DISCUSSION PAPER SERIES

IZA DP No. 2194

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July 2006

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
of Labor

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Discussion Paper No. 2194
July 2006

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IZA Discussion Paper No. 2194
July 2006

## ABSTRACT

# A Search Model of Unemployment and Inflation*

In this paper, I introduce money in the standard labor-matching model (Mortensen and Pissarides 1999, Pissarides 2000). A double coincidence problem makes Fiat Money necessary as a medium of exchange. In the long-run, a rise in the rate of money growth leads to higher inflation and higher unemployment, so the long-run Phillips curve is not vertical. The optimal monetary growth rate decreases with the workers' bargaining power, the level of unemployment benefits and the payroll tax rate.

JEL Classification: E24, E52, J64

Keywords: inflation, unemployment, search-matching, Friedman rule

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* This research project has started while Guillaume Rocheteau was visiting ERMES. I have very much benefited from discussions with him. I also wish to thank for their remarks Gabriele Cardullo, Mathias Hugerbühler, Ricardo Lagos, Sebastien Lotz, Randall Wright and participants at seminars in Paris 1 ERMES-Paris 2 and T2M 2005. The ideas expressed and the remaining errors are of my own responsibility.

# I Introduction

While it is recognized that inflation has distortionnary effects, how these distortions influence the labor market remains an open question. In particular, the persistence of unemployment at huge levels in some countries raises the issue of how monetary policy should be conducted to reduce unemployment, if it can. A first step is to address whether higher inflation has a long-run effect on unemployment: is the long-run Phillips curve vertical, as advocated by Friedman (1968), or do we have good reasons to believe that higher inflation influences the level of structural unemployment, and if the answer is positive, in which direction?

To investigate this issue, I extend a discrete-time version of the labor matching model of Mortensen and Pissarides (1999), and Pissarides (2000) (Henceforth MP). Jobs are created by the matching of unemployed workers and vacancies. This process is time-consuming and represented by a well-behaved matching function. Firms open vacancies until a free-entry condition is met. Workers and firms Nash bargain over wages. The departure from the MP setting is the introduction of frictions in the product market that makes fiat money necessary as a medium of exchange. For this purpose, I assume that the economy is composed of distinct goods, produced by distinct agents on separate islands. These goods are non storable, non transportable and are not consumed by their producers. These additional assumptions generate a double coincidence problem that gives money an essential role to play.

In this setting, a higher inflation rate induces a higher depreciation of money holdings trough an *inflation tax* mechanism. When inflation increases, a given amount of income at a given period enables to consume a lower amount of goods in the following period. Thus, the returns of economic activities are reduced, while search cost are not affected. Firms therefore post fewer vacancies and unemployment is eventually larger at the steady state. Hence, the long-run Phillips curve is upwards slopping in the inflation-unemployment space.

I then characterize what is the optimal monetary policy. In particular, I investigate whether or not the so-called *Friedman rule* (according to which prices deflate at a rate that makes the real return of money equal to the discount rate) is optimal. A departure from the Friedman rule is optimal if and only if employment is inefficiently high at the Friedman rule. It happens when workers' bargaining power is low compared to the Hosios (1990) condition, or when the labor market policies are too employment-friendly. In this retrospect, the optimal monetary policy is an explicit function of the labor market institutions and policies. The more employment-friendly are labor market policies, the higher the optimal inflation rate.

There are both empirical and theoretical investigations of the long-run effects of inflation on unemployment in the literature. A first empirical literature regresses the unemployment rate on various macroeconomic and institutional variables using country-panel datasets (e.g. Blanchard Wolfers 2000, Nickell et alii 2005). One typical result is that a higher real interest rate increases the unemployment rate (see also Pissarides and Valenti 2005). However, how monetary policy influences real interest rate remains unclear, so these studies are not very conclusive about the slope of long-run Phillips curve and the long-run effects of monetary policy. A second empirical literature uses VAR methods and focus on the following simultaneity problem: a positive long-run correlation between unemployment and inflation can also be explained by policymakers' desire to reduce unemployment in the short-run at the expense of larger inflation. This raises an identification issue. One popular strategy identifies structural innovations in monetary policies by assuming a vertical long-run Phillips curve. This assumption is by definition inappropriate to test the verticality of the long-run Phillips curve. King and Watson (1994, 1997) investigate the likelihood of long-run Phillips curve under alternative short-run and long-run identifying restrictions. While this literature (see Bullard 1999 for a survey) is informative, results remain contingent to the underlying identifying restrictions.

Theoretical approaches differ on how money is introduced and how the labor market works. Pissarides (1990, pp. 31-40) introduces a "dynamic IS-LM" structure *à la* Tobin (1965) in his MP model. At the steady-state, a rise in the monetary growth rate decreases the real interest rate and increases the inflation rate and the nominal interest. The former effect speeds up job creation, thereby decreasing the equilibrium unemployment rate. His consumption and money demand functions are exogenous reduced forms and lack micro-foundations. Cooley and Hansen (1989) and Cooley and Quadrini (1999 and 2004) introduce money through an explicit cash-in-advance assumption. In Cooley and Hansen (1999) labor supply is reduced when inflation is increased through a consumption-leisure substitution mechanism. Conversely, the labor market in Cooley Quadrini (1999, 2004) follows the MP setting. Cooley and Quadrini add a second production factor, namely intermediate input. They introduce a Cash-in-Advance constraint that applies to the purchase of this intermediate input only. A higher inflation rate induces firms to decrease their use of intermediate goods, which in turn decreases labor productivity, thereby increasing unemployment. Hence, the key mechanism is a labor / intermediate good trade off.

In my model, labor is the sole production factor, and the key mechanism is a consumption / search trade off. I further introduce unemployment insurance and labor payroll tax. This second departure enables me to consider how the optimal monetary policy should adjust to labor market policies, and how far the inflation tax is similar to a tax on labor. Finally, instead of assuming a cash-in-advance constraint, I define an environment that makes money essential for trades in the product market. In this retrospect, I follow the requirement of the monetary-search literature (see. Kiyotaki Wright 1993, Shi 1997, Lagos and Wright 2005 and Rocheteau and Wright 2005...) to build models where the frictions that justify the use of fiat money as a medium of exchange are made explicit. In this retrospect, my model is a proposal to bridge the gap between the monetary-search and the MP labor-search literatures.

The paper is organized as follows. The environment is described in the next section, while economic behaviors are solved in section III. The equilibrium is resolved in Section IV and optimal policies are described in Section V. The last section concludes.

## II The Economy

The economy is made of $n \geq 3$ symmetric "islands" indexed by $j \in \{1, ..., n\}$ [1]. An island is characterized by a specific consumption good that requires specific skills to be produced. In each island, there is a mass of type $j$ entrepreneurs (henceforth type $j$ firms) and a mass of $1/n$ type $j$ workers. These workers are either employed or unemployed. Type $j$ employed workers can only produce type $j$ good. Type $j$ firms can only hire type $j$ workers and be located in the $j^{\text{th}}$ island. There is also a government that gives unemployment benefits to unemployed workers and a lump-sum transfer to employed and unemployed workers. The government raises revenues from tax on labor and from money creation.

Time is discrete and indexed by $t \in \mathbb{N}$. The discount rate is $r > 0$. Each period (day) is divided into a labor market sub-period (the morning) and a product market sub-period (the afternoon). Matching process, wage bargaining and production take place in the morning, as in the MP setting. Trade in the product market, consumption and monetary transfers occur in the afternoon under perfect walrasian competition. The timing is displayed in Figure 1.

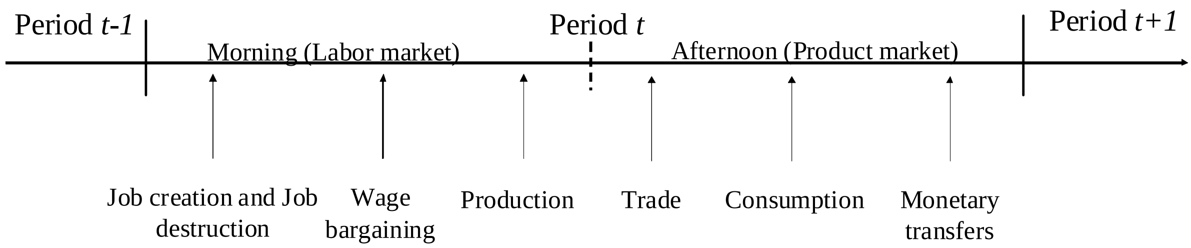


Figure 1: Timing of events

[1] I adopt the convention that $n + 1 = 1$. Put differently, I impose index $j$ to be defined modulo $n$.

I describe now the specific assumptions I impose to give money an essential role to play. Type $j$ individuals (workers and firms) do not want to consume the good they produce (type $j$) but instead desire to consume goods of type $j+1$. Hence, trade across islands is necessary. Consumption goods cannot be transported across islands. Therefore, barter is not feasible. For trade to occur, I assume the existence of a perfectly storable good. This good is divisible and intrinsically useless. I call it (fiat) money. Under the assumed perfect competition in product markets, trade is anonymous. Therefore, money is the only available medium of exchange (see Kocherlakota 1998 or the introductory survey by Rupert et alii 2000). I now detail what happens at each step of each subperiod.

## II.1 Labor market sub-period

Following Figure 1, the labor market subperiod is divided in three consecutive steps: job creation and destruction, wage bargaining, and production.

### II.1.1 Job creation and job destruction

Jobs are exogenously destroyed according to probability $s \in (0,1)$, so $s$ is also the fraction of preexisting jobs that are dissolved. Job creation is the outcome of a time-consuming matching process. Following MP, this process is represented by a matching function. Let ${}_ju_{t-1}$ and ${}_jv_{t-1}$ be respectively the number of unemployment workers and vacancies in island $j$ at the end of the period $t-1$ (therefore, at the very beginning of period $t$). The matching function $\mathcal{M}\left({}_ju_{t-1};{}_j v_{t-1}\right)$ gives the number of newly created jobs in island $j$. The total mass of employed and unemployed type $j$ workers is $1/n$, so employment in island $j$ is $1/n - {}_ju_t$, and the mass of job destroyed is $s\left(1/n - {}_ju_t\right)$. Therefore, unemployment in island $j$ evolves according to:

$$ {}_ju_t = s\left(\frac{1}{n} - {}_ju_{t-1}\right) - \mathcal{M}\left({}_ju_{t-1};{}_j v_{t-1}\right) \tag{1} $$

The matching function $\mathcal{M}(u;v)$ is identical across islands and time periods. Following the literature surveyed in Petrongolo and Pissarides (2001), I assume that the matching function exhibits constant returns to scale, is continuously differentiable, strictly increasing and strictly concave in both arguments. Unemployment and vacancies are necessary for job creation so:

$$ \mathcal{M}(0,v) = \mathcal{M}(u,0) = 0 $$

Finally, in the current discrete-time setting, the number of newly jobs is lower than the mass of vacancies and of unemployment. Therefore:

$$ \mathcal{M}(u,v) < \min(u,v) $$

Let ${}_j\theta_{t-1} = {}_jv_{t-1}/{}_ju_{t-1}$ be the tightness of the $j^{\text{th}}$ labor market at the end of period $t-1$. The *job-filling* probability for a vacancy to match with an unemployed worker is a function of tightness only: $q({}_j\theta_t) = \mathcal{M}({}_ju_{t-1}, {}_jv_{t-1})/{}_jv_{t-1} = \mathcal{M}(1/{}_j\theta_{t-1}, 1)$. Symmetrically, the *job finding* probability of an unemployed worker is a function of tightness: $\mathcal{M}({}_ju_{t-1}, {}_jv_{t-1})/{}_ju_{t-1} = \mathcal{M}(1, {}_j\theta_{t-1}) = {}_j\theta_{t-1}q({}_j\theta_{t-1})$. From the assumptions above, one has for any $\theta \in (0, +\infty)$ :

$$q(\theta) \in (0,1) \qquad \theta q(\theta) \in (0,1) \qquad q'(\theta) < 0 \qquad (\theta q(\theta))' > 0 \tag{2}$$
$$\lim_{\theta \longmapsto 0} q(\theta) = q^{\max} \in (0,1] \qquad \lim_{\theta \longmapsto +\infty} q(\theta) = 0$$

Finally, I denote $\eta(.)$ the elasticity of the job filling probability in absolute terms. Therefore, $\eta(\theta) \in (0,1)$ and

$$\eta(\theta) = -\frac{\theta \cdot q'(\theta)}{q(\theta)} = \frac{\mathcal{M}'_u(1,\theta)}{\theta q(\theta)} \qquad 1 - \eta(\theta) = \frac{\mathcal{M}'_v(1,\theta)}{q(\theta)} \tag{3}$$

### II.1.2 Wage bargaining

At each period, the worker and the firm Nash bargain over the nominal wage. This wage is negotiated in the morning but will only be paid in the afternoon once the production will be sold. The assumption that workers are paid after the production is consistent with reality where salaries for a given month are paid at the end of the month.

The negotiated wage may *a priori* depend on the firm and on the worker's money holdings. To rule out such possibility, I assume that an individual's level of money holding is private information and cannot be credibly communicated. Hence, the wage is island-specific and not match-specific. I denote ${}_jW_t$ the wage in island $j$. In the absence of money illusion, only the real wage matters. I denote ${}_jw_t = {}_jW_t/{}_{j+1}p_t$ the real wage on the $j^{\text{th}}$ island where the deflator is the price ${}_{j+1}p_t$ of the relevant consumption good. Since bargaining occurs at the plant level, the firm and the worker take the macroeconomic environment as given, and in particular the payroll tax rate $\tau$ and the lump-sum transfer (denoted $T_t$ in real terms) that is given to employed and unemployed workers.

### II.1.3 Production

Once an agreement is reached, production takes place. Each filled job produces $y > 0$ units of goods.

## II.2 The product market

Following Figure 1, the product market subperiod is divided in three consecutive steps: trade, consumption, and monetary transfers.

### II.2.1 Trade

There is a walrasian auctioneer in each island that sells the production of local firms. Type $j$ workers and type $j$ entrepreneurs move to island $j+1$. They choose how to split the money they hold at the beginning of the current period $m_t$ between consumption $c_t$ of good $j+1$ and money *hoarding* $\widehat{m}_t$. The $j+1^{\text{th}}$ auctioneer sets the price ${}_{j+1}p_t$ to clear the product market on the $j+1^{\text{th}}$ island. Since employment in island $j+1$ is $(1/n - {}_{j+1}u_t)$ and each filled job produces $y$ units of good at each period, the product market-clearing condition on island $j+1$ writes:

$$ {}_{j+1}p_t \cdot y \left( \frac{1}{n} - {}_{j+1}u_t \right) = {}_jM_t - {}_j\widehat{M_t} \tag{4} $$

where ${}_jM_t$ is the total amount of money hold by type $j$ individuals at the beginning of period $t$ and ${}_j\widehat{M_t}$ is the total amount of money hoarded by the same individuals at the end of the trade subperiod before receiving monetary transfers.

### II.2.2 Consumption

Once individuals have bought the amount of good they desire, they consume. A type $j$ worker who consumes $c$ units of type $j+1$ good enjoys utility $c$. Furthermore, firms decide at this point of time how many vacancies $v_t$ to open. Opening a vacancy implies a disutility cost[2] $\gamma > 0$. A type $j$ entrepreneur who consumes $c$ units of type $j+1$ good and open $v$ vacancies enjoys utility $c - \gamma \cdot v$.

### II.2.3 Monetary transfers

Once type $j$ individuals have consumed type $j+1$ goods in island $j+1$, they get back to the $j^{\text{th}}$ island. The $j^{\text{th}}$ auctioneer then gives to firms their money receipts from sales. Firms then pay their employees. Employed workers in turn pay their tax to the government. The government creates or destroys money, so the aggregate money stock becomes $M_{t+1}$ instead of $M_t$. Money creation (destruction) generates income (expenditures) for the government in terms of seigniorage. With these revenues, the government pays transfers to every workers and unemployment benefits to unemployed workers. Let $T_t$ and $z$ be respectively the real values of unconditional transfers and unemployment benefits.

Since islands are symmetric and of equal size, I henceforth drop index $j$. Let $u_t$ be the total mass of unemployed workers, the government's budget constraint writes:

$$ (1 - u_t) \cdot \tau \cdot W_t + M_{t+1} - M_t = p_t \left( T_t + u_t \cdot z \right) \tag{5} $$

[2] In the neighborhood of the symmetric steady state, it is equivalent to express vacancy cost in terms of disutility or in terms of goods to buy.

I choose an inflation pegging specification of the monetary policy. Instead of defining the monetary policy through the inflation rate, it is equivalent and more convenient (as it will shortly appear) to characterize it through:

$$i = (1 + r) \frac{p_{t+1}}{p_t} - 1 \tag{6}$$

To interpret $i$, imagine there were a perfect bound market in this economy, the equilibrium real interest rate would then be equal to the discount rate $r$ and the corresponding nominal interest rate would be $i$. Because preferences over consumption are linear, agents would be able to substitute current for future consumption perfectly, and any other level of the interest rate would not clear the bound market. In this retrospect, Equation (6) plays the role of a Fisher equation with a constant real interest rate.

The specific unemployment benefits $z$, the payroll tax rate $\tau$ and $i$ (equivalently the inflation rate) are the exogenous policy parameters. The money stock $M_t$ and the lump sum $T_t$ are endogenous variables that clears the budget constraint (5) and reach the target (6).

# III Economic Behaviors

In this section, I derive workers' and firms' behavior. Finally, I derive the wage setting equation.

## III.1 Workers

Let $V_t^{\mathrm{e}}(m_t)$ and $V_t^{\mathrm{u}}(m_t)$ be respectively the lifetime expected utility of an employed and of an unemployed worker. These values are functions of money holdings $m_t$ at the beginning of period $t$. Workers decide how to split their money holdings $m_t$ between consumption $c_t$ and money hoarding $\widehat{m}_t$, such that:

$$p_t \cdot c_t + \widehat{m}_t = m_t \qquad \Leftrightarrow \qquad c_t = \frac{m_t - \widehat{m}_t}{p_t} \tag{7}$$

Then, they receive some monetary transfers that come in addition to their money hoarding. Each employed workers receive wage $W_t$, transfers $p_t \cdot T_t$ and pay taxes $\tau \cdot W_t$. Therefore, their future money holdings are $m_{t+1} = \widehat{m}_t + (1 - \tau) W_t + p_t \cdot T_t$. Symmetrically, unemployed workers receive unemployment benefits and transfers, which amount to $p_t (z + T_t)$ units of money. Therefore, they start the next period with $m_{t+1} = \widehat{m}_t + p_t (z + T_t)$. Finally, employed (unemployed) workers loose their (find a) job according to probability $s$ ($\theta q(\theta)$). Using (7) to express consumption $c$ as a function of money holdings $m$ and money hoarding $\widehat{m}$, value functions therefore solve the following Bellman equations[3] for

[3] Index $t$ only states that the time-varying macroeconomic environment belongs to the list of state variables. Apart from this dependence, value functions are time-invariant.

any $m_t \in \mathbb{R}^+$:

$$\begin{array}{rcl} V_t^{\mathrm{e}}(m_t) & = & \max\limits_{\widehat{m}_t \geq 0} \dfrac{\frac{m_t - \widehat{m}_t}{p} + (1-s) V_{t+1}^{\mathrm{e}}(m_{t+1}) + s \cdot V_{t+1}^{\mathrm{u}}(m_{t+1})}{1+r} \\ \text{where} & : & m_{t+1} = \widehat{m}_t + (1-\tau) W_t + p_t \cdot T_t \end{array} \tag{8}$$

and

$$\begin{array}{rcl} V_t^{\mathrm{u}}(m_t) & = & \max\limits_{\widehat{m}_t \geq 0} \dfrac{\frac{m_t - \widehat{m}_t}{p} + (1-\theta_t q(\theta_t)) V_{t+1}^{\mathrm{u}}(m_{t+1}) + \theta_t q(\theta_t) \cdot V_{t+1}^{\mathrm{e}}(m_{t+1})}{1+r} \\ \text{where} & : & m_{t+1} = \widehat{m}_t + p_t (z + T_t) \end{array} \tag{9}$$

Applying the envelope theorem to (8) and (9), one gets for any $m_t \geq 0$ :

$$\frac{\partial V_t^{\mathrm{e}}(m_t)}{\partial m_t} = \frac{\partial V_t^{\mathrm{u}}(m_t)}{\partial m_t} = \frac{1}{1+r} \cdot \frac{1}{p_t}$$

so value functions are linear in money holdings:

$$V_t^{\mathrm{e}}(m) \equiv V_t^{\mathrm{e}} + \frac{1}{1+r} \cdot \frac{m_t}{p_t} \qquad V_t^{\mathrm{u}}(m) \equiv V_t^{\mathrm{u}} + \frac{1}{1+r} \cdot \frac{m_t}{p_t} \tag{10}$$

where $V^{\mathrm{j}} = V^{\mathrm{j}}(0)$ for j=e,u. The first-order conditions of (8) and (9) with respect to money hoarding $\widehat{m}$ are:

$$0 \geq -\frac{1}{p_t} + \frac{1}{(1+r) p_{t+1}} = -\frac{1}{p_t} \cdot \frac{i}{1+i} \qquad \text{with} = \text{ if } \widehat{m} > 0 \tag{11}$$

The last equality is derived from (6). Whenever $i > 0$, which I henceforth assume, programs (8) and (9) admits a corner solution[4] with $\widehat{m} = 0$.

The interpretation for this result is the following. Consider a decrease of $p_{t+1}(1+r)$ units of money hoarding $\widehat{m}$ and a corresponding increase in current consumption $c$. The former induces a decrease of $1+r$ units of consumption good for the following period. The corresponding discounted utility loss is unitary. Moreover, current consumption increases by $\frac{p_{t+1}}{p_t}(1+r) = 1+i$ units. Therefore, $i$ measures the net opportunity cost of carrying a unit of money across time. I henceforth refer to $i$ as the *cost of money holdings*.

At the so-called *Friedman rule*, prices evolve at a rate given by $p_{t+1}/p_t = 1/(1+r) < 1$, which corresponds to a negative inflation (deflation) rate. Under such a rule, money holdings have no cost ($i = 0$), and consumers are indifferent between consumption and money hoarding. If the inflation rate is higher than the *Friedman rule*, one gets $p_{t+1}/p_t > 1/(1+r)$ and therefore $i > 0$. In such case, individual wish to substitute current for

[4]Programs (8) and (9) should also include a non-negativity constraint on consumption $c$. Given (7), this equation writes $\widehat{m} \leq m$. I have solved these programs assuming that the non-negativity constraint on consumption is slack. Since the solution is $0 = \widehat{m} < m$, one has $c > 0$, so one can omit constraint $c \geq 0$ in the reasoning.

future consumption. So, workers minimize their money holdings, and $\widehat{m}_t = 0$. With this behavior in mind, and using the linearity of the value functions, we get from (8) and (9):

$$(1+r)\, V_t^{\mathrm{e}} = \frac{w_t (1-\tau) + T_t}{1+i} + (1-s)\, V_{t+1}^{\mathrm{e}} + s \cdot V_{t+1}^{\mathrm{u}} \tag{12}$$

$$(1+r)\, V_t^{\mathrm{u}} = \frac{z + T_t}{1+i} + (1-\theta_t q(\theta_t))\, V_{t+1}^{\mathrm{u}} + \theta_t q(\theta_t) \cdot V_{t+1}^{\mathrm{e}} \tag{13}$$

These equations in the present discrete-time setting correspond to usual asset equations for employed and unemployed workers in the continuous-time version of MP (see. e.g. equations 1.38 and 1.37 in Pissarides 2000) except for the presence of the *inflation tax* $1/(1+i)$ factor[5]. Receiving 1 additional unit of money at the end of the afternoon does not permit to consume $1/p_t$ additional unit of goods at the current period, but only $1/p_{t+1}$ units of goods next period. Because of discounting, the latter is valued $1+i$ times less than the former.

## III.2 Firms

As workers, entrepreneurs face the consumption/money hoarding trade off. At the end of the afternoon, an entrepreneur who has $\ell_t$ employed workers receives $p_t \cdot y_t \cdot \ell_t$ units of money that corresponds to her sales and pays $W_t = p_t \cdot w_t$ units of money to each of her $\ell_t$ employees. Hence, her value function depends on her money holding $m_t$ and on her number $\ell_t$ of employees. At each period a fraction $s$ of these jobs are dissolved. Each additional vacancy increases future employment by one unit with probability $q(\theta)$ but induces a disutility cost $\gamma$. Assuming that $v_t$ and $\ell_t$ are "large" enough, flows of newly created jobs and of destroyed jobs are deterministic[6] and respectively equal to $q(\theta) \cdot v_t$ and $s \cdot \ell_t$. Therefore, future employment is a deterministic variable and the firm's value function solves:

$$V_t^{\mathrm{f}}(m_t, \ell_t) = \max_{\widehat{m}_t \geq 0, v_t \geq 0} \frac{\frac{m_t - \widehat{m}_t}{p_t} - \gamma \cdot v_t + V_{t+1}^{\mathrm{f}}(m_{t+1}, \ell_{t+1})}{1+r} \tag{14}$$

$$\text{where} : m_{t+1} = \widehat{m}_t + p_t \cdot \ell_t (y_t - w_t)$$

$$\ell_{t+1} = (1-s)\, \ell_t + q(\theta_t) \cdot v_t$$

As for workers' programs (8) and (9), the envelope condition over money holdings

$$\frac{\partial V_t^{\mathrm{f}}}{\partial m_t} = \frac{1}{p_t} \cdot \frac{1}{1+r}$$

[5] To see this, rewrite (12) as

$$r \cdot V_t^{\mathrm{e}} = \frac{w_t (1-\tau) + T_t^0}{1+i} + s\,(V_{t+1}^{\mathrm{u}} - V_{t+1}^{\mathrm{e}}) + (V_{t+1}^{\mathrm{e}} - V_t^{\mathrm{e}})$$

The correspondence directly follows the approximation $(V_{t+1}^{\mathrm{e}} - V_t^{\mathrm{e}}) \simeq \dot{V}_t^{\mathrm{e}}$

[6] Therefore, the present model is an extension of what Pissarides (2000) call the "large firms" setting. In the case where either $\ell_t$ or $v_t$ is too low to apply the law of large number, the program includes uncertainty for a risk-neutral agent. Hence, results are unchanged.

implies that value function is linear in $m_t$. The first-order condition on money hoarding is again given by (11) and implies $\widehat{m}_t = 0$, whenever $i > 0$. The envelope condition over $\ell_t$ shows that the marginal value of filled job $\partial V_t^{\mathrm{f}} / \partial \ell_t$ is independent of money holdings $m_t$ and of employment $\ell_t$. Let then $J_t = \partial V_t^{\mathrm{f}} / \partial L_t$ be this marginal value. The envelope condition over $L_t$ gives:

$$(1+r) J_t = \frac{p_t (y - w_t)}{p_{t+1} (1+r)} + (1-s) J_{t+1} = \frac{y - w_t}{1+i} + (1-s) J_{t+1} \tag{15}$$

As for workers, equation (15) is a usual asset equation for a filled job augmented by the *inflation tax* factor $1/(1+i)$. The firm's value function is given by:

$$V_t^{\mathrm{f}} (m, \ell) = \ell \cdot J_t + \frac{1}{1+r} \cdot \frac{m}{p_t} \tag{16}$$

Finally, the first-order condition over vacancies $v$ expresses that firms open vacancies if and only if:

$$\gamma = q(\theta_t) \cdot J_{t+1} \tag{17}$$

Firms open vacancies as long as the expected gain of recruiting a worker is higher than the disutility of an additional vacancy $\gamma$. The former equals the value of filled job next period $J_{t+1}$ times the job filling probability $q(\theta_t)$ that a current vacancy finds an unemployed worker to hire at the beginning of the next period. For a given current number of unemployed workers $u_t$, the current mass of vacancies $v_t$ adjusts so that the current tightness in the labor market $\theta_t = v_t / u_t$ satisfies this free-entry condition. If there were too many (few) vacancies $v_t$, tightness $\theta_t$ would be too high (low), the *job-filling* probability $q(\theta_t)$ would be too low (high), which would induce firms to close (to open new) vacancies instantaneously. Tightness would therefore instantaneously decreases (increases) until (17) is satisfied.

## III.3 Wage Bargaining

Each worker Nash bargain with its employer over the current wage, taking as given the macroeconomic environment. For the worker (the firm), a successful negotiation generates a surplus equal to $V_t^{\mathrm{e}} - V_t^{\mathrm{u}}$ $(J_t)$. Notice from (10) and (16) that these surplus are independent of money holdings $m$. Let $\beta \in (0,1)$ denote the workers' bargaining power. The negotiated wage solves the following generalized Nash product[7]:

$$\max_{w_t} \quad \beta \log (V_t^{\mathrm{e}} - V_t^{\mathrm{u}}) + (1-\beta) \log J_t \tag{18}$$

taking $V_t^{\mathrm{u}}$ as given. Using (12) and (15), the first-order condition gives (see Appendix A):

$$\beta (1-\tau) J_t = (1-\beta) (V_t^{\mathrm{e}} - V_t^{\mathrm{u}}) \tag{19}$$

[7] Since each individual negotiation does not influence price on the product market, it is equivalent to bargain over nominal wage $W_t$ or over real wage $w_t = W_t / p_t$

In the absence of labor taxation (i.e. if $\tau = 0$), this condition stipulates that the worker (the firm) extracts a fraction $\beta$ $(1-\beta)$ of the total surplus generated by a match $V_t^{\mathrm{e}} - V_t^{\mathrm{u}} + J_t$. When the payroll tax rate is positive $\tau > 0$, a unit increase in the negotiated wage only yields a rise of $1-\tau$ units of wage for the worker, while the cost for the firm remains unitary. Because of this wedge, workers moderate their wage claims and therefore extract a lower share of the total surplus. This effect is very usual in wage bargaining models (see. e.g. Lockwood and Manning 1993, Pissarides 2000, or Cahuc and Zylberberg 2004). Conversely, the inflation tax does not affect the sharing rule. This is because the firm's and the workers' income are identically affected by the inflation tax. From the sharing rule (19), one can derive the wage equations from (12), (13), (15) and (17)(see again Appendix A):

$$w_t = \beta\left(y + (1+i)\,\gamma \cdot \theta_t\right) + (1-\beta)\,\frac{z}{1-\tau} \tag{20}$$

Wage positively depends on the utility firms derived from production $y/\left(1+i\right)$, on the utility unemployed workers derived from being unemployed $z/\left(1+i\right)$, and on the capital gain an unemployed worker expects from finding a job $\theta_t q\left(\theta_t\right)\left(V_{t+1}^{\mathrm{e}} - V_{t+1}^{\mathrm{u}}\right)$ (which is equal to $\gamma\left(\beta/\left(1-\beta\right)\right)\theta_t$, given 17) and (19). The two former terms are obtained in forms of additional money at the end of the day. They are therefore reduced when inflation increases. Conversely, the latter term is proportional to the cost of posting a vacancy $\gamma$ which is not expressed in terms of money and remains therefore unaffected by inflation. These are the reasons why a rise in inflation (thereby in the cost of money holdings $i$) decreases the utility obtained from wage payment $w_t/\left(1+i\right)$, but increases $w_t$. Finally, as usual in MP, the negotiated wage is an increasing function of productivity $y$, bargaining power $\beta$, payroll tax rate $\tau$, unemployment benefit $z$, vacancy cost $\gamma$ and tightness in the labor market $\theta_t$.

# IV Equilibrium

In this section, I characterize the equilibrium. The exogenous variables are the policy parameters, namely the marginal tax rate $\tau$, the (specific) unemployment benefits $z$ and the cost of money holding $i$. All remaining variables are endogenous. Moreover, the unemployment rate $u_t$ and the money supply $M_t$ are the only predetermined variables.

**Definition 1** *Given policy parameters $i$, $z$, $\tau$, and initial values of unemployment $u_0$ and of money supply $M_0$, an equilibrium is a sequence $\{J_t, V_t^e, V_t^u, \theta_t, w_t, u_t, M_t, p_t, T_t\}_{t\in\mathbb{N}}$ that satisfies:*

i*) The asset equations (12), (13), (15),*

*ii) The free-entry condition (17)*

iii*) The wage bargaining equation (20)*

iv*) The motion of unemployment (1)*

v*) The product market clearing conditions (4) together with the result that for any individual* $\widehat{m} = 0$.

vi*) The government's budget constraint (5).*

*vii) The unemployed workers receives a non-negative transfer* $z + T_t \geq 0$.

I first rewrite the Bellman equation for the value of marginal job (15) in terms of tightness $\theta_t$ thanks to the free-entry condition (17). Using (20) to eliminate the wage $w_t$ gives:

$$(1+r)\frac{\gamma}{q(\theta_{t-1})} = \frac{1-\beta}{1+i}\cdot\left(y - \frac{z}{1-\tau}\right) - \beta\cdot\gamma\cdot\theta_t + (1-s)\frac{\gamma}{q(\theta_t)} \tag{21}$$

As discussed in Blanchard and Fisher (1989, chapter 5), this kind of non-linear and forward-looking difference equation can lead to complex dynamics, including sunspots, bursting bubbles and cycles. Since the focus of this paper is on the long-run effect of monetary policy, I only consider *stationary* equilibria where tightness is constant over time ($\forall t,\ \theta_t = \overline{\theta}$). A stationary equilibrium value $\overline{\theta}$ solves:

$$\mathcal{F}\left(\overline{\theta}, \beta\right) = \frac{1}{1+i}\cdot\left(y - \frac{z}{1-\tau}\right) \qquad \text{where} \qquad \mathcal{F}(\theta, \beta) \equiv \left(\frac{r+s}{q(\theta)} + \beta\cdot\theta\right)\frac{\gamma}{1-\beta} \tag{22}$$

From (2), function $\mathcal{F}(., \beta)$ increases in $\theta$ from $(r+s)\gamma/\left(q^{\max}(1-\beta)\right)$ to $+\infty$. Therefore, if a stationary equilibrium tightness exists, it is unique. Moreover, a stationary tightness $\overline{\theta}$ exists only if:

$$\frac{1}{1+i}\cdot\left(y - \frac{z}{1-\tau}\right) > \frac{r+s}{1-\beta}\cdot\frac{\gamma}{q^{\max}} \tag{23}$$

A job should generate a joint surplus that is large enough at each period for firms to post vacancies. This condition is more likely to be satisfied if the productivity $y$ is sufficiently high and if the marginal taxation on labor $\tau$, the specific unemployment benefits $z$ or the cost of money holdings $i$ are sufficiently low. When condition (23) is not satisfied, creating a job is too costly and $\overline{\theta} = 0$. Otherwise, the unique equilibrium tightness $\overline{\theta}$ determines values $V^{\mathrm{e}}$, $V^{\mathrm{u}}$ $J$ and wages $w$, according to (12), (13), (15) and (20). Aggregation of (1) over islands together with $\theta q(\theta) = \mathcal{M}(u, v)/u$ gives:

$$u_t = s(1 - u_{t-1}) + \theta q(\theta) u_{t-1} \tag{24}$$

For a given initial unemployment rate $u_0$, (24) determines recursively a unique sequence of unemployment rate $\{u_t\}_{t\in\mathbb{N}}$. This sequence converges to $\overline{u}$ given by:

$$\overline{u} = \frac{s}{s + \overline{\theta} q\left(\overline{\theta}\right)} \tag{25}$$

Any policy that raises tightness $\overline{\theta}$ speeds up unemployed workers' entries into employment, thereby decreasing the steady-state unemployment rate $\overline{u}$. Since individuals have no incentive to hoard money, $\widehat{m} = 0$, aggregation of (4) across islands gives the equation of the quantity theory of Money:

$$\frac{M_t}{p_t} = (1 - u_t)\, y \tag{26}$$

Money growth is adjusted to peg an inflation rate, or equivalently given equation (6), to peg a cost of money holdings $i$. This induces the following policy rule for the money supply:

$$M_t = M_{t-1} \cdot \frac{1+i}{1+r} \cdot \frac{1-u_t}{1-u_{t-1}} \tag{27}$$

For a given initial level of Money supply $M_0$, (27) determines recursively a unique sequence of money supply $\{M_t\}_{t\in\mathbb{N}}$. In the long run, Money supply grows at the rate of inflation. Finally, at each period $t$ the unconditional transfer $T_t$ clears the government's budget constraint (5). An equilibrium exists only if at each period, the total transfers $z + T_t$ received by unemployed workers is non-negative.

From above, there exists at most a single stationary equilibrium. I can now derive the comparative statics. As in MP, the steady-state unemployment rate is a decreasing function of productivity $y$, but an increasing function of the bargaining power $\beta$, the payroll tax rate $\tau$, the unemployment benefit $z$ or the vacancy cost $\gamma$. The original property concerns the long-run effect of monetary policy.

**Proposition 1 (Long-run Phillips curve)** *Higher inflation increases unemployment in the long-run*

**Proof.** Given (6), a higher inflation rate $(p_{t+1}/p_t) - 1$ raises the cost of money holdings $i$. From (22) and $\mathcal{F}'_\theta > 0$, an increase in $i$ decreases the steady-state value of tightness $\overline{\theta}$. Finally, from (25), the steady state value of unemployment $\overline{u}$ is increased. ■

The intuition for this result is the following. The returns of a successful match is through additional money holdings. This is true both for the firm through additional sales, and for employees through wages. These additional money holdings cannot be spent instantaneously at price $p_t$, but only at price $p_{t+1}$ the following period. Given discounting, the latter is valued $1+i$ times less than the former. An increase in the inflation rate induces that monetary returns from economic activities are less valued. Conversely, the cost of posting vacancies remains unchanged. Firms thus create less vacancies, thereby reducing tightness on the labor market $\overline{\theta}$. Therefore, unemployment converge in the long-run to a higher steady-state level $\overline{u}$.

Monetary policy influences unemployment in this model through an *inflation tax* mechanism. It is therefore fruitful to compare the effects of inflation and the effects of a payroll tax $\tau$. A larger payroll tax $\tau$ reduces the total surplus, as does a larger inflation rate. This *surplus size* effect tends to reduce the value of a filled job $J$, inducing firms to post fewer vacancies. Additionally, a higher payroll tax reduces the worker's share of this surplus (see equation 19), which is not the case with the *inflation tax*. This *wage moderating* effect attenuates the reduction of tightness. In particular, if $z = 0$, the *wage moderating* effect completely offsets the *surplus size effect*, and tightness is independent of payroll tax rate[8].

In real worlds, lump-sum transfer does not exist. However, the combination of a linear payroll tax $\tau$, an unemployment benefit $z$ and a lump sum transfer $T_t$, is equivalent to a non-linear tax on labor $\mathcal{T}(w) = \tau \cdot w - T_t$ and a global unemployment benefits $z + T_t$. In this retrospect, one can interpret a positive $T_t$ as the indication that the overall tax schedule $\mathcal{T}(.)$ is progressive. To investigate the effect of a more progressive tax schedule on equilibrium, consider the following policy departure from an equilibrium with a positive $z > 0$. Consider then a change in policy such that global unemployment benefits $z + T_t$ and monetary policy $i$ are keep unchanged, specific unemployment benefits are nil $z = 0$, and the marginal tax rate $\tau$ is increased. According to (5) and (22), when $z = 0$, $\tau$ can be as high as necessary to obtained the same global unemployment benefits $z + T_t$ as before. This policy change induces a rise in equilibrium tightness (according to (22) since now $z = 0$), a decrease of unemployment in the long-run, and a rise in both $\tau$ and $T_t$. Hence, from any equilibrium, there exists a more progressive tax schedule that leave the same level of global unemployment benefits but with a lower unemployment rate in the long-run. This result is well known since Lockwood Manning (1993) (see also Pissarides 2000 or Cahuc and Zylberberg 2004). In the next section, I will also investigate how the monetary policy should respond to such policy change.

# V Social Optimum and Optimal policies

In this section, I investigate what is the optimal monetary policy $i^*$. For this purpose, I use a Utilitarian criterion. Aggregating (10) and (16) across all workers and firms, the social criterion $\Omega$ is defined as:

$$\Omega_t = (1 - u_t)\left(V_t^{\mathrm{e}} + J_t\right) + u_t \cdot V_t^{\mathrm{u}} + \frac{1}{1+r} \cdot \frac{M_t}{p_t}$$

In Appendix C is shown that:

$$(1 + r)\,\Omega_t = (1 - u_t)\,y - \gamma \cdot v_t + \Omega_{t+1} \tag{28}$$

[8]The same result occurs when the level of unemployment benefits is positive but proportional to the wage level (see. Pissarides 2000).

Let $L_t = 1 - u_t$ be the aggregate employment level whose dynamics is easily obtained from (24). The optimal allocation is therefore the solution of:

$$\Omega(L_t) = \max_{v_t} \quad \frac{L_t \cdot y - v_t \cdot \gamma + \Omega(L_{t+1})}{1+r} \qquad s.t : L_{t+1} = (1-s) L_t + \mathcal{M}(1 - L_t, v_t) \quad (29)$$

Appendix D shows that the optimal tightness $\theta^*$ solves:

$$\mathcal{F}(\theta^*, \eta(\theta^*)) = y \tag{30}$$

where function $\mathcal{F}(.,.)$ has been defined in (22). From (22), the equilibrium $\overline{\theta}$ and the optimal $\theta^*$ tightness coincide if and only if $F(\theta^*, \beta) = \left(y - \frac{z}{1-\tau}\right) / (1 + i^*)$. Given (30) this leads to:

$$i^* = \frac{1}{\mathcal{F}(\theta^*, \beta)} \left\{ \mathcal{F}(\theta^*, \eta(\theta^*)) - \mathcal{F}(\theta^*, \beta) - \frac{z}{1-\tau} \right\} \tag{31}$$

We therefore get the following proposition:

**Proposition 2 (Optimal monetary policy)** *The Friedman rule* $i = 0$ *decentralizes the optimum iff* $i^* = 0$

*If* $i^* < 0$*, the Friedman rule* $i = 0$ *is optimal but decentralizes only a second-best outcome.*

*If* $i^* > 0$*, the optimal monetary policy departs from the Friedman rule and decentralizes the social optimum.*

**Proof.** Consider the steady-state equilibrium at the Friedman Rule and let $\overline{\theta}(0)$ be the corresponding tightness. From (22), (30) and (31), $\overline{\theta}(0)$ solves:

$$\mathcal{F}\left(\overline{\theta}(0), \beta\right) = y - \frac{z}{1-\tau} = \mathcal{F}(\theta^*, \eta(\theta^*)) - \frac{z}{1-\tau} = (1 + i^*) \mathcal{F}(\theta^*, \beta)$$

Since $F'_\theta > 0$, we can distinguish three cases:

- If $i^* = 0$, one has $\overline{\theta}(0) = \theta^*$. Implementing the *Friedman rule* is then optimal since it decentralizes the optimal tightness.
- If $i^* < 0$, one has $\overline{\theta}(0) < \theta^*$, so $\overline{\theta}(0)$ is inefficiently low. $i = i^* < 0$ would be optimal but is not feasible. Therefore, only a second-best is implementable and this optimum requires the *Friedman rule* $i = 0$.
- If $i^* > 0$, $\overline{\theta}(0) > \theta^*$ and $\overline{\theta}(0)$ is inefficiently high. A marginal increase of inflation from the Friedman rule induces a decrease in tightness, which is welfare improving.

■

A departure from the Friedman rule is optimal only when equilibrium tightness at the steady state is inefficiently high. Then, a positive cost of money holdings decreases tightness. Total employment decreases, but total vacancies too. When tightness is inefficiently high, the latter reduction dominates the former, so total welfare increases.

Employment-friendly labor market environment make a departure from the Friedman rule more likely to be optimal. Three parameters matter: the workers' bargaining power, the unemployment benefits and the labor payroll tax. A rise in any of these three parameters reduces the equilibrium tightness, thereby making less desirable a departure from the Friedman rule. In the absence of taxes and transfers, a departure is optimal if and only if the bargaining power is higher than the one given by the Hosios (1990) condition. This result is in accordance with Cooley and Quadrini (2004) or Berentsen et alii (2006). The novelty of the present analysis is the role of labor market policies: positive unemployment benefits and payroll tax makes less likely the desirability of a departure from the Friedman rule. A striking result concerns how the monetary policy should respond to a more progressive tax schedule.

For this purpose I reconsider the policy change of the end of section IV. This change consists in a departure from an equilibrium with a positive $z > 0$ to an equilibrium with $z = 0$ and a higher $T_t$ and $\tau$ such that the global unemployment benefits $z + T_t$ is kept unchanged. As it has been shown then, such policy change corresponds to a rise in tax-progressivity that leads to a lower long-run unemployment. Hence, employment is more likely to be inefficiently high at the Friedman rule and a departure from the Friedman rule is therefore more likely to be optimal.

# VI Concluding remarks

In this paper, I extend the MP labor matching model by introducing frictions in the product market that makes money essential as a medium of exchange. I investigate what is the long run effect of inflation on unemployment. I find that at steady state, a higher inflation rate decreases the returns of economic activity, which makes firms more reluctant to post vacancies, thereby increasing unemployment. I then compute the optimal monetary policy. The *Friedman rule* is always optimal unless the workers' bargaining power, the unemployment benefits and the tax rate are very low or the global tax schedule is not too progressive.

The result such that a higher inflation increases unemployment in the long-run may look surprising, but is based on the property that a higher cost of money holdings is a real cost, and as such, penalizes unemployment. Hence, the key issue is how monetary policy should be conducted in the long-run to decrease the cost of money holdings. In my model, a higher growth rate of money increases inflation and therefore the cost of money holdings through a long-run adjustment. This logic follows the so-called Fisher equation according to which a unit increase in inflation should lead to a unit increase in nominal interest rate (thereby in the cost of money holdings) in the long-run. However, empirical estimations suggest that, at least in the short run, a higher growth rate of money decreases the nominal interest. Hence, the present model should be extended to introduce such short-run adjustments.

# A Wage Bargaining

From (12) and (15), maximizing the generalized Nash product (18) amounts to maximize

$$\max_{w_t} \quad \beta \log \left\{ \frac{(1-\tau)\, w_t + T_0^t}{1+i} + (1-s)\left(V_{t+1}^{\mathrm{e}} - V_{t+1}^{\mathrm{u}}\right) - (1-r)\, V_t^{\mathrm{u}} + V_{t+1}^{\mathrm{u}} \right\} + (1-\beta) \left\{ \frac{y - w_t}{1+i} + (1-s)\, J_{t+1} \right\}$$

the first order condition gives:

$$\frac{\beta\,(1-\tau)}{(1+r)\,(1+i)} \cdot \frac{1}{V_t^{\mathrm{e}} - V_t^{\mathrm{u}}} - \frac{1-\beta}{(1+r)\,(1+i)} \cdot \frac{1}{J_{t+1}} = 0$$

which gives (19). Moreover, we get:

$$\beta\,(1-\tau) \left\{ \frac{y - w_t}{1+i} + (1-s)\, J_{t+1} \right\} = (1-\beta) \left\{ \frac{(1-\tau)\, w_t + T_0^t}{1+i} + (1-s)\, V_{t+1}^{\mathrm{e}} + s \cdot V_{t+1}^{\mathrm{u}} - (1-r)\, V_t^{\mathrm{u}} \right\}$$

With (19) written for period $t+1$ and (13), this reduces to:

$$\beta\,(1-\tau)\, \frac{y - w_t}{1+i} = (1-\beta) \left\{ \frac{(1-\tau)\, w_t - z}{1+i} + \theta q\,(\theta) \left(V_{t+1}^{\mathrm{e}} - V_{t+1}^{\mathrm{u}}\right) \right\}$$

Using again (19) for period $t+1$ together with (17) gives

$$\beta\,(1-\tau)\, \frac{y - w_t}{1+i} = (1-\beta)\, \frac{(1-\tau)\, w_t - z}{1+i} + \theta\beta\,(1-\tau)\,\gamma$$

Multiplying by $(1+i)\,/\,(1-\tau)$ and rearranging terms gives (20).

# B Equilibrium Dynamics

Deriving (21) at the neighborhood of the steady state $\theta_{t-1} = \theta_t = \overline{\theta}$, one gets:

$$\frac{\partial \theta_{t-1}}{\partial \theta_t} = \frac{1}{1+r} \left\{ 1 - s - \frac{\beta}{\eta\,(\overline{\theta})}\, \overline{\theta} q\,(\overline{\theta}) \right\}$$

One has $\partial\theta_{t-1}/\partial\theta_t < 1$. Since $\theta_t$ is a forward-looking variable, its dynamics is locally determinate if and only if $\partial\theta_{t-1}/\partial\theta_t > -1$. This happens whenever

$$\beta < \eta\,(\overline{\theta})\, \frac{2 - s + r}{\overline{\theta} q\,(\overline{\theta})} \tag{32}$$

Then, the locally unique non-exploding dynamics implies that tightness instantaneously reaches its steady-state value $\theta$. However this local condition is not sufficient to eliminate cycles.

Under plausible parameters, condition (32) is satisfied. To see why, notice that under the Hosios condition $\beta = \eta\left(\overline{\theta}\right)$, one has: $\partial\theta_{t-1}/\partial\theta_t = \left(1 - s - \overline{\theta}q\left(\overline{\theta}\right)\right)/(1+r)$. In real worlds, the probability of being employed is higher for a currently employed worker than for a currently unemployed worker. So, $1 - s > \overline{\theta}q\left(\overline{\theta}\right)$, which implies $\partial\theta_{t-1}/\partial\theta_t > 0 > -1$.

# C Social criteria

From (12), (13) and (15), we get:

$$(1+r)\,\Omega_t = \frac{(1-u_t)\,y - (1-u_t)\,\tau\cdot w_t + u_t\cdot z + T_t}{1+i} + \frac{M_t}{p_t} + (1-s)\,(1-u_t)\,J_{t+1}$$
$$+\left[(1-s)\,(1-u_t) + \theta_t q\left(\theta_t\right)u_t\right]V^{\mathrm{e}}_{t+1} + \left[s\,(1-u_t) + \left(1-\theta_t q\left(\theta_t\right)\right)u_t\right]V^{\mathrm{u}}_{t+1}$$

Using (24) and (5), we get:

$$(1+r)\,\Omega_t = \frac{1}{1+i}\left\{(1-u_t)\,y + \frac{M_{t+1}-M_t}{p_t}\right\} + \frac{M_t}{p_t} + (1-u_{t+1})\,V^{\mathrm{e}}_{t+1} + u_{t+1}\cdot V^{\mathrm{u}}_{t+1} + (1-s)\,(1-u_t)\,J_{t+1}$$

Given (26)

$$(1+r)\,\Omega_t = (1-u_t)\,y + \frac{1}{1+i}\frac{M_{t+1}}{p_t} + (1-u_{t+1})\,V^{\mathrm{e}}_{t+1} + u_{t+1}\cdot V^{\mathrm{u}}_{t+1} + (1-s)\,(1-u_t)\,J_{t+1}$$

Using (17), (24) and $v_t\cdot q\left(\theta_t\right) = \theta_t q\left(\theta_t\right)\cdot u_t$,

$$(1+r)\,\Omega_t = (1-u_t)\,y - \gamma\cdot v_t + \frac{1}{1+i}\frac{M_{t+1}}{p_t} + (1-u_{t+1})\left(V^{\mathrm{e}}_{t+1} + J_{t+1}\right) + u_{t+1}\cdot V^{\mathrm{u}}_{t+1}$$

Finally, (6) induces $(1+i)\,p_t = (1+r)\,p_{t+1}$, so:

$$(1+r)\,\Omega_t = (1-u_t)\,y - \gamma\cdot v_t + \frac{1}{1+r}\frac{M_{t+1}}{p_{t+1}} + (1-u_{t+1})\left(V^{\mathrm{e}}_{t+1} + J_{t+1}\right) + u_{t+1}\cdot V^{\mathrm{u}}_{t+1}$$

which gives (28).

# D Optimal allocation

Taking (3) into account, the first-order condition of Program (29) is

$$\gamma = \Omega'\left(L_{t+1}\right)\cdot\left(1-\eta\left(\theta_t\right)\right)\cdot q\left(\theta_t\right)$$

while the envelope condition writes

$$(1+r)\,\Omega'\left(L_t\right) = y + \left(1 - s - \eta\left(\theta_t\right)\cdot\theta_t q\left(\theta_t\right)\right)\Omega'\left(L_{t+1}\right)$$

These two conditions imply:

$$(1+r)\,\frac{\gamma}{q\left(\theta_t\right)} = \left(1-\eta\left(\theta_t\right)\right)y + \left(1 - s - \eta\left(\theta_t\right)\cdot\theta_t q\left(\theta_t\right)\right)\frac{1-\eta\left(\theta_t\right)}{1-\eta\left(\theta_{t+1}\right)}\cdot\frac{\gamma}{q\left(\theta_{t+1}\right)}$$

A stationary solution to this recursive equation is implicitly defined by:

$$\left(\frac{r+s}{q\left(\theta^*\right)} + \eta\left(\theta^*\right)\cdot\theta^*\right)\gamma = \left(1-\eta\left(\theta^*\right)\right)y$$

which gives (30) directly.

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