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The Ambiguous Effect of Minimum Wages on Workers and Total Hours

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ABSTRACT

The Ambiguous Effect of Minimum Wages on Workers and Total Hours*

We model a competitive labour market where firms choose combinations of workers and hours per worker to produce output. If one assumes that the scale of production has no impact on hours per worker, then the change in the number of workers and hours per worker resulting from a minimum wage are inversely related. We demonstrate that total hours worked at the firm may rise for plausible parameter values if there are small fixed costs to hiring workers. Thus, in contrast to the conventional view, we show that the effect of minimum wages on employment is ambiguous.

JEL Classification: J22, J38

Keywords: minimum wages, hours, employment

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Section I: Introduction

While opinions appear to be split amongst economists and others on whether minimum wages are desirable or not and the reasons for supporting or opposing this policy are varied [see Klein and Gompers (2007)], evidence from minimum wage studies has frequently been used to assess whether labour markets are well approximated by the competitive model. The underlying rationale for this belief is that the competitive model predicts that a minimum wage has a negative impact on employment, while alternative models such as the monopsony model posit a positive relationship. For example, Neumark and Wascher (2007) note in their review of the empirical literature: " .. *we hope our review will help readers assess alternative models of the labour market*" (p.5).[2] However, Neumark and Wascher are careful to stress the limitations of the theoretical predictions noting that *"...even in the neoclassical model, the effect of the minimum wage on any given set of workers will depend on, among other things, the elasticities of substitution across different types of workers and cross elasticities of demand across different types of goods."*

In this paper we argue that in fact the results of empirical research on the employment effects of minimum wages can tell us little in terms of providing evidence for competing models of the labour market. To demonstrate this we examine the predictions of the standard competitive model where firms choose the number of workers and hours per worker and pay compensating differentials for different levels of hours per worker[3]. Our results show that the impact of minimum

[2] Neumark and Wascher in their introduction and throughout chapter 3.2 document several studies that suggest the sign of the employment effect from a minimum wage is evidence for these competing models.
[3] While there exists an abundance of studies empirically estimating the employment effects of the minimum wage [see Neumark and Wascher (2007) for a survey], this literature often estimates changes in the number of employees or sometimes the number of full-time equivalents. Moreover, the

wages on hours per worker, the number of workers, and total hours worked are indeed ambiguous.

The results from our model stand of course in stark contrast to the general view amongst economists that in a competitive labour market a minimum wage will reduce employment. For example, as Stigler (1946) noted: "The higher the minimum wage, the greater the number of covered workers who are discharged". Importantly, however, this statement is true only in a theoretical framework where the labour input can be thought of as total hours and total hours is defined the product of workers and hours per worker. One can arguably though think of a number of different reasons why firms may not take the labour input in the production function as the product of hours and workers. The most basic one is if hours per worker have a diminishing marginal product. Alternatively, firms may have different hours technologies – for example, a long haul trucking company with a couple of large trucks may want a small number of workers with long hours, while a local delivery service may be able to have a large number of workers using the same vehicle. It may also be that differences in the firms demand for the mix of bodies and hours come from the demand conditions facing the firm. For instance, a restaurant in an office district may be very busy for short periods and require a large number of part time workers, while a high street restaurant may be busy over longer periods that facilitate hiring a larger share of full-time workers. Importantly in this regard, we show here that once one explicitly models a firm's choice of workers and hours and allow firms to have a technology that puts more weight on the number of workers rather than hours per worker (or vice versa) in a standard neoclassical framework, then the impact of a minimum wage on hours per worker, the number of workers, and on total hours

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empirical evidence with regard to the effect of a minimum wage on hours per worker is rather mixed; see, for instance, Katz and Kruger (1992), Brown (1999), Zavodney (2000), and Gregory (2002).

worked becomes ambiguous.

One may want to note that even in non-competitive models of the labour market the effect of a minimum wage on employment is not clear. For instance, while Stigler (1946) noted the theoretical possibility that minimum wages may increase employment in a monopsony model, he discounted the importance of such models. A more recent literature argues that the Monopsony model is perhaps more relevant in modern labour markets [See Manning (2003)]. Bhaskar and To (1999), Walsh (2003) and Strobl and Walsh (2007) present more recent theoretical models showing the ambiguity of minimum wage employment effects under Monopsony. De Fraja (1999) shows that the employment effects of a minimum wage are small in a model with heterogeneity in workers preferences over wages and working conditions and Rebitzer and Taylor (1995) show that minimum wages may increase employment in an efficiency wage model where monitoring becomes more difficult as employment increases. Thus, we argue here that regardless of whether one takes a competitive or a non-competitive view of the labour market, the theoretically derived effect of a minimum wage on employment is ambiguous.

In the next section we outline the general framework of our model. In Section III we further illustrate our results by using the specific example of Cobb-Douglas type technology. The final section concludes.

Section II: The Model

The theoretical treatment of minimum wages in the literature when firms choose a combination of hours per worker and workers is rather limited.[4] In this paper

[4] Hamermesh (1993) develops a framework that deals with the firm's choice of workers and hours in a cost minimisation framework and includes a brief discussion of minimum wages, while Michl (2000) outlines a model where firms choose workers and hours and the wage does not increase with hours. Other studies, such as Stewart and Swaffield (2006), Zavodney (2000), Neumark and Schweitzer

we apply a minimum wage to Kinoshita's (1987) model which derives the equilibrium properties of a competitive labour market. Figure 1 illustrates the equilibrium in a compensating differentials model applied to hours worked graphically. The *u0..u2* curves represent indifference curves of workers with different preferences over hours worked[5], while *π1..π2* represent isoprofit curves for firms with different production functions where some may prefer workers to work longer hours. The equilibrium hourly wage hours locus *w(h)* is a set of tangencies where workers who wish to work longer hours match up with like minded firms. In equilibrium the supply and demand of each worker type are equal and no worker or firm can gain from deviating to another point on the locus. Compensating wage differentials are paid to workers for working a less desirable number of hours[6]. Firms are assumed to be able to hire as many workers as they wish at any level of hours *(h)*.[7] It may be worth emphasising that the fact that firms will choose the wage in the model below is not indicative of any market power. Firms are price takers and can hire as many workers as they wish at any given level of hours. In a competitive labour market though, the wage needed to induce different levels of hours will differ and firms must choose a point on the equilibrium wage hours locus.

(2000), and Connolly and Gregory (2002) contain more general discussions on how mimimum wages are related to hours.

[5] While we have drawn the indifference curves and the wage hours locus to have positive slopes, but theoretically they may slope downwards for some points since it is hourly wage rather than than total earnings on the vertical axis.

[6] The models of Lewis (1969) and Rosen (1986) are the precursors to this model.

[7] Of course in what has been referred to as "the canonical model of labour supply" it is sometimes assumed for simplicity that workers may choose to work any number of hours at a fixed wage. In practice many labour supply studies allow for non-linearities in the workers budget constraint that allow for non-constant hourly wages at different hours worked, see Blundel (1999)

While there must be a continuum of either worker or firm types (or both) to generate a continuous equilibrium locus of tangencies between worker indifference curves and firm's isoprofit curves, in our formal analysis we only look at the worker and firm who are located at the point on the locus where a minimum wage is just binding. That is in theory there may be other firms with lower wages prior to the minimum wage who are affected differently. Also, the model below is partial equilibrium, meaning that we ignore any potential impact on firms with lower wages and the possible impact of the minimum wage on the shape of the equilibrium wage hours locus and how this may affect employment[8]. On the other hand we show how firms respond directly to a minimum wage. One should also note that our results are not driven by general equilibrium effects or uncertainty about how the firm will substitute across different types of workers.

The firm's profit function is:

$$\Pi(n,h) = pf[n,h(w)] - wh(w)n - kn \tag{1.1}$$

The output price p is given to the firm and the production function $q = f(h,n)$ satisfies $f_n > 0$, $f_h > 0$, $f_{nn} < 0$, $f_{hh} < 0$, where h is hours per worker , n is the number of workers and q is output There are fixed costs k per worker[9]. The firm's choice of w and n at an interior solution satisfies the following first order conditions:

$$\begin{aligned} \Pi_w(w,n) &= pf_h[n,h(w)]h_w(w) - wh_w(w)n - h(w)n = 0 \\ \Pi_n(w,n) &= pf_n[n,h(w)] - wh(w) - k = 0 \end{aligned} \tag{1.2}$$

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[8]There is mixed evidence on the impact of minimum wages on the wage distribution. For instance, Card and Krueger (1995) find some spillover effects for the US while Dickens and Manning (2004) find these effects to be negligible for the U.K. for example. It is difficult to know, however, how such changes may impact on the equilibrium relationship between hours and hourly wages if at all.

[9] If training costs were convex the firm would effectively behave as a monopsonist since worker costs would increase with employment, [see Manning (2003) p34-35].

One can assess the impact of a minimum wage on the number of workers by totally differentiating the first order condition on *n*. Evaluating this differential at the initial equilibrium we get:

$$\frac{dn}{dw} = \frac{-\Pi_{nw}}{\Pi_{nn}} = \frac{[\frac{f_h}{n} - f_{nh}]}{f_{nn}} h_w \qquad (1.3)$$

Next we assume that the scale of production does not affect the optimal choice of hours per worker, other things equal. Hamermesh (1993) notes, for example, that *"...there is no evidence that weekly hours of full-time workers at General Motors differ substantially from hours of workers at the local steel fabricator"*(p.50). If this assumption holds, we show in the Appendix using the firms cost minimisation problem that the sign of impact of a minimum wage on the number of workers is the negative of the slope of the hourly wage hours locus [10]:

$$\frac{dn}{dw} = \frac{[\frac{f_h}{n} - f_{nh}]}{f_{nn}} h_w = -\frac{f_h}{f_n} h_w \qquad (1.4)$$

Figure 2, which in contrast to Figure one depicts isocost/isoquant graph for an individual firm, illustrates the intuition for this result.[11] The isocost curve gives the employment hours combination that are available at a fixed level of cost: $C_0 = wh(w)n + nk$. The isoquant shows the combinations of hours and workers that give a fixed level of output: $y_o = L(n,h)$ where y is the aggregate labour input. An arrow indicates the initial equilibrium [point (a)] where the isocost and isoquant are tangent at n_0 and h_0. When a minimum wage is imposed and the wage hours locus is

[10] In fact the assumption need only be true for a small deviation from equilibrium output for the analysis to go through. Many of the most commonly used functional forms used for the labour aggregator such as the class of functions $f(n,h) = An^{\alpha}x(h)$ satisfy this assumption. A and α are positive constants and x(h) is a positive function

[11] The graphical analysis used here draws on the analysis used in Hamermesh (1993).

upward sloping, the firm is forced to pay a higher wage, but gets a higher level of hours per worker in return. The firm substitutes from workers into hours moving to the point indicated by the arrow labelled "substitution effect" at point (b). If output was fixed one could be certain that the minimum wage would increase hours per worker and lower the number of workers. In fact, as can be seen, the original output would then lie on a higher isocost line and cost more so that one would expect output to adjust downward to a lower y_1 isoquant. To deal with the (remote) possibility that the number of hours per worker is an inferior input and that this reduction in output could be associated with a switch from hours to workers that could in theory more than offset the initial substitution away from workers, we assume that scale effects on hours are zero. Once one assumes this one can say for certain that the number of workers will fall as long as the wage hours locus slopes upward.

Equation (1.4) shows that if the hourly wage-hours per worker locus has a positive (negative) slope one would expect firms to use the minimum wage to increase (decrease) hours per worker and decrease (increase) the number of workers at a given level of output. A puzzling implication of (1.4) is that since much of the existing empirical evidence suggests a decline in hours from a minimum wage[12], then (1.4) suggests that affected firms decrease hours and increase the number of workers. This implies that workers are on a negatively sloped hourly wage-hours locus. There is no reason that this should not be so in the theory, but one may suspect that many economists would expect the contrary. For example Hamermesh (1993) assumes the equilibrium locus has a positive slope in his treatment of the theory of hours per workers, while Michl (2000) assumes the locus is flat. In any case equation (1.4) should make us reluctant to conclude that one can infer whether the labour market is

[12] See the review by Neumark and Wascher (2007) or Brown (1999).

competitive or not by looking at the results of studies that focus on the number of workers, as much of the literature does, since theory has no clear prediction on the change in hours per worker and predicts an offsetting change in the number of workers.[13]

While it has been pointed out in the literature that when one accounts for the possibility that hours per worker may fall in response to a minimum wage, this may be associated with an increase in the number of workers, even in the competitive model, the general belief is that total hours cannot increase. As Neumark and Wascher (2007) p166 note *"..although much of the literature has focused on the employment effects of the minimum wage, the predictions of theory tend to be about overall labour input rather than employment specifically..* (p.166)*".* The empirical studies that do try and estimate the impact on the overall labour input generally focus on total hours.[14]

We define the elasticity of output with respect to workers *(n)* and hours per worker *(h)*, respectively, as $f_h \frac{h}{f} = \varepsilon_{qh}$ and $f_n \frac{n}{f} = \varepsilon_{qn}$. It follows from the first order conditions (1.2) that:

$$h_w = \frac{h}{w}\left[\frac{1}{(1+\frac{k}{wh})\frac{\varepsilon_{qh}}{\varepsilon_{qn}} - 1}\right] \qquad (1.5)$$

If one thinks of employment as total hours *(nh)*, then using (1.4) and (1.5) the employment effect would be:

[13] See Neumark and Wascher (2007) for examples. Some studies do account for hours. For instance, Michl (2000) provides evidence and some theory to suggest that decreases in hours could explain the positive employment effects found in Card and Kruegers well known (1995)study of the New Jersey minimum wage increase.
[14] This is often approximated by measuring employment as the number of full-time equivalent workers.

$$\frac{d(nh)}{dw} = n\frac{dh}{dw} + h\frac{dn}{dw} = n(1 - \frac{\varepsilon_{qh}}{\varepsilon_{qn}})h_w = -\frac{nh}{w}[\frac{\frac{\varepsilon_{qh}}{\varepsilon_{qn}} - 1}{(1+\frac{k}{wh})\frac{\varepsilon_{qh}}{\varepsilon_{qn}} - 1}] \quad (1.6)$$

The elasticity of total hours *(nh)* with respect to the wage is:

$$\varepsilon_{nh,w} = -[\frac{\frac{\varepsilon_{qh}}{\varepsilon_{qn}} - 1}{(1+\frac{k}{wh})\frac{\varepsilon_{qh}}{\varepsilon_{qn}} - 1}] \quad (1.7)$$

Total hours will increase from a minimum wage if:

$$\frac{k}{wh} > \frac{\varepsilon_{qn} - \varepsilon_{qh}}{\varepsilon_{qh}} > 0 \quad (1.8)$$

One should note from (1.7) that when fixed costs *(k)* are equal to zero then $\varepsilon_{nh,w} = -1$ and a minimum wage reduces total hours proportionately. However (1.8) also indicates that when $\varepsilon_{qn} > \varepsilon_{qh}$, if fixed costs as a fraction of the wage bill lie above a certain threshold, then total hours will increase in response to a minimum wage[15]. Also, one does not need extreme values for the parameters for total hours to increase. For example, when $\varepsilon_{qn} > \varepsilon_{qh}$ but the elasticity of output with respect to workers and hours per worker are similar, the presence of small fixed costs will ensure a positive effect.

One can establish the following proposition:[16]

If $\varepsilon_{qn} > \varepsilon_{qh}$ a minimum wage will increase total hours worked if the hours per worker, hourly wage locus has a positive slope.

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[15] Feldstein (1967) and Michl (2000) amongst others explicitly assume $\varepsilon_{qn} > \varepsilon_{qh}$.

[16] A maple file solving the model explicitly and showing the model is well behaved over parameter ranges where total hours increase, where there are representative workers and firms with Cobb-Douglas utility and production functions is excluded because of space limitations but is available from the authors.

Proof: From (1.5) if the hourly wage locus has a positive slope the denominator of (1.7) is positive. This implies that (1.7) is positive since $\varepsilon_{qn} > \varepsilon_{qh}$.

We can use the graphical framework to illustrate how a minimum wage may increase total hours. Figure 3 is the same as Figure 2 but with the level curve H_0 (the thick dotted line added). Along the H0 level curve total hours (nh) are constant. We see that when the level curves for the labour aggregator are steeper than H0, then the original H0 level curve may lie below the new post minimum wage equilibrium at point (c), implying that the new equilibrium at point (c) would have higher total hours. We note that, other things equal, fixed costs on hiring workers will make the isocost curve (C0) steeper making the initial equilibrium more hours intensive

Section III: The Cobb-Douglas example

In this section we illustrate our results by assuming a Cobb-Douglas technology over hours and workers: $f(n,h) = h^a n^b$. A commonly used approach is to model the aggregate labour labour input as $L = n^{\eta} h^{\delta}$.[17] It also is important to remember that the technology above is for producing labour inputs and that one might expect that there is a diminishing marginal product associated with the labour input. One example would be $f(L) = L^{\beta} = h^{\delta\delta} n^{\beta\eta} = h^a n^b$[18] where both *a* and *b* are less than unity. Relative to *b*, a small value for *a* means a worker intensive production function and a large value for *a* means an hours intensive production function. Of course this Cobb-Douglas example is a very simple special case, but the point is to show that even in this very simple but reasonable special case one can illustrate the

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[17] See Hamermesh (1993) for example.

[18] Another way of looking at it is that there are s other inputs $x_1..x_s$ in addition to labour L and a Cobb-Douglas production function $f(x_1,..x_s, L) = x_1^{\alpha_1}...x_s^{\alpha_s} L^b$. The comparative static analysis would be much more complicated if we add more inputs, but the example shows that it would be plausible to have a production function where the weights on h and n would be less than unity.

possibility that the employment affects of minimum wages may be ambiguous when the weights on hours and workers differ. We note that in this case equation (1.8) becomes:

$$\frac{dn}{dw}=\frac{[\frac{f_h}{n}-f_{nh}]}{f_{nn}}\frac{dh}{dw}=-\frac{a}{b}\frac{n}{h}\frac{dh}{dw} \qquad (1.9)$$

In the Cobb-Douglas case the change in the number of workers from a minimum wage will always be the opposite to a change in hours. Firms who are on a positively/negatively sloped part of the hours wage locus will have an increase/decrease in hours per worker and a decrease/increase in the number of workers. Equation (5) becomes:

$$\frac{d(nh)}{dw}=n\frac{dh}{dw}+h\frac{dn}{dw}=n[\frac{b-a}{b}]\frac{dh}{dw} \qquad (1.10)$$

It may be worth noting that when fixed costs of hiring workers are zero this is always negative[19]. To sign (1.10) we need to solve for equilibrium hours. We continue by assuming that representative workers have the Cobb-Douglas utility function: u=cl=wh(t-h). At the equilibrium level of utility the indifference curve is:

$$w=\frac{u}{h(t-h)}. \quad (1.11)$$

The solutions for hours and the number of workers are given in Appendix 2. Substituting the solution for *h* from Appendix 2 into equation (A.2.1) we derive in the Appendix the following condition that determines the sign of $\frac{dh}{dw}$:

[19] If we take the first order condition on n and substitute it back into the profit function and totally differentiate over w and h we can show that the slope of the isprofit curve is: $\frac{dw}{dh}=\frac{a-b}{b}\frac{w}{h}$. As we illustrate in Figure 2 the isoprofit curve will be tangent to the wage hours locus in equilibrium so the sign of the slope of the isoprofit will be the same as the sign of $\frac{dh}{dw}$ in equation (7), so that (7) will always be negative.

Condition One: $\frac{dh}{dw} > 0$ if $\frac{b-a}{a} < \frac{kt}{2u}$ and $\frac{dh}{dw} < 0$ if $\frac{b-a}{a} > \frac{kt}{2u}$.

We note from Condition One above that when there are no fixed costs $\frac{dh}{dw}$ is positive when *b*<*a* and negative when *b*>*a*. When *a*=*b* the isoprofit lines are horizontal and the firm is indifferent over the number of hours at a given wage. Taking Condition One in conjunction with equation (7) one obtains the following condition :

Condition Two:

$$\frac{d(nh)}{dw} > 0 \quad \text{if} \quad 0 < \frac{b-a}{a} < \frac{kt}{2u}$$

From Condition Two above we see that when there are positive fixed costs there is a range of parameter values where the impact of a minimum wage on total hours is positive.[20] We also wish to ensure that the second order conditions are satisfied and the solutions for hours, workers, and profits are all positive. Given these considerations the easiest way to proceed is to simulate the model. We assume that *k*, *p*, *t*, and *u* are all equal to 1 and that *a* is 0.3. By inspecting Conditions One and Two above one can see that:

(a) $\frac{dh}{dw} > 0$ for $0 < b < 0.45$

(b) $\frac{dh}{dw} < 0$ for $0.45 < b < 1$

(c) $\frac{d(nh)}{dw} > 0$ for $0.30 < b < 0.45$

[20] It is possible to get a positive relationship between total hours and the minimum wage with no fixed costs with a more complicated production function, for example with a C.E.S production function over hours and workers.

(c) $\frac{d(nh)}{dw} > 0$ for $0 < b < 0.30$ or $0.45 < b < 1$

Figures 4(a).and (b) and 5(a) through 5(c) illustrate these terms for simulations using the parameter values assumed above. Accordingly, the second order conditions are satisfied, and profits, utility, hours and the number of workers are all positive for the parameter values in the graphs.[21]

To summarise for the Cobb-Douglas example, equation (6) and Condition One show that the impact of a minimum wage on hours per worker and the number of workers is ambiguous. Condition two shows that the impact on total hours is also ambiguous. We then solved the model explicitly for a range of parameter values over which all of the above three outcomes may increase or decrease in response to the minimum wage to ensure that the model is well behaved.

Section IV: Conclusion

The idea that minimum wages may lead to offsetting effects on hours per worker is generally recognised in the literature. However, given the prevalence of studies that focus solely on the number of workers and the willingness to make inferences on the underlying labour market from the results, we suspect that the fact that changes in hours per workers and the number of workers from a minimum wage can be either positive or negative and will typically be inversely related in a simple partial equilibrium competitive labour market is not yet well understood. In addition, the result that total hours may increase in response to a minimum wage when firms have even small fixed costs, should make researchers wary about using empirical

[21] We have not included all the graphs for these outcomes to save space, but a maple file to generate these simulations is available on request from the authors.

studies of minimum wages on employment to make inferences on the nature of the underlying labour market.

It is also clear from our analysis that different firms may respond differently in terms of their demand for total hours, workers or hours per worker. This means that while empirical analysis that focuses on a homogeneous group of low skilled workers (say in the fast food industry) will resolve a lot of estimation problems and may provide compelling results for that group of workers, the theoretical analysis implies that the results may not be representative of the impact of a minimum wage across all industries.

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Figure 1: The Equilibrium Wage hours locus

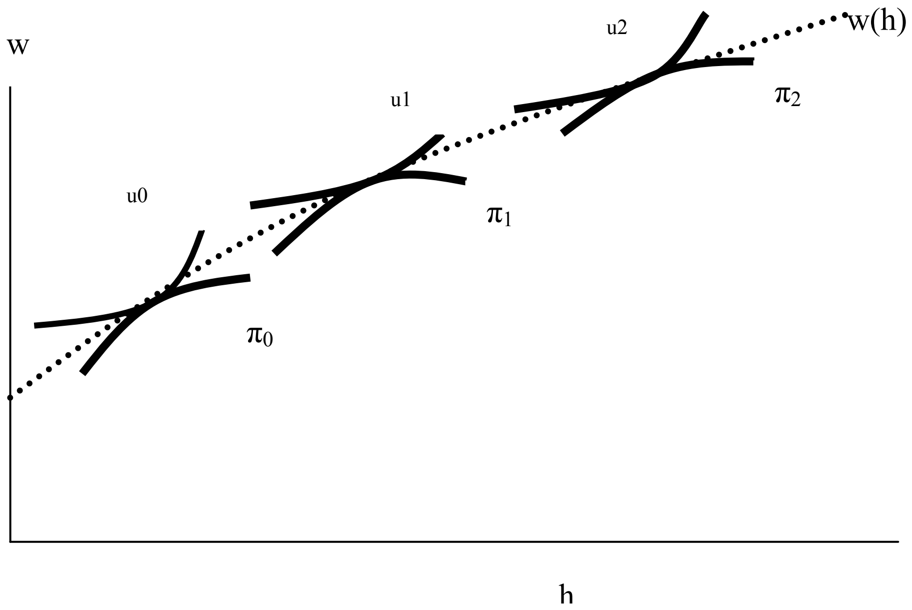


Figure 2: The Impact of a Minimum Wage on a Firms Hours and Workers

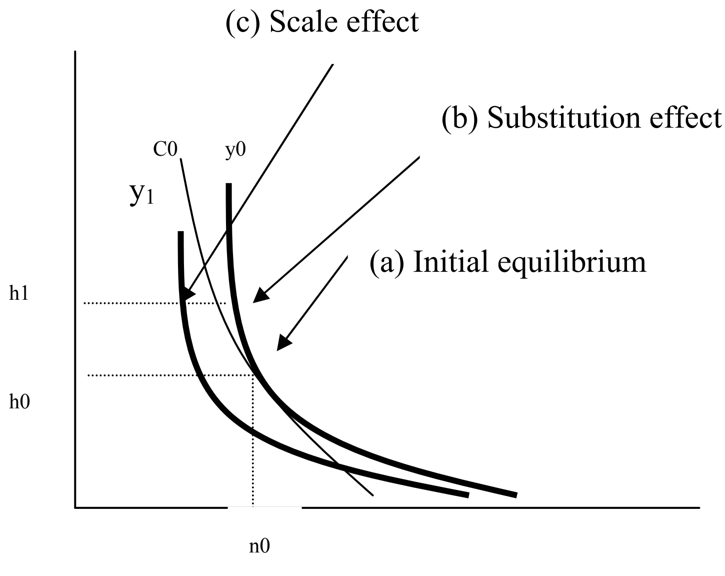


Figure 3: An Increase in a Firms Total Hours After a Minimum Wage

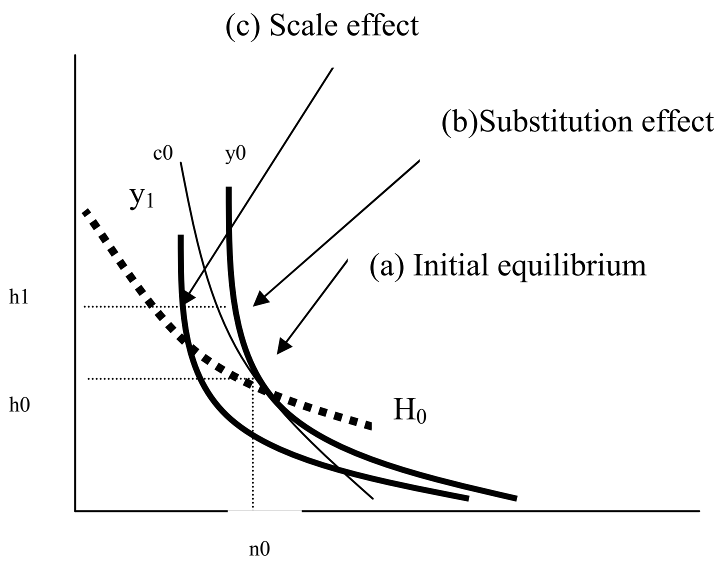


Figure 4(a)

The impact of the minimum wage on the number of workers (b=0.01..0.43)

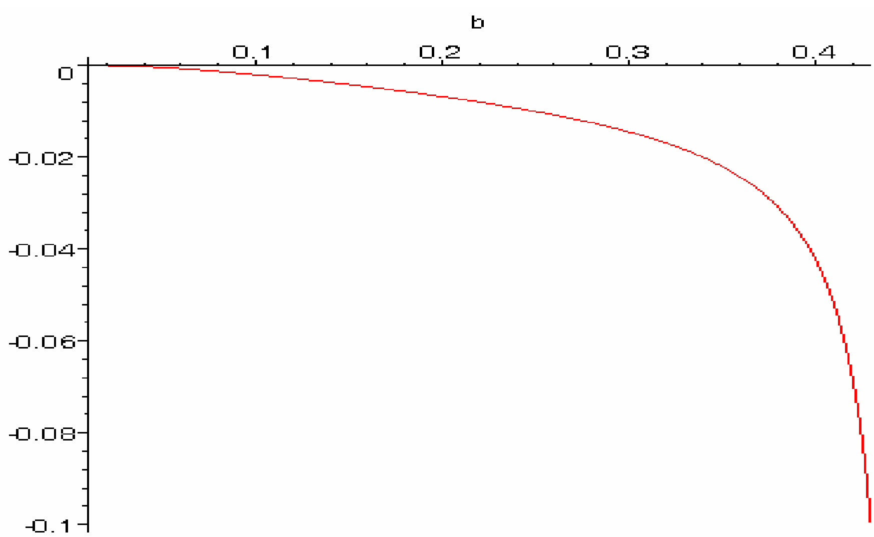


Figure 4(b)

The impact of the minimum wage on the number of workers (b=0.51..0.99)

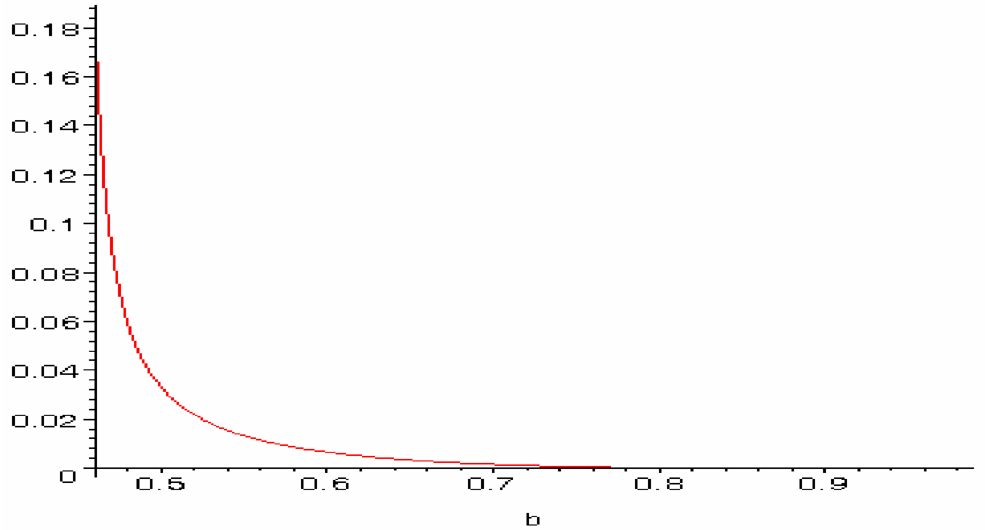


Figure 5(a)

The impact of the minimum wage on total hours worked (b=0.01..0.30)

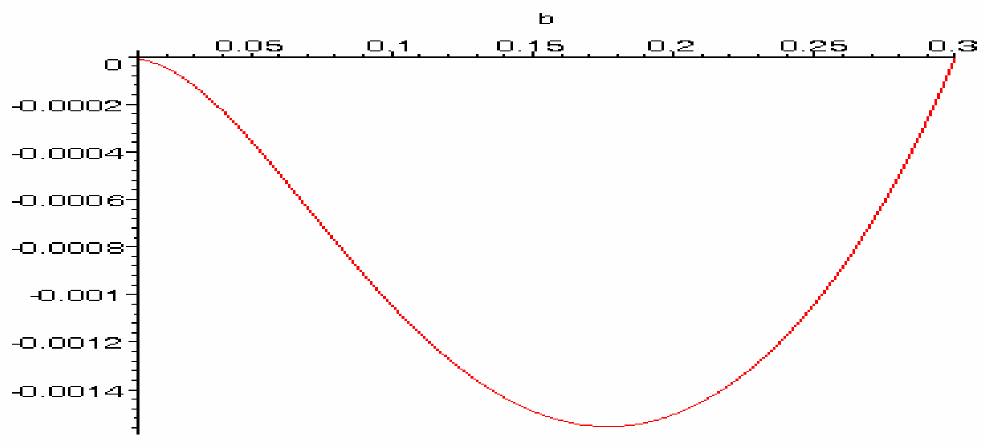


Figure 5(b)

The impact of the minimum wage on total hours worked (a=0.30..0.44)

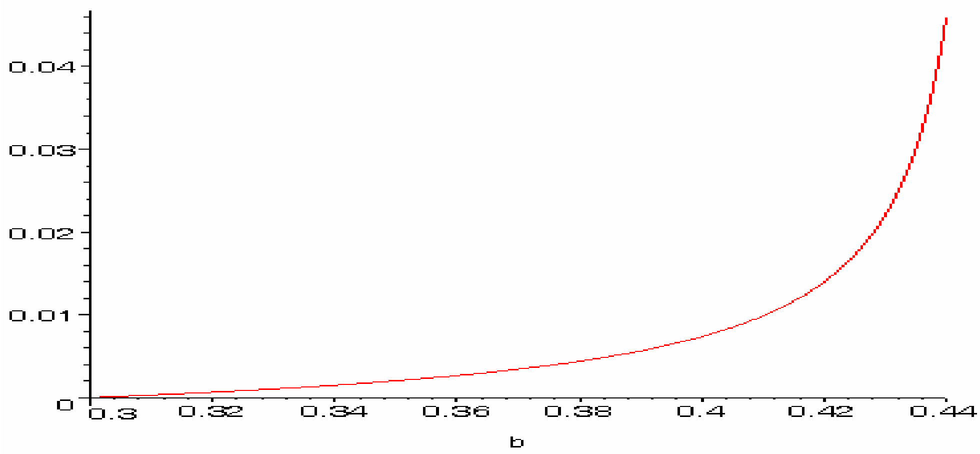
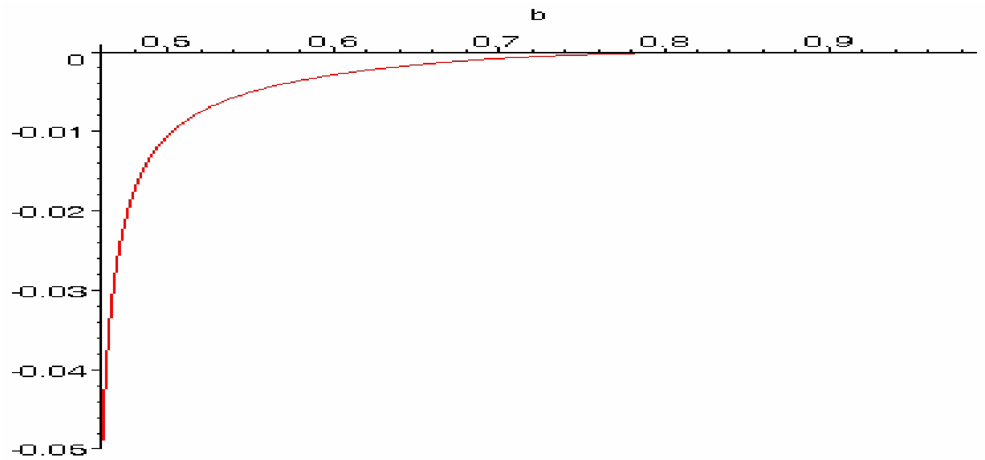


Figure 5(b)

The impact of the minimum wage on total hours worked (a=0.46..0.99)



Appendix 1:

Condition for scale effect on hours to equal zero

We minimise cost subject to the constraint that a given level of output q is produced using the lagrangean ι. The labour aggregator $f(h,n)$ turns combinations of workers (n) and hours per worker (h) into an amount (q) of output. The equilibrium hourly wage hours worked locus is: $w(h)$. A fixed cost k must be Paid per worker hired[22].

$$\iota = wh(w)n + nk + \lambda\{q - f[n, w(h)]\} \tag{A.1.1}$$

The first order conditions on h, n and λ respectively are:

$$\iota_w(n,h,\lambda) = wh_w(w)n + h(w)n - \lambda f_h[n,h(w)]h_w(w) = 0$$

$$\iota_n(n,h,\lambda) = wh(w) + k - \lambda f_n[n,h(w)] = 0$$

$$\iota_\lambda(n,h,\lambda) = q - f[n,h(w)] = 0 \tag{A.1.2}$$

Totally differentiating the first order conditions with respect to h,n,λ and q we get that if $\frac{\partial h}{\partial y} = 0$ then:

$$\iota_{wn}\iota_{n\lambda} - \iota_{w\lambda}\iota_{nn} = -\lambda f_{nn} f_h h_w - f_n(wh_w + h - \lambda f_{hn} h_w) = 0 \tag{A.1.3}$$

Note from this:

[22] We let the firm choose the level of hours here, which depends on the wage, rather than choose the wage which depends on hours as in the text. This makes no difference to the analysis but for exposition purposes it may be a little clearer to illustrate the scale affect when the firm chooses hours and to illustrate the minimum wage affect when the firm chooses the wage.

$$\frac{f_h}{f_n} = -\frac{(\frac{f_h}{n} - f_{hn})}{f_{nn}} \quad \text{(A.1.4)}$$

Appendix 2

Cobb-Douglas production function

We assume the firm has a Cobb-Douglas production function $f(n,h) = h^a n^b$.

Given the utility function u=cl=wh(t-h) the slope of the wage hours locus is just the slope of the indifference curve at u:

$$\frac{dh}{dw} = \frac{[h(t-h)]^2}{(-t+2h)u} \quad \text{(A.2.1)}$$

It follows that if t>2h then $\frac{dh}{dw} < 0$ and if if t<2h then $\frac{dh}{dw} > 0$. The representative firm's profit function is:

$$\pi(n,h) = h^a n^b - \frac{nu}{(t-h)} - kn \quad \text{(A.2.2)}$$

From the first order conditions on n and w we get the following quadratic form for h:

$$(ut + kt^2) - [(\frac{a+b}{a})u + 2kt]h + kh^2 = 0 \quad \text{(A.2.3)}$$

Noting that h<t the solution is:

$$h = t + (\frac{a+b}{a})\frac{u}{2k} - \frac{\sqrt{(a+b)^2 u^2 + 4ktabu}}{2ka} \quad \text{(A.2.4)}$$