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ABSTRACT

Child Labor and the Education of a Society*

We examine economic growth, inequality and education when the wellspring of growth is the formation of human capital through a combination of the quality of child-rearing and formal schooling. The existence of multiple steady states is established, including a poverty trap, wherein children work full-time and no human capital accumulation takes place, with continuous growth at an asymptotically steady rate as an alternative. We show that a society can escape from the poverty trap into a condition of continuous growth through a program of taxes and transfers. Temporary inequality is a necessary condition to escape in finite time, but long-run inequalities are avoidable provided sufficiently heavy, but temporary taxes can be imposed on the better-off. Programs aiming simply at high attendance rates in the present can be strongly non-optimal.

JEL Classification: H2, I2, O1, O41

Keywords: Child labor, growth and inequality, education, human capital, redistributive policies, poverty traps

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1 Introduction

The object of this paper is to analyze economic growth, inequality and education when the wellspring of growth is the formation of human capital through a combination of the quality of child-rearing and formal schooling. The importance of the mother's attributes and her role in determining a successful outcome for her children has been established beyond dispute. The nutrition and care she affords her child are, controlling for income, almost certain to be better if she is educated than if she is not. Throughout childhood, her use of language, the telling of stories and the reading of books all exert a powerful influence upon the child's linguistic development and intellectual abilities. The process of child-rearing thus involves an inter-generational transfer that, however unwilling, creates a certain potential in the child, which, if fully realized through the instilling of existing social knowledge, can lead to the continuous growth of human capital construed in the broad sense and hence of individual productivity.

Instilling existing social knowledge, beginning with literacy and numeracy, is the province of formal education, and without it, the child will almost surely attain only a small part of the potential bequeathed to her during child-rearing, modest though this might be if her parents happen to be unlettered and poor. If they are sufficiently poor and the child can work to supplement the family's income, they may decide to deny her any schooling at all, thereby perpetuating their own condition of ignorance and low productivity. Those children who are substantially engaged in part-time work are arguably so sapped by the effort that their studies suffer when they do attend school, so that in this case, too, the formation of human capital is hindered. From a long-term perspective, the ensuing failure to build human capital must be counted as the main social cost of child labor.

It is quite possible, therefore, that the setting described here can yield a 'poverty trap', in the form of a stationary, locally stable equilibrium in which productivity is low. At the same time, there is also the more cheerful possibility that continuous growth can occur, once a family's needs for current consumption are no longer so pressing that it can afford to send its children to school. Establishing whether multiple equilibria exist is the task of the first half of the paper. Engineering a 'bootstrap' operation, whereby a whole society that is initially mired in a state of ignorance and low productivity raises itself into a condition of literacy and continuous growth through a program of taxes and transfers, is the focus of the second half.

The main conclusions to be drawn from the latter are: First, even without outside aid, an escape from the poverty trap may be possible through various programs of redistributive taxes and subsidies. Second, temporary inequality in post-tax incomes and human capital is a necessary condition to escape from poverty in finite time, but long-term inequalities can be avoided, provided the government enjoys enough freedom in choosing taxes on incomes. An inability to tax the well-to-do sufficiently heavily, however, can result not only in a delay in attaining universal and full education, but also in persistent long-run inequality. Third, programs that minimize the time needed to educate the society as a whole do not maximize school attendance rates in any particular period; rather they reflect the connection between the schooling received by succeeding members of the lineages of children who are educated today and the expansion of taxable capacity and the avoidance of subsidies in the future. Indeed, maximization of the school attendance rate in a particular period can induce a slide back into the poverty trap. Fourth, outside aid must be targeted in accordance with the same principles. If, plausibly, it is not so generous as to effect an escape from the trap in a single period, then it must be preceded by suitable internal transfers if it is to be exploited to the full.

There is a rapidly growing literature on the economics of child labor, which has been newly surveyed by Basu (1999). The possibility of multiple equilibria was first established by Basu and Van (1998) and discussed by Swinnerton and Rogers (1999). The setting of these and other recent theoretical contributions is usually one-period and static, with a high-wage equilibrium in which the children do not work, and a low-wage one in which they do. One exception is the two-period structure employed by Baland and Robinson (1998), in which parents expect to receive support from their children in old-age, and their decisions concerning education can fall prey to time-inconsistency. Another is the recent contribution by Dessy (2000), in which an infinitely lived "dynasty" has preferences over the sequences of consumption levels and family sizes. The approach has something in common with that adopted here, and yields the possibility of a poverty trap, with an unstable equilibrium separating a low-productivity, high-fertility steady state from a high-productivity, low-fertility steady state. Furthermore, Jafarey and Lahiri (2000) examine to what extent credit opportunities such as "food for education" or "investment in education quality" can lower the incidence of child labor and increase the effectiveness of educational investments. There appears, however, to be no contribution that deals directly with the possibility of both stationary and growth equilibria, as is done here. Nor does there seem to be any corresponding work on tax-and-transfer programs – as opposed to compulsory education – to put the whole society on the path from poverty and child labor to literacy and growth.[1]

Recent empirical work has identified both similarities and differences in the nature of child labor and schooling across developing countries [see Grootaert and Kanbur (1995) and Basu (1999) for surveys and Maitra and Ray (2000) for a new comparative study]. Edmonds (2000) points to a close relationship between child labor and home production, which can affect the policy recommendations drawn from models of child labor with parental preferences or credit constraints. Emerson and Souza (2000) provide evidence concerning the nature and persistence of child labor.

The plan of the paper is as follows. Section 2 lays out the basic model, which draws on the idea in Uzawa (1965) and is of the OLG variety. Starting with the technology for producing human capital and output, it then proceeds to analyze the household's behavior, the final step being the derivation of the system's dynamics. Section 3 takes up the policy problem in three variations. All involve finding a program of taxes and subsidies that will minimize the time needed for all households to attain a level of human capital such that either all children attend school full-time or sustainable growth is assured, subject to certain constraints on the degree of inequality that can be tolerated. In section 4 we analyze the minimum-time program to educate the whole society. Section 5 discusses some aspects of development policy in the light of these results, and concludes by identifying a number of open issues.

1 There exists a large literature on growth and inequality, recently surveyed in Aghion and Williamson (1998).This does not, however, deal with the problems of finding a path from one steady state to another, as we do here. The work of Galor and Zeira (1993) shows that the distribution of wealth matters for economic growth. At this fundamental level, our paper deals with a similar issue to the growth and inequality literature: do short-term or long-run tradeoffs exist between growth and inequality?

2 The Basic Model

Consider an OLG-model in which individuals live for two periods, which will be labeled childhood and adulthood, respectively. Each generation consists of a continuum of households represented by $[0, 1]$. A household is indexed by i or j, where $i, j \in [0, 1]$. In the basic model all households are alike and we drop indices. Each household, or "family", comprises one adult and one child. Let the proportion of childhood devoted to education in period t be denoted by $e_t \in [0, 1]$, the residual being allocated to work. Adults spend all of their time working.

2.1 The Technology

Human capital is assumed to be formed in a process whereby child-rearing is combined with formal education in the following way. Let an adult in period t possess λ_t efficiency units of labor, where $\lambda_t \in [1, \infty)$ is a natural measure of her human capital, and the condition $\lambda = 1$ for the society as a whole can be thought of as state of backwardness. In the course of rearing her daughter, an activity that is assumed to claim a fixed amount of time, the adult gives the child a certain capacity to build human capital for adulthood. The amount of this contributing factor is assumed to be a fixed fraction $z \in (0, 1]$ of the adult's own endowment of efficiency units of labor. The adult's gift will be unavailing to preserve the child from the state of $\lambda = 1$ as an adult, however, unless it is complemented by some formal education, in which the basic cultural capabilities of reading, writing and calculating can be learned. With these assumptions, the child's endowment of efficiency units of labor on reaching adulthood at time $t + 1$ is given by[2]

$$\lambda_{t+1} = h(e_t)(z\lambda_t) + 1 \tag{1}$$

where $h(.)$ is assumed to be a continuous, increasing and differentiable function on $[0, 1]$, with $h(0) = 0$. Eq.(1) implies that the gift of rearing and formal education are both necessary if human capital is to exceed the basic level $\lambda = 1$ in the next generation.[3]

2 The fixed amount of time devoted to child-rearing has been netted out.

3 The evolution of human capital described in equation (1) is the simplest formulation, since it does not involve any persistence. Persistence can be introduced by extending equation (1) to

For any sequence of formal education $\{e_t\}_{t=0}^{\infty}$, the intergenerational growth rate of the adult's human capital in a given household in period t, g_t, is given by

$$1 + g_t = \lambda_{t+1}/\lambda_t = zh(e_t) + (1/\lambda_t) \tag{2}$$

The level of λ is momentarily stationary, i.e. $\lambda_{t+1} = \lambda_t$, for all pairs (λ_t, e_t) satisfying

$$[1 - zh(e_t)]\lambda_t = 1 \tag{3}$$

One such pair is $(\lambda_t = 1, e_t = 0)$, namely, backwardness. In order to describe the evolution of human capital, which will be needed later, suppose that e_t takes the value unity for all $\lambda_t \geq \lambda^a$. It is then seen from (2) that if the system starts from some value of $\lambda \geq \lambda^a$, and if the "technology" for (re)producing human capital is sufficiently productive, in the sense that it satisfies the condition $zh(1) \geq 1$, then λ_t will tend asymptotically to steady growth at the rate $[zh(1) - 1]$. In the special case $zh(1) = 1$, λ_{t+1} is still strictly monotonically increasing, but its growth rate tends asymptotically to zero. If $zh(1) < 1$, the largest steady state value of λ is denoted by λ^* and we need to distinguish between two cases. First, if in addition, $\lambda^a \leq zh(1)\lambda^a + 1$ and $\lambda^a \leq \lambda^*$, where $\lambda^* = \frac{1}{1-zh(1)}$, we have a stable fixed point at λ^* $\forall$ $\lambda_t \geq \lambda^a$. Second, if $\lambda^a > zh(1)\lambda^a + 1$ and $\lambda^* < \lambda^a$, then human capital will decrease if it starts at the level λ^a. Without further information about the relationship between e_t and λ_t for $\lambda_t < \lambda^a$, nothing can be said about stability or the possibility of chaotic behavior of human capital accumulation in this case.

Having established these pregnant possibilities, we turn to the technology for producing output, the latter taking the form of an aggregate consumption good. With our sights on steady-state growth, and in view of the fact that human capital in the above account is a produced factor of production, let there be a proportional relationship between output and inputs of labor measured in efficiency units. All output will then accrue to the household as income.

The child's contribution to the household's income is given as follows: Without any education, the child will supply at most one efficiency unit of labor, because of the

$\lambda_{t+1} = h(e_t)(z\lambda_t) + 1 + \rho\lambda_t$. For ρ sufficiently small, our analysis should still hold. In future research, we hope to say more about the case where persistence is large.

complementarity between the gift received during child-rearing and formal education. Indeed, it is plausible that the child's efficiency will be somewhat lower, ceteris paribus, on grounds of age alone. To reflect these considerations, let the child be able to supply at most γ efficiency units of labor, where $\gamma \in (0,1)$ and this upper limit is reached when the child works full-time. The household therefore supplies a total of $[\lambda_t + (1-e_t)\gamma]$ efficiency units of labor to the production of the aggregate good. Under the above assumption on the technology, the level of output produced by a household that has endowment λ_t and chooses e_t is

$$y_t = \alpha[\lambda_t + (1-e_t)\gamma] \tag{4}$$

where $\alpha \in (0,\infty)$ is the (constant) productivity of an efficiency unit of labor. Thus, we have a so-called AK-model, the form of which has a certain affinity to Uzawa's (1965) learn-or-do model. Recalling the conditions for λ to grow (asymptotically) at a steady rate, it is seen at once that, should they hold, output per family will grow in the same manner, e_t being unity $\forall\, \lambda_t > \lambda^a$.

2.2 The Household's Behavior

Following Basu and Van (1998), it is assumed that all allocative decisions lie in the adult's hands. We rule out any bequests at death, so that the whole of current income, as given by (4), is consumed. The gift of the factor $z\lambda_t$ through rearing is one form of transfer inter vivos. According to (1), however, the second form, namely, sending the child to school at least part of the time ($e_t > 0$), is also necessary if the child is to enjoy $\lambda_{t+1} > 1$ as an adult. Since current consumption is maximized by choosing $e_t = 0$, it follows that the adult's sense of altruism towards her child must be sufficiently strong if she does choose $e_t > 0$.[4]

For the sake of simplicity, let the child's consumption be a fixed fraction $\beta \in (0,1]$ of the adult's, the latter being denoted by c_t. From (4), we then obtain the family's

4 An alternative possibility is that children are expected to support their parents in old-age, with the contributions becoming more generous with a rising level of the children's income as adults. For an analysis of endogenous fertility (without child labor) along these lines, see Raut and Srinivasan (1994).

budget line[5] in the space of (c_t, e_t):

$$(1+\beta)c_t + \alpha\gamma e_t = \alpha(\lambda_t + \gamma) \tag{5}$$

In this section, it will be convenient to choose e_t as the numéraire, in which case, (5) is rewritten as

$$pc_t + e_t = \mu_t \tag{6}$$

where

$$p \equiv \frac{1+\beta}{\alpha\gamma} \tag{7}$$

is the (constant) relative price of current consumption and

$$\mu_t \equiv 1 + \lambda_t/\gamma \tag{8}$$

is the household's full income measured in terms of the numéraire, where $\mu > 2$ by virtue of $\gamma \in (0,1)$ and $z \in (0,1]$. Observe that μ_t is increasing in λ_t. It will also be useful to define

$$\overline{c}_t(\lambda_t) \equiv \frac{\alpha(\lambda_t + \gamma)}{1+\beta} \tag{9}$$

and

$$\underline{c}_t(\lambda_t) \equiv \frac{\alpha \cdot \lambda_t}{1+\beta} \ , \tag{10}$$

which correspond to the adult choosing $e_t = 0$ and $e_t = 1$, respectively. With this preliminary settled, the adult's preference relation $\succ$ is defined on the set

$$S = \{(c_t, e_t) : c_t \geq 0, 0 \leq e_t \leq 1\}. \tag{11}$$

Given the endowment λ_t, her feasible set in the space of (c_t, e_t) is the subset of S defined as

$$S(\lambda_t) = \{(c_t, e_t) : pc_t + e_t \leq \mu_t, c_t \geq 0, 0 \leq e_t \leq 1\}. \tag{12}$$

These sets are depicted in figure 1. In addition to completeness and transitivity, the preference relation $\succ$ is assumed to be strictly increasing.

5 We assume that it is impossible for an adult to borrow against the future income of the child.

It is clear from (6) and (8) that an increase in λ_t enlarges the feasible set by inducing a parallel shift of the budget line. If λ_t is very small, the family's existence will be so precarious that there may be no option but to put the child to work full-time. At sufficiently high values, however, it seems plausible that altruism will be operative, in the sense that the parent chooses $e_t > 0$. It is plausible, therefore, to assume that the following conditions hold:

$$(\overline{c}_t, 0) \succ (c_t, e_t) \in S(\lambda_t) \backslash (\overline{c}_t, 0) \qquad \text{if } \lambda_t \leq \lambda^S \tag{13}$$

$$\exists \text{ a bundle } (c_t', e_t') \in S(\lambda_t) \text{ such that } (c_t', e_t') \succ (\overline{c}_t, 0) \qquad \text{if } \lambda_t > \lambda^S \tag{14}$$

and

$$\exists\, \lambda^a > \lambda^S \text{ such that } (\underline{c}_t, 1) \succ (c_t, e_t) \in S(\lambda_t) \backslash (\underline{c}_t, 1) \qquad \text{if } \lambda_t > \lambda^a \tag{15}$$

The critical value λ^S is a limit that must be exceeded if altruism is to be operative. It is highly plausible that $\lambda^S > 1$; for in a general state of backwardness, all are likely to be living mostly from hand to mouth, with little thought for the morrow. Associated with λ^S is the level of consumption.

$$\overline{c}(\lambda^S) = \frac{\alpha(\lambda^S + \gamma)}{1 + \beta} \tag{16}$$

The value λ^a can be thought of as marking the beginning of such affluence that the whole of childhood is spent at the school-desk.

Conditions (13) and (14) imply, respectively, that the adult will choose $e_t = 0$ for all $\lambda_t \leq \lambda^S$ and $e_t > 0$ for all $\lambda_t > \lambda^S$. In order to analyze how her choice of e_t will vary with λ_t in the latter case, some further assumptions are needed. Let the preference ordering be representable by the continuous, strictly increasing, differentiable, strictly quasi-concave function $u(c_t, e_t)$, and consider the problem

$$\max_{(c_t, e_t) \in S(\lambda_t)} \{u(c_t, e_t)\} \tag{17}$$

In view of the assumptions on $u(.)$, this problem has a unique solution, denoted by $(e^0(\lambda_t), c^0(\lambda_t))$, which is also continuous in λ_t. Since an increase in λ_t shifts the budget line, but leaves the relative price between e_t and c_t unchanged[6], the solution $e^0(\lambda_t)$ is the adult's Engel function and its image is the income expansion path. We assume that both goods are non-inferior and, therefore that, $\frac{\partial e^0}{\partial \lambda_t} \geq 0$ for $\lambda_t \in [\lambda^S, \lambda^a]$. Under the foregoing assumptions, therefore, the "income expansion path" curve, as induced by changes in λ_t, takes the following form:

$$(c_t, e_t) = \begin{cases} (c_t^0, e_t^0) = (\overline{c}(\lambda_t), 0) & \forall\, \lambda_t \leq \lambda^S \\ (c_t^0, e_t^0) & \forall\, \lambda_t \in (\lambda^S, \lambda^a) \\ (c_t^0, e_t^0) = (\underline{c}(\lambda_t), 1) & \forall\, \lambda_t \geq \lambda^a \end{cases} \tag{18}$$

where the locus (c_t^0, e_t^0) is increasing in λ_t for all $\lambda_t \in (\lambda^S, \lambda^a)$.

Before using (18) to analyze the difference equation (1), a remark should be made about the choice of the domain of the adult's preferences. As represented above, she cares directly only about her current consumption and about the level of education her child receives. An alternative assumption would involve preferences over current consumption and the child's future human capital or, more generally, utility. In the latter case, she cares directly about the well-being of her daughter, though she knows that her daughter, as an adult, will care about her own child, and so forth. As in Barro (1974), therefore, all generations are effectively connected. In such a setting, by increasing the time devoted to school, the adult in period t enlarges her daughter's feasible set in such a way that the latter is always better-off as a result, given that the daughter, too, will be confronted with the same problem of balancing current consumption in period $t+1$ against her own daughter's well-being in period $t+2$. In this case, an adult's current utility depends upon the whole sequence of utilities achieved by her descendants. We expect that this alternative formulation would yield qualitatively similar results to those obtained here. Our formulation of preferences appears to be equally plausible and is much easier to handle in a technical sense.

6 If the child's productivity as a child-worker increases with her mother's productivity, then p will decline with λ and there will be a substitution effect that works against sending the child to school. This possibility is not pursued here.

2.3 Dynamics

Returning to (1) in the light of (18), we obtain

$$\lambda_{t+1}^i = \begin{cases} 1 & \forall\, \lambda_t^i \leq \lambda^S \\ zh(e_t^{i0}(\lambda_t^i))\lambda_t^i + 1 & \forall\, \lambda_t^i \in (\lambda^S, \lambda^a) \\ zh(1)\lambda_t^i + 1 & \forall\, \lambda_t^i \geq \lambda^a \end{cases} \tag{19}$$

The following qualitative results are immediate. In view of the (plausible) assumption that $\lambda^S > 1$, it follows from the first part of (19) that the state of backwardness ($\lambda = 1$) is a locally stable equilibrium. The dynamical system in (19) can exhibit four different patterns, which we describe first in detail for the case where $h(e_t(\lambda_t))\lambda_t$ is concave, and then proceed to the case where it is convex.

Proposition 1

Suppose that $h(e_t(\lambda_t))\lambda_t$ is strictly concave in $[\lambda^S, \lambda^a]$. Then the system exhibits the following four patterns of dynamic behavior:

(i) $zh(1) \geq 1$:

there are two steady states, that associated with $\lambda = 1$ being stable and that with $\lambda = \lambda^*$ being unstable.

(ii) $zh(1) < 1$, *with* $\lambda^a > zh(1)\lambda^a + 1$ *and there exists a* $\lambda_t = \lambda^* \in (1, \lambda^a)$ *such that* $zh(e_t(\lambda_t))\lambda_t + 1 = \lambda_t$:

there are three steady states, those associated with $\lambda = 1$ and $\lambda = \lambda^*$ being stable and that with $\lambda = \lambda^{**}$, with $\lambda^S < \lambda^{**} \leq \lambda^*$, being unstable.[7]

(iii) $zh(1) < 1$, *with* $\lambda^a > zh(1)\lambda^a + 1$ *and there exists no* $\lambda_t \in (1, \lambda^a)$ *such that* $zh(e_t(\lambda_t))\lambda_t + 1 = \lambda_t$:

there is only one steady state, namely $\lambda = 1$, which is stable.

(iv) $zh(1) < 1$, *with* $\lambda^a \leq zh(1)\lambda^a + 1$:

there are three steady states, those associated with $\lambda = 1$ and $\lambda = \lambda^*$, being stable and that with $\lambda = \lambda^{**}$ being unstable.

7 In the special case where $\lambda^* = \lambda^{**}$, stability depends on the starting point. Only if $\lambda_t > \lambda^*$ can λ_t converge to λ^*.

Proposition 2

Suppose that $h(e_t(\lambda_t))\lambda_t$ is strictly convex in $[\lambda^S, \lambda^a]$. Then the system exhibits the following three patterns of dynamic behavior:

(i) $zh(1) \geq 1$*: there are two steady states, that associated with $\lambda = 1$ being stable and that with $\lambda = \lambda^*$ being unstable.*

(ii) $zh(1) < 1$*, with $\lambda^a > zh(1)\lambda^a + 1$ and there exists no $\lambda_t \in (1, \lambda^a)$ such that $zh(e_t(\lambda_t))\lambda_t + 1 = \lambda_t$:*
there is only one steady state, namely, that associated with $\lambda = 1$, which is stable;

(iii) $zh(1) < 1$*, with $\lambda^a \leq zh(1)\lambda^a + 1$:*
there are three steady states, those associated with $\lambda = 1$ and $\lambda = \lambda^* = \frac{1}{1-zh(1)}$ being stable and that with $\lambda = \lambda^{**}$ being unstable.

Note that $h(e)$ is increasing in e, and that, by normality, $e^0(\lambda)$ is increasing in λ. Hence, $h[e^0(\lambda)] \cdot \lambda$ is increasing in λ.

Propositions (1) and (2) do not cover all possible cases. For instance, if $h(e_t(\lambda_t))$ is concave, then $h(e_t(\lambda_t))\lambda_t$ can have one or several points of inflection, with convex and concave sections, and the system's dynamical behavior becomes much more complex.

In the following, we will illustrate the dynamic patterns of cases (i) and (ii) of proposition (1) graphically (henceforth the **growth case** and **non-growth case**, respectively).

In the **growth case**, starting from any $\lambda_t \geq \lambda^a$, λ_t (and hence y_t) will grow indefinitely, approaching the steady growth rate of $[zh(1)-1]$ asymptotically; λ^* is an unstable fixed point. In the **non-growth case**, steady-state growth is not possible: In the case where

$$\left[1 - zh\big(e^0(\lambda_t)\big)\right]\lambda_t = 1 \tag{20}$$

the dynamical system possesses only the roots $\lambda_t = 1$ and λ^*. Then, for any $\lambda_0 \geq \lambda^*$ as starting value, λ_t will converge to the finite value λ^*. If (20) possess a third root, denoted by λ^{**}, then λ_t will converge towards λ^* in the interval $(\lambda^{**}, \lambda^*)$, too.

In order to see what all this portends for the behavior of λ_t, we illustrate the trajectory of λ_t for the **growth case** in figure 2. The 45^o line through the origin is labeled a; the line g represents $\lambda_{t+1} = zh(1)\lambda_t + 1$, where $g's$ slope is $zh(1) \geq 1$. The horizontal segment BC reflects the fact that $\lambda_{t+1} = 1$ for all $\lambda_t \in [1, \lambda^S]$. The rising segment that passes through D and then meets g at λ^a arises from $\lambda_{t+1} = zh(e_t^0(\lambda_t))\lambda_t + 1$ for $\lambda^S < \lambda_t < \lambda^a$. The instability of the stationary state corresponding to $\lambda_t = \lambda^*$ is evident from the trajectories drawn to the left and right of D, respectively. Figure 3 shows the trajectory in the **non-growth case** with two steady states, λ^{**} and λ^*, in addition to $\lambda_t = 1$. The latter, depicted by E, occurs at $\lambda_t = \lambda^*$, which is a stable stationary state. The former corresponds to D, where $\lambda_t = \lambda^{**}$, which is unstable.

2.3.1 An Example

We conclude this section by deriving the middle part of (19) as an explicit difference equation in λ in a special case. Starting with preferences, we follow Basu and Van (1998) in assuming that these are of the (symmetric) Stone-Geary kind:

$$u(c_t, e_t) = \begin{cases} (c_t - c_{min})e_t, & \text{if} \quad c_t \geq c_{min} \\ c_t - c_{min} & \text{otherwise} \end{cases} \tag{21}$$

It is readily checked that the solution to problem (17) in this case is

$$c_t^0 = \frac{1}{2}\left[c_{min} + \frac{\alpha\gamma}{1+\beta}\right] + \frac{\alpha\lambda_t}{2(1+\beta)}, \quad \forall\ \lambda_t \in (\lambda^S, \lambda^a) \tag{22}$$

$$e_t^0 = \frac{1}{2}\left[1 - c_{min}\frac{1+\beta}{\alpha\gamma}\right] + \frac{\lambda_t}{2\gamma}, \quad \forall\ \lambda_t \in (\lambda^S, \lambda^a) \tag{23}$$

Thus, current consumption and e_t^0 are both linear in λ_t, where $e_t^0(\lambda^S) = 0$ and $e_t^0 = 1$ for $\lambda_t \geq \gamma + \frac{1+\beta}{\alpha}c_{min} \equiv \lambda^a$. Hence, (1) specializes to the form

$$\lambda_{t+1} = zh\left[\frac{1}{2}[1 - c_{min}\frac{1+\beta}{\alpha\gamma}] + \frac{\lambda_t}{2\gamma}\right]\lambda_t + 1, \quad \forall\ \lambda_t \in (\lambda^S, \lambda^a). \tag{24}$$

It is common in the literature on growth to assume that various elements of the technology are isoelastic. In that case, let $h(e) = b \cdot e^\theta$, where $b > 0$ and $\theta > 0$ are parameters. Substitution into (24) then yields the further specialization

$$\lambda_{t+1} = z\,b(\xi + \frac{\lambda_t}{2\gamma})^\theta\lambda_t + 1, \qquad \forall\ \lambda_t \in (\lambda^S, \lambda^a) \tag{25}$$

where $\xi = \frac{1}{2}[1 - \frac{1+\beta}{\alpha\gamma}c_{min}]$. It is clear that the r.h.s. of (25) is strictly convex in λ_t, so that proposition 2 applies.

3 The Policy Problem

In this section we formulate the policy problem associated with an initial state of backwardness, where the broad objective of policy is to liberate all lineages from this condition for good. The case for intervention in the present setting rests on the externalities that arise when the improvements in all future generations' welfare that would stem from a better education of today's children are not fully reflected in the preferences of today's parents, who are assumed to make the relevant decisions. Suppose, for example, that parents care only about their own consumption and their children's welfare, and that their preferences over the same are additively separable. It can then be shown that the parent in period t will maximize a (discounted) stream of future felicity levels, in which all members of the lineage from t onwards are counted just once, so that all generations are effectively connected. If, however, she were to care directly not only about her child, but also about her grandchild, then future members of the lineage would be counted more than once in making up the corresponding stream, so that the sacrifice involved in longer schooling would result in greater benefits, as she calculates them, and hence make longer schooling more attractive than if she were to care only about her own child. The longer the chain of generations considered, the more completely the exernalities will be internalized. If, as is arguable, the government has a longer horizon than individual households, then the case for intervention to promote longer schooling at the expense of child labor is, in principle, established.

The instruments available to the government for this purpose are assumed to be taxes and subsidies. For the most part, the only restriction imposed on the net tax due from any household is that its current subsistence requirements be met. The government is assumed to be able to identify each household (or lineage), an ability which is vital to the payment of subsidies in an efficient way, and to assess its current level of full income. Since an adult's income is fixed, a tax thereon is effectively lump-sum in nature, so that given the government's general objective, first-best allocations are feasible. The effects of imposing further restrictions on the tax schedule will be briefly discussed after the main results have been derived.

3.1 Taxation and Subsidization

We assume that the whole society is initially ($t = 0$) in a state where $\lambda_0 = 1$ and $e^0 = 0$. Moreover, we assume that $zh(1) > 1$ (**the growth case**) and that $h(e_t(\lambda_t))\lambda_t$ is convex in λ_t. Thus, by proposition 2, the dynamical system in (19) has exactly two steady states, namely, $(1, 0)$ and $(\lambda^*, e^0(\lambda^*))$, where the latter is unstable, and the assumptions imply $\lambda^a > \lambda^*$.

For simplicity, we assume that only the income of adults is taxable.[8] Let $\tau_t(\alpha\lambda_t^i)$ denote the tax levied in period t on household i. Some fraction of the population will be subsidized out of the ensuing revenues. We denote by $s_t^i(\alpha\lambda_t^i)$ the subsidy household i will receive in period t if the adult has income $\alpha\lambda_t^i$, where subsidies should be interpreted in a broad sense. For instance, they may consist of infra-structural support in a particular, small region. We denote by w_t^i the net income of household i in period t, measured in units of output:

$$w_t^i = \alpha\lambda_t^i + \alpha(1 - e_t^i)\gamma + s_t^i(\alpha\lambda_t^i) - \tau_t(\alpha\lambda_t^i) \equiv w_t^{ia} + \alpha(1 - e_t^i)\gamma \tag{26}$$

where w_t^{ia} denotes the net disposable income accruing to the adult in question. The household's net tax burden is defined by

$$v_t^i(\alpha\lambda_t^i) \equiv \tau_t(\alpha\lambda_t^i) - s_t^i(\alpha\lambda_t^i).$$

Since the adult chooses e_t^i on the basis of the household's potential full income (equivalently, on w_t^{ia}),[9] the evolution of human capital accumulation follows the same logic as in equation (19) and is given by

$$\lambda_{t+1}^i = \begin{cases} 1 & \forall\, \lambda_t^i \leq \lambda^S + \frac{v_t^i}{\alpha} \\ zh(e_t^{i0})\lambda_t^i + 1 & \forall\, \lambda_t^i \in (\lambda^S + \frac{v_t^i}{\alpha}, \lambda^a + \frac{v_t^i}{\alpha}) \\ zh(1)\lambda_t^i + 1 & \forall\, \lambda_t^i \geq \lambda^a + \frac{v_t^i}{\alpha} \end{cases} \tag{27}$$

8 This may be justified by the easiness of tax evasion for child income. It is unlikely that allowing household income to be taxable would change the main results of the paper.

9 Since the potential full income of a household amounts to $w_t^{ia} + \alpha\gamma = \alpha\lambda_t^i - v_t^i(\alpha\lambda_t^i) + \alpha\gamma$, the optimal choice of e_t^i can be written as a function of $w_t^{ia} + \alpha\gamma$. Since $\alpha\gamma$ is a constant, we can equivalently use $e_t^{i0}(w_t^{ia})$ to describe the household's selection of schooling time, and do so in the remainder of the paper.

Notice that taxation need not differ across households that have the same taxable income. Subsidization, however, can and must be made dependent on income *and* the particular type of household. Although, in the end, only the net tax $v_t^i(\alpha\lambda_t^i)$ matters for household i, the distinction between taxation and subsidization will be useful in illustrating the working of different policies and discussing the impact of foreign aid.

The optimal educational choice $e_t^{i0}(w_t^{ia})$ is monotonically increasing in adult income w_t^{ia} in the interval $(\alpha\lambda^S, \alpha\lambda^a)$, with $e_t^{i0}(\alpha\lambda^S) = 0$ and $e_t^{i0}(\alpha\lambda^a) = 1$. Note that the simple structure of the system in equation (27) follows from the assumption that the domain of adults' preferences encompasses their own consumption and the formal education of their children. Other formulations of preferences will imply a more complex evolution of human capital. For instance, if adults have preferences over consumption and the future human capital of their children, the optimal education level will depend, in general, on both full income and λ_t^i; for the former determines the size of the budget set and the latter the level of λ_{t+1}^i for a given e_t^i.

Where taxable capacity is concerned, we assume that there exists a subsistence level c^{sub} for the consumption of adults and βc^{sub} for children which must be ensured under all circumstances. The tax burden of household i is therefore assumed to be constrained by:

$$\alpha\lambda_t^i - \tau_t(\alpha\lambda_t^i) + \alpha\gamma \geq (1+\beta)c^{sub} \quad \forall\, i \tag{28}$$

In particular, the tax schedule must fulfill the condition:

$$0 \leq \tau_t(\alpha) \leq \alpha(1+\gamma) - (1+\beta)c^{sub} \equiv \tau^{ba} \tag{29}$$

where it is plausible that τ^{ba} is small, since households with $\lambda_t = 1$ may already be close to the subsistence level. The society may also have access to additional fiscal resources $\overline{B}_t$ in period t, where $\overline{B}_t$ may take the form of government revenues from other sources, be it from taxation of income generated in other and richer parts of a country, seigniorage or revenues from natural resources. $\overline{B}_t$ could also be foreign aid; in this case, our model will be used to discuss the optimal use of transfers to educate a society. To formulate the budget constraints of the society as a whole, we reinterpret the indexation of households as a real valued function on $[0, 1]$ which assigns every household its human capital in a particular period. Then total government revenues in period t are denoted by B_t. The budget constraint is given by:

$$B_t = \overline{B}_t + \int_0^1 \tau_t(\alpha\lambda_t^i)\, di \;\geq\; \int_0^1 s_t^i(\alpha\lambda_t^i)\, di \tag{30}$$

3.2 Policies

A program of taxation and subsidization is to be chosen in order to bring the society out of backwardness in a sustainable way. There are several ways to formulate such a policy. Starting in period 0, let T denote the number of periods needed to bring all adults to at least the efficiency level λ^a.[10] We also denote by $\tilde{\Delta}_t$ the maximal difference in disposable incomes across households in period t. $\tilde{\Delta}_t$ is given by

$$\tilde{\Delta}_t = \max_{i,j}\Big\{ \big[w_t^{ia} + \alpha\gamma(1 - e_t^{i0}(w_t^{ia}))\big] - \big[w_t^{ja} + \alpha\gamma(1 - e_t^{j0}(w_t^{ja}))\big] \Big\} \tag{31}$$

Since $w_t^{ia}+\alpha\gamma(1-e_t^{i0}(w_t^{ia}))$ is monotonically increasing in w_t^{ia}, both goods being normal, we can work with a simpler measure of income inequality, relying only on differences in adults' disposable incomes. We define[11]

$$\Delta_t = \max_{i,j}\Big\{ w_t^{ia} - w_t^{ja} \Big\} \tag{32}$$

All the formulations of the policy problem considered here involve minimizing T subject to some upper bound on the degree of inequality a society is prepared to tolerate. They can therefore be regarded as "turnpike" programs. The first is

$$\text{P1}: \quad \min_{\tau_t(\alpha\lambda_t^i),\, s_t^i(\alpha\lambda_t^i)} \{T\}$$

$$s.t. \quad \Delta_t \leq \overline{\Delta}, \;\; (28), (29) \text{ and } \; \lambda_T^i \geq \lambda^a \;\; \forall i$$

where $\overline{\Delta}$ is the said upper bound on Δ_t. A special case of the policy problem (P1) is that where $\overline{\Delta} = \infty$, which seeks the fastest unconstrained path to the state in which

10 As will become clear, the policy problems are formulated in such a way that this level is also indefinitely sustainable after completion of the programs in question. The requirement to bring the whole society to $(\lambda^* + \epsilon)$ for some $\epsilon > 0$ would yield the same qualitative result.

11 As will become clear shortly, inequality will arise only from differences across groups, and the number of groups will most likely be quite small; so that taking the range of the distribution as a measure of inequality is defensible.

all adults have at least the efficiency level λ^a. Observe that when all adults attain λ^a and none is subject to taxation, then all of their offspring will enjoy full-time schooling, and so attain at least λ^a.

Whereas the policy program (P1) focuses on inequality constraints while the program is in operation, an alternative set of programs can be formulated which focus on the degree of long-run inequality that prevails after the program has been completed. Such an alternative policy problem can be stated as follows:

$$\text{P2}: \quad \min_{\tau_t(\alpha\lambda_t^i), s_t^i(\alpha\lambda_t^i)} \{T\}$$

$$s.t. \quad \Delta_T \leq \overline{\Delta}, \ (28), \ (29) \text{ and } \lambda_T^i \geq \lambda^a \ , \ \forall i$$

The policy problem (P2) seeks the optimal speed program such that inequality is at most $\overline{\Delta}$ after the society has been fully educated. A special case of (P2) is $\Delta_T = 0$, where no long-run inequality is allowed.

Other policy formulations are possible. In particular, we think that the following problem is relevant:

$$\text{P3}: \quad \min_{\tau_t(\alpha\lambda_t^i), s_t^i(\alpha\lambda_t^i)} \{T\}$$

$$s.t. \quad \Delta_t \leq \overline{\Delta}, \ (28), \ (29) \text{ and } \lambda_T^i \geq \lambda^a \ , \ \forall i$$

$$e_{t+1}^i \geq e_t^i \ , \ \forall i, t$$

In (P3) we impose the additional constraint that the time spent in school by members of a particular lineage does not decline over time. This appears to be a sensible implicit restriction on the tax schedule, both on economic and on political-economic grounds, and its imposition can increase the time a society needs to escape backwardness.

3.3 The Inequality - Speed Dilemma

The above formulation of the policy problem involves a potential, underlying inequality - speed dilemma, which we now investigate. To start with, we assume that $\overline{B}_t = 0 \ \forall t$, i.e., no outside resources are available and all subsidies must be financed by current taxation. Without loss of generality, we assume that every household, viewed as a lineage, receives positive subsidies only once.[12] That being so, our first observation is obvious:

Fact 1

In an initial state of backwardness, equal treatment of citizens with respect to taxes and subsidies leaves the whole society in that state. In particular, if $\overline{\Delta} = 0$ *in (P1), then* $T = \infty$.

Equal treatment in period 0 would imply that all households would have adult disposable incomes equal to α, which leads to $e_0^{i0} = 0$ and hence $\lambda_1^i = 1\ \forall i$. No household can escape from backwardness at any time under such a policy.

Having observed that creating inequality, if only temporarily, is a necessary condition to increase human capital when $\overline{B}_t = 0$, we next discuss how much inequality is needed if the whole society is to be educated. The next proposition presents our first important result.

Proposition 3

If $\overline{\Delta} < \alpha(\lambda^* - 1)$ *and* $\lambda_0^i = 1\ \forall i$*, then the policy problem (P1) yields* $T = \infty$.

Proof :

In order to educate the whole society, each lineage must achieve at least human capital λ^* at some point in time; for otherwise its human capital would stay at, or revert to, $\lambda = 1$, λ^* being unstable. Since $\overline{\Delta} < \alpha(\lambda^* - 1)$ and subsidization of some individuals necessarily requires taxation of others, it is impossible to grant sufficiently generous subsidies that the children of those households receiving them will attain λ^* in the very next period, which also rules out succeeding generations of those lineages attaining λ^* in any period.

■

We can generalize the above result to the following constellation, in which we allow a policy program under which some households are at the subsistence level while others

12 However, the proofs of the propositions rely crucially on this assumption. While the overall conclusions appear to remain robust if we drop the assumption, the proofs become extremely tedious.

are simultaneously at the disposable adult income $\alpha\lambda^*$.

Proposition 4

If $\overline{\Delta} \leq \alpha\lambda^* - [(1+\beta)c^{sub} - \alpha\gamma]$ *and* $\lambda_t^i \leq \lambda^*$ *for all* t *and* i*, then the policy problem (P1) yields* $T = \infty$.

Proof :

Note that condition (29) implies $\alpha(1+\gamma) > (1+\beta)c^{sub}$, and hence $\alpha\lambda^* - [(1+\beta)c^{sub} - \alpha\gamma] > \alpha(\lambda^* - 1)$. Thus, we allow for a greater tolerance of income inequality than in proposition 3. For any subsidized household in period $t = 0$, an optimal policy requires

$$zh[e(\alpha + s_0^i)] + 1 = \lambda^* \tag{33}$$

Since the program must be self-financing, only a fraction of all households can be subsidized, while the others must be taxed. Denote by $\tilde{s}_0$ the solution of the implicit equation (33). Let the share of households whose children will have human capital λ^* in $t = 1$ be denoted by δ. This share is constrained by the government's budget identity

$$\delta_0\tilde{s}_0 = (1 - \delta_0)(\alpha(1+\gamma) - (1+\beta)c^{sub}) \tag{34}$$

where δ_0 denotes the maximal value of δ that can be attained in $t = 0$. Equation (34) determines δ_0 uniquely. Let us now consider the economy in $t = 1$. The fraction δ_0 of the adult population has human capital λ^*. If the corresponding households are to choose a level of education such that their children attain at least λ^*, then they cannot be taxed. For

$$\lambda^* = zh(e_t^{i0}(\alpha\lambda^*))\lambda^* + 1, \tag{35}$$

and any net transfer from these households to the state would lower the disposable income of the adult below $\alpha\lambda^*$, which, in turn, would yield $\lambda < \lambda^*$ in the next generation. Therefore, to move a fraction of the remaining households, denoted by δ_1, to the knowledge level λ^* in the next period ($t = 2$), the budget constraint of the government implies

$$\delta_1(1-\delta_0)\tilde{s}_0 = (1-\delta_1)(1-\delta_0)\left[\alpha(1+\gamma) - (1+\beta)c^{sub}\right] \tag{36}$$

Obviously, we obtain $\delta_0 = \delta_1$. Repeating our argument for every period yields:

$$\delta_t = \delta_{t-1} = ... \ \delta_0 \equiv \tilde{\delta} \tag{37}$$

Therefore, after any finite number of periods, say n, a positive measure $(1 - \tilde{\delta})^n$ of households in the population remains in the state of backwardness, which in turn implies $T = \infty$.

■

Propositions (3) and (4) have profound implications for the tradeoff between speed and income inequality. The society must be prepared to tolerate a greater degree of inequality in incomes than $\overline{\Delta} = \alpha(\lambda^* - 1)$, and some households must possess human capital above λ^* while others remain temporarily in the state $\lambda = 1$ if all are to escape wholly from backwardness in finite time.

Propositions (3) and (4) are in the nature of "impossibility" results. They have been derived under the assumption that the society under consideration cannot rely on resources from outside, which leads to the question: can such resources make a difference? It is readily verified that outside resources in an amount smaller than $\tilde{s}_0$ and only temporarily available at the beginning of the education process ($t = 0$) will invalidate neither proposition, because the logic thereof would apply to the population not attaining λ^* the next period.[13] Thus, we have a third "impossibility" result:

Corollary 1

Suppose $\tilde{s}_0 > \overline{B}_0$ and $\overline{B}_t = 0 \ \forall t \geq 1$. Then the policy problem (P1) with $\overline{\Delta} \leq \alpha(\lambda^* - 1)$ yields $T = \infty$.

In striking contrast to corollary 1, however, outside help in an amount smaller than $\tilde{s}_0$ but at a later point in time can provide an escape, provided a tax-and-transfer program that (temporarily) raises λ for some households by an allowable amount has been put into effect in previous periods. For with a sufficiently large, one-time injection of

13 If the adult's net income is $\alpha\lambda^*$, then λ^* will be realized in the next period. Suppose, therefore, that every household with $\lambda = 1$ receives a subsidy of $\tilde{s}_0$ from outside. Then all will choose e^0 such that λ^* is realized in the next period. If, in period 0, the total outside subsidy exceeds $\int_0^1 \tilde{s}_0^i di = \tilde{s}_0$, then the problem will indeed be solved in one period.

outside resources, albeit smaller than $\tilde{s_0}$, it will then be possible to lift all households to λ^* in one swoop by giving each a donation such that the adult's net income exceeds $\alpha\lambda^*$. This, in turn, will trigger human capital accumulation that will yield λ^a in finite time. We summarize this observation in the following "possibility" result:

Corollary 2
Suppose $\tilde{s}_0 > \overline{B}_{t'} > 0$ for some t' and $\bar{B}_t = 0$ in the remaining periods. Then there exists a period, denoted by $\bar{t}$, such that (P1) with $\lambda_t^i \leq \lambda^*$, $\forall t$ and $\forall i$, yields $T < \infty$ if $t' > \bar{t}$.

The interesting point here is that if outside resources alone do not suffice at the outset, then domestic policy must prepare the ground by creating some temporary inequality in advance of the outside transfer.

Another "possibility" result is a direct consequence of Proposition 3:

Corollary 3
There exist a $\bar{t} \geq 1$ and a $\bar{B}_t \in (0, \tilde{s}_0)$ for some t, with $\bar{B}_t = 0$ in all remaining periods, such that $(P1)$ with $\bar{\Delta} \leq \alpha(\lambda^* - 1)$ yields $T < \infty$ if $t \geq \bar{t}$.

The essential point here is that if a suitable program of taxes and subsidies is introduced before the receipt of outside funds, then the latter can be smaller than $\tilde{s_0}$ and yet still suffice to bring about λ^* in all lineages within the period in which they are received. Note that the receipt of exactly $\tilde{s_0}$ at any time will, by definition, solve the problem at once.

A brief discussion of the effects of introducing further restrictions on the tax schedule is now called for. Propositions 3 and 4 and Corollary 1, being "impossibility" results, evidently remain valid if the government's ability to raise taxes is restricted beyond the basic requirement that all lineages can at least survive, as expressed by condition (29). Corollaries 2 and 3 also remain valid; for both rely solely on the government's ability to distribute subsidies in the appropriate way, given the restrictions on revenue-raising implied by (28) and (29).

4 Minimum Time

In this section, we examine the special case of the policy problem (P1) where no upper limit on inequality is imposed ($\overline{\Delta} = \infty$). That is, we seek a policy yielding the swiftest possible attainment of a fully educated society. We assume throughout that $\overline{B}_t = 0 \; \forall t$, i.e. the society has to rely fully on its own resources to educate itself. For this purpose, we assume that

$$zh(1) + 1 \geq \lambda^a \tag{38}$$

Condition (38) implies that if an adult in a state of backwardness were to educate her child fully, the child would choose full-time education for her own child, provided she herself were not taxed as an adult. This condition simplifies the analysis, but is not essential to our argument. It ensures that (P1) could, in principle, be solvable in two periods: if those households singled out in period 0 for promotion from backwardness receive a subsidy sufficiently large to induce them to choose $e_0 = 1$, then their offspring will attain at least λ^a, and so provide a new tax base in period 1. If this tax base is large enough to subsidize all other households to the point where the latter choose $e_1 = 1$ in period 1, then (P1) will have been solved in two periods.

The next observation follows from the fact that a marginal tax rate of 100 per cent will have no effect on the formation of human capital if the adult's post-tax income does not fall below $\alpha\lambda^a$:

Fact 2

A (maximal) speed policy under (P1) implies

$$\tau_t^*(\alpha\lambda_t^i) = \alpha\lambda_t^i - \alpha\lambda^a \;\; \textit{if} \quad \lambda_t^i \geq \lambda^a \tag{39}$$

To see this, suppose that adult i in period t has gross income $\alpha\lambda_t^i$, with $\lambda_t^i \geq \lambda_t^a$. If we require $e_t^i = 1$, the maximal tax that can be imposed on her is $\tau_t(\alpha\lambda_t^i) = \alpha\lambda_t^i - \alpha\lambda^a$. Since a lower tax has no positive effect on λ_{t+1}^i, and hence on the future tax base of the entire society, setting current taxes at this limit will (weakly) increase the speed with which the entire society can be educated. Therefore $\tau_t^*(\alpha\lambda_t^i) = \alpha\lambda_t^i - \alpha\lambda^a$ for $\lambda_t^i \geq \lambda^a$ is a feature of an optimal tax policy, as expressed in Fact 2.

Observe that Fact 2 respects conditions (28) and (29), but implies a marginal tax rate of 100 per cent for all $\lambda_t^i \geq \lambda^a$, which is scarcely politically plausible. We are led, therefore, to consider further restrictions on the government's ability to tax. It is reasonable to suppose that the marginal tax rate at all levels of income lies in the interval $[0, \bar{\rho}]$, where $\bar{\rho} < 1$, so that post-tax income is everywhere strictly increasing in pre-tax income. That being so, the tax schedule would take the following form

$$\tau_t(\alpha\,\lambda_t^i) = \tau^{ba} + \phi[\alpha\,(\lambda_t^i - 1)], \tag{40}$$

where $\phi(0) = 0$ and $\phi'(\cdot) \in [0,\, \bar{\rho}]\ \forall\ \lambda_t^i$. With $\bar{B}_t = 0\ \forall\, t$, (30) can be rewritten as

$$B_t = \tau^{ba} + \int_0^1 \phi\left[\alpha\,(\lambda_t^i - 1)\right] di \geq \int_0^1 s_t^i\,(\alpha\,\lambda_t^i)\, di, \tag{41}$$

where

$$\int_0^1 \phi\left[\alpha\,(\lambda_t^i - 1)\right]\, di\ \leq\ \bar{\rho}\cdot\alpha\left[\int_0^1 \lambda_0^1\, di - 1\right] \tag{42}$$

To make the analysis more tractable, it will be assumed in what follows that the only restriction on the function $\phi(\cdot)$ is that condition (29) not be violated. It should be emphasized that this simplification in no way invalidates the method of forward induction that underpins the analysis.

The tax-subsidy problem is complex and no closed-form solution exists which covers all possible periods T for which (P1) has a solution. Therefore, we proceed by considering the optimal policy for a given number of periods within which the entire society can be educated, ascending from $T = 1$. We establish conditions and optimal policies under the assumption that a particular period, say $\overline{T}$, is a solution under (P1). If $\overline{T}$ is not the solution, we move to $\overline{T} + 1$ and look once more. Starting from $T = 1$ and using this induction method, we will be able to characterize sequentially the solution under (P1). In the next section, we explore the case where all lineages can escape from backwardness within two generations. The extension to the case where the process takes three or more generations is set out in a technical appendix.

4.1 Minimum Time: $T = 2$

We first note that $T = 1$ can never be a solution under (P1) because in $t = 0$ only a fraction of the society can receive positive net transfers such that the adults in $t = 1$ will have achieved λ^a. The first possibility, then, is to escape from backwardness in two periods. In $t = 0$, suppose that the share of households with $i \in [0, \delta_0]$ will be subsidized. The total subsidy to this group of families is given by:

$$s_0^i = \frac{(1-\delta_0)\left(\alpha(1+\gamma) - (1+\beta)c^{sub}\right)}{\delta_0} \equiv s_0 \tag{43}$$

which implies the following pattern of human capital accumulation:

$$\lambda_1^i = \begin{cases} zh\left(e(\alpha + s_0)\right) + 1 & \text{if } i \in [0, \delta_0] \\ 1 & \text{if } i \in (\delta_0, 1] \end{cases} \tag{44}$$

To educate the whole society within $T = 2$ periods, the tax-subsidy scheme in $t = 1$ must fulfill both

$$\lambda_2^i = \begin{cases} zh\left(e(\alpha\lambda_1^i - \tau_1(\alpha\lambda_1^i))\right)\lambda_1^i + 1 = \lambda^a & \text{if } i \in [0, \delta_0] \\ zh\left(e(\alpha + s_1)\right) + 1 = \lambda^a & \text{if } i \in (\delta_0, 1] \end{cases} \tag{45}$$

and the financing constraint in period 1:

$$\delta_0 \tau_1(\alpha\lambda_1^i) \geq (1-\delta_0)s_1. \tag{46}$$

To formulate the above conditions as an optimization problem, we note, first, that the solution of the upper branch of equation (45), which is denoted by $\hat{\tau}_1(\alpha\lambda_1^i)$, is uniquely determined by λ_1^i, and hence by the choice of δ_0 through s_0 and the upper branch of (44). Second, observe that s_1 is a given number, as given by the lower branch of (46). Consider, therefore, the following problem:

$$\max_{0 \leq \delta_0 \leq 1} \left\{\delta_0 \hat{\tau}_1\left(\alpha\lambda_1^i\left(s_0(\delta_0)\right)\right) - (1-\delta_0)s_1\right\} \quad i \ \in \ [0, \delta^0] \tag{47}$$

Since the objective function is continuous and $\delta_0 \in [0, 1]$, a solution exists, and is denoted by $\hat{\delta}_0$. We obtain:

Proposition 5
If $\hat{\delta}_0\hat{\tau}_1\left(\alpha\lambda_1^i\left(s_0(\hat{\delta}_0)\right)\right) - (1-\hat{\delta}_0)s_1 \geq 0$, then $T = 2$ is the solution of the policy problem (P1). In particular, the subsidy $\hat{s}_0 = \frac{(1-\hat{\delta}_0)\left(\alpha(1+\gamma)-(1+\beta)c^{sub}\right)}{\hat{\delta}_0}$ to the households $i \in [0, \hat{\delta}_0]$ in the first period ensures that $T = 2$.

The preceding analysis highlights the point that two considerations enter into the determination of $\hat{\delta}_0$ or, equivalently, $\hat{s}_0$. First, how large is the taxable capacity in the next period yielded by subsidizing a fraction of households in the present period, a capacity expressed by $\delta_0 \tau_1 (\alpha \lambda_1^i (s_0(\delta_0)))$? Second, how large is the burden of future subsidies needed to promote the rest, as expressed by $(1 - \delta_0)s_1$?

Proposition 5 carries a number of further implications. First, if $T = 2$ is the solution under (P1), there will not, in general, be a unique optimal tax and subsidy policy. For suppose that the budget surplus in the second period is positive: $\hat{\delta}\, \tau_1 \big(\alpha\, \lambda_1^i\, (s_0(\hat{\delta}))\big) - (1 - \hat{\delta})s_1 > 0$, and denote by $\overline{\delta}_0$ one value of the share of subsidized persons in the first period for which the above financing constraint, (46), holds as an equality.[14] Then there exists a continuum of solutions for δ_0 and $s_0(\delta_0)$ with $\delta_0 \in [\overline{\delta}_0, \delta_0^*]$ and s_0 given by equation (43).

Second, in order to resolve this non-uniqueness, additional considerations can be brought into the reckoning. On the one hand, we can impose the constraint that inequality in the first period be minimized. On the other hand, we can require that there be no budget surplus at the end of the policy program. While the first requirement yields uniqueness, the second one does not, since setting the budget surplus to zero can be accomplished in various ways.

Third, although income inequality is temporarily created in order to escape from the state of low productivity, inequality in human capital, and hence in future income, has disappeared at the end of the two-period program. The program in proposition 5, therefore, has the desirable feature that while initially identical households are treated differently with respect to taxes and subsidies in the first period, their offspring become identical once more in the next period, and enter the stage of continuous growth as such. Hence, the solutions of (P1) and (P2) are identical. In other words, the constraint $\Delta_T = 0$ is not binding for (P2) if $\overline{T} = 2$. This observation can easily be generalized to the case $\overline{T} \geq 3$, as we now show.

14 There may be other solutions. In this case we take one for which the budget surplus is positive in the interval $[\overline{\delta}_0, \delta_0^*]$.

4.2 The equivalence between (P1) and (P2)

We claim that if (28) and (29) are the only restrictions placed on taxes, then maximal speed programs can always avoid long-run inequality, in the sense that all lineages attain the same level of human capital at the conclusion of such programs.

Corollary 4

Suppose that $\{\tau_t^*(\alpha\lambda_t^i), s_t^{*i}(\alpha\lambda_t^i)\}$ *is a solution of (P1) for* $\overline{\Delta} = \infty$*, with minimum time* $\overline{T}$*. Then* $\overline{T}$ *is also the minimum time of (P2) with* $\Delta_T = 0$*.*

Proof :

Since $\overline{T}$ is feasible under (P1) with $\overline{\Delta} = \infty$, all households will have attained at least λ^a at the end of $\overline{T}$ periods. Suppose that household i attains $\lambda_{\overline{T}}^i > \lambda^a$ after $\overline{T}$ periods, having reached $\lambda_{\overline{T}-1}^i$ in period $\overline{T} - 1$. Since $zh(e_t)\lambda_t$ is continuous and monotonic in e_t and λ_t, and $zh(0)\lambda = 0 < \lambda^a$, there exists a tax $\overline{\tau}_{\overline{T}-1}(\alpha\lambda_{\overline{T}-1}^i) > \tau_{\overline{T}-1}^*(\alpha\lambda_{\overline{T}-1}^i)$ such that $\lambda_{\overline{T}}^i = \lambda^a$. Since additional taxation of this kind increases the funds available to subsidize the poor without jeopardizing the condition $\lambda_{\overline{T}}^i \geq \lambda^a$, the solution of (P1) with the modified tax scheme still yields $\overline{T}$. Since these considerations apply to any household which attains more human capital than λ^a in period $\overline{T}$, the tax scheme $\overline{\tau}_{\overline{T}-1}(\alpha\lambda_{\overline{T}-1}^i)$ can be made independent of a particular household i. Therefore, by appropriate taxation in period $\overline{T} - 1$, all households will reach λ^a in period $\overline{T}$, which proves the corollary.

■

4.3 Persistent Inequality

The preceding corollary shows that long-run inequalities can be avoided, but it implies that the schooling of children of educated adults in period $\overline{T}-1$ must be so limited that their human capital as adults will be exactly λ^a in period $\overline{T}$. In many cases, this might be politically implausible or even impossible, because the implied range of income over which the marginal tax rate is 100 per cent, and hence the average tax rate, could be very large. The policy problem (P3) addresses this difficulty by imposing the constraint that e_t be non-decreasing within each lineage. This is equivalent to imposing suitable restrictions on the tax function $\phi(\cdot)$ and the maximal marginal rate of taxation, $\overline{\rho}$. It will now be shown that the optimal program for policy problem (P3) may require more time to attain the goal of educating the whole population than does problem (P2), as well as resulting in income inequalities that persist beyond the program's horizon.

Consider the case of symmetric Stone-Geary preferences in the form

$$u = \begin{cases} (c-g)e + (g - c^{sub}) & \text{if } c \geq g \\ c - s^{sub} & \text{if } g \geq c \geq c^{sub} \\ -\infty & \text{otherwise.} \end{cases}$$

Consumption must be at least c^{sub} for "survival", and full income must exceed the amount g if schooling if to occur. Consider the following constellation of parameter values: $\alpha = 0.16$, $\beta = 1$, $\gamma = 0.7$, $c^{sub} = 0.1$, $g = 0.14$ and $z = 0.7$. In the absence of any intervention, it is readily checked that all households choose $c = 0.136$ and $e = 0$: all are in the poverty trap. The tax $\tau^{ba} = 0.072$ per household is the basis of an escape. Recalling that λ^a is the minimum value of λ such that the household will choose $e = 1$ in the absence of intervention, it is also readily checked that these parameter values yield $\lambda^a = 2.45$.

Turning to the educational technology, let this take the form $h(e) = 5.05e^2$. For those lineages receiving subsidies in period 1, it then follows from the lower branch of equation (45) that $e_{12} = 0.64$, so that the required subsidy is $s_1 = 0.151$, which must be financed by taxes on those receiving subsidies in period 0. Let the latter households be the fraction $\delta_0 = 0.237$ of the whole population. Then it can be verified from equation (43) that the subsidy $s_0 = 0.232$, which yields $e_{01} = 0.999$ and hence $\lambda_{11} = 4.528$. From equation (46), each of this first group to receive subsidies must pay a tax of 0.487 in period 1 to finance s_1. With this tax burden, they will choose $e_{11} = 0.304$, and their offspring will attain $\lambda = 2.45$ in period 2, as required.

In this example, therefore, all lineages attain λ^a in period 2 and the government's budget is exactly balanced in periods 0 and 1. Any deviation from $\delta_0 = 0.237$ will cause the whole process to be delayed by at least one period.

The other snag is that $e_{11} = 0.304 < 0.999 = e_{01}$, so that the above program does not solve (P3). It is instructive to examine the average tax rates so implied. For any household in group 2 (in the state of backwardness), τ^{ba} is 26.5 per cent of total income. In period 1, each household in group 1 pays 0.487 in taxes, which is no less than 60.7 per cent of its total earnings. Suppose, therefore, that the marginal rate on the adult's income were limited to 50 per cent, with the tax function taking the form $\phi = \tau^{ba} + 0.5 \max[0, \alpha\lambda - (1+\beta)g]$. With δ_0 remaining at 0.237, it can be verified that $e_{11} = 1$, the associated tax in period 1 being 0.294. This yields a subsidy $s_1 = 0.091$, which induces $e_{12} = 0.408$. Hence, in period 2, we have a startling difference in human capital: $\lambda_{21} = 17$ and $\lambda_{22} = 1.588$. Given the restriction on tax rates, the only way to combat inequality is to distribute the subsidy more widely in period 0, i.e. to raise δ_0, but at the cost of delaying the universal attainment of λ^a.

4.4 An Algorithm for Maximal Speed

We complete this section by generalizing the procedure for finding programs to educate the whole society in the least possible time.

The previous sections indicate how maximal speed programs have to be designed. Ascending from $T = 1$, one can establish necessary and sufficient conditions that the least time is a particular period, say $\overline{T}$. We summarize the procedure as follows:

Characterization of Maximal Speed Programs

A least-time optimal program starting from $t = 0$ spans $\overline{T}$ periods if and only if there exists a sequence of tax-and-transfer schemes such that:

(i) all households attain at least λ^a in period $\overline{T}$;

(ii) there exists a budget surplus (or balanced budget) in period $\overline{T} - 1$ and balanced budgets in all previous periods;

(iii) no such program exists for $\overline{T} - 1$ periods.

A formal description of the algorithm is given in the technical appendix. While the principle underlying the procedure for deriving speed programs is outlined above, there are a number of further considerations when we move to longer time horizons.

An important feature of maximal speed programs is that school attendance rates in all but the last period are not maximized. Indeed, the preceding results imply that distributing subsidies more broadly to increase the number of children at school in any particular period can lead to a return to poverty in the future, since the succeeding cohorts of these lineages will not attend school to the same extent, if at all. A second feature of such programs is that the indeterminacy of tax-and-transfer schemes increases as $\overline{T}$ increases, which opens the door to the use of additional criteria for judging inequality.

5 Conclusions

This investigation of the nexus of child labor, economic growth and inequality is based on the assumption that parents decide on their children's schooling, an assumption that can be justified by the enormous difficulties of monitoring and enforcing school attendance and labor standards (see, for example, Basu and Van [1998]). Granted this much, the design of policies is constrained by parental decisions. Our analysis has yielded several findings, while leaving a number of issues unsettled.

The main conclusions are, first, that a whole society or group can be mired in a stable state of ignorance and low productivity, wherein all children work full-time to supplement their family's income and, in missing out on schooling, thereby perpetuate as adults the condition into which they themselves were born. Second, an escape from this poverty trap into a state of universal literacy and continuous economic growth is possible through various programs of redistributive taxes and subsidies, even without outside aid. Third, the catch here is that in the absence of outside aid on a sufficiently large scale, such programs necessarily involve some temporary inequality, and they may result in long-run inequality, if the government is unable to tax the better-off sufficiently heavily. The key point is that the very limited taxable capacity of the economy in its original state demands that the subsidies thus financed, which are designed to induce voluntary schooling, must be targeted in such a way that additional taxable capacity is created in the future. This implies that the beneficiaries' offspring must attain a level of education such that their lineages do not slide back into the poverty trap. Fourth, outside aid, unless on a grand scale, must also be targeted, and for exactly the same reasons. Fifth, the necessity of temporary inequality speaks against policies aimed at bringing about high attendance rates in the short term without giving due consideration to the long-term perspective within which the problem is to be solved.

The case where the set of policies is widened to encompass compulsory schooling as a complement to taxes and subsidies is not considered here, but it raises some interesting issues where the optimal package of measures is concerned. Suppose, for example, that child labor can be limited to the point where the family can just afford the subsistence level of consumption in the absence of any taxation of income. If the resulting schooling is sufficiently limited, such a partial ban will not by itself yield an escape from the trap, but a combination of even more limited schooling and some taxes, with suitably targeted subsidies, might do so. Nor is it clear that universal compulsory schooling would eliminate the necessity of some degree of temporary inequality as an element of a program aimed at bringing about a fully educated society in minimum time, since inequality can increase taxable capacity. While there is no space to explore the consequences of a partial ban on child labor in this paper, these matters will be important in situations where some monitoring and enforcement are possible under governmental or international standards.

The list of open issues does not end with those arising from compulsory education. First, what other concepts of inequality, both within and across generations, commend themselves in the evaluation of alternative paths to full literacy? Second, how should foreign aid be allocated in such a setting? Third, how do the necessity of employing distortionary taxes and the possibility of tax evasion affect the complexion, or even the feasibility, of an optimal program? Fourth, how can credit opportunities for individual households or for the country as a whole help to decrease the time to educate a society? These and other pressing issues in the area of child labor and education are left as topics for further research.

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6 Figures

Figure 1:

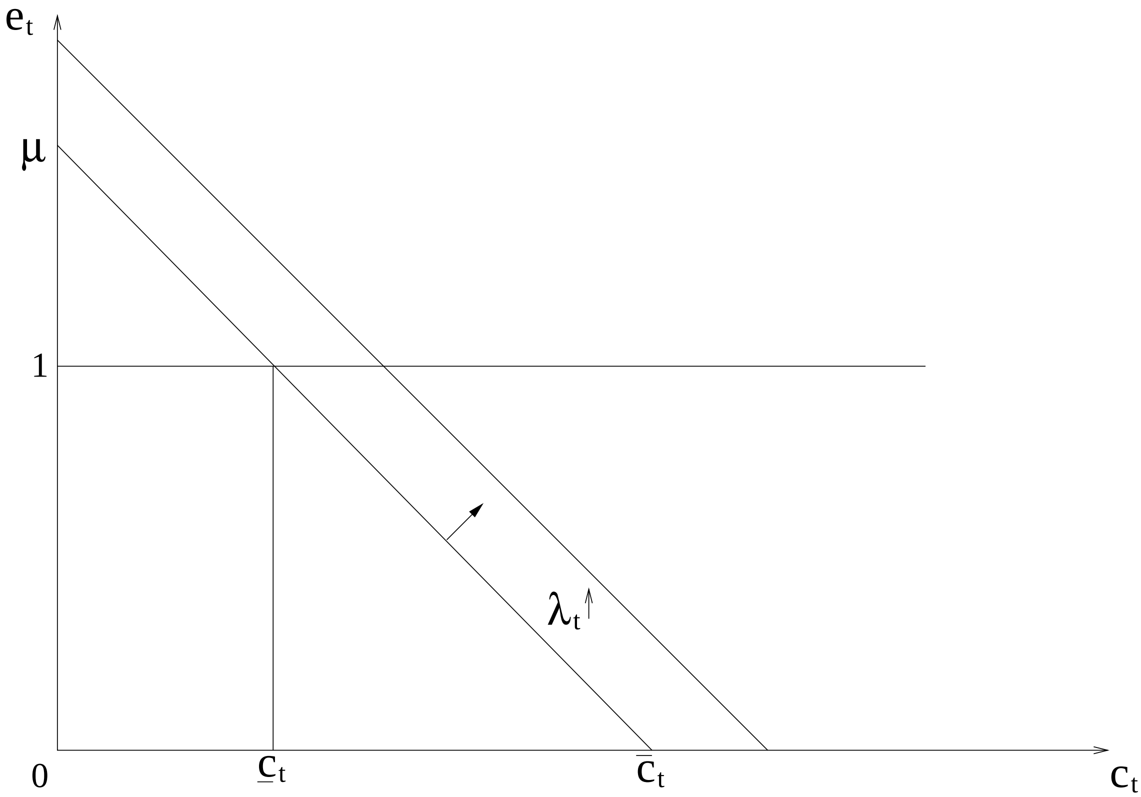


Figure 2:

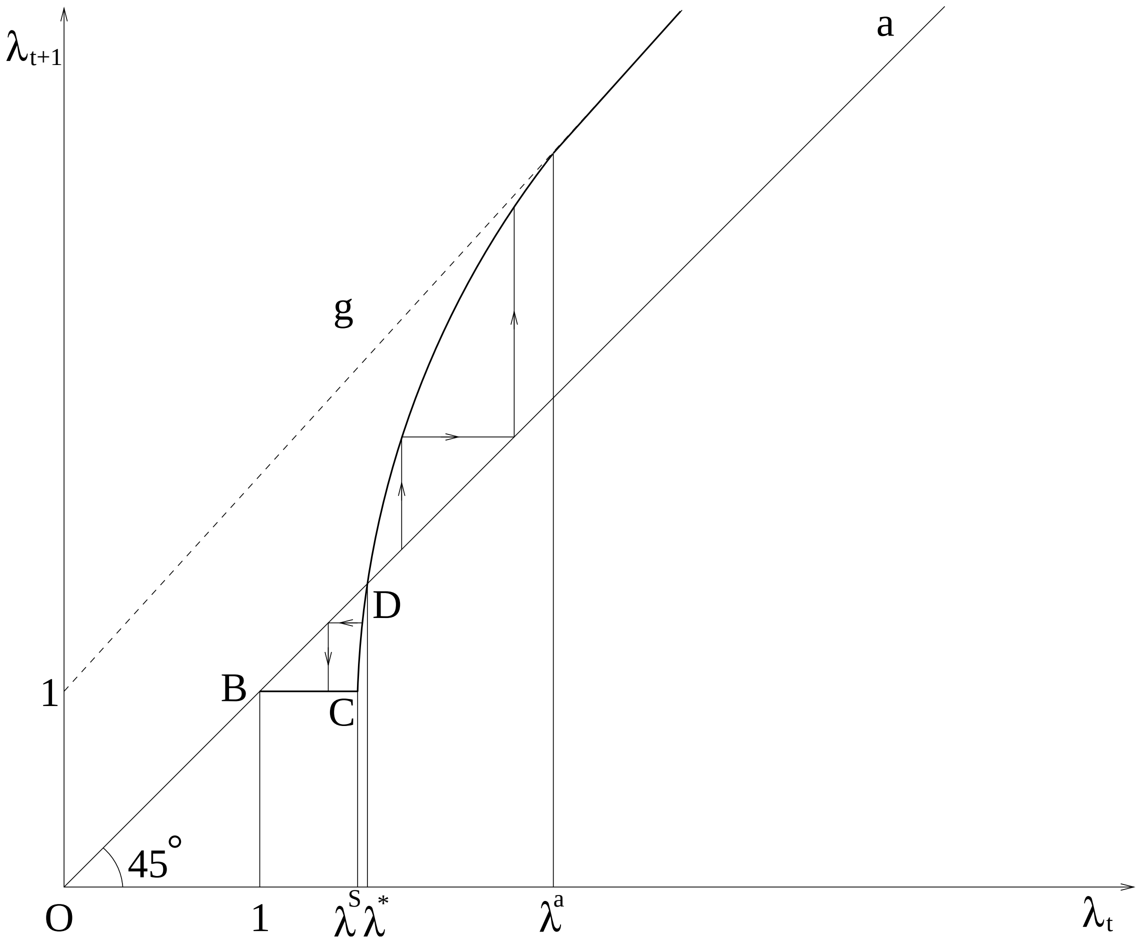
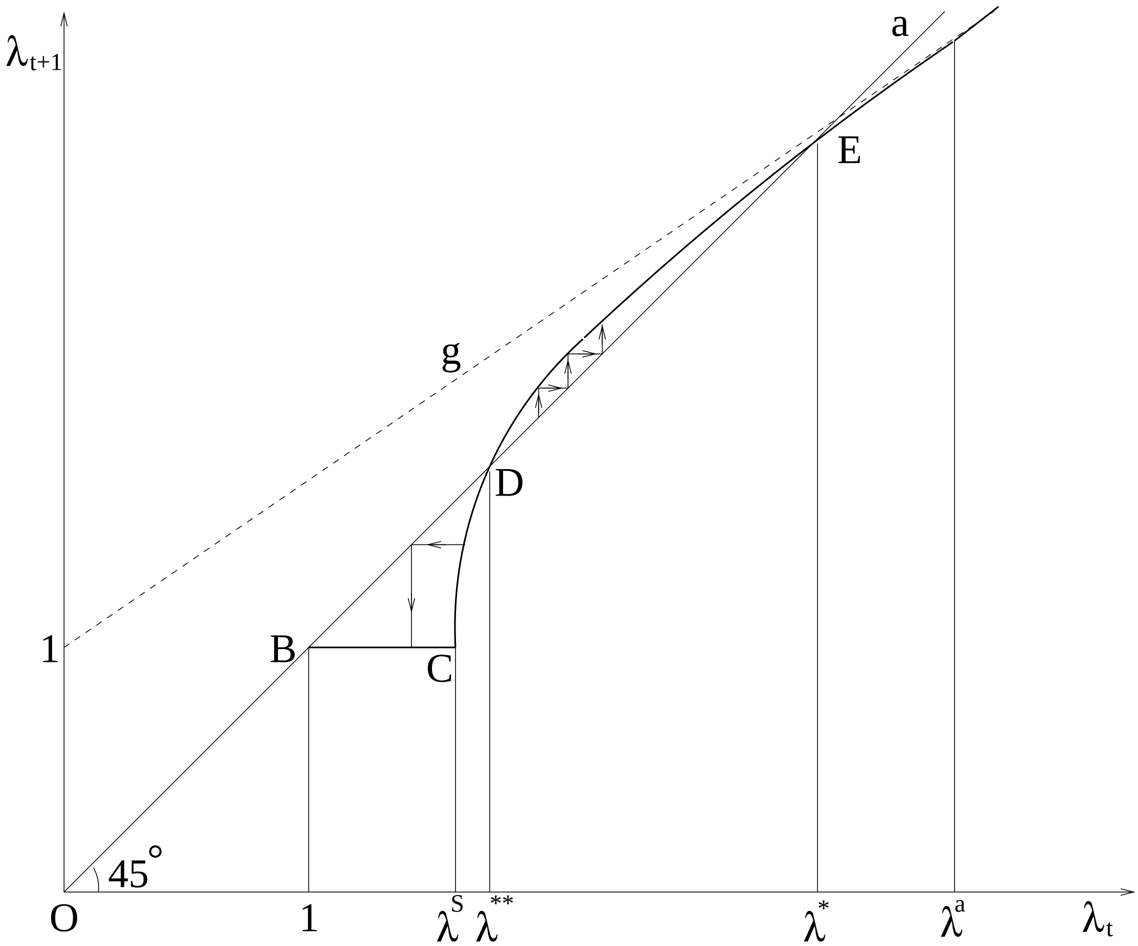


Figure 3:



7 Technical Appendix

This technical appendix sets out the algorithm to find the minimum time needed to educate a society formally in the case where $T \geq 3$. In the text, we have argued that in order to find the smallest value of T needed to escape from a general state of backwardness, one must examine the sequence $T = 2$, $T = 3$, ... , $T = n$ step by step until a feasible tax-subsidy scheme is found for the first time. The case where $T = 2$ is feasible is treated there in some detail. In the following, we begin by describing the procedure for $T = 3$ and then go on to generalize it to the case of $T = n$. To simplify the exposition, we employ the following notation:

- s^a, the net subsidy a household a state of in backwardness needs for its child to reach λ^a in the following period: s^a is given by the implicit equation $\lambda^a = zh(e(\alpha + s^a)) + 1$
- δ_t, the share of all households still in backwardness in period t that receive a subsidy in that period: $\delta_t \in [0, 1]$
- μ_k, the share of all households that have received subsidies during the periods up to and including period k: $\mu_k = \delta_0 + \sum_{l=1}^{k} \delta_l \left[\prod_{m=0}^{l-1}(1 - \delta_m)\right], k \in N,\ \mu_0 = \delta_0$

7.1 $T = 3$

In period $t = 0$, the whole society is in a state of backwardness ($\lambda_0^i = 1 \quad \forall i \in [0, 1]$). If the fraction δ_0 of all households are subsidized, the tax paid by household i is

$$\tau_0^i = \begin{cases} 0 & i \in [0, \delta_0] \\ \tau^{ba} & i \in (\delta_0, 1] \end{cases} \tag{48}$$

which yields total tax revenues

$$B_0 = \int_0^1 \tau_0^i di = (1 - \delta_0)\tau^{ba} \tag{49}$$

and the associated subsidy to household i:

$$s_0^i = \begin{cases} \frac{B_0}{\delta_0} & i \in [0, \delta_0] \\ 0 & \text{otherwise} \end{cases} \tag{50}$$

This yields the resulting evolution of human capital in period 1:

$$\lambda_1^i = \begin{cases} zh(e(\alpha + s_0^i)) + 1 & i \in [0, \delta_0] \\ 1 & i \in (\delta_0, 1] \end{cases} \tag{51}$$

A remark in connection with the upper branch of (51) is now in order. Recall that, by assumption, a lineage can be subsidized only once. Let those which receive a subsidy in period 0 $(i \in [0, \delta_0])$ attain a λ in period 1 denoted by λ_1^i. If, in the absence of taxes in periods 1 and 2, $\lambda_3^i > \lambda^a$, then some taxable capacity will be present, so that $\tau_1^i (i \in [0, \delta_0])$ can take a positive value to finance subsidies in period 1. Denoting by λ^2 the value λ_1^i must take in order that λ^a be attained in the absence of taxation in periods 1 and 2, it then follows that s_0^i must be chosen such that

$$\lambda_1^i = zh\left(e(\alpha + s_0^i)\right) + 1 \geq \lambda^2 \quad i \in [0, \delta_0].$$

An analogous condition applies to each of the relevant subsets of lineages in subsequent periods.

In period 1, another set of households will be subsidized. The tax paid by household i is given by

$$\tau_1^i = \begin{cases} \tau_1(\alpha\lambda_1^i) = \tau_1^{\delta_0} & i \in [0, \delta_0] \\ 0 & i \in (\delta_0, \delta_0 + (1 - \delta_0)\delta_1] \\ \tau^{ba} & i \in (\delta_0 + (1 - \delta_0)\delta_1, 1] \end{cases} \tag{52}$$

which yields total tax revenues

$$B_1 = \int_0^1 \tau_1^i di = \delta_0 \tau_1^{\delta_0} + (1 - \delta_0)(1 - \delta_1)\tau^{ba} \tag{53}$$

and the associated subsidy

$$s_1^i = \begin{cases} \frac{B_1}{(1-\delta_0)\delta_1} & i \in (\delta_0, \delta_0 + (1 - \delta_0)\delta_1] \\ 0 & \text{otherwise} \end{cases} \tag{54}$$

Observe that $\tau_1^{\delta_0} \leq \alpha(\lambda_1^i - \lambda^2)$ and that analogous restrictions apply to taxes in subsequent periods. Substituting as before, we obtain the human capital attained by lineage i in period 2:

$$\lambda_2^i = \begin{cases} zh(e(\alpha\lambda_1^i - \tau_1^{\delta_0}))\lambda_1^i + 1 & i \in [0, \delta_0] \\ zh(e(\alpha + s_1^i)) + 1 & i \in (\delta_0, \delta_0 + (1 - \delta_0)\delta_1] \\ 1 & i \in (\delta_0 + (1 - \delta_0)\delta_1, 1] \end{cases} \tag{55}$$

In the third and last period ($t = 2$), all remaining households still in backwardness must be (sufficiently) subsidized in order that the **whole** society reach at least λ^a after three periods. This implies $\delta_2 = 1$ and the following schedule of tax payments:

$$\tau_2^i = \begin{cases} \tau_2(\alpha\lambda_2^i) = \tau_2^{\delta_0} & i \in [0, \delta_0] \\ \tau_2(\alpha\lambda_2^i) = \tau_2^{\delta_0+(1-\delta_0)\delta_1} & i \in (\delta_0, \delta_0 + (1-\delta_0)\delta_1] \\ 0 & i \in (\delta_0 + (1-\delta_0)\delta_1, 1] \end{cases} \tag{56}$$

These yield tax revenues

$$B_2 = \delta_0 \tau_2^{\delta_0} + (1-\delta_0)\delta_1 \tau_2^{\delta_0+(1-\delta_0)\delta_1} \tag{57}$$

and the associated pattern of subsidies:

$$s_2^i = \begin{cases} s^a & i \in (\delta_0 + (1-\delta_0)\delta_1, 1] \\ 0 & \text{otherwise} \end{cases} \tag{58}$$

If $T = 3$ is a solution, human capital accumulation must fulfill the condition

$$\lambda_3^i = \begin{cases} zh(e(\alpha\lambda_2^i - \tau_2^{\delta_0}))\lambda_2^i + 1 & \geq \lambda^a & i \in [0, \delta_0] \\ zh(e(\alpha\lambda_2^i - \tau_2^{\delta_0+(1-\delta_0)\delta_1}))\lambda_2^i + 1 & \geq \lambda^a & i \in (\delta_0, \delta_0 + (1-\delta_0)\delta_1] \\ zh(e(\alpha + s^a)) + 1 & = \lambda^a & i \in (\delta_0 + (1-\delta_0)\delta_1, 1] \end{cases} \tag{59}$$

where the budget surplus in the last period is $B_2 - (1 - \delta_0 - (1-\delta_0)\delta_1)s^a \geq 0$.
If we impose equalities in (59), the only variables in this scheme are (δ_0, δ_1), so that the optimization problem may be formulated as

$$\max_{\delta_0,\delta_1} \{B_2 - (1 - \delta_0 - (1-\delta_0)\delta_1)s^a\} \tag{60}$$

subject to: (59) and the implicit restrictions that arise from the assumption that a lineage receives a subsidy only once. If the objective function is non-negative for the *argmax* values of (δ_0, δ_1), then $T = 3$ is indeed a solution of the program to educate all lineages in minimum time.

7.2 $T = n$

The generalization to $T = n$ periods proceeds forward as follows

$t = 0:$

$$\lambda_0^i = 1 \quad \forall i \in [0, 1] \tag{61}$$

$$\tau_0^i = \begin{cases} 0 & i \in [0, \mu_0] \\ \tau^{ba} & i \in (\mu_0, 1] \end{cases} \tag{62}$$

$$B_0 = \int_0^1 \tau_0^i di = (1 - \delta_0)\tau^{ba} \tag{63}$$

$$s_0^i = \begin{cases} \frac{B_0}{\mu_0} & i \in [0, \mu_0] \\ 0 & \text{otherwise} \end{cases} \tag{64}$$

$t = 1$:

$$\lambda_1^i = \begin{cases} zh(e(\alpha + s_0^i)) + 1 & i \in [0, \mu_0] \\ 1 & i \in (\mu_0, 1] \end{cases} \tag{65}$$

where $zh(e(\alpha + s_0^i)) + 1 \geq \lambda^{T-1}$, and λ^{T-1} denotes the value λ_1^i must take in order that λ^a be attained at the end of the program in the absence of taxation in period 1,... $T - 1$.

$$\tau_1^i = \begin{cases} \tau_1(\alpha\lambda_1^i) = \tau_1^{\mu_0} & i \in [0, \mu_0] \\ 0 & i \in (\mu_0, \mu_1] \\ \tau^{ba} & i \in (\mu_1, 1] \end{cases} \tag{66}$$

where $\tau_1^{\mu_0} \leq \alpha(\lambda_1^i - \lambda^{T-1}),\ i \in [0,\ \mu_0]$

$$B_1 = \int_0^1 \tau_1^i di = \mu_0 \tau_1^{\mu_O} + (1 - \delta_0)(1 - \delta_1)\tau^{ba} \tag{67}$$

$$s_1^i = \begin{cases} \frac{B_1}{\mu_1 - \mu_0} & i \in (\mu_0, \mu_1] \\ 0 & \text{otherwise} \end{cases} \tag{68}$$

$t = n - 1$:

$$\lambda^i_{T-1} = \begin{cases} zh(e(\alpha\lambda^i_{T-2} - \tau^{\mu_0}_{T-2}))\lambda^i_{T-2} + 1 & i \in [0, \mu_0] \\ zh(e(\alpha\lambda^i_{T-2} - \tau^{\mu_1}_{T-2}))\lambda^i_{T-2} + 1 & i \in (\mu_0, \mu_1] \\ & . \\ & . \\ & . \\ zh(e(\alpha\lambda^i_{T-2} - \tau^{\mu_{T-3}}_{T-2}))\lambda^i_{T-2} + 1 & i \in (\mu_{T-4}, \mu_{T-3}] \\ zh(e(\alpha\lambda^i_{T-2} + s^i_{T-2})) + 1 & i \in (\mu_{T-3}, \mu_{T-2}] \\ 1 & i \in (\mu_{T-2}, 1] \end{cases} \tag{69}$$

$$\tau^i_{T-1} = \begin{cases} \tau_{T-1}(\alpha\lambda^i_{T-1}) = \tau^{\mu_0}_{T-1} & i \in [0, \mu_0] \\ \tau_{T-1}(\alpha\lambda^i_{T-1}) = \tau^{\mu_1}_{T-1} & i \in (\mu_0, \mu_1] \\ & . \\ & . \\ & . \\ \tau_{T-1}(\alpha\lambda^i_{T-1}) = \tau^{\mu_{T-2}}_{T-1} & i \in (\mu_{T-3}, \mu_{T-2}] \\ 0 & i \in (\mu_{T-2}, 1] \end{cases} \tag{70}$$

$$B_{T-1} = \mu_0\tau^{\mu_0}_{T-1} + \sum_{k=1}^{T-2}(\mu_k - \mu_{k-1})\tau^{\mu_k}_{T-1} \tag{71}$$

$$s^i_{n-1} = s^a = \begin{cases} \frac{B_{T-1}}{\mu_{T-1}-\mu_{T-2}} & i \in (\mu_{T-2}, \mu_{T-1}] \\ 0 & \text{otherwise} \end{cases} \tag{72}$$

Note that since $n = T$, $\mu_{n-1} = \mu_{T-1} = 1$. In period n, all households attain at least λ^a.

t = n:

$$\lambda^i_T = \begin{cases} zh(e(\alpha\lambda^i_{T-1} - \tau^{\mu_0}_{T-1}))\lambda^i_{T-1} + 1 & = \lambda^a \quad i \in [0, \mu_0] \\ zh(e(\alpha\lambda^i_{T-1} - \tau^{\mu_1}_{T-1}))\lambda^i_{T-1} + 1 & = \lambda^a \quad i \in (\mu_0, \mu_1] \\ & . \\ & . \\ & . \\ zh(e(\alpha\lambda^i_{T-1} - \tau^{\mu_{T-2}}_{T-1}))\lambda^i_{T-1} + 1 & = \lambda^a \quad i \in (\mu_{T-3}, \mu_{T-2}] \\ zh(e(\alpha + s^a)) + 1 & = \lambda^a \quad i \in (\mu_{T-2}, 1] \end{cases} \tag{73}$$

The associated optimization problem is

$$\max_{\delta_0,\delta_1,\ldots,\delta_{T-2}} \{B_{T-1} - (1 - \mu_{T-2})s^a\} \tag{74}$$

subject to (73) and the implicit restrictions imposed by the assumption that a lineage receives a subsidy but once during the n periods.

Proposition 6

If there exists a vector $(\delta_0, \delta_1, \ldots, \delta_{T-1})$ *such that the value of the objective function in problem (74) is non-negative, then there exists a solution such that every lineage has human capital accumulation of exactly* λ^a *in* $T = n$.

Proof:

B_T is irrelevant, because thereafter the human capital of all lineages is increasing without fiscal intervention. Since every household has reached at least λ^a in period n, (71) is linear in $\tau_{T-1}^{\mu_k}$. Hence, a necessary condition for $\{B_{T-1} - (1 - \mu_{T-2})s^a\}$ to take a maximum is that each and every $\tau_{T-1}^{\mu_k}$ be maximized subject to (73). Hence, $\tau_{T-1}^{\mu_k} = \alpha(\lambda_{T-1}^i - \lambda^1)$, which implies $\lambda_T^i = \lambda^a \; \forall i \in [0, 1]$. ■

Remark:

Observe that if all lineages have exactly λ^a in any period, then all will have not only increasing, but also identical, levels of human capital and income thereafter.

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