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ABSTRACT

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We develop a model with two asymmetric countries. Firms choose the number and the location of plants that they operate. The production of each firm increases when trade costs fall. The fall also induces multinationals to repatriate their production into a single country, which is likely to be the large country because of the home market effect. The net effect on total output is favorable in the large country and ambiguous in the small country. We extend the model to endogenize country sizes and we show that in an equilibrium with multinationals only, a rent can be taxed by governments.

JEL Classification: F12, F15, F23, R12, R30

Keywords: globalization, economic geography, trade costs, multinational firms, home market effect

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1 Introduction

According to the home market effect, a large country hosts a more than proportionate share of firms from an industry that produces differentiated goods, and this effect is amplified at lower trade costs. (see Krugman, 1980; Helpman and Krugman, 1985; Head, Mayer and Ries, 2002; Ottaviano and Thisse, 2004; Yu, 2005). To understand this result, consider single-plant firms in two countries with prohibitive trade costs. Under reasonable assumptions, profits are proportional to earnings, which, in autarchy, are themselves proportional to the number of individuals living in each country. At the location equilibrium, firms equalize their profits. Thus they locate in proportion to the populations and there is no home market effect under prohibitive trade costs. Now consider a fall in trade costs, making it possible for consumers of a country to buy goods in the other country more easily. As a consequence, firms now have to cope with home competitors as well as with foreign competitors. The resulting change in competition is not the same in each country. In the small country, firms now face the competition of the large number of firms located in the large country, which strongly affects their profits. By contrast profits of firms located in the large country are less affected because the number of new competitors from the small country is small. As a result, some firms have an incentive to relocate from the small country to the large country: firms disproportionately locate in the large country and this disproportion increases with trade openness.

The above reasoning considers countries of different sizes but abstracts from multinationals. Let us now consider multinationals of the horizontal type and abstract from differences in country sizes.[1] It is well understood that for prohibitive trade costs, firms duplicate their production in each country and serve each market from their local plant. For lower trade costs, the costs of exporting production from a single plant are lower than the cost of duplication. Therefore, firms concentrate their production into a single country (see e.g. Brainard, 1993, 1997; Markusen and Venables, 2000).

The current paper presents a model that combines these two strands of the literature. It considers a model with two countries of unequal sizes and with a sector producing a good under increasing returns to scale. It examines how the firms' organization (multinational versus single-plant), the firms' location, the firms' production and the regional output respond to a change in trade costs and to a change in the population imbalance.[2]

[1] Multinationals are of the "horizontal" type if they produce the same good in different countries. They are of the "vertical" type if they separate production by stages, with their headquarter in one country and their production in another country.

[2] Berhens and Picard (2007) use a model with quadratic preferences to examine firms lo-

We adopt a footloose capital model according to which one unit of capital is required to run a firm, the total stock of capital is fixed worldwide and the returns of that capital are distributed among capital owners who are spread across both countries. Thus, there is fixed entry. This modeling creates a rich set of equilibria, which we can derive analytically:[3] (i) for large trade costs, all firms are multinationals; (ii) for smaller trade costs, some firms are multinationals and the others are single-plant firms in the large country; (iii) for smaller trade costs, some firms are multinationals and the others are single-plant firms in the large or in the small country; (iv) for smaller trade costs, some firms are single-plant firms in the large or in the small country; (v) for smaller trade costs, all firms are single-plant firms in the large country.

As in Raybaudi-Massilia (2000), Ekholm and Forslid (2001), and Behrens and Picard (2007), our analysis confirms that the home market effect is attenuated by multinationals. For large trade costs all varieties are produced in both countries (all firms are multinationals) and the small country does not suffer in terms of varieties produced locally. Moreover, we show that the share of total production in one country is equal to its share in population. The home market effect vanishes. For lower trade costs, some firms concentrate their production in a single-plant and the home market effect comes into effect: the share of single-plant firms and the share of production in the small country are always smaller than its share in population. A reduction of trade costs increases this effect because it affects competition in the small country more strongly than in the large country.

Concerning the levels of production (and employment) in each country, we show that a reduction in trade costs induces all firms to produce more. A reduction in trade costs also induces firms to change their organization and their location. In many equilibria, this change is detrimental to the small country because firms tend to concentrate their production into the large country. Therefore, total production and employment in the increasing returns to scale sector increase in the large country but they are likely to

cation and organization. The present paper uses standard CES-Cobb-Douglas preferences and emphasizes how the production of each firm evolves. The present paper also contains a section that endogenizes countries sizes, which drives the model towards economic geography.

[3] In our framework, the number of firms is exogenously determined by the amount of capital, which is fixed. An alternative would be to let the number of firms change with the level of trade costs. Unfortunately, this alternative assumption prevents equilibria with the coexistence of single-plant firms and multinationals; moreover a change in trade costs would provoke catastrophic changes in firms structure (see Brainard, 1987; Elberfeld, Götz and Stähler, 2005). Our assumption limits the analysis because the number of varieties is fixed, but it allows a deeper analysis because it allows smooth changes in firm's structure when trade costs change.

decrease in the small country.

Still, we show that when multinationals and single-plant firms in both countries coexist (equilibrium (iii)), total production (and total employment) can increase in the small country as trade costs decrease. Indeed, some multinationals concentrate their production in the small country, which raises production in that country. Other multinationals concentrate their production in the large country, which reduces production in the small country. However, the reduction in production is attenuated because with trade costs, the new single-plant firms in the large country sell less in the small country than they were used to sell when they were multinationals. As a consequence, we show that total production may increase in the small country. Improving freeness of trade may increase production and employment in a small country, even though the home market effect favors the large market.

We also extend the model to account for migrations of agents and we identify "weakly stable" equilibria in which all firms are multinationals. At one of these equilibria, a small migration of agents defines a new partition of the population across countries, which is also an equilibrium in which all firms are multinationals. Finally we show that when all firms are multinationals they benefit from higher profits than if they were single-plant firms. This is a rent that can be taxed away by governments without inducing any change in firms' location and organization. This result is also emphasized in Behrens and Picard (2005) and it complements the literature on tax competition in models of economic geography (see Baldwin and Krugman, 2004; Kind, Midelfart Knarvik and Schjelderup, 2000; Ludema and Wooton, 2000).

We present the model in the next section. Then we assume that firms cannot split their production in two plants (no multinational), which allows us to present in a simple way the intuition behind the home market effect. Then we solve the complete model with multinationals and we consider some extensions.

2 The model

The model builds on Dixit and Stiglitz (1977) with two countries, r and s. The population of country r (resp. s) is denoted L_r (resp. L_s). Without loss of generality, it is assumed that country r is larger than country s, $L_r \geq L_s$ with $L_r + L_s = L$. These values are exogenously given in the first part of the paper whereas they are endogenized in Section 4.3.

Technology There are two sectors. The *traditional sector* transforms labor into an homogenous good under constant returns to scale. The marginal

product of labor is normalized to one so that individuals who work in that sector earn a unit wage. There are no costs to trade this good across countries and the good is used as the numeraire.

The *modern sector* produces differentiated varieties under increasing returns to scale. A firm active in that sector first creates its own variety, then chooses the number of plants in which the variety will be produced and finally transforms one unit of labor into one unit of the variety. We assume that individuals can work in both sectors. Thus the existence of both sectors in a country ensures that workers earn the same (unit) wage. To build a plant, a firm spends a fixed cost f which is paid in terms of the numeraire good. A *multinational firm* duplicates its production in two plants whereas a *single-plant firm* chooses the country in which it will operate its production. To create its own variety, a firm must use one unit of a special input which we call capital. The total fixed costs are thus equal to the costs of one unit of capital to which we add f times the number of plants of the firm.

There are N units of capital available worldwide so that the number of varieties is equal to N.[4] The capital is mobile across countries and its ownership is equally distributed among the population with each individual owning N/L units of capital. This is the *footloose capital model* introduced by Martin and Rogers (1995). Profits are exhausted by capital costs because firms compete for the available units of capital. We denote Π the worldwide capital revenues that are shared among the individuals.

There are no costs to trade the traditional good between both countries. The modern good is costly to trade with a proportion $\phi \in (0,1)$ of the output shipped from one country arriving in the other country (the standard iceberg trade cost hypothesis). This is the usual setup in economic geography (see e.g. Baldwin et al., 2003).

Preferences An individual who consumes C_T units of the traditional goods and $C(i)$ units of variety i ($i \in [0,N]$) gets the following utility

$$U = C_M^{\mu} C_T^{1-\mu}, \text{ where } C_M \equiv \left(\int_0^N C(i)^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}, \ 0 < \mu < 1 < \sigma$$

Thus, she spends a share μ of her income on the modern goods and a share $1-\mu$ on the traditional good. The modern good is a composite made of a continuum of differentiated varieties i with a constant elasticity of substitution between varieties, σ.

[4] See Picard, Thisse and Toulemonde (2004) for a general discussion on the role played by the location of capital owners on firms' location.

The income available in country r is denoted Y_r.[5] Accordingly, the demand for variety i in country r is iso-elastic:

$$C_r(i) = \mu Y_r \left(P_r\right)^{\sigma-1} \left[p'_r(i)\right]^{-\sigma} \quad \text{where } P_r \equiv \left[\int_0^N p'_r(i)^{-(\sigma-1)} di\right]^{-1/(\sigma-1)} \tag{1}$$

The income available in country r is equal to the labor income plus the country's share of capital revenues, $Y_r = L_r + \Pi * L_r/L$. The price index of the varieties sold in that country is P_r and $p'_r(i)$ is the consumer price of variety i in country r,i.e., the price paid by a household located in country r for one unit of variety i. This price is equal to the mill price, $p_r(i)$, if variety i is produced in country r but it is larger than the mill price if variety i is produced in the other country s: $p'_r(i) = p_s(i)/\phi$ where $p_s(i)$ is the mill price of a variety i produced in country s. For notational convenience we define $\Phi \equiv \phi^{\sigma-1}$, which Baldwin et al. (2003) call freeness of trade.

Price setting There are N_r firms producing only in country r, N_s firms producing only in country s and N_t multinational firms that have a plant in both countries, with $N_r + N_s + N_t = N$.[6]On each market, the firm sets its price as a constant markup over its marginal costs because the demand is iso-elastic and because each firm considers the price indexes as given under the assumption of a continuum of firms. The marginal cost of production is equal to one for any type of firms. Therefore, the mill price is equal to $p \equiv \sigma/(\sigma-1)$ for all varieties and the price index in country r can be written as

$$P_r = p\left(N_r + N_t + \Phi N_s\right)^{\frac{-1}{\sigma-1}} \tag{2}$$

In the following, we first examine the firms' behavior when multinationals are forbidden. This allows us to isolate the home market effect precisely. Then we analyze the behavior of firms when they can split their production between the two countries.

3 Firms location with single-plant firms only

Consider a single-plant firm i that is located in country r. The firm sets a price $p = \sigma/(\sigma-1)$ and earns a profit, gross of capital costs, $\pi_r(i) = q_r(i)/(\sigma-1) - f$ where $q_r(i)$ is its production. The production is sold to the consumers from r whose demand is $C_r(i)$, and to the consumers from s

[5] In the following, it suffices to replace the subscript r with a subscript s for the analysis of country s.

[6] t stands for **t**wo plants or **t**ransnational.

who buy $C_s(i)/\phi$ units to consume $C_s(i)$ units. We use (1) to write $q_r(i) = \mu p^{-\sigma}\left(Y_r P_r^{\sigma-1} + \Phi Y_s P_s^{\sigma-1}\right)$. At an interior equilibrium (with some firms in each country), firms must earn the same profits in both countries. This condition requires

$$\pi_r(i) = \pi_s(i) \iff q_r(i) = q_s(i) \iff \frac{Y_r}{Y_s}\left(\frac{P_r}{P_s}\right)^{\sigma-1} = 1$$

We use (2) with $N_t = 0$, $N_s = N - N_r$, and the definition of nominal earnings, $Y_r = L_r + \Pi * L_r/L$, which implies $Y_r/Y_s = L_r/L_s$, to re-write this condition as

$$\frac{N_r}{N} = \frac{L_r}{L} + \frac{\Phi}{1-\Phi}\frac{L_r - L_s}{L} \tag{3}$$

For such a proportion of firms in country r, profits are equalized across countries. This is the interior location equilibrium. The equilibrium is feasible only if the proportion of firms is smaller than 1, that is, only if $\Phi \leq L_s/L_r$. For $\Phi > L_s/L_r$, the output of a firm located in r is larger than its potential output in the small country s; in such a case, the profit is larger in country r and all firms locate in the large country, $N_r/N = 1$.

From (3), it is clear that the large country hosts a more than proportionate share of firms. This is the home market effect. Moreover, the easier it is to trade, the larger is this effect: the proportion of firms in the large market increases with Φ.

To understand this result, note that the difference between the two countries is twofold: a difference in sizes ($Y_r/Y_s = L_r/L_s$ for any location of firms and for any trade costs) and a difference in competition if the mass of firms differs across countries. Consider prohibitive trade costs ($\Phi = 0$). In country k ($k \in \{r, s\}$), each firm sells $\mu Y_k/N_k$. The numerator μY_k denotes the market size of country k. The denominator denotes the market share of each firm active on market k (the strength of competition on market k). These firms do not compete with the firms located on the other market because trade costs are prohibitive. Obviously, the output per firm is equalized across countries if and only if the partition of firms between countries is equal to the partition of earnings (or populations). In that case, firms located in the small country face a weak competition in a small market whereas firms located in the large country face a fierce competition in a large market.

Now, consider a decrease in trade costs (an increase in Φ). This does not change the market sizes but this affects the competition on each market. In the small market, competition increases significantly because firms located on this market now face the competition of the large number of firms located on the large market. By contrast, competition increases less in the large

market because there are only few firms in the small market that can now sell in the large market. As a result, everything else equal, output per firm falls more in the small market than in the large market and the large market becomes more profitable than the smaller market. The opening of trade between a small and a large country increases substantially competition in the small market and affects the large market to a lesser extent. Therefore, some firms from the small market move to the large market, which decreases competition in the small market and increases it in the large market. Hence, the equalization of profits across countries is restored through a movement of firms from the small to the large market which now hosts a more than proportionate mass of firms.

4 Firms location with any types of firms

We now extend the analysis and allow firms to become multinational (or to stay single-plant firms). We begin with the analysis of firms location. Then we examine the effects of a reduction in trade costs on total production and employment. Finally we discuss two extensions of the model to take into account economic geography and tax competition.

4.1 Firms location

The analysis requires the comparison of the profits made by the different firms. A single-plant firm in country r earns $\pi_r(i) = q_r(i)/(\sigma-1) - f$ where $q_r(i) = \mu p^{-\sigma}\left(Y_r P_r^{\sigma-1} + \Phi Y_s P_s^{\sigma-1}\right)$. A multinational firm sells more because it saves on trade costs, but it also has to build two plants; its profits is $\pi_t = q_t(i)/(\sigma-1) - 2f$ where $q_t(i) = \mu p^{-\sigma}\left(Y_r P_r^{\sigma-1} + Y_s P_s^{\sigma-1}\right)$.[7] It is the trade-off between the size of sales (proximity) and the costs of duplicating plants that determines firms' organization and location. To check the optimal firms' location and firms' organization, we use (2) to compare the profits for any partition $(N_r,\, N_s,\, N_t)$.

$$\begin{aligned}
\pi_k &= \frac{1}{\sigma}\left(\frac{Y_k}{N_k + N_t + \Phi N_l} + \Phi\frac{Y_l}{N_l + N_t + \Phi N_k}\right) - f, \qquad k,l \in \{r,s\} \text{ and } k \neq l, \\
\pi_t &= \frac{1}{\sigma}\left(\frac{Y_r}{N_r + N_t + \Phi N_s} + \frac{Y_s}{N_s + N_t + \Phi N_r}\right) - 2f. \qquad (4)
\end{aligned}$$

[7] This is the same output as a single-plant firm that could operate under no trade costs ($\Phi = 1$).

Also, we use (4) to find $\Pi = N_r\pi_r + N_s\pi_s + N_t\pi_t = \mu\left(Y_r + Y_s\right)/\sigma - (N + N_t)\, f$, which we combine with the definition $Y_k = L_k + \Pi * L_k/L$ to find

$$Y_k = \sigma L_k \frac{L - (N + N_t)\, f}{(\sigma - \mu)\, L} \tag{5}$$

Plugging this expression in (4) gives the profits as functions of the partition of firms (N_r, N_s, N_t).

We will distinguish two cases in the analysis. One in which the two countries do not differ too much in size, and the other in which they substantially differ in size. The critical condition for separating the two cases is the following:

$$\frac{L_r - L_s}{L} < \frac{\sigma - \mu}{\mu} \frac{Nf}{L - Nf} \tag{6}$$

The comparison of profits is done in Appendices 7.1 and 7.2. The critical values of freeness to trade (Φ_1, ..., Φ_4) are given in Appendix 7.1. The results show that under (6), five types of equilibria may exist:

- (i) for large trade costs ($\Phi \leq \Phi_1$), all firms are multinationals;
- (ii) for smaller trade costs ($\Phi \in]\Phi_1, \Phi_2]$), some firms are multinationals and the others are single-plant firms in the large country;
- (iii) for smaller trade costs ($\Phi \in]\Phi_2, \Phi_3]$), some firms are multinationals and the others are single-plant firms in the large or in the small country;
- (iv) for smaller trade costs ($\Phi \in]\Phi_3, \Phi_4]$), some firms are single-plant firms in the large or in the small country;
- (v) for smaller trade costs ($\Phi > \Phi_4$), all firms are single-plant firms in the large country.

If (6) does not hold, then equilibria (iii) and (iv) vanish whereas (ii) holds for $\Phi \in]\Phi_1, \Phi_2']$ and (v) holds for $\Phi > \Phi_2'$. Figure 1a summarizes the results when (6) holds whereas Figure 1b represents firms' location otherwise.

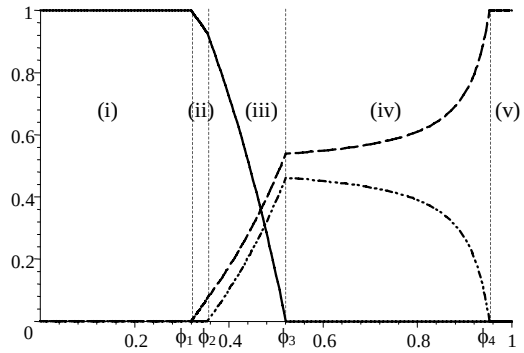

Figure 1a: firms' location as a function of trade openness ($\sigma = 5$, $\mu = .7$, $N = 1$, $f = .2$, $L_s = 2$, $L_r = 2.1$) Plain: multinationals; dash: national firms in r; dots: national firms in s.

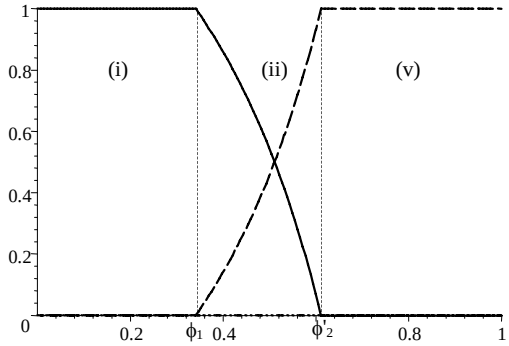

Figure 1b: firms' location as a function of trade openness ($\sigma = 5$, $\mu = .7$, $N = 1$, $f = .2$, $L_s = 2$, $L_r = 4$) Plain: multinationals; dash: national firms in r; no national firms in s.

We first examine the link between trade openness and firms' organization and location. Then we focus on the number of varieties produced in each country. Finally, we examine the link between population imbalances, trade openness and firms' organization and location.

As expected (and formally shown in Appendix 7.2), the number of multinationals decreases with trade openness, whereas the number of single-plant firms in the large country increases with openness. Interestingly, we distinguish two cases for the evolution of the number of single-plant firms in the small country. For small differences between the two countries, the number of single-plant firms in the small country increases with trade openness when openness is low whereas it decreases with trade openness when openness is

large. When the difference between both countries is strong enough, the small country never hosts single-plant firms.

The location equilibrium results from a trade-off between two forces. On the one hand, firms tend to switch from multinationals to single-plant firms when trade costs are reduced because it becomes less costly to export the production. On the other hand, a reduction in trade costs exacerbates the home market effect: as we have seen in previous section, single plant firms tend to agglomerate in the large market because the small market suffers more from the extra competition induced by the fall in trade costs. This second effect is stronger, the larger is the difference between the sizes of the two countries[8] and the smaller are trade costs. If the second effect is not too strong (i.e. the two countries do not differ too much in sizes), then a reduction in trade costs first induces some multinationals to become single-plant firms, some of them locate in the small country. However, a further reduction in trade costs makes competition so strong in the small country that all single-plant firms move to the large country. If the two countries differ substantially in sizes, then multinational firms always locate in the large country when they switch to single-plant firms.

Let us now focus on the number of varieties produced in each country. For large trade costs, all firms are multinationals. Hence, both countries produce the same number of varieties. For lower trade costs, the number of multinationals decreases; the number of single-plant firms in the small country may increase, but one can check that it increases by a smaller amount.[9] Thus, trade openness reduces the number of varieties produced in the small country. Moreover, when single-plant firms exist, they are over-represented in the large country: $N_r/N_s > L_r/L_s$, which is reminiscent of the home market effect (see the Appendix 7.3 for a formal proof).

We have represented the location equilibria as function of freeness of trade. We can also represent them as a function of the population imbalance, though the analysis is a little bit more tedious to perform. For that purpose, we use the dependence of the bounds Φ_j on L_r: Φ_1, Φ_2' and Φ_4 decrease with L_r, whereas Φ_2 increases with L_r and Φ_3 is independent of L_r. Moreover, Φ_2 is equal to Φ_1 at $L_r = L/2$ and it is equal to Φ_2' when (6) holds with equality. These properties allow us to depict the location equilibria in the axes (L_r, Φ) as in Figure 2.[10] The figure shows that for large trade costs,

[8] When countries have approximately the same size, they face more or less the same competition.

[9] The only regime under which $N_s + N_t$ could in principle increase is regime (iii) and it is readily checked with the values of N_s and N_t in the Appendix that $N_s + N_t$ decreases with Φ.

[10] Note that the Figure extends the result to consider also $L_r < L/2$.

firms are multinationals only if the population imbalance is not too large (the grey area). If the population imbalance is intermediate, then some firms are multinationals whereas the others are single-plant firms in the large country (the area with vertical lines). Finally, if population imbalance is large, all firms are single-plant firms in the large country (the area with dots). For a small range of intermediate trade costs, we have a similar ranking with the exception that if population imbalance is small, the three types of firms coexist (instead of being all multinationals) (the black area). Finally, small trade costs are incompatible with multinationals; we find the same equilibria as in standard models of economic geography: coexistence of single-plant firms in both countries for small population imbalances (the white area) and single-plant firms in the large country otherwise (the area with dots).

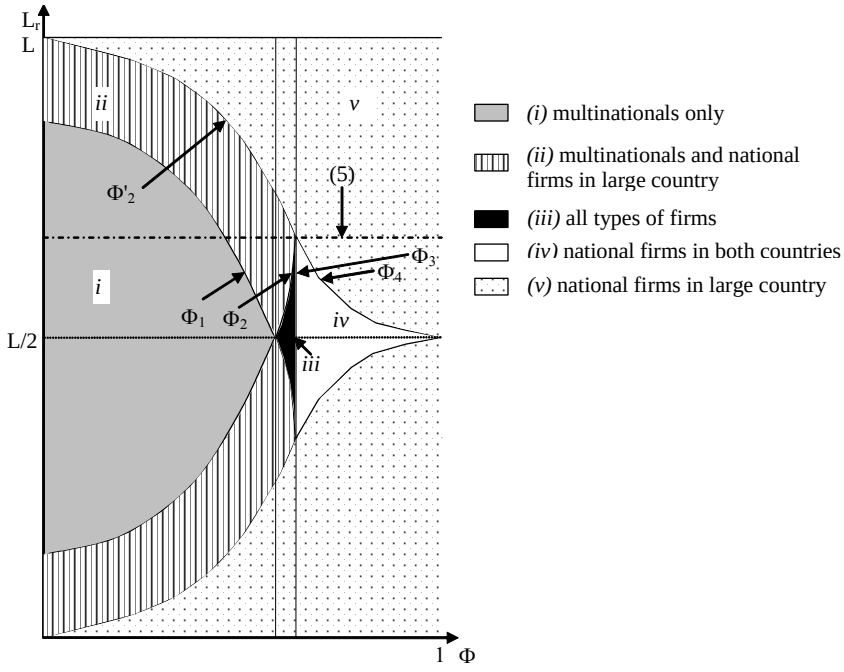

Figure 2: firms behavior as a function of freeness of trade and population imbalances.

4.2 Production and employment

In this section, we focus on the production in each country. We first start with the production per inhabitant, which allows us to emphasize the home

market effect; we then follow with the production per firm and we finish with the total production.

4.2.1 Production per inhabitant

A plant located in country r sells $\mu p^{-\sigma} Y_r P_r^{\sigma-1}$ to its domestic market. This is the sole production of the plant if it is a subsidiary of a multinational. By contrast, a single-plant firm also sells $\Phi\mu p^{-\sigma} Y_s P_s^{\sigma-1}$ to the foreign market. The total production in country r is $\mu p^{-\sigma}[(N_r + N_t) Y_r P_r^{\sigma-1} + N_r \Phi Y_s P_s^{\sigma-1}]$. Dividing this expression by L_r gives production *per inhabitant* in country r. We can use (2) and (5) to check that production per inhabitant is larger in the large country r if and only if

$$N (L_s N_r - L_r N_s) + N_r N_s (1 - \Phi) (L_r - L_s) \geq 0$$

This condition always holds.[11] Thus, production per inhabitant is always at least as large in the large country as in the small country. This is the home market effect. Note however that if all firms are multinationals ($N_r = N_s = 0$), the condition holds with equality. That is, the home market effect vanishes and the production per inhabitant is the same in each country.

4.2.2 Production per firm

We examine how the output per firm responds to a reduction in trade costs. A reduction in trade costs has several effects: (1) it makes the foreign country more accessible, which induces single-plant firms to produce more for the export; (2) it increases competition from foreign firms, which reduces the market share (and the production) of each plant on its domestic market; (3) it induces some firms to change their organization, and/or their location. The balance between these three effects depends on the regime in which firms operate.

Table 1 presents the main results that are developed in this section. A fall in trade costs raises the production of each plant. This increase is balanced by the loss of firms in the small country whereas it is (in most cases) accompanied by an increase in the mass of firms in the large country. As a result, production may increase or decrease in the small country whereas it increases in the large country.

[11] Using the result that $L_s N_r - L_r N_s \geq 0$ under all regimes (see Appendix 7.3).

		Δ production				
		(i)	*(ii)*	*(iii)*	*(iv)*	*(v)*
small country	- production of a single-plant	n.a.	n.a.	+	0	n.a.
	- production of a multinational	0	+	+	n.a.	n.a.
	- total production	0	±	±	-	n.a.
large country	- production of a single-plant	n.a.	+	+	0	0
	- production of a multinational	0	+	+	n.a.	n.a.
	- total production	0	+	+	+	0

0 : unchanged + : increase - : decrease n.a.: non applicable

Table 1: effects of a decrease in trade costs on production.

In the following we examine successively the production of multinationals and single-plant firms under the various regimes.

Production of a multinational plant The production of a multinational plant in country k is $\mu p^{-\sigma} Y_k P_k^{\sigma-1}$. Using (2) and (5), we can check that this expression is proportional to

$$\frac{L - (N + N_t) f}{N_k + N_t + \Phi N_l} \tag{7}$$

which is a useful shortcut for production of a multinational plant in country k. The numerator denotes the worldwide resources available for consumption; a proportion L_k/L of them are available in country k. The denominator represents the strength of competition within country k. For prohibitive trade costs ($\Phi = 0$), plants in country k compete only with plants located in that country (i.e., with $N_k + N_t$ plants). In the absence of trade costs ($\Phi = 1$), firms in country k compete with all the other firms (i.e., with $N_k + N_t + N_l$ plants). More generally, competition with foreign firms increases when trade costs decrease.

Under **regime (i)**, all firms are multinational. Then it is straightforward to check that (7) is independent of Φ. The production of a multinational plant in any country k is independent of trade costs because these plants do not export.

Regime (ii) is more intricate: some single-plant firms are located in the large country, all the other firms are multinationals, and a reduction in

trade costs induces some multinational firms to concentrate their production in a single-plant firm located in the large country. We first consider the production of a multinational plant located in the large country; then we consider the small country.

In the large country r, one can write the production of a multinational plant in r, (7), as $[L-(N+N_t)f]/[N_r+N_t]$. The numerator denotes the worldwide resources available for consumption. It increases when fewer firms chose to be multinational because fewer resources are used in the duplication of plants. Under regime (ii), the reduction in trade costs induces fewer firms to be multinationals, so that resources available for consumption and production are increased. The denominator is indicative of the strength of competition among firms located in the large country. Under regime (ii), all firms have plants located in the large country: N_r+N_t is constant and equal to N.[12] Hence, the market share of each firm is unchanged in the large market. Combining all the effects, we check that a reduction in trade costs increases the production of a multinational plant in the large country.

In the small country s, (7) can be written as $[L-(N+N_t)f]/[\Phi N_r+N_t]$. As in the above analysis with the large country, firms can sell more if trade costs fall because fewer resources are used in the duplication of multinational plants; this can be seen from the numerator. The denominator, indicates the strength of competition in the small country. In contrast to the case of a large country, it is not constant because not all firms are active in the small country: the single-plant firms are located in the large country only and they suffer from trade costs when they sell in the small country. A fall in trade costs has contrasting effects on competition. It makes the small country more accessible to single-plant firms located in the large country. The *direct effect* of this fall is thus an increase in competition. However, the fall also induces some multinational firms to concentrate their production into the large country. The plant that is closed in the small country had a perfect access to the small market whereas the plant where production is concentrated in the large country has a poor access to this market. Hence, competition is decreased in the small market and plants of multinationals gain market shares in the small market. These two effects on competition can be traced by differentiating the denominator with respect to Φ: $N_r+\partial N_t/\partial\Phi+\Phi\partial N_r/\partial\Phi = N_r+(1-\Phi)\,\partial N_t/\partial\Phi$. The first term is the direct effect whereas the last is the indirect effect through a change in firms' organization. The net effect on competition is unknown. Still, the combination of the ambiguous com-

[12] When trade costs decrease, the multinational firms that close their plant in the small country to concentrate their production in the large country do not change the strength of competition in the large country because they were already active in that country.

petition effect and the positive demand effect gives non-ambiguous results. Using the values of N_r and N_t given in Appendix 7.2 and differentiating (7) with respect to Φ, gives $(\sigma - \mu) Lf / \left[\mu L_s (1 - \Phi)^2\right]$ which is clearly positive. In the small country, the plant of a multinational produces more when trade costs fall.

The analysis of **Regime (iii)** is similar to the small country analysis in regime (ii). The reduction in trade cost induces more firms to become single-plant firms, which raises resources available for consumption because fewer resources are spent in fixed costs. Multinational plants suffer now more from competition of the single-plant firms located in the other country. However, fewer firms are directly active in each country because some firms have concentrated their production in the other country. As a result the net effect on competition in each country is unknown. To find the net effect on the production of a multinational plant, we use the values of N_r, N_s and N_t given in Appendix 7.2 and we compute the derivative of (7) with respect to Φ, which gives $(\sigma - \mu) Lf / [\mu L_l (1 - \Phi)]$ for country $k \neq l$. This is clearly positive, which means that plants of multinational firms produce more.

Production of a single plant firm The production of a single-plant firm in country k is $\mu p^{-\sigma} Y_k P_k^{\sigma-1} + \Phi \mu p^{-\sigma} Y_l P_l^{\sigma-1}$. Using (2) and (5), we can check that this expression is proportional to

$$\frac{L - (N + N_t) f}{N_k + N_t + \Phi N_l} \left(1 + \Phi \frac{L_l}{L_k} \frac{N_k + \Phi N_l + N_t}{N_l + \Phi N_k + N_t}\right) \tag{8}$$

Under **regime (ii)** the sales of a single plant firm in its own country are the same as the sales of a plant of a multinational in the same country. We have analyzed above the effects of a fall in trade costs. On the one hand, there are fewer multinationals, which allows to save fixed costs and to consume more. On the other hand, competition is unchanged in the home country as the multinationals that concentrate their production in the foreign country were already active in that country. It results that single-plant firms sell more to their own country. In the foreign country, the fall in trade costs allows consumers to spend more on consumption, which increases sales. Competition in that country is affected by two opposite effects. On the one hand, the foreign country becomes more accessible, which raises competition and decreases the market share of each firm. On the other hand, there are fewer firms located in the foreign country, which reduces competition. The net effect on competition is a priori unknown. Using the value of N_r given in Appendix 7.2 and differentiating the sales on the foreign market with respect

to Φ gives $\Phi(\sigma-\mu)Lf/[\mu L_r(1-\Phi)]$ which is clearly positive. Single plant-firms raise the sales to the foreign country. As a result total sales of a single plant firms increase as trade costs are reduced.

We now examine the response of sales of a single-plant firm to a fall in trade costs under **regime (iii).** On the one hand, sales to both countries increase because fewer resources are used in the duplication of plants. On the other hand competition is affected by opposite forces: the foreign country is more accessible but fewer firms have a plant located there; the home country is also more accessible, which increases the competition from single-plant firms located in the foreign country, but fewer plants are located in the home country, which reduces competition. Despite these opposite forces, we can check that the net effect is positive. To check this formally, it suffices to plug the values of N_r, N_s and N_t shown in Appendix 7.2 in (8) and to differentiate the expression with respect to Φ. This gives $(1+\Phi)(\sigma-\mu)Lf/[\mu L_k(1-\Phi)]$, for country $k \neq l$, which clearly increases with Φ.

Under **regime (iv)**, a reduction in trade costs makes both markets more accessible. Single-plant firms sell more easily to the foreign market but they suffer more from competition on their home market. Using in (8) the values $N_t = 0$ and the values of N_r, N_s given in Appendix 7.2, one can show that the production of a single plant firm is proportional to $L(L-fN)/NL_k$, for country $k \neq l$, which is clearly independent of Φ. Thus, trade openness does not alter the production of single-plant firms in this regime.

Finally, under **regime (v)**, single-plant firms from the large country gain a better access to the small country when trade costs are reduced. However, they compete more severely to access these consumers. As a result of these two forces they do not change their production as can be seen by plugging $N_r = N$ and $N_s = N_t = 0$ in (8): the resulting expression is independent of Φ. Note however that consumers from the small country increase their consumption because fewer goods are lost in transport. In this regime, trade liberalization benefits the consumers rather than firms.

4.2.3 Total production

So far the analysis shows that firms (multinationals and single-plant firms) never decrease their production when trade costs fall. This does not imply that *total production* in a country never decrease, as the number of firms responds to trade costs.

We first examine the response of total production (employment) in the modern sector to trade liberalization in *the large country.* It does not respond under regimes (i) and (v) in which neither the production per plant

nor the number of plants in the large country respond to a fall in trade costs. Employment clearly rises under regime (iv): the large country hosts more single-plant firms whose individual production is unchanged. It also rises under regime (ii): the production of each plant increases when trade costs decrease; some multinational firms concentrate their production into the large country. Since single-plant firms produce more than the plant of a multinational, total production is raised in the large country. It is less straightforward to analyze regime (iii). The production of each plant increases; as under regime (ii) but the multinational plants are not replaced one for one by single-plant firms in the large country; some of them are replaced by single-plant firms in the small country. Computations (shown in Appendix 7.4) show that the net effect is nevertheless positive: total production increases in the large country under regime (iii).

The response of total production (employment) in the modern sector to trade liberalization in the *small country* is different. It does not respond under regimes (i) and (v) in which neither the production per plant nor the number of plants respond to a fall in trade costs. Employment clearly falls under regime (iv): the small country hosts fewer single-plant firms whose individual production is unchanged. Regimes (ii) and (iii) cannot be fully treated analytically. Under regime (ii) the plants of multinational firms produce more but fewer of them locate in the small country. The net effect is ambiguous but simulations suggest that it is more likely to be negative. The situation is less unfavorable in regime (iii). In that case, the small country hosts more single-plant firms that produce more than the plants of multinational firms. The net effect of a fall in trade costs is ambiguous but simulations suggest that it is likely to be positive, as shown in the next figure.

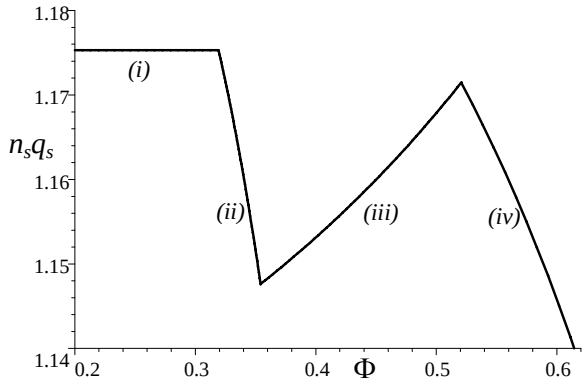

Figure 3: total production in the small country as a function of trade openness. ($\sigma = 5$, $\mu = .7$, $N = 1$, $f = .2$, $L_s = 2$, $L_r = 2.1$)

4.3 Economic geography

In models of economic geography, agglomeration of economic activity in a country raises the attractiveness of that country and promotes further agglomeration (see Baldwin et al. (2003)). This circular causality is absent from the model that we have developed in which (partial) agglomeration is caused by the home market effect only. Since we have assumed that agents are immobile, the agglomeration of firms does not induce agents to migrate to the country where firms agglomerate. In this section, we remove this assumption and we assume that agents can move across countries.

In standard models of economic geography, it is assumed that some agents can move but other agents cannot move: in Krugman (1991) workers are mobile but farmers are immobile; in Forslid and Ottaviano (2003) skilled workers are mobile but unskilled are not. The population that cannot move creates a dispersion force that ensures that agglomeration of economic activity is not the sole stable equilibrium. In our model, we have considered only one type of agent, the workers who are also the owners of capital. Thus the model does not allow to distinguish the mobility of distinct agents. Therefore, in this section, we have to assume that all agents are mobile. The dispersion force of a pool of immobile agents is removed. As a result, agglomeration of firms and of agents will be a stable equilibrium for any value of trade costs. Still we show that this is not the only stable equilibrium. The existence of multinational firms creates other equilibria for large trade costs.

When moving, an agent considers her earnings in both countries. By construction, the wage is the same across countries and an agent earns the same share of profits wherever she locates. Therefore, the nominal earnings are equal across countries. By contrast, real earnings may differ because the price levels can be different. The price of the good produced under constant returns to scale is normalized to one in both countries, but the price level of the goods produced under increasing returns to scale will be different if the number of varieties produced in a country is not the same as in the other country: P_s is larger than P_r if and only if $N_s < N_r$. Therefore, individuals will move towards the country that hosts more single-plant firms.

A location is an equilibrium if the agents do not want to migrate to the other country and if firms do not want to move or to change their status (single-plant versus multinational). The equilibrium is *unstable* if the migration of a small mass of agents induces other agents to migrate in the same direction. It is *stable* if the migration of a small mass of agents induces other agents (or the same agents) to migrate in the reverse direction. We will

see that the existence of multinationals creates another type of equilibrium which we call "*weakly stable*" and which we define as a situation in which the migration of a small mass of agents does not induce any other agents to move.

As a first type of equilibrium, we consider agglomeration of agents in one country (say country r, $L_r = L$ and $L_s = 0$). Then, it is readily checked that all firms choose to be single-plant firms in country r ($\Phi_4 = 0$ and $\Phi_2' \to 0$): $N_r = N > N_s = 0$. Therefore, the price level is smaller in country r than in s and the agents do not want to migrate to country s. Agglomeration of agents and firms is a stable equilibrium for any value of freeness of trade, Φ.

As a second type of equilibrium, we consider a case in which the mass of single-plant firms is strictly positive. This partition of firms is an equilibrium only if the price levels are equalized across countries, that is, only if the mass of single-plant firms is the same in both countries. However such an equilibrium is unstable because the migration of a small mass of agents to country r, induces more firms to become single-plant firms in r and it induces fewer firms to stay single-plant firms in s (it is readily checked that under equilibria of types (iii) and (iv), N_r increases with L_r, see the expressions of N_r in Appendix 7.2). Thus, the price level decreases in country r, which induces more agents to migrate to that country.

Finally, we consider a case where all firms are multinationals. The price levels are then equal in both countries, and this partition of firms is compatible with an equilibrium because none of the agents want to move. As can be seen from Figure 2, for a given value of freeness of trade, there is a range of the partition of agents that supports an equilibrium with multinationals (the grey area). Within this range, the migration of a small mass of agents does not induce firms to change their status. Since firms remain multinationals, the price levels are still equalized across countries and agents do not want to follow or to counter the initial migration. This type of equilibrium is weakly stable.

To sum up, the stable equilibria of the model with workers mobility are depicted by full agglomeration of firms and workers (the top and bottom borders of Figure 2)[13] and the weakly stable equilibria are depicted by the grey area in which all firms are multinationals. The model includes strong agglomeration forces, so that agglomeration of economic activity is always

[13]Note that if we assume that only some agents are mobile, then the equilibrium with agglomeration would be replaced with an equilibrium in which all mobile agents are agglomerated in a single country whereas firms are not necessarily agglomerated. In terms of Figure 2, the firms behavior would be represented by an horizontal line at the value of L_r equal to the sum of the mass of immobile workers from r and the mass of mobile workers.

stable. Still, the possibility for firms to split their production across countries attenuates the agglomeration force. For large trade costs and for a partition of agents that is not too unequal, there exists weakly stable equilibria in which firms locate plants in each country. In this simple model, these equilibria are clearly not pareto-efficient. The reason is that there is no costs associated to agglomeration: no congestion costs and no costs of migrations. The number of varieties produced under agglomeration is the same as under a weakly stable equilibrium, so that agents benefit from the same price level in both types of equilibria. However agglomeration requires fewer resources to be spent in the operation of plants, so that agents have a larger consumption under agglomeration than when all firms are multinationals.

4.4 Tax competition

In models of economic geography, it has been emphasized that the country where firms agglomerate (the core) can keep all firms even if it imposes a larger tax rate than the periphery (see Baldwin and Krugman, 2004; Kind, Midelfart Knarvik and Schjelderup, 2000; Ludema and Wooton, 2000). The intuition is simple: firms agglomerate in the core because they make profits that are strictly higher than in the periphery. A small increase of the tax on their profits will not reverse the inequality even if the tax in the periphery is unchanged. The government just taxes the locational rent. Thus it is possible to tax firms in the core more than in the periphery. This result holds for values of parameters that allow for agglomeration, i.e. for small trade costs.

In this paper, it is straightforward to check that a rent is also created at large trade costs because of the existence of multinational firms (see Behrens and Picard, 2005, for a similar result). Indeed, for large trade costs and for countries that do not differ too much in sizes, all firms have plants in both countries because their profit is strictly larger than under concentration of their production in a single plant. This profit differential is an organizational rent that can be taxed by a government. Indeed, a country can capture parts of the profits made by a plant located on its territory without inducing the multinational to close this plant and to concentrate its production in the other country.

5 Conclusion

The paper develops a model with two asymmetric countries. Firms choose the number and the location of plants that they operate. We show how firms

location, firms organization and firms production depend on trade costs and on the population imbalance.

All firms (multinational firms and single-plant firms) raise (do not decrease) their production when trade costs fall. However, a fall in trade costs also induces some firms to change their location or organization. Fewer firms choose to organize themselves as multinationals; more firms choose to have a single-plant located in the large country. There is also a range of trade costs for which some firms choose to concentrate their production in a single-plant located in the small country. As a result, total production (employment) never decrease with trade costs in the large country, whereas it may increase or decrease in the small country. A fall in trade costs is unambiguously beneficial to the large country that benefits from the home market effect. It may, or may not, benefit the small country.

The effect of the population imbalance on the location and organization of firms depends on the level of trade costs. We distinguish three cases. First, for large trade costs, a small population imbalance induces firms to choose a multinational organization. A larger imbalance induces some of the firms to concentrate their production into the large country whereas the other firms are multinationals. Finally, a huge imbalance induces all firms to be single-plant firms into the large country. Second, for lower trade costs, a small imbalance induces some firms to be multinationals whereas the other firms are single-plant firms in the large or in the small country. A larger imbalance favors the large country in which more single-plant firms locate. Third, for small trade costs, all firms are single-plant firms whatever the population imbalance. Some single-plant firms are active in the small country if the imbalance is small whereas they are all active in the large country if the imbalance is large.

We extend the model in two directions. First we endogenize country sizes: individuals migrate towards the country where real earnings are larger. The model does not have a strong dispersion force (such as an immobile mass of agents) and it is therefore not surprising that agglomeration of agents and firms is a stable equilibrium for any level of trade costs. Still there exists another equilibrium for large trade costs: all firms are multinationals and the population imbalance is not too strong. In that equilibrium a small migration of agents does not induce firms to change their organization and it does not entail any change in the real earnings of the agents. Therefore, the migration is not followed by any force that induces some agents to follow the initial migration or that induces the agents who have migrated to migrate back. This is a "weakly stable" equilibria where a small deviation from the initial equilibrium immediately ends up in a new equilibrium where all firms remain multinationals.

Second, we show that the equilibrium in which all firms are multinationals creates a rent. A government can tax this rent away without inducing the firms to change their location.

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7 Appendix

7.1 Critical values of Φ

$$\begin{aligned}
\Phi_1 &\equiv 1-\frac{Nf(\sigma-\mu)}{\mu(L-L_r)}\frac{L}{L-2Nf}\\
\Phi_2' &\equiv \left[1+\frac{Nf(\sigma-\mu)}{\mu(L-L_r)}\frac{L}{L-Nf}\right]^{-1}\\
\Phi_2 &\equiv 1-\frac{Nf}{\mu L_r}\frac{\sigma L-2\mu L_r}{L-2Nf}\\
\Phi_3 &\equiv 1-\frac{2(\sigma-\mu)Nf}{\mu(L-2Nf)+Nf\sigma}\\
\Phi_4 &\equiv \frac{L_s}{L_r}
\end{aligned}$$

7.2 Location equilibria with any type of firms

Equilibria of type (i). We set $N_t = N$, $N_r = 0$ and $N_s = 0$ in (4) and (5). Then we check that $\pi_t > \pi_r$ and $\pi_t > \pi_s$ if and only if $\Phi < \Phi_1$.[14]

Equilibria of type (ii). We set $N_s = 0$ in (4) and (5), which we plug in π_r, π_s and π_t. To have coexistence of multinationals and single-plant firms in the large country, it must be that $\pi_t = \pi_r$, which requires

$$\frac{N_t}{N} = 1-\frac{1}{\sigma L-\mu L_r}\left[\frac{(\sigma-\mu)L}{1-\Phi}-\frac{\mu L_s(L-2fN)}{Nf}\right],\ \frac{N_r}{N}=1-\frac{N_t}{N}$$

It is readily checked that $N_r > 0$ if and only if $\Phi > \Phi_1$ and $N_t > 0$ if and only if $\Phi < \Phi_2'$. Also, to have $N_s = 0$ it must be that, at the equilibrium, $\pi_r > \pi_s$ which requires $\Phi < \Phi_2$. Finally, $\Phi_2' > \Phi_2$ if and only if (6) is fulfilled. Hence, under (6), equilibria of type (ii) exists if and only if $\Phi \in (\Phi_1, \Phi_2)$. Note that it is easily checked that N_r increases with Φ whereas N_t decreases with Φ.

Equilibria of type (iii). It must be that $\pi_t = \pi_r = \pi_s$. These two conditions requires that

$$\begin{aligned}
\frac{N_r}{N} &= \frac{L_r}{L}+\frac{\Phi}{1-\Phi}\frac{L_r-L_s}{L}-\frac{L_s}{L}\frac{N_t}{N}\\
\frac{N_s}{N} &= \frac{L_s}{L}+\frac{\Phi}{1-\Phi}\frac{L_s-L_r}{L}-\frac{L_r}{L}\frac{N_t}{N}\\
\frac{N_t}{N} &= 1-\frac{2(\sigma-\mu)}{\sigma(1-\Phi)}+\frac{\mu(L-2fN)}{\sigma Nf}
\end{aligned}$$

[14] Note that the second inequality is always fulfilled under the first inequality.

It is readily checked that $N_s > 0$ if and only if $\Phi > \Phi_2$ and that $N_r > N_s$. Also, $N_t > 0$ if and only if $\Phi < \Phi_3$. Hence, all types of firms coexist if $\Phi_2 < \Phi < \Phi_3$. Note that the interval exists ($\Phi_3 > \Phi_2$) if and only if (6) is fulfilled.

Finally, it is readily checked that N_r and N_s increase with Φ whereas N_t decreases with Φ.

Equilibria of type (iv). We set $N_t = 0$. To have single-plant firms in both countries, it must be that $\pi_r = \pi_s$, which requires

$$\frac{N_r}{N} = \frac{L_r}{L} + \frac{\Phi}{1-\Phi}\frac{L_r - L_s}{L}, \; \frac{N_s}{N} = 1 - \frac{N_r}{N}$$

N_r is always positive whereas N_s is positive if and only if $\Phi < \Phi_4$. Finally, the profit of a multinational must be smaller than that of a single-plant firm: $\Phi > \Phi_3$. Hence, single-plant firms coexist if $\Phi_3 < \Phi < \Phi_4$ where it can be checked that $\Phi_4 > \Phi_3$ if and only if (6) is fulfilled. Note that it is readily checked that N_r increases with Φ whereas N_s decreases with Φ.

Equilibria of type (v). We set $N_r = N$, $N_s = 0$, and $N_t = 0$. We check that $\pi_r > \pi_s$ if and only if $\Phi > \Phi_4$ whereas $\pi_r > \pi_t$ if and only if $\Phi > \Phi_2'$. Moreover, $\Phi_4 > \Phi_2'$ if and only if (6) is fulfilled.

7.3 Proof that $N_r/N_s > L_r/L_s$

Under regimes (ii) and (v), the result is trivial ($N_s = 0$ and $N_r > 0$). Under regime (iii), we can use the equilibrium values of N_r, N_s and N_t to show that $N_r L_s - N_s L_r = (L_r - L_s)\left[N_t(1-\Phi) + N\Phi\right]/(1-\Phi) > 0$. Under regime (iv), $N_s = N - N_r$, and we can use the value of N_r to establish that $N_r L_s - N_s L_r = (L_r - L_s) N\Phi/(1-\Phi) > 0$.

7.4 Proof that the total production in the large country increases under regime (iii)

Total production in the large country is $\mu p^{-\sigma}[(N_r + N_t) Y_r P_r^{\sigma-1} + N_r \Phi Y_s P_s^{\sigma-1}]$. Using (2) and (5), we can check that this expression is proportional to

$$\left(N_r + N_t + N_r \Phi \frac{L_s}{L_r}\frac{N_r + \Phi N_s + N_t}{N_s + \Phi N_r + N_t}\right)\frac{L - (N + N_t) f}{N_r + N_t + \Phi N_s}$$

We use the values of N_r, N_s, N_t in regime (iii) from Appendix 7.2 to show that this expression is equal to

$$\frac{(1-\Phi) L + 2Nf\Phi}{(\Phi - 1)^2 L_r}\frac{\sigma - \mu}{\sigma}(L_r - \Phi L_s)$$

The derivative of this expression with respect to Φ is

$$\frac{\sigma-\mu}{\sigma L_r\left(1-\Phi\right)^3}\left[\left(L_r-L_s\right)L\left(1-\Phi\right)+2Nf\left(L_r+\Phi L_r-2\Phi L_s\right)\right]>0$$

Under regime (iii), total production increases when trade costs fall.