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# Offshoring and Unemployment: The Role of Search Frictions and Labor Mobility

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# ABSTRACT

# Offshoring and Unemployment: The Role of Search Frictions and Labor Mobility*

In a two-sector, general-equilibrium model with labor-market search frictions, we find that wage increases and sectoral unemployment decreases upon offshoring in the presence of perfect intersectoral labor mobility. If, as a result, labor moves to the sector with the lower (or equal) vacancy costs, there is an unambiguous decrease in economywide unemployment. With imperfect intersectoral labor mobility, unemployment in the offshoring sector can rise, with an unambiguous unemployment reduction in the non-offshoring sector. Imperfect labor mobility can result in a mixed equilibrium in which only some firms in the industry offshore, with unemployment in this sector rising.

JEL Classification: F11, F16, J64

Keywords: trade, offshoring, search, unemployment

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# 1 Introduction

"Offshoring" is the sourcing of inputs (goods and services) from foreign countries. When production of these inputs moves to foreign countries, the fear at home is that jobs will be lost and unemployment will rise. In the recent past, this has become an important political issue. The remarks by Greg Mankiw, when he was Head of the President's Council of Economic Advisers, that "outsourcing is just a new way of doing international trade" and is "a good thing" came under sharp attack from prominent politicians from both sides of the aisle. Recent estimates by Forrester Research of job losses due to offshoring equaling a total of 3.3 million white collar jobs by 2015 and the prediction by Deloitte Research of the outsourcing of 2 million financial sector jobs by the year 2009 have drawn a lot of attention from politicians and journalists (Drezner, 2004), even though these job losses are only a small fraction of the total number unemployed, especially when we take into account the fact that these losses will be spread over many years.[1] Furthermore, statements by IT executives have added fuel to this fire. One such statement was made by an IBM executive who said "[Globalization] means shifting a lot of jobs, opening a lot of locations in places we had never dreamt of before, going where there is low-cost labor, low-cost competition, shifting jobs offshore", while another statement was made by then Hewlett-Packard CEO Carly Fiorina in her testimony before Congress that "there is no job that is America's God-given right anymore" (Drezner, 2004). The alarming estimates by Bardhan and Kroll (2003) and McKinsey (2005) that 11 percent of our jobs are potentially at risk of being offshored have provided anti-offshoring politicians with more ammunition for their position on this issue.

While the relation between offshoring and unemployment has been an important issue for politicians, the media and the public, there has hardly been any careful theoretical analysis of this relationship by economists. In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two-sector, general-equilibrium model in which unemployment is caused by search frictions a la Pissarides (2000).[2] There is a single factor of production, labor. Firms in one sector, called sector $Z$, use labor to produce two inputs which are then assembled into output. The production of one of these inputs (production input) can be offshored, but the other input (headquarter services) must be produced using domestic labor only. There is another sector, $X$, that uses only domestic labor to produce its output. Goods $Z$ and $X$ are combined to produce the consumption good $C$.

An important result of this paper is that in the presence of perfect intersectoral labor mobility, offshoring leads to wage increases and unemployment reductions in both sectors. The very basic intuition is that there will be gains from international trade which in this case takes the form of offshoring. In a truly single-factor model, this would mean that this factor of production gains from trade, and that explains why, when labor is

[1] The average number of gross job losses per week in the US is about 500,000 (Blinder, 2006). Also see Bhagwati, Panagariya and Srinivasan (2004) on the plausibility and magnitudes of available estimates of the unemployment effects of offshoring.

[2] For a comprehensive survey of the search-theoretic literature on unemployment, see Rogerson, Shimer and Wright (2005).

intersectorally perfectly mobile, real wage increases and unemployment declines. When there are impediments to intersectoral labor mobility, it is possible for unemployment to increase in the $Z$ sector (offshoring sector), however, unemployment in the $X$ sector must decrease. The very basic intuition is that with impediments to labor mobility, we are effectively moving away from a one-sector model. Thus, even with overall gains from trade, we can have winners and losers. In the extreme case of labor being totally immobile across sectors, we truly have a two-factor model, and both factors need not necessarily be winners from offshoring (trade). Since offshoring is similar to a technological improvement in the $Z$ sector, the relative supply of $Z$ increases and its relative price falls as a result (the relative price of $X$ rises). Given that $X$-sector labor has to win from trade due to the positive relative price effect in its favor, the only possible loser, if at all there is one, is $Z$-sector labor.

Moving from the very basic to more detailed intuition, offshoring reduces the cost of production and hence the relative price of good $Z$, since one of the inputs is offshored and is cheaper. The resulting increase in the relative price of the non-offshoring sector $X$ leads to greater job creation and hence reduced unemployment there. The impact of offshoring on $Z$-sector unemployment depends on the relative strengths of two mutually opposing forces, namely the decrease in the relative price of $Z$, and the increase in the productivity of workers engaged in headquarter activities there (with each such worker now working with more production input, since it is cheaper). In the presence of perfect labor mobility, the no arbitrage condition ensures that the second effect dominates and that increases job creation and wages in sector $Z$. Even though offshoring of the production input destroys the jobs of workers engaged in the production of this input in the $Z$ sector, additional $Z$-sector headquarter jobs and $X$-sector jobs, in excess of the production jobs offshored, are created.

In the imperfect labor mobility case, it is possible for the negative relative price effect to dominate the positive productivity effect in the $Z$ sector. The relative price effect may be weaker or stronger in the imperfect mobility case (compared to perfect mobility) depending on whether labor is required to move out of the $Z$ sector or into it. (As explained later in the paper, the direction of movement of labor upon offshoring depends on relative strengths of substitution elasticities in $C$ and $Z$ production.) If labor ends up moving from sector $X$ to sector $Z$ upon offshoring, then the relative price effect is weaker in the case of imperfect mobility compared to the perfect mobility case, and hence offshoring leads to a reduction in the unemployment in the $Z$ sector. In the more plausible case of labor movement from sector $Z$ to sector $X$, the negative relative price effect is stronger with imperfect mobility, and can dominate the positive productivity effect in the $Z$ sector. In this case, we show the possibility of an incomplete offshoring equilibrium (mixed equilibrium) where some firms offshore and others do not. That is, firms are indifferent between offshoring and not offshoring because the domestic cost of producing the offshorable input gets equalized to the cost of the offshored input. This, in sector $Z$, brings the domestic wage down and the unemployment up relative to autarky. Incomplete offshoring makes domestic labor in the $Z$ sector and the offshored input substitutes at the margin. This channel of competitive pressure on the domestic price of labor in the $Z$ sector goes away when there is complete offshoring. In the case of

complete offshoring equilibrium with labor moving from sector $Z$ to sector $X$, the impact of offshoring on the unemployment and wage in the $Z$ sector becomes ambiguous.

The impact of offshoring on aggregate unemployment depends on autarky sectoral unemployment rates, changes in sectoral unemployment rates, and direction of labor movement consequent upon offshoring. In the case of perfect labor mobility, since the sectoral unemployment rates fall upon offshoring, the aggregate unemployment rate falls as well if labor moves from the high unemployment to the low unemployment sector. Since labor moves from the offshoring sector $Z$ to the non-offshoring sector $X$ for most of the parameter space, aggregate unemployment for those parameter values will fall if the offshoring sector has the higher search cost and consequently higher unemployment rate in autarky. In the imperfect mobility case, since the impact of offshoring on sectoral unemployment rates itself is ambiguous, the impact on aggregate unemployment is ambiguous as well.

Our theoretical results are consistent with the empirical results of Amiti and Wei (2005a, b) for the US and the UK. They find no support for the "anxiety" of "massive job losses" associated with offshore outsourcing from developed to developing countries.[3] Using data on 78 sectors in the UK for the period 1992-2001, they find no evidence in support of a negative relationship between employment and outsourcing. In fact, in many of their specifications the relationship is positive. In the US case, they find a very small, negative effect of offshoring on employment if the economy is decomposed into 450 narrowly defined sectors which disappears when one looks at more broadly defined 96 sectors. Alongside this result, they also find a positive relationship between offshoring and productivity. These results are consistent with opposing effects on employment (and unemployment) created by offshoring. In this context, Amiti and Wei (2005a) write: "On the one hand, every job lost is a job lost. On the other hand, firms that have outsourced may become more efficient and expand employment in other lines of work. If firms relocate their relatively inefficient parts of the production process to another country, where they can be produced more cheaply, they can expand their output in production for which they have comparative advantage. These productivity benefits can translate into lower prices generating further demand and hence create more jobs. This job creation effect could in principle offset job losses due to outsourcing." This intuition is consistent with the channels in our model and the reason driving the possibility of a reduction in sectoral unemployment as a result of offshoring.

A discussion of the related theoretical literature is useful here, as it puts in perspective the need for our analysis. While the relationship between offshoring and unemployment has not been analytically studied in detail before by economists, there is now a vast literature on offshoring and outsourcing.[4] All the models in that literature, following the tradition in standard trade theory, assume full employment. In spite of this assumption

[3] The offshoring variable they use, which they call offshoring intensity, is defined as the share of imported inputs (material or service) as a proportion of total nonenergy inputs used by the industry.

[4] See Helpman (2006) for a review of this literature.

in the existing literature, it is important to note that our results are similar in spirit to those in an important recent contribution by Grossman and Rossi-Hansberg (2008) where they model offshoring as "trading in tasks" and show that even factors of production whose tasks are offshored can benefit from offshoring due to its productivity enhancing effect. Our paper is also closely related to the fragmentation literature which analyzes the economic effects of breaking down the production process into different components, some of which can be moved abroad.[5] In this literature, the possibility of fragmentation leading to the equivalent of technological improvement in an industry has been shown.[6]

Also closely related to our work is a recent paper by Davidson, Matusz and Shevchenko (forthcoming) that uses a model of job search to study the impact of offshoring of high-tech jobs on low and high-skilled workers' wages, and on overall welfare. While their emphasis is on the effect of offshoring on relative wages, they also briefly discuss and derive the impact on unemployment in the short-run (when the number of firms is held fixed). Since job prospects for domestic high-skilled workers do not look as promising upon offshoring some of their jobs, they are willing to accept low-skill jobs and in turn increase the competition for such jobs among workers. Therefore, in the short run, labor-market tightness goes down and unemployment goes up. In the long-run, however, there is a confounding factor, namely the entry of new firms arising out of an increase in profitability. Our paper differs in many respects as follows: Firstly, the main focus of our paper is on the impact on unemployment, both at the sectoral level and aggregate levels. Secondly, in our paper, the role of the extent of intersectoral labor mobility and its interaction with offshoring is analyzed. Thirdly, we look at the general-equilibrium effects on the rest of the economy where offshoring does not take place. Finally, we also look at the role of substitution elasticities in production and consumption in determining the impact of offshoring on unemployment.

Another paper looking at the impact of offshoring on the labor market is Karabay and McLaren (2006) who study the effects of free trade and offshore outsourcing on wage volatility and worker welfare in a model where risk sharing takes place through employment relationships. Bhagwati, Panagariya and Srinivasan (2004) also analyze in detail the welfare and wage effects of offshoring. Neither Karabay and McLaren, nor Bhagwati, Panagariya and Srinivasan incorporate unemployment in their analysis.

It is also important to note that there does exist a literature on the relationship between trade and search induced unemployment (e.g. Davidson and Matusz (2004), Moore and Ranjan (2005), Helpman and Itskhoki (2007)). The main focus of this literature, as discussed in Davidson and Matusz, has been the role of efficiency in job search, the rate of job destruction and the rate of job turnover in the determination of comparative advantage.[7] Using an imperfectly competitive set up, Helpman and Itskhoki look at how gains from trade and

[5] See for instance Arndt (1997), Jones and Kierzkowski (1990 and 2001) and Deardorff (2001a and b).

[6] See for instance Jones and Kierzkowski (2001).

[7] See also the influential and well cited paper by Davidson, Martin and Matusz (1999) for a careful analysis of these relationships under very general conditions.

comparative advantage depend on labor market rigidities as captured by search and firing costs and unemployment benefits, and how labor-market policies in a country affect its trading partner. Moore and Ranjan, whose focus is quite different from the rest of the literature on trade and search unemployment, show that the impact of skill-biased technological change on unemployment can be quite different from that of globalization. None of these models deals with offshoring.

The rest of the paper is organized as follows. In section 2 we set up the basic model and derive the autarky equilibrium. In section 3 we describe the offshoring equilibria with perfect and imperfect mobility of labor. Section 4 studies the implications of offshoring for wages and unemployment. Section 5 discusses some possible extensions and robustness issues. Section 6 concludes.

# 2 The Model

## 2.1 Preferences

All agents share the identical lifetime utility function from consumption given by

$$\int_t^\infty \exp^{-r(s-t)} C(s)ds, \tag{1}$$

where $C$ is consumption, $r$ is the discount rate, and $s$ is a time index. Asset markets are complete. The form of the utility function implies that the risk-free interest rate, in terms of consumption, equals $r$.

Each worker has one unit of labor to devote to market activities at every instant of time. The total size of the workforce is $L$. The final consumption good $C$ is produced under CRS using two goods $Z$ and $X$ as inputs (or equivalently can be considered to be a composite basket of these two goods) as follows:

$$C = F(Z, X) \tag{2}$$

We choose the final consumption good $C$ as numeraire. Let $P_z$ and $P_x$ be the prices of $Z$ and $X$, respectively. Since the price of $C = 1$, we get

$$1 = g(P_z, P_x) \tag{3}$$

where the unit cost function for $C$, denoted by $g$, is increasing in both $P_z$ and $P_x$. Therefore, an increase in $P_z$ is associated with a decrease in $P_x$. Also, (2) implies that the relative demand for $Z$ is given by

$$\left(\frac{Z}{X}\right)^d = f(\frac{P_z}{P_x}); f' < 0 \tag{4}$$

In addition to the utility from consumption, workers also have idiosyncratic preferences for working in a particular sector which is captured by a per-period utility (or disutility) to individual-$j$ of $\varepsilon_i^j$ from being part

of the labor force in sector-$i$.[8] This can arise from individual-specific preference for the region in which this industry is located or from the individual specific costs of updating one's human capital that may differ across sectors.[9] This is our way of introducing mobility costs in this model. Define $\varphi^j \equiv \varepsilon_z^j - \varepsilon_x^j$. If $\varphi^j > 0$, then $\varphi^j$ is the cost to worker $j$ of moving from sector $Z$ to sector $X$. Similarly, if $\varphi^j < 0$, it is costly for worker $j$ to move from sector $X$ to sector $Z$. $\varphi^j = 0$, $\forall j$, will capture perfect mobility.

Let us assume that $\varepsilon_z$ and $\varepsilon_x$ are independent of each other and each follows the same extreme value distribution as in Artuc, Chaudhuri and McLaren (2008), which is represented by the following special case of the Gumbel cumulative distribution function:

$$F(\varepsilon_i, i = x, z) = \exp\left(-\exp\left(-\frac{\varepsilon_i}{\alpha} - \gamma\right)\right), \varepsilon_i \in (-\infty, \infty)$$

where $\gamma = 0.5772$ is Euler's constant and $\alpha$ is the scale parameter. The mean of $\varepsilon_i$ is zero and variance is $\pi^2\alpha^2/6$ (where the constant, $\pi \approx 3.14$). In this case, $\varphi = \varepsilon_z - \varepsilon_x$ follows a symmetric distribution with mean zero and a variance equal to $\pi^2\alpha^2/3$, and this distribution, denoted by $G(\varphi)$, is given by

$$G(\varphi) = \frac{\exp(\varphi/\alpha)}{1 + \exp(\varphi/\alpha)}, \varphi \in (-\infty, \infty) \tag{5}$$

As $\alpha$ decreases, the distribution of $\varphi$ becomes more concentrated around the mean of zero. In the limit, when $\alpha \to 0$, the distribution collapses at $\varphi = 0$, which captures our perfect labor mobility case.

## 2.2 Goods and labor markets

Production of good $X$ is undertaken by perfectly competitive firms. To produce one unit of $X$ a firm needs to hire one unit of labor.

$Z$ is also produced by competitive firms, but using a slightly more sophisticated technology involving two separate stages which are then combined. The production function for $Z$ is given as follows.

$$Z = (\tau m_h^\rho + (1 - \tau) m_p^\rho)^{\frac{1}{\rho}} \tag{6}$$

where $m_h$ is the labor input into certain core activities (say headquarter services) which have to remain within the home country and $m_p$ is the labor input for production activities which can potentially be offshored. Parameter $\tau$ captures the headquarter intensity and $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution between headquarter services and production services.

If we denote the total amount of labor employed by a firm by $N$, then we have

$$N = m_h + m_p \tag{7}$$

[8] In the case of an extra utility, $\varepsilon_i^j > 0$, while in the case of an extra disutility, $\varepsilon_i^j < 0$.

[9] As a simplifying assumption, one can assume full obsolescence or depreciation of one's human capital or skills each period. In order to work or search each period in a particular sector, an individual has to incur costs each period to acquire the updated sector-specific human capital. These costs can be assumed to be individual- and sector-specific.

To produce either $X$ or $Z$, a firm needs to open job vacancies and hire workers. The cost of vacancy in terms of the numeraire good is $c_i$ in sector $i = x, z$.[10] Let $L_i$ be the total number of workers who look for a job in sector $i$. Define $\theta_i = \frac{v_i}{u_i}$ as the measure of market tightness in sector $i$, where $v_i L_i$ is the total number of vacancies in sector $i$ and $u_i L_i$ is the number of unemployed workers searching for jobs in sector $i$. The probability of a vacancy filled is $q(\theta_i) = \frac{m(v_i,u_i)}{v_i}$ where $m(v_i, u_i)$ is a constant returns to scale matching function[11]. Since $m(v_i, u_i)$ is constant returns to scale, $q'(\theta_i) < 0$. The probability of an unemployed worker finding a job is $\frac{m(v_i,u_i)}{u_i} = \theta_i q(\theta_i)$ which is increasing in $\theta_i$. Any job in either sector can be hit with an idiosyncratic shock with probability $\delta$ and be destroyed.

## 2.3 Determination of Unemployment

Denoting the rate of unemployment in sector-$i$ by $u_i$, in steady-state the flow into unemployment must equal the flow out of unemployment:

$$\delta(1 - u_i) = \theta_i q(\theta_i) u_i;\ i = x, z$$

The above implies

$$u_i = \frac{\delta}{\delta + \theta_i q(\theta_i)};\ i = x, z \tag{8}$$

The above is the standard Beveridge curve in Pissarides type search models where the rate of unemployment is positively related to the probability of job destruction, $\delta$, and negatively related to the degree of market tightness $\theta_i$.

## 2.4 Firm's optimization problem

We solve the firm's problem in two stages. In the first stage, employment and the number of vacancies are chosen, correctly anticipating the wages denoted by $w_i$. Then given the employment levels chosen in the first stage, the wage rate for each worker is determined by a process of bilateral Nash bargaining with the firm separately.[12]

[10] The robustness of our results to alternatively defining and fixing vacancy costs in terms of good $Z$ or in terms of labor is discussed in the penultimate section of this paper.

[11] Our framework, that (as seen later) nests different degrees of intersectoral labor mobility, requires search by a potential worker to be directed towards one sector at a time. The number of workers that want to be part of the labor force of each sector (will work or search in that sector) will be determined in equilibrium. However, restricting to perfect intersectoral labor mobility (no mobility costs), the assumption regarding search can be altered to general search by each worker across the two sectors simultaneously. Since wages and market tightness in the case of identical search parameters are equalized across the two sectors even under directed search, it is easy to see in such a case that, with general search, our results regarding the impact of offshoring on aggregate unemployment and wages will be unchanged. If vacancy costs are different across sectors, then with general search (unified labor market), workers in the two sectors will get different wages and the bargaining problem will be somewhat more complicated.

[12] Allowing for intra-firm wage bargaining, along the lines of Stole and Zweibel (1996), with the possibility of employment choice in stage 1 affecting wages in stage 2, results in a solution for wage and employment that is equivalent to the one where the firm in

Denote the number of vacancies posted by a firm in the $Z$ sector by $V$. Assuming that each firm is large enough to employ and hire enough workers to resolve the uncertainty of job inflows and outflows, the dynamics of employment for a firm is

$$\dot{N(t)} = q(\theta_z(t))V(t) - \delta N(t) \tag{9}$$

Therefore, the profit maximization problem for an individual firm can be written as

$$\underset{V(s),m_h(s),m_p(s)}{Max} \int_t^{\infty} e^{-r(s-t)} \left\{P_z(s)Z(s) - w_z(s)N(s) - c_z V(s)\right\} ds \tag{10}$$

The firm maximizes (10) subject to (6), (7), and (9). We provide details of the firm's maximization exercise in the appendix. Since we are going to study only the steady state in this paper, we suppress the time index hereafter. From the first-order conditions of the firm's maximization problem, the optimal mix of headquarter and production labor is given by

$$\frac{m_h}{m_p} = \left(\frac{\tau}{1-\tau}\right)^{\sigma} \tag{11}$$

which in turn makes the output effectively linear in the total employment of the firm as follows:

$$Z = \tau' N; \text{ where } \tau' \equiv \left[\tau^{\sigma} + (1-\tau)^{\sigma}\right]^{\frac{1}{\sigma-1}} \tag{12}$$

The key equation from the firm's optimal choice of vacancy, derived in the appendix, is given by

$$\frac{\tau' P_z - w_z}{(r+\delta)} = \frac{c_z}{q(\theta_z)} \tag{13}$$

The expression on the left-hand side is the marginal benefit from creating a job which equals the present value of the stream of the value of marginal product net of wage of an extra worker after factoring in the probability of job separation each period. The expression on the right-hand side is the cost of creating a job which equals the cost of posting a vacancy, $c_z$, multiplied by the average duration of a vacancy, $\frac{1}{q(\theta_z)}$. The left hand side of (13) is also the asset value of an extra job for a firm which will be useful in the wage determination below. An alternative way to write (13) is

$$\tau' P_z = w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \tag{14}$$

That is, the value of marginal product of a worker is equal to the marginal cost of hiring a worker (wage plus the annuitized value of recruitment cost). This is the modified pricing equation in the presence of search frictions where in addition to the standard wage cost, expected search cost is added to compute the marginal cost of hiring a worker. This equation is also known as the job creation condition in the literature.

Since the $X$ sector uses one unit of labor to produce one unit of output, the marginal revenue product of labor in the $X$ sector simply equals $P_x$, and therefore, the profit maximization by firms in the $X$ sector yields the following analogue of (14)

$$P_x = w_x + \frac{(r+\delta)c_x}{q(\theta_x)} \tag{15}$$

stage 1 takes wage as given (at the perfectly foreseen level that will obtain in stage 2). This is due to the constancy of marginal product of labor that obtains in our set up. See Cahuc and Wasmer (2001) for a formal proof.

## 2.5 Wage Determination

Wage is determined for each worker through a process of Nash bargaining with his/her employer. Workers bargain individually and simultaneously with the firm. Rotemberg (2006) justifies this assumption by viewing it as a situation where each worker bargains with a separate representative of the firm. Thus each worker and the representative that he bargains with assume at the time of bargaining that the firm will reach a set of agreements with the other workers that leads these to remain employed.

Denoting the unemployment benefit in terms of the final good by $b$, it is shown in the appendix that the expression for wage is the same as in a standard Pissarides model and is given by

$$w_i = b + \frac{\beta c_i}{1-\beta}[\theta_i + \frac{r+\delta}{q(\theta_i)}];\ i = x, z \tag{16}$$

where $\beta$ represents the bargaining power (weight) of the worker relative to the employer (See appendix). The above wage equation along with (8) and (14) derived earlier are the three key equations determining $w_z$, $\theta_z$, and $u_z$ for a given $P_z$. For the $X$ sector, the three key equations are (8), (15), and (16).

For each of the two sectors, for a given price we can determine the wage, $w_i$ and the market tightness, $\theta_i$ as follows. Equation (16) represents the wage curve, $WC$ which is clearly upward sloping in the $(w, \theta)$ space in Figure 1. The greater is the labor market tightness, the higher is the wage that emerges out of the bargaining process (as the greater is going to be the value of each occupied job). Note that the position of this curve is independent of the price, $P_i$. The job creation curve, $JC$, depicting (14) for sector $Z$ and (15) for sector $X$, is downward sloping in the $(w, \theta)$ space. The recruitment cost, $\frac{c_i}{q(\theta_i)}$, is increasing in market tightness, $\theta_i$. The tighter the market the longer it takes to fill up a vacancy. Therefore, for a given value of the marginal product of labor, there is a tradeoff between the wage and the market tightness. The intersection of $WC$ and $JC$ gives the partial equilibrium levels of $w_i$ and $\theta_i$ for a given $P_i$. As the price, $P_i$, increases, $JC$ shifts up, leading to an increase in $w_i$ and $\theta_i$, and thus from the Beveridge curve a reduction in unemployment.

## 2.6 Sectoral choice of workers

Since unemployed workers can search in either sector, they search in the sector where their expected utility is higher. As shown in equation (38) in the appendix, the asset value of unemployed worker-$j$ searching in sector-$i$ is given by $rU_i^j = \varepsilon_i^j + b + \frac{\beta}{1-\beta} c_i \theta_i$. Recall that $\varepsilon_i^j$ is the per-period utility for worker-$j$ from being affiliated with sector-$i$, while the market tightness variable $\theta_i$ positively affects the wage and job finding rate in sector-$i$.

Since $\varphi^j \equiv \varepsilon_z^j - \varepsilon_x^j$, the sectoral choice of workers is given as follows.

$$\begin{aligned} \text{If } \varphi^j &\geq \frac{\beta}{1-\beta}(c_x\theta_x - c_z\theta_z) \text{ then search in sector-}Z \\ \text{If } \varphi^j &< \frac{\beta}{1-\beta}(c_x\theta_x - c_z\theta_z) \text{ then search in sector-}X \end{aligned}$$

Given the above relationship, the equilibrium sectoral choice is determined by a cutoff value of $\varphi$ denoted by $\widehat{\varphi}$ where

$$\widehat{\varphi}(\theta_x, \theta_z) = \frac{\beta}{1-\beta}(c_x\theta_x - c_z\theta_z) \tag{17}$$

such that a fraction $1 - G(\widehat{\varphi})$ of workers are affiliated with sector $Z$, while the remaining fraction $G(\widehat{\varphi})$ are affiliated with sector $X$. That is,

$$L_z = (1 - G(\widehat{\varphi}))L; L_x = G(\widehat{\varphi})L \tag{18}$$

In the case of perfect mobility ($\varphi^j = 0$ for all $j$), all workers must be indifferent between the two sectors, which would imply the following no arbitrage condition

$$c_x\theta_x = c_z\theta_z \tag{19}$$

Having specified the building blocks of the model, we next derive the relative supply curve in order to solve for autarky equilibrium.

## 2.7 The relative supply curve (under autarky)

In order to derive the relative supply corresponding to each relative price $p = \frac{P_z}{P_x}$ obtain the values of $P_z$ and $P_x$ from (3) which is the zero profit condition ($ZPC$) for the numeraire good, $C$. Next, for these values of $P_i$ determine $w_i$ and $\theta_i$ from the intersection of $WC$ and $JC$ for sector $i$ as shown in Figure 1. Having determined $\theta_i$, find the corresponding $\widehat{\varphi}$ from (17). Denote $\widehat{\varphi}$ as a function of $p$ in the case of autarky by $\widehat{\varphi}^A(p)$. Using (12) and (18), the relative supply of $Z$ can be written as

$$\left(\frac{Z}{X}\right)^s = \frac{\tau'(1-u_z)L_z}{(1-u_x)L_x} = \frac{\tau'(1-u_z)}{(1-u_x)\exp(\widehat{\varphi}^A(p)/\alpha)} \tag{20}$$

where $u_i$ which is a decreasing function of $\theta_i$ is given by (8). To see what happens to the relative supply when $p$ increases, note from (3) that our choice of numeraire implies an increase in $p$ leads to an increase in $P_z$ and a decrease in $P_x$. This also implies an increase in $\theta_z$ and a decrease in $\theta_x$, which in turn implies a decrease in $u_z$ and an increase in $u_x$. As well, note from (17) that there is a decrease in $\widehat{\varphi}$. Therefore, $\frac{d\widehat{\varphi}^A(p)}{dp} < 0$, which is shown in figure 2a. What it says is that more people search for jobs in sector $Z$ as $P_z$ goes up. Therefore, the relative supply of $Z$ is increasing in its relative price $p$. We depict this relative supply curve in Figure 2b.

Recall that $\widehat{\varphi}^A$ depicted in Figure 2a is solely a function of $p$ and independent of $\alpha$. Therefore, relative supply is increasing in $\alpha$ when $\widehat{\varphi}^A > 0$ and decreasing in $\alpha$ when $\widehat{\varphi}^A < 0$. At $\widehat{\varphi}^A = 0$, it is clear from (20) that relative supply becomes independent of $\alpha$. Denote the solution to $\widehat{\varphi}^A(p) = 0$ by $p^A$. It is easy to see that the relative supply curves given by (20) for different values of $\alpha$ all pass through the same point at $p = p^A$. This is shown in Figure 2b (in which and in all subsequent figures, we normalize the unemployment benefit, $b$ to zero for simplicity). For $p < p^A$, we have $\widehat{\varphi}^A > 0$, and hence the relative supply curve for higher $\alpha$ lies to the right

of the one for lower $\alpha$, while for $p > p^A$, the relative supply curve for higher $\alpha$ lies to the left of the one for lower $\alpha$. Thus, as $\alpha$ goes down, the relative supply curve rotates clockwise around $p = p^A$ (Figure 2b). Clearly around that point, labor mobility goes up with a decrease in $\alpha$, i.e., at that point any given price shock leads to a bigger movement in labor from one sector to another, the smaller is $\alpha$. In the limit, when $\alpha \to 0$, $\varphi^j \to 0$ $\forall j$. In this case the relative supply is zero for any $p < p^A$ because no one wants to work in the $Z$ sector, and it becomes horizontal at $p = p^A$ since all workers are indifferent between working in the two sectors. This is the case of perfect labor mobility. The relative supply curves with 2 different degrees of mobility $\alpha_1$ and $\alpha_2$ ($\alpha_2 > \alpha_1 > 0$) are shown in Figure 2b and denoted by $RS(\alpha_1, A)$ and $RS(\alpha_2, A)$, respectively. The perfect mobility horizontal relative supply curve is denoted by and $RS^p(A)$.

### 2.8 Equilibrium under autarky

Having derived the relative supply curve, the autarky equilibrium can be determined by bringing in the relative demand curve given in (4) which is downward sloping. The intersection of the relative demand curve with the relative supply curve determines the autarky equilibrium. Note that in the case of perfect labor mobility, since the relative supply curve is horizontal at $p = p^A$ where $p^A$ solves $\widehat{\varphi}^A(p) = 0$, the autarky equilibrium price is necessarily $p^A$. At $p^A$ the no arbitrage condition (19) is satisfied, and therefore, all workers are indifferent between being in the two sectors (since $\varphi^j = 0$ for all workers).

Autarky equilibrium price with imperfect mobility can be higher or lower than $p^A$ depending on the position of the relative demand curve. To facilitate comparison of the autarky equilibrium with the offshoring equilibrium in the presence of various degrees of labor mobility, we will assume that the technology that yields $C$ in terms of $Z$ and $X$ is such that the relative demand curve, $RD$, passes through the common point of intersection of the autarky relative supply curves with varying degree of intersectoral labor mobility (Figure 2b). That is, the relative demand is such that the autarky equilibrium price for various degrees of labor mobility is $p^A$. Denote the corresponding values of other endogenous variables of interest by $P_z^A, P_x^A, w_z^A, w_x^A, \theta_z^A, \theta_x^A, u_z^A, u_x^A$.

## 3 Offshoring

Now, suppose firms in the $Z$ sector have the option of procuring input $m_p$ from abroad (which we call offshoring in this paper) instead of producing them domestically.[13] The per unit cost of offshored input is $w_s$ in terms of the numeraire good $C$, and this country takes this per unit cost as given.[14] This includes transportation cost,

[13] The assumption here is that one unit of home (domestic) labor can produce one unit of the production input. Therefore, we use $m_p$ to denote both the number of units of the imported input in the offshoring case as well as the number of units of production labor in the autarky case.

[14] The assumption that $w_s$ is fixed is effectively a small country assumption. However, as argued in an earlier version of this paper, there is no loss of generality resulting from it. Large amounts of labor used in the production of a numeraire consumption

tariffs, foreign wage costs and possible search costs, all of which, for analytical tractability, we assume to be proportional to the amount of the input offshored. If and when offshoring takes place, the final good $C$ will be exported to pay for the imports of $m_p$.

We use the following notational simplification in the rest of the paper.

Definition 1: $\widetilde{w_z} \equiv w_z + \frac{(r+\delta)c_z}{q(\theta_z)}; \omega \equiv \frac{\widetilde{w_z}}{w_s}$

In the above definition $\widetilde{w_z}$ is the total cost of hiring a labor in the $Z$ sector which includes wage and the recruitment cost. $\omega$ is the cost of of a unit of domestic labor in the $Z$ sector relative to the cost of an offshored production input. In an offshoring equilibrium it must be the case that $\omega \geq 1$. Starting from an autarky equilibrium with relative price $p^A$ and associated cost of employing a worker in sector $Z$ given by $\widetilde{w_z}^A$, it must be the case that $w_s < \widetilde{w_z}^A$, so that offshoring the production input is cheaper than producing it domestically. We assume this to be the case in the analysis of offshoring and state it explicitly in the assumption below.

**Assumption 1**: *Cost of offshoring input, $w_s$, is less than the autarky equilibrium labor cost in sector $Z$, $\widetilde{w_z}^A$.*

### 3.1 Offshoring firm's problem

For a firm offshoring its production input, the production function specified in (6) can be written as $Z = (\tau N^\rho + (1-\tau)m_p^\rho)^{\frac{1}{\rho}}$, where $N$ is the domestic labor used for headquarter services. An offshoring firm's first stage problem is given by

$$\underset{V(s),N(s),m_p(s)}{Max} \int_t^\infty e^{-r(s-t)}\{P_z(s)Z(s) - w_z(s)N(s) - w_s m_p(s) - c_z V(s)\}ds$$

Again, the firm anticipates the wage correctly while choosing its employment and the quantity of offshored production input. [15]

Using the notation in definition 1, the ratio in which an offshoring firm uses headquarter and production inputs in steady state is given by

$$\frac{N}{m_p} = \left(\frac{\tau}{(1-\tau)\omega}\right)^\sigma \tag{21}$$

good in the South (country to which input production is offshored), which forms a large share in the household budget, fixes wage and the unemployment rate also in input production there. One can here easily work out the implications of offshoring for the South.

[15] If firms can freely adjust the amount of offshored input, $m_p$ even after the wage-bargaining stage, then $w_s$ must equal the value of marginal product of offshored input. Given the CRS production function this also pins down the value of marginal product of headquarter workers, and hence the solution for wage and employment is equivalent to the one where the firm takes the wage as given in stage 1 as shown by Cahuc and Wasmer (2001). This is important to note since we are effectively assuming that the quantity of the offshored input is freely adjustable by firms.

From the first order conditions of an offshoring firm's optimization problem we get

$$P_z = \left(\tau^\sigma \left(\widetilde{w_z}\right)^{1-\sigma} + (1-\tau)^\sigma w_s^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{22}$$

The expression on the right hands side above is the marginal cost of producing a unit of $Z$.

Since in steady-state the value of a headquarter job in the $Z$ sector must still equal the recruitment cost, $\frac{c_z}{q(\theta_z)}$, the Nash bargained wage is still given by

$$w_z = b + \frac{\beta c_z}{1-\beta}[\theta_z + \frac{r+\delta}{q(\theta_z)}] \tag{23}$$

Also, as a result, the asset value of unemployed worker-$j$ searching in sector-$i$ is still given by $rU_i^j = \varepsilon_i^j + b + \frac{\beta}{1-\beta} c_i \theta_i$, and therefore, the equilibrium sectoral choice is determined by a cutoff value of $\varphi$ denoted by $\widehat{\varphi}$ defined in (17) earlier, which we recall is

$$\widehat{\varphi}(\theta_x, \theta_z) = \frac{\beta}{1-\beta}(c_x \theta_x - c_z \theta_z) \tag{24}$$

## 3.2 Productivity Effect of Offshoring

Before deriving the offshoring relative supply curve which allows us to derive offshoring equilibrium, we identify the productivity effect of offshoring as follows. Rewrite (22) as

$$\left(\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}\right)^{\frac{1}{\sigma-1}} P_z = \widetilde{w_z} \tag{25}$$

The left hand side is the value of marginal product of domestic labor in headquarter activity, which must equal the cost of hiring domestic labor inclusive of the recruitment cost. This is the job creation condition for headquarter jobs for offshoring firms. At $\omega = 1$ the expression above reduces to the job creation condition (14) derived in autarky. Note that at the autarky equilibrium price $P_z^A$ and the autarky equilibrium labor cost, $\widetilde{w_z}^A$, in the $Z$ sector the l.h.s of (25) exceeds the r.h.s because $\widetilde{w_z}^A > w_s$ by assumption. That is, the value of marginal product of a headquarter worker exceeds its hiring cost (wage plus the annuitized value of recruitment cost), which would lead to more job creation in headquarter activity by offshoring firms. This increase in the value of marginal product of headquarter labor in the $Z$ sector is the productivity effect of offshoring. Now, if $P_z$ was unchanged at the autarky level, as would be the case in a one sector model, then to satisfy the job creation condition (25), $\widetilde{w_z}$ must increase which implies from (23) a higher $w_z$ and $\theta_z$ as well. We can show this diagrammatically in Figure 1. For each $P_z$, the $JC'$ curve representing (25) lies to the right of the $JC$ curve representing (14) because $\widetilde{w_z}$ satisfying (25) is higher than the $\widetilde{w_z}$ satisfying (14). Since the wage bargaining curve remains unchanged, $w_z$ and $\theta_z$ are higher in the offshoring case. A higher $\theta_z$ implies a lower unemployment as well. That is, the productivity effect of offshoring by itself creates greater job creation and lower unemployment. This gives rise to the lemma below.

**Lemma 1:** *Holding product prices constant, offshoring implies an increase in $\theta_z$ and a decrease in $u_z$.*

Below we will see that in our two sector model, the productivity effect can be offset by the relative price effect, the strength of which depends on the extent of intersectoral mobility of labor and the elasticities of substitution in production and consumption.

### 3.3 Offshoring relative-supply curve

To derive the offshoring equilibrium, we first derive the offshoring relative supply at each relative product price $p$ as follows. For any $p$ if the labor cost in the $Z$ sector in the absence of offshoring, $\widetilde{w_z}$, is below the cost of offshored input $w_s$ then there is no offshoring. Since in the absence of offshoring $\widetilde{w_z}$ is monotonically increasing in $p$, for any $w_s$, there exists a $\underline{p}(w_s)$ such that for $p < \underline{p}(w_s)$, $\widetilde{w_z} < w_s$, and hence in this case the relative supply with the possibility of offshoring coincides with the autarky relative supply curve. At $p = \underline{p}(w_s)$, $\widetilde{w_z}$ exactly equals $w_s$. Denote the price of good $Z$ corresponding to $\underline{p}(w_s)$ by $\underline{P_z}(w_s)$From (14) it is clear that if $P_z = \underline{P_z}(w_s) = \frac{w_s}{\tau^r}$, then $\widetilde{w_z} = w_s$, that is, at $p = \underline{p}(w_s)$, firms are indifferent between offshoring the production input and sourcing it domestically. Therefore, at $p = \underline{p}(w_s)$ there is a horizontal segment in the offshore relative supply curve with no firms offshoring determining the left boundary of the horizontal segment and all firms offshoring determining its right boundary.

For $p > \underline{p}(w_s)$, $\widetilde{w_z}$ in the absence of offshoring exceeds $w_s$, therefore, all firms offshore. It can be easily verified that due to the productivity effect discussed earlier, offshoring relative supply curve lies to the right of the autarky relative supply curve in this range. In the appendix we formally prove the following lemma on the shift in relative supply in the case of imperfect mobility of labor.

**Lemma 2**: *There is a step shift in the offshoring relative supply curve compared to autarky. For $p < \underline{p}(w_s)$ the offshoring supply curve corresponds to the autarky supply curve. For $p = \underline{p}(w_s)$, the offshoring supply curve has a horizontal segment and for $p > \underline{p}(w_s)$, the offshoring supply curve lies to the right of the autarky supply curve.*

The offshoring relative supply curve in the case of perfect labor mobility can be obtained as the limiting case of offshoring relative supply curve with imperfect mobility when $\alpha \to 0$ ($\alpha$ is the variance parameter of the distribution of $\varphi$). Denote the cutoff $\widehat{\varphi}$ as a function of $p$ with the possibility of offshoring by $\widehat{\varphi}^o(p)$. The following lemma is proved in the appendix.

**Lemma 3**: *The offshoring relative supply curve in the case of perfect labor mobility is horizontal at $p^o$ where $p^o \in (\underline{p}(w_s), p^A)$ is the solution to $\widehat{\varphi}^o(p) = 0$.*

Note that since $p^o$ is the solution to $\widehat{\varphi}^o(p) = 0$, at $p^o$, $c_x\theta_x = c_z\theta_z$, and hence all workers are indifferent between the two sectors as is required in the case of perfect mobility.

Figure 3 depicts the positions of the offshoring relative supply curves for $\alpha_1$ and $\alpha_2$ such that $\alpha_2 > \alpha_1$, and for the the perfect mobility case. They are denoted by $RS(\alpha_1, O)$, $RS(\alpha_2, O)$, and $RS^p(O)$, respectively. Analogous to autarky, offshoring relative supply curves with various degrees of labor mobility all pass through

the same point at $p = p^o$. It is worth pointing out that while the horizontal relative supply in the case of perfect labor mobility arises due to the indifference of workers between the two sectors, the horizontal segment in the case of imperfect mobility arises due to the indifference of firms in the $Z$ sector between offshoring and domestic sourcing of production input.

Having derived the offshoring relative supply curve, we are ready to discuss the possible offshoring equilibria in the model.

### 3.4 Offshoring equilibrium

Given the offshoring relative supply curve described in lemma 2 above, there are two possible types of offshoring equilibria in the imperfect mobility case.

1) *Complete Offshoring Equilibrium.* If the relative demand curve intersects the offshoring relative supply curve on the right-hand rising part, then we get a complete offshoring equilibrium with all firms offshoring. Figure 3 shows complete offshoring equilibria for two different values of $\alpha$, $\alpha_1$ and $\alpha_2$. The offshoring equilibrium prices are $p^o(\alpha_1)$ and $p^o(\alpha_2)$, respectively.

2) *Mixed Offshoring Equilibrium.* If the relative demand curve intersects the horizontal part of the offshoring relative supply curve, then we get a mixed equilibrium where only some firms in the industry offshore and others remain fully domestic. This equilibrium is shown in Figure 4. In this case the equilibrium price is necessarily equal to $\underline{p}(w_s)$.

From lemma 3 it is clear that there cannot be a mixed equilibrium in the case of perfect labor mobility since $p^o > \underline{p}(w_s)$. Therefore, we get a complete offshoring in this case, which is depicted in Figure 3, where the equilibrium price is $p^o$.

## 4 Impact of offshoring on the domestic labor market

In an offshoring equilibrium- with perfect and imperfect mobility of labor- the relative price of $Z$ is lower than in autarky. This is the 'relative price effect' of offshoring mentioned in the introduction. An increase in the relative price of $Z$ also implies an increase in the price of $Z$, $P_z$, in terms of the final consumption good and a decrease in the price of $X$, $P_x$, in terms of the final consumption good. Below we discuss the implications of offshoring for sectoral labor markets.

### 4.1 Impact on sectoral unemployment and wages in sector $X$

An increase in $P_x$ increases job creation in the $X$ sector because the recruitment cost that needs to be paid is fixed in terms of the numeraire good. In terms of Figure 1, there is a rightward shift in the $JC$ curve in the

$X$ sector, while the $WC$ curve remains unchanged. Therefore, $w_x$ and $\theta_x$ increase relative to autarky while $u_x$ decreases.

## 4.2 Impact on sectoral unemployment and wages in sector $Z$

The impact of offshoring on unemployment in the $Z$ sector depends on two opposing forces. The productivity effect discussed earlier increases job creation in the headquarter activities in the $Z$ sector, and thereby leads to lower unemployment. The relative price effect, by lowering the price of good $Z$ in terms of the numeraire good reduces job creation in the $Z$ sector and hence increases unemployment. The net effect depends on the relative strengths of these opposing forces. In terms of Figure 1, the positive productivity effect shifts the $JC$ curve to the right for a given $P_z$, however, a decline in $P_z$ (the relative price effect) shifts it to the left. The net shift in the $JC$ curve is ambiguous in general, however, we obtain unambiguous results in the following two cases: 1) perfect mobility of labor; and 2) mixed equilibrium with imperfect mobility of labor

If labor is perfectly mobile across sectors, the result that $\theta_x$ increases implies from the no arbitrage condition (19) that $\theta_z$ must increase as well. That is, the positive productivity effect must dominate the negative relative price effect, and hence there must be an increase in the wage and a decrease in unemployment in the $Z$ sector. In terms of Figure 1, the productivity effect takes the $JC$ curve to the right to $JC'$ and the price effect shifts it back in the other direction to $JC$" but not all the way back up to $JC$.

In the case of imperfect labor mobility, when there is a mixed equilibrium, the equilibrium price is $\underline{p}(w_s) < p^A$. The offshoring equilibrium domestic labor cost in the $Z$ sector, $\widetilde{w_z}$, corresponding to $p = \underline{p}(w_s)$, equals $w_s$, which by assumption 1 above is less than $\widetilde{w_z}^A$. Therefore, both $w_z$ and $\theta_z$ decrease relative to autarky, and hence the unemployment rate is higher in the $Z$ sector. Looking at the job creation condition, (25), note that $\omega = 1$ in the case of mixed equilibrium, and hence a reduction in $P_z$ leads to a definite decrease in the value of marginal product of labor in the $Z$ sector and consequently a decline in $Z$-sector wage and an increase in $Z$-sector unemployment. That is, the negative relative price effect more than offsets the positive productivity effect in the case of mixed equilibrium.

Finally, in the case of complete offshoring equilibrium with imperfectly mobile labor, the impact of offshoring on unemployment and wage in the $Z$ sector is ambiguous. The results are summarized below.

**Proposition 1** *A) In the case of perfect labor mobility only a complete offshoring equilibrium is possible, and sectoral wages are unambiguously higher and sectoral unemployment rates unambiguously lower in an offshoring equilibrium compared to the autarky equilibrium.*

*(B) In the case of imperfect labor mobility*

*(i) the unemployment rate in the non-offshoring sector goes down and the wage rate goes up, relative to what we obtain in the autarky equilibrium,*

*(ii) in the offshoring sector, (a) the unemployment rate goes up and the wage rate goes down in a mixed offshoring equilibrium, but (b) the impact is ambiguous in a complete offshoring equilibrium.*

Even though the impact of offshoring on unemployment in the offshoring sector is ambiguous in a complete offshoring equilibrium with imperfect mobility of labor, we can get some additional insights by comparing it with the equilibrium obtained in the perfect mobility case. Denoting the endogenous variables in an offshoring equilibrium with superscript $o$, using a continuity argument, we derive the following corollary for the imperfect mobility case.

**Corollary 1:** *For any $w_s < \widetilde{w_z}^A$, there exists an $\alpha^*(w_s)$ such that for $\alpha < \alpha^*(w_s)$, $w_z^o > w_z^A$ and $\theta_z^o > \theta_z^A$.*

The Corollary above implies that with a sufficient degree of labor mobility, the sectoral unemployment rates decrease in both sectors. More generally, since the productivity effect dominates the relative price effect in the perfect mobility case, the same happens in the imperfect mobility case as long as the negative relative price effect is weaker than in the perfect mobility case, i.e., whenever the equilibrium relative price under offshoring with imperfect labor mobility is higher than $p^o$. On the other hand, if the offshoring equilibrium relative price with imperfect mobility is lower than $p^o$, then the negative relative price effect is stronger than in the case of perfect labor mobility, and hence the impact of offshoring on unemployment in the $Z$ sector is ambiguous. This latter case is depicted in Figure 3. We can see that offshoring leads to a bigger fall in $p$ under imperfect labor mobility than under perfect mobility.

Whether the relative price effect in the imperfect labor mobility case is weaker or stronger than in the perfect mobility case is tied to the issue of direction of intersectoral labor movement as a consequence of offshoring, which in turn depends on the fundamental parameters of the models such as the elasticities of substitution in production and consumption as is discussed in detail below. Intuitively speaking, if the parameters are such that labor is required to move out of the offshoring sector and into the non-offshoring sector in the perfect mobility case, then fewer people will move from the $Z$ sector to the $X$ sector in the imperfect mobility case, leading to a greater decline in the relative price of good $Z$. In this case, the negative relative price effect is stronger with imperfect mobility of labor. In the reverse case the negative relative price effect would be weaker with imperfect mobility of labor.

## 4.3 Determinants of the direction of intersectoral movement of labor

Providing analytical results on the movement of labor consequent upon offshoring is not feasible in the case of imperfect mobility, however, in the case of perfect mobility the no arbitrage condition allows us to derive analytical results which we provide below. Assume a constant elasticity of substitution production function for $C$ where the elasticity of substitution is $\phi$. Recall that the elasticity of substitution between headquarter and production labor in $Z$ production is $\sigma$. We prove the following lemma in the appendix for the perfect mobility case.

**Lemma 4**: *When $c_x = c_z$, except when $\phi > 1$ and $\sigma < 1$, labor moves from the $Z$ sector to the $X$ sector as a result of offshoring. When $\phi > 1$ and $\sigma < 1$, it is possible for labor to move from the $X$ sector to the $Z$ sector as a result of offshoring.*

Intuitively, since production jobs are lost in the $Z$ sector, while there is greater job creation in the $X$ sector, workers are likely to move from $Z$ sector to the $X$ sector. As well, cheaper offshored production input can be substituted for more expensive domestic headquarter labor leading to further movement of workers to the $X$ sector. Countering these effects is the increase in the relative demand for good $Z$ resulting from a decrease in its relative price. The latter effect on the derived demand for labor is normally dominated by the former effects. However, if the elasticity of substitution between $X$ and $Z$ in the production of the consumption good $C$ is very high ($\phi > 1$) and the elasticity of substitution between headquarter and production labor in the production of $Z$ is relatively low ($\sigma < 1$), then workers could move from the $X$ sector to $Z$ sector upon offshoring. A high $\phi$ implies a large increase in the relative quantity of $Z$ demanded for a small decrease in the relative price of $Z$. A low $\sigma$ implies fewer headquarter jobs can be substituted by cheaper production jobs. Therefore, with $\phi > 1$ and $\sigma < 1$ workers may end up moving to the $Z$ sector.[16] While lemma 4 discusses labor movement for all possible values of $\phi$ and $\sigma$, it is reasonable to think that the elasticity of substitution between headquarter and production input is less than 1. In that case we can say that labor moves from $Z$ to $X$ if $\phi \leq 1$ and may move from $X$ to $Z$ if $\phi > 1$.

Even though the analytical result in Lemma 4 obtains for $c_x = c_z$, using a continuity argument we claim that it will hold if $c_x$ and $c_z$ are not too different. Numerical simulations confirm that the result on $L_z$ decreasing upon offshoring is valid even when $c_x \neq c_z$ ($c_x$ and $c_z$ are fairly far apart) except in the case of very high $\phi$ and very low $\sigma$. Also, the same parameters determine the direction of labor movement in the imperfect mobility case.

## 4.4 Impact of offshoring on aggregate unemployment

While we have derived results on the impact of offshoring on sectoral unemployment rates, the economywide unemployment rate is a weighted average of the sectoral unemployment rates with the weights being the share of each sector in the total labor force. Now, even if the sectoral unemployment rates go down, economywide

[16] With perfect intersectoral labor mobility, it is worth noting that if we get rid of all the labor market frictions in this model and the labor market is made perfectly competitive, the labor force allocation across the two sectors will be exactly the same as in the case of $c_x = c_x$ in our labor-market search model (with perfect intersectoral labor mobility). That is, in the absence of frictions in the labor market, offshoring will lead to movement of workers from sector $Z$ to sector $X$ except when $\phi > 1$ *and* $\sigma < 1$. This can be easily verified in the proof of labor allocation in the appendix. Since there will be full employment when search frictions are absent, there will be no change in unemployment as a result of offshoring. The wage increase and the sectoral unemployment reduction that we get upon offshoring in the presence of search frictions will, in the absence of these frictions, be translated into just a wage increase.

unemployment rate may increase if workers move from low unemployment sector to high unemployment sector. Alternatively, even if the unemployment rate in the $Z$ sector increases upon offshoring (as happens in a mixed equilibrium), economywide unemployment rate may go down if workers move to the low unemployment sector upon offshoring. Since the impact of offshoring on the unemployment rate in the $Z$ sector is ambiguous with imperfect labor mobility, the impact on the aggregate unemployment rate is ambiguous as well. In the case of perfect mobility, we can get some clear cut results depending on the sectoral search costs, which are discussed below.

**Case I:** In the special case of $c_x = c_z$, no arbitrage condition (19) implies $\theta_x = \theta_z$ and hence $u_x = u_z$. Since offshoring reduces sectoral unemployment rates, the aggregate unemployment rate must fall as well.

When $c_x \neq c_z$, we have $\theta_x \neq \theta_z$, and therefore, the two sectors have different unemployment rates. Now, the impact of offshoring on economywide unemployment also depends on the direction of labor movement, that is whether labor moves to the high unemployment sector or low unemployment sector, which in turn depend on parameters as described in lemma 4 above. To avoid discussing too many cases, we discuss the results in the more plausible case of $\sigma < 1$.

**Case II:** $c_x < c_z$. In this case, no arbitrage condition (19) implies $\theta_x > \theta_z$, and hence $u_x < u_z$. For $\phi \leq 1$ labor moves from $Z$ sector to $X$ sector, and hence there is an unambiguous decrease in aggregate unemployment. In the case of $\phi > 1$ labor may move from $X$ to $Z$, in which case the impact on aggregate unemployment would be ambiguous.

**Case III:** $c_x > c_z$. For $\phi \leq 1$ labor moves from $Z$ sector to $X$ sector, and hence the impact on aggregate unemployment is ambiguous. If $\phi > 1$, then labor may move from $X$ to $Z$, in which case there would be an unambiguous decrease in aggregate unemployment.

The result on aggregate unemployment is summarized in a proposition below.

**Proposition 2** *(A) In the case of imperfect mobility of labor, the impact of offshoring on aggregate unemployment rate is ambiguous.*

*(B) With perfect mobility, however,*

*(i) there is a decrease in aggregate unemployment if labor moves from the high unemployment sector to the low unemployment sector. (When vacancy costs are higher in sector $Z$ than in $X$, this happens when the elasticity of substitution between $Z$ and $X$ in yielding $C$ is not high relative to the elasticity of substitution between headquarter and production inputs in $Z$ production.)*

*(ii) the impact is ambiguous if labor moves from the low unemployment sector to the high unemployment sector. (When vacancy costs are higher in $Z$, this can happen when the elasticity of substitution in $C$ is relatively high.)*

# 5 Possible Extensions and Discussion

While we have discussed offshoring in a model with perfect or imperfect intersectoral mobility of labor, there was perfect intra-sectoral mobility of labor. Below we discuss how the results would change with alternative descriptions of intra and intersectoral labor mobility.

## 5.1 A model with skilled and unskilled labor

Suppose there is no labor mobility across the two types of jobs in the $Z$ sector but there is mobility of production labor between the two sectors, i.e., headquarter jobs require skilled workers, while production jobs require unskilled or relatively less skilled workers who can also work in the $X$ sector. After offshoring, the production input cost in sector $Z$ equals $w_s$, and all the domestic production labor moves to sector $X$. Holding product prices constant at the autarky level, the value of marginal product of headquarter labor rises due to the productivity effect discussed earlier. Thus, upon offshoring, at autarky product prices, unemployment falls for skilled workers who work in the headquarter activities in the $Z$ sector, while it remains unchanged in sector $X$. More headquarter labor is employed as a result in sector $Z$. In addition, at autarky prices, since the ratio of production input to headquarter labor has gone up, employment of production input (now all offshored) and therefore the output of $Z$ have also gone up. Holding product prices at the autarky level, the $X$-sector labor force actually increases upon offshoring since all the domestic production labor from $Z$ actually flows into $X$. Thus, both the outputs of $X$ and $Z$ go up at autarky product prices and as a result, the impact on relative supply $Z/X$ is ambiguous (depends on parameters, including $w_s$). These parameters will determine how much production labor is released from the $Z$ sector to go to the $X$ sector upon offshoring and how large the increase is in the marginal product of headquarter labor. Thus, the offshoring equilibrium relative price of $Z$ could be higher or lower than in autarky. If the relative price of $Z$ is lower in the offshoring equilibrium, then this negative price effect counteracts the positive productivity effect, rendering the impact of offshoring on the unemployment of headquarter labor ambiguous. An increase in the price of $X$ in this case implies a reduction in the unemployment of production labor all of which is absorbed in the $X$ sector labor force. If the parameters are such that the relative price of $Z$ increases upon offshoring, then headquarter unemployment goes down and production labor unemployment goes up.[17]

The general result for within-sector immobility of labor across job types (i.e, with two types of labor) discussed above in this subsection is that upon offshoring, unemployment cannot rise at the same time for both types of labor, but can fall for both. At least, one type of labor will experience a fall in its unemployment rate. While in the main part of the paper, we have worked out the consequences of imperfect intersectoral

[17] It is important to note that in the case of the positive price effect, a mixed equilibrium is possible, where simultaneously some amount of domestic production labor is used in the $Z$ sector and some amount of offshoring takes place. (The derivation of results in this subsection can be obtained from the authors upon request.)

mobility, in this section we explored the implications of within sector immobility. If we had both intersectoral immobility and within-sector immobility across job types in the same model, it is easy to see in that case that unemployment of production workers in the $Z$ sector would go up as a result of offshoring for the following reason: These workers would have to compete with the cheap input coming from abroad, while they would have no alternative domestic employment opportunities.

### 5.2 Alternative ways of modeling vacancy costs

We next focus on the modeling of vacancy cost in this paper. We have modeled vacancy cost, $c$, in terms of the numeraire good which seemed natural given the two sector structure of the model. One could alternatively model the vacancy cost either in terms of labor or foregone output. In the former case, the vacancy cost would be $c_i w_i$ for sector $i = X, Z$, where $w_i$ is the sectoral wage. In the latter case, it would be $c_i p_i$. We find that, under fairly plausible and reasonable conditions, the qualitative results would be unchanged. The key to obtaining our result on unemployment is that productivity changes should not be fully absorbed by wage changes, which will obtain with alternative specifications of search costs as well.

## 6 Conclusions

In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two-sector general-equilibrium model in which unemployment is caused by search frictions. Our model incorporates imperfections in labor mobility across sectors. Perfect labor mobility is a special case of our framework. For this case, we find that, contrary to general perception, wage increases and sectoral unemployment decreases due to offshoring when labor is intersectorally perfectly mobile. This result can be understood to arise from the dominance of the productivity enhancing (cost reducing) effect of offshoring over its negative relative price effect on the offshoring sector. This result is consistent with the recent empirical results of Amiti and Wei (2005a, b) for the US and UK, where, when sectors are defined broadly enough, they find no evidence of a negative effect of offshoring on sectoral employment.

When parameters are such that they result in substantial impediments to intersectoral labor mobility, the negative relative price effect mentioned above can dominate the positive productivity effect of offshoring, and unemployment can increase (and wage can decrease) in the sector which is subject to offshoring. This happens when the substitution elasticity between the two intermediate goods in the production of the final consumption good is small relative to the substitution elasticity between offshorable and non-offshorable inputs within the offshoring sector. In the other (intermediate good) sector, offshoring has a stronger unemployment reducing effect in the absence of perfect intersectoral labor mobility. With imperfect labor mobility, there is also the possibility of a mixed equilibrium (incomplete offshoring). When a mixed offshoring equilibrium emerges, we

know for sure that unemployment has gone up in the offshoring sector relative to autarky.

There is also a parameter configuration which could lead to a smaller negative price effect on the offshoring sector under imperfect labor mobility, and therefore sectoral unemployment unambiguously goes down in that case due to offshoring.

The above are results pertaining to sectoral unemployment. While it is difficult to characterize the effects of offshoring on aggregate or overall unemployment, it is possible to some extent to derive results on aggregate unemployment for the special case of perfect labor mobility. In this case, even though both sectors have lower unemployment post-offshoring, there is an additional determinant of the overall unemployment rate. It is whether the sector with the lower unemployment or higher unemployment expands. If the search cost is identical in the two sectors, this additional factor obviously goes away, implying identical rates of unemployment across sectors, in which case the economywide rate of unemployment declines unambiguously after offshoring. Alternatively, even if the search cost is higher in the sector which experiences offshoring (implying a higher wage as well as higher rate of unemployment in that sector), the economywide rate of unemployment most likely decreases because, under the relatively more plausible parameter configurations, workers move from the higher unemployment sector to the lower unemployment sector. This means that, in this case, the additional sectoral composition factor works in the same direction as the impact of offshoring on sectoral unemployment.

We have two main messages. Firstly, how offshoring will affect unemployment will depend on the alternative opportunities available for workers with offshored jobs. If these workers can freely start searching for alternative jobs in the same or another sector, we see a reduction in the unemployment rates for all types of workers. Secondly, with imperfect mobility (across sectors and/or across jobs), unemployment for some workers can go up with offshoring. However, unemployment rates for all types of workers is unlikely to go up at the same time.

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# 7 Appendix

## 7.1 Maximization problem of the firm in the autarky case

The firm maximizes (10) subject to (9), and (7). Denoting the Lagrangian multiplier associated with (9) by $\lambda$, and with (7) by $\phi$, the current value Hamiltonian for each firm can be written as

$$H = P_z Z - w_z N - c_z V + \lambda[q(\theta_z)V - \delta N] + \phi[N - m_h - m_p]$$

where $Z$ is given in (6). The first order conditions for the above maximization are follows.

$$m_h \quad : \quad P_z \tau m_h^{\rho-1}(\tau m_h^\rho + (1-\tau)m_p^\rho)^{\frac{1}{\rho}-1} = \phi \tag{26}$$

$$m_p \quad : \quad P_z (1-\tau) m_p^{\rho-1}(\tau m_h^\rho + (1-\tau)m_p^\rho)^{\frac{1}{\rho}-1} = \phi \tag{27}$$

$$V \quad : \quad c_z = \lambda q(\theta_z) \tag{28}$$

$$N \quad : \quad w_z + \lambda\delta - \phi = \dot{\lambda} - r\lambda \tag{29}$$

Now, (26) and (27) imply

$$\frac{m_h}{m_p} = \left(\frac{\tau}{1-\tau}\right)^{\frac{1}{1-\rho}} \tag{30}$$

using the above in (26) gives

$$\tau' P_z = \phi \tag{31}$$

Since the value of marginal product of labor, given by $\tau' P_z$, is constant, using the result from Cahuc and Wasmer (2001) mentioned in footnote 11, we have treated wage to be exogenous in deriving (29) above.

Next, note from (28) that for a given $\theta_z$, $\lambda$ is constant. Using $\dot{\lambda} = 0$, (28), and (31) in (29) we get

$$\tau' P_Z - w_z = (r+\delta)\lambda = \frac{(r+\delta)c_z}{q(\theta_z)} \tag{32}$$

$\lambda$ is the shadow value of an extra job.

## 7.2 Wage Determination

Let $U_z^j$ denote the income of the unemployed worker-$j$ searching for a job in the $Z$ sector. The asset value equation for the unemployed in this sector is given by

$$rU_z^j = \varepsilon_z^j + b + \theta_z q(\theta_z)[E_z^j - U_z^j] \tag{33}$$

where $E_z^j$ is the expected income from becoming employed in the $Z$ sector, which is the sum of the idiosyncratic benefit, $\varepsilon_z^j$, the unemployment benefit $b$, and the expected capital gain from the possible change in state from unemployed to employed.

The asset value equation for employed worker-$j$ in sector $Z$ is given by

$$rE_z^j = \varepsilon_z^j + w_z^j + \delta(U_z^j - E_z^j) \Rightarrow E_z^j = \frac{\varepsilon_z^j}{r+\delta} + \frac{w_z^j}{r+\delta} + \frac{\delta U_z^j}{r+\delta} \tag{34}$$

Again the return on being employed is the sum of the idiosyncratic benefit, $\varepsilon_z^j$, the wage, and the expected change in the asset value from a change in state from employed to unemployed. Next, (34) implies that

$$E_z^j - U_z^j = \frac{\varepsilon_z^j}{r+\delta} + \frac{w_z^j}{r+\delta} - \frac{rU_z^j}{r+\delta} \tag{35}$$

Assume the rent from a vacant job to be zero which is ensured by no barriers to the posting of vacancy. Now, denote the surplus for a firm from a job occupied by worker-$j$ by $J_z^j$. From (32) above,

$$J_z^j = \frac{\tau' P_z - w_z^j}{(r+\delta)} \tag{36}$$

The Nash-bargained wage is obtained by

$$\arg\max_{w_z^j} \left(E_z^j - U_z^j\right)^{\beta} \left(J_z^j\right)^{1-\beta}$$

where $E_z^j - U_z^j$ is given in (35) and $J_z^j$ is given by (36). The first-order condition for the bargained wage is

$$E_z^j - U_z^j = \frac{\beta}{1-\beta} J_z^j = \frac{\beta}{1-\beta} \frac{c_z}{q(\theta_z)} \tag{37}$$

where the last equality follows from the fact that the value of an occupied job, $J_z^j$, equals $\frac{c_z}{q(\theta_z)}$ as discussed in (13) in the text. Plugging the value of $E_z^j - U_z^j$ from above into the asset value equation for the unemployed in (33) we have a simplified version of this asset value equation

$$rU_z^j = \varepsilon_z^j + b + \frac{\beta}{1-\beta} c_z \theta_z \tag{38}$$

Use (37) to substitute out $E_z^j - U_z^j$ and (38) to substitute out $rU_z^j$ in (35) to get the following simplified wage equation:

$$w_z^j = b + \frac{\beta c_z}{1-\beta}[\theta_z + \frac{r+\delta}{q(\theta_z)}]$$

Note that $w_z^j$ is the same for all $j$.Similarly, in the case of the $X$ sector, we obtain $w_x = b + \frac{\beta c_x}{1-\beta}[\theta_x + \frac{r+\delta}{q(\theta_x)}]$.

## 7.3 Proofs of Lemmas 2 and 3

At $p = \underline{p}(w_s)$ the relative supply of $Z$ is given by

$$\frac{\left(\tau^{\sigma} + (1-\tau)^{\sigma}\right)^{\frac{1}{\sigma-1}} \left[L_z(1-u_z) - N^o\right] + \tau^{-\sigma}\left(\tau^{\sigma} + (1-\tau)^{\sigma}\right)^{\frac{\sigma}{\sigma-1}} N^o}{L_x(1-u_x)} \tag{39}$$

where the total domestic employment of the offshoring firms is denoted by $N^o$ and $N^o \in [0, L_z(1-u_z)]$, and $L_x$ and $L_z$ are given in (18). Since $\widehat{\varphi}$ is a function of $p$, $L_z$ and $L_x$ are functions of $p$ as well. Therefore, the denominator remains constant while the numerator increases with $N^o$.

To find the offshoring relative supply when $p > \underline{p}(w_s)$, we need to obtain the amount of labor affiliated with each sector, which in turn depends on $\widehat{\varphi}$ given in (24). Denote the $\widehat{\varphi}$ with the possibility of offshoring by $\widehat{\varphi}^o(p)$, where the superscript $o$ stands for offshoring, and as in autarky, $\widehat{\varphi}$ is a function of $p$. For $p < \underline{p}(w_s)$, allowing for offshoring leaves $\widehat{\varphi}(p)$ unchanged. For $p > \underline{p}(w_s)$, $P_z$ and $P_x$ are still given by (3). Therefore, $\theta_x$ and $w_x$ remain unchanged from autarky for each $p$. However, $\theta_z$ and $w_z$ are now determined by (25). We know from lemma 1 that $\theta_z$ and $w_z$ are higher than in autarky. Since $\theta_z$ is higher while $\theta_x$ is unchanged for each $p > \underline{p}(w_s)$, (24) implies that the $\widehat{\varphi}^o(p)$ curve lies to the left of the $\widehat{\varphi}^A(p)$ curve as is shown in Figure 2a.

Note that the expressions for the amount of labor going to each sector in the case of offshoring are still given by (18) with $\widehat{\varphi}^A$ being replaced by $\widehat{\varphi}^o$. The relative supply for each $p > \underline{p}(w_s)$ is given by

$$\left(\frac{Z}{X}\right)^s = \frac{\tau^{-\sigma}\left(\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}(1-u_z)}{(1-u_x)\exp(\widehat{\varphi}^o(p)/\alpha)} \tag{40}$$

where $u_i$ , $\omega$, and $\widehat{\varphi}^o$ are functions of $p$. For each $p > \underline{p}(w_s)$, $\omega > 1$, $\widehat{\varphi}^o(p) < \widehat{\varphi}^A(p)$, $u_x$ is unchanged from autarky, while $u_z$ is lower than in autarky, therefore, the expression on the r.h.s above exceeds the expression on the r.h.s of (20). This proves lemma 2.

In the limit, when $\alpha \to 0$, $\varphi^j \to 0$ $\forall j$. In this case the relative supply is zero for any $p < p^o$ because $c_x\theta_x > c_z\theta_z$, and hence no one wants to work in the $Z$ sector, and it becomes horizontal at $p = p^o$ since all workers are indifferent between working in the two sectors. Also, $\widehat{\varphi}^o(p) < \widehat{\varphi}^A(p)$ implies $p^o < p^A$. Next we prove that $p^o > \underline{p}(w_s)$. Note that, there is no offshoring for $p < \underline{p}(w_s)$. Therefore, $p^o \geq \underline{p}(w_s)$. Suppose $p^o = \underline{p}(w_s)$. Now, by assumption $w_s < \widetilde{w_z}^A$, which leads to offshoring (Recall that the superscripts "$o$" and "$A$" denote the equilibrium values of variables under offshoring and autarky, respectively). At $p = \underline{p}(w_s)$ we have $\widetilde{w_z}^o = w_s < \widetilde{w_z}^A$. This implies, from the wage curve equation for $Z$, that $\theta_z^o < \theta_z^A$. Additionally, $\widetilde{w_z}^o < \widetilde{w_z}^A$ in conjunction with the numeraire condition (or the zero-profit condition for the numeraire good $C$) implies that $\widetilde{w_x}^o > \widetilde{w_x}^A$, which in turn from the wage curve equation for $X$, gives us $\theta_x^o > \theta_x^A$. By the no arbitrage condition, we start in autarky from a situation where $\theta_z^A = \theta_x^A$. Given that $\theta_z^o < \theta_z^A$ and $\theta_x^o > \theta_x^A$, this implies $\theta_z^o < \theta_x^o$. Thus the no arbitrage condition is not satisfied under offshoring. This is a contradiction because at $p^o$ the no arbitrage condition must be satisfied by definition. Therefore, $p^o > \underline{p}(w_s)$. This proves lemma 3.

The expression in (40) makes it clear that the relative supply is independent of $\alpha$ for $\widehat{\varphi}^o(p) = 0$. Therefore, the offshoring relative supply curves with different values of $\alpha$ all pass through the same point at $p = p^o$. Using the same argument as in the case of autarky, we can verify that a decrease in $\alpha$ leads to a clockwise rotation of the relative supply curve at $p = p^o$. This pins down the relative positions of the offshoring relative supply curves corresponding to $\alpha_1$ and $\alpha_2$, respectively, in Figure 3.

## 7.4 Proof of Lemma 4

Assume $c_x = c_z$ and the following production function for $C$

$$C = \left(\gamma Z^{\frac{\phi-1}{\phi}} + (1-\gamma)X^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$$

where $\phi$ is the elasticity of substitution between $X$ and $Z$. The production function for $C$ implies the following cost function.

$$\left(\gamma^{\phi}\left(P_z\right)^{1-\phi} + (1-\gamma)^{\phi}\left(P_x\right)^{1-\phi}\right)^{\frac{1}{1-\phi}} \tag{41}$$

Since $C$ is the numeraire, the unit cost of $C$ must equal 1. Note that the relative demand (4) for $Z$ when the production function for $C$ is of the CES type is given by

$$\left(\frac{Z}{X}\right)^{d} = \left(\frac{\gamma P_x}{(1-\gamma)P_z}\right)^{\phi} \tag{42}$$

The relative demand for $Z$ equal to relative supply in autarky equilibrium can be written as

$$\frac{\tau' L_z}{L - L_z^A} = \left(\frac{\gamma P_x}{(1-\gamma)P_z}\right)^{\phi} \frac{(1-u_x)}{(1-u_z)} \tag{43}$$

Next, $c_x = c_z$ implies $\theta_x = \theta_z$, which in turn implies $w_x = w_z$, and hence $P_x = \tau' P_z$ where $\tau' \equiv [\tau^{\sigma} + (1-\tau)^{\sigma}]^{\frac{1}{\sigma-1}}$. Also, $\theta_x = \theta_z$ implies $u_x = u_z$. Therefore, from (43)

$$\frac{L_z^A}{L - L_z^A} = \frac{1}{\tau'}\left(\frac{\gamma\tau'}{(1-\gamma)}\right)^{\phi} \tag{44}$$

where $L_z^A$ is the amount of labor in the $Z$ sector in autarky equilibrium. Note that if there was no labor market friction in the model, the expression for $L_z$ in autarky would be exactly the same as in (44).

Similarly, the relative demand equals relative supply in the offshoring equilibrium can be written as

$$\frac{\tau^{-\sigma}\left(\tau^{\sigma} + (1-\tau)^{\sigma}\omega^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} L_z}{L - L_z} = \left(\frac{\gamma P_x}{(1-\gamma)P_z}\right)^{\phi} \frac{(1-u_x)}{(1-u_z)} \tag{45}$$

Again, $c_x = c_z$ implies $\theta_x = \theta_z$ and hence $u_x = u_z$. Also, $\theta_x = \theta_z$ and $w_x = w_z$ imply $P_x = \left(\tau^{\sigma} + (1-\tau)^{\sigma}\omega^{\sigma-1}\right)^{\frac{1}{\sigma-1}} P_z$. Therefore, (45) can be written as

$$\frac{L_z^o}{L - L_z^o} = \frac{\tau^{\sigma}}{\left(\tau^{\sigma} + (1-\tau)^{\sigma}\omega^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}}\left(\frac{\gamma\left(\tau^{\sigma} + (1-\tau)^{\sigma}\omega^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1-\gamma)}\right)^{\phi} \tag{46}$$

where $L_z^o$ is the amount of labor in the $Z$ sector in the offshoring equilibrium. Again, if there is no labor market friction then the expression for $L_z$ in an offshoring equilibrium would be the same as in (46). The only difference would be that $\omega$ would be the ratio of domestic wage to foreign wage rather than being the ratio of domestic labor cost to foreign wage.

Comparing (44) and (46) note that $L_z^A > (<) L_z^o$ if the following inequality holds.

$$\left(\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}}\right)^{\frac{\phi-1}{\sigma-1}} > (<) \frac{\tau^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} \tag{47}$$

We get the following possibilities:

Case I: $\phi = 1$. In this case the l.h.s of (47) exceeds the r.h.s if $\frac{\tau^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} < 1$, if which is true for any $\sigma$. Therefore, if the production function for $C$ is Cobb-Douglas, then irrespective of the elasticity of substitution in $Z$ production, labor always moves from $Z$ to $X$ upon offshoring.

Case II: $\phi = \sigma$. In this case the l.h.s of (47) exceeds the r.h.s if $1 + \left(\frac{1-\tau}{\tau}\right)^\sigma > 1$, which is always true implying $L_z^A > L_z^o$.

Case III: $\phi < 1$, $\sigma > 1$. In this case the l.h.s of exceeds 1 since $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} < 1$ and $\frac{\phi-1}{\sigma-1} < 0$, while the r.h.s is less than 1. Therefore, again $L_z^A > L_z^o$.

Case IV: $\phi < 1$, $\sigma < 1$. Again, the l.h.s of (47) exceeds 1 because $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} > 1$ and $\frac{\phi-1}{\sigma-1} > 0$. Therefore, again $L_z^A > L_z^o$.

Case V: $\phi > 1$, $\sigma > 1$. Again, the l.h.s of (47) exceeds 1 because $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} > 1$ and $\frac{\phi-1}{\sigma-1} > 0$. Therefore, again $L_z^A > L_z^o$.

Case VI: $\phi > 1$, $\sigma < 1$. In this case $\sigma < 1$ implies $\frac{\tau^\sigma + (1-\tau)^\sigma}{\tau^\sigma + (1-\tau)^\sigma \omega^{\sigma-1}} > 1$,but $\phi > 1$, $\sigma < 1$ implies $\frac{\phi-1}{\sigma-1} < 0$. Therefore, the l.h.s of (47) is less than 1. Since both the l.h.s and the r.h.s are less than 1, it is possible for the r.h.s to exceed l.h.s in which case $L_z^A < L_z^o$.

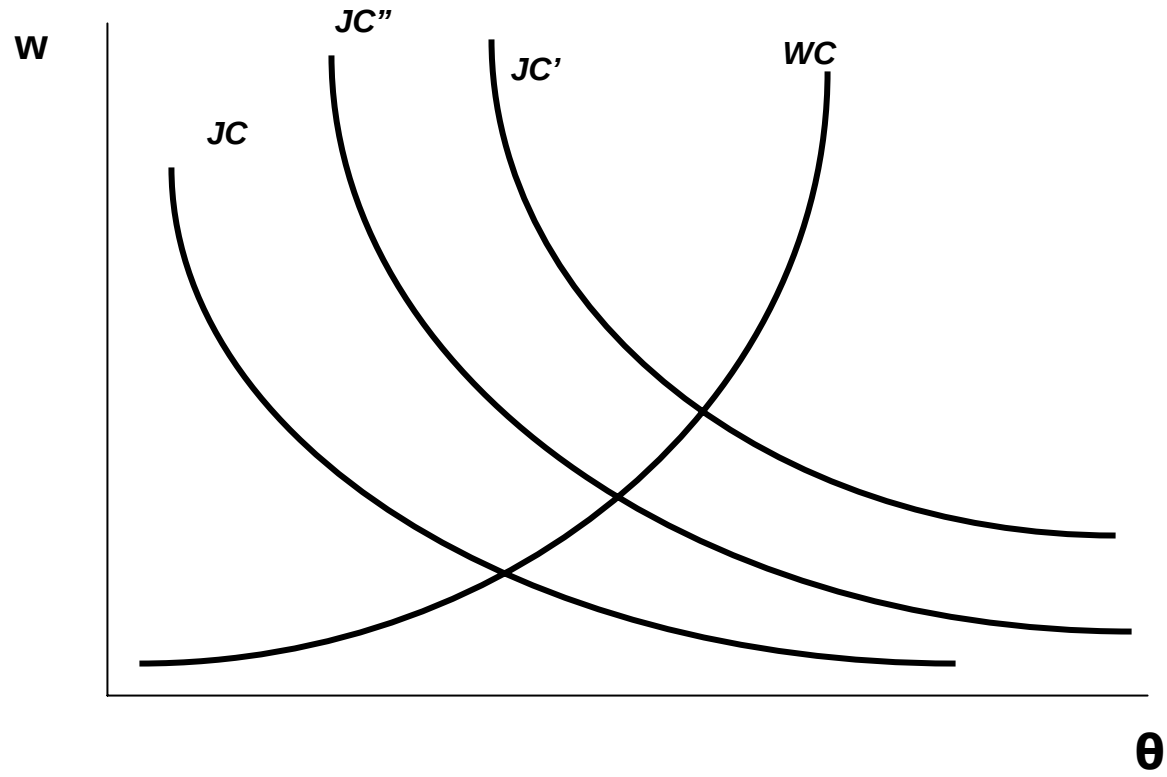


**Figure 1: Partial Equilibrium**

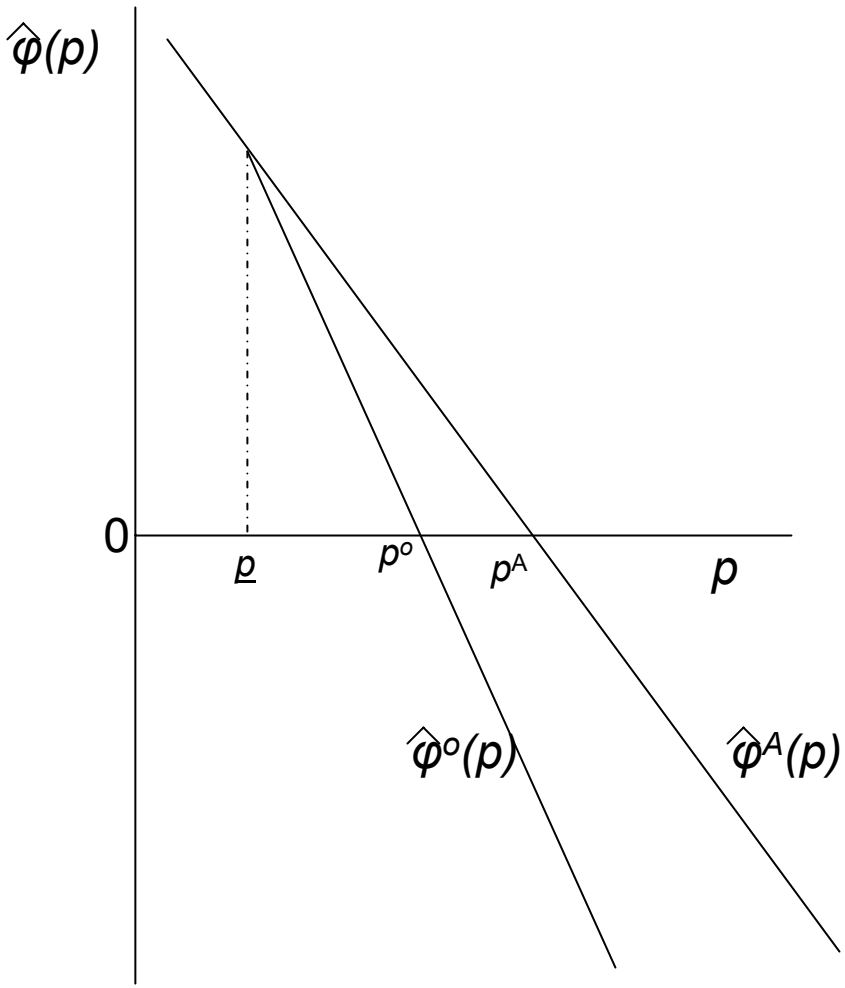


**Figure 2a: Equilibrium Prices**

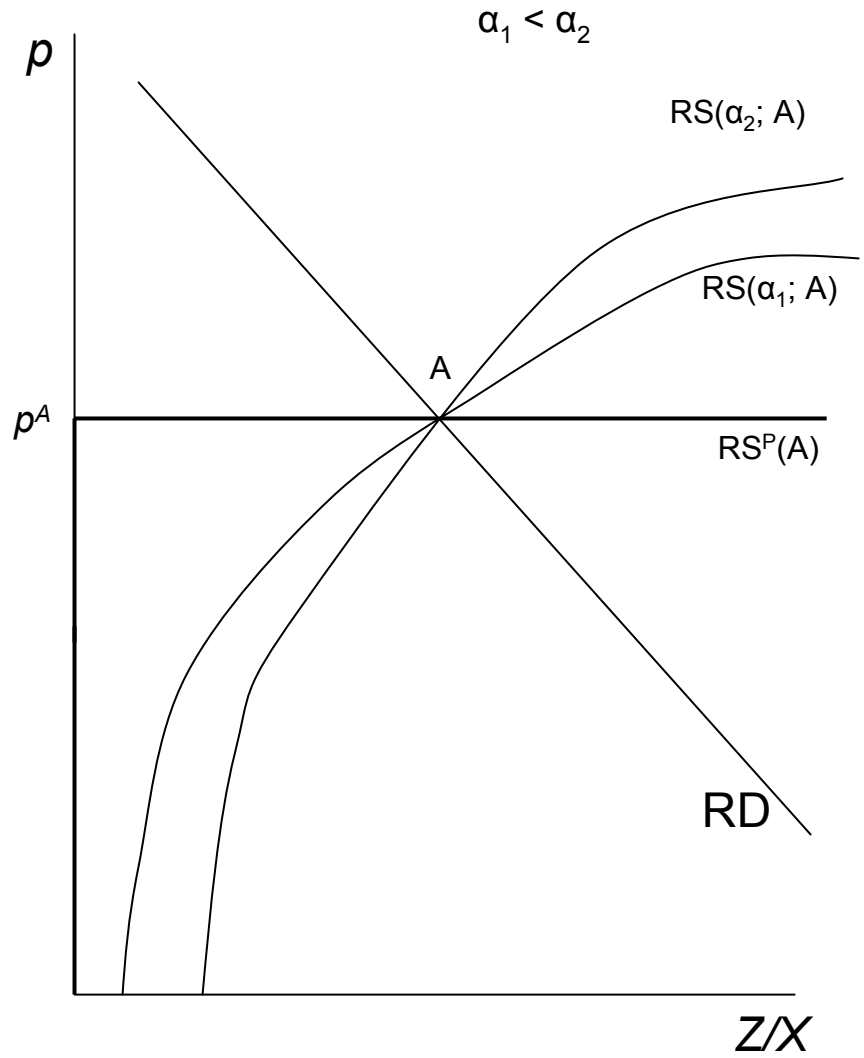


**Figure 2b: Autarky Equilibrium**

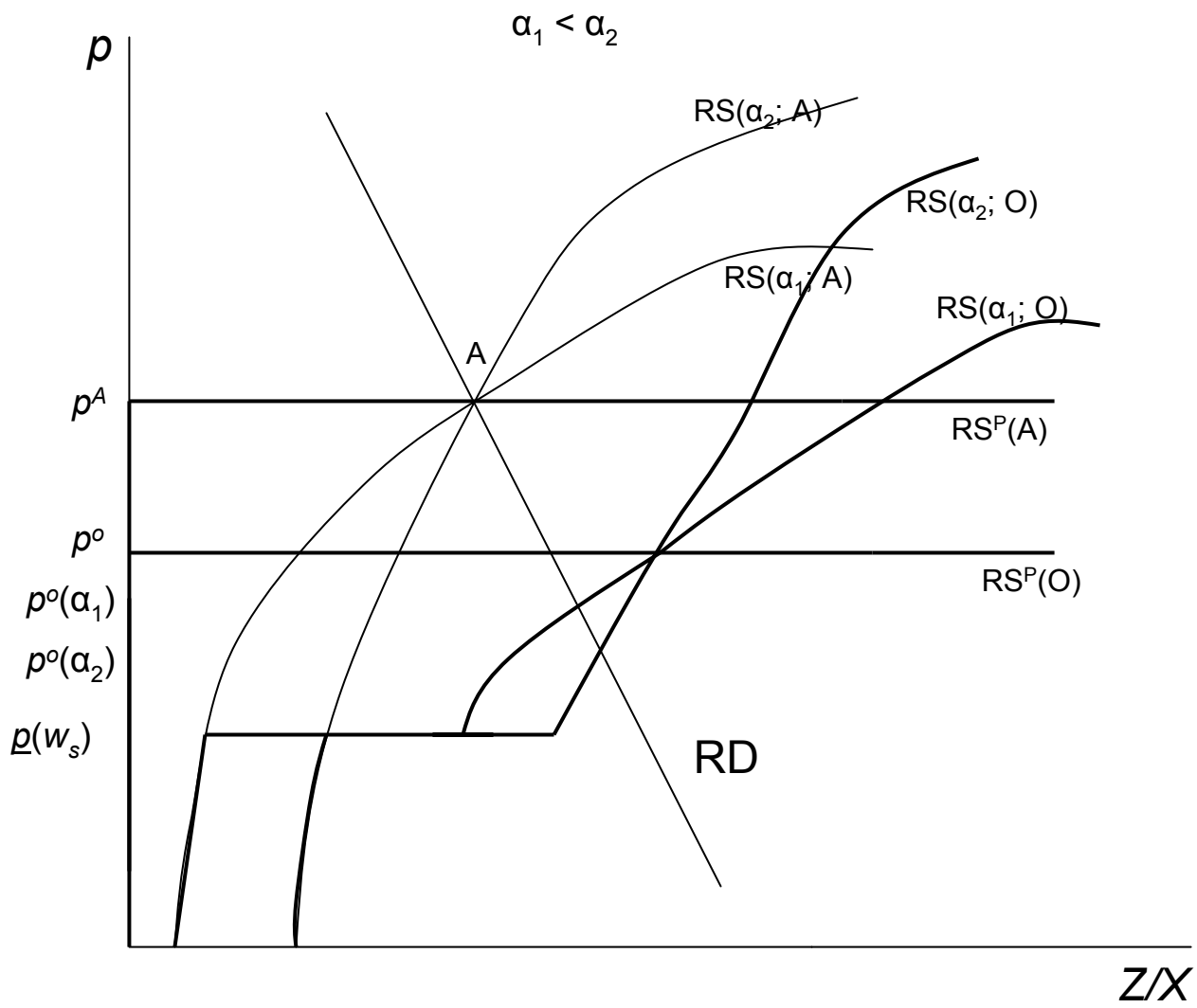


**Figure 3: Complete Offshoring Equilibrium**

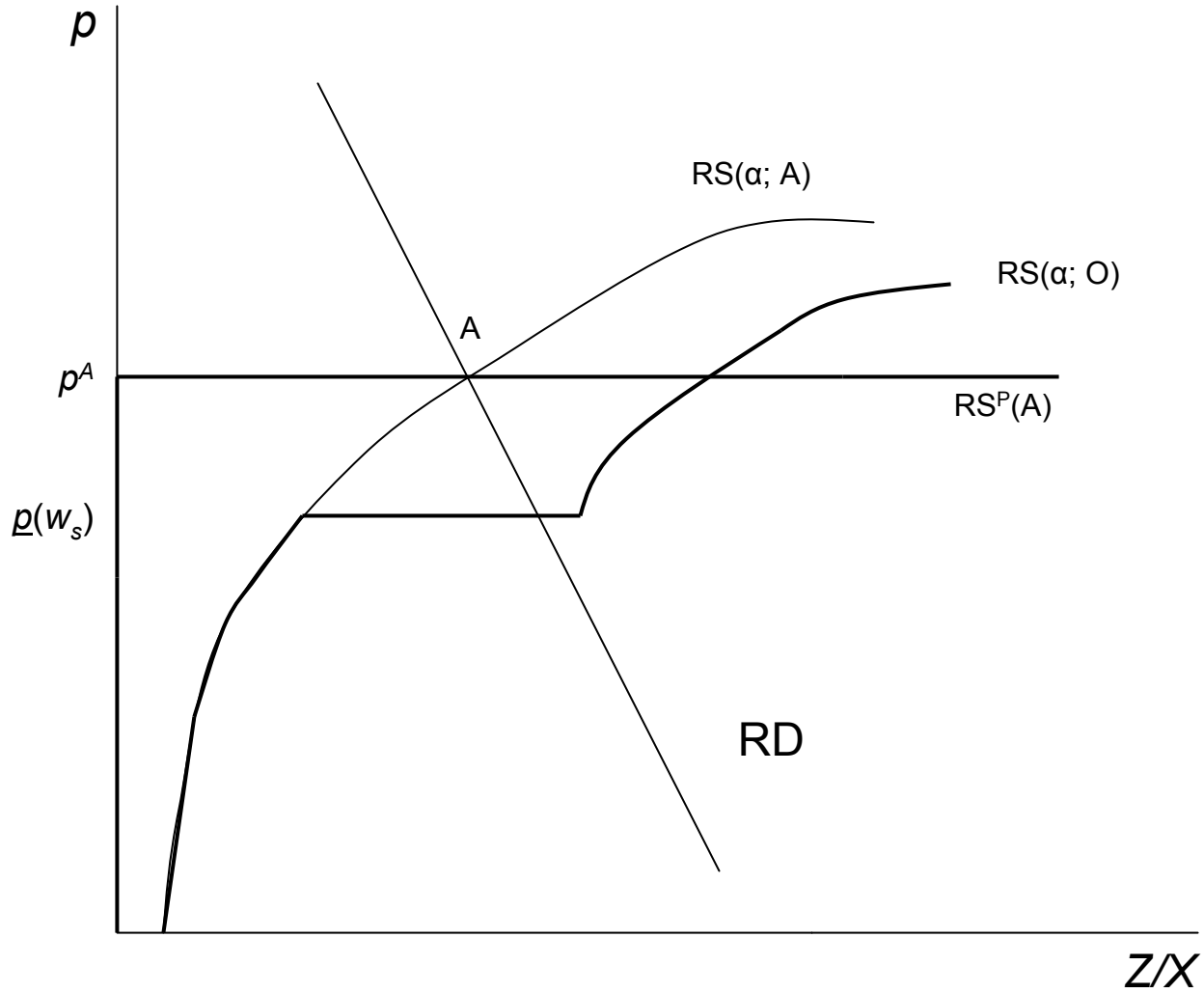


**Figure 4: Mixed Offshoring Equilibrium**