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ABSTRACT

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Contrary to widespread belief, we show that low-pay workers might not generally prefer that the minimum wage rate be increased to a level where the labor demand is unitary elastic. Rather, there exists a critical value of elasticity of labor demand such that increases in the minimum wage rate make low-pay workers better off for higher elasticities, but worse off for lower elasticities. We demonstrate that the critical value decreases with the workers' income-equivalent wage rate and increases with their risk aversion. It is also shown that there may not exist an optimal minimum wage rate, and if it does exist, may not be unique.

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1 Introduction

A common assumption in the minimum-wage literature is that low-wage workers as a group benefit from an increase in their total wage income. These workers are therefore best off when the minimum wage rate is set at a level where the aggregate demand for low-wage labor is unitary elastic, and are made better off by increases in the minimum wage rate as long as the aggregate demand for labor is inelastic. As a result, there is a close connection between the success of the minimum wage rate and the inelasticity of the demand for low-wage labor.[1] Indeed, the motivation for many empirical studies of the demand for low-wage labor is the desire to evaluate the efficacy of a legislated minimum wage for combatting poverty.[2]

In the face of a downward-sloping demand curve for labor, a minimum-wage legislation that raises workers' wages above the competitive level will inevitably lead to job losses for some of the workers.[3] The standard analysis assumes that the workers' income will then be reduced to zero and that leisure has no value. In reality, however, workers are likely to collect unemployment benefits or obtain a lower-paying job in an uncovered sector, and they may value leisure. Although they would be worse off than if they were employed at the minimum wage rate, they are not quite as bad off as assumed. That is, the income-equivalent wage

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[1] For example, according to Sobel (1999 p. 761) one of

> "the most popularly stated goals of minimum-wage policy [is to choose] the minimum-wage rate at which the relevant labor demand is unitary elastic – maximizing the total earnings of minimum-wage workers".

See also Freeman (1995), Deere et al. (1996), and Addison and Blackburn (1999). Economics textbooks follow the lead. A recent example is Kaufman and Hotchkiss (2006, p. 285):

> "Has the minimum wage law achieved its objective of raising the income of low-wage workers? The answer to this depends on the elasticity of the demand curve in the covered sector. If labor demand is *inelastic* the answer is yes; if it is elastic the answer is no."

[2] See Brown (1988), Neumark and Wascher (1992), Deere et al. (1995), and Burkhauser et al. (2000).

[3] Card (1992) and Card and Krueger (1994) question whether a minimum wage rate always reduces employment. For ensuing discussions, see Machin and Manning (1994), Neumark and Wascher (1994, 2000), Dickens et al. (1999), and Card and Krueger (2000).

rate for unemployed workers may very well be positive.

The standard analysis also assumes that workers care only about their expected wage income (the income at the minimum wage rate multiplied by the employment probability) and thus are risk neutral. More realistically, however, workers are likely to be risk averse and hence negatively affected by the uncertainty associated with whether they will earn the minimum wage rate or will have to make do with the income-equivalent wage rate.

The purpose of this paper is to examine how the income-equivalent wage rate and risk aversion affect the range of labor demand elasticities for which increases in the minimum wage rate benefit the working poor. We first show that for each minimum wage rate there exists a critical value of the elasticity of labor demand – generally different from minus unity – such that an increase in the minimum wage rate makes workers better off if labor demand is less elastic than the critical value, but worse off if labor demand is more elastic than the critical value. We then prove that the critical value of the elasticity of labor demand decreases with the income-equivalent wage rate and increases with the workers' risk aversion. Finally, we demonstrate that there may not exist an optimal (for workers) minimum wage rate, or that an optimal minimum wage rate may not be unique.

2 The Model

We consider a low-wage labor market that is competitive except for a legislated minimum wage rate. There are N identical workers, each endowed with one unit of indivisible time. The minimum wage rate is denoted by m, and the utility of a worker that is employed at the minimum wage rate is $U(m)$, where $U' > 0$ and $U'' \leq 0$. A worker that does not find employment at the minimum wage rate obtains the utility $U(y)$, where $y \in [0, m)$ is the income-equivalent wage rate that stems from a combination of unemployment benefits, home production or lower-paid temporary jobs, and leisure. Hence, $U(m) - U(y)$ is the utility gain from being employed at the minimum wage rate.

The (aggregate) labor demand at the minimum wage rate is given by $L(m)$, where $L' < 0$.

It is assumed that the minimum wage rate is set above the market-clearing level, but not so high as to eliminate all employment, that is, $L(m) \in (0, N)$. Consequently, only part of the workers find employment at the minimum wage rate. Each worker has the same probability $L(m)/N$ of finding employment, so a worker's expected utility is

$$U(m)\frac{L(m)}{N} + U(y)\left[1 - \frac{L(m)}{N}\right].$$

Differentiating with respect to m, we obtain that the effect of an increase in the minimum wage rate on a worker's expected utility is

$$\begin{aligned} & U'(m)\frac{L(m)}{N} + [U(m) - U(y)]\frac{L'(m)}{N} \\ = & \frac{L(m)}{Nm}[U(m) - U(y)](e_L + e_U), \end{aligned} \tag{1}$$

where $e_L \equiv mL'(m)/L(m)$ is the (negative) elasticity of the labor demand with respect to m, and $e_U \equiv mU'(m)/[U(m) - U(y)]$ is the elasticity of the utility gain from employment at the minimum wage rate with respect to m. Thus, according to expression (1), the effect of an increase in the minimum wage rate on a worker's expected utility has the same sign as the sum of these two elasticities, i.e., as $e_L + e_U$. This also follows directly by recognizing that an increase in the minimum wage rate has the same effect on a worker's expected utility gain from employment ($[U(m) - U(y)]L(m)/N$) as on a worker's total expected utility.

For every value of e_U there exists a unique critical value of e_L which is denoted by e_L^* and defined by $e_L^* + e_U = 0$. Hence, a worker's expected utility is maximized if $e_L = e_L^*$.[4] Furthermore, a worker's expected utility increases with the minimum wage rate if $e_L > e_L^*$. On the other hand, the minimum wage rate is set so high that workers would benefit from a decrease in the minimum wage rate if $e_L < e_L^*$.

[4] It is assumed that $e_L = e_L^*$ indicates a maximum. Differentiating expression (1) with respect to m shows that the second-order condition for a maximum is $-L''(m)/L'(m) + U''(m)/U'(m) + 2e_L^* < 0$.

3 The Critical Value of e_L

In the special case that the income-equivalent wage rate for unemployed workers is zero ($y = 0$) and workers are risk neutral ($U'' = 0$), then $e_U = 1$. It follows that the critical value of e_L is minus unity. The reason is that a worker's expected utility is simply $mL(m)/N$, which is maximized at a minimum wage rate for which the demand for labor is unitary elastic ($e_L = -1$), as presumed in much of the literature. Furthermore, workers benefit from an increase in the minimum wage rate as long as the demand for labor is inelastic ($e_L > -1$), but are harmed by an increase in the minimum wage rate if the demand for labor is elastic ($e_L < -1$).

Turning now to the general case, the income-equivalent wage rate for unemployed workers may be positive and the workers may be risk averse. In that case, it is likely that e_U will differ from unity, and hence the critical value of e_L will differ from minus unity. In the following we establish how e_L^* depends on the income-equivalent wage rate and the workers' risk aversion.

3.1 The Income-Equivalent Wage Rate

We first consider the effect of the income-equivalent wage rate for unemployed workers. By the definition of e_U, the income-equivalent wage rate for unemployed workers affects e_U positively and therefore e_L^* negatively. The reason is that a higher income-equivalent wage rate reduces the utility gain from being employed at the minimum wage rate. The workers are therefore more willing to accept the adverse employment consequences of a higher minimum wage rate. Hence, the higher y is, the lower are the values of e_L for which the workers would still benefit from an increase in the minimum wage rate. That is, e_L^* decreases with the income-equivalent wage rate.

One particular consequence of the negative relationship between y and e_L^* is that if the income-equivalent wage rate is positive and workers are risk neutral, then $e_L^* < -1$. In other words, if $y > 0$, the expected utility of risk-neutral workers not only increases with the minimum wage rate if the demand for labor is inelastic; it also increases with the minimum wage rate if the demand for labor is unitary elastic and if it is elastic as long as the elasticity exceeds $e_L^* = -m/(m-y)$. Furthermore, if the income-equivalent wage rate is positive, then, independent of the workers' attitude towards risk, e_U approaches infinity as the minimum wage rate approaches the income-equivalent wage rate. It follows that e_L^* is very small for minimum wage rates that are close to the income-equivalent wage rate, and that an increase in the minimum wage rate can benefit workers even if the labor demand is very elastic.

3.2 Risk Aversion

We next consider the effect of the workers' risk aversion. Since an increasing strictly concave transformation of the workers' utility function would decrease $U'(m)$ relative to $U(m)-U(y)$, the effect of the workers' risk aversion on e_U is the opposite of that of the income-equivalent wage rate for unemployed workers, and would affect e_U negatively and therefore the critical value of e_L positively. The logic is that the more risk averse the workers are, the more concerned they will be about the risk of not being able to find employment at the minimum wage rate and the more reluctant they will be to bear the negative employment consequences of a higher minimum wage rate. Consequently, the more risk averse the workers are, the larger must e_L be for workers to be made better off by an increase in the minimum wage rate. Hence, the critical value of e_L increases with the workers' risk aversion. In particular, if the income-equivalent wage rate is zero, then $e_L^* > -1$ and risk-averse workers may not benefit from an increase in the minimum wage rate even if the labor demand is inelastic. In order to benefit, the labor demand must be sufficiently inelastic that $e_L > e_L^*$.

4 The Optimal Minimum Wage Rate

If there exists an optimal minimum wage rate for which $L(m) \in (0, N)$, then it satisfies $e_L + e_U = 0$. Equivalently, the elasticity of labor demand at the optimal minimum wage rate satisfies $e_L = e_L^*$. However, an optimal minimum wage rate may not exist, and if it does exist, it may not be unique.

To illustrate these different possibilities, assume that the income-equivalent wage rate is zero and the workers' utility function exhibits constant relative risk aversion less than unity, i.e., that $U(m) = m^{1-S}$, where $S < 1$ is the relative risk aversion. It follows then that $e_U = 1 - S$, and hence that $e_L^* = S - 1$. If it is assumed that the labor demand is given by $L(m) = \alpha - m$, where $\alpha \in (0, (2 - S)N)$, then $e_L = -m/(\alpha - m)$. In this case, a unique optimal minimum wage rate exists and is determined by $-m/(\alpha - m) = S - 1 \Rightarrow m = \alpha(1 - S)/(2 - S)$.

If, instead, it is assumed that the labor demand is given by $L(m) = m^{-\beta}$, where $\beta > 0$, then $e_L = -\beta$. Accordingly, if $\beta > 1 - S$, then workers would always benefit from increases in the minimum wage rate. No optimal minimum wage rate therefore exists. On the other hand, if $\beta < 1 - S$, then workers would always be hurt by increases in the minimum wage rate, so that there is no optimal minimum wage rate above the competitive wage rate (which is $N^{-1/\beta}$, since $L(N^{-1/\beta}) = N$). In fact, the workers would favor abolishing the minimum wage rate altogether and being paid only the competitive wage rate. Lastly, if $\beta = 1 - S$, then workers would not be affected by increases in the minimum wage rate and any minimum wage rate that is at least equal to the competitive wage rate would be optimal.

5 Conclusion

Contrary to widespread belief, this paper has shown that low-pay workers might not generally prefer that the minimum wage rate be increased to a level where the labor demand is unitary elastic. Rather, there exists a critical value of the elasticity of labor demand so that increases in the minimum wage rate make low-pay workers better off for higher elasticities, but worse off for lower elasticities. The paper demonstrates that this critical value decreases with the workers' income-equivalent wage rate and increases with their risk aversion. It is also shown that there may not exist an optimal minimum wage rate, and that if an optimal minimum wage rate exists, it may not be unique.

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