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Fertility, Female Labor Supply and Public Policy

Patricia Apps
Ray Rees

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Patricia Apps
University of Sydney, Economics Program, RSSS, ANU and IZA, Bonn

Ray Rees
University of Munich and University of York

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IZA

P.O. Box 7240
D-53072 Bonn
Germany

Tel.: +49-228-3894-0
Fax: +49-228-3894-210
Email: iza@iza.org

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ABSTRACT

Fertility, Female Labor Supply and Public Policy*

Historically, in virtually all developed economies there seems to be clear evidence of an inverse relationship between female labor supply and fertility. However, particularly in the last decade or so, the relationship across countries has been positive: for example countries like Germany, Italy and Spain with the lowest fertility rates also have the lowest female participation rates. We accept the hypothesis that the reason for this lies in the combined effects of a country's tax system and system of child support, and we have sought to clarify this theoretically, using an extended version of the Galor-Weil model. The results suggest that countries with individual rather than joint taxation, and which support families through improved availability of alternatives to domestic child care, rather than through direct child payments, are likely to have both higher female labor supply and higher fertility. These results are strengthened when we take account of the heterogeneity among households that undoubtedly exists.

JEL Classification: H31, H53, J13, J22

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Patricia Apps
University of Sydney
173-175 Phillip Street
Sydney NSW 2000
Australia
Tel.: +61 2 9351 0241
Fax: +61 2 9351 0200
Email: pfapps@law.usyd.edu.au

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1 Introduction

A recent paper by Oded Galor and David N Weil (1996) makes an important contribution to our understanding of the interrelationships among female labor supply, fertility choices and growth in real wages, by presenting a tractable overlapping generations (OLG) model in which these can be formally analysed as endogenous variables. The core of the model is the idea that increasing capital per worker raises the relative wage of women, this reduces family size, since child-rearing and female labour supply are substitutes[1], and this in turn increases capital per worker. The wages of men and women differ because women supply only labor that is complementary to capital, whereas men supply not only this but also labor that is neither a complement nor a substitute to capital. This allows the fact of an increase in the women's wage relative to that of men over time, the diminishing "gender gap", to be captured in a relatively simple way.

The data presented in Table 1 for a group of developed economies certainly confirms, for each country individually, the idea of a negative relationship between female employment and fertility over time. What is also immediately apparent, however, is that this negative relationship no longer holds across countries. As Figures 1 and 2 show, it existed in 1970 but not in 1990. For example, in 1990 Germany, the Netherlands, Italy and Spain have substantially lower female employment, but also very much lower fertility rates, than the US, Canada, the UK, France and the Scandinavian countries. In the latter group of countries a larger growth in female employment over the period 1970-1990 was accompanied by much smaller falls in fertility than in the former group[2].

Detailed empirical work[3] has suggested that a likely explanation for these across-country differences is to be found in the effects of their tax and social security contribution structures, in combination with the cost and availibility of child care outside the home[4]. These parameters determine the terms of

[1] There is of course a positive effect of real income growth on fertility, but the model is so structured that this is outweighed by the substitution effect.

[2] There is a strong presumption, in at least some of the countries with low female employment, that this is actually the desired policy outcome. The irony is that the low fertility in these countries is the precise opposite of what is supposed to happen: making female labor force participation difficult does not result in higher fertility, but simply in lower levels of both labor supply and fertility.

[3] See for example Gustafsson (1985), and Fenge and Ochel (2001).

[4] For example, in Germany married women entering the labor force face: joint taxation

the trade-off between family size and female labor supply, with sharp differences among countries[5] in the nature of this trade-off and the corresponding equilibrium choices of these variables. The aim of this paper is to extend the Galor-Weil model to allow a formal analysis of the way the public finance regime and child care substitution possibilities interact to determine the relationship between female employment and fertility[6].

Table 1 and Figures 1,2 about here

An important feature of the Galor-Weil model is the very simple child care production function. There is a fixed requirement of the mother's time per unit of child care. In this paper we extend this by allowing for substitution between parental time and a bought-in child care input in a standard production function. This simple and, we believe, realistic extension has surprisingly important implications. There need no longer be an inverse relationship between female labor supply and fertility. This suggests that the undoubtedly negative relationship that has existed empirically, and which is captured effectively in the Galor-Weil model, really does result, as the empirical evidence suggests, from inadequate possibilities of substitution between different forms of child care.

A further empirical fact, not captured in the Galor-Weil model, is the significant variation in female labor supply and fertility across households within a given economy. This has lead some influential writers in this area to advocate the stick of tax penalties for households that choose low fertility, in addition to the carrot of higher transfer payments per child, as policies to reverse the fertility decline. At the same time, the system of taxation and child-related transfers may imply substantial income redistributions across households, and, as we show below, there are reasons to believe that these may involve serious horizontal inequities. It seems to us useful therefore to

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(income splitting), implying a relatively high marginal tax rate; social security contributions well in excess of their incremental actuarial value; very scarce and expensive preschool child care facilities; and half-day schooling, implying that typically children return home from school at midday.

[5]Across-country differences in the parameters of the Galor-Weil model – preferences for children relative to consumption, time required per unit of child care, and relative male and female labor productivities - could of course be proposed as accounting for the variation in outcomes, but this seems *a priori* implausible given the broad similarities of the economic characteristics of the two groups of countries.

[6]For a substantial body of work which considers issues of family taxation, including its effect on fertility, from the more usual public finance standpoint, see Balestrino et al (2002), Cigno (1986), (2001), and Cigno and Pettini (2001).

extend the model to include households of different types, so that the issues arising out of differences in household choices can be formally analysed.

We therefore extend the Galor-Weil model in three main ways. First, we introduce a public sector, which taxes labor supplies, pays a fixed grant per child, and may subsidise bought-in child care. Secondly, we extend the technology of child care, replacing the fixed parental time input by a production function that allows smooth substitution between parental time and a market child care input. Finally, we distinguish between household types according to their productivities in child care, thus allowing explicit analysis of variations in household choices. Our primary concern in this paper is with the effect of the tax regime and child support system on the steady state of the model, and so, in contrast to the Galor-Weil paper, our analysis of the model's dynamics is confined to showing that a stable steady state exists. In the next section we set out the model with only one household type. In section 3 we derive the main policy propositions. In section 4 we extend these for the case of different household types. Section 5 concludes.

2 The Model

We take fertility and female labor supply decisions as endogenous, and analyse the effects of taxation and policies toward child care and child support in a 3-period OLG model, in which explicit account is taken of child care costs. Individuals spend the first period of their lives as children, being cared for by their parents, the second period of their lives working, saving, and caring for their children, and the third period in retirement, when they consume the proceeds of their saving[7]. We call the type of labor supplied by both men and women type-1 labor, and that supplied only by men type-2 labor[8], with the corresponding (gross of tax) wage rates w_{1t} and w_{2t} respectively.

[7] For simplicity, we ignore the existence of state pensions on retirement. The formal reason is that, since the present value of the pension will affect saving and fertility decisions, these then become functions of the interest rate.This in turn implies that the basic equation of motion of the system becomes a second order nonlinear difference equation, which complicates the analysis of the dynamics, without adding anything of interest in the present context. In Apps and Rees (2001), we use such a model to examine issues of pensions policy.

[8] Galor-Weil call these types of labor "mental" and "physical" labor respectively. Perhaps better would be "services" and "manufacturing", as long as it is understood that the economy produces only one physical output.

2.1 The household model

As in the Galor-Weil model, the household utility function takes the form[9]

$$u_t = \gamma \ln n_t + (1-\gamma)\ln c_{t+1} \tag{1}$$

where c_{t+1} is consumption in the retirement phase and n_t denotes the number of children[10], both chosen by the household at time t. Let $z_t \in [0,1]$ denote the time the female earner in a household spends in child care, so that $1-z_t$ is her market labor supply, and x_t the amount of a bought-in market good used in child care. The male earner in a household is assumed to supply 1 unit of time inelastically to each type of labor, and none to child care. The production function for child care[11]

$$n_t = f(z_t, x_t) \tag{2}$$

is assumed linear homogeneous, continuously differentiable and strictly quasi-concave.

We assume initially that there is a single tax rate τ_t. A male worker earns the net of tax wage

$$w_{mt} = (1-\tau_t)(w_{1t} + w_{2t}) \tag{3}$$

and a female the net of tax wage

$$w_{ft} = (1-\tau_t)w_{1t} \tag{4}$$

The household's budget constraint is then

$$s_t + (1-\sigma_t)x_t = w_{mt} + w_{ft}(1-z_t) + g_t n_t \tag{5}$$

where s_t is the household's saving, and so

$$c_{t+1} = (1 + r_{t+1})s_t \tag{6}$$

[9] It is, as Galor and Weil argue, a harmless simplification to take consumption in the working period to be zero.

[10] As in the Galor-Weil model, this is actually the number of pairs of children. $n_{it} = 1$ implies that the population of two-person households just reproduces itself.

[11] No confusion should result from treating the number of children and the quantity of "child care output" as identical.

with r_{t+1} the one-period interest rate, paid at $t+1$. The price of the bought-in child care good is 1, but there may be a subsidy from the government of $\sigma_t > 0$ per unit. Moreover, the government pays a grant[12] of $g_t > 0$ per child.

One formulation of the household's optimisation problem would be to maximise (1) subject to (2) - (4), but it is convenient instead to solve the problem in two stages. First, the household takes an efficient child care production decision by solving[13]

$$\min C_t = w_{ft}z_t + (1-\sigma_t)x_t \tag{7}$$

$$s.t. \quad n_t = f(z_t, x_t) \tag{8}$$

yielding the child care cost function

$$C_t = p(w_{ft}, \sigma_t)n_t \tag{9}$$

and child care input demand functions[14]

$$z_t^* = \hat{z}(w_{ft}, \sigma_t)n_t \tag{10}$$

$$x_t^* = \hat{x}(w_{ft}, \sigma_t)n_t \tag{11}$$

where $\hat{z}_t = \hat{z}(w_{ft}, \sigma_t)$ and $\hat{x}_t = \hat{x}(w_{ft}, \sigma_t)$ are given by the respective derivatives of the unit cost function $p(w_{ft}, \sigma_t)$

$$\frac{\partial p(w_{ft}, \sigma_t)}{\partial w_{ft}} = \hat{z}_t; \quad -\frac{\partial p(w_{ft}, \sigma_t)}{\partial \sigma_t} = \hat{x}_t \tag{12}$$

and are the input requirements for one unit of child care.

The household's problem[15] can then be written[16]

$$\max \gamma \ln n_t + (1-\gamma)\ln c_{t+1} = u_t \tag{13}$$

[12] Clearly, both this grant and the child care subsidy could be related to household income, either by means-testing, or by making them part of taxable income. Alternatively they could take the form of tax allowances. For example, Australia has "family tax benefits" that are means tested on household income and spouse's income. In Canada, the secondary earner is allowed to set child care expenditures, up to a cerain maximum, against taxable income. Here for simplicity we assume they are independent of income.

[13] We assume an interior solution. Extensions of the argument to corner solutions is straightforward.

[14] Throughout this paper, an asterisk denotes an optimal value of a choice variable.

[15] Note that it must be assumed that the household has perfect foresight at t of the interest rate r_{t+1}, which is not determined until time $t+1$. This is a standard assumption in OLG models.

[16] We assume throughout that $p(w_{2t}, \sigma_t) > g_t$, the child payment is always less than the unit cost of a child.

$$s.t.\ [p(w_{ft},\sigma_t)-g_t]n_t+\frac{c_{t+1}}{(1+r_{t+1})} = w_{mt}+w_{ft}\equiv W_t \tag{14}$$

where W_t is household *net full income*[17], as compared to its *net labor income,* $w_{mt}+w_{ft}(1-z_t)$. The solution to this problem yields fertility, consumption and saving functions

$$n_t^* = \frac{\gamma W_t}{p(w_{ft},\sigma_t)-g_t} \tag{15}$$

$$c_{t+1}^* = (1-\gamma)W_t(1+r_{t+1}) \tag{16}$$

$$s_t^* = (1-\gamma)W_t \tag{17}$$

Note that, as a result of the Cobb-Douglas utility function, neither fertility nor saving are functions of the interest rate. This, a feature shared with the Galor-Weil model, implies a substantial simplification of the model's dynamics.

As we shall see, an important role is played in the policy analysis by the relationship between fertility and the female net wage, w_{ft}. In the Galor-Weil model, this is always negative. The generalisation we have made to permit substitutability between parental time and bought-in child care has a quite significant impact on this result, as the following proposition shows.

Proposition 1. *If the time spent in domestic child care is sufficiently small relative to the strength of the preference for children, in a sense to be defined below, fertility increases with the female net wage.*

The exact conditions on which this proposition holds depend on whether the change in the female net wage is due to a change in the gross wage, or to a change in her marginal tax rate. In the latter case it matters whether the tax rates of both spouses change, or whether we consider changes in the tax rates separately. It is useful for the later policy analysis to give the results for each of the possibilities.

(a) *Change in the gross wage.* Differentiating in (15) with respect to w_{1t}, recalling (3), (4) and (14) gives

$$\frac{\partial n_t^*}{\partial w_{1t}}=\frac{(1-\tau_t)(2\gamma-z_t^*)}{p_t-g_t} \tag{18}$$

Thus

$$\frac{\partial n_t^*}{\partial w_{1t}}>0\Leftrightarrow 2\gamma>z_t^* \tag{19}$$

[17] That is, the after tax income it would earn if all time was used for market labor supply.

Recall that γ is the exponent of n_t in the utility function, while $z_t^* \in [0,1]$. Thus this condition is certainly satisfied if $\gamma > 0.5$, and γ can be smaller, for given n_t^*, the smaller is $\hat{z}_t$, the domestic time input per child, and so the larger is $\hat{x}_t$, the bought-in child care.

(b) *Change in the marginal tax rate of both spouses.* Then we have

$$\frac{\partial n_t^*}{\partial \tau_t} = \frac{w_{1t} z_t^* - \gamma(2w_{1t} + w_{2t})}{p_t - g_t} \tag{20}$$

Thus

$$\frac{\partial n_t^*}{\partial \tau_t} < 0 \Leftrightarrow (2 + \frac{w_{2t}}{w_{1t}})\gamma > z_t^* \tag{21}$$

It is therefore more likely that fertility increases with a rise in the female net wage in this case, because the male net wage also rises, and this has an additional income effect. We can think of the ratio w_{2t}/w_{1t} as a measure of the "gender gap", because w_{2t} is the difference between male and female wage rates, and so this condition is more likely to be satisfied, the larger the gender gap.

There is an interesting alternative way of expressing this condition. Clearly the sign of $\partial n_t^*/\partial \tau_t$ is unchanged when we multiply through (20) by $(1-\tau_t)$, and so we can write

$$(1-\tau_t)\frac{\partial n_t^*}{\partial \tau_t} = \frac{(1-\tau_t) w_{1t} z_t^*}{p_t - g_t} - n_t^* \tag{22}$$

So[18]

$$\frac{\partial n_t^*}{\partial \tau_t} < 0 \Leftrightarrow (1-\tau_t) w_{1t} z_t^* < n^*(p_t - g_t) \Leftrightarrow n^* g_t < (1-\sigma_t) x_t^* \tag{23}$$

Thus a fall in the tax rate, increasing the female net wage, will increase fertility if the cost of the mother's time spent in child-rearing is less than the total cost of the children, net of child payments, or equivalently if the cost of bought-in child care is greater than the total value of child payments.

(c) *Only the wife's tax rate changes.* Let τ_f and τ_m denote the female and male tax rates respectively. Then the fertility demand function becomes

$$n_t^* = \frac{\gamma[(1-\tau_f)w_{1t} + (1-\tau_m)(w_{1t} + w_{2t})]}{p[(1-\tau_f)w_{1t}, \sigma] - g_t} \tag{24}$$

[18] Recall that $p_t = (1-\tau_t)w_{1t}\hat{z}_t + (1-\sigma_t)\hat{x}_t$.

and

$$\frac{\partial n_t^*}{\partial \tau_f} = \frac{w_{1t}(z_t^* - \gamma)}{p_t - g_t} \tag{25}$$

Thus

$$\frac{\partial n_t^*}{\partial \tau_f} < 0 \Leftrightarrow \gamma > z_t^* \tag{26}$$

Again by rewriting this condition we obtain an interesting alternative interpretation. Multiplying through (25) by $(1 - \tau_f)/n^*$ will not change its sign, and, defining $\omega_t \equiv (1 - \tau_f)w_{1t}/W_t$, the female's proportional contribution to net household full income[19], we can rewrite the condition as

$$\frac{\partial n_t^*}{\partial \tau_f} < 0 \Leftrightarrow \frac{\omega_t}{1 - \omega_t}[(1 - \sigma_t)\hat{x}_t - g_t] > (1 - \tau_f)w_{1t}\hat{z}_t \tag{27}$$

Thus the larger the wife's proportional contribution to net household full income (the smaller the "gender gap") and the larger the expenditure on bought-in child care per child, the more likely it is that an increase in the female net wage, induced by a fall in her tax rate, will increase fertility.

(d) *Only the husband's tax rate changes.* Then we have simply

$$\frac{\partial n_t^*}{\partial \tau_m} = \frac{-\gamma(w_{1t} + w_{2t})}{p_t - g_t} < 0 \tag{28}$$

Since the husband does not in this model contribute time to child care, a change in his tax rate has only an income effect.

The intuition for these results rests on the comparison of income and price effects. When, as in the Galor-Weil model, the only input into child-rearing is the woman's time, while she contributes only a proportion of the household's income, then the effect of an increase in her net wage in raising the unit cost of a child always outweighs the effect on household income - the negative price effect dominates the positive income effect. When however there is a second input into child-rearing, so that the cost of the woman's time is only a proportion of the total cost, then this need no longer be true. For example, (27) shows that where the woman is contributing close to half the net household full income, there is no child care subsidy nor child payment, and the cost of bought-in child care exceeds the cost of the mother's

[19] Note, full income, not labor income, and so this depends only on wage and tax rates, and not on female labor supply.

time sufficiently, then increasing her net wage by reducing only her tax rate increases fertility - the income effect outweighs the negative price effect.

This is very relevant to the across-country comparisons made in the introduction to this paper. Given the positive elasticity of female labor supply with respect to the net wage, the less strongly negative is the fertility elasticity with respect to the wage, the weaker will be the negative association between female employment and fertility in an economy characterized by growth in the female relative wage over time.

2.2 The aggregate economy

The number of active households, N_t, evolves according to

$$N_t = n_{t-1}^* N_{t-1} \tag{29}$$

We adopt the aggregate production function of the Galor-Weil model

$$Y_t = A[\alpha K_t^a + (1-\alpha)(L_t^f)^a]^{\frac{1}{a}} + BN_t \quad A, B > 0, \alpha \in (0,1), a \in (-\infty, 1) \tag{30}$$

where the elasticity of substitution $\varepsilon = 1/(1-a)$. Since each male supplies one unit of type 2 labor inelastically, the total supply of this is N_t, the number of active households at time t. Since each male also supplies one unit of type 1 labor inelastically, while each female supplies $(1 - z_t^*)$ units, we have

$$L_t^f = N_t(2 - z_t^*) \tag{31}$$

The production function can then be written in per-household terms as

$$y_t = A[\alpha k_t^a + (1-\alpha) l_t^a]^{\frac{1}{a}} + B \tag{32}$$

where

$$l_t = \frac{L_t^f}{N_t} = (2 - z_t^*) \tag{33}$$

Given the standard competitive market assumptions we have

$$w_{2t} = B \tag{34}$$

$$w_{1t} = (1-\alpha) l_t^{a-1} A[\alpha k_t^a + (1-\alpha) l_t^a]^{\frac{1}{a}-1} \tag{35}$$

The capital stock in each period is determined by the saving of the active households in the previous period, so that

$$K_{t+1} = N_t s_t^* \tag{36}$$

Finally, we have the government budget constraint[20], written in per-household terms,

$$\tau_t[w_{1t}(2 - z_t^*) + w_{2t}] - g_t n_t^* - \sigma_t x_t^* = 0 \tag{37}$$

This says simply that tax revenue on labor incomes must cover expenditures on child payments and subsidies on child care, if any.

2.3 Existence of a stable steady state

The basic equation that determines the dynamics of the system is

$$k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{N_t s_t^*}{N_{t+1}} = \frac{(1-\gamma)W_t}{n_t^*} \tag{38}$$

This gives rise to a first order nonlinear difference equation.

Since the purpose of this paper is to examine the nature of the steady state, we shall deal rather briefly with the model's dynamics.[21] All we really need to confirm is that a stable steady state of the model exists. The rest of the paper will then be concerned with the comparative statics of this steady state. Furthermore, in showing that a stable steady state exists we shall focus on the basic model without a public sector, the main difference to the Galor-Weil model then being that as the wage rate rises, households can smoothly substitute bought-in for domestic child care.

Substituting from (15) into (38) we can write the basic equation as

$$k_{t+1} = \frac{(1-\gamma)}{\gamma} p(w_{1t}) \tag{39}$$

Since

$$l_t = 2 - z_t^* = 2 - \hat{z}(w_{1t}) n_t^* = 2 - \hat{z}(w_{1t}) \frac{\gamma W_t}{p(w_{1t})} \equiv l(w_{1t}) \tag{40}$$

We assume $l'(w_{1t}) > 0$, i.e. the labor supply curve is never backward bending. We can write (35) as

$$w_{1t} = m(l(w_{1t}), k_t) \tag{41}$$

[20]Note that this government budget constraint rules out borrowing: the budget must balance in each period.

[21]The dynamic behavior of the model was of course the central concern of the Galor-Weil paper.

>From this we obtain the derivative

$$\frac{dw_{1t}}{dk_t} = \frac{\frac{\partial m}{\partial k_t}}{(1 - l'(w_{1t})\frac{\partial m}{\partial l_t})} > 0 \tag{42}$$

The sign follows since, from the CES production function, $\frac{\partial m}{\partial l_t} < 0$, (diminishing marginal productivity of type 1 labor) and $\frac{\partial m}{\partial k_t} > 0$, (capital and type 1 labor are complements). Thus (41) defines w_{1t} as a strictly increasing function of k_t, which we write as $\mu(k_t)$, with $\mu^{'}(k_t) > 0$ given by (42). Substituting into (39) then gives

$$k_{t+1} = \frac{(1-\gamma)}{\gamma} p[\mu(k_t)] = \psi(k_t) \tag{43}$$

as the difference equation we need.

It is obvious from (35) that $\psi(0) > 0$. We also have

$$\psi^{'}(k_t) = \frac{(1-\gamma)}{\gamma} \hat{z}_t \mu^{'}(k_t) > 0 \tag{44}$$

A sufficient condition for the existence of a stable steady state is $\lim_{k_t \to \infty} \psi^{'}(k_t) = 0$. Consider first $\mu^{'}(k_t)$. Now, $\hat{z}_t$ is bounded below by zero, and l_t above by 2, thus suggesting it is reasonable to assume $\lim_{k_t \to \infty} l^{'}(w_{1t}) = 0$ - ultimately, an increasing wage cannot increase a household's labor supply further. From the CES production function, we can show that $\lim_{k_t \to \infty} \frac{\partial m}{\partial k_t} = 0$ and $\lim_{k_t \to \infty} \frac{\partial m}{\partial l_t} \leqslant 0$. Thus we can conclude that $\lim_{k_t \to \infty} \mu^{'}(k_t) = 0$. As k_t and w_{1t} increase $\hat{z}_t$ falls due to a substitution effect. We have not specified a functional form, but $\hat{z}_t$ must always lie in $[0, 1]$, and it is reasonable to assume that in the limit $\hat{z}_t \geqslant 0$. Thus we have $\lim_{k_t \to \infty} \psi^{'}(k_t) = 0$, as required.

3 Fertility, female labor supply and policy in the steady state

In this section we want to examine the way in which the tax system and the system of child support, in conjunction with the cost and availability of child care, influence the relationship between fertility and female labor supply in the steady state. We denote a steady state value of a variable simply by deleting the t subscript. Then we have the following steady state system:

$$n^* = \frac{\gamma W}{p(w_f, \sigma) - g} \tag{45}$$

$$z^* = \hat{z}(w_f, \sigma) n^* \tag{46}$$

$$x^* = \hat{x}(w_f, \sigma) n^* \tag{47}$$

$$w_1 = (1-\alpha) l^{a-1} A[\alpha k^a + (1-\alpha) l^a]^{\frac{1}{a}-1} \tag{48}$$

$$w_f = (1-\tau) w_1 \tag{49}$$

$$k = \frac{(1-\gamma)W}{n^*} \tag{50}$$

$$l = (2 - z^*) \tag{51}$$

$$0 = \tau[w_1(2-z^*) + w_2] - gn^* - \sigma x^* \tag{52}$$

$$W = (1-\tau)(2w_1 + w_2) \tag{53}$$

Given the values of two of the three policy variables in (52), the remaining policy variable and all endogenous variables are determined (recall that $w_2 = B$, a constant).

We wish to explore how the tax structure and the system of child support determine the relationship between fertility and female labor supply. We do this by carrying out a comparative statics analysis on the steady state equilibrium, to see how changes in the policy parameters affect fertility n^* and female labor supply $(1-z^*)$. We give the nature of the changes and summarise the results here. The details of the analysis are given in the Appendix. To sharpen the results we assume first that $\sigma = 0$, which is empirically reasonable in most countries, and secondly, that we can ignore the general equilibrium feedback effects of changes in the capital-labor ratio and the gross wage w_1. This latter is a strong assumption, but is made so that we can focus on the policy issues of most concern for this paper. The general equilibrium effects are typically ambiguous, in that the relevant derivatives can usually not be uniquely signed, and this multiplies considerably the number of cases that have to be considered without adding anything of substantive interest by way of results.

(i) *Child payments and taxation.* Intuitively, we would think that the higher the level of the child payment g, the higher must be the level of fertility in an economy. Even if we take account of the fact that this would generally imply higher taxation, the Galor-Weil model leads us to expect that, since this reduces the female net wage, this would reinforce the increase in fertility.

We would expect this to be at the expense of female labor supply. An increase in g increases the demand for children and the required domestic time input, and so female labor supply falls. Thus in two otherwise identical economies, the one with higher child payments and taxes might be expected to have lower female labor supply and higher fertility. However, we saw in Proposition 1 that, under reasonable conditions, raising the tax rate could reduce fertility. Moreover, reducing female labor supply reduces the tax base, and makes the increase in tax rate required to fund a given increase in g higher than where this effect is absent. It is therefore possible that an economy with higher tax levels and child payments could actually have *both* lower female labor supply *and* lower fertility. We confirm this possibility in

Proposition 2: *Given an initial steady state equilibrium, revenue neutral increases in g and τ reduce fertility if and only if*

$$\frac{\tau}{\hat{z}}\frac{\partial \hat{z}}{\partial \tau} > \frac{(1-\gamma)}{z^*}(2+\frac{w_2}{w_1}) \tag{54}$$

The left hand side of this condition is the elasticity of domestic child care per child with respect to the tax rate. In the Galor-Weil model this is necessarily zero, while in the present model this is positive and larger, the larger the elasticity of substitution between domestic and bought-in child care, which in turn will depend on the quality of the latter. The right hand side will be smaller the greater the preference for children relative to consumption, the smaller the gender gap, and the more time women currently spend looking after children.

(ii) *Effects of the system of child support.* In this model, child support takes the form of a child payment g and possibly a subsidy for bought-in child care σ. Though increases in both can be expected to increase fertility, they have very different effects on female labor supply. An increase in g can be expected to reduce female labor supply, since it does not change the relative prices of the different forms of child care. An increase in σ on the other hand will induce a substitution of bought-in for domestic child care, and so this will tend to offset the effect of increasing fertility in reducing female labor supply. Moreover, the requirement that substitution of one form of support for the other be revenue neutral has important implications for the relative sizes of the changes that can be made. The overall outcome of these effects is given by

Proposition 3: *An increase in the subsidy to bought-in child care, σ, financed by a reduction in the child payment, g, increases both female labor* *supply and fertility.*

We can show[22] that the total derivative of fertility with respect to g is

$$\frac{dn^*}{dg} = -\frac{\tau w_1 n^*}{(p-g)}\frac{\partial \hat{z}}{\partial \sigma}\Delta^{-1} \tag{55}$$

where $\Delta < 0$ is the marginal cost[23] to the government of an increase in σ. Since $\partial \hat{z}/\partial \sigma < 0$, this expression is negative, implying that fertility increases with a reduction in g. We see that this increase is larger, the higher the female wage, the higher the tax rate, the greater the reduction in domestic child care resulting from the subsidy increase, and the smaller the marginal cost of the subsidy.

This result tells us that in two otherwise identical economies, the one which places more weight on subsidising bought-in child care and less on direct child payments will have both higher fertility and higher female labor supply.

(iii) *Interaction of tax and child support systems.* Next, we consider the effects of a reduction in the female tax rate financed by a reduction in the child payment g. Since the male tax rate remains unchanged, this can be thought of as a move away from a joint taxation system. Intuitively, one would think this must result in a reduction in fertility, especially given the *a priori* expectation that fertility and the female net wage are negatively related. However, Proposition 1 earlier showed that this need not be the case, and moreover, the revenue neutrality requirement can lead to counter-intuitive results because of the induced effects on female labor supply. Thus we have[24]

Proposition 4: *Revenue neutral reductions in the female tax rate and child payment certainly increase female labor supply, and increase fertility if and only if*

$$n^* \tau_f \frac{\partial \hat{z}}{\partial \tau_f} > 1 - \gamma \tag{56}$$

The intuition for this result is as follows. The larger is n^*, the more revenue is released by a cut in g, and so the larger is the revenue neutral cut in the

[22] See the Appendix.

[23] This consists of the direct cost of the subsidy, plus the public expenditures arising out of the induced increase in fertility, but net of the increased tax revenue resulting from the increased female labor supply.

[24] See the Appendix for the derivation

female tax rate.The greater the effect of the tax cut in reducing domestic time input per child, the larger will be the increase in bought-in child care and the more likely it is that both fertility and female labor supply will increase. Finally, the stronger the relative preference for children in the household utility function, as measured by γ, the more likely it is that fertility increases.

(iv) *Effects of the tax structure.* Suppose that initially joint taxation is the case, so that male and female tax rates are equal, but that a move is made in the direction of progressive individual taxation by reducing the female's tax rate τ_f and increasing that of the male, τ_m, in a revenue neutral way. We then have

Proposition 5: *A revenue neutral increase in the male tax rate and decrease in the female tax rate certainly increases female labor supply, and increases fertility if and only if*

$$\frac{\tau_f}{\hat{z}}\frac{\partial \hat{z}}{\partial \tau_f} > \frac{1-\gamma}{\gamma} \tag{57}$$

The left hand side is the elasticity of per unit domestic child care with respect to the tax rate. This is positive, and larger, the greater the elasticity of substitution between types of child care[25]. The right hand side is smaller, the greater the relative preference for children in the household utility function. There is certainly no reason in general why the condition may not hold.

The intuition underlying this condition is straightforward. An increase in the male tax rate reduces fertility by an income effect, a reduction in the female tax rate increases fertility by an income effect, while the increase in the implicit price of a child tends to reduce fertility. Consideration of the fertility demand function alone could then lead one to expect that the net effect on fertility would be negative. However, given the revenue neutrality requirement, the fact that female labor supply, and therefore the tax base, increases, means that the reduction in the female's tax rate can be larger than the increase in the male's tax rate[26], and so, for a high enough female labor supply elasticity, the overall effect can be a rise in fertility. Female

[25] The value of this elasticity of substitution will of course depend on the quality of bought-in child care, which we take as given, although, arguably, this is also a policy variable. Generally, anything which improves the quality of child care outside the home strengthens the conclusions of this paper.

[26] Also important here is the fact that the male's labor income is likely to be much larger than the female's.

labor supply can increase, even though fertility increases, because of the substitution of bought-in for domestic child care.

These results imply that in two otherwise identical economies, we would be likely to observe both higher fertility and higher female labor supply in an economy with progressive individual as opposed to joint taxation.

4 Heterogeneous households

Empirically there is a high degree of heterogeneity across households in two respects that are of interest for this paper. Households differ significantly in market labor supply of the female spouse, even after controlling for wage rates and demographic characteristics, and they also differ in the number of children, with a tendency for female labor supply and family size to be inversely related. We now want to explore some of the issues raised by this heterogeneity.

First we have to build into the model a reason for the heterogeneity. We follow an argument of Gary Becker (1976), to the effect that differences in physical and human capital across households can cause differences in productivity in domestic production - here, child care - and that specialisation will be in the direction of greater productivity. We therefore assume that there are two types of households, indexed $i = 1, 2$, that differ in their underlying child care production functions. Denoting the household that has higher productivity in child care as type 1, this is assumed to imply the inequalities[27]:

$$p_1(w_f, \sigma) < p_2(w_f, \sigma) \tag{58}$$

$$\hat{z}_1(w_f, \sigma) > \hat{z}_2(w_f, \sigma) \tag{59}$$

$$\hat{x}_1(w_f, \sigma) < \hat{x}_2(w_f, \sigma) \tag{60}$$

at all pairs of values (w_f, σ). The households are otherwise identical, in that they have the same preferences and face the same net wage rates, prior to any policy change. These assumptions imply in turn that $n_1^* > n_2^*$, $z_1^* > z_2^*$. The situation in respect of total bought-in child care is ambiguous, because although type-2 households are assumed to use more of this per child, they

[27] A simple example of a production function that has these results is $n_i = B_i z_i^{b_i} x_i^{1-b_i}$, $i = 1, 2$, with $B_1 > B_2$, $b_1 > b_2$.

have fewer children. However, we *assume* $x_1^* < x_2^*$. Thus we have two household types, one of which has more children, lower female labor supply and lower bought-in child care, both per child and in total. In this section we examine how the relative proportions of the two household types in the economy influence the way in which the tax structure and system of child support affect the relationship between fertility and female labor supply.

The extension to two household types has important implications for the interpretation of the tax system. Type-1 households have a lower income from market labor supply, and so, under a tax system in which the marginal tax rate depends on joint market income, they will face a lower tax rate and would have a higher net wage than type-2 households. Since we wish to assume, for simplicity, that the two household types face the same pre-change net wage, the tax rate τ will be interpreted as a flat rate income tax. First, we consider in this section the effects of an increase in the child payment, financed by an increase in the flat rate of income tax, which of course "rewards" higher fertility households and "punishes" lower fertility households by redistributing income from the latter to the former, as advocated by some policy analysts.We consider next the effects of a revenue neutral reduction in the tax rate on women and increase in that on men, which can be interpreted as a move towards progressive individual taxation. Finally we examine the effects of a revenue neutral reduction in the tax rate for type-1, and an increase in that for type-2 households, which can be interpreted as a move toward joint taxation with a tax rate depending on household market labor income. The aim in each of these comparative statics exercises is to see how the tax and child support systems interact to determine the relationship between female labor supply and fertility, this time in the presence of across-household heterogeneity.

It is important to note that, in this model, type-1 households have higher utility than type-2 households, because as (14) shows, the household with the lower implicit price of children will have a higher budget constraint. Thus the indirect utility function for a household of type i is

$$u_i = \gamma \ln \frac{\gamma W}{p_i(w_f, \sigma) - g} + (1-\gamma)\ln(1-\gamma)W(1+r_{t+1}) \tag{61}$$

The households have the same net full income W and receive the same payment g per child, and so

$$u_1 > u_2 \Leftrightarrow p_1(w_f, \sigma) < p_2(w_f, \sigma) \tag{62}$$

Recall that type 1 households have the lower net household income from market labor supply. This underlines the inadequacy of the latter as a welfare indicator in the presence of household production. It also means that the result in (62) would be strengthened if the tax system were progressive on the basis of joint market income, rather than flat rate as assumed here.

In the steady state, let N_{it} denote the number of households of type i, and $\phi_i \equiv N_{it}/N_t$ the proportion of type i households, which is assumed to stay constant over time.[28] N_{it} evolves according to

$$N_{it} = \phi_i N_t = \phi_i \sum_{j=1}^{2} n_j^* N_{j,t-1} \quad \phi_i \in (0,1), \quad \sum_i \phi_i = 1 \tag{63}$$

The government budget constraint in the steady state now becomes,

$$\sum_i N_{it}\{\tau[w_1(2 - z_i^*) + w_2] - g n_i^* - \sigma x_i^*\} = 0 \tag{64}$$

This says simply that tax revenue on labor incomes must cover expenditures on child payments and subsidies on child care, if any. Since $N_{it} = \phi_i N_t$, we can write the government budget constraint in average per household terms as

$$\tau[w_1(2 - \bar{z}^*) + w_2] - g\bar{n}^* - \sigma\bar{x}^* = 0 \tag{65}$$

where $\bar{z}^* \equiv \sum_i \phi_i z_i^*$, $\bar{n}^* \equiv \sum_i \phi_i n_i^*$, and $\bar{x}^* \equiv \sum_i \phi_i x_i^*$.

This government budget constraint implies, since $z_1^* > z_2^*$, and $n_1^* > n_2^*$, that type-1 households pay less tax and receive more in child payments than type-2 households[29]. The overall distributional effect depends also on the last component of the transfer. Empirically however, σ is typically very small, if not zero. Overall we conclude that there is a net transfer from type 2 to type 1 households, which is regressive, since the latter have higher utility. We now consider how the tax structure and child support system interact to determine the relationship between fertility and female labor supply in the case of heterogeneous households.

(i) *Increase in the child payment, financed by an increase in the tax rate.* The first point to note is that in this model this is regressive, since it increases

[28]This implies that not all the children of one household type form households of that type.

[29]This conclusion would be strengthened if the tax system were progressive on joint income rather than flat rate.

the size of the transfer from type-2 to type-1 households. To isolate the effect on fertility, total differentiation of the government budget constraint gives

$$d\tau = \lambda dg \tag{66}$$

where

$$\lambda \equiv \frac{\tau w_1 \frac{\partial \bar{z}^*}{\partial g} + \bar{n}^* + g\frac{\partial \bar{n}^*}{\partial g}}{w_1(2-\bar{z}^*) + w_2 - w_1\frac{\partial \bar{z}^*}{\partial \tau} - g\frac{\partial \bar{n}^*}{\partial \tau}} > 0 \tag{67}$$

λ represents the balanced budget marginal rate of substitution between τ and g, and is clearly larger, the larger the fall in female labor supply resulting from the increased tax rate. We then have the effect on fertility in each household type:

$$\begin{aligned} dn_i^* &= \frac{\partial n_i^*}{\partial \tau} d\tau + \frac{\partial n_i^*}{\partial g} dg & (68) \\ &= (\lambda \frac{\partial n_i^*}{\partial \tau} + \frac{\partial n_i^*}{\partial g}) dg & (69) \end{aligned}$$

Inserting the specific expressions for the two partial derivatives and rearranging gives

$$\frac{dn_i^*}{dg} < 0 \Leftrightarrow p_i - g > \frac{1-\tau}{\lambda} + (1-\tau)w_1\hat{z}_i \tag{70}$$

or

$$\hat{x}_i > g + \frac{1-\tau}{\lambda} \tag{71}$$

Thus if the expenditure on bought-in child care is sufficiently large, the net effect of the policy is to reduce fertility. This expenditure can be smaller, the lower the child payment, the higher the tax rate, and the higher the rate at which the tax rate must be increased to finance a given increase in the child payment. Taking the set of all households, this condition may be satisfied for all of them, type-2 households only (since $\hat{x}_2 > \hat{x}_1$), or none of them. In the first case average fertility certainly falls, in the second case we require the condition

$$\frac{d\bar{n}^*}{dg} = \sum_i \phi_i \frac{dn_i^*}{dg} < 0 \Leftrightarrow -\frac{dn_2^*}{dg} > \frac{\phi_1}{\phi_2}\frac{dn_1^*}{dg} \tag{72}$$

which is more likely to be satisfied, the higher the expenditure on bought-in child care in both household types and the greater the proportion of type-2 households in the population. We summarise this discussion in:

Proposition 6: *An increase in the child payment, funded by raising taxation, as well as being regressive, also reduces average per household fertility if (a) expenditure on bought-in child care is sufficiently large, in the sense of condition (71), for all households, or (b) this holds only for type-2 households, and the proportion of them in the population is sufficiently large in the sense of condition (72)*

(ii) *A move toward individual taxation, in which secondary earners pay a lower tax rate than primary earners.* The effect of this is to increase the net income of type-2 households and reduce that of type 1 households. Here we want to focus on the overall impact on fertility.

First, totally differentiating through the government budget constraint gives

$$d\tau_f = -\mu d\tau_m \tag{73}$$

where

$$\mu \equiv \frac{(w_1 + w_2) - \sum_i \phi_i(\tau_f w_1 \hat{z}_i + g)\frac{\partial n_i^*}{\partial \tau_m}}{w_1(1 - \bar{z}^*) - \sum_i \phi_i(\tau_f w_1 \frac{\partial z_i^*}{\partial \tau_f} + g\frac{\partial n_i^*}{\partial \tau_f})} \tag{74}$$

Since $\partial n_i^*/\partial \tau_m < 0$, the numerator is positive and gives the sum of the marginal increase in tax from increasing the male tax rate, and the increase in net revenue which results from the induced fall in fertility. The latter is the sum of the gain in tax revenue from the diversion of the mother's time away from child care, and the direct payment per child. The denominator is almost certainly positive and less than the numerator. The first term is the marginal effect on tax revenue of a change in the female tax rate, and is smaller, the smaller the average household female labor supply. It is certainly less than $w_1 + w_2$. The term $\tau_f w_1 \frac{\partial z_i^*}{\partial \tau_f} > 0$, if female labor supply increases with the net wage, which seems empirically to be the case, and is larger, the more elastic is female labor supply with respect to the net wage. Finally, we know from the preceding section[30] that $\frac{\partial n_i^*}{\partial \tau_f} > \frac{\partial n_i^*}{\partial \tau_m}$, and so the last term in the denominator is certainly larger than the corresponding term in the numerator. Thus we conclude $\mu > 1$. Moreover, μ will be larger, the smaller the existing average female labor supply per household and the larger the elasticity of average household female labor supply with respect to the net wage.

[30] Clearly $w_1(z_i^* - \gamma) > -\gamma(w_1 + w_2)$.

Turning now to average fertility $\bar{n}^*$, we have

$$d\bar{n}^* = \sum_i \phi_i(\frac{\partial n_i^*}{\partial \tau_m} d\tau_m + \frac{\partial n_i^*}{\partial \tau_f} d\tau_f) \quad (75)$$

$$= \sum_i \phi_i(\frac{\partial n_i^*}{\partial \tau_m} - \mu \frac{\partial n_i^*}{\partial \tau_f}) d\tau_m \quad (76)$$

A sufficient condition for $d\bar{n}^* > 0$ is that the term in brackets, dn_i^*, be positive for both i. A necessary condition is that it be positive for at least one i. To evaluate this term we substitute for the partial derivatives and rearrange to obtain the condition

$$dn_i^* > 0 \Leftrightarrow \mu > \frac{1 + \frac{w_2}{w_1}}{1 - \frac{z_i^*}{\gamma}} \quad (77)$$

We can intepret the numerator on the right hand side as a measure of the "gender gap", since w_2 is the difference between the male and female wage. We saw in Proposition 1 that the smaller is z_i^*/γ, the more likely it is that a rise in the female net wage increases fertility. Finally, by definition μ measures the rate at which τ_f can be reduced as τ_m increases. Since $z_1^* > z_2^*$, it is more likely that this condition will be satisfied for type 2 households than for type 1. We summarise in

Proposition 7: *The smaller is the gender gap, the lower the level of domestic child care relative to the strength of preference for children, the larger the rate of substitution between primary and secondary earner tax rates, and the larger the proportion of type-2 households in the economy, the more likely is it that a revenue neutral move toward individual taxation will increase average fertility.*

(iii) *The effects on average fertility of revenue neutral changes in tax rates on households.* We consider the effects of revenue neutral changes $d\tau_2 > 0 > d\tau_1$, where τ_i is the tax rate paid by household of type $i = 1, 2$, and the rates are equal initially. The balanced budget requirement implies that

$$d\tau_2 = -\theta \frac{\phi_1}{\phi_2} d\tau_1 \quad (78)$$

where

$$\theta \equiv \frac{w_1(2 - z_1^*) + w_2 - \tau_1 w_1 \frac{\partial z_1^*}{\partial \tau_1} - g\frac{\partial n_1}{\partial \tau_1}}{w_1(2 - z_2^*) + w_2 - \tau_2 w_1 \frac{\partial z_2^*}{\partial \tau_2} - g\frac{\partial n_2}{\partial \tau_2}} > 0 \quad (79)$$

The change in average fertility is

$$d\bar{n}^* = \phi_1 \frac{\partial n_1^*}{\partial \tau_1} d\tau_1 + \phi_2 \frac{\partial n_2^*}{\partial \tau_2} d\tau_2 \tag{80}$$

$$= \phi_1 \left(\frac{\partial n_1^*}{\partial \tau_1} - \theta \frac{\partial n_2^*}{\partial \tau_2}\right) d\tau_1 \tag{81}$$

Then we have

Proposition 8: *if the condition in (21) of Proposition 1 is satisfied for type 2 households and not for type-1 households, then regardless of the proportions of the two types in the population, average fertility falls when* $d\tau_2 > 0 > d\tau_1$.

The reason for this is simply that an increase in the net female wage reduces fertility in type-1 households, as in the Galor-Weil model, while a fall in the net wage also reduces fertility of type 2 households, if (21) holds, and so fertility must fall, i.e. in (74) $d\bar{n}^* < 0$,because $\partial n_1^*/\partial \tau_1 > 0$, $\partial n_2^*/\partial \tau_2 < 0$ and $d\tau_1 < 0$.

5 Conclusions

Historically in virtually all developed economies there seems to be clear evidence of an inverse relationship between female labor supply and fertility. However, particularly in the last decade or so, the relationship across countries has been positive: for example countries like Germany, Italy and Spain with the lowest fertility rates also have the lowest female participation rates. We accept the hypothesis that the reason for this lies in the combined effects of a country's tax system and system of child support, and we have sought to clarify this theoretically, using an extended version of the Galor-Weil model. The results do suggest that countries with individual rather than joint taxation, and which support families through improved availibility of alternatives to domestic child care, rather than through direct child payments, are likely to have both higher female labor supply and higher fertility. These results are strengthened when we take account of the heterogeneity among households which undoubtedly exists. The presence of a significant proportion of households with relatively high levels of child care outside the home increases the likelihood of a positive relationship between fertility and female labor supply. Thus a reversal of the trend in fertility, which many regard as vital to resolve the problems for social security programs presented by ageing populations, need not be bought at the cost of significant reductions in female labor supply.

Appendix

In order to focus on the primary concerns of this paper, it is useful to begin by excluding the effects of changes in w_1 and k, which are in any case usually ambiguous. We assume therefore that w_1 stays constant throughout the comparative statics analysis[31]. We also assume that initially $\sigma = 0$, which is reasonably realistic and reduces the dimensionality of the system to be studied. This means that we can work with a relatively simple steady state system:

$$n^* - \frac{\gamma[(1-\tau_f)w_1 + (1-\tau_m)(w_1+w_2)]}{p[(1-\tau_f)w_1, \sigma] - g} = 0 \tag{82}$$

$$z^* - \hat{z}[(1-\tau_f)w_1, \sigma]n^* = 0 \tag{83}$$

$$\tau_f w_1(1-z^*) + \tau_m(w_1+w_2) - gn^* - \sigma x^* = 0 \tag{84}$$

where under a flat rate taxation system $\tau_f = \tau_m = \tau$. The relevant partial derivatives are

$$\frac{\partial n}{\partial \tau_f} = \frac{w_1 n\hat{z} - \gamma w_1}{p-g} \mathsf{R}\ 0 \tag{85}$$

$$\frac{\partial n}{\partial \tau_m} = \frac{-\gamma(w_1+w_2)}{p-g} < 0 \tag{86}$$

$$\frac{\partial n}{\partial \tau} = \frac{\partial n}{\partial \tau_f} + \frac{\partial n}{\partial \tau_m} \mathsf{R}\ 0 \tag{87}$$

$$\frac{\partial n}{\partial \sigma} = \frac{x}{p-g} > 0 \tag{88}$$

$$\frac{\partial n}{\partial g} = \frac{n}{p-g} > 0 \tag{89}$$

$$\frac{\partial \hat{z}}{\partial \tau_f} = -w_1 \frac{\partial \hat{z}}{\partial(1-\tau_f)w_1} > 0 \tag{90}$$

$$\frac{\partial \hat{z}}{\partial \sigma} = -\frac{\partial \hat{z}}{\partial(1-\sigma)} < 0 \tag{91}$$

[31] The assumption of constant gross wage rates is quite usual in the analysis of tax policy.

We analyse the cases (i) - (iv), and give proofs for the corresponding Propositions 2 - 5, in turn.

Case (i): Here we take an exogenous increase in g, with n^*, z^* and τ endogenous, and τ increasing to fund the increase in child payment. The comparative statics results are derived from the linear system

$$\begin{bmatrix} 1 & 0 & -\frac{\partial n}{\partial \tau} \\ -\hat{z} & 1 & -n^* \frac{\partial \hat{z}}{\partial \tau} \\ -g & -\tau w_1 & w_1(2-z^*)+w_2 \end{bmatrix} \begin{bmatrix} dn^* \\ dz^* \\ d\tau \end{bmatrix} = \begin{bmatrix} \frac{\partial n}{\partial g} \\ 0 \\ n^* \end{bmatrix} dg \tag{92}$$

The determinant of the left hand matrix is

$$\Delta = w_1(2-z^*) + w_2 - \tau w_1 n^* \frac{\partial \hat{z}}{\partial \tau} - \frac{\partial n}{\partial \tau}(\tau w_1 \hat{z} + g) \tag{93}$$

This expression is the net effect on tax revenue of an increase in the tax rate, and so it is reasonable to assume $\Delta > 0$. We then have

$$\frac{dn^*}{dg} = \frac{\frac{\partial n}{\partial g}(w_1(2-\hat{z}) + w_2 - \tau w_1 n^* \frac{\partial \hat{z}}{\partial \tau}) + n^* \frac{\partial n}{\partial \tau}}{\Delta} \tag{94}$$

$$\frac{dz^*}{dg} = \frac{\frac{\partial n}{\partial g}[\hat{z}(w_1(2-z^*)+w_2) + gn^* \frac{\partial \hat{z}}{\partial \tau}] + n^*(\hat{z}\frac{\partial n}{\partial \tau} + n^* \frac{\partial \hat{z}}{\partial \tau})}{\Delta} \tag{95}$$

By inserting the specific expressions for the derivatives into these equations and rearranging, we obtain the conditions

$$\frac{dn^*}{dg} < 0 \Leftrightarrow \frac{\tau}{\hat{z}}\frac{\partial \hat{z}}{\partial \tau} > \frac{(1-\gamma)}{z^*}(2 + \frac{w_2}{w_1}) \tag{96}$$

$$\frac{dz^*}{dg} > 0 \Leftrightarrow p\frac{n^*}{\hat{z}}\frac{\partial \hat{z}}{\partial \tau} + (1-\gamma)(2w_1 + w_2) > 0 \tag{97}$$

The latter condition is obviously satisfied since all terms on the right are positive. The former condition is discussed in the text.

Case (ii): Here we take an exogenous change in g, treating n^*, z^* and σ as endogenous. In this case we have the system

$$\begin{bmatrix} 1 & 0 & -\frac{\partial n}{\partial \sigma} \\ -\hat{z} & 1 & -n^* \frac{\partial \hat{z}}{\partial \sigma} \\ -g & -\tau_f w_1 & -x^* \end{bmatrix} \begin{bmatrix} dn^* \\ dz^* \\ d\sigma \end{bmatrix} = \begin{bmatrix} \frac{\partial n}{\partial g} \\ 0 \\ n^* \end{bmatrix} dg \tag{98}$$

The determinant of the left hand matrix is

$$\Delta = -(x^* + (g + \tau_f w_1 \hat{z})\frac{\partial n}{\partial \sigma}) - \tau_f w_1 n^* \frac{\partial \hat{z}}{\partial \sigma} \tag{99}$$

This is the net impact on the government budget of an increase in the subsidy σ, and it is reasonabe to assume $\Delta < 0$, despite the fact that the last term is positive because $\frac{\partial \hat{z}}{\partial \sigma} < 0$. This last term gives the increase in tax revenue arising from the increase in female labor supply induced by an increase in the subsidy.

It is straightforward to show that

$$\frac{dn^*}{dg} = \frac{-\tau_f w_1 n^* \frac{\partial \hat{z}}{\partial \sigma}\frac{\partial n}{\partial g}}{\Delta} < 0 \tag{100}$$

so that a revenue neutral increase in σ and reduction in g increases fertility. Similarly one can show that

$$\frac{dz^*}{dg} = \frac{(n^* + g\frac{\partial n}{\partial g})\frac{\partial \hat{z}}{\partial \sigma}}{\Delta} > 0 \tag{101}$$

so that female labor supply certainly rises when g falls. In both cases the effects are greater, the larger is $\frac{\partial \hat{z}}{\partial \sigma}$ in absolute value, *i.e.*the larger the elasticity of substitution between the two types of child care.

Case (iii): Here we consider a balanced budget reduction in both g and τ_f, *i.e.* a reduction in the female tax rate funded by cutting the child payment. Thus we take an exogenous dg, with n^*, z^* and τ_f endogenous. We have the system

$$\begin{bmatrix} 1 & 0 & -\frac{\partial n}{\partial \tau_f} \\ -\hat{z} & 1 & -n^* \frac{\partial \hat{z}}{\partial \tau_f} \\ -g & -\tau_f w_1 & w_1(1 - z^*) \end{bmatrix} \begin{bmatrix} dn^* \\ dz^* \\ d\tau_f \end{bmatrix} = \begin{bmatrix} \frac{\partial n}{\partial g} \\ 0 \\ n^* \end{bmatrix} dg \tag{102}$$

This yields the total derivatives

$$\frac{dz^*}{dg} = \frac{\frac{\partial n}{\partial g}(w(1 - z^*)\hat{z} + gn^* \frac{\partial \hat{z}}{\partial \tau_f}) + n^*(\hat{z}\frac{\partial n}{\partial \tau_f} + n^* \frac{\partial \hat{z}}{\partial \tau_f})}{\Delta} \tag{103}$$

$$\frac{dn^*}{dg} = \frac{\frac{\partial n}{\partial g}(w(1 - z^*) - \tau_f w_1 n^* \frac{\partial \hat{z}}{\partial \tau_f}) + n^* \frac{\partial n}{\partial \tau_f}}{\Delta} \tag{104}$$

where $\Delta > 0$ as in case (i). Inserting the specific expressions for the partial derivatives and cancelling terms then gives

$$\frac{dz^*}{dg} = \frac{(1-\gamma)w_1 + p\frac{n^*}{\hat{z}}\frac{\partial \hat{z}}{\partial \tau_f}}{\Delta} > 0 \tag{105}$$

so that reducing g certainly increases female labor supply, and

$$\frac{dn^*}{dg} = \frac{(1-\gamma) - n^*\tau_f \frac{\partial \hat{z}}{\partial \tau_f}}{\Delta} \gtreqless 0 \tag{106}$$

For a reduction in g to increase fertility we require this to be negative, which implies the condition given and discussed in the text.

Case (iv). Here we take an exogenously given $d\tau_m$, with n^*, z^* and τ_f endogenous. The corresponding differentials dn^*, dz^*, $d\tau_f$ must satisfy the government budget constraint[32], *i.e.* must be revenue neutral. The comparative statics results are derived from the linear system

$$\begin{bmatrix} 1 & 0 & -\frac{\partial n}{\partial \tau_f} \\ -\hat{z} & 1 & -n^*\frac{\partial \hat{z}}{\partial \tau_f} \\ -g & -\tau_f w_1 & w_1(1-z^*) \end{bmatrix} \begin{bmatrix} dn^* \\ dz^* \\ d\tau_f \end{bmatrix} = \begin{bmatrix} \frac{\partial n}{\partial \tau_m} \\ 0 \\ -(w_1+w_2) \end{bmatrix} d\tau_m \tag{107}$$

The determinant of the left hand matrix is

$$\Delta = w_1(1-z^*) - \tau_f w_1 n^* \frac{\partial \hat{z}}{\partial \tau_f} - (g + \tau_f w_1 \hat{z})\frac{\partial n}{\partial \tau_f} \tag{108}$$

This term is the net marginal tax revenue from an increase in τ_f, taking into account the loss of revenue resulting from a reduction in female labor supply (the second term) and the cost associated with an increase in fertility (the third term). We assume that $\Delta > 0$, since otherwise tax revenue could be increased by reducing tax rates. We then have that

$$\frac{dn^*}{d\tau_m} = \frac{\frac{\partial n}{\partial \tau_m}(w_1(1-z^*) - \tau_f w_1 n^* \frac{\partial \hat{z}}{\partial \tau_f}) - (w_1+w_2)\frac{\partial n}{\partial \tau_f}}{\Delta} \tag{109}$$

$$\frac{dz^*}{d\tau_m} = \frac{\frac{\partial n}{\partial \tau_m}(w_1(1-z^*)\hat{z} + gn^*\frac{\partial \hat{z}}{\partial \tau_f}) - (w_1+w_2)(\hat{z}\frac{\partial n}{\partial \tau_f} + n^*\frac{\partial \hat{z}}{\partial \tau_f})}{\Delta} \tag{110}$$

Inserting the relevant partial derivatives and rearranging then gives the results reported in Proposition 5 of the text.

[32] We can ignore dx^* because of the assumption $\sigma = 0$.

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Figure 1: TFR and %female employment 1970

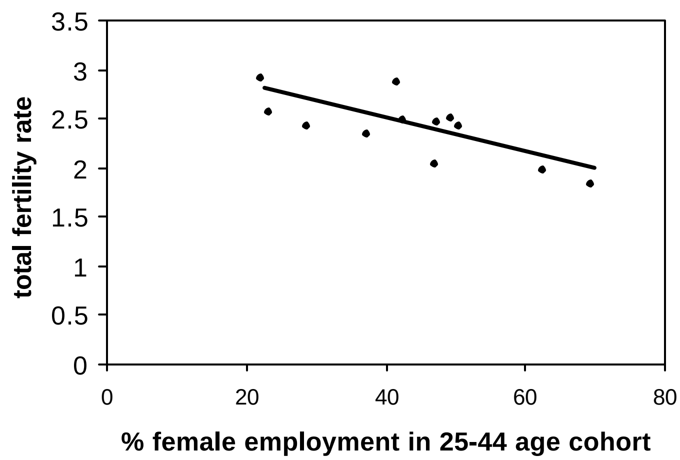

Figure 2:TFR and %female employment 1990

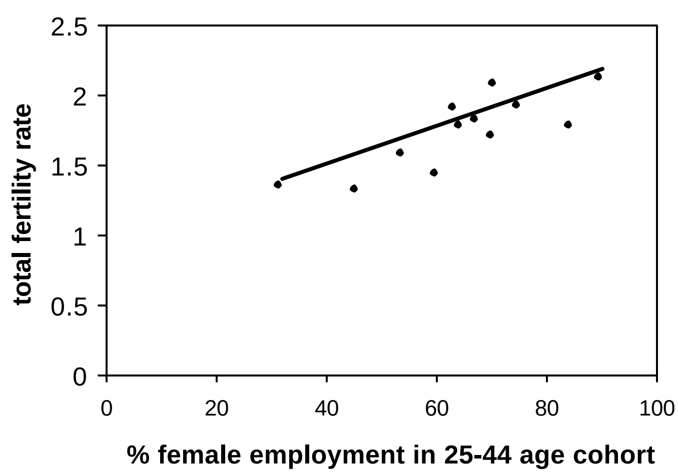

Table 1: % female employment and TFR

Year	1970		1990	
	% fem emp 25-44*	**TFR****	**% fem emp 25-44***	**TFR****
Germany	47.15	2.03	59.50	1.45
Australia	41.60	2.86	63.05	1.91
UK	50.50	2.43	67.05	1.83
US	42.55	2.48	70.05	2.08
Canada	37.15	2.33	69.75	1.71
NL	23.30	2.57	53.30	1.59
Sweden	62.55	1.97	89.45	2.13
Norway	49.45	2.50	74.70	1.93
Finland	69.50	1.83	84.15	1.78
France	47.35	2.47	63.85	1.78
Italy	28.80	2.42	44.95	1.33
Spain	22.10	2.9	31.25	1.36

*Source: OECD Labour Force Statistics: from tables for participation and unemployment for women aged 25 to 44 years

**TFR: average number of children a woman would expect to have if she were to experience all of the age-specific birth rate occurring in that year.

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