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ABSTRACT

Employment Subsidies and Substitutable Skills: An Equilibrium Matching Approach*

The search-matching model is well suited for an equilibrium evaluation of labor market policies. When those policies are targeted on some groups, the usual juxtaposition of labor markets is however a shortcoming. There is a need for a setting where workers' productivity depends on employment levels in all markets. This paper provides such a theoretical setting. We first develop a streamlined model and then show that it can be extended to deal with interactions among various labor market and fiscal policies. Simulation results focus on the effects of employment subsidies and in-work benefits and on their interactions with the profile of unemployment benefits and with active labor market programs.

JEL Classification: E24, J3, J41, J64, J65, J68

Keywords: unemployment, search-matching equilibrium, wage bargaining, reductions of social security contributions, unemployment insurance, labor market programs

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"Countries can engineer a reduction of unemployment *without* a sacrifice of low-end pay and a rise in low-end pay *without* a sacrifice of employment" (Phelps , 2003, p. 11)

1 Introduction

To boost employment among the relatively low-paid, several countries have introduced employment subsidies, in-work benefits or cut in payroll taxes. In frictional labor markets, these fiscal instruments change the quasi-rents that accrue to employers and workers who have matched. This induces various effects on firms' and workers' decisions (vacancy creation, job-search effort, investment in training and the like). Developing a comprehensive view of these effects is essential to evaluate whether these fiscal instruments can alleviate the unemployment problem. The equilibrium matching model is admittedly a powerful setting for such an evaluation. However, it remains rather simplistic for two reasons. First, policies targeted on specific groups require a model in which interactions *between* labor markets are sufficiently rich. However, the state-of-the-art model juxtaposes the various labor markets. Second, tax reforms do not take place in isolation. They interact with other existing policies. Some papers looked at interactions with employment protection (see e.g. Mortensen and Pissarides, 2003, henceforth 'M-P'). Our paper introduces passive and active labor market policies.

To avoid a juxtaposition of labor markets while keeping the model tractable, we consider an economy with a final consumption good produced with two substitutable intermediate goods. Each of them is produced with a single input, namely labor of a given skill. The marginal product of labor is constant. An additional vacancy accessible to one type of worker eventually raises employment and the quantity of the corresponding intermediate good. This decreases its productivity in the production of the final good and raises the productivity of the other intermediate good. These changes in productivity modify the marginal value of labor and hence the quasi-rents that accrue to employers and workers in all the economy. The decision to open new vacancies and the effort to search for a job are therefore affected, too. These new interactions complement the standard matching externalities ("caused by the congestion that searching firms and workers cause for each other during trade", Pissarides, 2000, p.8).

A natural question is to what extent these extensions to the M-P model lead to different properties. To answer that question, as M-P, we introduce a tax-subsidy schedule $a + \tau \cdot w$, where w is the net wage rate and τ is a positive proportional tax rate. In the absence of untaxed income (such as home production), the marginal tax τ is absorbed entirely by workers (through the net wages). If a is negative, it can be interpreted as a lump-sum employment subsidy (the interpretation retained below), an in-work benefit or a cut in payroll taxes. The employment subsidy raises the surplus created when a worker and a vacancy have matched. Trough bargaining, the employment subsidy is therefore partly used to raise net wages and partly to raise employment. We show that the effects of these tax instruments are deeply modified in our enlarged setting. Take the case of a lump-sum employment subsidy targeted on the low-skilled workers. We show that the state-of-the-art matching model overestimates (respectively, underestimates) its effect on low-skilled (respectively, high-skilled) employment. A simulation exercise yields order of magnitudes of these various effects. The elasticity of substitution between skilled and unskilled labor lies between 1 and 2 (Cahuc and Zylberberg, 2004, p. 211). Consider an employment subsidy on the low-skilled amounting to an ex ante reduction of 12% of their wage cost. With an elasticity of substitution close to 1, the state-of-the-art matching model overestimates the impact on low-skilled employment by 5% and underestimates the impact on high-skilled employment by 0.7%. The differences are more important in terms of job-search effort and utility levels. Altogether, this leads to very different normative conclusions. The optimal low-skilled employment subsidy (i.e. the one maximizing net output) is 63% larger in the state-of-the-art matching model.

This framework is then further generalized to deal with some labor market policies ('LMPs') extensively used in OECD economies. Our aim is to show how a generalization of the M-P setting can be used as an evaluation instrument. We focus on the interactions between employment subsidies and other LMPs. First, we introduce a two-tier benefit system (a stylized representation of many unemployment schemes). As many authors (see e.g. Fredriksson and Holmlund, 2001, or Albrecht and Vroman, 2005), we assume that the fall from the 'high' to the 'low' benefit occurs at a Poisson rate. Compared to a flat rate, time-varying unemployment benefits have different effects on job-search and on the wage bargain (Cahuc and Lehmann, 2000, Fredriksson and Holmlund, 2001, and Coles and Masters). Second, we add short-duration active labor market programs (counseling, job clubs, among others) that enhance the matching effectiveness of the participants. They influence job-search intensity (see Van der Linden, 2005) and wage formation (see Holmlund and Lindén, 1993). However, by assumption, this kind of active programs does not modify workers productivity. More generally, the model takes the distribution of skills as given. On the role of wage subsidies on human capital, see Heckman, Lochner and Cossa (2002) and Blundell, Costas Dias and Meghir (2003).

In our computational experiments, participation to the labor market is endogenous and a budget constraint of the State closes the model. Contrary to what is often done, we do not contrast highly stylized European and Nord-American economies. Instead, we calibrate and then simulate the model for a specific country plagued with a large low-skill problem (Belgium). As other countries of Western Europe, Belgium extensively uses reductions of employers' social security contributions targeted on low-skilled workers.

Economists are nowadays more and more conscious that labor market reforms should be comprehensive. Theoretical analyses of complementarities can be found in Coe and Snower (1997), Orszag and Snower (1998), Chapter 4 of OECD (2003) and Boone and van Ours (2004). Dealing with imperfectly substitutable skills and endogenous search allows to extend this literature (see Joseph, Pierrard and Sneessens, 2004, and Pierrard, 2005). Empirical analyses, such as Nickell and Layard (1999) and Belot and van Ours (2004), conclude that particular combinations of labor market institutions and policies can be responsible of good or bad performances on the labor market. These analyses are however constrained by the availability of data. Some features such as the profile of unemployment benefits can at best be proxied by some aggregate indicators. Simulations of an equilibrium matching framework seem to be preferable. To the best of our knowledge, the literature has paid scant attention to the complementarities between employment subsidies on the one hand and the time-profile of unemployment benefits and active labor market policies on the other hand.

This paper is organized as follows. Section 2 introduces the model. A streamlined version is first presented and then generalized to deal with active and passive programs. Section 3 provides some descriptive information about the structure of taxes on earnings and about Belgium. Section 4 explains how the model has been calibrated and validated. Section 5 presents simulation results and Section 6 concludes the paper.

2 The framework

2.1 A middle ground

Consider a continuous-time model with a continuum of infinitely-lived and risk-neutral workers who have perfect foresight. Each firm is made of a single (filled or vacant) job. There are frictions on the labor market. Other markets are instead frictionless (perfect competition prevails). Assume two skill groups (high-skilled indexed by h and low-skilled indexed by l) and skill-specific technologies. Let y_n denote the fixed marginal products of labor ($y_l < y_h$) and E_n the employment levels ($n \in \{l, h\}$). To study the effects of reemployment bonus programs, Davidson and Woodbury (1993) developed a matching model with different types of workers where the total number of jobs was given. So, their model can be seen as imposing $E_l + E_h = \text{constant}$. This extreme assumption has since then been abandoned. The number of jobs is the consequence of firms' and workers' decisions. The market for the final good always clears and the corresponding price can be chosen arbitrarily. This is equivalent to a setting where firms supply their optimal amount of a final good facing a perfectly elastic demand. In this alternative setting, total output supplied and exchanged, $y_l\ E_l + y_h\ E_h$, can take any value. Since the price of the final good can be normalized to one, the marginal value of an additional type-n employee is constant and equal to y_n. Consequently, the parameters of the cost function are the only determinants of supply and supply decisions taken by one firm do not affect those of the other ones. This alternative to the assumption of Davidson and Woodbury (1993) is nowadays standard in the literature. Mortensen and Pissarides (1999) and Mortensen and Pissarides (2003) are examples.[1] This alternative, henceforth the "*M-P assumption*", remains quite restrictive however.

The present paper develops a more flexible and more realistic model inspired by Acemoglu (2001), Joseph, Pierrard and Sneessens (2004) and Pierrard (2005). The novelty is very simple: Total output is now a convex combination of $y_l\ E_l$ and $y_h\ E_h$. The interpretation is the following. A single final good (the numeraire) is produced with two intermediate goods. Let Q_l (respectively, Q_h) denote the amount of the low-skilled intermediate good (respectively, the high-skilled intermediate good). Keeping y_n constant, we have $Q_n = E_n \cdot y_n$. The final good production function is homogeneous of degree one. Total output is now given by:

$$F(Q_l, Q_h), \quad \text{with } \frac{\partial F}{\partial Q_n} > 0 \text{ and } \frac{\partial^2 F}{\partial {Q_n}^2} < 0, n \in \{h, l\}. \tag{1}$$

The two inputs are p-substitutes ($0 < \frac{\partial^2 F}{\partial Q_l \partial Q_h} < +\infty$).[2] Compared to the "M-P assumption", there are two main differences. First, the elasticity of substitution between the two skills can take any positive value. The higher the elasticity of substitution, the closer we are to the "standard model".[3] Second, the marginal value of labor now varies with the number of workers in *both* sectors. Let p_n denote the real price of the intermediate good n. Profit maximization in the final good firm implies that

$$p_n = \frac{\partial F(E_l y_l, E_h y_h)}{\partial E_n y_n}, n \in \{h, l\}\,. \tag{2}$$

The marginal value product of labor depends negatively on the number of workers employed in the sector (since $\partial p_n / \partial E_n < 0$)[4] and positively on the number of workers employed in the other sector ($\partial p_n / \partial E_m > 0, n \neq m$).

[1] A similar assumption is made when technologies are not skill-specific (see Albrecht and Vroman, 2002).

[2] We also assume Inada conditions: $\lim_{Q_n \to 0} \frac{\partial F}{\partial Q_n} + \infty$ and $\lim_{Q_n \to +\infty} \frac{\partial F}{\partial Q_n} = 0$.

[3] The elasticity of substitution between skilled and unskilled labor lies between 1 and 2 (Cahuc and Zylberberg, 2004, p. 211).

[4] A similar property could also be achieved with "large" firms and diminishing returns to labor. However, we here avoid the complex intra-firm bargaining issues (see Stole and Zwiebel, 1996, and Cahuc and Wasmer, 2001).

The other assumptions are standard. Workers are able to direct their search. The flow of hires, M_n is a function of the number of vacancies, V_n and the number of job-seekers measured in efficiency units, $s_n \cdot U_n$, where s_n designates the job-search effort of the U_n unemployed. L_n denotes the size of the labor force. The matching function is written $M_n = m(s_n \cdot U_n, V_n)$. The function $m(.,.)$ is assumed to be increasing, concave and homogeneous of degree 1. Tightness is measured in efficiency units, namely $\theta_n \equiv V_n/(s_n U_n)$ or equivalently, after division by the exogenous and constant labor force L_n, $\theta_n \equiv v_n/(s_n u_n)$. The rate at which vacant jobs become filled is $q(\theta_n) \equiv M_n/V_n, q'(\theta_n) < 0$. A job-seeker moves into employment according to a Poisson process with rate $s_n \cdot \alpha(\theta_n) \equiv s_n \cdot \theta_n \cdot q(\theta_n)$, with $\alpha'(\theta_n) > 0$. Moreover, it is assumed that $\lim_{\theta_n \to 0} q(\theta_n) = +\infty$ and $\lim_{\theta_n \to 0} \alpha(\theta_n) = 0$.

The model is developed in steady state and in continuous time. The equality between separations (occurring at an exogenous rate ϕ_n) and entries lead to an increasing relationship between employment on the one hand, search and tightness on the other

$$E_n = \mathbb{E}(\theta_n, s_n) \equiv \frac{s_n \, \alpha(\theta_n)}{\phi_n + s_n \, \alpha(\theta_n)} L_n \tag{3}$$

Individuals have no access to capital markets. Let r be the discount rate common to all agents. For a worker endowed with skill n, the discounted present value in employment, $V_{E,n}$ verifies :

$$rV_{E,n} = w_n + \phi_n (V_{U,n} - V_{E,n}), \tag{4}$$

where w_n is the net wage in the nth intermediate sector (working time is normalized to 1) and $V_{U,n}$ represents the discounted expected lifetime income of an unemployed. We assume that the instantaneous utility in unemployment is the sum of unemployment benefits (proportional to the net wages) and the imputed value of leisure, z_n,[5] net of the cost of job-search $d(s_n)$ (with $d(0) = 0, d' > 0$ and $d'' > 0$). Denoting the replacement ratio by ρ_n, $V_{U,n}$ verifies the following Bellman equation:

$$rV_{U,n} = \max_{s_n} \{\rho_n \, w_n + z_n - d(s_n) + s_n \, \alpha(\theta_n)(V_{E,n} - V_{U,n})\}. \tag{5}$$

In a very standard way, at each point in time, the unemployed chooses the best level of job-search taking tightness and the net intertemporal gain as given. The first-order (and sufficient) condition balances the marginal cost of search and the corresponding marginal gain:

$$d'(s_n) = \alpha(\theta_n)(V_{E,n} - V_{U,n}) \tag{6}$$

[5]This should be understood as the monetary value of activities taking place during working hours, that an unemployed has to give up when (s)he moves to employment. The importance of these depends on institutional features of unemployment insurance. It should also be noticed that the nonmonetary effects of unemployment are not taken into account.

Let $\Pi_{E,n}$ denote the firm's discounted expected return from an occupied job if the firm produces the nth intermediate good (and recruits workers endowed with skill n). For simplicity, taxation is linear. Let $a_n + \tau_n w_n$ be the amount of taxes paid if the net wage is w_n ($\tau_n \geq 0$). According to its sign, a_n is an employer tax or subsidy. It does not matter which side of the market pays or receives a_n. So, the latter can also be interpreted as a lump-sum in-work tax or subsidy. Each filled vacancy yields y_n units of output times the price p_n of the intermediate good. At an exogenous rate ϕ_n the job is destroyed and its value becomes nil. The discounted expected return of a vacant job in sector n is denoted by $\Pi_{V,n}$. Let k_n be the flow cost of posting a vacancy. The discounted expected returns satisfy the following conditions:

$$r\Pi_{E,n} = p_n \cdot y_n - a_n - (1+\tau_n)w_n + \phi_n \left(-\Pi_{E,n}\right), \tag{7}$$

$$r\Pi_{V,n} = -k_n + q(\theta_n)\left(\Pi_{E,n} - \Pi_{V,n}\right). \tag{8}$$

There is free entry of vacancies. In equilibrium, $\Pi_{V,n}$ then equals 0 in each sector. From (7) and (8), the demand side of the market can be summarized by the following "vacancy-supply curve" relating the wage and tightness on the labor market for skill n:

$$w_n = VS_n(\theta_n) \equiv \frac{p_n\, y_n - a_n - (r+\phi_n)(k_n/q(\theta_n))}{1+\tau_n},\ VS_n{}' < 0 \tag{9}$$

Higher tax parameters (a_n or τ_n) shift the VS_n curve downward.

When a worker and an employer form a match, the surplus $V_{E,n} - V_{U,n} + \Pi_{E,n}$ is shared through bargaining. As usual in this literature, we assume a Nash bargain. If β_n denotes the exogenous bargaining power of the type-$n$worker ($0 < \beta_n < 1$), the solution to the game can be written as

$$V_{E,n} - V_{U,n} = \beta_n(V_{E,n} - V_{U,n} + \Pi_{E,n}) \tag{10}$$

This property, the Bellman equations (4) and (5) and the free-entry condition ($\Pi_{E,n} = k/q(\theta_n)$) lead then to the following "wage-setting curve":

$$w_n = WS_n(\theta_n, s_n) \equiv \frac{1}{1-\rho_n}\left[z_n - d(s_n) + \frac{\beta_n}{1-\beta_n}\frac{k_n}{1+\tau_n}\left(s_n\,\theta_n + \frac{r+\phi_n}{q(\theta_n)}\right)\right], WS_n{}' > 0, \tag{11}$$

A rise in τ_n shifts the WS_n curve downwards. However, the effect is less than proportional since the instantaneous income in unemployment contains an untaxed component $z_n - d(s_n)$ which in general is neither zero nor inversely proportional to $1+\tau_n$.

Under free entry and taking (10) into account, the optimality condition (6) becomes:

$$d'(s_n) = \frac{\beta_n}{1-\beta_n}\frac{k_n}{1+\tau_n}\theta_n \tag{12}$$

This defines an implicit increasing relationship between s_n and θ_n. Conditional on tightness, a rise in the tax rate τ_n lowers the equilibrium return of search and hence search effort. From (12), it is obvious that a marginal change in job-search effort does not shift the wage-setting curve.

The relationship $VS_n - WS_n = 0$ can be written as:

$$\begin{aligned}\mathbb{G}_n \equiv\ & (1-\rho_n)(1-\beta_n)\{p_n\, y_n - a_n\} - (1-(1-\beta_n)\rho_n)\frac{(r+\phi_n)k_n}{q(\theta_n)} \\ & -\beta_n\, s_n\, k_n\, \theta_n - (1-\beta_n)(1+\tau_n)(z_n - d(s_n)) = 0 \qquad (13)\end{aligned}$$

in which p_n is a function of both employment levels and hence a function of tightness and search effort in both sectors. If the price of the intermediate good p_n remained constant, $\mathbb{G}_n = 0$ would be an implicit function of θ_n only. Here, $\mathbb{G}_n$ is a function of tightness in both sectors. Taking the implicit relationship (12) between search effort and tightness into account, differentiating $\mathbb{G}_n$ yields:

$$\frac{\partial \mathbb{G}_n}{\partial \theta_n} = A_n + B_n < 0 \qquad (14)$$

$$\frac{\partial \mathbb{G}_n}{\partial \theta_m} = C_{n,m} > 0 \qquad (15)$$

in which

$$A_n = (1-(1-\beta_n)\rho_n)\frac{(r+\phi_n)k_n}{q(\theta_n)^2}q'(\theta_n) - \beta_n\, s_n\, k_n < 0, \qquad (16)$$

$$B_n = (1-\rho_n)(1-\beta_n)\frac{\partial p_n}{\partial y_n E_n}\, y_n \left[\frac{\partial \mathbb{E}_n}{\partial \theta_n} + \frac{\partial \mathbb{E}_n}{\partial s_n}\frac{\partial s_n}{\partial \theta_n}\right] < 0, \qquad (17)$$

$$C_{n,m} = (1-\rho_n)(1-\beta_n)\frac{\partial p_n}{\partial y_m E_m}\, y_m \left[\frac{\partial \mathbb{E}_m}{\partial \theta_m} + \frac{\partial \mathbb{E}_m}{\partial s_m}\frac{\partial s_m}{\partial \theta_m}\right] > 0. \qquad (18)$$

In these expressions, $A_n < 0$ is the effect found in the standard matching model. A higher tightness raises the exit rate out of unemployment (pushing bargained wages upwards) and increases the expected duration needed to fill a vacancy (reducing the wage that firms can afford under free entry). B_n is a new negative term that captures the effects of a higher tightness in sector n on employment in this sector and hence on the price of the corresponding intermediate good.[6] As the labor market becomes more tight, employment increases. In addition, a higher job finding rate raises search effort

[6] The partial derivative of the price $\partial p_n/\partial y_n E_n$ is computed from (2) and is negative. $\partial \mathbb{E}_n/\partial \theta_n$ and $\partial \mathbb{E}_n/\partial s_n$ are computed from (3) and are positive. Finally, $\partial s_n/\partial \theta_n$ is computed from (12) and is positive, too.

which in turn raises employment. These combined positive effects on employment lower the marginal product of the corresponding intermediate good in the production of the final good. Hence, the equilibrium price p_n shrinks and this depresses the creation of vacancies. Finally, $C_{n,m}$ captures a positive cross effect.[7] Increasing tightness in sector m raises employment in this sector. As the two intermediate goods are substitutes, the marginal product of the other intermediate good increases and this eventually stimulates the opening of vacancies in the other sector (n).

In steady-state, the equilibrium pair(s) (θ_l, θ_h) verify the system of equations $\mathbb{G}_l = \mathbb{G}_h = 0$. Each of these equalities define an increasing implicit relationship between θ_l and θ_h. It is therefore far from obvious that an equilibrium exists and is unique. Cardullo (2005) shows this property. Figure 1 illustrates the equilibrium. Knowing the levels of tightness, the values of net wages and of employment follow immediately respectively from (11) and (3).

We now look at the effects on tightness levels of changes in the lump-sum employment tax/subsidy, a_n, and in the tax rate, τ_n. We only consider the case $n = l$. The comparative static analysis starts from a unique equilibrium (θ_l, θ_h). Under the M-P assumption, the price of the intermediate goods is assumed to be fixed. We compare the changes in tightness levels in this setting with those coming out when these prices are endogenous.

Effects of the lump-sum employment tax/subsidy

Consider a marginal change in a_l. Under the M-P assumption, the real price of the two intermediate goods being constant, the effect on tightness would be:

$$\frac{d\theta_l}{da_l} = -\left(\frac{d\mathbb{G}_l}{da_l}\right) / \left(\frac{d\mathbb{G}_l}{d\theta_l}\right) = (1-\rho_l)(1-\beta_l)/A_l < 0 \tag{19}$$

$$\frac{d\theta_h}{da_l} = 0 \tag{20}$$

Taking the endogeneity of prices into account, one gets after some manipulation:

$$\frac{d\theta_l}{da_l} = \mu_l\,(1-\rho_l)(1-\beta_l)/A_l < 0, \tag{21}$$

$$\frac{d\theta_h}{da_l} = -\frac{(1-\rho_l)(1-\beta_l)C_{h,l}}{(A_l+B_l)(A_h+B_h) - C_{l,h}\,C_{h,l}} < 0, \tag{22}$$

[7]The expressions used to compute the various partial derivatives are explained in the previous footnote.

where, exploiting Euler's formula for linear homogeneous function,[8] one has:

$$\mu_l = \frac{A_l(A_h + B_h)}{(A_l + B_l)(A_h + B_h) - C_{l,h}\, C_{h,l}} = \frac{A_l(A_h + B_h)}{A_l(A_h + B_h) + B_l\, A_h} < 1.$$

Figure 1 illustrates these effects (see the interrupted line). So, compared to the case where the real prices of the two intermediate goods are taken as constant, $d\theta_l/da_l$ is less negative. Two opposite effects are present. First, if the employment tax is augmented in a given sector, say l, there is at given prices p_n a reduction in tightness and hence in employment in this sector. Less employment implies a rise in the marginal product of workers and this leads to a higher price for the corresponding intermediate good Q_l. More vacancies are therefore posted. This attenuates the initial drop in employment. Second, less employment in sector l, where the employment tax is augmented, implies a lower marginal product of the other intermediate good Q_h. Less vacancies are therefore created in sector h. And this in turn depress job creation in sector l. One easily see that this chain of effects creates a multiplicative effect which tends to amplify the initial decline in tightness θ_l. Since $0 < \mu_l < 1$, the first effect dominates.

This discussion implies that an employment subsidy in the low-skill sector will rise tightness (and hence employment) in this sector less than under the M-P assumption. The induced effect of the employment subsidy is moreover positive in the skilled sector (while it is zero under the M-P assumption). The quantitative importance of these differences will be studied in Section 5.

It should also be noticed that contrary to the standard M-P model (in which marginal variations in search effort do not affect tightness in equilibrium), endogeneizing search effort matters here. For, marginal changes in search effort affect the level of employment (see (3)) and hence the prices of the intermediate goods. The equilibrium levels of tightness are therefore eventually modified. Consequently, endogeneizing search effort changes the impacts of the tax paramerters a_n and τ_n in our model.

Effects of the tax rate

Consider a marginal change in the tax parameter τ_l. Totally differentiating $\mathbb{G}_l = \mathbb{G}_h = 0$ with respect to θ_l, θ_h and τ_l, it can be verified that the sign of the variation in both θ_l and θ_h is given by the sign of $d\mathbb{G}_l/d\tau_l$. Remembering (12), we have the following effects:

$$\frac{d\mathbb{G}_l}{d\tau_l} = (1 - \beta_l) \left[y_l(1 - \rho_l) \frac{\partial p_l}{\partial y_l E_l} \frac{\partial \mathbb{E}_l}{\partial s_l} \frac{d s_l}{d\tau_l} - (z_n - d(s_l)) \right], \tag{23}$$

in which $\partial s_l/\partial\tau_l$ is computed from (12) and is negative. Two mechanisms are at work *conditional on the levels of tightness*. First, rising the tax rate τ_l reduces search effort

[8] Namely, $\frac{\partial^2 F}{\partial^2 Q_n} \frac{\partial^2 F}{\partial^2 Q_m} = \left[\frac{\partial^2 F}{\partial Q_m \partial Q_n} \right]^2$.

and hence employment in Sector l. This raises the equilibrium price for the intermediate good sold by this sector. So, the first product between brackets in (23) is nonnegative. According to this first effect, a higher tax rate *raises* equilibrium tightness in both sectors. The second mechanism which is not new (see Holmlund, 2001) is due to the existence of untaxed activities. Raising taxes affects the level of bargained wages (see (11)). With constant replacement ratios, the change in unemployment benefits is by definition proportional. However, the net income of the unemployed is also influenced by two components that are not influenced by the tax rate, namely z_n and $d(s_n)$. This introduces a form of real wage rigidity. Therefore, bargained wages do not fully adjust when the tax rate rises. Hence, tightness is affected by proportional taxation. If $z_l - d(s_l) > 0$ (respectively, < 0), according to this second mechanism only, a rise in τ_l would reduce (respectively, increase) equilibrium tightness θ_l. This is the conclusion under the M-P assumption. In our more flexible framework, because of the first mechanism, the marginal effect $d\theta_l/d\tau_l$ has an ambiguous sign if $z_l - d(s_l) > 0$.

2.2 Generalizing the model to encompass other LMPs

Employment subsidies do not take place in isolation. They are typically introduced in labor markets where so-called active and passive LMPs are also present. We now show how the framework of the previous subsection can be further extended to evaluate the interactions between these policies in a general equilibrium setting. Some hypotheses will be chosen with Continental Europe in mind.

In accordance with institutions in many OECD countries, a two-tiered benefit system is assumed to prevail. An insured unemployed whose 'high' benefits has expired enters a state where (s)he indefinitely can benefit from a lower unemployment benefit. The latter could be an assistance benefit. High benefits expire at an exogenous rate $\pi_n > 0$. For jobless individuals, three states are identified: Insured unemployment with high benefits (U_n), insured unemployment with low benefits (X_n) and participation (T_n) in a short-duration active labor market policy (ALMP) organized by the Public Employment Services (PES). We have in mind counseling programs, job clubs or very brief training schemes. By assumption, these policies do not change the productivity of the participants.[9] These upper-case symbols will designate both the states and the number of individuals occupying them in steady state. The corresponding intertemporal discounted values will be denoted by $V_{U,n}$, $V_{X,n}$ and $V_{T,n}$. Figure 2 displays the various states and the flows in this economy. A growing literature shows that dura-

[9]Assuming a fixed distribution of skills and skill-specific matching, it should be stressed that this paper does not deal with (long-duration training) schemes that intend to enhance skills (see Albrecht, van den Berg and Vroman, 2004, and Boone and van Ours, 2004) or to enlarge the set of occupations that are accessible (see Masters, 2000).

tion dependence is largely spurious in Continental Europe (see Machin and Manning, 1999). True duration dependence is therefore assumed to be a negligible phenomenon in this economy.

Let $s_{U,n}, s_{X,n}$ and $s_{T,n}$ denote search intensities in the various states. A unique *exogenous* matching effectiveness parameter c_n will be associated to states U_n and X_n. For ALMP participants, this parameter can be different and will be denoted $c_{T,n}$. It is assumed that $c_{T,n} > c_n > 0$.[10] So, in the matching function $m(S_n, V_n)$, $S_n \equiv c_n\,(s_{U,n}\,U_n + \; s_{X,n}\,X_n) + c_{T,n}\,s_{T,n}\,T_n$ and tightness is defined as $\theta_n \equiv V_n/S_n$.

The unemployed receive an offer to take part to the ALMP at an exogenous rate $\gamma_n \; \geq 0$.[11] The unemployed have then to decide whether they enter the program (right away) or not. Two cases will be distinguished. First, this offer is not used to verify the availability of the unemployed. Then, the intertemporal value of those who receive an offer in state U_n is $\overline{V}_{U,n} = \max(V_{T,n}; V_{U,n})$. In state X_n, it is $\overline{V}_{X,n} = \max(V_{T,n}; V_{X,n})$. Second, this offer is on the contrary used as a way of monitoring the unemployed. Someone in state U_n can be sanctioned if (s)he refuses to take part to the program. Let the sanction be an immediate entry in state X_n. Then, $\overline{V}_{U,n} = \max(V_{T,n}; V_{X,n})$. It is assumed that those in state X_n cannot be sanctioned by entering a lower position than X_n, which then plays the role of a minimum income guarantee. In addition, participation to the ALMP can be unsuccessful in some case. More precisely, it is assumed that the program fails at an exogenous rate $\lambda_n \; \geq 0$.

In steady state, the stocks of individuals in each position (U_n, X_n,...) are constant. Equalities between entries and exits in each state determine the level of employment E_n among workers endowed with skill n. E_n increases with tightness θ_n and search effort levels $\mathbb{S}_n \equiv (s_{U,n}, s_{X,n}, s_{T,n})$ (for details, see Appendix B).

If the wage negotiation took place at the individual level, the wage would be different according to the state of origin, at least just after entry into the firm. Having Continental Europe in mind, we assume instead that the wage in sector n is bargained over by incumbent employees on behalf of all workers of this sector. The fall-back position of these "insiders" is the intertemporal discounted utility of an unemployed entering state U_n, denoted by $V_{U,n}$. Then, there is a single skill-specific wage. The discounted value

[10] The ALMP can intrinsically improve the effectiveness of search effort. Other explanations can be suggested, too. As job-entry rates are often used in the assessment of labor programs, the PES can for instance give priority to participants to these programs, in particular in the case of a closed treatment of job offers. This refers to the case where the PES select those who are suitable for vacancies in their register.

[11] Conditioning the access to an ALMP on the level of unemployment benefits would be considered as discriminatory. So, this possibility is ruled out here. As it is observed in several countries, participation to active programs is a sufficient condition to become eligible to high benefits again. Relaxing this assumption would substantially complicate the mathematical expressions used below and in the appendices without adding much insight.

of holding a job still verifies (4). We keep the hypothesis of constant replacement ratios and assume the following very plausible ranking: $1 > \rho_{T,n} \geq \rho_{U,n} > \rho_{X,n} > 0$. Let $v_{\iota,n} \equiv \rho_{\iota,n} \cdot w_{\iota,n} + z_n - d(s_{\iota,n}))\, \iota \in \{U, X, T\}$. We impose that $v_{\iota,n}$, with $\iota \in \{U, X, T\}$, is always positive. For jobless people endowed with skill n, the intertemporal utility levels solve the following Bellman equations:

$$rV_{U,n} = \max_{s_{U,n}}\{v_{U,n} + c_n s_{U,n}\alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n})\}, \tag{24}$$

$$rV_{X,n} = \max_{s_{X,n}}\{v_{X,n} + c_n\, s_{X,n}\, \alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\overline{V}_{X,n} - V_{X,n})\}, \tag{25}$$

$$rV_{T,n} = \max_{s_{T,n}}\{v_{T,n} + c_{T,n}\, s_{T,n}\, \alpha(\theta_n)(V_{E,n} - V_{T,n}) + \lambda_n(V_{U,n} - V_{T,n})\}. \tag{26}$$

Under the assumptions made so far, Appendices A and B show that the intertemporal values can always be ranked ($V_{E,n} > V_{T,n} > V_{U,n} > V_{X,n}$) under an additional sufficient condition, namely $\lambda_n > \phi_n$.[12] Consequently, all the unemployed choose to take part to the program and $\overline{V}_{U,n} = \overline{V}_{X,n} = V_{T,n}$. The optimal levels of search effort $\mathbb{S}_n$ solve first-order conditions that are similar to (6). They are stated in Appendix B. They imply that $s_{X,n} > s_{U,n}$ because the unemployed in the second tier gain more from searching ($V_{E,n} - V_{X,n} > V_{E,n} - V_{U,n}$). On the contrary, $s_{T,n}$ and $s_{U,n}$ cannot be ranked. The treated are induced to search harder because search effort is more efficient ($c_{T,n} > c_n$). However, when search is successful, the net gain is lower for the treated: $V_{E,n} - V_{T,n} < V_{E,n} - V_{U,n}$.

Job creation is modelled in the same way as in Section 2.1. Thus, the vacancy-supply curve (9) remains unchanged. Since the expression relating $V_{U,n}$ to the endogenous variables and the parameters is much more complex than in Section 2.1, the "wage-setting curve" is more involved, too (see (48) in Appendix B). However, the properties found earlier remain. The net wage w_n is an increasing function of tightness θ_n. Marginal changes in job-search effort do not shift the wage-setting curve. The equations that characterize search effort levels in equilibrium are much more complex than (12). They are stated in Appendix B. It remains true that search effort increases with tightness and decreases with the tax rate τ_n.

Eliminating the net wage from the wage-setting and the vacancy-supply curves yields a system of equations $\mathbb{G}_l = \mathbb{G}_h = 0$. As in Section 2.1, each of these equalities define an increasing implicit relationship between θ_l and θ_h and the equilibrium is unique. It can easily be seen that the employment tax-subsidy a_n and the tax rate τ_n

[12] In Continental Europe, it is quite natural to assume that the expected length of an employment spell (taking all types of contracts into account) is longer than the expected duration of the short-duration ALMP. Consequently, $1/\lambda_n < 1/\phi_n, \forall n$.

play *qualitatively* the same role as in Section 2.1.[13]

The equilibrium effects of the parameters characterizing the unemployment insurance system and the ALMP have already been developed under the M-P assumption.[14] Here, we briefly summarize theses effects and then explain how the comparative statics changes in our more flexible setting. Due to space limitation, we focus on two major parameters only: π_n and γ_n.

In a two-tiered unemployment benefit scheme, Van der Linden (2003a) shows that a marginal increase in the rate (π_n) at which jobless workers flow from the "high" to the "low" benefit level has a positive direct (i.e. conditional on θ_n) effect on employment E_n since $s_{X,n} > s_{U,n}$[15] and a positive impact on labor market tightness through a decline in the fall-back position of workers. A more tight labor market also stimulates search effort. However, a marginal increase in π_n has in addition a negative direct effect on search in the second tier and in the ALMP. This is due to an unfavorable "entitlement effect" (see Mortensen, 1977): The gain of a successful search activity also depends on the utility if the new job is lost. This gain is negatively affected by π_n. Therefore, the net effect on E_n cannot be signed. These are the analytical conclusions under the M-P assumption. In our more flexible setting, not only the effect of employment but probably also the one on tightness are ambiguous in sector n. The above-mentioned direct effect and the entitlement effect influence E_n in opposite ways. If the entitlement effect is dominated by the other one,[16] the rise in employment lowers the price of the intermediate good. So, the vacancy-supply curve $w_n = VS(\theta_n)$ shifts downwards. This move and the downward shift of the wage-setting curve explain why the net effect on θ_n is now ambiguous. In addition, the variation of E_n has an induced effect on tightness in the other sector. This mechanism has already been explained in Section 2.1. If the entitlement effect is dominated by the other one, a rise in π_n would unambiguously increase equilibrium tightness in the other sector.[17] Recall that this cross-effect does not appear under the M-P assumption.

Van der Linden (2005) shows that a marginal rise in the rate of entry into the ALMP, γ_n, has a direct positive effect on employment conditional on tightness if, as we assume for the rest of this section, the "matching effectiveness" $c_{T,n} \cdot s_{T,n}$ is sufficiently

[13]In some countries, like France, the wage of the low-skilled is not bargained over but equal to the legal minimum wage. The latter is periodically adjusted upwards to keep relative wages approximately constant. We have verified that the qualitative properties of the employment subsidy remain when w_h is negotiated and $w_l = a\, w_h$, where a is an exogenous parameter ($0 < a < 1$).

[14]See Van der Linden (2003a), Van der Linden (2003b) and Van der Linden (2005).

[15]Intuitively, this creates a relative increase in the number of unemployed in the second tier where search effort is higher.

[16]That is, if $\frac{\partial E_n}{\partial \pi_n} + \sum_{\iota \in U,X,T} \frac{\partial E_n}{\partial s_{\iota,n}} \frac{\partial s_{\iota,n}}{\partial \pi_n} > 0$. Although there is no formal proof, this sounds plausible because the entitlement effect is a delayed effect if the worker returns in unemployment.

[17]The proof which makes use of Euler's formula is available upon request.

higher than $c_n \cdot s_{X,n}$ (see Appendix C). In the model, the matching effectiveness of those in the first tier (U_n) can be raised either by sending them into the active program or by letting them enter the second tier (X_n). Independently of the induced effects, the active program cannot raise employment if the former channel is less effective than the latter (i.e. if $c_{T,n} \cdot s_{T,n} < c_n \cdot s_{X,n}$). As far as indirect effects are concerned, increasing the rate γ_n has a wage-push effect (since $V_{T,n} > V_{U,n}$) and no effect on the vacancy-supply curve under the M-P assumption. So, equilibrium tightness θ_n declines. This induces a negative adjustment in equilibrium search. Both counteract the direct effect on employment E_n. The rate γ_n has also a direct effect on search effort in states X_n and T_n. It can be shown that the former is negative while the latter is positive.[18] To sum up, putting more jobless individuals in this ALMP has a clear-cut negative impact on tightness but an unclear net effect on employment.[19]

Now, in our more flexible setting, not only the effect of employment but also the one on tightness are in general ambiguous because, under the M-P assumption, the net effect of γ_n on E_n is ambiguous (see Appendix C). However, if this effect is nonnegative, Appendix C shows that one net impact on tightness can be signed in our more flexible setting: $d\theta_l/d\gamma_l < 0$ and $d\theta_h/d\gamma_h < 0$.

3 Taxes, subsidies and the low-skilled problem

Several countries (Belgium, France and Germany) with high tax wedges (measured as the ratio between the wage cost and the net wage) on low-paid workers have recently reduced the tax burden on the low-paid relative to the one of the highly-paid (see Figures 3 and 4). In Belgium, the country considered from now on, Figure 4 shows a slow downward tendency. This can be explained by the combination of two reforms.

First, reductions of social security contributions amounted to 0.5% of GDP in 1998 and 1.2% in 2004. These reductions vary with the wage. Figure 5 shows that the total rate of social security contributions is substantially reduced on low-wage workers (in the private sector). The highest reduction is at the legal minimum wage (about 1050 Euro/month at the age of 18). The phased-out region is spread over a wide interval of gross wages (from the minimum wage to about 2000 Euros/month). About 28% of men and 55% of women are currently employed in the phased-out region. The phased-out region contains most of the minimum wages bargained at the sectoral level. Above 2000 Euros/month, reductions of social security contributions are first completely flat and they increase again above a monthly wage of about 4000 Euros.

Second, a tax reform has been introduced in Belgium in 2001. Despite a small

[18] For an explanation, see Appendix B.

[19] The same kind of reasoning holds in case of a decline in the failure rate, λ_n.

individualized income tax credit at the bottom of the income distribution, the "effects of the reform in terms of increase in disposable income appear to be strongest for the middle to upper class " (Orsini, 2005, p. 42).

So, in recent years, Belgian authorities have used two instruments to reduce the tax wedge on the labor income of almost all types of workers with only a limited emphasis on low-wages. Figure 4 illustrates this conclusion. Other countries like France appear to have implemented a clearer strategy.

Long-term unemployment is a major problem in Belgium. During the last thirty years or so, more than 50% of the stock has typically been unemployed for more than a year. The median duration in the stock amounts to about 2 years. In Belgium, negative duration dependence is very strong but Cockx and Dejemeppe (2005) have shown that it is largely spurious in the South. On data covering the period 1995 - 2004, Heylen and Bollens (2005) find positive duration dependence for men (nearly no dependence for women) in the North of the country. Their result for men is in accordance with our theoretical model. The level of skill (understood as education) is one of the key individual characteristics that affect the hiring rate. In each region of the country and for each gender, the unemployment rate of the less-educated (at most a lower-secondary degree) is for many years two to four times higher than the one of those with post-secondary education.

The following simulation exercise will quantify the effects of reductions in the wedge and see to what extent these effects are sensitive to the choice of the eligible population. Moreover, we will measure to what extent they are sensitive to the design of other LMPs.

4 Calibration, validation and extensions

We take the month as unit of time. Data refer to the 1997-1998 period where the stocks were fairly stable in Belgium. It should be stressed at the outset that we do not have access to individual data about (non-)participants to LMPs nor to a pilot-study. Due to statistical availability, only two levels of skill are distinguished. The low-skill population is assumed to hold at most a lower-secondary degree. The low-skilled represent 34% of the active population, 30% of salaried employment and 64% of the stock of unemployed. Table 1 presents the calibrated values and the rates of people in the various states. The low-skilled total unemployment rate is about 20% against 6.5% for the skilled workers. e designates the ratio between salaried employment and the active population. p is the participation rate. Both are much lower for the low-skilled.

To calibrate the model, we first exploit relationships derived from the model (equilibrium of flows in steady-state, the wage-setting curves, the optimality conditions). We also make use of various surveys[20], published statistics[21], other statistics collected for the purpose of this study, and results found in the literature. A sensitivity analysis is conducted on some parameters.

We take $a_h = a_l = 0$. Data on wage costs and net wages are used to fix the tax rate (including social security contributions) τ_n. In accordance with the evidence shown in Section 3, these are high ($\tau_l = 1, \tau_h = 1.17$). Administrative data indicate that less than 2.5% of the unemployed do not receive unemployment benefits (UBs) in Belgium. Neglecting this phenomenon, there is first a period of one year where UBs stay constant. With the month as unit of time, π_n is therefore equal to 0.083. For about two-third of the insured unemployed, the level of benefits decreases afterwards. In 1998, less than 2% of the unemployed have lost their entitlement to (after a very long spell of unemployment). This phenomenon is therefore neglected, too. The time-profile of skill-specific UBs is an average computed from administrative data. The replacement ratios are displayed in Table 1. At the end of the nineties, many beneficiaries of active programs participated (often simultaneously) to a combination of three interventions:[22] Individual advice and guidance[23], job-search assistance (such as job clubs, tips on finding jobs and writing a successful resume) and short-duration vocational training[24]. Due to constraints on data, those policies are taken as an aggregate and henceforth called 'counseling programs'.

As many other papers, let us assume the following Cobb-Douglas matching function (see Petrongolo and Pissarides, 2001): $m(S_n, V_n) \equiv m_0 S_n^{0.5} V_n^{0.5}$. Parameter m_0 is a scaling factor for the various c_ι's and for k_n.[25] The discount rate is fixed at 0.004 (5% on an annual basis). Annual reports of the PES allow to fix parameters ϕ_n, λ_n and γ_n (see Table 1).

The expected duration of a vacancy (2.5 month) and the share of the low-skilled in the total number of recruitments (0.38) is used to calibrate the θ's. The aggregate production function is a C.E.S. Due to a lack of appropriate time-series for Belgium,

[20] Simoens, Denys and Denolf (1998), Denolf, Denys and Simoens (1999) and Delmotte, Van Hootegem and Dejonckheere (2001).

[21] Published by national and regional PES in Belgium and by Eurostat (2002a) and Eurostat (2002b).

[22] See Vos, Struyven and Bollens (2000).

[23] "Plan d'accompagnement des chômeurs" i.e. a small number of meetings with a member of the PES during a period of four months.

[24] According to annual reports of the regional PES, there exist very short programs mixing counseling and short-lived training that lasted about 100 hours on average.

[25] Assuming that $m_0 = 0.5$ yields reasonable values. A sensitivity analysis has been conducted. We consider an alternative matching function, inspired by the results of Cockx and Dejemeppe (2002), namely $m_0 S_n^{0.4} V_n^{0.6}$. Unreported simulation results show that the effects of changes in the tax wedge are similar.

we use a French study (Biscourp and Gianella, 2001) to fix the elasticity of substitution to 1.1. The "vacancy-supply curves" (9) are then used to calibrate the k's. k_n also affects the wage-setting curve and hence the calibrated value of the bargaining power. The marginal products $p_n \cdot y_n$ are chosen so as to produce sensible values for the ratio of the share of the wage bill in output.

We assume an iso-elastic cost of job-search $d(s) = \psi_n \cdot s^{\xi_n}/\xi_n$, with $\psi_n > 0$ and $\xi_n > 1$. In the absence of relevant information, we impose $z_n = 0$. The products $c_\iota s_\iota$, $\iota = \{T, n\}, \{X, n\}, \{U, n\}, n \in \{l, h\}$ can be computed from the flow equilibrium conditions. Conditional on these products, the calibration then fixes the c_ι's, the s_ι's, ξ_n, and the bargaining power of the workers β_n. This part of the calibration is based on Equations (48), (49), (50) and (51) in Appendix A. This system is solved conditional on the assumption $\psi_l = \psi_h = 7.4$. Raising this parameter induces a proportional increase in $c_{T,n}$ and c_n and a proportional reduction in all search-effort levels without affecting the other parameters. From Table 1, an increase in γ_n has a direct positive effect on employment. Skilled workers search more intensively. As expected, they have higher matching effectiveness parameters.

The bargaining power of the skilled workers verifies the Hosios condition, i.e. their level of unemployment would be efficient in an economy without taxes, transfers and subsidies.[26] Under the same assumptions, the calibrated bargaining power would lead to an inefficiently high level of unemployment for the low-skilled. One can wonder why β_l is somewhat higher than β_h in Table 1. Union density is not the unique determinant of the bargaining power. Nevertheless, part of difference between the β's can be related to differences in union density. The latter is more important among blue-collar workers than among white-collar ones. In addition, Belgian unions have a strong distaste for inequality.

To check the validity of this calibration, we first look at two properties of the model that were not used during the calibration and about which some data are available. Then, we compute two major elasticities and compare them to standard values found in the literature. In 1997, the average stock of vacancies registered by the PES amounted to 24,500. With a market share of the PES in the range $[0.4, 0.5]$, the calibrated stock of vacancies (53,000) is an acceptable order of magnitude. With the calibrated parameters, the expected duration of an unemployment spell amounts to 11 months for the skilled and 31 months for the low-skilled. Weighted by the share of each skill in the inflow into unemployment, the mean duration would then be equal to 19 months, a result that is in line with the computations of Dejemeppe (2005).[27]

[26]Cardullo (2005) shows that the Hosios condition guarantees efficiency in the setting of Section 2.1.

[27]From her analysis of unemployment dynamics in Belgium, the average unemployment duration in 1992 was equal to 2 years in the South of Belgium and to 1.5 years in the North.

The computed wage elasticity of salaried employment (job-search effort remaining fixed) amounts to low but reasonable values, namely - 0.54 for low-skilled workers and - 0.33 for skilled ones. The estimations of the elasticity of substitution being rather dispersed, we also consider later an elasticity of 2 instead of 1.1. After a new calibration, the two elasticities of labor demand become respectively equal to - 0.65 and - 0.34.

Finally, the elasticity of unemployment duration with respect to the level of UBs (tightness remaining fixed) is equal to 0.39 for the high-skilled and 0.26 for the low-skilled. The latter elasticities are relatively low but they remain acceptable (according to Meyer (2002), an elasticity of 0.5 is a standard order of magnitude).

The model of Section 2.2 is in addition extended to deal with the extensive margin (participation decisions). Furthermore, the government budget constraint is added to close the model. Participation is modeled in a very simple way (see Pissarides, 2000). Inactive people have an arbitrage condition: Staying inactive or entering state X_n.[28] Let $\mathcal{P}_n$ be the exogenous size of the working age population ($\mathcal{P} \equiv \sum_n \mathcal{P}_n$). Let $[V_{1,n}, V_{2,n}]$ be the finite support of the distribution of intertemporal utility levels in inactivity, $V_{I,n}$. With a uniform distribution, the participation rate is simply defined as

$$p_n \equiv \frac{L_n}{\mathcal{P}_n} = \frac{V_{X,n} - V_{1,n}}{V_{2,n} - V_{1,n}}. \tag{27}$$

Following Immervoll, Kleven, Thurstup Kreiner and Saez (2004), the elasticity of the participation rate p_n with respect to w_n is fixed to 0.4 for the low-skilled and 0.2 for the high-skilled. These assumptions and the participation rates allow to calibrate the boundaries $V_{1,n}$ and $V_{2,n}$ introduced in (27). Let lower case letters e_n, u_n, x_n, t_n and v_n be the rates obtained by dividing the absolute numbers by L_n (e.g. $e_n \equiv \frac{E_n}{L_n}$). The budget of the State scaled by $\mathcal{P}$ can be written as follows:

$$\sum_n (\,\rho_{U,n} u_n + \,\rho_{X,n} x_n + (\rho_{T,n} + C) t_n)\, p_n\, \mathcal{P}_n = \sum_n (a_n + \tau_n\, w_n)\, e_n\, p_n\, \mathcal{P}_n, \tag{28}$$

where C is the average cost of the program.[29] When the budget constraint is binding in the simulation exercises, *both tax rates τ_n are adjusted proportionally to fulfill* (28).

[28] Alternatively, they could enter uninsured unemployment (i.e. start an unemployment spell without any benefit). However, in Belgium, people who are ready to take a job and have no income are eligible to a minimum income guarantee. The latter is in a way or another related to the lowest level of UBs, $\rho_{X,n} \cdot w_n$. So, the simplifying assumption made here is not a substantial limitation.

[29] Data in Eurostat (2002a, 2002b) allow to estimate that the average cost C of these programs amounted to 130 Euro per worker and per month (net of transfers to beneficiaries of these programs). To Equation (28) we also add an exogenous level Q of net public expenses that solves this equation for the calibrated values of the parameters and of the endogenous variables. This level is kept fixed during simulations.

5 Simulation results

In this section, we illustrate by how much the effects of an employment subsidy on the low-skilled, a_l, change when the M-P assumption is replaced by our more flexible setting. Then, we look for the optimal level employment subsidy. Finally, we consider the interactions between an employment subsidy and other LMPs.

Table 2 considers an employment subsidy $a_l = -300$ Euro/month, i.e. 12% of the calibrated wage cost. Comparing the case where the marginal values of labor are fixed (the M-P assumption) to the one where they vary lead to relatively small differences in tightness but large ones in search effort. Assuming fixed marginal values of labor leads to overestimate the level of employment E_l by about 5% and to underestimate E_h by 0.7% (see the two first lines of Table 2). Taking into account the differences in terms of the net wages, the over- and underestimations are more substantial in the case of intertemporal discounted values. As expected, the differences shrink when the elasticity of substitution, σ, increases (compare the two last lines of Table 2 with the two first ones). The magnitude of the over- and underestimations vary with the size of the employment subsidy. According to the indicator, doubling the level of a_l multiplies the differences provided in Table 2 by 1.4 to 1.8.

Next, we look for the optimal employment subsidy taking the other parameters of the model unchanged. This task can be divided in two steps. First, the choice of the eligible population. Second, conditional on this choice, the level of the employment subsidy a_n. All simulations made lead to one first conclusion: Targeting the employment subsidy on the low-skilled is the best thing to do. To illustrate this assertion, consider the following comparison. An employment subsidy scheme $(a_l = -300, a_h = 0)$ Euro/month is compared to a scheme that has the same cost ex ante (wages and employment being fixed) and a structure similar to current practices in Belgium, namely $(a_l = -110, a_h = -81)$. Taking the budget constraint (28) into account, the first scheme creates two times more employment than the second one. The intertemporal discounted income levels of all groups are also higher in the case of the first scheme.[30] So, from now on, we put a_h to zero and focus on a_l only.

With risk-neutral agents and in the absence of a concern for redistribution, we consider a benevolent planner who at any moment t maximizes

$$\int_t^{+\infty} e^{-r(\tilde{t}-t)} W(\tilde{t}) d\tilde{t}, \tag{29}$$

[30] It is not obvious that even the high skilled prefer the first scheme to the second one. For, the latter would a priori raise their bargained wage and their employment rate more than the first scheme. However, the lower global effect of the second scheme on (un)employment leads to higher tax rates $\tau_n, \forall n \in \{l, h\}$. And this effect turns out to outweigh the others.

where $W(\tilde{t})$ is the sum of the instantaneous utility of the individuals (weighted by their numbers) and of profits made by the final and the intermediate firms. When the policies are financed (i.e. (28) is fulfilled) and if the discount rate tends to zero, this benevolent planner actually maximizes net output in steady state:

$$F(E_l\, y_l, E_h\, y_h) - \sum_n (U_n\, d(s_{U,n}) + X_n\, d(s_{X,n}) + T_n\, d(s_{T,n})) - \sum_n k_n\, V_n. \tag{30}$$

Expression (30) scaled by the size of the population is denoted by Y in Figure 6.[31] Y reaches a maximum when a_l is close to - 1490 Euro/month. We also computed net output assuming fixed marginal values of labor and found a maximum for a_l close to -2430 Euro/month. So, the normative conclusion appears to be very different whether marginal values of labor are assumed to be fixed or not.

The optimal employment subsidy looks extremely large compared to the calibrated value of the net wage (1229 Euro/month). Through bargaining, the employment subsidy is to some extent appropriated by the low-skilled workers. When $a_l = -1490$, the net monthly wage amounts to 1677 Euro (+36%). Thanks to a cut in unemployment (the low-skilled unemployment rate is halved), the rise in the tax rate needed to finance the subsidy (11%) is not huge. Altogether, the optimal low-skilled wage cost is equal to 2053 Euro (16% lower than the calibrated wage cost, 2258 Euro). The optimal total amount of taxes paid on low-skilled work equals 376 Euro (i.e. 70% lower than without the employment subsidy). By comparison with current policies, consider a low-skilled single person without children. The corresponding average total amount of taxes is about 850 Euro/month in 2005. In the case of a couple with two children and a single (low-skilled) wage, the average total amount of taxes is about 620 Euro/month in 2005. This comparison suggests that the gap between current and optimal levels of taxes on low-skilled workers is still large.

Adopting a political economy perspective, it is however doubtful that the optimal value of a_l would be implemented. For, the skilled workers, who represent two-third of the active population, first benefit from the employment subsidy but start losing below $a_l \approx -600$ Euro (See Figure 6). The relationship between the tax rates τ_n and a_l is U-shaped.[32] For sufficiently small values of a_l, the employment subsidy is so effective that the tax rates τ_n can be slightly reduced. Above an employment subsidy ≈ 370 Euro/month, the tax rates τ_n start rising. So, on the one hand there is the favorable effects of a_l on tightness θ_h explained in Section 2.1 and on the other the rise in the tax rate τ_h eventually reduces the net wage and the employment level of the high-skilled. Their intertemporal discounted income starts shrinking, too. This conclusion could be

[31] Since our calibration uses a discount rate of 0.004, a fully rigorous analysis would require to look also at the adjustment path towards the steady-state values.

[32] As both tax rates τ_n follow the same evolution, only τ_l is displayed on the figure.

sensitive to the way of financing the employment subsidy. However, we leave this issue for further research.

The previous simulation exercise illustrates that "countries can engineer a reduction of unemployment *without* a sacrifice of low-end pay and a rise in low-end pay *without* a sacrifice of employment" (Phelps , 2003, p. 11). We now consider interactions between fiscal instruments and other LMPs and raise the question: Could reforms to LMPs improve the effectiveness of employment subsidies? We have seen in Section 3 that the tax wedge is relatively high in Belgium. Public expenditures on LMPs represented 3.75% of GDP in Belgium in 2003. Is it possible to engineer reforms to LMPs that reduce public expenditures but reinforce the effects of an employment subsidy and are welfare improving (at least in steady state)? To answer that question, we consider a reform that induces steeper time-profile of UBs (a rise in parameter π_n) and another that lowers the rate γ_n of entry into the active program. Below, we consider an employment subsidy of $a_l = -300$ Euro/month. According to Figure 6, such a subsidy improves the intertemporal utility of all individuals. Moreover, with $a_l = -300$, the ex ante cost of the subsidy equals 1% of GDP, i.e. the total amount of reductions in employers' payroll taxes in Belgium in 2003. Given the huge public debt of this country, keeping total (ex ante) expenses constant looks reasonable.

At given tax levels, Section 2.2 indicated that a rise in π_n would have a clear-cut negative effect on net wages and ambiguous effects on tightness and employment. If this reform allows to reduce the tax wedge, the effect on the wage could however be reversed. This is illustrated by the following simulation. Let us compare the calibrated value, $\pi_n = 1/12$, to $\pi_n = 1/3$, $\forall n \in \{l, h\}$. This reform divides the expected duration of "high" benefits by four. When $a_l = -300$, total net output, Y, is 2% higher with the reform: 2137 Euro instead of 2095. The tax rates τ_n can be reduced (for instance, τ_h equals 1.07 after the reform versus 1.13).[33] Without this tax reduction, the low-skilled, whatever their current position, would be worse-off after the reform, while the high-skilled would be better-off. Table 3 shows that the reform becomes a steady-state Pareto improvement when tax rates are adjusted (downwards) to fulfill the budget constraint (28). Unreported simulations results show that this conclusion is robust to changes in the discount rate. To check this, we considered an annual discount rate of 20% (instead of 5%), we calibrated the model again and then simulated it. Remember however that the model assumes risk neutrality.[34]

[33] By the way, since the tax wage $\tau_h = 1.17$ when $a_l = 0$, there is a kind of Laffer effect: the efficiency of the employment subsidy leads to tax cuts. Recall however that we only focus on steady-state effects.

[34] The same kind of reform of the profile of UBs is analyzed under risk aversion by Fredriksson and Holmlund (2001), Fredriksson and Holmlund (2003), Heer (2003), Van der Linden (2003b) and Coles and Masters (2006).

Van der Linden (2005) studies the affect of an active program that enhances matching effectiveness. In a framework where the marginal values of labor are fixed, his simulation exercise for Belgium leads to mixed conclusions. When taxation is kept fixed, (un)employment deteriorates with γ_l while the low-skilled intertemporal indicators of welfare are improving. This conflict still holds when the financing of the program is taken into account. In addition, since the tax rate of both skill groups has to adjust to finance the active program, the welfare of the high-skilled is negatively affected by a rise in the size of the program. Given these results, we here assume that this active program is abandoned and see how an employment subsidy performs in that case. When $a_l = -300$, net output is larger (2131 Euro versus 2095 with the active program). The tax rate is reduced ($\tau_h = 1.08$ instead of 1.13)). Table 4 indicates that again net wages, employment and utility levels can simultaneously increase if the active program analyzed here disappears when the employment subsidy is introduced.

These simulation exercises have illustrated the existence of reforms to LMPs that at the same time reduce public expenditures and improve the effects of an employment subsidy.

6 Conclusion

This paper has shown that the equilibrium search-matching model can be enriched to become an instrument of evaluation of policies targeted on specific groups. Instead of assuming a juxtaposition of labor markets, we have modelled interactions between them. The marginal value of labor then varies with the number of workers in all sectors. This paper shows that the model remains tractable. Several analytical conclusions can still be derived. For policy evaluations, the model has afterwards been extended to deal with institutional features and various labor market policies.

Using this framework, computational experiments have shown that employment subsidies targeted on low-skilled workers perform well. At least in countries with large tax wedges, they can simultaneously raise employment, wages and intertemporal income of all groups. This conclusion is in accordance with those of Drèze and Malinvaud *et al.* (1994), Phelps (1997) and Mortensen and Pissarides (2003). We have also illustrated that the efficiency of employment subsidies can be reinforced by reforms to active and passive labor market policies. We have developed an extensive sensitivity analysis which suggests that these conclusions are robust.

There are some caveats to add concerning the following limitations of our theoretical framework. First, employment subsidies influence training and schooling decisions made by individuals and firms (Blundell, Costas Dias and Meghir, 2003). Second, employment subsidies affect job destruction rates (Mortensen and Pissarides, 2003).

If they are targeted on low productivity jobs, such subsidies have a clear-cut negative effect on job destruction rates. Third, there is evidence that skilled workers supply labor on less-skilled labor markets (ladder effect) and that this phenomenon reduces the effectiveness of employment subsidies (Sneessens, 2005). These two last features have been combined in the model of Pierrard (2005), who concludes that employment subsidies targeted on low-paid workers have substantial positive effects on employment and on welfare. Fourth, it has been argued that employment subsidies targeted on low-skilled workers lock them in low-paid jobs. A model with on-the-job search and additional skill categories could take such an effect into account. Finally, for several years now, countries such as France, Germany, The Netherlands and Belgium have played a non-cooperative game to maintain their competitiveness. Cuts in payroll taxes are among the instruments used. Our analysis has only been conducted for a single (closed) economy.

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Appendixes

A Proof that $V_{T,n} > V_{U,n} > V_{X,n}$

Let us recall the assumptions: $1 > \rho_{T,n} \geq \rho_{U,n} > \rho_{X,n},\ c_{T,n} > c_n,\ \gamma_n \geq 0, \lambda_n \geq 0, \pi_n > 0$. Let $b_{U,n} = \rho_{U,n} \cdot w_n + z_n$. $b_{X,n}$ and $b_{T,n}$ are defined similarly. We now distinguish the case where participation to the ALMP is chosen freely and the one where there is a treat to be sanctioned in case of refusal.

Case 1: Free participation ($\overline{V}_{U,n} = \max(V_{T,n}, V_{U,n})$)

Let us prove that $V_{U,n} > V_{X,n}$. If $V_{U,n}$ was lower or equal to $V_{X,n}$ and $s_{U,n}$ was optimally chosen by the unemployed, the following inequalities would hold:

$$\begin{aligned} rV_{U,n} =& b_{U,n} - d(s_{U,n}) + c_n s_{U,n}\alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) \\ >& b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n} + V_{X,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) \\ &+ \pi_n(V_{X,n} - V_{U,n}) \\ \geq& b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) \end{aligned} \tag{31}$$

Several possibilities should be considered. Each of them should lead to a contradiction.

Case 1.1. $V_{T,n} \geq V_{X,n} \geq V_{U,n}$

Then, $\overline{V}_{U,n} = \overline{V}_{X,n} = V_{T,n}$. The RHS of (31) is higher than or equal to

$$\begin{aligned} & b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\overline{V}_{U,n} - V_{X,n}) \\ =& b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\overline{V}_{X,n} - V_{X,n}) \\ =& rV_{X,n} \end{aligned} \tag{32}$$

if $s_{X,n}$ is the optimal level of search (this condition is not recalled below). This leads to a contradiction. Therefore, $V_{U,n} > V_{X,n}$.

Case 1.2. $V_{X,n} \geq V_{U,n} \geq V_{T,n}$

Then, $\overline{V}_{U,n} = V_{U,n}$ and $\overline{V}_{X,n} = V_{X,n}$. The RHS of (31) is then equal to

$$b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}),$$

which is again $rV_{X,n}$ because those in the second tier refuse to take part to the program. So, again, there is a contradiction. Therefore, $V_{U,n} > V_{X,n}$.

Case 1.3. $V_{X,n} > V_{T,n} > V_{U,n}$

The RHS of (31) is then equal to

$$\begin{aligned} & b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(V_{T,n} - V_{U,n}) \\ \geq\, & b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) = rV_{X,n} \end{aligned} \tag{33}$$

This leads once more to a contradiction. Therefore, $V_{U,n} > V_{X,n}$.

Knowing that $V_{U,n} > V_{X,n}$, let us now prove that $V_{T,n} > V_{U,n}$. If $V_{U,n}$ was higher than or equal to $V_{T,n}$ and $s_{T,n}$ was optimally chosen by the trainee, the following inequalities would be verified:

$$\begin{aligned} rV_{T,n} &= b_{T,n} - d(s_{T,n}) + c_{T,n}\, s_{T,n}\, \alpha(\theta_n)(V_{E,n} - V_{T,n}) + \lambda_n(V_{U,n} - V_{T,n}) \\ &\geq b_{T,n} - d(s_{U,n}) + c_n\, s_{U,n}\, \alpha(\theta_n)(V_{E,n} - V_{U,n} + V_{U,n} - V_{T,n}) \\ &> b_{U,n} - d(s_{U,n}) + c_n\, s_{U,n}\, \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) \end{aligned} \tag{34}$$

As $V_{T,n} \leq V_{U,n}$, $\overline{V}_{U,n} = V_{U,n}$. So, adding $\gamma_n(\overline{V}_{U,n} - V_{U,n})$ to the RHS of (34) does not modify this expression. The RHS of (34) is then equal to $rV_{U,n}$. This leads to a contradiction. So, $V_{T,n} > V_{U,n}$.

Case 2: Participation under threat of a sanction in the first tier ($\overline{V}_{U,n} = \max(V_{T,n}, V_{X,n})$)

Then, $\overline{V}_{U,n} = \overline{V}_{X,n}$. Let us prove that $V_{U,n} > V_{X,n}$. If $V_{U,n}$ was lower or equal to $V_{X,n}$ and $s_{U,n}$ was optimally chosen by the unemployed, the following inequalities would hold:

$$\begin{aligned} rV_{U,n} =\, & b_{U,n} - d(s_{U,n}) + c_n s_{U,n}\alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) \\ >\, & b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n} + V_{X,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) \\ & + \pi_n(V_{X,n} - V_{U,n}) \\ \geq\, & b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\overline{V}_{U,n} - V_{X,n} + V_{X,n} - V_{U,n}) \\ \geq\, & b_{X,n} - d(s_{X,n}) + c_n\, s_{X,n}\alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\overline{V}_{X,n} - V_{X,n}) = rV_{X,n}, \end{aligned} \tag{35}$$

which leads to a contradiction. So, $V_{U,n} > V_{X,n}$.

To show that $V_{T,n} > V_{U,n}$, let us start from (34). Under the assumption that $V_{T,n}$ is lower than or equal to $V_{U,n}$, two sub-cases should be distinguished: $V_{T,n} \leq V_{X,n} < V_{U,n}$ and $V_{X,n} < V_{T,n} \leq V_{U,n}$.

Case 2.1 $V_{T,n} \leq V_{X,n} < V_{U,n}$

Since $V_{T,n} \leq V_{X,n}$, $\overline{V}_{U,n} - V_{U,n} = V_{X,n} - V_{U,n} < 0$. All the unemployed reject the offers to take part to the program. Therefore, the RHS of (34):

$$\begin{aligned} & b_{U,n} - d(s_{U,n}) + c_n\, s_{U,n}\ \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) \\ \geq\ & b_{U,n} - d(s_{U,n}) + c_n\, s_{U,n}\ \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) \\ =\ & rV_{U,n}. \end{aligned}$$

There is again a contradiction. So, $V_{T,n} > V_{U,n}$.

Case 2.2. $V_{X,n} < V_{T,n} \leq V_{U,n}$

Then $\overline{V}_{U,n} = \max(V_{T,n}; V_{X,n}) = V_{T,n}$. Those who are in the first tier prefer to take part to the program but they lose since $\overline{V}_{U,n} - V_{U,n} = V_{T,n} - V_{U,n} < 0$. Therefore the RHS of (34):

$$\begin{aligned} & b_{U,n} - d(s_{U,n}) + c_n\, s_{U,n}\ \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) \\ \geq\ & b_{U,n} - d(s_{U,n}) + c_n\, s_{U,n}\ \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n}) + \gamma_n(\overline{V}_{U,n} - V_{U,n}) \\ =\ & rV_{U,n}. \end{aligned}$$

There is once more a contradiction. So, $V_{T,n} > V_{U,n}$.

To show that $V_{E,n} > V_{T,n} > V_{U,n} > V_{X,n}$, we still have to prove the first inequality. This will be done in Appendix B.

B Precise specification of various equations

The steady-state relationship defining the employment level E_n is:

$$\begin{aligned} E_n = \mathcal{E}(\theta_n, \mathbb{S}_n) \equiv L_n\, & [[c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n]\,(c_n s_{U,n}\alpha(\theta_n)\,[c_n s_{X,n}\alpha(\theta_n) + \gamma_n] \\ & + \pi_n c_n s_{X,n}\alpha(\theta_n)) + \gamma_n c_{T,n}s_{T,n}\alpha(\theta_n)\,[\pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n]]\Delta_{0,n}^{-1}, \end{aligned} \tag{36}$$

where,

$$\begin{aligned} \Delta_{0,n} \equiv & [c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n]\,([c_n s_{U,n}\alpha(\theta_n) + \phi_n]\,[c_n s_{X,n}\alpha(\theta_n) + \gamma_n] \\ & + \pi_n\,[c_n s_{X,n}\alpha(\theta_n) + \phi_n]) + \gamma_n\,[c_{T,n}s_{T,n}\alpha(\theta_n) + \phi_n]\,[\pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n]\,. \end{aligned} \tag{37}$$

Let

$$\begin{aligned} \Delta_{1,n} \ \equiv\ & (r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\,[[r + c_n s_{U,n}\alpha(\theta_n) + \phi_n][r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n] \\ +\ & \gamma_n[r + c_{T,n}s_{T,n}\alpha(\theta_n) + \phi_n]] + \pi_n[[r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n][r + c_n s_{X,n}\alpha(\theta_n) + \phi_n] \\ +\ & \gamma_n[r + c_{T,n}s_{T,n}\alpha(\theta_n) + \phi_n]], \\ \Delta_{2,n} \ \equiv\ & r + \pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n, \\ \Delta_{3,n} \ \equiv\ & r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n + \gamma_n. \end{aligned} \tag{38}$$

Let $\delta_{ET,n} \equiv w_n - v_{T,n} > 0$ and $\delta_{\iota\iota',n} \equiv v_{\iota,n} - v_{\iota',n}, \iota, \iota' \in \{U, X, T\}, \iota \neq \iota'$. The following differences can be derived from Equations (4), (24), (26) and (25):

$$V_{E,n} - V_{U,n} = [(r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n)\,[(r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\,(w_n - v_{U,n}) \\ +\pi_n(w_n - v_{X,n})] + \gamma_n\,(r + \pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\,w_n - v_{T,n})]\Delta_{1,n}^{-1}, \quad (39)$$

$$V_{U,n} - V_{X,n} = [\delta_{UX,n} + c_n(s_{U,n} - s_{X,n})\alpha(\theta_n)(V_{E,n} - V_{U,n})]\,\Delta_{2,n}^{-1}, \quad (40)$$

$$V_{T,n} - V_{U,n} = [(r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\,(\delta_{TU,n} + (c_{T,n}s_{T,n} - c_n s_{U,n})\alpha(\theta_n))(V_{E,n} - V_{U,n})) \\ +\pi_n\,(\delta_{TX,n} + (c_{T,n}s_{T,n} - c_n s_{X,n})\alpha(\theta_n))(V_{E,n} - V_{U,n}))][\Delta_{2,n}\Delta_{3,n}]^{-1}, \quad (41)$$

$$V_{E,n} - V_{T,n} = [\delta_{ET,n} + (\lambda_n - \phi_n)(V_{T,n} - V_{U,n})]\,\Delta_{4,n}^{-1}. \quad (42)$$

So, if $\phi_n < \lambda_n$, one has $V_{E,n} > V_{T,n}$.

Search-effort levels verify the following (sufficient) conditions:

$$d'(s_{U,n}) = c_n\,\alpha(\theta_n)(V_{E,n} - V_{U,n}), \quad (43)$$

$$d'(s_{X,n}) = c_n\,\alpha(\theta_n)(V_{E,n} - V_{X,n}), \quad (44)$$

$$d'(s_{T,n}) = c_{T,n}\,\alpha(\theta_n)(V_{E,n} - V_{T,n}). \quad (45)$$

Under free-entry, the Nash bargain over wages leads to:

$$V_{E,n} - V_{U,n} = \mathcal{V}(\theta_n) \equiv \frac{\beta_n}{1-\beta_n}\frac{k_n}{q(\theta_n)}\frac{1}{1+\tau_n}, \; \frac{\partial \mathcal{V}}{\partial \theta_n} > 0. \quad (46)$$

Equation (4) can be used to replace $V_{E,n} - V_{U,n}$ by $(w_n - rV_{U,n})/(r + \phi_n)$. So,

$$w_n = rV_{U,n} + (r + \phi_n)\mathcal{V}(\theta_n). \quad (47)$$

Finally, one has to replace $rV_{U,n}$ in the previous equality. This task is more complex because the number of possible positions on the labor market is larger than in Section 2.1. It leads to the following explicit function for the wage:

$$w_n = WS(\theta_n, \mathbb{S}_n) \equiv \frac{\sum_\iota \Omega_{\iota,n}\,(z_n - d(s_{\iota,n}) + c_{\iota,n}s_{\iota,n}\alpha(\theta_n)\mathcal{V}(\theta_n)) + (r + \phi_n)\mathcal{V}(\theta_n)}{1 - \sum_\iota \Omega_{\iota,n}\rho_{\iota,n}}, \quad (48)$$

with $\iota \in \{U, X, T\}$ and

$$\Omega_{U,n} \equiv [r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n]\,[r + \gamma_n + c_n s_{X,n}\alpha(\theta_n)]\,/\Delta_{5,n}$$

$$\Omega_{T,n} \equiv \gamma_n\,[r + \gamma_n + c_n s_{X,n}\alpha(\theta_n) + \pi_n]\,/\Delta_{5,n}$$

$$\Omega_{X,n} \equiv \pi_n\,[r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n]\,/\Delta_{5,n}$$

$$\Delta_{5,n} \equiv [r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n + \gamma_n]\,[r + \gamma_n + c_n s_{X,n}\alpha\,(\theta_n) + \pi_n]$$

and $\Omega_{U,n} + \Omega_{T,n} + \Omega_{X,n} = 1$. As in the model of Section 2.1, it can be shown that this curve is not affected by marginal changes in search effort levels.

In a symmetric equilibrium, Expression (46) can be substituted for $V_{E,n} - V_{U,n}$ in the first-order conditions (43), (44) and (45) in which $V_{U,n} - V_{X,n}$ has first been replaced by (40) and $V_{T,n} - V_{U,n}$ by (41). After some manipulation, this leads for each n to:

$$\Sigma_U(\theta_n, s_{U,n}) \equiv d'(s_{U,n}) - c_n\,\alpha(\theta_n)\mathcal{V}(\theta_n) = 0, \tag{49}$$

$$\Sigma_X(\theta_n, s_{U,n}, s_{X,n}) = 0 \tag{50}$$

with $\Sigma_X \equiv \Delta_{2,n}\, d'(s_{X,n}) - c_n\,\alpha(\theta_n)\left[\delta_{UX,n} + \left(\Delta_{2,n} + c_n\,[s_{U,n} - s_{X,n}]\,\alpha(\theta_n)\right)\,\mathcal{V}(\theta_n)\right],$

$$\Sigma_T(\theta_n, s_{U,n}, s_{X,n}, s_{T,n}) = 0 \tag{51}$$

with $\Sigma_T \equiv \Delta_{2,n}\,\Delta_{3,n}\, d'(s_{T,n}) - c_{T,n}\,\alpha(\theta_n)\,[(\Delta_{2,n}\,\Delta_{3,n} - [r + c_n\,s_{X,n}\,\alpha(\theta_n) + \gamma_n]$
$[c_{T,n}\,s_{T,n} - c_n\,s_{U,n}]\alpha(\theta_n) - \pi_n[c_{T,n}\,s_{T,n} - c_n\,s_{X,n}]\alpha(\theta_n))\,\mathcal{V}(\theta_n)$
$- (r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\;\delta_{TU,n} - \pi_n\,\delta_{TX,n}].$

Totally differentiating equations (49), (50) and (51), it can be checked that $\frac{\partial \Sigma_\iota}{\partial s_{\iota'}} = 0\ \forall \iota, \iota' \in \{\{T,n\},\{X,n\},\{U,n\}\}, \iota \neq \iota'$. Moreover, the levels of search effort of type-n workers increase with tightness θ_n and decrease with the tax rate τ_n.

Expression (44) implies that $s_{X,n}$ increases with the gain $V_{E,n} - V_{X,n} = V_{E,n} - V_{U,n} + V_{U,n} - V_{X,n} = \mathcal{V}(\theta_n) + V_{U,n} - V_{X,n}$. As $V_{T,n} > V_{U,n} > V_{X,n}$, those in state X_n gain more from the ALMP than those in U_n. Therefore, Van der Linden (2005) shows that $V_{U,n} - V_{X,n}$ shrinks with γ_n and so does $s_{X,n}$ (conditional on θ_n). From (45), $s_{T,n}$ increases with the gain $V_{E,n} - V_{T,n} = \mathcal{V}(\theta_n) - (V_{T,n} - V_{U,n})$. Van der Linden (2005) shows that $V_{T,n} - V_{U,n}$ shrinks with γ_n (conditional on θ_n). And so, the direct effect of γ_n on $s_{T,n}$ is positive.

C Comparative static analysis

The equilibrium effect of a marginal change in γ_l can be measured by differentiating the following system where (θ_l, θ_h) are the endogenous variables and γ_l is the parameter of interest here:

$$\begin{array}{l} VS_l(\theta_l, \theta_h \mid \gamma_l) - WS_l(\theta_l \mid \gamma_l) = 0 \\ VS_h(\theta_l, \theta_h \mid \gamma_l) - WS_h(\theta_h) = 0 \end{array} \tag{52}$$

In these equations,

$$VS_n(\theta_n, \theta_m) \equiv \frac{p_n\,y_n - a_n - (r + \phi_n)(k_n/q(\theta_n))}{1 + \tau_n}$$

in which p_n is written as a function of both tightness levels. More precisely, remembering (36) in Appendix B,

$$p_n = \frac{\partial F(\mathcal{E}(\theta_l, \mathbb{S}_l)y_l, \mathcal{E}(\theta_h, \mathbb{S}_h)y_h)}{\partial E_n y_n}$$

in which $\mathbb{S}_n$ is a function of θ_n and, if $n = l$, of γ_l (see (49) to (51)).[35] Moreover, the function $\mathcal{E}(\theta_l, \mathbb{S}_l)$ is influenced by γ_l (see again (36)). In System (52), the WS_n functions are defined by (48).[36] As can be seen from this definition, γ_l influences WS_l conditional on θ_l.

Differentiating with respect to θ_l, θ_h and γ_l leads to:

$$\frac{d\theta_l}{d\gamma_l} = -\frac{det \begin{bmatrix} \frac{\partial VS_l}{\partial \gamma_l} - \frac{\partial WS_l}{\partial \gamma_l} & \frac{\partial VS_l}{\partial \theta_h} \\ \frac{\partial VS_h}{\partial \gamma_l} & \frac{\partial VS_h}{\partial \theta_h} - \frac{\partial WS_h}{\partial \theta_h} \end{bmatrix}}{det \begin{bmatrix} \frac{\partial VS_l}{\partial \theta_l} - \frac{\partial WS_l}{\partial \theta_l} & \frac{\partial VS_l}{\partial \theta_h} \\ \frac{\partial VS_h}{\partial \theta_l} & \frac{\partial VS_h}{\partial \theta_h} - \frac{\partial WS_h}{\partial \theta_h} \end{bmatrix}} < 0. \tag{53}$$

Consider first the matrix at the numerator. We know that $\frac{\partial WS_l}{\partial \gamma_l} > 0$. Moreover,

$$\frac{\partial VS_l}{\partial \gamma_l} = \frac{y_l}{1+\tau_{1,l}} \frac{\partial p_l}{\partial (E_l\, y_l)} \left[\frac{\partial \mathcal{E}(\theta_l, \mathbb{S}_l)}{\partial \gamma_l} + \sum_{\iota \in U,X,T} \frac{\partial \mathcal{E}(\theta_l, \mathbb{S}_l)}{\partial s_{\iota,l}} \frac{\partial s_{\iota,l}}{\partial \gamma_l} \right]. \tag{54}$$

Looking at equation (36) we have that $\frac{\partial E_l}{\partial \gamma_l} > 0$ if $c_{T,l} s_{T,l}$ is "sufficiently larger" than $c_l s_{U,l}$ and $c_l s_{X,l}$.[37] However, the direct effects of γ_l on $s_{U,l}$ is nil, on $s_{X,l}$ is negative and on $s_{T,l}$ is positive (see the end of Appendix B). So, in (54), the sign of the sum

[35] Looking at these equations, it turns out that conditional on tightness, $s_{U,n}$ is not affected by γ_n.

[36] In which again $\mathbb{S}_n$ is a function of θ_n but this does not matter since marginal changes in search effort do not shift the wage-setting curve.

[37]

$$\begin{aligned} \frac{\partial E_n}{\partial \gamma_n} = {} & \frac{\phi_n(c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n)}{\Delta_{0,n}^2} \Big[\pi_n(c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n + \gamma_n)\, c_n(s_{U,n} - s_{X,n})\alpha(\theta_n) \\ & + (\pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n)\Big((c_n s_{X,n}\alpha(\theta_n) + \gamma_n)(c_{T,n}s_{T,n} - c_n s_{U,n})\alpha(\theta_n) \\ & + \pi_n(c_{T,n}s_{T,n} - c_n s_{X,n})\alpha(\theta_n)) \Big) \Big] \\ & \geq 0 \quad \text{if } c_{T,n}s_{T,n} \text{ is sufficiently larger than } c_n s_{U,n} \text{ and } c_n s_{X,n} \\ & < 0 \quad \text{if } c_{T,n}s_{T,n} < c_n s_{U,n} \text{ and } c_{T,n}s_{T,n} < c_n s_{X,n} \end{aligned}$$

between brackets is ambiguous. Therefore, it can be checked that the sign of $d\theta_l/d\gamma_l$ is in general ambiguous, too.

Let us however assume that the expression between brackets in (54) is nonnegative. Then, $\frac{\partial VS_l}{\partial \gamma_l}$ is nonpositive since $\frac{\partial p_l}{\partial E_l} < 0$ and so $\frac{\partial VS_l}{\partial \gamma_l} - \frac{\partial WS_l}{\partial \gamma_l} < 0$. Moreover then, $\frac{\partial VS_h}{\partial \gamma_l}$[38] is nonnegative since $\frac{\partial p_h}{\partial E_l} > 0$. It can easily be checked that $\frac{\partial VS_l}{\partial \theta_h} > 0$ and $\frac{\partial VS_h}{\partial \theta_h} - \frac{\partial WS_h}{\partial \theta_h} < 0$. Since, using again Euler's formula, we can prove that

$$\frac{\partial VS_l}{\partial \gamma_l}\frac{\partial VS_h}{\partial \theta_h} > \frac{\partial VS_h}{\partial \gamma_l}\frac{\partial VS_l}{\partial \theta_h},$$

the numerator in (53) is then positive, too. So does the denominator. Therefore, we conclude that $\frac{d\theta_l}{d\gamma_l}$ is negative if the direct effect of γ_l on E_l (i.e. the expression between brackets in (54)) is nonnegative.

To check the sign of $\frac{d\theta_h}{d\gamma_l}$ we follow the same procedure. The numerator is equal to:

$$\underbrace{\left(\frac{\partial VS_l}{\partial \theta_l} - \frac{\partial WS_l}{\partial \theta_l}\right)}_{-}\underbrace{\frac{\partial VS_h}{\partial \gamma_l}}_{+\text{ or }0} - \underbrace{\frac{\partial VS_h}{\partial \theta_l}}_{+}\underbrace{\left(\frac{\partial VS_l}{\partial \gamma_l} - \frac{\partial WS_l}{\partial \gamma_l}\right)}_{-}. \tag{55}$$

We are not able to sign the determinant at the numerator. So, the marginal effect of γ_l on θ_h is ambiguous.

[38] $\frac{\partial VS_h}{\partial \gamma_l} = \frac{y_h}{1+\tau_{1,h}}\frac{\partial p_h}{\partial E_l}\left[\frac{\partial \mathcal{E}(\theta_l,\mathbb{S}_l)}{\partial \gamma_l} + \sum_{\iota \in U,X,T}\frac{\partial \mathcal{E}(\theta_l,\mathbb{S}_l)}{\partial s_{\iota,l}}\frac{\partial s_{\iota,l}}{\gamma_l}\right].$

Parameters	l	h
ϕ	0.009	0.006
$p \cdot y$ (Euro/month)	3300	4200
k	18211	41442
$\rho_U = b_U/w$	0.55	0.55
$\rho_X = b_X/w$	0.43	0.40
τ	1.00	1.17
γ	0.006	0.02
λ	0.1	0.1
π	0.083	0.083
ψ	7.4	7.4
ξ	1.142	1.139
β	0.56	0.50
c	0.17	0.31
c_T	0.19	0.34
Endogenous var.		
u	0.056	0.031
x	0.139	0.027
t	0.008	0.006
e	0.61	0.75
p	0.54	0.72
s_U $(c\, s_U\, \alpha(\theta))$	0.16 (0.03)	0.39 (0.08)
s_X $(c\, s_X\, \alpha(\theta))$	0.20 (0.04)	0.49 (0.10)
s_T $(c_T\, s_T\, \alpha(\theta))$	0.26 (0.06)	0.55 (0.12)
θ	2.22	0.83
$V/(U+X+T)$	0.09	0.14
w (Euro/month)	1229	1512

Table 1. Calibration: Parameters and levels of endogenous variables in steady state.

		θ	E	w	s_U	s_X	$r\,V_E$	$r\,V_U$	$r\,V_X$
$\sigma = 1.1$	l	+1.95	+4.97	+3.90	+11.43	+11.43	+5.15	+6.10	+6.13
	h	-0.43	-0.66	-1.39	-2.40	-2.44	-1.64	-1.86	-1.87
$\sigma = 2$	l	+1.25	+3.18	+2.53	+7.22	+7.22	+3.33	+3.93	+3.95
	h	-0.28	-0.43	-0.91	-1.58	-1.61	-1.08	-1.22	-1.23

Table 2. Properties when the marginal values of labor are fixed (the M-P assumption) compared to those when they vary (relative differences in %): The case of an employment subsidy $a_l = -300$ Euro/month when the budget of the State (28) is ignored. The elasticity of substitution of the aggregate production function, σ, takes two values: 1.1 and 2.

		e	w	$r\,V_E$	$r\,V_U$	$r\,V_X$
$\pi_n = 1/12$	l	0.647	1337	960	792	788
$\pi_n = 1/3$	l	0.659	1368	981	809	807
$\pi_n = 1/12$	h	0.754	1562	1306	1136	1130
$\pi_n = 1/3$	h	0.762	1617	1354	1178	1176

Table 3. Properties of an employment subsidy $a_l = -300$ Euro/month when the budget of the State (28) is balanced and the expected duration of "high" benefits, $1/\pi_n$, equals 12 or 3 months.

		e	w	$r\,V_E$	$r\,V_U$	$r\,V_X$
With active program	l	0.647	1337	960	792	788
Without	l	0.658	1365	981	810	805
With active program	h	0.754	1562	1306	1136	1130
Without	h	0.761	1606	1344	1169	1163

Table 4. Properties of an employment subsidy $a_l = -300$ Euro/month when the budget of the State (28) is balanced and the active program is either present or abandoned.

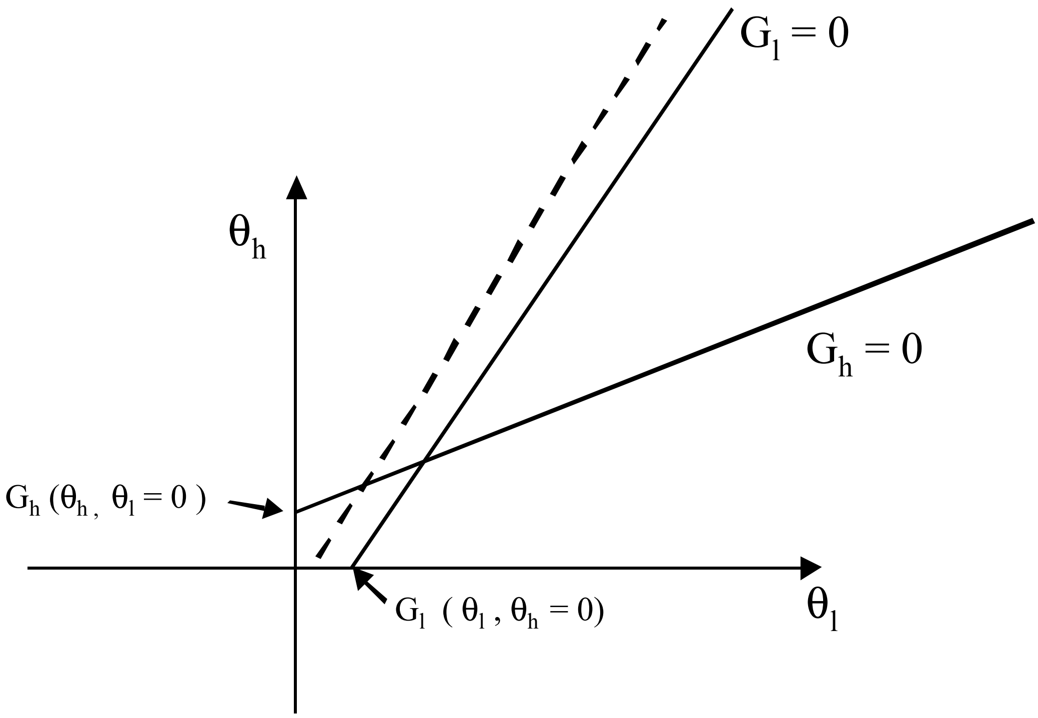

Figure 1: The unique steady-state equilibrium.

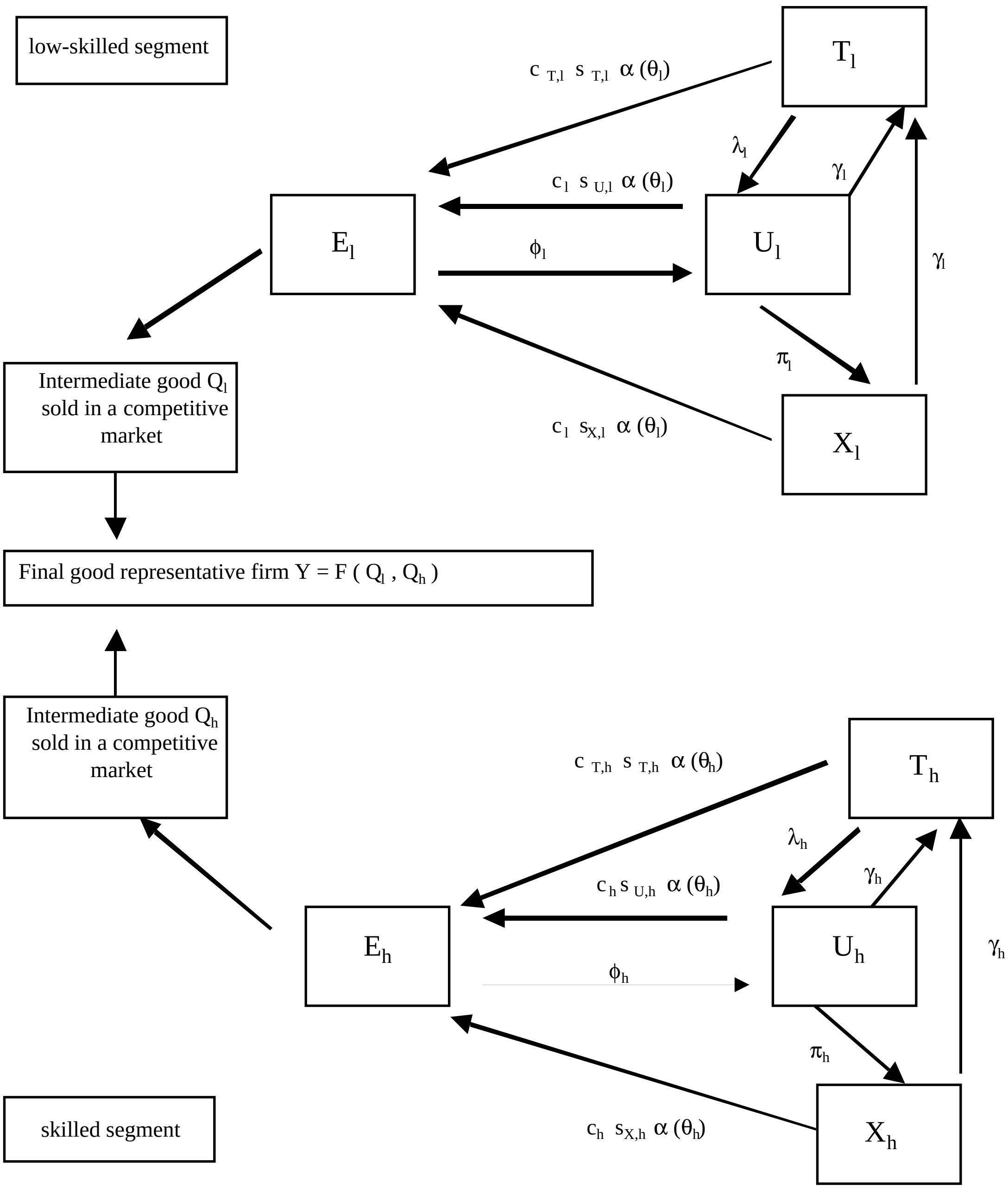

Figure 2: Labor market flows.

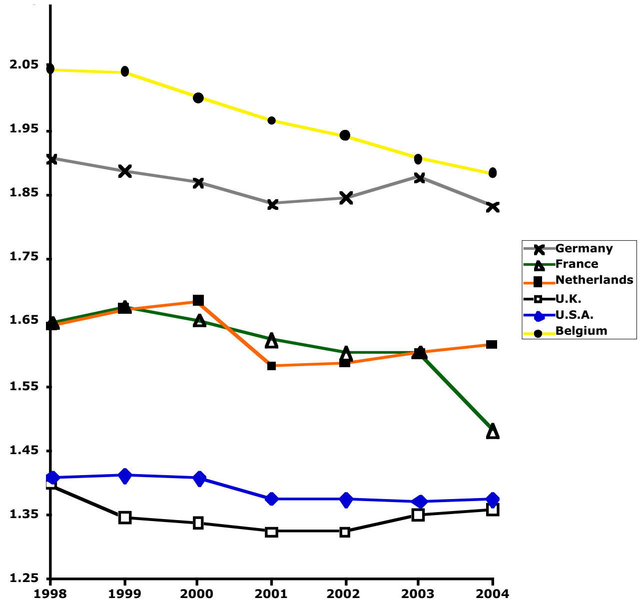

Figure 3: The tax wedge (measured as the ratio between the wage cost and the net wage) on single persons without children at 67% of average earnings. Source: OECD *Taxing wages*.

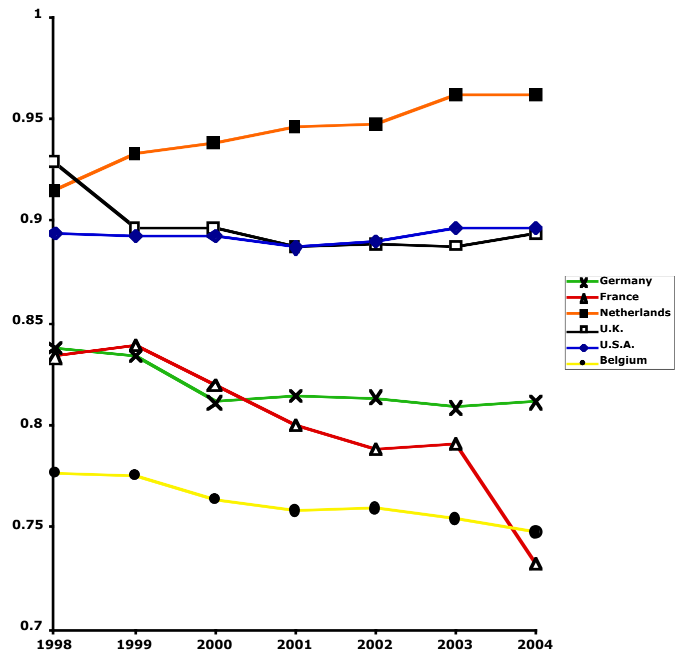

Figure 4: The relative tax wedge (measured as the ratio between the wage cost and the net wage): Single persons without children at 67% of average earnings compared to those at 167%. Source: OECD *Taxing wages*.

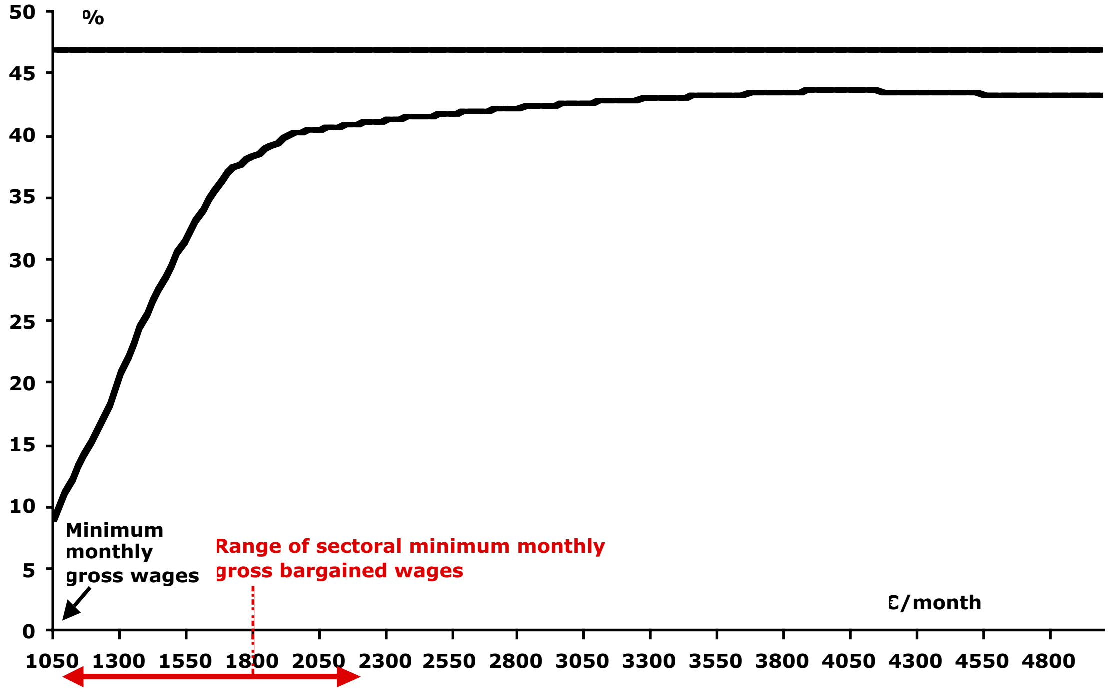

Figure 5: Total rate of social security contributions in % of the gross wage as a function of the monthly gross wages (truncated at 5000 Euros/month): Belgium in 2005, the standard rate (close to 47%) and the rate after the reductions.

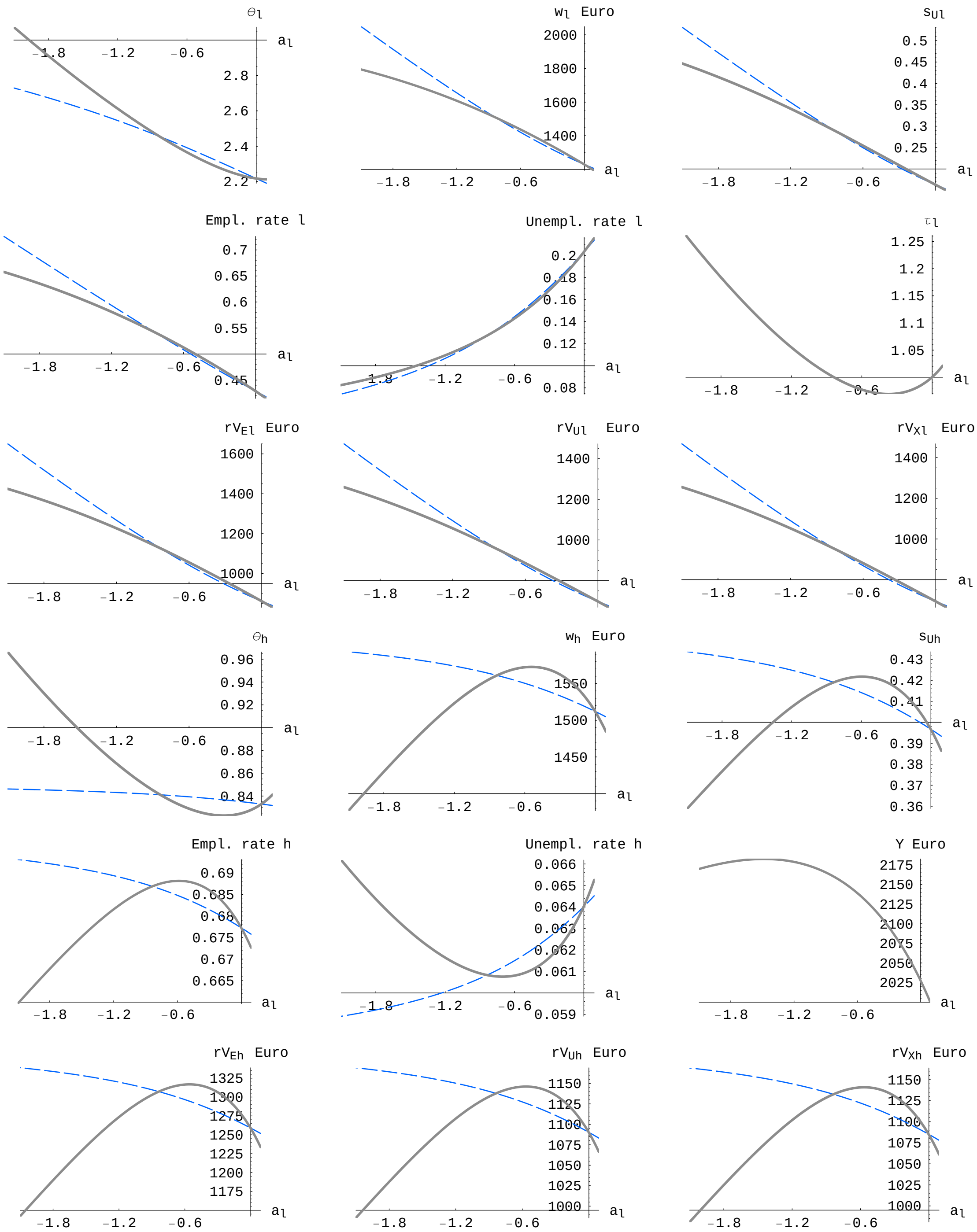

Figure 6: Various indicators as a function of a_l in 1000 Euro. Interrupted (resp., thick) lines: The case where the budget constraint (28) is ignored (resp. binding).