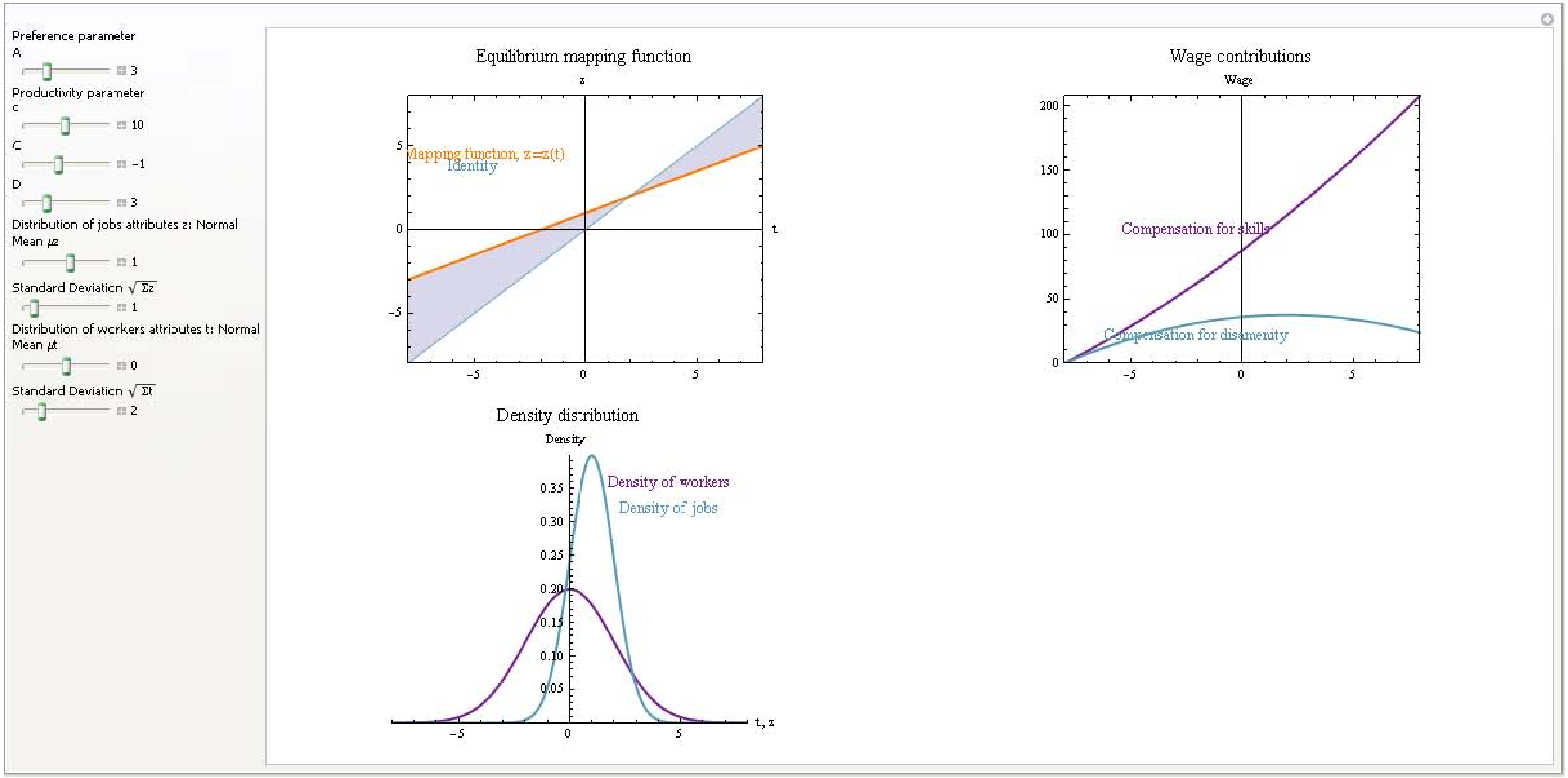
The first order conditions for utility maximization read as:

$$\frac{\partial k(z,t)}{\partial z} = p'(z) \tag{27}$$

$$\frac{\partial j(z',t)}{\partial z} = \frac{w(z',t)}{\partial z} \tag{28}$$

Let $t(z)$ be the implicit function that solves Equation 27. This function indicates the attributes of workers consuming good of quality z in equilibrium, given product tastes $k(.,.)$, and the equilibrium price function. Let $z'(t)$ be the implicit function that solves Equation 28. This function indicates the optimal job to choose for workers with attributes t given job tastes $j(.,.)$ and the equilibrium wage function $w(z, t)$.

In equilibrium, the worker that consumes good of quality z might not necessarily be the one that produces z, i.e. $t(z) \neq t'(z, y)$ in general.



Quadratic-Normal example of the unidimensional unified hedonic model with $\Sigma_z = 1$, $\Sigma_t = 4$, $\mu_z = 1$, $\mu_t = 0$, and $c = 10$, $C = -1$, $D = 3$ and $A = 3$.

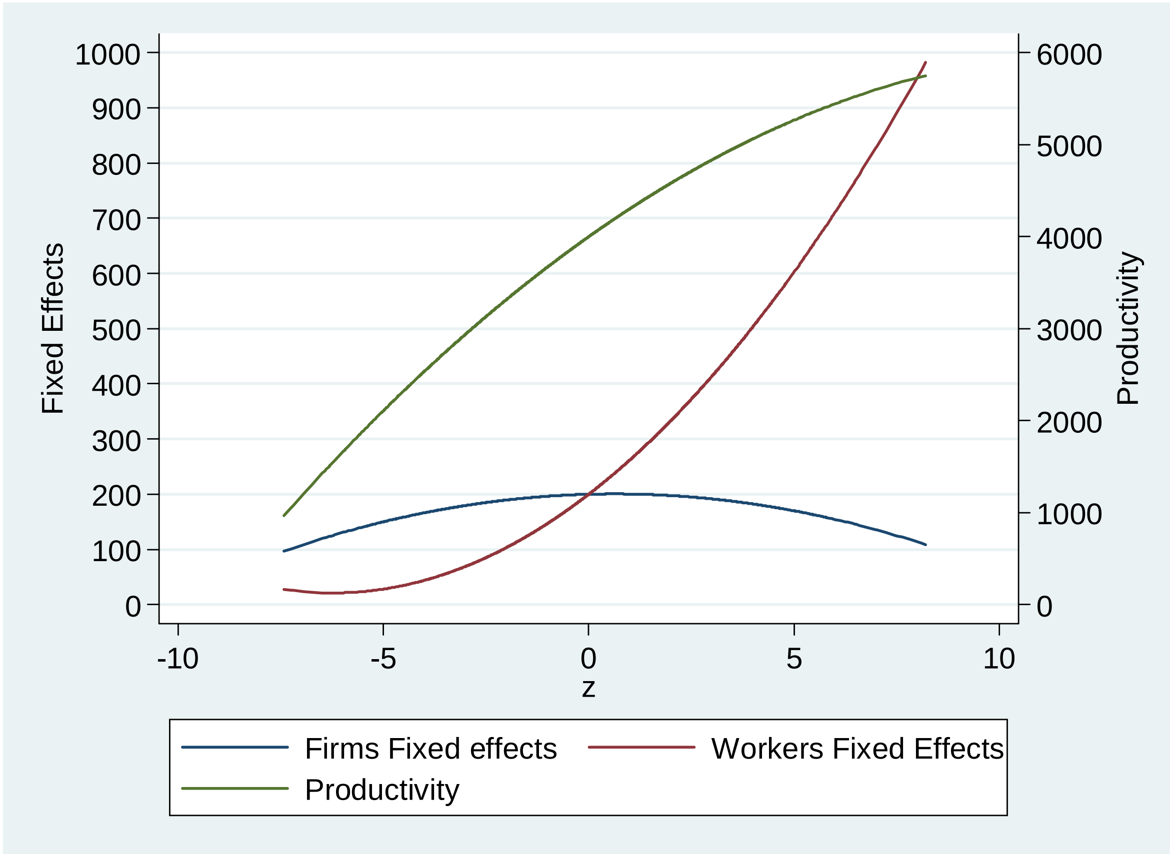


Figure 1: A calibration (see Table 1) of the unidimensional quadratic-normal unified hedonic model. This calibration generates data where both the contribution of workers' attributes to wages and that of jobs' attributes correlate positively with firms' productivity but not with with each other.

Table 1: A calibration of the unidimensional quadratic-normal unified hedonic model.

Parameters		$COV(p,\vartheta)$	$COV(p,\kappa)$	$COV(\vartheta,\kappa)$
		0.93	0.40	0.04
A	8.00			
c	40.80			
C	-1.00			
b	260.44			
B	-45.44			
D	8.00			
Σ_z	5.00			
Π_1	1.40			
$=> \Sigma_t$	9.80			
μ_z	0.15			
π_0	-0.25			
$=> \mu_t$	-0.04			