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ABSTRACT

Raising Juveniles*

This paper investigates how families make decisions about the education of juveniles. The decision problem is analyzed in three variations: a 'decentralized' scheme, in which the parents control the purse-strings, but the children dispose of their time as they see fit; a 'hierarchical' scheme, in which the parents can enforce a particular level of schooling by employing a monitoring technology; and the cooperative solution, in which the threat point is one of the two noncooperative outcomes. Adults choose which game is played. While the subgame perfect equilibrium of the overall game is Pareto-efficient when viewed statically, it may yield less education than the hierarchical scheme. Regulation in the form of restrictions on child labor and compulsory schooling generally affects both the threat point and the feasible set of bargaining outcomes, and families may choose more schooling than the minimum required by law.

JEL Classification: D13, J13, J22, J24

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1 Introduction

The importance of educating juveniles well is beyond dispute, not least because the formation of human capital is crucial for economic growth. Here, the family plays a central role. For the levels of education individuals have attained on reaching adulthood are not the result of investment decisions made by a collection of rational Robinson Crusoes; rather they reflect their families' choices as they were growing up. This fact is well-recognized in the literature that examines investment in education as the outcome of family decision-making and its implications for economic development. In the usual formulation, how much education the children receive basically depends on how much the parents are able and willing to finance. Children have the same preferences as their parents, or their parents can force them to go to school against their will without exerting any effort. In reality, however, the family faces a more complicated problem, in that juveniles may have quite different preferences from their parents, and how they spend their time depends on what inducements their parents provide and their efforts at enforcement.[1] Although conflicts between parents and their children due to differences in preferences have been analyzed in the literature on the 'rotten kid theorem' and the 'samaritan's dilemma' (Becker, 1974; Bergstrom 1989), there are no direct inferences about the long-run consequences of different ways of resolving the conflict.

The object of this paper is to combine these two strands of the literature in order to examine how family decision-making affects investment in education, and hence the economy's long-run growth, when juveniles may also devote their time to work or the pursuit of leisure. As formulated here, parents care about current consumption and their children's future human capital, but the juveniles themselves are also keen on 'fun'. Thus, juveniles may wish to play long and hard, even though their parents are willing to finance a fuller education. It is also possible that the parents are keen to consume, in which case the juveniles' ability to earn, and so contribute to the common pot, constitutes a reason for the former to deny them all the education they desire. As in the literature on family decision-making and economic development, the trade-off between education and work has a prominent place, but we go a step further by allowing juveniles to allocate their time in ways that may run counter to their parents' wishes. This step towards realism enables us to arrive at interesting results on the connection between the static resolution of the family's conflict and its long-term consequences.

The decision problem is analyzed in three variations: first, a 'decentralized' solution,

[1] Broadly related, though different, bargaining problems arise between spouses. Pollak (2007) provides an extensive account, with an emphasis on joint taxation.

in which the parents control the purse-strings, but the children dispose of their time as they see fit, subject to their parents' willingness to finance their choice of education; second, a 'hierarchical' scheme, in which the parents can enforce, at some cost, more schooling than their children desire; and third, the cooperative solution, in which the threat point is one of the two noncooperative outcomes. The parents choose which game is played. We prove that the sub-game perfect equilibrium almost always involves the cooperative solution. The main result is that for some constellations of preferences and technologies there exists a tradeoff between static efficiency, which always holds in a cooperative solution of the family's current conflict, and long-run growth. The reason is that in the hierarchical decision scheme, whose outcome is always pareto-inefficient, the parents possess the means to force their children to attend school, but not to make them work, so that the latter might spend their time just hanging about and having fun, which the parents do not value. Hence, if the parents have preferences for schooling that are sufficiently stronger than their children's, but not too strong, and the marginal costs of enforcement are not too high, they would choose extensive schooling under hierarchy; but they would do still better in the bargaining outcome. For the savings in enforcement costs would be enjoyed by both parties, partly in the form of more consumption, with the juveniles contributing to the household's budget by working more in exchange for less schooling and perhaps some pocket money to pursue fun, even if there were less time for it.

In this setting, it is natural to ask how legal restrictions on child labor and compulsory schooling will affect families' decisions. If the main aim is to foster the formation of human capital, the principle of targeting indicates that enforcing extensive schooling is the right way to intervene. Our analysis reveals, however, that there are subtle effects of restrictions on child labor when juveniles can while away their time instead of attending school: these restrictions can induce high levels of education, even when the compulsory minimum does not bind. Joint regulation may, therefore, be advantageous.

There is a large literature on the economics of the family (for an overview, see Neuwirth and Haider [2004]). The family's role in educational decisions is usually emphasized when there are borrowing constraints (Banerjee, 2003), especially in the context of developing countries. As for the motives for such investments and who has the final say, whilst the assumption that parents are in charge of financial decisions is almost universal, there are various formulations of the parties' utility functions. They range from the parents and children sharing a single, unified utility function (Becker, 1981; Loury, 1981) to that in which parents' only incentive to lend to their children is because the latter will care for them in old age (Cox, 1987; Cremer, Kessler and Pestieau, 1992;

Barham et al., 1995; Cox and Jakubson, 1995). There are also formulations in which parents have some altruism towards their children, expressed by the former putting some weight on the latter's income, consumption or human capital.

In the non-cooperative setting of this paper, the fundamental family conflict is also related to the literature on the 'rotten kid theorem' and the 'samaritan's dilemma'. In summary, in a sequential game with altruistic parents and a selfish child, the parents can achieve their first-best by moving first (samaritan's dilemma) or last (rotten kid theorem), whereby the right choice depends on the specific form of the utility functions (see Dijkstra [2007] for a good overview). These problems are usually two-dimensional, with each party deciding over one dimension. In most settings, the parents decide over the level of an income transfer to the children, who control a variable that is often interpreted as work effort. The setting employed here, in which the parties dispose of a variety of alternatives, is five-dimensional, with three degrees of freedom. It is correspondingly richer in its implications.

The paper is organized as follows. Section 2 describes the family's structure, endowments, and activities. Its decision problem is set out in Section 3, in the three variations described above, and the sub-game perfect equilibrium is characterized. The effects of legal restrictions on work and schooling are examined in Section 4, followed by a numerical, illustrative example in Section 5. Some concluding remarks are drawn together in Section 6.

2 The Structure

The youthful members[2] of the family (identical juveniles) split their time among education, e, work, w, and leisure, l. They consume an aggregate good directly, together with a good that will be called 'fun', which is produced by combining inputs of leisure and the aggregate good. They also have a certain say in family decision-making. Their parents (the adults) work full time, consume only the aggregate good, and have their say in family decision-making. The number of family members in group $a\,(= 1, 2)$ is denoted by n^a, where there are $n^2 = 2$ adults.[3] Each member of the family is endowed with one unit of time. A juvenile's time budget therefore satisfies

$$e + l + w = 1, \quad (e, l, w) \geq \mathbf{0}, \tag{1}$$

[2] According to the U.N.'s definition, 'youth' are those aged 12 to 24.

[3] In principle, we could also allow for the possibility that one parent dies before the children reach puberty. As premature adult mortality is not the focus of the paper, we rule out such heterogeneity.

there being no other way to use time. Let λ^a denote the human capital possessed by a member of group a.

Three technologies are involved.

Assumption 1
The aggregate good is produced under constant returns to scale by means of human capital alone, with the factor of proportionality α.

Assumption 2
The technology for producing fun is represented by

$$\zeta = \zeta(l, m), \tag{2}$$

where m is the level of a youth's complementary expenditure on the aggregate good, all of which falls on the family budget, and the function $\zeta(\cdot)$ is increasing, concave and differentiable in both arguments. In contrast to pocket money, leisure is essential in the production of fun: $\zeta(0, m) = 0 \; \forall m$ and $\zeta(l, 0) > 0 \; \forall l > 0$.

The third technology is that for producing human capital. In general, one can write the level of human capital attained by juveniles when they will have reached full adulthood, λ^2_{+1}, as a function of their parents' human capital and the time spent in schooling.

Assumption 3

$$\lambda^2_{+1} = \Gamma(\lambda^2, e), \tag{3}$$

where Γ is an increasing, concave and differentiable function of e.

It is assumed that a social rule governs the distribution of the consumption of the aggregate good within the family, with each juvenile receiving the fixed fraction β of an adult's consumption level, c.[4] Hence, the family's budget constraint is

$$(2 + \beta n^1)c + (\sigma e + \alpha\lambda^1 l + m)n^1 = \alpha \cdot (2\lambda^2 + n^1\lambda^1), \tag{4}$$

where σ is the cost (including the opportunity cost of a juvenile's time) of full-time education, $(\alpha\lambda^1 l + m)$ is the combined expenditure on fun made by each youthful member, and the expression on the RHS is the family's full income, measured in units of the aggregate good. It should be remarked that in writing (4) as a strict equality,

[4]Note that future human capital plays no role in generating income today.

we have ruled out free disposal of full income, for example, by making grants to other families. To do otherwise would introduce another dimension to the parties' action spaces, thereby further complicating the analysis of the non-cooperative games, in which the parents control the purse strings.

3 The Household's Problem

The family has to choose an allocation, in the sense of choosing a feasible vector (c, e, l, m, w). This means a tussle between the youthful members of the family and their parents, for their interests are partly opposed. The outcome must respect the household's budget constraint (4) and the juveniles' time budget (1). As both hold with equality, there are effectively three degrees of freedom. We assume that juveniles have preferences over βc, ζ and λ^2_{+1}, whereas their parents have preferences over c and λ^2_{+1} alone. We further assume that their respective preferences can be represented by increasing, strictly concave and differentiable utility functions $U^a\,(a = 1, 2)$, that all goods are normal and the aggregate good is necessary in consumption.

In general, the possible solutions to the family's problem take three forms: (i) a 'decentralized equilibrium', in which the parents control all expenditures on the aggregate good and the juveniles decide how they will spend their time, subject to their parents' willingness to finance the outlays on education; (ii) a 'hierarchical' decision scheme, in which parents have some means of setting e directly when they regard the juveniles' choice thereof as too low; and (iii) the Nash bargaining solution, which will be realized only if both parties are willing to negotiate. Henceforth, these are denoted by D, H and B, respectively. We assume that parents are free to choose between the first two arrangements, since they can decide whether they want to incur the costs of monitoring and enforcing the level of the juveniles' schooling. The household's entire decision process can therefore be depicted by the following overall game structure.

Stage 1: The parents decide among {D, H, B}.

Stage 2: If the parents chose B, bargaining will take place. Otherwise the parents' choice of either D or H will be executed.

Stage 3: If bargaining fails to yield agreement, the parents decide between D and H and their choice will be executed.

In principle, D, H, and B are themselves games. Accordingly, the overall game can be viewed as the decision process whereby the parents and the juveniles choose the particular game they want to play in order to solve the household's allocation problem.

By backward induction, the overall game is solved by both the parents and the juveniles calculating the outcomes of D and H. The outcome that yields the parents the higher level of utility is the threat point in the bargaining game, as the parents are free to choose among the non-cooperative games in the event that bargaining fails. It is then possible to determine the outcome of the cooperative game. Once the solutions to D, H, and B have been determined, we are in a position to characterize the subgame perfect equilibria of the overall game. For simplicity, we add the following assumption:

Assumption 4
If the parents are indifferent between choosing B and one of the non-cooperative games, they will go with the latter.

The reason for this assumption is that, in principle, the parents could always choose B at stage 1 even if they already knew that bargaining would fail. This is because the parents can still decide between D or H at the last stage and bargaining is assumed not to involve any costs. Introducing a minimal cost of bargaining would have the same effect as assumption 4. We now have the following lemma:

Lemma 1
Under assumption 4, the parents' first-stage choice will be executed in every subgame perfect equilibrium.

Proof. By assumption 4, the parents will choose B at the first stage only if they prefer the bargaining outcome to D or H and the juveniles will accept the bargaining outcome. In this case, the bargaining solution will be implemented. In all other cases, the parents' first-stage choice will be executed by definition. □

We now characterize the solution to the household's problem, proceeding as described above. In particular, we shall describe and solve the sub-games, D, H and B, and then determine the outcome of the overall game.

Before turning to these sub-games, however, we can state the following result, which is an immediate consequence of the assumption that the parents place no value on the juveniles' 'fun'.

Lemma 2

If parents have unfettered control over the purse strings, but juveniles decide how they will spend their time no later than their parents decide how to spend the budget, then in all non-cooperative arrangements with no repetitions of play, parents will grant no allowances.

The consequences for the juveniles' allocation of time depends crucially on the forms of $U^a(\cdot)$ and $\zeta(\cdot)$. Suppose, for example, that although the juveniles are keen on fun, they spend no time having fun. The reason must lie in a sufficiently strong degree of complementarity between the size of the allowance and leisure in the production of 'fun'. With substantial substitutability, however, juveniles will spend time in leisure despite receiving no pocket money. Whether, and if so how much, juveniles work depends on how their parents will respond to the resulting larger budget.

3.1 The 'Decentralized' Solution

Under this arrangement, the parents have control over the variables (c, m) and the young are free to manage their own time (e, l, w), subject to the parents' willingness to finance e. We begin by demonstrating that this is not quite complete as a description of their respective action spaces. The juveniles' earnings increase the total amount that the parents can allocate to all forms of expenditure. In particular, with control over the purse strings, the parents are in a position to restrict the juveniles' educational choices. That being so, the actual outcome where e is concerned will be the minimum of what juveniles want and what the parents are prepared to finance. The parents' desired level of e is, moreover, a function of total income and hence of the juveniles' working time. Thus, each party's set of feasible actions depends on the other's actual action.

To establish why this may lead to a problem, consider the following example. Both players move simultaneously, the parents maximize over (c, m) given (e, l, w), and the juveniles over (e, l, w) given (c, m). Scrutiny of the juveniles' decision problem reveals that, if (c, m) is given, working will bring them nothing, so that they would spend their time lounging about or going to school. As the game is one of common knowledge, however, the juveniles know that by working they would contribute to total income; and by Lemma 2, they also know that the parents will choose $m = 0$. Since both players are assumed to be rational, the parents would respond to additional income, given e, by increasing c. Under the usual assumption that both players' rationality is common knowledge, this implies that the juveniles know that if they hold e constant and substitute a certain amount of work for leisure, then (c, m) would change, which contradicts the hypothesis that the juveniles take (c, m) as given when choosing (e, l, w).

These two issues require special attention when setting up the game structure. To reflect the contribution of w to family income, we rewrite the budget constraint as

$$(2 + \beta n^1)c + (\hat{\sigma} e + m)n^1 = \alpha(2\lambda^2 + wn^1\lambda^1) \equiv y(w) \tag{5}$$

where $\hat{\sigma}$ is the direct cost of full-time education, i.e., without the opportunity costs of time. Observe that given $(n^1, 2)$ and the social rule expressed by β, only two of c, e and m can be chosen independently for each choice of w.

The problem of determining e is solved as follows. The juveniles decide on (e^1, l, w) subject to (1), and the parents choose (c, m) and the education level, e^2, which they would like the juveniles to achieve, given the available budget. If $e^1 \leq e^2$, the outcome is $e = e^1$, as the adults cannot control e directly. If $e^1 > e^2$, however, the parents can veto the juveniles' education decision by refusing to finance it. These possibilities are recognized by both parties before play commences.

The formal specification of the game involves two stages, whereby the juveniles move first. If $e^1 \leq e^2$, the parents accept the juveniles' choice of e^1. It follows at once from (5) and Lemma 2 that they choose

$$(c, m) = \left(\frac{y(w) - n^1 \hat{\sigma} e^1}{2 + \beta n^1}, 0 \right). \tag{6}$$

At the first stage, therefore, the juveniles' decision problem is

$$\begin{array}{ll} \max_{(e^1,\, l,\, w)} & U^1(\beta c, \lambda^2_{+1}(e^1), \zeta) \\ \text{s. t.} & (1),\ (6),\ (e^1, l, w) \geq \mathbf{0}. \end{array} \tag{7}$$

Since $U^1(\cdot), \zeta(\cdot)$ and $\Gamma(\cdot)$ are all concave functions and the constraints define a convex feasible set, the solution to this problem is unique, and hence also is the solution of this non-cooperative game. The assumption that the aggregate good is necessary in consumption ensures that $c > 0$ at the juveniles' optimum.[5] To complete the characterization of the solution, let $e^1 < 1$ and $w > 0$, so that the corresponding first-order

[5]If, contrary to Assumption 2, $\zeta(l, 0) = 0$, the juveniles cannot have any fun, and so choose $l = 0$.

condition is

$$-\frac{\partial U^1}{\partial(\beta c)} \cdot \frac{\beta(y' + n^1\hat{\sigma})}{2+\beta n^1} + \frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot \frac{\partial \lambda^2_{+1}}{\partial e} \leq 0,\ e^1 \geq 0,\ \text{compl.} \tag{8}$$

This may be expressed more intuitively as, using (5) to substitute for y',

$$-MRS^1_{1,2} \equiv \frac{\partial U^1/\partial(\beta c)}{\partial U^1/\partial \lambda^2_{+1}} \geq \frac{2+\beta n^1}{\beta n^1} \cdot \frac{\partial \lambda^2_{+1}/\partial e}{\alpha\lambda^1 + \hat{\sigma}}\ ,\ e^1 \geq 0,\ \text{compl.} \tag{9}$$

where $MRS^1_{1,2}$ denotes the marginal rate of substitution between the first and second arguments of U^1, and the ratio of the marginal yield of schooling (in terms of future human capital) to its combined direct and opportunity costs is adjusted by the claims on the common pot where consumption is concerned.

If $e^2 < e^1$, where e^1 solves problem (7), the parents can implement e^2. Their decision problem is

$$\begin{array}{ll} \max_{(c,\, m,\, e^2 | e^1,\, l,\, w)} & U^2(c, \lambda^2_{+1}(e^2)) \\ \text{s. t.} & (5),\ (c, m, e^2) \geq \mathbf{0}. \end{array} \tag{10}$$

The foregoing assumptions ensure that for each and every $(e^1,\ l,\ w)$, this problem has a unique solution, with $c > 0$ and $m = 0$. Inspection of (5) with $m = 0$ reveals that the upper frontier of the feasible set in the space of (c, λ^2_{+1}) shifts parallel to the right as w increases. The assumption that all goods are normal then ensures that λ^2_{+1}, and hence also $e^2(w)$, is increasing in w so long as $e^2(w) < e^1$.

With the first stage in mind, we now introduce a further assumption:

Assumption 5
$e^2(w)$ is a concave function of w $\forall e^2(w) < e^1$.

The juveniles' problem at the first stage is now:

$$\begin{array}{ll} \max_{\ (e^1,\, l,\, w)} & U^1(\beta c, \lambda^2_{+1}(e^1), \zeta) \\ \text{s. t.} & (1),\ (5),\ m = 0,\ e^1 \leq e^2(w),\ (e^1, l, w) \geq \mathbf{0}. \end{array} \tag{11}$$

Together with the earlier assumptions, assumption 5 ensures that the solution to this problem, and hence the solution to this variant of the non-cooperative game, is unique. It is clear that $e^1 = e^2(w)$ holds at the optimum. In what follows, the solution to the decentralized game will be denoted by $(c^d, m^d, e^d, l^d, w^d)$, where $m^d = 0$.

3.2 The Hierarchical Decision Scheme

In addition to controlling just the purse strings, parents can try to control the time spent at school directly, which is potentially attractive whenever $e^2 > e^1$ in D. The drawback is that monitoring and enforcement are normally costly. Let $k(e)$ denote the costs of enforcing e for each juvenile. The budget constraint (5) becomes

$$(2 + \beta n^1)c + n^1(\hat{\sigma}e + k(e)) + n^1 m = y(w). \tag{12}$$

The hierarchical decision scheme differs from D mainly in that the parents have the possibility of fixing the level of education at the first stage, before the juveniles decide on the allocation of the remainder of their time between leisure and work time at the second. Finally, at the last stage, the parents split the budget net of education costs between consumption and the allowance. By lemma 2, this structure implies that the allowance is zero. Using the budget constraint, we obtain:

$$(c, m) = \left(\frac{y(w) - n^1(\hat{\sigma}e + k(e))}{2 + \beta n^1}, 0 \right). \tag{13}$$

Knowing that all the remaining budget will be consumed and they will not receive any pocket-money, the juveniles solve the following problem at the second stage:

$$\begin{array}{ll} \max_{(l,\ w)} & U^1(\beta c, \lambda^2_{+1}(e), \zeta) \\ \text{s. t.} & (1),\ (13),\ (w,\ l) \geq \mathbf{0}. \end{array} \tag{14}$$

The solution to this problem is leisure $l^{h1}(e)$ and work time $w^{h1}(e)$ as functions of the education level, which is decided upon by the parents.[6] Their first-stage problem is written as:

$$\begin{array}{ll} \max_{(e)} & U^2(c, \lambda^2_{+1}(e)) \\ \text{s. t.} & c = (y(w) - n^1(\hat{\sigma}e + k(e)))/(2 + \beta n^1), \\ & w = w^{h1}(e),\ 0 \leq e \leq 1. \end{array} \tag{15}$$

We impose restrictions on the juveniles' preferences such that the following holds:

Assumption 6
$w^{h1}(e)$ is concave.[7]

[6] It is assumed that parents are unable to take out loans in order to finance education.
[7] The details of the associated conditions on U^1 and ζ are available upon request.

In view of the fact that the juveniles go to school voluntarily under the decentralized arrangement, we make the following assumptions about $k(\cdot)$:

Assumption 7
$k(e)$ is an increasing, convex and twice-differentiable function $\forall e > e^1$, with $k'(e^1) = 0$, and $k(e) = 0$ otherwise.

It follows from assumption 6 and the convexity of $k(\cdot)$ that there is a unique solution to problem (14). Let the solution to the game be denoted by $(c^h, m^h, e^h, l^h, w^h)$. Since the aggregate good is necessary in consumption and $y(0) > 0$, $c^h > 0$.

Of particular interest is whether the parents will choose $e^h > e^d$; for otherwise they opt, in effect, for the decentralized scheme, in which enforcement is unnecessary. The corresponding first-order condition associated with problem (15) is

$$\frac{\partial U^2}{\partial c} \cdot \left(\frac{\partial c}{\partial y} \cdot y' \cdot \frac{dw^{h1}}{de} + \frac{\partial c}{\partial e} \right) + \frac{\partial U^2}{\partial \lambda^2_+} \cdot \frac{\partial \lambda^2_+}{\partial e} \leq 0, \; e \geq 0, \; \text{compl.} \tag{16}$$

From (13), $\frac{\partial c}{\partial y} \cdot y' > 0$, but dw^{h1}/de is likely negative. Thus, whether $e^h > e^d$ depends not only on the parents' preferences and the education and enforcement technologies, but also on the juvenile's preferences, which influence w^{h1}. Analogously to the juveniles' decision problem (7) under D, the first-order condition may be expressed as

$$-MRS^2_{1,2} \geq \frac{2 + \beta n^1}{n^1} \cdot \frac{\partial \lambda^2_{+1}/\partial e}{-\alpha \lambda^1 \cdot dw^{h1}/de + \hat{\sigma} + k'}, \; e \geq 0, \; \text{compl.} \tag{17}$$

a comparison of which with (9) reveals that, for any given e, the adjusted ratio of marginal pay-off to marginal cost here is not necessarily more favorable than under D. Parents may, of course, have much stronger tastes for their children's future capital than the children themselves, which is precisely H's attraction.

Observe that if $\Gamma(\lambda^2, e)$ satisfies the lower Inada condition with respect to e, then $e^h > 0$ is assured, but not $e^h > e^d$. A necessary and sufficient condition for the latter to hold is obtained as follows. Let the solution to problem (7) be denoted by $\mathbf{s}^1 = (c^1, m^1, e^1, l^1, w^1)$, where (c^1, m^1) is given by (6). Then, recalling that $k'(e^1) = 0$ by assumption 7, a necessary and sufficient condition for $e^h > e^d$ is

$$\frac{\partial U^2}{\partial c} \cdot \left(\frac{\alpha n^1 \lambda^1}{2 + \beta n^1} \cdot \frac{dw^{h1}}{de} - \frac{n^1 \hat{\sigma}}{2 + \beta n^1} \right) + \frac{\partial U^2}{\partial \lambda^2_+} \cdot \frac{\partial \lambda^2_+}{\partial e} > 0, \tag{18}$$

where all derivatives are evaluated at $\mathbf{s}^1$. For if $e^d < e^1$, the parents will have vetoed the juveniles' choice of schooling under scheme D and so will choose $e^h = e^d$, with $k = 0$ under H. Condition (18) may be expressed as

$$-MRS^2_{1,2}(\mathbf{s}^1) < \frac{2+\beta n^1}{n^1}\left(\frac{\partial\lambda^2_{+1}/\partial e}{-\alpha\lambda^1 \cdot dw^{h1}/de + \hat{\sigma}}\right)_{\mathbf{s}^1} . \tag{19}$$

This condition also determines whether parents prefer H to D, as we now demonstrate. By choosing $k = 0$ under H, parents effectively choose D and so obtain U^{2d}. Since $k = 0$ at $e = e^1$, condition (19) implies that at the parents' optimum under H, $e^h > e^d$, and hence $U^{2h} > U^{2h}(k = 0) = U^{2d}$. If condition (19) fails to hold, parents effectively choose D. This establishes

Proposition 1
Parents prefer H to D iff condition (19) holds.

3.3 The Pareto-inefficiency of D and H

Non-cooperative games often yield pareto-inefficient outcomes, and since the threat point in the cooperative game arises from D or H as alternatives, it is important to establish whether the latter are inefficient in this sense. That the hierarchical decision scheme is indeed so follows at once from the fact that the parents could allocate the expenditures on control as pocket money to the juveniles or to the common consumption pot, given that e^h stays constant. That D is, in general, likewise is seen as follows. All pareto-efficient allocations arise as solutions to the following problem:

$$\max_{(c,e,l,m,w)} U^1 \quad \text{s.t. } (1),\ (5),\ U^2 \geq \bar{U}^2, \tag{20}$$

where $\bar{U}^2$ can vary parametrically over some feasible interval. It is almost never the case that the solution invariably involves $m = 0$, and whenever it does not, the outcome under D will be pareto-inefficient. To complete the argument, we examine the outcome under D directly, when $m = 0$. It can be proved that, with very limited exceptions, it is indeed pareto-inefficient (see Appendix).

To summarize:

Proposition 2
All allocations under D are pareto-inefficient under the assumptions set out above, with the following exceptions:
(i) Unanimity on full-time child labor ($e^1 = e^2(w = 1) = 0$) is pareto-optimal.

(ii) Unanimity on full-time education ($e^1 = e^2 = 1$) *is pareto-optimal unless leisure and pocket money are sufficiently poor substitutes in producing fun.*
(iii) If the juveniles just lounge about ($l^1 = 1$) *and the parents desire some education* ($e^2(w = 0) > 0$)*, then the additional assumption that the* $|MRTS|$ *between leisure and pocket money in producing 'fun' at* $(l, w) = (1, 0)$ *is smaller than a juvenile's marginal productivity is strongly sufficient to ensure that this outcome is pareto-efficient.*
All allocations under H, which necessarily involves $e^h > e^1$*, are pareto-inefficient.*

3.4 The Nash Bargaining Solution

A third possibility is that the allocation is the outcome of a bargaining game between the two groups. The distinction between the direct costs of education and the time actually spent in the classroom is a potentially important one where the description of the game is concerned. If agreement can be reached on e, this implies that the adults commit themselves to pay the corresponding fees and the juveniles not to play truant once the fees have been paid, with each party taking the other's commitment to be credible. No monitoring is necessary, so that the set of feasible allocations is

$$S = \{s \in \mathbb{R}^5_+ |\, s \text{ satisfies (1), (5)}\},$$

with the typical element $s = (c, m, e, l, w)$. Then

$$X = \{(x^1, x^2) \in \mathbb{R}^2 |\, (x^1, x^2) = (U^1(s), U^2(s)), s \in S\}$$

is the set of feasible utility pairs.[8]

The threat point plays a central role in the definition of any bargaining problem. Since the adults may choose between the above two non-cooperative games in the event that the negotiations are unsuccessful, their choice will determine the threat point. Rational as they are, they will choose that which yields the higher level of utility; for in the absence of any means to pre-commit themselves to one or the other for the purposes of negotiation, this will be the best course of action should there be no agreement. Denote by s^d and s^h the allocations chosen in the decentralized and hierarchical decision schemes, respectively, and U^{2d} and U^{2h} the parents' corresponding

[8] Since the utility and production functions are concave, X is convex.

utility levels. Then the threat point can be written as

$$\xi = (x^{1\xi}, x^{2\xi}) = \begin{cases} (U^1(s^d), U^2(s^d)), & U^{2d} \geq U^{2h} \\ (U^1(s^h), U^2(s^h)), & U^{2d} < U^{2h} \end{cases}$$

where $U^{2d} = U^2(s^d)$ and $U^{2h} = U^2(s^h)$.

We are now in a position to define the bargaining problem:

Definition 1
(X, ξ) *represents the bargaining problem between the juveniles and their parents.*

The Nash-solution to the bargaining problem is

$$\arg \max_{(x^1,\, x^2) \geq \xi} (x^1 - x^{1\xi})(x^2 - x^{2\xi}), \quad (x^1, x^2) \in X,$$

or, equivalently, $(U^1(s^n), U^2(s^n))$, where

$$\begin{aligned} s^n &= \arg\max_{s \in S} (U^1(s) - x^{1\xi})(U^2(s) - x^{2\xi}), \\ &\quad \text{s.t. } (U^1(s), U^2(s)) \geq \xi. \end{aligned}$$

Let the parties' respective gains from cooperation be denoted by $U^{1+} \equiv U^1(s) - x^{1\xi}$ and $U^{2+} \equiv U^2(s) - x^{2\xi}$, and observe that with the exceptions noted in Proposition 2, there will be strictly positive gains for both parties.

We now turn to characterization. The first-order conditions, whose solution is s^n, are

$$\left(\frac{\partial U^1}{\partial c} \cdot U^{2+} + \frac{\partial U^2}{\partial c} \cdot U^{1+}\right) - \mu(2 + \beta n^1) \leq 0, \ c \geq 0 \tag{21}$$

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial m} \cdot U^{2+} - \mu n^1 \leq 0, \ m \geq 0 \tag{22}$$

$$\left(\frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda^2_{+1}} \cdot U^{1+}\right) \cdot \frac{\partial \lambda^2_{+1}}{\partial e} - \mu \hat{\sigma} n^1 - \nu \leq 0, \ e \geq 0 \tag{23}$$

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial l} \cdot U^{2+} - \nu \leq 0, \ l \geq 0 \tag{24}$$

$$\mu \alpha n^1 \lambda^1 - \nu \leq 0, \ w \geq 0 \tag{25}$$

where μ and ν are the Lagrange multipliers associated with (5) and (1) , respectively.[9]

[9] There are also pairs of complementary inequalities corresponding to the corner solutions involving $e = 1, l = 1$ and $w = 1$, whose form is evident, and so omitted to save space.

Since U^i is increasing in c ($i = 1, 2$), it follows that $\mu > 0$ at s^n; for an increase in $y(w)$ can always be allocated to consumption of the aggregate good. For the same reason, $\nu > 0$ at s^n, since an increase in the time available to juveniles can be allocated to work, so raising $y(w)$. That the lower Inada-condition holds for U^i with respect to c ($i = 1, 2$) and $y(0) > 0$ also imply that $c^n > 0$, so that the first part of (21) holds as an equality at s^n. Hence, the shadow price of family income is

$$\mu = \left(\frac{\partial U^1}{\partial c} \cdot U^{2+} + \frac{\partial U^2}{\partial c} \cdot U^{1+} \right) / (2 + \beta n^1).$$

It is the weighted sum of the parties' marginal utilities of consumption at s^n, spread over $(2 + \beta n^1)$ effective consumers, where the weights are the other party's gain from cooperation.

Corner solutions for the remaining variables cannot be ruled out without further assumptions. If a small investment in formal education at $e = 0$ produces a large enough improvement in the juveniles' human capital upon attaining adulthood, which both parties value, then $e^n > 0$ and the first part of (23) will hold as an equality at s^n when $e^n < 1$. Now suppose that $l^n > 0$, so that ν follows from the first part of (24). Recalling from assumption 2 that pocket money is of no use unless $l > 0$, consider $(l^n, m^n) > \mathbf{0}$. It then follows from (24) and (22) that $|MRTS_{lm}| = \nu/\mu n^1$, so that if $(\partial \lambda^2_{+1}/\partial e)_{e=0}$ is sufficiently large, the first part of (23) may be written

$$\left(\frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda^2_{+1}} \cdot U^{1+} \right) \cdot \frac{\partial \lambda^2_{+1}}{\partial e} - \mu n^1 (\hat{\sigma} + |MRTS_{lm}|) = 0.$$

As a final step, let $w^n > 0$, so that there is a full interior solution, $s^n > \mathbf{0}$. It follows from (25) that $\nu = \mu \alpha n^1 \lambda^1$ and hence that $|MRTS_{lm}| = \alpha \lambda^1$, which is the market opportunity cost of a juvenile's time. Substituting for $|MRTS_{lm}|$, we have

$$\left(\frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda^2_{+1}} \cdot U^{1+} \right) \cdot \frac{\partial \lambda^2_{+1}}{\partial e} - \mu n^1 (\hat{\sigma} + \alpha \lambda^1) = 0.$$

This states that the cost of a (small) unit of education, including the juveniles' opportunity costs in the labor market, when multiplied by the shadow price of family income, be equal to the weighted sum of the marginal utilities of human capital induced by that unit of education.

To complete the characterization of a full interior solution, note that (24) and (25) yield

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial l} \cdot U^{2+} = \alpha \lambda^1 \mu n^1 = \nu.$$

This reveals that the change in a juvenile's utility brought about by a little more leisure time in the production of fun, when multiplied by the adults' gain from cooperation, must be equal to the shadow price of family income times the amount of the aggregate forgone by working that much less.

3.5 Solution of the Overall Game

Having characterized the solutions of the (sub-)games D, H, and B, we are now in a position to examine the solution of the overall game. From Lemma 1, we know that the parents' first-stage choice will rule in a subgame perfect equilibrium. To determine this choice, one must solve the overall game by backward induction. The first step is to determine the parents' threat point in the bargaining game. Equivalently, we can ask, when will parents resort to H if bargaining fails? Proposition 1 supplies the answer. If, moreover, the parents do choose H, the juveniles will be worse off ($U^{1h} < U^{1d}$). For under D, the latter are free to choose e^1 and they do not have to share in the costs of enforcing $e^h > e^1$. That $U^{2h} > U^{2d}$ implies $U^{1h} < U^{1d}$ enables us to say something about the distribution of the gains from cooperation. If the parents choose H as their threat in the event of a disagreement, they will also do better in the cooperative game than if they were to choose D instead – a threat that is not credible in view of $U^{2h} > U^{2d}$.

As all players in the game possess perfect information, they know the outcomes of all the games D, H, B; in particular, the parents know whether the juveniles would accept the bargaining outcome. We can now state our main result:

Proposition 3
(i) The subgame perfect equilibrium of the overall game is pareto-efficient from a static perspective.
(ii) H will never be implemented, D almost never.
(iii) The subgame perfect equilibrium of the overall game may yield a lower level of human capital formation than H.

The proof is given in the appendix. The intuition runs as follows. Parts (i) and (ii) follow from the fact that if H and D are inefficient, parents will choose the bargaining outcome, for then both parties will be better off in B than in the non-cooperative games. Hence, the overall outcome will be efficient. As emphasized in part (iii), however, static efficiency does not necessarily yield the greatest possible dynamic gains, through education. The result in (iii) stems from the fact that the only possibility for parents directly to influence juveniles' allocation of time is by forcing them to go to school. If, for example, juveniles like to spend a lot of time in leisure – which adds nothing to the parents' utility – their parents can enforce time in education but not in work. In the bargaining solution, the juveniles can offer to work more at the expense of schooling. If the parents' preferences for consumption relative to the juveniles' human capital are sufficiently strong, they will accept this offer.

4 Regulation

Thus far, all decisions have been made free of any legal restrictions on how much juveniles may work or the levels of their schooling. In affluent countries, such work is fairly strictly regulated, and compulsory education is likewise strongly enforced, so it is interesting to examine the effects of such restrictions, severally and jointly.

4.1 The labor market

Denote by $\bar{w}$ the legal upper limit of a juvenile's working time. This will affect the outcome under D if and only if $e^d + l^d > 1 - \bar{w}$. Since $y(w^d) > y(\bar{w})$, at least one of c and e must be lower. If the parents continue not to exercise their veto, the juveniles' first-order conditions yield, when $e^1 > 0$ and $l^1 > 0$,

$$-\frac{\partial U^1}{\partial(\beta c)} \cdot \frac{\beta n^1 \hat{\sigma}}{2 + \beta n^1} + \frac{\partial U^1}{\partial \lambda_{+1}^2} \cdot \frac{\partial \lambda_{+1}^2}{\partial e} = \frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial l} < \frac{\partial U^1}{\partial(\beta c)} \cdot \frac{\beta \alpha \lambda^1}{2 + \beta n^1},$$

where all terms are evaluated at $w = \bar{w}$, with $\zeta = \zeta(l, 0)$. By hypothesis, the regulation so reduces the feasible set in the space of $(\beta c, \lambda_{+1}^2(e), \zeta(l, 0))$ that $\mathbf{s}^d$ is no longer attainable. Hence, given the normality of all goods, c will surely be lower if the regulation binds sufficiently tightly; and the fact that education involves direct as well as opportunity costs works in favor of the juveniles increasing leisure more than schooling. Turning to the possibility that the parents would veto e^1 in the absence of the regulation, its introduction produces two effects. On the one hand, there is an *income effect* which simply induces a reduction in $e^2 \,(= e^d)$, so as to moderate the fall in c that would otherwise result from the reduction in y. On the other hand, there is also a *substitution effect* which influences the opportunity costs of education for the parents.

Without the regulation, the parents may have vetoed e^1 because they knew that the juveniles would then spend some additional time at work. When the labor restriction binds, this is no longer possible, and the juveniles will spend the additional time in leisure. In contrast to education, the juveniles' leisure yields the parents no utility, so this effect works in favor of increasing e^2.

Under H, the regulation will not bind if $e^h > 1 - \bar{w}$, since $l \geq 0$; and hence it has no effect, provided $e^h > e^d(w = \bar{w})$, as is very likely. If the regulation does bind, the juveniles choose $w = \bar{w}$ and $l \geq 0$, with the latter holding with equality iff $e^h = 1 - \bar{w}$. The parents therefore solve problem (15), with $w = \bar{w}$ and $y = y(\bar{w}) < y(w^h)$. As before, the reduction in y will work in favor of a reduction in e, for both goods are normal and this will lower outlays on enforcement, and so moderate the fall in c. Against this, however, there is also the substitution effect: now the juveniles just hang about, whereas without the restriction, they spent some time at work. There is also a direct *enforcement cost effect* with respect to education. This effect results from changes in the juveniles' voluntary school attendance in response to the restriction on working. In principle, this effect can go either way. Hence, whether education increases or decreases under H as a result of labor regulation depends on the particular strength of these three effects.

Let H provide the threat point under B, regulation or no, and let $e^h > 1 - \bar{w}$ and $e^n < e^h$ in the absence of the regulation. Its imposition will affect the outcome under B if it causes the feasible set S to contract in the neighborhood of $\mathbf{s}^n$ by limiting total income to $y(\bar{w})$. By hypothesis, the threat point is unchanged, so that at least one party will be worse off as a result. If the parents come out relatively better, this will involve an increase in e^n at the expense of w and, possibly, l; for in such allocations, the parents have relatively strong preferences for their children's future human capital. It should be remarked that with a complete ban on juvenile labor ($\bar{w} = 0$), the regulation will bind in all subgames. If $e^h < 1$, it follows that $e^n(\bar{w})$ may exceed e^h; for with $w = 0$ and given the fixed sharing rule for consumption, the only other adjustment in the parents' favor after the distribution of the savings in enforcement costs is for juveniles to sacrifice some of their leisure time.

4.2 Schooling

The regulation is interesting only if the juveniles have some spare time to devote to leisure or work; for otherwise the family would have no latitude of any kind, with family income being fixed, at $y(0)$, and the residual after meeting the direct costs of education and enforcement, if any, going to consumption. Hence, we assume that the compulsory level of education is fixed at $\underline{e} < 1$, with families enjoying the discretion to choose more if they wish.

An immediate question is, who bears the costs of enforcing $\underline{e}$, should this be necessary? At one extreme, the authorities could charge parents with the task of enforcing $\underline{e}$, with the threat of severe sanctions if they do not. D is then no longer an option if $e^d < \underline{e}$, in which event, the parents must make (unwilling) outlays of $k(\underline{e})$ on enforcement. The regulation has no effect on the allocation under H if $e^h \geq \underline{e}$; but if $e^h < \underline{e}$, complying with it will entail additional outlays on enforcement. Hence, if $e^d < \underline{e}$ or $e^h < \underline{e}$, we have, by lemma 2, $c = (y(w) - n^1(\hat{\sigma}\underline{e} + k(\underline{e}))/(2 + \beta n^1), e = \underline{e}$ and $m = 0$.

The juveniles take this choice as given and solve

$$\begin{aligned} &\max_{(l,w)} U^1(c, \lambda^2_{+1}, \zeta) \\ \text{s.t.} \quad &(1),\ c = (y(w) - n^1(\hat{\sigma}\underline{e} + k(\underline{e}))/(2 + \beta n^1),\ e = \underline{e},\ m = 0,\ (l, w) \geq \mathbf{0}. \end{aligned}$$

Denote their choice by (l^{r1}, w^{r1}). The following marginal condition will hold at an interior optimum:

$$-MRS^1_{1,3} \equiv \frac{\partial U^1/\partial c}{\partial U^1/\zeta} = \frac{(2 + \beta n^1) \cdot \partial\zeta/\partial l}{\alpha n^1 \lambda^1}\,, \tag{26}$$

where all derivatives are evaluated at the said values of (c, e, m).

At the other extreme, the school authorities could enforce at least $\underline{e}$,[10] with the family bearing any additional costs under H if parents choose $e^h > \underline{e}$. Again, D in its unrestricted form is no longer an option if $e^d < \underline{e}$. Since $e^d < e^h$, it follows nevertheless that if $e^h < \underline{e}$, the non-cooperative game is effectively D, but with $e = \underline{e}$ and no enforcement costs for the family. The parents decide on the allocation of the net income that remains after meeting the direct costs of education, which are non-discretionary, and the juveniles how to allocate their time between work and leisure. By lemma 2, $m = 0$ and $c = (y(w) - \hat{\sigma}\underline{e}n^1)/(2 + \beta n^1)$. The remainder of the solution is characterized by (26), evaluated at these values of (c, e, m).

It must be borne in mind, however, that the fact that the authorities bear enforcement costs in the amount of $n^1 k(\underline{e})$ itself makes education yet more attractive to parents under H, so that e^h must be derived accordingly. In problem (15), we now have $c = (y(w) - n^1(\hat{\sigma}\underline{e} + k(e) - k(\underline{e}))/(2 + \beta n^1)$, with $e \geq \underline{e}$ and $m = 0$. If the solution

[10]For simplicity, we ignore the fact that such public expenditures must be financed by taxation.

involves $e^{rh} > \underline{e}$, it is quite possible that the 'income effect' of $n^1k(\underline{e})$ results in $e^{rh} > e^h$, the level of e chosen under H in the absence of regulation. In any event, condition (26), evaluated at the corresponding values of (c, e, m), continues to hold at the juveniles' interior optimum.

We turn to the effects of compulsory education on the parties' utilities. If, in the absence of regulation, the parents choose D and desire $e^2 > e^1$, they will welcome the school authorities' efforts to enforce $\underline{e}$, even if the family has to bear all the costs, provided $\underline{e}$ does not go too far beyond e^2. The juveniles will certainly be worse off as a result. Thus, the parents' bargaining position will be strengthened under B, and this will normally lead to a higher level of e^n than that ruling in the absence of regulation, even if $e^n > \underline{e}$. If, instead, the parents choose H in the absence of regulation, with $e^h < \underline{e}$, and the authorities bear the costs of enforcing $\underline{e}$, the former still may be better off under the regulation. The same holds for the juveniles, who will also benefit in the form of higher consumption, but the net effect is still likely to be a strengthening of the parents' bargaining position under B.

It is clear that, under B, the effects of imposing compulsory education stem in part from its effects, if any, on the threat point, even when the regulation does not bind under B itself. If B is chosen, the solution is characterized by (21), (22), (24) and (25), with $e = \underline{e}$ when the regulation binds; otherwise, (23) applies as before. Since the agreement within the family is voluntary, there are no enforcement costs for any party, even when the outcome without regulation would be $e^n < \underline{e}$. In this connection, it should be remarked that the regulation will not overturn proposition 2. If D is chosen in the absence of regulation, with $e^2 > e^1$, and the school authorities bear the costs of enforcing $\underline{e}$ $(> e^1)$, D with $e = \underline{e}$ will hold under the regulation. If, however, H is chosen in the absence of regulation, with $e^h < \underline{e}$, the regulation will not necessarily result in D with $e = \underline{e}$; for $e^{hr} > \underline{e}$ is compatible with $e^h < \underline{e}$. Recalling proposition 2, compulsory education rules out full-time work and is irrelevant if there is unanimity on the desirability of full-time education. The only remaining possibility involves $l^d = 1$, but that too is ruled out by $\underline{e} > 0$.

4.3 Joint regulation

The level of compulsory education may be such that, despite the restrictions on juvenile labour, both bind in one or more of the subgames. In that event, the juveniles will have no effective choices left, with $l = 1 - \underline{e} - \bar{w} \geq 0$, family income $y(\bar{w})$ is given, and enforcement costs, if any, likewise. What, then, is there left to bargain over in B? The answer is m, and if $l = 1 - \underline{e} - \bar{w} \geq 0$ and the threat point is pareto-inefficient, the juveniles will indeed enjoy some pocket money in the subgame perfect equilibrium.

4.4 Summary and discussion

A novel feature of our analysis is that juveniles desire leisure in order to have 'fun'. Where regulation is concerned, this adds effects that are not present in the previous literature on family decision-making and development. In the latter, it is oftentimes emphasized that the negative income effect of an upper limit on child labor may lead to less schooling. By explicitly including the juveniles' decision problem, we identify additional effects of limiting child labor, in particular, the substitution effect and the enforcement-cost effect. As established above, and as the example in the next section illustrates, these effects are important when evaluating the efficiency of different measures such as banning child labor and compulsory schooling in fostering long-run growth by promoting human capital formation.

The principle of targeting tells us that it is almost always optimal to attack a distortion as close to its source as possible. If private decisions yield levels of schooling that are deemed socially to be too low and, as here, the capital market plays no role, it is therefore tempting to conclude that the right form of intervention is setting and enforcing the desired level of compulsory education. At the same time, it is important to note that although families almost always arrive at a cooperative solution (recall Proposition 2), changes in regulation can also affect the threat point and hence the outcome under bargaining. A move to longer compulsory schooling, if fully enforced, will necessarily increase actual schooling if families are currently choosing less; but by influencing the threat point in the parents' favor, the more stringent regulation may increase schooling even when families' current choices exceed the minimum it stipulates.

What role is left for child labor laws? None, if leisure is ruled out. In practice, however, adolescents, in particular, find various ways of spending time in leisure, so that families have another margin to work with in response to changes in regulation. We have established that tighter restrictions on child labor alone will have ambiguous effects on the level of schooling in the non-cooperative games. What counts, however, is the finding that in the bargaining outcome, which almost always rules and in which the juveniles normally enjoy some leisure, such restrictions may even lead to more schooling than under hierarchy. For any contraction in the set of feasible allocations that favors the parents will result in the juveniles sacrificing some leisure, this being the only margin left to them when they may not work as much as they would like.

The above arguments take no heed of possible differences in the state's costs of enforcing the two forms of regulation, which must be financed by taxation in some form or other. The most effective way to foster human capital formation and growth may therefore involve the joint regulation of child labor and education, whereby the costs of administration, subsidies and the taxes to pay for them must be integrated into the analysis.

5 An Example

In this section we provide a numerical example to illustrate our results. The functional forms for the production of human capital, the costs of enforcing education and the production of the good 'fun' are as follows:

$$\begin{aligned} \lambda^2_{+1} &= z\lambda^2 e^{\phi} + 1, \\ k(e) &= \begin{cases} A(e-e^1)^{\kappa}, & \text{if } e > e^1, \\ 0, & \text{otherwise,} \end{cases} \\ \zeta(l,m) &= B\left[ql^{-\rho} + (1-q)m^{-\rho}\right]^{-\frac{1}{\rho}}. \end{aligned}$$

The parties' preferences are given by

$$\begin{aligned} U^1 &= (\beta c)^{a^1}\left\{\left[\omega^1\zeta^{-\gamma} + (1-\omega^1)(\lambda^2_{+1})^{-\gamma}\right]^{-\frac{1}{\gamma}}\right\}^{(1-a^1)}, \\ U^2 &= c^{a^2}(\lambda^2_{+1})^{(1-a^2)}. \end{aligned}$$

The following parameter values characterize our "standard scenario". In particular, note that the juveniles have stronger tastes for consumption than their parents ($a^1 > a^2$).[11]

a^1	a^2	ω^1	γ	β	z	ϕ	A	κ	q	ρ	B	α	n^1	$\hat{\sigma}$	λ^1	λ^2
$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0.6	1.2	$\frac{1}{2}$	1	1.2	0.6	$-\frac{1}{2}$	2	2	2	1	$\frac{1}{2}$	2

The allocations in the three decision schemes in the standard scenario are given in Table 1. Enjoying their freedom in D, the juveniles spend about two thirds of their time in leisure and split the rest almost equally between education and work. In this

[11] In violation of assumption 2, $\zeta(0,m) > 0\,\forall m > 0$. In this section, however, all allocations involve $l > 0$, so that the said condition plays no role.

constellation, however, parents can do better under H, in which e^h is much higher at close to one-half, at the expense of juveniles' time at play and, to a lesser extent, at work. Enforcing this level of schooling claims 0.2301 of the aggregate good; together with the partial loss of the juveniles' earnings, the sacrifice of consumption is substantial. In the bargaining solution, in which H constitutes the threat-point, the juveniles agree to a sharp reduction in their leisure time and an even bigger increase in time at work in exchange for some pocket money and a little less schooling than the rigors of e^h. It is also worth noting that the juveniles will not reach the utility level they would have achieved in D, had that been available.

Table 1: Outcomes in the standard scenario

Scheme	e	$k(e)$	w	l	m	c	U^2	U^1
D	0.1664	0	0.1749	0.6587	0	2.5053	2.2267	1.3537
H	0.4603	0.2301	0.1006	0.4391	0	2.1314	2.3669	1.2492
B	0.4069	0	0.4253	0.1679	0.0745	2.4649	2.4976	1.3380

Labor regulation takes the form of a complete ban ($\bar{w} = 0$). Under D, the juveniles devote the time they would otherwise have spent working almost wholly to leisure, and their parents do not veto the minimal increase in e^1. The latter, however, continue to do better under H, despite the loss of the juveniles' earnings. Indeed, they step up e^h, with somewhat higher associated enforcement costs of 0.2794. Recalling section 4, this increase arises from the changed opportunity costs of enforcement offsetting the direct income effect, which by itself would lead to a lower e^h. That the substitution effect is very large – under D, the juveniles would spend almost all of the additional time in leisure, which yields the parents no utility – appears to be particularly relevant in this example. Higher enforcement costs and parents' preferences tilted more towards consumption may result in a dominant income effect and thus a lower education level with labor regulation. To give such an example, let $A = 1.5$, $\kappa = 1.1$, and $a^2 = \frac{2}{3}$: these yield $e^h = 0.2232$ without regulation and $e^h = 0.2213$ with $\bar{w} = 0$.

Table 2: Outcomes in the standard scenario with a ban on child labor

Scheme	e	$k(e)$	w	l	m	c	U^2	U^1
D	0.1691	0	0	0.8309	0	2.3943	2.1811	1.3497
H	0.5147	0.2794	0	0.4854	0	2.0037	2.3353	1.2197
B	0.5513	0	0	0.4487	0.0340	2.1342	2.4366	1.3096

The outcome of the game is, as before, the allocation under B. With the juveniles unable to work, the only way for them to increase their parents' utility is to agree to more schooling: e^n now exceeds e^h, and both are larger than their counterparts in the absence of regulation. The savings in enforcement costs suffice to permit an increase in consumption, and with so much leisure time at their disposal, the juveniles make do with less pocket money than they would receive without the ban on working. Both parties are worse off under the regulation.

Turning to compulsory schooling, let this stipulate the minimum level $\underline{e} = 0.5$. Table 3 reports the outcomes both when there is direct enforcement of $\underline{e}$ by the state and when the parents have to pay all enforcement costs. Note that when $\underline{e}$ exceeds the level chosen in D and enforcement has to be provided by the parents, the imposition of the regulation on the latter is identical to its imposition on H when $e^h < \underline{e}$. In this particular constellation, only in H when the state enforces $\underline{e}$ does the level of education exceed the required minimum. This accords with intuition, since part of the parents' enforcement costs are then fully subsidized. Comparing these outcomes with those without regulation, imposing higher education levels by assumption favors the parents, whose ranking of D and H is left unchanged. The juveniles are worse off, except H when $\underline{e}$ is enforced by the state. The parents' tastes for consumption are sufficiently strong that the income effect of the subsidy dampens the attractions of education so much that the juveniles, whose tastes for consumption are stronger still, are more than compensated. The actual outcome of the overall game is B under both arrangements for bearing the costs of enforcement. In this constellation, this particular regulation is welcomed by the parents, but it is a cause for complaints by the juveniles.

Table 3: Outcomes in the standard scenario with compulsory schooling ($e \geq \underline{e}$)

Scheme	e	$k(e)$	w	l	m	c	U^2	U^1
	$\underline{e}$ enforced by state							
D	0.5	0	0.0409	0.4591	0	2.2131	2.4431	1.2940
H	0.5370	0.0191	0.0243	0.4378	0	2.1666	2.4455	1.2780
B	0.5	0	0.3301	0.1699	0.0755	2.3466	2.5157	1.3172
	$\underline{e}$ enforced by parents							
H	0.5	0.2678	0.0857	0.4144	0	2.0752	2.3658	1.2290
B	0.5	0	0.3336	0.1664	0.0730	2.3498	2.5174	1.3163

As argued in Section 4.4, no comparison of the two forms of regulation is complete without a full specification of the associated costs of implementing them. For ignoring such costs, one could simply set $\underline{e} = 1$, if the aim be to promote human capital formation to the fullest – though the old saying that 'all work and no play makes Jack (Jill) a dull boy (girl)' suggests that a slightly milder regime is desirable. It is still interesting, however, to examine the outcomes when the above regulations are jointly implemented and the state enforces $\underline{e} = 0.5$ directly, which are set out in Table 4. Joint regulation promotes education still further, as the juveniles cannot evade education by working. They are, morever, even worse off; and their parents would prefer to have compulsory education alone. Note that if parents have to bear all enforcement costs, the outcomes would be the same as in H and B in our example with the complete ban ($\bar{w} = 0$) only.[12] The reason is that under the ban, the parents will enforce education above $\underline{e}$ voluntarily.

Table 4: Outcomes in the standard scenario with joint regulation ($\bar{w} = 0$, $\underline{e} = 0.5$)

scheme	e	$k(e)$	w	l	m	c	U^2	U^1
D	0.5	0	0	0.5	0	2.1875	2.4180	1.2937
H	0.6105	0.0711	0	0.3895	0	2.0730	2.4410	1.2422
B	0.6360	0	0	0.3630	0.0201	2.0899	2.4678	1.2761

6 Concluding remarks

How juveniles split their time among schooling, work and play involves a clash of interests with their parents; for the parties almost surely have different preferences, whatever be the common bonds of family. The structure developed and analyzed here places this clash at the center of the family's allocation problem, which includes choosing how much to produce of the household good we have called 'fun'. Subject only to the social 'rule' that juveniles receive a fixed fraction of an adult's consumption of the aggregate good, the cooperative solution is always pareto-efficient viewed statically; but under the 'hierarchical' arrangement, in which the parents devote resources to enforce their desired level of schooling, juveniles may receive more education, which will lead to faster long-term growth.

This result depends on the assumption that in the absence of an agreement, parents face prohibitively high costs of enforcing juveniles' time at work, leaving enforcement of schooling as the only option. In the bargaining solution, however, the parents' enforcement costs are relevant only through their influence on the threat point, and the structure of the relative costs of education and work time depends on the parties'

[12] Again, the decentralized decision effectively drops out, because the parents have to enforce the minimum education level.

preferences. If the juveniles' opportunity costs of a marginal unit of time at work relative to one at school are lower than the relative enforcement costs in the hierarchical scheme and the parents' preferences for consumption are sufficiently strong, though weaker than the juveniles', the outcome under bargaining will involve more work at the expense of education. The assumption that parents find it less costly to enforce their children's schooling than their time at work seems reasonable in many countries, particularly those where there is a tight and strictly enforced cap on how much juveniles may work. With a reversal of the parents' costs of enforcing work relative to education, the same line of argument yields the result that the bargaining solution is likely to involve more time in school at the expense of work.

Our analysis also highlights another point. Even when a bargaining solution is available, it may not be realized if enforcement costs are so high as to make hierarchy less attractive to parents than the decentralized alternative. For we have established that the latter may be pareto-optimal, though the circumstances in which it is so are rather limited. At all events, only if the costs of enforcing schooling are sufficiently small can one be sure that there will be a credible threat point to the cooperative solution that leaves room for bargaining. Thus, the absolute level of enforcement costs plays a role in determining whether bargaining – if available – actually takes place, whereas the relative structure of the enforcement costs determines how the outcomes under hierarchy and bargaining differ.

The hierarchical scheme is always pareto-inefficient, and if the decentralized scheme is likewise, the outcome will be the bargaining solution. As argued in section 4, a change in the enforcement technologies affects the threat points of the cooperative game, but not the feasible set of bargaining outcomes. Regulation in the form of restrictions on child labor or compulsory schooling, in contrast, can affect both. A ban on child labor may reduce schooling in the non-cooperative schemes, but as the only means then left to the juveniles is to agree to more education at the expense of their leisure, the ban may lead to an increase in education in the bargaining solution, which actually rules. Indeed, such a ban may well be more effective in promoting education than a sufficiently modest level of compulsory schooling. For if juveniles are not effectively restricted in how much they may work, they can bargain for less schooling by offering to work more, with an outcome that may still comply with the minimum level of schooling. This indicates that, in general, joint regulation is called for in order to foster fuller education. Establishing the best form of regulation, however, requires that the costs of administration, subsidies and the taxes to pay for them must be integrated into the analysis. That is a task for another paper.

Appendix

A Proof of Proposition 2

That the solution scheme H is inefficient is established in the main text. Thus we only have to consider parts (i)-(iii).

We begin with allocations in which no parental veto is exercised.

Case (a): $0 < e^1 < e^2$. If, at the juveniles' optimum, $l > 0$, they can always reduce it slightly in favor of e^1. This has only a second-order effect on U^1, but it yields a first-order improvement in U^2. If $l = 0$, then $w > 0$. A small increase in e^1 at the expense of w will have only a second-order effect on U^1, but it will yield a first-order improvement in U^2 by virtue of $e^2 > e^1$, which implies that the parents are willing to sacrifice some current consumption in favor of education at $e = e^1$ and given $y(w)$.

Case (b): $0 < e^1 = e^2 < 1$. There is agreement about e, but note that a small increase in w at the expense of l will have only a second-order effect on U^1, while yielding a first-order improvement in U^2.

Case (c): $0 = e^1 < e^2$. If, at the juveniles' optimum, $0 < l < 1$, a small increase in w will have only a second-order effect on U^1, but will yield a first-order improvement in U^2. If $l = 1$, it suffices to show that $e^1 = m = w = 0$ does not solve problem (20) when $\bar{U}^2 = U^2\left(y(0)/(2 + \beta n^1), \lambda^2_{+1}(0)\right)$. Writing down the first-order conditions in the form of complementary inequalities and employing the hypothesis $l = 1, m = w = 0$, a little manipulation yields

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta(1,0)}{\partial l} > \mu > \nu \alpha n^1 \lambda^1 > \alpha \lambda^1 \cdot \frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta(1,0)}{\partial m},$$

where μ and ν are the multipliers associated with (1) and (5), respectively. Hence, if the |MRTS| between leisure and pocket money in producing 'fun' at (1,0) is, in fact, smaller than a juvenile's marginal productivity, $\alpha\lambda^1$, we have a contradiction. Since lounging about the whole day without any pocket money is not especially attractive, imposing the condition $|\text{MRTS}|_{l=1,m=0} < \alpha\lambda^1$ does not seem very restrictive. Be that as it may, we have established part (iii) of the proposition.

Case (d): $e^1 = e^2 = 1$. With $l = w = 0$ and both parties preferring λ^2_{+1} to consumption at the margin defined by $e = 1$, there is no way of improving one party's position without worsening the other's unless the juveniles enjoy some fun, which involves $l > 0$, and the parents enjoy more consumption, which involves $w > 0$, both to compensate for the corresponding reduction in e^1. Since, by hypothesis, the juveniles choose $l = w = 0$

in view of $m = 0$, the solution to problem (20) when $\bar{U}^2 = U^2\left(y(0)/(2+\beta n^1), \lambda^2_{+1}(1)\right)$ involves $m > 0$ only if l and m are sufficiently poor substitutes. Otherwise, $e^1 = e^2 = 1$ is pareto-optimal. This establishes part (ii) of the proposition.

We turn to the remaining allocations, in which the parents impose $e = e^2(w)$ at the second stage and hence a (binding) constraint on the juveniles' choices at the first.
Case (e): $e^1 > e^2 > 0$. A small increase in e^2 at the expense of c will have only a second-order effect on U^2, but, by the envelope theorem, it will yield a first-order improvement in U^1, with corresponding optimal adjustments in l and w.
Case (f): $e^2 = 0$. If $1 > l > 0$, a small reduction in l in favor of w will have only a second-order effect on U^1, while yielding a first-order improvement in U^2. If $l = 1$ and $e^1 > 0$, devoting a little time to work under the condition that the whole of the proceeds go to financing education would make the juveniles better off at no cost to the parents. Such transfers are ruled out in D, however. Finally, in the extreme case $w = 1$, the allocation is pareto-optimal. For $l = 0$ yields the largest feasible set from the parents' point of view, and if they desire $e^2 = 0$ when $w = 1$, they cannot possibly do better under any other arrangement; so that $e^1 = e^2 = 0$ solves problem (20) when $\bar{U}^2 = U^2\left(y(0)/(2+\beta n^1), \lambda^2_{+1}(0)\right)$. This establishes part (i) of the proposition. □

B Proof of Proposition 3

(i) The bargaining solution will be implemented if and only if the threat point is inefficient. For if the threat point is efficient, there is no possibility of making one party better off without making the other worse off. If, on the other hand, the threat point is inefficient, both parties will gain from cooperation; for transfers of the aggregate good are possible and perfectly divisible. Hence, B will always be agreed upon in such a situation. As the Nash bargaining solution is pareto-optimal by definition and will fail to be implemented only in situations in which the alternative non-cooperative game yields an efficient outcome, the subgame perfect equilibrium of the overall game must be pareto-optimal from a static perspective.

(ii) By proposition 2, H is not pareto-optimal. Hence, the parents never propose it at the first stage of the overall game, as they will be better off by bargaining. The same applies, almost always, to D.

(iii) Let the constellation of preferences and technologies be such that the first part of (17) holds as an equality with $e^h = 1$, where the derivative dw^{h1}/de is evaluated as $e \to 1$ from below. If the juveniles have strong tastes for consumption, they will also cut back on leisure (without pocket money), if they had chosen any, as they are forced to undergo more schooling when e^h is close to 1. Hence, $dw^{h1}/de \geq -1$. In such an allocation, the parents would also accept a very small increase in consumption, financed by the juveniles working a little bit at the expense of their education.

From (23) and (25), it is seen that the corresponding first-order condition under B is

$$\Omega \equiv \frac{U^{2+} \cdot \partial U^1/\partial c + U^{1+} \cdot \partial U^2/\partial c}{U^{2+} \cdot \partial U^1/\partial \lambda^2_{+1} + U^{1+} \cdot \partial U^2/\partial \lambda^2_{+1}} \leq \frac{2+\beta n^1}{n^1} \cdot \frac{\partial \lambda^2_{+1}/\partial e}{\alpha\lambda^1 + \hat{\sigma}}, \; e \leq 1, \text{ compl.}$$

Observe that $\Omega > -MRS^2_{1,2}$ iff $-MRS^1_{1,2} > -MRS^2_{1,2}$. Substituting from (17) under the said hypothesis about the allocation under H, we have

$$\Omega \leq -MRS^2_{1,2}(s^h(e^h=1)) \cdot \frac{(-\alpha\lambda^1 \frac{dw^{h1}}{de} + \hat{\sigma} + k')_{e=1}}{\alpha\lambda^1 + \hat{\sigma}} \cdot \frac{\partial \lambda^2_{+1}/\partial e}{(\partial \lambda^2_{+1}/\partial e)_{e=1}}, e \leq 1, \text{ compl.}$$

Now suppose also that $e^n = 1$, so that $l^n = w^n = m^n = 0$. Since $k(1) > 0$, $c^n > c^h$ and hence $-MRS^2_{1,2}(s^n(e^n=1)) < -MRS^2_{1,2}(s^h(e^h=1))$, though the difference will be small if $k(1)$ is sufficiently small. If juveniles have sufficiently strong tastes for consumption ($-MRS^1_{1,2} > -MRS^2_{1,2}$), $\Omega|_{e=1} > -MRS^2_{1,2}(s^n(e^n=1))$. Inspection of the RHS of the above weak inequality reveals, however, that if $k(1)$ and $k'(1)$ are sufficiently small, and $(dw^{h1}/de)_{e=1}$ exceeds -1 by a sufficient margin, this will contradict $\Omega|_{e=1} > -MRS^2_{1,2}(s^n(e^n=1))$.

By continuity, the argument will also go through if e^h is sufficiently close to 1, or if, at $e^h = 1$, (17) holds as an inequality in reverse, provided the RHS exceeds $-MRS^2_{1,2}$ by a sufficiently small margin. □

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