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Precautionary Mismatch

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Precautionary Mismatch*

Abstract

We document that poorer workers are more mismatched with their jobs. We develop a model with two-sided heterogeneity, search frictions, and incomplete markets, in which precautionary motives lead poor workers to exit unemployment faster by accepting a wider range of jobs—*precautionary mismatch*. Calibrating the model to the US economy, we find that mismatch lowers aggregate labor productivity by about 3%, of which precautionary mismatch accounts for roughly one-fifth.

JEL classification

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Keywords

sorting, mismatch, incomplete markets, search and matching

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1 Introduction

How does the labor market allocate heterogeneous workers and jobs? This question is central to understanding wages, productivity, and aggregate output, and has been studied by an influential literature under risk neutrality. Yet transitions between employment and unemployment are among the largest risks households face. Growing evidence shows that wealth, as a key source of self-insurance, shapes risk-averse workers' job search behavior. This paper fills the gap by studying how consumption and savings decisions interact with equilibrium labor market sorting. We derive the distributional implications for wage and wealth inequality as well as aggregate implications for labor productivity.

To this end, we develop and quantify a general equilibrium random search and matching model with two-sided productive heterogeneity and incomplete markets. This framework combines three canonical classes of models: an assignment model à la Becker (1973), a DMP search and matching model (Diamond, 1982; Pissarides, 1985; Mortensen and Pissarides, 1994), and an incomplete markets model in the spirit of Bewley (1986)-İmrohoroğlu (1989)-Huggett (1993)-Aiyagari (1994). It is well known that analyzing this model is difficult.[1] This paper takes a first step toward this challenging research agenda and makes progress both analytically by deriving several theoretical results and quantitatively by building upon the computational insights from Achdou, Han, Lasry, Lions, and Moll (2022).

At the micro level, workers and firms face a trade-off between match quality and the speed of forming a match. This is because in a frictional labor market, waiting for a better match takes time. Crucially, asset holdings shift this trade-off. To see why, consider a worker with little wealth. Because this worker has a high marginal utility of consumption, she is willing to accept a mediocre match rather than remain unemployed to search for a better match. In equilibrium, a relatively low wage is sufficient to induce her to accept the job, making it profitable for the firm to hire her even when match quality is poor. Therefore, lower wealth leads workers and firms to agree on a wider range of acceptable matches. We call this phenomenon *precautionary mismatch*.

Moreover, this micro behavior generates misallocation in the aggregate.[2] A poor worker accepts a job to which she is mismatched, but that same job might have been a good match for someone else. This misallocation reduces aggregate productivity. As a result, wealth inequality

[1] For example, Krusell, Mukoyama, and Şahin (2010) note that "the computation of our model, however, is considerably more complex than for standard incomplete-markets models," even in the absence of two-sided heterogeneity and labor market sorting. More recently, Eeckhout and Sepahsalari (2024) argue that "given the difficulties analyzing sorting in the random search model (Shimer and Smith, 2000), there is little hope to address sorting on assets in random search with risk aversion."

[2] Our notion of "misallocation" differs from that typically used in the macro development and firm dynamics literature, which refers to the quantity misallocation of homogeneous factor inputs (Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008). Here, we emphasize the quality misallocation of heterogeneous inputs.

and labor market allocation interact: the wealth distribution affects aggregate productivity through precautionary mismatch, and labor market outcomes in turn feed back into wealth accumulation.

The model generates three testable predictions. First, precautionary mismatch implies that poorer workers are more mismatched with their jobs. Second, poor workers have higher job-finding rates because they are less selective in their job choices. Third, conditional on productivity, wealthier workers earn higher wages not only because of better outside options but also because of smaller mismatch. Using data from the NLSY79 and O*NET, we find empirical support for all three predictions.

To take the model to the data, we calibrate it to the US labor market and validate it against the empirical relationship between asset holdings, job-finding rates, and mismatch. Using the calibrated model, we find that precautionary mismatch generates substantial earnings and productivity inequality between wealth-poor and wealth-rich workers of the same productivity type. In particular, among the highest-skilled group, workers in the bottom wealth percentile exhibit about 30% lower earnings and 40% lower output per worker than those in the top wealth percentile. This suggests that efficient reallocation of mismatched workers could yield meaningful productivity gains. Indeed, we find that aggregate labor productivity would be about 3% higher if workers and jobs were optimally reallocated. This provides an estimate of the productivity loss due to mismatch from all sources. To isolate the role of precautionary mismatch, we conduct a counterfactual that shuts down this channel. We find that precautionary mismatch accounts for roughly 20% of the total productivity loss.

We then extend the model to allow for on-the-job search. Quantitatively, this extension has only a modest effect on the productivity implications. The reason is that on-the-job search affects mismatch through two countervailing forces. First, workers can move up the job ladder over time, gradually improving match quality. Second, anticipating future outside offers, workers initially accept wider ranges of jobs, increasing mismatch in transitions out of unemployment. The net effect is therefore a race between the ex post reallocation effect and the ex ante anticipation effect. As on-the-job search intensity rises, the reallocation effect increasingly dominates the steady state. This is because when outside offers arrive frequently, workers can correct early mismatches quickly and initial acceptance decisions become less consequential. At the empirically plausible level of on-the-job search intensity, however, the two forces largely offset each other, and the aggregate productivity implications remain close to the baseline model.

Related Literature

Theoretically, this paper analyzes frictional sorting under incomplete markets, building on two foundational frameworks: the random-search sorting model of Shimer and Smith (2000) and the incomplete-markets search model of Krusell, Mukoyama, and Şahin (2010). In doing so, we connect two strands of literature.

The first strand studies frictional sorting in labor markets, including Shimer and Smith (2000); Gautier and Teulings (2006, 2015); Eeckhout and Kircher (2011); Lise, Meghir, and Robin (2016); Lise and Robin (2017); Hagedorn, Law, and Manovskii (2017); Bagger and Lentz (2019); Lise and Postel-Vinay (2020). We contribute to this literature by introducing self-insurance and precautionary motives into a frictional sorting environment. This delivers new implications for how wealth shapes equilibrium sorting and mismatch. Particularly related is Marimon and Zilibotti (1999), who show that more generous unemployment insurance can reduce mismatch by allowing workers to wait for more suitable jobs. Our mechanism is complementary but operates through endogenous self-insurance: wealthier workers can afford to be more selective, whereas poorer workers accept a wider set of matches.

The second strand studies labor search under incomplete markets. Early contributions emphasize how self-insurance affects search effort (Lentz and Tranæs, 2005; Chetty, 2008), while Lise (2013); Chaumont and Shi (2022) study its interaction with on-the-job search. Krusell, Mukoyama, and Şahin (2010); Krusell, Luo, and Ríos-Rull (2022) embed incomplete-markets search into general equilibrium. Acemoglu and Shimer (1999); Eeckhout and Sepahsalari (2024) allow for firm-side productivity heterogeneity, while Griffy (2021) introduces worker heterogeneity in human capital. We contribute to this literature by modeling two-sided productive heterogeneity, which allows us to study how wealth affects worker–job sorting and, consequently, allocative efficiency in the labor market. The closest paper to ours is Herkenhoff, Phillips, and Cohen-Cole (2024), which also features risk-averse workers and two-sided heterogeneity, but adopts directed search and an exogenous wage rule.

Empirically, our paper relates to a large literature documenting relationships between asset holdings and job search behavior, including Algan et al. (2003); Rendon (2006); Card, Chetty, and Weber (2007); Basten, Fagereng, and Telle (2014) and, more recently Herkenhoff (2019); Fontaine, Jensen, and Vejlin (2024); Clymo, Denderski, and Harvey (2022), among others. These papers study how households' ability to smooth consumption affects unemployment duration, reemployment wages, and job tenure. To our knowledge, we are the first to document the effect of workers' asset holdings on a direct measure of worker–job mismatch.

Computationally, we extend the continuous-time method developed by Achdou et al. (2022) for incomplete-markets models to an environment with frictional sorting and endogenous wage

determination. We derive a tractable wage expression in terms of current value and policy functions, thereby avoiding the additional guess-and-update procedure for the wage schedule required in Krusell, Mukoyama, and Şahin (2010).

The rest of the paper is organized as follows. Section 2 presents the model and theoretical results on the links between wealth, job search behavior, and labor market outcomes. Section 3 describes the data, the construction of the mismatch measure, and empirical evidence on the relationship between liquid wealth, mismatch, and wages. Section 4 presents the calibration and model validations, and quantifies the productivity implications of precautionary mismatch. Section 5 studies the model with on-the-job search. Section 6 concludes.

2 Model

This section develops and characterizes a model that combines three key elements. First, workers and firms are ex-ante heterogeneous in productive types, allowing us to study sorting. Second, the labor market is frictional with random meetings, so it takes time for workers and jobs to form good matches. Third, workers are risk averse and face incomplete financial markets, so they accumulate precautionary savings to self-insure against unemployment risk.

2.1 Environment

Time is continuous.

Demography. The demographic structure follows a perpetual-youth economy. Workers exit the economy at Poisson rate δ and are replaced one-for-one by newborns who enter unemployment with initial assets drawn from a distribution $h(a)$.

Preferences. Workers are risk averse, with increasing and concave flow utility $\mathfrak{u}(c)$, and maximize expected lifetime utility discounted at rate ρ. Firms are risk neutral and maximize the present value of expected profits discounted at the risk-free rate r.

Production. Upon entry, workers draw a productivity type $x \in \mathbb{X}$ from density $d_w(x)$ and jobs draw a productivity type $y \in \mathbb{Y}$ from density $d_j(y)$. A match between a worker of type x and a job of type y produces flow output $f(x, y)$. Matches are destroyed exogenously at Poisson rate σ. In addition, at Poisson rate $\tilde{\sigma}$ the match draws an opportunity to separate endogenously.[3] An unemployed worker of type x produces flow payoff $b(x)$, capturing unemployment benefits and home production.

[3] In discrete-time models with endogenous separation, the opportunity to separate is implicitly available once each period, which corresponds to setting $\tilde{\sigma} = 1$.

Search and matching. The labor market is frictional: unemployed workers and vacancies meet randomly according to a meeting function $M(u, v)$, where u is the measure of unemployed workers and v the measure of vacancies.[4] Assuming constant returns to scale (CRS), the meeting rate for an unemployed worker is $p(\theta) \coloneqq M(u, v)/u = M(1, \theta)$ and the meeting rate for a vacancy is $q(\theta) \coloneqq M(u, v)/v = M(\theta^{-1}, 1) = p(\theta)/\theta$, where $\theta := v/u$ denotes labor-market tightness. Wages are determined by Nash bargaining, with worker bargaining power η.

Incomplete markets. Households cannot trade a complete set of Arrow securities and instead have access only to a risk-free bond, subject to a borrowing constraint $a \geq \underline{a}$. An annuity market redistributes the wealth of exiting workers to the living: the insurer collects the assets of those who exit and pays survivors an annuity flow δa, proportional to their asset holdings. As a result, the effective return on assets is $r + \delta$.

2.2 Characterization

2.2.1 Distribution

Before characterizing the value functions, it is useful to introduce the distributions that describe the economy. For notational convenience, we use the first letter of each object: matches m, employed workers e, unemployed workers u, producing jobs p, and vacant jobs v. For example, the density of matches is $d_m(a, x, y) : \mathbb{R} \times \mathbb{X} \times \mathbb{Y} \to \mathbb{R}^+$. The associated marginal densities are defined in the natural way. The density of employed workers is $d_e(a, x) = \int d_m(a, x, y)\,\mathrm{d}y$ and the density of producing jobs is $d_p(y) = \iint d_m(a, x, y)\,\mathrm{d}a\,\mathrm{d}x$. The density of unemployed workers is $d_u(a, x)$ and the density of vacant jobs is $d_v(y)$. The aggregate unemployment and vacancy rates are denoted by $u = \iint d_u(a, x)\,\mathrm{d}a\,\mathrm{d}x$ and $v = \int d_v(y)\,\mathrm{d}y$, respectively. Table 1 summarizes these accounting identities.

2.2.2 Hamilton-Jacobi-Bellman Equations

The consumption, saving, matching, and separation decisions of workers and firms are characterized by a system of Hamilton–Jacobi–Bellman (HJB) equations, which provide the first set of partial differential equations characterizing the equilibrium. We relegate derivations to Appendix I.2.1.

Worker values. Employed workers choose consumption and saving and, when given the opportunity, whether to separate from their current job. As a worker accumulates assets, a match

[4] We distinguish *meetings* from successful *matches*. After a meeting, the worker and the job decide whether to form a match. Some meetings do not result in production because one or both parties prefer to continue searching. The acceptance decision is a central equilibrium object.

Table 1: Distribution Add-up Properties

Description	Add-up Property
Workers	$d_w(x) = \int d_u(a,x)\,\mathrm{d}a + \iint d_m(a,x,y)\,\mathrm{d}y\,\mathrm{d}a$
Unemployment	$u = \iint d_u(a,x)\,\mathrm{d}x\,\mathrm{d}a$
Jobs	$d_j(y) = d_v(y) + \iint d_m(a,x,y)\,\mathrm{d}x\,\mathrm{d}a$
Vacancies	$v = \int d_v(y)\,\mathrm{d}y$

Notes: The table summarizes the add-up properties relating $d_u(a,x)$, $d_v(y)$, u, v, and $d_m(a,x,y)$.

that was previously acceptable may become unacceptable, inducing endogenous separation. Let $U(a,x)$ denote the value of an unemployed worker of type x with assets a, and let $W(a,x,y)$ denote the value of an employed worker of type x with assets a matched to a firm of type y. The HJB equation for an employed worker is

$$\begin{aligned}(\rho+\delta)W(a,x,y) = &\max_c \mathfrak{u}(c) + \sigma\left[U(a,x) - W(a,x,y)\right] + \tilde{\sigma}\left[U(a,x) - W(a,x,y)\right]^+ \\ &+ \dot{a}W_a(a,x,y) \\ \text{s.t.}\quad &\dot{a} = (r+\delta)a + \omega(a,x,y) - c \\ &a \geq \underline{a}\end{aligned} \tag{1}$$

where $[\bullet]^+ \coloneqq \max\{\bullet, 0\}$. An employed worker receives flow interest and annuity $(r+\delta)a$ and wage $\omega(a,x,y)$. Matches separate exogenously at Poisson rate σ and, at rate $\tilde{\sigma}$, receive an opportunity for endogenous dissolution. The term $\dot{a}W_a$ captures the change in value due to asset accumulation. The optimal consumption policy satisfies the first-order condition

$$\mathfrak{u}'(c^e) = W_a(a,x,y). \tag{2}$$

Unemployed workers choose consumption and saving and decide whether to accept job offers. The HJB equation for an unemployed worker is

$$\begin{aligned}(\rho+\delta)U(a,x) = &\max_c \mathfrak{u}(c) + p(\theta)\int \frac{d_v(y)}{v}\left[W(a,x,y) - U(a,x)\right]^+ \mathrm{d}y + \dot{a}U_a(a,x) \\ \text{s.t. }\ &\dot{a} = (r+\delta)a + b(x) - c \\ &a \geq \underline{a}\end{aligned} \tag{3}$$

An unemployed worker produces $b(x)$ at home and receives flow interest and annuity $(r+\delta)a$. At

Poisson rate $p(\theta)$, she meets a vacant job drawn from the vacancy distribution. Upon meeting, she decides whether to accept the offer by comparing the value of employment, $W(a,x,y)$, to the value of remaining unemployed, $U(a,x)$. Note that the matching set in the unemployed workers' HJB equation is the complement of the separation set in the employed workers' HJB equation. The optimal consumption policy satisfies the first-order condition

$$\mathfrak{u}'(c^u) = U_a(a,x). \tag{4}$$

Firm values. Let $V(y)$ denote the value of a vacant job of type y, and let $J(a,x,y)$ denote the value of a producing job of type y matched with a worker of type x who holds assets a. The HJB equation for a producing job is

$$\begin{aligned} rJ(a,x,y) = f(x,y) - \omega(a,x,y) + (\sigma+\delta)[V(y) - J(a,x,y)] + \tilde{\sigma}[V(y) - J(a,x,y)]^+ \\ + \dot{a}^e(a,x,y) J_a(a,x,y), \end{aligned} \tag{5}$$

where $\dot{a}^e(a,x,y) \coloneqq (r+\delta)a + \omega(a,x,y) - c^e(a,x,y)$ is the worker's optimal asset-accumulation policy while employed. The firm receives flow output net of wage paid to the worker. The job becomes vacant following either an exogenous separation shock or an endogenous separation decision.[5] Because wages and separation decisions depend on the worker's asset position, the job's continuation value depends on a. Firms take the worker's saving policy as given.

The value of a vacant job is

$$rV(y) = q(\theta) \iint \frac{d_u(a,x)}{u} [J(a,x,y) - V(y)]^+ \,\mathrm{d}a\,\mathrm{d}x. \tag{6}$$

The vacancy meets a searcher at rate $q(\theta)$. Conditional on a meeting, the worker's state (a,x) is drawn from the unemployment distribution.

The total mass of jobs is determined at an ex-ante entry stage that precedes the search economy. Potential jobs can be created by paying a sunk entry cost κ. Upon entry, each job draws its type y from the exogenous distribution G and enters the labor market as a vacancy. Free entry requires

$$\kappa = \int V(y)\,\mathrm{d}G(y), \tag{7}$$

which pins down the equilibrium mass of jobs. After entry, the total stock of jobs remains fixed, although individual jobs may transition between vacant and producing states. Thus, conditional on the mass of jobs determined at entry, the subsequent search economy is equivalent to one

[5] See Qiu (2022) for evidence that worker separations can leave positions vacant, leading employers to repost them in search of replacement workers.

with an exogenously fixed stock of jobs.

2.2.3 Wage Determination

As is standard in the search-and-matching literature, we assume wages are determined by Nash bargaining. With linear utility, the total match surplus is independent of the wage, and Nash bargaining yields the familiar surplus-sharing rule. With risk-averse workers, this is no longer the case. As worker values are measured in utils and firm values are measured in dollars, the surplus is no longer invariant to the wage. We show that, in continuous time, the bargaining problem nevertheless admits a simple generalized surplus-sharing representation.

Proposition 1. *Nash bargaining implies the following generalized surplus-sharing rule:*

$$\eta \frac{J(a,x,y) - V(y)}{1 - J_a(a,x,y)} = (1-\eta) \frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)}, \tag{8}$$

where $\eta \in (0,1)$ *denotes worker bargaining power.*

Appendix I.2.2 provides the formal derivation; here we give intuition. The match surplus is defined as the sum of the worker's surplus and the job's surplus:

$$S(a,x,y) \coloneqq W(a,x,y) - U(a,x) + J(a,x,y) - V(y).$$

With concave utility, worker and firm values are measured different units, so a meaningful surplus-sharing rule requires converting the two surpluses into a common metric. The relevant conversion factors turn out to be W_a and $1 - J_a$, which motivates defining the adjusted match surplus as

$$\hat{S}(a,x,y) \coloneqq \frac{1}{W_a(a,x,y)} [W(a,x,y) - U(a,x)] + \frac{1}{1 - J_a(a,x,y)} [J(a,x,y) - V(y)]. \tag{9}$$

Equation (8) states that Nash bargaining splits this adjusted surplus according to the bargaining weight η. In particular, the worker's and firm's surpluses have the same sign, so the two parties agree on whether a match should form or separate.

Why are W_a and $1 - J_a$ the appropriate conversion factors? The role of W_a is immediate: it is the marginal value of one dollar to the worker, converting utils into dollars. To see why $1 - J_a$ is the appropriate adjustment for the firm, consider transferring one additional dollar from the worker to the firm. The firm gains one dollar directly, but the worker's lower asset position also affects the firm's value indirectly through wages and separation decisions, summarized by $-J_a$. Hence the firm's total marginal gain from a one-dollar transfer is $1 - J_a$.

Second, Appendix I.2.2 solves the Nash bargaining problem using value functions under a deviating wage, denoted by tildes (e.g., $\tilde{W}(w,a,x,y)$ and $\tilde{J}(w,a,x,y)$). We show that

$$\frac{-\tilde{J}_w(w,a,x,y)}{\tilde{W}_w(w,a,x,y)} = \frac{1-J_a(a,x,y)}{W_a(a,x,y)},$$

which implies that the adjusted surplus can equivalently be written as

$$\hat{S}(a,x,y) \coloneqq \frac{1}{\tilde{W}_w}\left[W(a,x,y)-U(a,x)\right] + \frac{1}{-\tilde{J}_w}\left[J(a,x,y)-V(y)\right].$$

This representation provides additional intuition. The term $\tilde{W}_w$ captures the marginal value to the worker of a one-dollar increase in the wage, and $-\tilde{J}_w$ captures the marginal value to the firm of a one-dollar decrease in the wage (i.e., paying one dollar less). These two derivatives therefore convert each side's surplus into a common metric, and Nash bargaining splits the resulting adjusted surplus between the worker and the firm.

We derive a wage equation that yields substantial computational tractability. Appendix I.2.2 derives an expression for wages:

$$\begin{aligned}\omega(a,x,y) =& \eta\frac{f(x,y)+\left((r+\delta)a-\tilde{c}^e\right)J_a(a,x,y)+(\rho-r)J(a,x,y)-\rho V(y)}{1-J_a(a,x,y)} \\ &-(1-\eta)\frac{\mathfrak{u}(\tilde{c}^e)+\left((r+\delta)a-\tilde{c}^e\right)W_a(a,x,y)-(\rho+\delta)U(a,x)}{W_a(a,x,y)}. \end{aligned} \tag{10}$$

Krusell, Mukoyama, and Şahin (2010) work in discrete time and emphasize that Nash bargaining with risk-averse workers can be computationally burdensome, even without two-sided heterogeneity. The difficulty is that one has to guess a wage schedule and iteratively update it, effectively adding an extra fixed-point loop on top of the value-function iteration.[6] Our continuous-time formulation circumvents this problem, thanks to the above wage equation that expresses the wage function in terms of contemporaneous value and policy functions. Hence, given a current guess or update of the value and policy functions, wages can be evaluated directly, without the need to solve the Nash bargaining problem state by state through a separate guess-and-update routine. By contrast, the discrete-time analogue generally involves continuation values and future objects, so a similar expression is not available.

Finally, notice that as the curvature of the utility function goes to 0, i.e., as utility becomes linear, $W_a \to 1$ and $J_a \to 0$. In this limit, the adjusted surplus reduces to the standard surplus, the generalized surplus-sharing rule collapses to the familiar surplus-sharing rule, and the wage

[6] Krusell, Mukoyama, and Şahin (2010) point out that their model "cannot be computed with an algorithm that reduces to a one-dimensional fixed-point problem; instead one needs to solve a functional fixed-point problem. The reason is that the consumer needs to know not just two prices but the entire wage function, $\omega(a)$, to maximize utility."

equation converges to a similar form in, e.g., Lise and Postel-Vinay (2020).

Discussion of the Nash bargaining assumption. We assume that wages are determined by Nash bargaining, following a large literature in the DMP tradition, including Krusell, Mukoyama, and Şahin (2010) and Shimer and Smith (2000), both of which provide foundational benchmark frameworks nested within our model. Adopting Nash bargaining for wage setting therefore facilitates comparisons with the existing literature and helps isolate the additional implications of combining frictional sorting with incomplete markets.

A potential concern is that Nash bargaining requires employers to observe workers' wealth, raising questions about how our results would change under asymmetric information. The bargaining literature provides microfoundations for Nash bargaining in environments with heterogeneity and incomplete information, where agents use strategic delay between offers and counteroffers to signal their types (Admati and Perry, 1987; Gul and Sonnenschein, 1988; Cramton, 1992). Intuitively, a high-wealth worker would decline the initial offer and delay counteroffers to credibly signal her type. In the limit as the time between offers becomes infinitesimal, the outcome converges to the Nash solution. Consequently, even if we model workers' wealth as privately observed, equilibrium wages would still vary with wealth through the outside-option channel captured by Nash bargaining. We therefore view the perfect-information assumption as a simplifying device that is unlikely to materially affect our findings.

2.2.4 Kolmogorov Forward Equations

We focus on a stationary equilibrium in which all distributions are time-invariant. The steady-state distribution is characterized by a system of Kolmogorov forward (KF) equations. Let $\Phi(a, x, y)$ denote the acceptance rule, where $\Phi(a, x, y) = 1$ if a meeting between a worker with assets a and type x and a job of type y results in production, and $\Phi(a, x, y) = 0$ otherwise. Accordingly, $1 - \Phi(a, x, y)$ captures the endogenous separation decision. The first set of KF equations describes the inflow-outflow balance for employed workers:

$$\begin{aligned} 0 = &-\frac{\partial}{\partial a}\left[\dot{a}^e(a, x, y)\, d_m(a, x, y)\right] - \{\sigma + \delta + \tilde{\sigma}\left[1 - \Phi(a, x, y)\right]\} d_m(a, x, y) \\ &+ d_u(a, x)\, p(\theta)\, \frac{d_v(y)}{v} \Phi(a, x, y), \quad \forall\, a, x, y. \end{aligned} \tag{11}$$

The first term captures drift in assets among employed workers, and the second term captures outflows from employment state (a, x, y) due to exit and separation (both exogenous and endogenous). The final term represents inflows from unemployment.

The second set of KF equations describes the inflow-outflow balance for unemployed workers:

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}^u(a,x)\, d_u(a,x)\right] - p(\theta)\int \frac{d_v(y)}{v}\Phi(a,x,y)\, d_u(a,x)\,\mathrm{d}y - \delta d_u(a,x) + \sigma\int d_m(a,x,y)\,\mathrm{d}y + \tilde{\sigma}\int\left[1-\Phi(a,x,y)\right] d_m(a,x,y)\,\mathrm{d}y + \delta d_w(x)h(a), \quad \forall\ a,x. \tag{12}$$

The first three terms capture outflows from the unemployment state (a,x) due to asset drift, job finding, and exit, respectively. The remaining terms capture inflows into unemployment due to exogenous separations, endogenous separations, and newborn entrants.

Lastly, the density functions satisfy add-up properties:

$$1 = \iiint d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y + \iint d_u(a,x)\,\mathrm{d}a\,\mathrm{d}x. \tag{13}$$

Moreover, the marginal distributions of worker and job types must coincide with the exogenous type densities:

$$d_w(x) = \iint d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}y + \int d_u(a,x)\,\mathrm{d}a, \tag{14}$$

$$d_j(y) = \iint d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x + d_v(y). \tag{15}$$

That is, for each worker type x, the mass of employed and unemployed workers sums to $d_w(x)$, and for each job type y, the mass of producing and vacant jobs sums to $d_j(y)$.

2.3 Equilibrium

We study a stationary sorting equilibrium. We assume an open asset market, so the economy takes the interest rate as given.[7]

2.3.1 Formal Equilibrium Definition

A stationary equilibrium consists of (i) value functions $\{W(a,x,y), U(a,x), J(a,x,y), V(y)\}$ for employed workers, unemployed workers, producing jobs, and vacant jobs, respectively; (ii) policy functions for consumption $\{c^e(a,x,y), c^u(a,x)\}$ and matching acceptance $\Phi(a,x,y)$; (iii) a wage function $\omega(a,x,y)$; (iv) invariant distributions of matches $d_m(a,x,y)$ and unemployed workers $d_u(a,x)$; and (v) labor market tightness θ such that:

[7]Because our focus is on sorting between workers and firms, we impose this assumption for simplicity. In the quantitative exercise, we map model assets to liquid wealth in the data. To the extent that liquid wealth constitutes only a small share of aggregate assets, treating the interest rate as exogenous is a reasonable approximation.

1. The value functions and policy functions solve the worker and firm optimization problems, as characterized by the HJB equations (1, 3, 5, 6) and FOCs (2,4);
2. The matching acceptance decision is $\Phi(a,x,y) \coloneqq \mathbb{1}\left\{\hat{S}(a,x,y) \geq 0\right\}$;
3. The Nash bargained wage satisfies the generalized surplus sharing rule (8);
4. The stationary distributions satisfy the KF equations (11–12) and add-up conditions (13–15);
5. Market tightness adjusts so that the free entry condition (7) holds.

2.3.2 Model Outputs

The model delivers a joint characterization of employment, wages, and the wealth distribution through equilibrium objects.

Labor market flows. The aggregate separation rate is

$$\pi_{eu} = \sigma + \tilde{\sigma}\frac{\iiint [1-\Phi(a,x,y)]\, d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y}{\iiint d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y},$$

where the first term is the exogenous separation rate and the second term is the endogenous separation rate. The aggregate job-finding rate is the average job-finding rate across unemployed workers:

$$\pi_{ue} = \iint \frac{d_u(a,x)}{u}\pi_{ue}(a,x)\,\mathrm{d}a\,\mathrm{d}x,$$

where the job-finding rate of an unemployed worker with assets a and type x is

$$\pi_{ue}(a,x) = p(\theta)\int \frac{d_v(y)}{v}\Phi(a,x,y)\,\mathrm{d}y.$$

Joint distribution of wages and wealth. The model also pins down the joint distribution of wages and wealth. For employed workers, the joint density of wages and wealth is

$$f_{a,w}(a,w) = \frac{1}{e}\iint d_m(a,x,y)\,\delta_D(w-\omega(a,x,y))\,\mathrm{d}x\,\mathrm{d}y.$$

where e is the measure of employed workers and δ_D is the Dirac delta.

Aggregate output and productivity. Aggregate output and measure of employment are

$$Y = \iiint f(x,y)\, d_m(a,x,y)\, \mathrm{d}a\, \mathrm{d}x\, \mathrm{d}y,$$
$$e = \iiint d_m(a,x,y)\, \mathrm{d}a\, \mathrm{d}x\, \mathrm{d}y,$$

respectively, so output per worker (or labor productivity) is $\overline{Y} = Y/e$. These expressions make clear that aggregate productivity depends on $d_m(a,x,y)$, the equilibrium sorting between workers and jobs. We use these statistics to compare labor productivity in counterfactual economies.

2.3.3 Limiting Economies

The model nests several prominent benchmark theories and provides a unified framework that combines two-sided heterogeneity and sorting (Becker, 1973), search-and-matching frictions (Diamond, 1982; Mortensen and Pissarides, 1994), and incomplete markets (Bewley, 1986; Huggett, 1993; Aiyagari, 1994). Two limiting cases are worth highlighting. First, as the curvature of the flow utility vanishes, workers become risk neutral. In this limit, the consumption-savings margin no longer affects match formation and separation decisions, and the equilibrium collapses to a frictional sorting model with linear utility, as in Shimer and Smith (2000). Second, if productive heterogeneity on both worker and firm sides is shut down, there is no scope for sorting. The economy then reduces to an incomplete-markets model with search-and-matching frictions, as studied by Krusell, Mukoyama, and Şahin (2010).

2.4 Theoretical Results

This section develops the model's key theoretical implications for the interaction between wealth, job search, and wages. In particular, we characterize how wealth affects labor market sorting and output, and how the model generates an endogenous joint distribution of wealth and wages. For these theoretical results, we additionally assume decreasing absolute risk aversion (DARA), a standard condition in search-and-savings models that is satisfied by the CRRA preferences used in our quantitative analysis.

Proposition 2 (**Precautionary Mismatch**). *Define the asset-specific matching set* $\mathbb{M}(a) := \{(x,y) : \Phi(a,x,y) = 1\}$. *Then, for* $a < a'$, *we have* $\mathbb{M}(a) \supseteq \mathbb{M}(a')$. *The inclusion is strict if some worker–job pair has an interior acceptance threshold in* (a, a').

Proof. See Appendix I.2.3. The proof builds on the cross-state consumption ordering of Rendon (2025): for acceptable matches, consumption is higher in employment than in unemployment,

and the marginal value of wealth is higher in unemployment than in employment. This yields a single-crossing property of match surplus with respect to wealth. □

Proposition 2 states that the matching set is wider for workers with lower assets. Importantly, this result does not depend on any particular form of productive heterogeneity or sorting pattern. Figure 1 demonstrates this generality using two conceptually distinct production structures. In Panel (1a), heterogeneity is vertical: workers and firms can both be fully ranked, like rungs on a ladder, with productivity increasing in the worker and firm types, and complementarity between the two makes higher-type workers especially productive at higher-type jobs. In Panel (1b), heterogeneity is horizontal: worker and job types are locations on a unit circle, and match output depends only on the distance between them. No worker or job is intrinsically more productive than another; productivity depends solely on match quality, and every worker attains the same maximal productivity when matched with the closest job type. Appendix I.1 describes the two production functions in detail.

Figure 1 shows the matching sets (highlighted in yellow) under these two production structures. In both panels, matches cluster around the 45-degree line, with workers on the horizontal axis and firms on the vertical axis. A frictionless competitive benchmark would deliver perfect matches along this line ($x = y$): under vertical heterogeneity, this corresponds to positive assortative matching (PAM), with high-type workers matched to high-type firms; under horizontal heterogeneity, it corresponds to pairing each worker with the closest job type. With search frictions, such perfect matches are costly to find and may take time to materialize, so both workers and firms are willing to accept a neighborhood of imperfect matches around the ideal one. Within each panel, the left figure shows the matching set for workers at the lowest wealth level in the model, while the right figure shows the matching set for workers with assets equal to five times average annual earnings. In both cases, workers with lower wealth have wider matching sets, regardless of whether heterogeneity is vertical or horizontal. We refer to this mechanism as "precautionary mismatch."

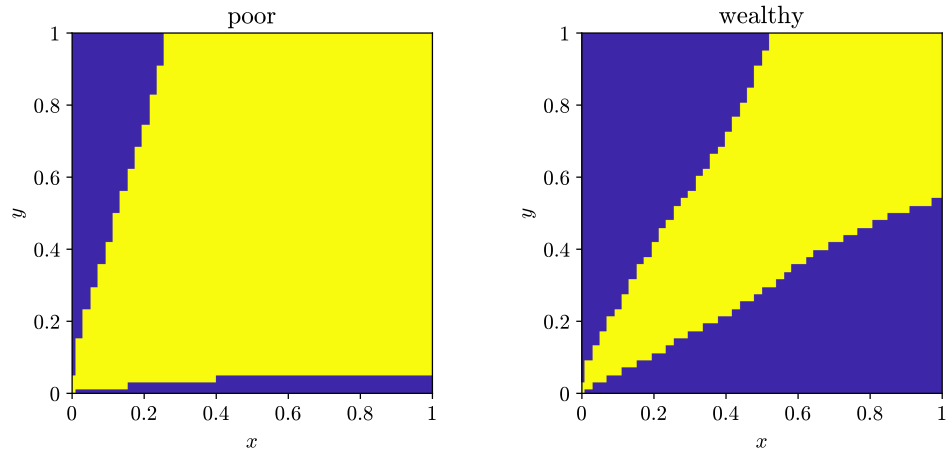
Proposition 3 (**Wealth and Job-Finding Rate**). *The job-finding rate* $\pi_{ue}(a, x)$ *is decreasing in wealth* a.

Proposition 3 is a direct corollary of Proposition 2: as workers with lower wealth have wider matching sets, they are more likely to find successful matches, thereby leading to higher job-finding rates and shorter unemployment durations. See Appendix I.2.4 for the proof.

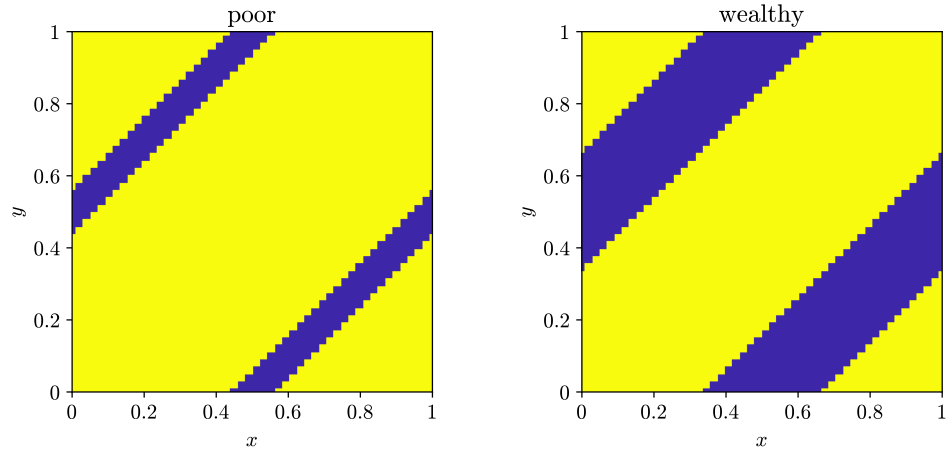
Wealth and reemployment wages. Wealth also affects wages upon reemployment through two distinct channels. To make this distinction precise, fix worker type x and let $B(a, x) \coloneqq \{y : \Phi(a, x, y) = 1\}$ denote the set of jobs accepted by a worker with wealth a. The average wage of

Figure 1: Matching Sets

(a) Vertical Heterogeneity



(b) Horizontal Heterogeneity



Notes: This graph shows the matching sets between workers and firms of different productivity types. Panel (1a) shows the case where agents are heterogeneous vertically and Panel (1b) shows the case where agents are heterogeneous horizontally.

new hires is

$$\bar{\omega}(a,x) = \frac{\int_{B(a,x)} \omega(a,x,y)\, d_v(y)\, \mathrm{d}y}{\int_{B(a,x)} d_v(y)\, \mathrm{d}y}.$$

Consider two wealth levels $a_2 > a_1$. By Proposition 2, $B(a_2,x) \subseteq B(a_1,x)$. The difference in average reemployment wages can then be decomposed exactly as

$$\begin{aligned}\bar{\omega}(a_2,x) - \bar{\omega}(a_1,x) = &\underbrace{\mathbb{E}\left[\omega(a_2,x,y) - \omega(a_1,x,y) \mid y \in B(a_2,x)\right]}_{\text{bargaining effect}} \\ &+ \underbrace{\mathbb{E}\left[\omega(a_1,x,y) \mid y \in B(a_2,x)\right] - \mathbb{E}\left[\omega(a_1,x,y) \mid y \in B(a_1,x)\right]}_{\text{composition effect}},\end{aligned}$$

where expectations are taken with respect to the vacancy distribution.

The first term captures the effect of wealth on the wages holding fixed the set of matches that remain acceptable at the higher wealth level. As in Krusell, Mukoyama, and Şahin (2010), greater wealth strengthens the worker's outside option relative to employment, creating a force toward higher bargained wages for a given match (x,y). The second term captures the change in the composition of accepted jobs. Precautionary mismatch implies that wealthier workers are more selective and accept a smaller set of jobs. If the matches they shed are lower-paying, on average, than those they retain, this composition effect further raises reemployment wages by shifting new hires toward better matches.

The theory does not, however, sign the wage effect in full generality. We therefore treat the relationship between wealth and reemployment wages as a quantitative implication of the model and examine its empirical counterpart below.

3 Data and Empirical Results

Our empirical analysis uses panel data from the 1979 National Longitudinal Survey of Youth (NLSY79), a nationally representative survey of US residents born between 1957 and 1964. At the time of the initial interview in 1979, respondents were between 14 and 22 years old. We merge respondents' work histories and asset information from the NLSY79 with occupation-level skill measures from the Occupational Information Network (O*NET), which reports scores on skill content for 923 occupations. The merge creates a panel dataset with both worker and job characteristics. We construct a measure of worker skills to proxy for worker type x, and a measure of occupational skill requirements to proxy for job type y. The remainder of this section describes the data sources, the construction of these skill measures, and the empirical findings.

3.1 Data Sources and Skill Measures

NLSY. We construct a monthly panel from the NLSY79 work history. Following Farber and Gibbons (1996); Guvenen et al. (2020); Lise and Postel-Vinay (2020), we focus on work histories of a representative sample of individuals attached to the labor market. Accordingly, we begin each worker's work history in the first year after school completion in which they work at least 1,200 hours, exclude workers with more than two years of work experience prior to school completion, and drop individuals with military service.

We construct a measure for worker skills using several test scores from the Armed Services Vocational Aptitude Battery (ASVAB), including Arithmetic Reasoning, Mathematics Knowledge, Word Knowledge, and Paragraph Comprehension. The literature commonly uses the composite "AFQT score" built from these subtests as a proxy for cognitive ability. Since the tests were administered before most respondents entered the labor market, this measure is less likely to be affected by reverse causality from labor-market outcomes to skills. We aggregate these test scores using principal component analysis (PCA) and retain the first principal component, thereby reducing a set of scores to a single index. We then rescale this index linearly into $[0, 1]$, which we use as our measure of worker type x.

In addition to work histories and test scores, we use the NLSY79 asset module to construct annual measures of liquid wealth. Since the survey did not begin collecting detailed wealth information until 1985 (when respondents were roughly 20–28 years old), wealth is typically first observed after labor-market entry. In the NLSY79, we measure liquid wealth as the total value of all assets other than housing, farms, businesses, and vehicles, net of all debts other than those associated with housing, farms, businesses, and vehicles.[8] Because asset information is not collected in every survey wave for all respondents, we linearly interpolate between observed years to construct an annual wealth series for each individual.

O*NET. O*NET provides information on nearly 1,000 occupations representative of the US labor market. We map these O*NET-SOC codes into NLSY79 occupation codes (1970 and 2000 Census classification) using the crosswalk of Sanders (2012). Each occupation is rated along 277 attributes ("descriptors"), which can be grouped into nine broad categories: skills, knowledge, abilities, work activities, work context, required education, job interests, work styles, and work values. Following Guvenen et al. (2020), we use 24 descriptors from the abilities,

[8]The NLSY79 asset and debt questionnaires become more disaggregated only from the 2004 wave onward. To maximize coverage and comparability across waves, we construct liquid wealth using categories that are available consistently throughout the panel. Specifically, liquid assets are the sum of "Total amount of money assets like savings accounts of R/spouse" and "Total market value of all other assets each worth more than $500," net of "Total amount of other debts over $500 R/spouse owe." The results are robust to additionally including "Total market value of stocks/bonds/mutual funds," which is available from 1988 onward.

skills, and knowledge categories that capture its cognitive and technical demands, such as mathematical reasoning, reading comprehension, and mechanical knowledge.[9] As with our worker-skill measure, we summarize these occupational descriptors using PCA and keep the first principal component as our measure of occupational skill requirement. We compute the principal component over all person-months in our NLSY79 sample, so that occupations are weighted by employment. We then rescale this component linearly into $[0, 1]$, which we use as our measure of job type y.

Mismatch. Let x_i denote the skill type of individual i and y_j the skill requirement of job j. We define the mismatch between individual i and job j as

$$m_{i,j} := y_j - x_i,$$

so that $m_{i,j} > 0$ indicates that worker i is under-qualified (or over-matched) for job j, while $m_{i,j} < 0$ indicates the opposite. We define the magnitude of mismatch as $mm_{i,j} = |m_{i,j}|$. Note that this definition is match-specific rather than aggregate; it characterizes the degree of misalignment for a given worker–job pair. Nevertheless, aggregate labor market misallocation manifests through these micro mismatches. In the frictionless benchmark with perfect sorting, worker types would align exactly with job types so that $m_{i,j} = 0$ holds for all matches.

3.2 Empirical Results

In Section 2.4, we derive a set of theoretical implications linking wealth, job search, and wages. We now use the merged NLSY79 and O*NET data to assess the empirical relevance of these implications.

3.2.1 Wealth and Mismatch

We begin by documenting the relationship between workers' liquid wealth and mismatch, a central prediction of the paper. Figure 2a presents a binscatter plot for workers who have just found a job. The vertical axis shows the standardized mismatch measure (so the unit is a standard deviation of mismatch), and the horizontal axis shows workers' liquid wealth.[10] The

[9] The 24 descriptors are, from the abilities category: Oral Comprehension, Written Comprehension, Deductive Reasoning, Inductive Reasoning, Information Ordering, Mathematical Reasoning, and Number Facility; from the skills category: Reading Comprehension, Science, Technology Design, Equipment Selection, Installation, Operation and Control, Equipment Maintenance, Troubleshooting, and Repairing; and from the knowledge category: Computers and Electronics, Engineering and Technology, Building and Construction, Mechanical, Physics, Chemistry, Biology, and English Language. For each descriptor we use the O*NET level rating, divided by its maximum of 7.

[10] Liquid wealth in Figure 2a is converted into real 1982–1984 dollars using the regional CPI-U.

figure residualizes both variables with respect to a rich set of controls, including a quadratic polynomial in age, cumulative labor-market experience, and occupational experience, as well as race, gender, education, and worker skill types. We also control for the national unemployment rate and the region fixed effects. The red curve shows the best quadratic fit. We find a systematic negative relationship between wealth and the magnitude of mismatch: a $1,000 increase in liquid wealth is associated with a 0.008-standard-deviation decline in the magnitude of mismatch, statistically significant at the 1% level. This finding provides evidence for Proposition 2, which predicts that wealthier workers have a more selective set of acceptable jobs, leading to less mismatch upon reemployment.

Empirical work commonly uses job tenure as a proxy for match quality (e.g., Card, Chetty, and Weber, 2007; Nekoei and Weber, 2017). Figure 2b presents a binscatter plot of job tenure (in months) against workers' wealth at the start of each employment spell, controlling for the same covariates as in the previous regression. Note that we keep one observation per employment spell, and measure wealth in the first month of employment to mitigate reverse causality (i.e., longer tenure leading to greater wealth accumulation). We find that higher wealth is associated with longer job tenure. OLS estimates imply that a $1,000 increase in wealth holdings is associated with a 0.4-month increase in job tenure.

3.2.2 Wealth and Job-Finding Rate

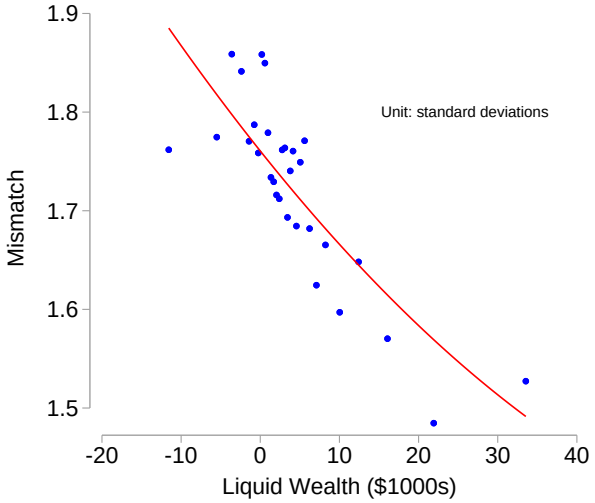
Next, we examine the empirical relationship between wealth and job finding, as in Proposition 3. Figure 2c presents a binscatter plot of unemployment duration against workers' liquid wealth holdings, using the same set of controls as above. Results from an OLS regression indicate that on average, a $1,000 increase in workers' liquid wealth is associated with a 0.4-week increase in unemployment duration, statistically significant at the 1% level. This is consistent with empirical findings in Card, Chetty, and Weber (2007), Chetty (2008), Lise (2013), among others. This finding aligns with Proposition 3, which predicts that wealthier workers are more selective and therefore take longer to accept a job.

3.2.3 Wealth and Wages

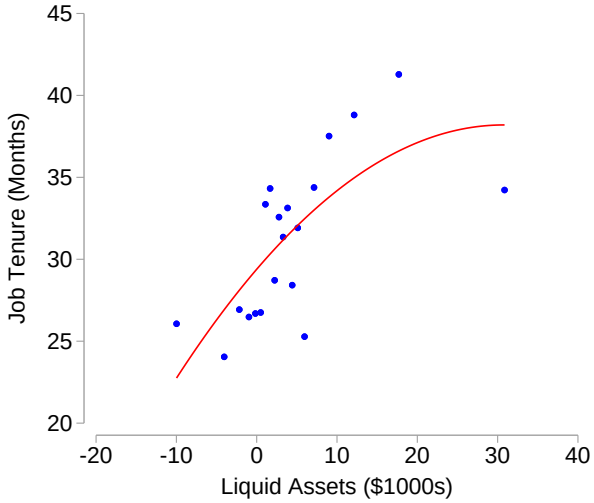
Finally, we examine the relationship between wealth and wages upon reemployment. Figure 2d presents a binscatter plot of log wages at reemployment against workers' liquid wealth. In addition to the covariates used in the previous regressions, we further control for the wage on the previous job. OLS estimates indicate that a $1,000 increase in liquid wealth is associated with a roughly 1% higher reemployment wage, statistically significant at the 1% level. Prior research such as Rendon (2006) has documented that workers' reservation wages rise with wealth

Figure 2: Labor Market Impacts of (Liquid) Wealth

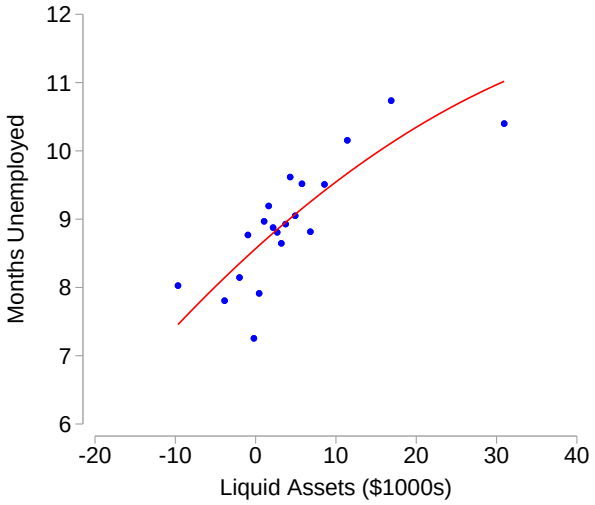
(a) Mismatch and Wealth



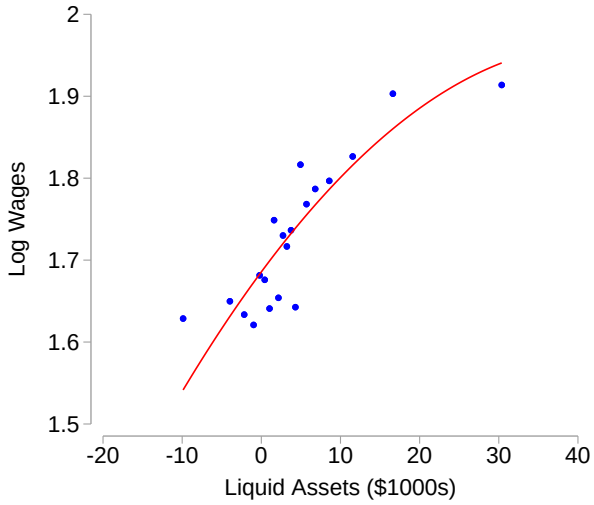
(b) Job Tenure and Wealth



(c) Unemployment Duration and Wealth



(d) Wages and Wealth



Notes: Demographic controls include race, gender, education, and a quadratic function of age. Experience and skill controls include quadratic functions of cumulative labor market experience and occupational experience, and the ASVAB score. Economic controls include region fixed effects and the national unemployment rate. Wage specifications additionally control for the previous-job wage; the results are similar without this control.

holdings. Nekoei and Weber (2017) find that greater UI generosity raises reemployment wages, consistent with this prediction.[11]

Appendix III reports the corresponding regression coefficients of the above binscatters. We also examine over- and under-match separately (i.e., $x < y$ versus $x > y$) at reemployment. We find that wealth primarily reduces under-match, with little effect on over-match, consistent with the quantitative results in Section 4.4.

4 Quantitative Results

4.1 Parameterization

We adopt standard functional-form assumptions to facilitate the numerical analysis. Workers' flow utility function exhibits constant relative risk aversion (CRRA):

$$\mathfrak{u}(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \quad , \quad \gamma > 0.$$

The aggregate meeting function is Cobb-Douglas:

$$M(u, v) = \chi u^{\alpha} v^{1-\alpha}.$$

We represent worker and job types, x and y, by their percentile ranks, so by construction they are uniformly distributed on $[0, 1]$. The production function $f(x, y)$ is therefore defined over percentiles. We adopt the production function specification from Hagedorn, Law, and Manovskii (2017):

$$f(x, y) = f_0 + f_1 \left(x^{\xi} + y^{\xi}\right)^{\nu/\xi},$$

where $\nu > 0$ governs returns to the joint scale of worker and job skill, while ξ governs their degree of complementarity. The production function is supermodular whenever $\nu > \xi$.[12] A lower ξ strengthens complementarity between worker and job types. A larger ν increases returns to skill and amplifies productivity differences across skill levels.

We parameterize home production $b(x)$ as a fraction of each worker type's lowest market output, i.e., $b(x) = b_0 \cdot f(x, 0)$, where $b_0 \in [0, 1]$. This parsimoniously captures the fact that

[11] Card, Chetty, and Weber (2007) find statistically insignificant effects of UI generosity on reemployment wages in Austria. Nekoei and Weber (2017) reconcile these results through two opposing forces: a selectivity effect that raises accepted wages and a duration effect that lowers wages due to negative duration dependence. The empirical finding of Nekoei and Weber (2017) also provides empirical support for the selectivity mechanism in our model.

[12] The cross-partial is $f_{xy} = f_1 \nu (\nu - \xi) \left(x^{\xi} + y^{\xi}\right)^{\nu/\xi - 2} x^{\xi-1} y^{\xi-1}$, so that $f_{xy} > 0$ if and only if $\nu > \xi$ for $x, y > 0$ and $\nu, f_1 > 0$.

unemployment benefits rise with pre-unemployment earnings and thus with skill, and, more broadly, that the value of nonmarket activities increases with skill. This is standard in quantitative frictional-sorting models (e.g., Lise, Meghir, and Robin, 2016; Lise and Robin, 2017; Bagger and Lentz, 2019; Lise and Postel-Vinay, 2020). This assumption also ensures that no worker or job type goes unmatched in equilibrium.

For the numerical solution, we discretize the state space using 200 asset grid points, 20 worker types, and 20 job types. See Appendix II for details of the computation algorithm.

4.2 Calibration

We normalize market tightness to $\theta = 1$ in the stationary equilibrium. This normalization is without loss of generality because for any θ there exists a unique vacancy posting cost that satisfies free entry and rationalizes that equilibrium. We normalize the time unit to be a quarter. Table 2 reports the parameter values in quarterly units.

4.2.1 External Calibration

We set the quarterly interest rate r to match an annual real rate on liquid assets of 2%, and set the coefficient of relative risk aversion to $\gamma = 2$. We impose a borrowing constraint at $\underline{a} = 0$. For the quantitative analysis, we also assume that newborn workers enter with zero assets, so h is degenerate at $a = 0$. We set the matching elasticity α and workers' bargaining power η to 0.72 (Shimer, 2005; Krusell, Mukoyama, and Şahin, 2010). Note, however, that the Hosios condition does not in general restore constrained efficiency with incomplete markets (Davila et al., 2012) and heterogeneous worker–job matches (Shimer and Smith, 2001). The worker exit rate δ is calibrated to imply an expected labor-market lifetime of 45 years (ages 20–65). We set the arrival rate of opportunities for voluntary separation to $\tilde{\sigma} = 3$, so that workers receive an opportunity to quit once per month on average.[13]

4.2.2 Internal Calibration

We calibrate the remaining parameters to match key moments from the US labor market. Although the calibration uses all moments jointly, so no single moment uniquely identifies a single parameter, the model's structure implies that certain moments are especially informative

[13]In discrete-time models with monthly frequency and endogenous separation, the opportunity to separate is implicitly available once each period, which corresponds to $\tilde{\sigma} = 3$ under our quarterly normalization. We vary $\tilde{\sigma}$ from 3 (monthly opportunities) to 12 (weekly opportunities) and find that the key model outputs are insensitive to this choice.

about particular parameters. We therefore provide intuition for why each targeted moment is useful for pinning down the corresponding parameter.

Separation and matching. We set the exogenous separation rate σ so that the model matches a total separation rate (exogenous plus endogenous) of 0.034 per month (Shimer, 2005). The matching efficiency parameter χ scales the job-finding rate through the meeting function, so we calibrate it to match a job-finding rate of 0.45 per month (Shimer, 2005).

Production function. The production function contains four parameters, f_0, f_1, ξ, ν. We normalize the intercept $f_0 = 1$ without loss of generality. The remaining three parameters are disciplined by features of the wage distribution.

The parameter ξ governs the strength of complementarity between worker and job types, and thus the return to assortative matching. Stronger complementarity makes mismatch more costly, which narrows matching sets in equilibrium. This reduces frictional wage dispersion, i.e., wage variation among identical workers arising solely from matching with different job types. Therefore, we target the mean-min ratio proposed by Hornstein, Krusell, and Violante (2011), which captures wage dispersion that cannot be explained by worker characteristics. To compute this statistic in the model, we simulate workers from the stationary distribution and residualize log wages with worker-type fixed effects by running the regression

$$\log(\omega_i) = \beta_0 + D_{x(i)} + \epsilon_i,$$

where the left-hand side is log wage for worker i, and the right-hand side includes an intercept, worker type fixed-effects, and a residual. We then compute the mean-min ratio of residualized wages,

$$\exp\left(\overline{\epsilon_i} - \min_i(\epsilon_i)\right).$$

A higher mean-min ratio indicates greater frictional wage dispersion. In the model, smaller ξ strengthens sorting and compresses residual wage dispersion, providing identification.

The parameter ν controls the curvature of output with respect to skill, and thus shapes between-type wage inequality. Higher ν amplifies productivity differences between high- and low-type workers, producing greater wage inequality and a more concave wage Lorenz curve. We therefore calibrate ν to match the Lorenz curve of the US wage distribution. Intuitively, ξ primarily disciplines wage dispersion within worker types, whereas ν disciplines wage dispersion across worker types.

The scale parameter f_1 governs the importance of the skill-dependent component of production relative to the constant term f_0. For high-type workers, who are typically matched with

high-type jobs, the skill-dependent component $(x^\xi + y^\xi)^{\nu/\xi}$ is large, so output and hence wages are scaled nearly proportionally by f_1. For low-type workers, typically in low-type jobs, this term is small, so output and wages are anchored closer to the constant f_0. As a result, f_1 primarily stretches the upper tail of the wage distribution while leaving the bottom relatively anchored. We therefore target the 90–10 wage ratio to discipline f_1.

Finally, we calibrate the home production parameter b_0 so that, on average, home production replaces 40 percent of labor market income (Shimer, 2005; Krusell, Mukoyama, and Şahin, 2010).

Discount rate. The discount rate ρ governs households' willingness to shift consumption across time and hence their steady-state wealth accumulation: a lower ρ implies greater patience, more saving, and higher wealth holdings. We therefore set ρ to match the ratio of aggregate net liquid wealth to aggregate annual household labor earnings in the Survey of Consumer Finances (SCF).[14]

We use the 1998–2019 survey waves, apply population weights, and average across the five implicates. Following Kaplan, Violante, and Weidner (2014), we measure net liquid wealth as the sum of imputed cash holdings, checking and saving accounts, money market and call accounts, and directly held mutual funds, stocks, and bonds, net of credit card balances. We exclude retirement accounts, housing, vehicles, and private business equity because these assets are relatively illiquid and therefore less readily available for consumption smoothing during an unemployment spell. All dollar values are converted to 2019 dollars using the R-CPI-U-RS series.

We impose three sample restrictions to align the SCF with the population and assets represented in the model. First, we retain households whose head is aged 20–65, corresponding to the working-age population in the model and its expected 45-year working life. Second, we retain households with non-negative net liquid wealth and labor earnings. The restriction on liquid wealth is consistent with the model's zero borrowing constraint, while the restriction on labor earnings is consistent with the model, in which labor earnings are non-negative.[15] Third, we exclude households in the top 5 percent of the total net worth distribution, whose wealth accumulation is more likely to reflect mechanisms outside the scope of the model, such

[14] We use the SCF rather than the NLSY79 for this calibration because the two datasets serve different purposes here. The NLSY79 provides the panel work histories and pre-market ability measures needed to study how labor market outcomes vary with wealth, whereas the SCF provides the detailed, representative portfolio information better suited to measuring aggregate liquid wealth holdings.

[15] This non-negativity restriction applies only to the SCF sample used to construct the calibration target. We do not impose the same restriction in the NLSY79 analysis in Section 3, which retains workers with zero or negative liquid wealth. This difference is deliberate: the NLSY79 is used to document how labor-market outcomes vary over the observed wealth distribution, whereas the SCF moment is restricted to the asset support represented in the quantitative model.

as entrepreneurship, bequest motives, and heterogeneous asset returns.[16]

Table 2: Calibration

Parameter	Value	Source
External Calibration		
interest rate	$r = 0.005$	annual interest rate on liquid asset 2%
relative risk aversion	$\gamma = 2$	common parameterization
bargaining power	$\eta = 0.72$	Shimer (2005)
matching elasticity	$\alpha = 0.72$	Shimer (2005)
worker exit rate	$\delta = 0.0056$	45-year expected working life
separation opportunity rate	$\tilde{\sigma} = 3$	monthly adjustment
Internal Calibration		
matching efficiency	$\chi = 4.7$	monthly job-finding rate 0.45
exogenous separation	$\sigma = 0.1$	monthly separation rate 0.034
discount rate	$\rho = 0.014$	liquid wealth/annual earnings ratio 0.57 (SCF)
home production	$b_0 = 0.75$	average 40% replacement rate
production function	$f_0 = 1$	normalization
production function	$f_1 = 1.78$	wage 90-10 ratio = 10
production function	$\xi = 0.8$	M-m ratio (Hornstein, Krusell, and Violante, 2011)
production function	$\nu = 3$	US wage Lorenz curve (Lise, 2013)

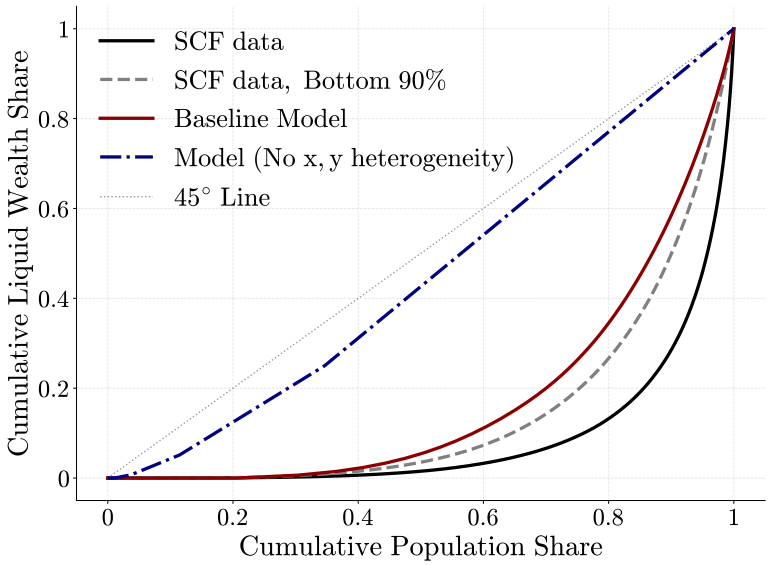
Notes: The table summarizes the calibrated parameters and their sources. The unit of time is normalized to a quarter.

4.3 Additional Model Validations

Section 3 evaluates the model's central theoretical prediction on wealth and mismatch. This subsection provides additional validations on the model's quantitative performance.

[16] See, for example, Quadrini (2000); Cagetti and De Nardi (2006); De Nardi (2004); Fagereng et al. (2020); Bach, Calvet, and Sodini (2020).

Figure 3: Liquid Wealth Distribution, Model vs. Data



Notes: This figure compares Lorenz curves of liquid wealth for the SCF sample, the SCF sub-sample restricted to the bottom 90% of the liquid wealth distribution, the baseline model, and a model variant without worker skill x and job y heterogeneity. The Lorenz curves plot cumulative population shares, ordered by liquid wealth, against the corresponding cumulative shares of liquid wealth.

4.3.1 Wealth Distribution

The model endogenously generates a stationary distribution of wealth. We compare this model-implied wealth distribution to its empirical counterpart in the SCF. For comparability with the model as well as consistency, we use the same SCF sample as in the calibration of ρ.

Figure 3 plots Lorenz curves for liquid wealth in the model and in the SCF. The Lorenz curve plots cumulative population shares, ordered by liquid wealth, against the corresponding cumulative shares of liquid wealth. The black solid curve reports the empirical distribution in our SCF sample, while the gray dashed curve restricts the sample to the bottom 90% of the liquid wealth distribution. The red curve corresponds to the baseline model with two-sided heterogeneity, and the blue dash-dotted curve corresponds to a version of the model without worker and job heterogeneity, in which all employed workers earn a common wage equal to the average wage in the baseline model.

The baseline model reproduces the pronounced convexity of the empirical Lorenz curves: the curve is nearly flat at the bottom, reflecting the large share of households that hold very little liquid wealth, and steepens sharply toward the upper end, reflecting the concentration of wealth among wealthier households. Importantly, this distributional shape is not targeted by the calibration, which targets only the mean ratio of liquid wealth to earnings. By contrast, shutting down production heterogeneity produces a Lorenz curve much closer to the 45-degree

line, implying far less wealth inequality than the data. The difference is because two-sided heterogeneity creates persistent earnings dispersion: high-type workers in high-type jobs earn substantially more than low-type workers in low-type jobs, and this earnings inequality cumulates into wealth inequality over time. Without heterogeneity, workers face similar income dynamics, and precautionary saving alone cannot generate the degree of wealth dispersion in the data. The model understates concentration at the very top, which is unsurprising given that the calibration targets earnings dispersion only between the 10th and 90th percentiles. Nevertheless, it reproduces much of the overall dispersion in liquid wealth without directly targeting the wealth distribution.

Section 2 establishes how wealth impacts labor market outcomes through bargaining and matching. We now examine the feedback in the opposite direction: labor market behavior in turn generates heterogeneity in saving motives. The Euler equations make these forces clear:

$$\frac{\dot{c}^e}{c^e} = \frac{1}{\gamma}\left\{r - \rho + \omega_a + \sigma\left[\frac{\mathfrak{u}'\left(c^u\right)}{\mathfrak{u}'\left(c^e\right)} - 1\right]\right\},$$
$$\frac{\dot{c}^u}{c^u} = \frac{1}{\gamma}\left\{r - \rho - p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[1 - \frac{\mathfrak{u}'\left(c^e\right)}{\mathfrak{u}'\left(c^u\right)}\right] \mathrm{d}y\right\},$$

where arguments of $\omega(a,x,y), c^u(a,x), c^e(a,x,y)$ are suppressed for brevity, and $B\left(a,x\right) := \{y : \Phi(a,x,y) = 1\}$ is the acceptance set of worker (a,x). Appendix I.2.5 provides the derivation. The term $r-\rho$ captures the standard intertemporal substitution motive. For employed workers, two additional motives arise. First, ω_a reflects the wealth effect on bargained wages. Since higher wealth improves the worker's bargaining position, employed workers have an additional incentive to accumulate assets to increase future wages. Second, the bracketed term captures precautionary savings against job loss. This motive is particularly strong when wealth is low or the current wage is high, so that the marginal utility gap $\mathfrak{u}'\left(c^u\right)/\mathfrak{u}'\left(c^e\right)$ is large. For unemployed workers, the prospect of finding a job creates a dissaving motive, as expected future wage relaxes the need for precautionary savings today. Importantly, this dissaving motive depends on the worker's acceptance set $B\left(a,x\right)$. These Euler equations resemble those in Lise (2013), with two key differences: (i) the ω_a term reflecting that the bargained wage depends on wealth, as in Krusell, Mukoyama, and Şahin (2010); and (ii) the acceptance set $B\left(a,x\right)$, which itself depends on wealth in our model.

Figure 4 illustrates the resulting heterogeneity in saving rates across the (x,y) space for low-wealth (left) and high-wealth (right) workers. We define the saving rate as $\dot{a}/\left(\omega + (r+\delta)a\right)$, where the denominator is total flow income. Several patterns emerge. First, comparing across panels, saving rates are uniformly higher at low wealth. This is because workers closer to the borrowing constraint face greater consumption risk from job loss and therefore save more. Second, within each panel, holding worker type fixed, saving rates vary with job type because

Figure 4: Saving Rate Heterogeneity



Notes: The figure shows the saving rates of workers of different levels of productivity x employed at firms of different types y. The left panel shows the saving rates with respect to x and y when wealth is low (close to 0) and the right panel shows the case when wealth is high. Note that matches form only within the equilibrium matching set. We therefore omit (x, y) pairs outside the region.

wages differ across jobs. Third, holding match quality fixed (e.g., along the diagonal of perfect matches), saving rates increase with worker type, because high-x workers suffer larger income losses from separation.

4.3.2 Wealth and Labor Market Outcomes: Model versus Data

Another key external validation for the model is the relationship between job-finding rates and liquid wealth. This moment captures how workers' precautionary motives affect their search behavior. We estimate it using monthly labor market histories from the NLSY79. Specifically, we estimate a latent-index model on the sample of unemployed workers, where the transition from unemployment to employment is determined by

$$U2E_{i,m} = \mathbf{1}\left\{U2E^*_{i,m} > 0\right\}, \quad U2E^*_{i,m} = \alpha_1 + \beta_1 g\left(a_{i,m}\right) + \mathbf{X}'_{i,m}\gamma_{1,x} + \epsilon_{1,i,m},$$

where $U2E_{i,m}$ is an indicator equal to one if unemployed worker i transitions from unemployment in month m to employment by the following month and $\epsilon_{1,i,m}$ follows a logistic distribution. Equivalently,

$$\Pr(U2E_{i,m} = 1 | a_{i,m}, \mathbf{X}_{i,m}) = F\left(\alpha_1 + \beta_1 g\left(a_{i,m}\right) + \mathbf{X}'_{i,m}\gamma_{1,x}\right), \tag{16}$$

where $F(\cdot)$ is the logit CDF and $g(a_{i,m})$ denotes a function of liquid wealth. The control vector $\mathbf{X}$ includes quadratic functions of age and labor market experience, as well as controls for race, gender, education, ASVAB score, and the national unemployment rate.

The relationship between wealth and wages upon reemployment provides another validation moment. We estimate it using the following regression, restricting the sample to months in which a worker transitions from unemployment to employment:

$$\log w_{i,m} = \alpha_2 + \beta_2 g(a_{i,m}) + \mathbf{X}'_{i,m}\gamma_{2,x} + \epsilon_{2,i,m} \quad \text{if } U2E_{i,m} = 1 \tag{17}$$

where $w_{i,m}$ is the accepted wage in the reemployment month.

We consider three specifications for $g(\cdot)$: (i) an indicator for non-positive liquid wealth, $\mathbb{1}(a \leq 0)$,[17] (ii) log liquid wealth among workers with strictly positive liquid wealth, $\log(a)$ for $a > 0$, and (iii) the inverse hyperbolic sine of liquid wealth, $\text{asinh}(a) = \log\left(a + \sqrt{a^2+1}\right)$, which accommodates zero and negative values while allowing for nonlinear effects of wealth.[18] Together, these specifications characterize the wage–wealth and job-finding–wealth relationships at the borrowing constraint, along the intensive margin among workers with positive wealth, and across the full wealth distribution. We estimate the same specifications on data simulated from the model, controlling for worker skill type to mirror the empirical specification.

Table 3 reports the estimates from the data and the calibrated model. The model reproduces the sign of the wealth gradient in every specification and for both outcomes: workers at the borrowing constraint exit unemployment more quickly and accept lower-paying jobs, and both relationships attenuate as wealth rises. The model overstates the job-finding gradient in all three specifications but reproduces the wage gradient reasonably closely (-0.102 versus -0.121 in Panel A, 0.067 versus 0.070 in Panel B, and 0.139 versus 0.190 in Panel C). Because reemployment wages, conditional on worker type, reflect both bargaining and match quality, the close fit of the wage–wealth gradient indicates that the model captures how wealth shapes accepted matches, which is central to the productivity losses from precautionary mismatch quantified in Section 4.5.

The model generates substantial mass at the borrowing constraint: 20.7 percent of workers in the stationary equilibrium hold zero wealth. In the NLSY79, where liquid wealth may also be negative, 19.5 percent of workers in the U–E estimation sample and 21.5 percent in the reemployment-wage sample have non-positive liquid wealth. Thus, the model generates a

[17] Because the model imposes a zero borrowing constraint, $\mathbb{1}\{a \leq 0\}$ is equivalent to $\mathbb{1}\{a = 0\}$ in the model-simulated sample. In the NLSY79 sample, the same indicator includes both zero and negative liquid wealth, since the empirical analysis retains the full observed wealth distribution.

[18] Unlike the indicator and the logarithm, the inverse hyperbolic sine is not scale-invariant. We therefore apply the same normalization to wealth in the model and in the data, expressing the transformation relative to the standard deviation of wealth in each sample.

Table 3: Wealth and Labor Market Outcomes: Model versus Data

	U-E transition		Log reemployment wage	
	Model	NLSY79	Model	NLSY79
	(1)	(2)	(3)	(4)
Panel A: Non-positive-wealth indicator, $\mathbb{1}(a \leq 0)$				
β	0.389	0.120***	-0.102	-0.121***
	(-)	(0.037)	(-)	(0.019)
Share $a \leq 0$	0.207	0.195	0.207	0.214
Observations	-	58,245	-	6,193
Panel B: Log wealth, $\log(a)$ *for* $a > 0$				
β	-0.243	-0.074***	0.067	0.070***
	(-)	(0.010)	(-)	(0.005)
Observations	-	46,871	-	4,868
Panel C: Inverse hyperbolic sine, $\text{asinh}(a)$				
β	-0.457	-0.252***	0.139	0.190***
	(-)	(0.031)	(-)	(0.017)
Observations	-	58,245	-	6,193

Notes: * p<0.10, ** p<0.05, *** p<0.01. Each entry reports the estimate of β_1 or β_2, respectively, from a separate estimation of equations (16) and (17), with standard errors in parentheses. Columns (1) and (3) use simulated data from the stationary equilibrium of the baseline model. Because coefficients are estimated on a large simulated panel, standard errors are negligible and therefore omitted, as are significance levels. Columns (2) and (4) use the NLSY79 sample described in Section 3.1. The model specifications in Columns (1) and (3) control for worker type. The empirical specifications in Columns (2) and (4) control for quadratic functions of age and work experience, as well as race, gender, education, ASVAB score, and the national unemployment rate. In Panel C, wealth is normalized by the standard deviation of liquid wealth computed separately in the model and in the data. Thus, the argument of the inverse hyperbolic sine transformation is expressed in standard-deviation units.

prevalence of workers with very limited liquid resources that is broadly comparable to the data. Neither these lower-tail wealth shares nor the wealth gradients are targeted in the calibration. The model's ability to reproduce them therefore provides additional support for the mechanism.

For transparency, Appendix Figure A-3 reports the model-implied counterparts to Figure 2, constructed from a large simulated sample from the stationary equilibrium using the same plotting procedure as in the data. Because the model imposes a borrowing constraint at zero, the simulated wealth distribution has no support on the negative-wealth region. Despite this limitation, the model reproduces the qualitative patterns in the data.[19]

[19]The levels of job tenure and unemployment duration differ, reflecting differences between the aggregate labor-market moments targeted in the calibration and those observed in the NLSY79 sample. In particular, the model is calibrated to the aggregate US job-finding and separation rates in Shimer (2005), whereas the NLSY79 sample exhibits substantially longer unemployment spells and shorter job tenure.

4.4 Distributional Implications of Precautionary Mismatch

Proposition 2 establishes that poorer workers of any given productivity type tend to be more mismatched with their jobs. How does this mechanism vary across the skill distribution? Which workers are most affected by precautionary mismatch? We now examine these distributional implications.

Figure 5 compares labor market outcomes for workers at the top and bottom of the within-type wealth distribution. Panel (a) plots the average matched firm type y for workers at the 99th percentile of wealth (red solid line) versus the 1st percentile (blue dashed line) within each worker type x. Across most of the skill spectrum, low-wealth workers are on average matched with lower-productivity jobs than equally skilled but wealthier workers. This gap widens with worker productivity.

Panels (b) and (c) translate these allocative differences into earnings and output. The gaps are small for low-skilled workers but widen dramatically at higher skill levels. This is because (i) poor high-skilled workers experience more under-matching, and (ii) the cost of under-matching is larger for them due to the supermodularity of the production function. For the most productive workers ($x = 1$), the earnings gap between the wealth-poor and the wealth-rich is 31.5% and the output gap is 40.8%.

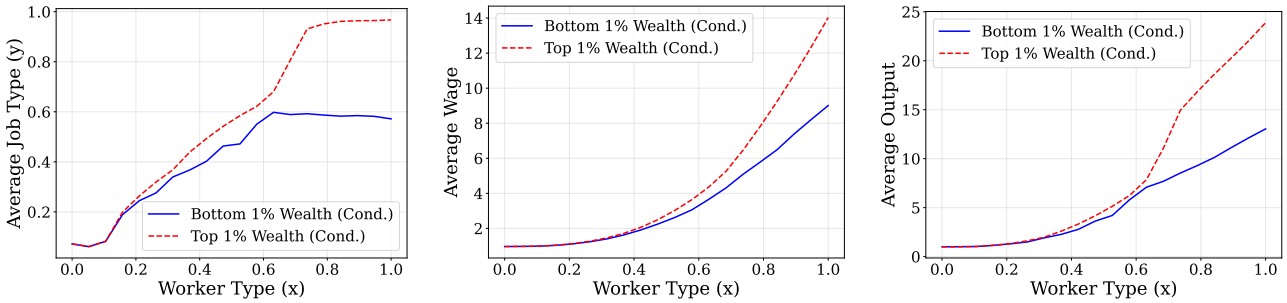
These findings highlight the link between wealth inequality and labor market allocations. Lack of wealth pushes workers to accept worse matches, lowering their earnings and output. The effects are concentrated among high-skilled workers, those with the largest potential contribution to aggregate output. This suggests that policies alleviating liquidity constraints could generate efficiency gains by improving the allocation of talent to jobs. We quantify these gains in the next section.

4.5 Aggregate Implications of Precautionary Mismatch

In this section, we quantify the aggregate cost of precautionary mismatch, i.e., how much output is lost because wealth-constrained workers accept mismatched jobs. We address this question through two exercises. The first is an accounting benchmark. It asks, given the workers and jobs currently active in equilibrium, how much output could be gained by reassigning workers to jobs. This measures the gains from eliminating all mismatch. The second exercise asks how much output would be gained if workers search as if they are risk neutral. This isolates the output loss attributable specifically to precautionary mismatch, separating it from mismatch that arises purely from search frictions.

Figure 5: Effect of Wealth on Allocations, Earnings, and Output

(a) Average Matched y (b) Average Earnings (c) Average Output



Notes: The figure shows, among workers of each production type x, expected labor market outcomes for workers in the top and bottom 1 percent of wealth distribution conditional on x. From Panel (5a) to Panel (5c), the labor market outcomes are average firm types, average (annualized) earnings and output respectively. Earnings and output are expressed in model units, in which the flow output of a match between the lowest-skill worker and the lowest-quality job is normalized to one ($f_0 = 1$).

Constrained Optimal Assignment. We first compute the output-maximizing assignment of workers to jobs, holding fixed the marginal distributions of employed workers and producing jobs as in the baseline equilibrium. This exercise asks what is the maximal output that could be achieved by reshuffling matches. Formally, the constrained optimal assignment solves:

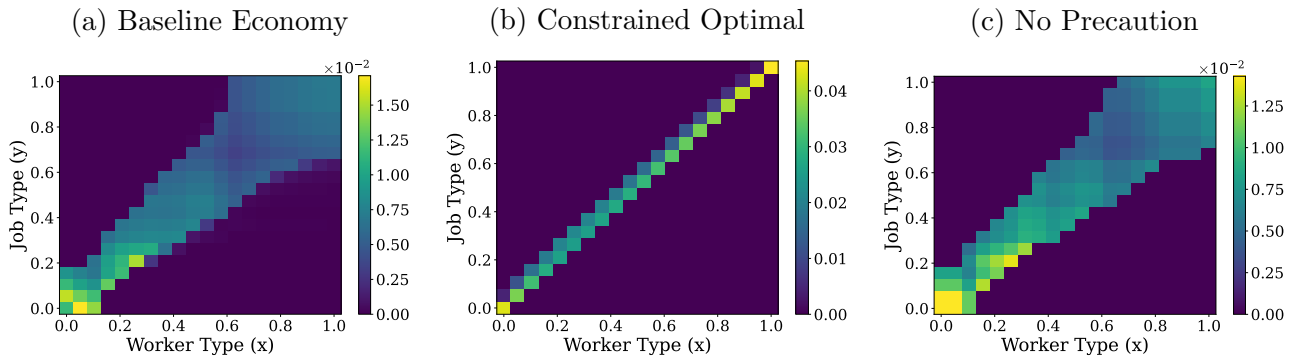
$$\max_{m(x,y)} \int f(x,y)\, m(x,y)\, dxdy$$
$$\text{s.t.} \quad \int m(x,y)\, dy = d_e(x), \quad \int m(x,y)\, dx = d_p(y)$$

where $d_e(x)$ and $d_p(y)$ denote the equilibrium distribution of matched workers and jobs in the baseline economy. This problem is a linear program.

Figure 6 compares the decentralized equilibrium allocation (panel (a)) with the constrained output-maximizing allocation (panel (b)). The horizontal axis represents worker type x and the vertical axis job type y. Lighter colors indicate higher match density. The constrained optimum assigns higher-type workers to higher-type jobs, concentrating mass tightly along the diagonal. By contrast, the decentralized equilibrium allocation exhibits a thicker dispersion of matches around the diagonal. Column (1) of Table 4 compares the two allocations and finds that labor market misallocation reduces productivity and output by 2.94%.

Removing Precautionary Motives. The constrained optimal assignment provides an upper bound on the gains from eliminating mismatch from all sources. To isolate the contribution of precautionary mismatch, we conduct a second counterfactual that shuts down precautionary motives. Specifically, we assume that each worker type x adopts the search strategy of the

Figure 6: Labor Market Allocation in Counterfactual Economies



Notes: This figure plots the match density over worker types x (horizontal axis) and job types y (vertical axis) in the baseline equilibrium (left), the constrained output-maximizing assignment (middle), and a counterfactual economy with no precautionary mismatch (right). Colors indicate match density, with the scale reported in the color bars.

wealthiest workers of that type. For these workers, the shadow value of additional liquidity and the consumption consequences of income fluctuations are negligible, so precautionary considerations play little role in job acceptance. We therefore use their acceptance behavior as an approximation to the search strategy that would prevail absent precautionary motives. Comparing this counterfactual economy to the baseline therefore isolates the role of precautionary mismatch from mismatch due to search frictions alone.

Panel (c) in Figure 6 plots the match distribution in this counterfactual. Relative to the baseline in panel (a), matches are slightly more concentrated along the diagonal, indicating stronger sorting between worker and job types. In the baseline economy, precautionary motives induce low-wealth workers to accept jobs farther away from their productivity type. Removing precautionary motives eliminates this additional source of mismatch and shifts matches somewhat closer to the optimal assignment. Table 4 shows that removing precautionary motives raises productivity by 0.58%. However, this improved sorting comes at a cost. Table 4 reports that employment falls by 1.36%. This is because workers in the no-precaution counterfactual are more selective and wait longer for better matches. On net, aggregate output declines by 0.79%, despite higher productivity per worker.

Comparing Columns (1) and (2) of Table 4 indicates that precautionary behavior accounts for roughly one-fifth (0.58/2.94) of the total productivity loss due to mismatch. The remaining four-fifths reflects mismatch generated by search frictions themselves: even risk-neutral workers cannot perfectly sort when matching is frictional. The takeaway is that precautionary mismatch is a quantitatively meaningful contributor to labor market mismatch, but search frictions per se remain the dominant source.

The decline in output highlights an important distinction between sorting and employment.

Table 4: Comparison Across Counterfactual Economies: Deviations from Baseline

	(1) Constrained Optimal Assignment	(2) No Precautionary Motive
Employment (%)	+0.00	−1.36
Productivity (%)	+2.94	+0.58
Output (%)	+2.94	−0.79

Notes: Entries are percentage changes relative to the baseline economy. Column (1): constrained optimum, which solves the output-maximizing assignment of employed workers to active jobs, holding fixed the marginal distributions. Column (2): a counterfactual economy that shuts down precautionary motives.

Removing precautionary motives improves sorting and therefore raises output per worker, but it also makes workers more selective, increasing unemployment and reducing employment. The latter effect dominates in aggregate output. Thus, this counterfactual should not be interpreted as a welfare comparison: it isolates the quantitative contribution of precautionary motives to mismatch, while simultaneously changing the extensive margin of employment.

5 On-the-Job Search

A salient feature of the data is substantial worker mobility across jobs without intervening unemployment spells. For example, the monthly job-to-job transition rate in the US is about 2% (Fallick and Fleischman, 2004; Fujita, Moscarini, and Postel-Vinay, 2024). Motivated by this fact, we extend the baseline model to allow for on-the-job search (OJS) to study how this margin affects the interaction between precautionary motives and job acceptance, equilibrium sorting, and the resulting productivity losses.

5.1 Model Extension

The baseline environment is unchanged except that employed workers can search while on the job. Specifically, we allow employed workers to search on the job with relative intensity λ. As a result, the relevant measure of effective searchers is $s = \lambda e + u$ and the effective labor market tightness becomes $\theta = v/(\lambda e + u)$. An unemployed worker receives offers at rate $p(\theta)$ and an employed worker receives outside offers at rate $\lambda p(\theta)$.

When an employed worker of type x with assets a at a job of type y meets a vacancy of type y', she compares the value of working at the new job with the value of staying at the current job. If the new match yields a higher value, the worker transitions directly from y to y' with-

out an intervening unemployment spell. Wages are determined by the same Nash bargaining protocol as in the baseline model.[20] On-the-job search adds an additional continuation value to employment: the worker can continue searching while earning the current wage and switch if a better match arrives. The baseline model is nested as the special case $\lambda = 0$. To save space, we relegate the HJB equations and Kolmogorov forward equations for the OJS extension to Appendix I.3.

5.2 Results

5.2.1 Matching Set

We first examine how on-the-job search changes the matching sets. Figure 7 compares the matching sets in the baseline model without OJS and the extended model with OJS. The left panel plots the comparison for low-wealth workers and the right panel plots the comparison for high-wealth workers.

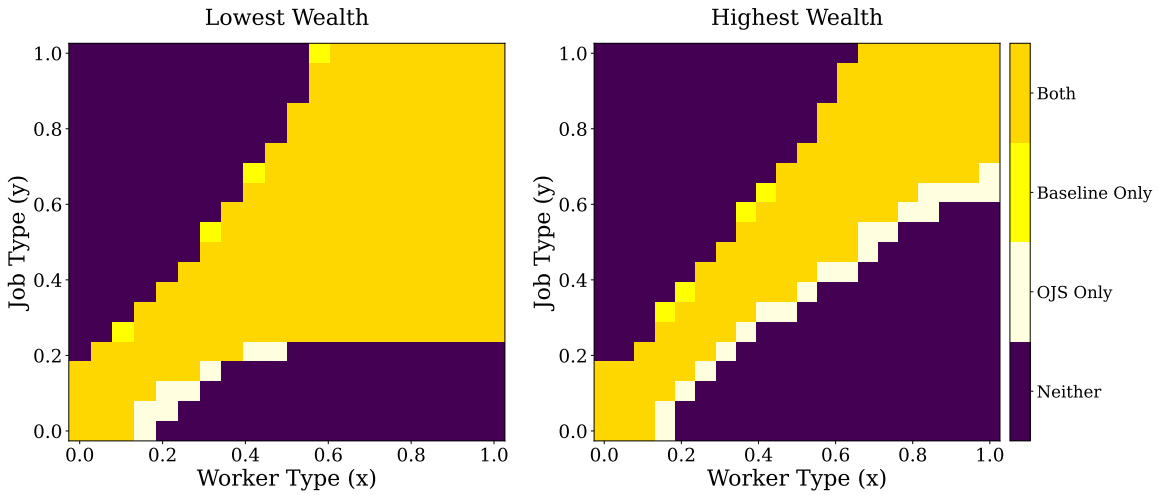
First, OJS expands the matching set in the bottom-right region (high x, low y). That is, workers accept more under-matches. This is because, with OJS, by accepting a job today, the worker can continue searching while employed and move to a better match if one arrives. This option value of OJS lowers the worker's reservation match quality and makes worse jobs acceptable relative to the baseline.

Second, the impact of OJS on the matching set is asymmetric. OJS shrinks the matching set in the top-left region (low x, high y). That is, jobs accept fewer under-qualified hires. This is because, when employed workers search as well, each vacancy draws from a larger pool of potential applicants and can poach workers from other firms. This increases the firm's outside option and makes firms more selective. Quantitatively, however, the contraction of the top-left region is smaller than the expansion in the bottom-right region.

Third, the impact of OJS on the matching set differs across wealth. The change in the matching set is more pronounced for high-wealth workers. For low-wealth workers, precautionary motives already generate wide matching sets in the baseline, leaving less room for OJS to further relax selectivity. By contrast, for high-wealth workers, who are more selective absent OJS, the option value of searching while employed leads to a more noticeable expansion in acceptable under-matches.

[20] Wages are continuously renegotiated via Nash bargaining, with unemployment as the worker's outside option (i.e., we abstract from counteroffers). Gottfries (2026) shows that continuous renegotiation resolves the Shimer (2006) critique and preserves the applicability of the axiomatic Nash bargaining solution.

Figure 7: Matching Sets: Baseline vs. On-the-Job Search



Notes: The figure compares matching sets between the baseline model (without on-the-job search) and the extended economy (with on-the-job search), for workers at the lowest asset level (left panel) and highest asset level (right panel). Orange indicates matches accepted in both economies; yellow indicates matches accepted only in the baseline; white indicates matches accepted only with OJS; and blue indicates matches rejected in both economies.

5.2.2 Equilibrium Matching

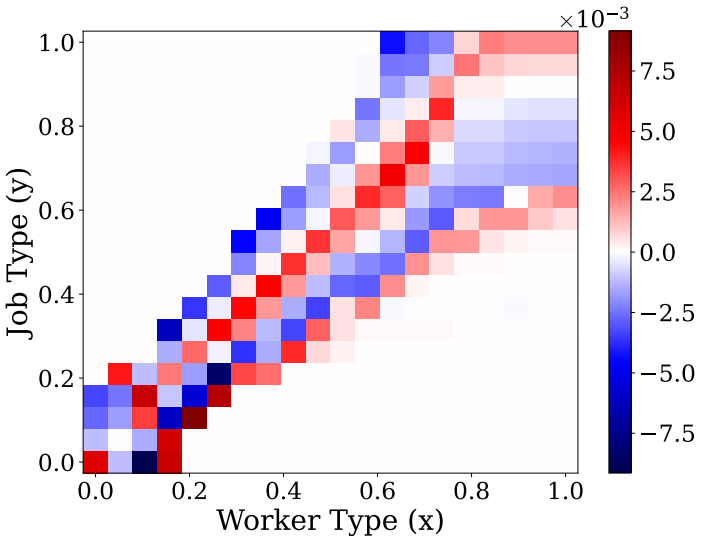
Figure 8 compares the equilibrium match distributions in the model with OJS and the baseline by plotting, for each (x, y) pair, the difference in match density between the two models. The figure reveals how OJS changes the equilibrium sorting pattern.

First, changes in the matching sets carry over directly into equilibrium matching. As shown in Figure 7, OJS expands the matching set in the bottom-right region and shrinks the matching set in the top-left region. Therefore, the model with OJS features more matches in the bottom-right region (high-x workers at low-y jobs) and fewer matches in the top-left region (low-x workers at high-y jobs) relative to the baseline.

Second, OJS allows employed workers to continue receiving offers. As a result, some workers who start in bad matches move to better matches over time. This dynamic reallocation shifts match density toward the diagonal, which tends to tighten sorting in steady state.

Overall, OJS affects mismatch through two countervailing forces. Red cells along the diagonal in Figure 8 (i.e., more perfect matches) reflect an ex post "reallocation effect": continued search while employed shifts match mass toward better matches. At the same time, red cells in the bottom-right region (i.e., more under-matches) reflect an ex ante "anticipation effect": unemployed workers are more willing to accept initially worse jobs because they expect to receive better offers. These forces work in opposite directions, so their net impact on sorting and productivity is ultimately a quantitative question.

Figure 8: Equilibrium Matching: Difference Between OJS and Baseline



Notes: The figure plots the difference in match density between the extended model with on-the-job search and the baseline model. Red indicates matches with greater mass under OJS; blue indicates matches with greater mass in the baseline.

5.2.3 Aggregate Implications

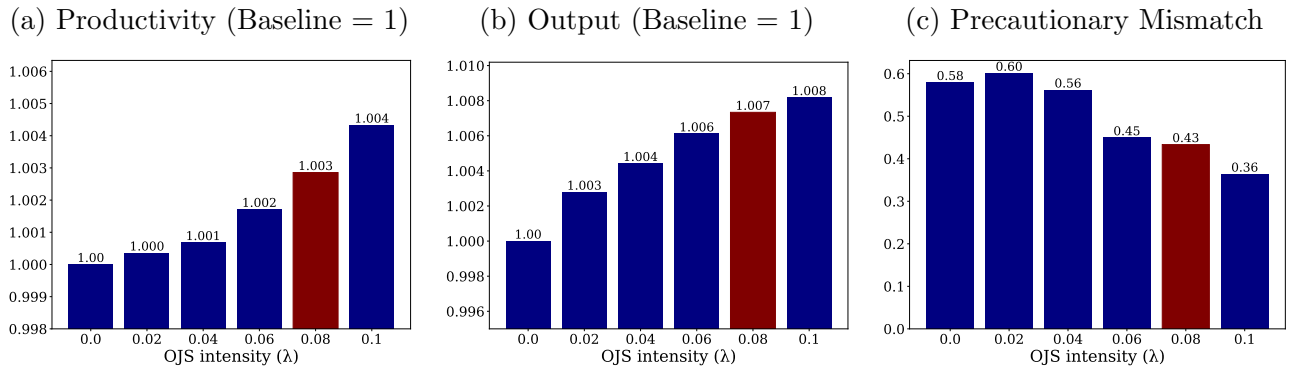
We now examine how OJS affects aggregate outcomes. Figure 9 reports productivity, total output, as well as productivity loss from precautionary mismatch, across different values of OJS intensity λ. The baseline economy without OJS corresponds to $\lambda = 0$. Calibrating the model with OJS to match a monthly job-to-job transition rate of 2% implies an empirically relevant OJS intensity of $\lambda = 0.08$, which we highlight in red.

Panel (a) shows that productivity rises with OJS intensity, but only very modestly. This muted response reflects the two opposing forces discussed above: an ex post reallocation effect, whereby workers move from worse to better matches over time, and an ex ante anticipation effect, whereby the prospect of future outside offers makes unemployed workers more willing to accept worse matches initially. The small net increase indicates that reallocation slightly dominates. At the empirically relevant value of $\lambda = 0.08$, productivity is 0.3% higher than in the baseline model without OJS.

Panel (b) shows that output also rises with OJS intensity, and by more than productivity. The larger increase reflects higher employment: the anticipation effect makes unemployed workers less selective, so they accept jobs more quickly. At $\lambda = 0.08$, output is 0.7% higher than in the baseline model without OJS.

Panel (c) reports how OJS intensity affects the productivity loss due to precautionary mismatch, measured as the productivity gap between the decentralized equilibrium with OJS and

Figure 9: Comparative Statics in OJS Intensity



Notes: The figure plots the productivity loss due to precautionary motives (relative to the risk-neutral benchmark) for varying levels of on-the-job search intensity λ. The red bar indicates the calibration $\lambda = 0.08$ that leads to an empirically relevant job-to-job transition rate. The baseline economy without OJS corresponds to $\lambda = 0$.

its "no precaution" counterfactual, in which all workers adopt the acceptance rule of the wealthiest workers of their respective types. This shuts down the precautionary motive and allows us to isolate the productivity losses attributable to precautionary mismatch.

In the baseline model without OJS ($\lambda = 0$), precautionary mismatch lowers productivity by 0.58%, consistent with Table 4. Introducing a small OJS intensity of $\lambda = 0.02$ increases the productivity loss to 0.60%. As OJS becomes more intense, the productivity loss decreases. At the empirically relevant value of $\lambda = 0.08$, eliminating precautionary mismatch raises productivity by 0.43% under OJS.

Why do we see the non-monotonic pattern? Precautionary motives operate through the ex ante anticipation channel, which governs job acceptance. With OJS, this anticipation effect is amplified: because workers expect more opportunities to climb up the job ladder, they are even more willing to accept bad matches initially. This pushes the economy toward more mismatch. However, the productivity loss is determined by where workers end up in the stationary equilibrium, not directly by which jobs they initially accept. At high λ, workers receive outside offers frequently and can correct initial mismatches quickly. As a result, the equilibrium distribution of matches becomes less sensitive to initial acceptance decisions, and the productivity loss from precautionary mismatch decreases. In the extreme where workers receive infinitely many outside offers instantly on the job, the initial mismatch becomes irrelevant.

6 Conclusion

This paper studies how wealth affects the allocation of workers to jobs. We develop a framework with two-sided heterogeneity, search frictions, and incomplete markets. We empirically document and theoretically analyze *precautionary mismatch*, whereby wealth-poor workers accept worse matches. This behavior reduces unemployment risk but comes at the cost of lower match quality at the micro level and reduced allocative efficiency in the aggregate.

Quantitatively, labor market mismatch is economically significant. Relative to a constrained output-maximizing reassignment of workers and jobs active in equilibrium, mismatch reduces aggregate labor productivity by about 3%. A counterfactual that shuts down precautionary motives reveals that precautionary mismatch accounts for roughly one-fifth of this productivity loss. These conclusions are robust to allowing for on-the-job search. OJS affects mismatch through two opposing channels—an ex post reallocation effect that strengthens sorting over time and an ex ante anticipation effect that weakens initial selectivity—which largely offset each other at the empirically relevant OJS intensity.

Several directions remain for future research. First, extending the analysis to multidimensional skills, as in Lise and Postel-Vinay (2020) and Guvenen et al. (2020), could shed light on how wealth shapes mismatch across different dimensions of comparative advantage. Second, we abstract from human capital accumulation; if match quality affects skill formation, early-career mismatch may have persistent consequences that amplify the long-run costs of precautionary mismatch. Third, it would be valuable to develop a directed-search version of our framework, which would require extending Eeckhout and Sepahsalari (2024) to incorporate worker-side productive heterogeneity. We leave these extensions for future work.

Data Availability Statement

The data and code underlying this article are available on Zenodo at `https://doi.org/10.5281/zenodo.23073818`.

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I Theoretical Appendix

I.1 Production Functions

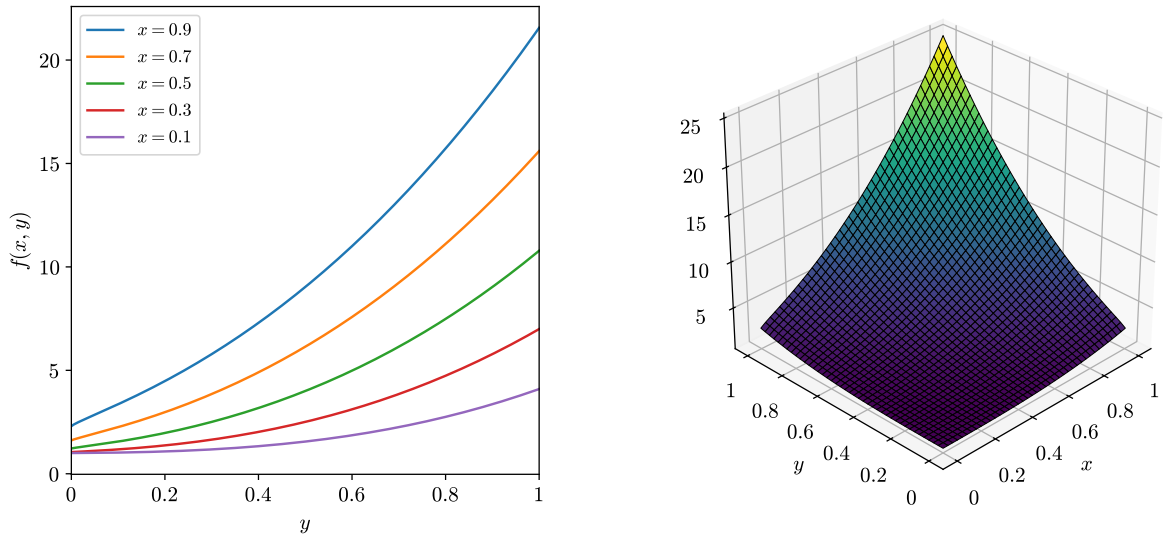
Production Function with Vertical Heterogeneity

We specify production using the CES form:

$$f(x,y) = f_0 + f_1 \left(x^{\xi} + y^{\xi}\right)^{\nu/\xi}, \quad x, y \in [0,1]$$

where f_0 controls the minimum level of production, f_1 controls its scale, ξ governs the degree of complementarity between worker type x and job type y, and ν governs the curvature of the production function with respect to type. We focus on parameterization in which output is increasing in both x and y, so workers and jobs can be vertically ranked by productivity. Complementarity between the two types generates positive assortative matching, with higher-type workers particularly productive at higher-type jobs. Figure A-1 illustrates this production function.

Figure A-1: Production Function $f(x,y) = f_0 + f_1 \left(x^{\xi} + y^{\xi}\right)^{\nu/\xi}$



Notes: This figure illustrates the production function with vertical heterogeneity. The left panel plots output as a function of job type y for different worker types x. The right panel shows the function in three dimensions.

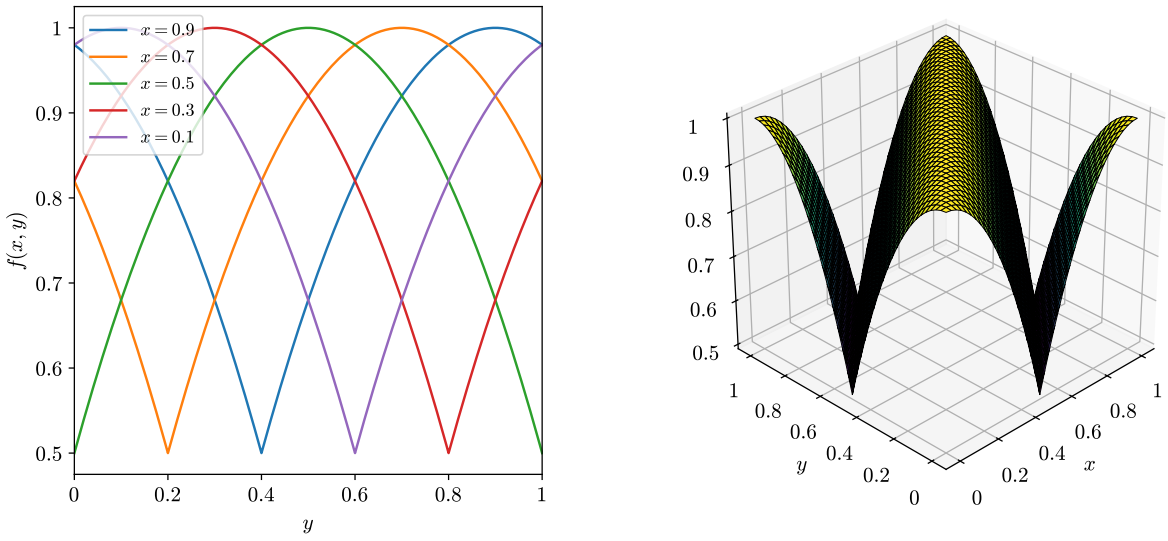
Production Function with Horizontal Heterogeneity

We specify production as

$$f(x,y) = f_0 - f_1 \min\left(|x-y|, |1+x-y|, |1+y-x|\right)^2, \quad x,y \in [0,1)$$

In this case, worker and job types are locations on a unit circle, with 0 and 1 representing the same point. Unlike the case of vertical heterogeneity, no worker or job type is intrinsically more productive than another. Instead, output depends only on the distance between the worker and the job on the circle. Each worker attains maximal output when matched with the job at the same location, and output declines as the distance between their types increases. Thus, heterogeneity is entirely in match quality rather than in the absolute productivity ranking of workers or jobs. Figure A-2 illustrates this production function.

Figure A-2: Production Function $f(x,y) = f_0 - f_1 \min\left(|x-y|, |1+x-y|, |1+y-x|\right)^2$



Notes: This figure illustrates the production function with horizontal heterogeneity. The left panel plots output as a function of job type y for different worker types x. The right panel shows the production function in three dimensions.

I.2 Proofs

I.2.1 Derivation of HJB Equations

We begin from the discrete-time problem with period length Δ:

$$W(a,x,y) = \max_{c} \mathfrak{u}(c)\,\Delta + \frac{1}{1+\rho\Delta}\Big\{ (1-\Delta\sigma)(1-\Delta\tilde{\sigma})\,W(a',x,y) + \underbrace{\Delta\sigma U(a',x)}_{\text{exogenous separation}} + (1-\Delta\sigma)\,\Delta\tilde{\sigma}\,\underbrace{\max\left[W(a',x,y), U(a',x)\right]}_{\text{endogenous separation}}\Big\}$$
$$\text{s.t.} \quad a' = a + (ra + \omega(a,x,y) - c)\,\Delta.$$

Over the interval, the worker separates exogenously with probability $\Delta\sigma$ and, conditional on remaining, receives an opportunity to reconsider the match with probability $\Delta\tilde{\sigma}$, exercising it only when unemployment dominates.

Multiplying both sides by $(1+\rho\Delta)$, subtracting W, and then dividing by Δ gives

$$\rho W(a,x,y) = \mathfrak{u}(c^e)(1+\rho\Delta) + \frac{1}{\Delta}\left[W(a',x,y) - W(a,x,y)\right] + \sigma\left[U(a',x) - W(a',x,y)\right] + (1-\Delta\sigma)\,\tilde{\sigma}\max\left[0, U(a',x) - W(a',x,y)\right],$$

where c^e denotes optimal consumption in employment.

Taking $\Delta \to 0$, the difference quotient converges to the drift term $\dot{a}\,W_a$ with $\dot{a} = ra + \omega - c^e$, the factors $(1+\rho\Delta)$ and $(1-\Delta\sigma)$ tend to one, and $a' \to a$. This yields the employed worker's HJB equation:

$$\rho W(a,x,y) = \mathfrak{u}(c^e) + \underbrace{(ra + \omega(a,x,y) - c^e)}_{\dot{a}} W_a(a,x,y) + \sigma\left[U(a,x) - W(a,x,y)\right] + \tilde{\sigma}\left[U(a,x) - W(a,x,y)\right]^+,$$

where $[\bullet]^+ \coloneqq \max\{\bullet, 0\}$. Allowing for a constant mortality rate δ is straightforward: under actuarially fair annuities it raises the return on wealth to $r+\delta$ and, since death delivers a continuation value of zero, the effective discount rate to $\rho+\delta$, matching equation (1). The remaining value functions are derived analogously.

I.2.2 Nash Bargaining

We start from the discrete-time problem with period length Δ. Consider an employed worker of type x with assets a at a job of type y who receives an arbitrarily deviating flow wage w for

one period, after which the wage reverts to the equilibrium bargained wage. The associated value satisfies

$$\tilde{W}(w,a,x,y) = \max_{c} \mathfrak{u}(c)\,\Delta + \frac{1}{1+\rho\Delta}\left\{(1-\Delta\sigma)(1-\Delta\delta)\,W(a',x,y) + \Delta\sigma U(a',x) + \Delta\delta\cdot 0\right\}$$
$$\text{s.t.} \quad a' = a + (\tilde{r}a + w - c)\,\Delta,$$

where $\tilde{r} := r + \delta$ is the annuitized return and death (at rate δ) delivers a continuation value of zero. Denote the optimal consumption policy by $\tilde{c}^e(w,a,x,y)$. The envelope condition with respect to w gives

$$\tilde{W}_w(w,a,x,y) = \frac{\Delta}{1+\rho\Delta}\{(1-\Delta\sigma)(1-\Delta\delta)\,W_a(a',x,y) + \Delta\sigma U_a(a',x)\}.$$

Similarly, the value of a producing job that pays the deviating wage w is

$$\tilde{J}(w,a,x,y) = f(x,y)\,\Delta - w\Delta + \frac{1}{1+r\Delta}\left[(1-\Delta(\sigma+\delta))\,J(a',x,y) + \Delta(\sigma+\delta)V(y)\right],$$

where the firm discounts at r and, from its point of view, the worker's consumption policy $\tilde{c}^e(w,a,x,y)$ is taken as given, so that $\partial a'/\partial w = (1-\tilde{c}^e_w)\,\Delta$. The envelope condition then yields

$$\tilde{J}_w(w,a,x,y) = -\Delta + \frac{\Delta}{1+r\Delta}\left[1-\Delta(\sigma+\delta)\right]J_a(a',x,y)\left[1-\tilde{c}^e_w(w,a,x,y)\right].$$

Under Nash bargaining, the wage policy solves

$$\omega(a,x,y) = \arg\max_{w}\left[\tilde{W}(w,a,x,y) - U(a,x)\right]^{\eta}\left[\tilde{J}(w,a,x,y) - V(y)\right]^{1-\eta}.$$

with first-order condition

$$\eta\left[\tilde{J}(w,a,x,y) - V(y)\right]\tilde{W}_w(w,a,x,y) + (1-\eta)\left[\tilde{W}(w,a,x,y) - U(a,x)\right]\tilde{J}_w(w,a,x,y) = 0.$$

To pass to continuous time, we characterize the limiting ratio of the two marginal values.

Lemma I.1.

$$\lim_{\Delta\to 0}\frac{-\tilde{J}_w(w,a,x,y;\Delta)}{\tilde{W}_w(w,a,x,y;\Delta)} = \frac{1-J_a(a,x,y)}{W_a(a,x,y)}.$$

Proof. Dividing numerator and denominator by Δ and substituting the envelope expressions,

$$\frac{-\tilde{J}_w}{\tilde{W}_w} = \frac{1-\frac{1}{1+r\Delta}(1-\Delta(\sigma+\delta))\,J_a(a',x,y)\,(1-\tilde{c}^e_w(w,a,x,y))}{\frac{1}{1+\rho\Delta}\{(1-\Delta\sigma)(1-\Delta\delta)\,W_a(a',x,y) + \Delta\sigma U_a(a',x)\}}$$

As $\Delta \to 0$ we have $a' \to a$ and $\tilde{c}^e_w \to 0$ (Lemma I.2), while every factor of the form $(1 - \Delta\cdot)$ and $(1 + \Delta\cdot)$ tends to one and the term $\Delta\sigma\, U_a$ vanishes. Hence

$$\frac{-\tilde{J}_w}{\tilde{W}_w} \longrightarrow \frac{1 - J_a(a,x,y)}{W_a(a,x,y)}.$$

□

Lemma I.2. $\lim_{\Delta\to 0} \tilde{c}^e_w(w,a,x,y;\Delta) = 0$.

Proof. Optimal consumption is characterized by its first-order condition

$$\mathfrak{u}'(\tilde{c}^e) = \frac{1}{1+\rho\Delta}\left\{(1-\Delta\sigma)(1-\Delta\delta)\, W_a(a',x,y) + \Delta\sigma U_a(a',x)\right\}.$$

Taking $\Delta \to 0$ gives the limiting condition $\mathfrak{u}'(\tilde{c}^e) = W_a(a,x,y)$, so that $\tilde{c}^e \to (\mathfrak{u}')^{-1}(W_a(a,x,y))$, which does not depend on w. Under standard smoothness conditions permitting an interchange of limit and derivative,

$$\lim_{\Delta\to 0} \frac{\partial \tilde{c}^e}{\partial w}(w,a,x,y;\Delta) = \frac{\partial}{\partial w} \lim_{\Delta\to 0} \tilde{c}^e(w,a,x,y;\Delta) = \frac{\partial}{\partial w}(\mathfrak{u}')^{-1}(W_a(a,x,y)) = 0.$$

□

Taking $\Delta \to 0$ in the first-order condition, and using $\tilde{W} \to W$, $\tilde{J} \to J$ together with Lemma I.1, the bargaining condition becomes

$$\eta\left[J(a,x,y) - V(y)\right] + (1-\eta)\left[W(a,x,y) - U(a,x)\right]\frac{J_a(a,x,y) - 1}{W_a(a,x,y)} = 0,$$

which rearranges to the surplus-sharing rule

$$\eta\frac{J(a,x,y) - V(y)}{1 - J_a(a,x,y)} = (1-\eta)\frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)}. \tag{A1}$$

To express (A1) in terms of primitives, multiply both sides by $(\rho + \sigma + \delta)$. This particular scaling is convenient because it collapses the continuation terms of both value functions: substituting the flow (HJB) representations

$$\begin{aligned}(r+\sigma+\delta)\, J(a,x,y) &= f(x,y) - \omega(a,x,y) + \dot{a}\, J_a(a,x,y) + (\sigma+\delta)\, V(y),\\ (\rho+\sigma+\delta)\, W(a,x,y) &= \mathfrak{u}(\tilde{c}^e) + \dot{a}\, W_a(a,x,y) + \sigma\, U(a,x), \qquad \dot{a} \coloneqq (r+\delta)\, a + \omega - \tilde{c}^e,\end{aligned}$$

we obtain, after cancellation of the separation and death terms,

$$(\rho+\sigma+\delta)\left[J-V\right]=f\left(x,y\right)-\omega+\dot{a}\,J_{a}+\left(\rho-r\right)J-\rho V,$$
$$(\rho+\sigma+\delta)\left[W-U\right]=\mathfrak{u}\left(\tilde{c}^{e}\right)+\dot{a}\,W_{a}-\left(\rho+\delta\right)U.$$

Substituting these into the rescaled version of (A1) gives

$$\eta\frac{f\left(x,y\right)-\omega+\dot{a}\,J_{a}\left(a,x,y\right)+\left(\rho-r\right)J\left(a,x,y\right)-\rho V\left(y\right)}{1-J_{a}\left(a,x,y\right)}$$
$$=\left(1-\eta\right)\frac{\mathfrak{u}\left(\tilde{c}^{e}\right)+\dot{a}\,W_{a}\left(a,x,y\right)-\left(\rho+\delta\right)U\left(a,x\right)}{W_{a}\left(a,x,y\right)}.$$

Isolating the wage terms via $\dot{a}\,J_{a}=\left(\left(r+\delta\right)a-\tilde{c}^{e}\right)J_{a}+\omega J_{a}$ and $\dot{a}\,W_{a}=\left(\left(r+\delta\right)a-\tilde{c}^{e}\right)W_{a}+\omega W_{a}$, the ω terms combine as $\eta\omega+\left(1-\eta\right)\omega=\omega$, yielding the wage equation

$$\omega\left(a,x,y\right)=\eta\frac{f\left(x,y\right)+\left(\left(r+\delta\right)a-\tilde{c}^{e}\right)J_{a}\left(a,x,y\right)+\left(\rho-r\right)J\left(a,x,y\right)-\rho V\left(y\right)}{1-J_{a}\left(a,x,y\right)}$$
$$-\left(1-\eta\right)\frac{\mathfrak{u}\left(\tilde{c}^{e}\right)+\left(\left(r+\delta\right)a-\tilde{c}^{e}\right)W_{a}\left(a,x,y\right)-\left(\rho+\delta\right)U\left(a,x\right)}{W_{a}\left(a,x,y\right)}. \tag{A2}$$

I.2.3 Proof of Proposition 2

The proposition follows from a single-crossing property of the adjusted surplus with respect to wealth. Fix a worker type x and a job type y, and define $\Delta\left(a;x,y\right)\coloneqq W\left(a,x,y\right)-U\left(a,x\right)$.

Sign of the surplus. From the Nash bargaining solution,

$$\frac{W\left(a,x,y\right)-U\left(a,x\right)}{W_{a}\left(a,x,y\right)}=\eta\,\hat{S}\left(a,x,y\right).$$

Worker optimization implies $W_{a}\left(a,x,y\right)=\mathfrak{u}'\left(c^{e}\left(a,x,y\right)\right)>0$. Since $\eta\in\left(0,1\right)$, the adjusted match surplus and the worker's surplus have the same sign:

$$\operatorname{sign}\hat{S}\left(a,x,y\right)=\operatorname{sign}\Delta\left(a;x,y\right).$$

It therefore suffices to characterize how $\Delta(a;x,y)$ crosses zero as wealth changes.

Slope of the worker surplus. Consider any wealth level a at which the match (x,y) is acceptable. Under DARA, the cross-state consumption ordering established by Theorem 2.2.3 in Rendon (2025) implies that consumption is higher in employment than in unemployment for

acceptable jobs: $c^e(a,x,y) > c^u(a,x)$. Hence

$$\Delta_a(a;x,y) = W_a(a,x,y) - U_a(a,x) = \mathfrak{u}'(c^e(a,x,y)) - \mathfrak{u}'(c^u(a,x)) < 0.$$

Therefore, among acceptable jobs, the worker's surplus from a given match is strictly decreasing in wealth.

Single crossing. The preceding result implies that $\Delta(a;x,y)$ can cross zero at most once. Therefore, for each pair (x,y), there is at most one threshold $a^*(x,y)$ such that $\Delta(a^*(x,y);x,y) = 0$, with the match accepted below the threshold and rejected above it:

$$\Delta(a;x,y) \geq 0 \quad \Longleftrightarrow \quad a \leq a^*(x,y),$$

whenever an interior threshold exists. If no threshold exists, the match is either accepted or rejected at all wealth levels, which is also consistent with the same nesting property.

Since $\hat{S}$ and Δ have the same sign, it follows that if a match (x,y) is acceptable at wealth a', it is also acceptable at any lower wealth level $a < a'$. Hence

$$\mathbb{M}(a) \supseteq \mathbb{M}(a'), \qquad a < a'.$$

The inclusion is strict whenever there exists a type pair (x,y) with an interior wealth threshold $a^*(x,y) \in (a,a')$, so that the match is acceptable at a but not at a'.

I.2.4 Proof of Proposition 3

Consider two wealth levels $a > a'$. By Proposition 2, the matching set shrinks with wealth, $\mathbb{M}(a) \subseteq \mathbb{M}(a')$. Equivalently, for every job type y, $\Phi(a,x,y) \leq \Phi(a',x,y)$. Therefore,

$$\pi_{ue}(a,x) = p(\theta) \int \frac{d_v(y)}{v} \Phi(a,x,y)\,\mathrm{d}y \leq p(\theta) \int \frac{d_v(y)}{v} \Phi(a',x,y)\,\mathrm{d}y = \pi_{ue}(a',x).$$

I.2.5 Derivation of Euler Equations

We derive the Euler equations for interior asset choices and away from the boundaries of the acceptance and separation regions.

Employed workers. Applying the envelope theorem to employed workers' HJB with respect to assets a gives

$$(\rho+\delta)W_a(a,x,y) = \sigma\left[U_a(a,x) - W_a(a,x,y)\right] + \left[r+\delta+\omega_a(a,x,y)\right]W_a(a,x,y) + \dot{a}^e W_{aa}(a,x,y).$$

Canceling the δW_a terms yields

$$\rho W_a(a,x,y) = \sigma\left[U_a(a,x) - W_a(a,x,y)\right] + \left[r+\omega_a(a,x,y)\right]W_a(a,x,y) + \dot{a}^e W_{aa}(a,x,y).$$

Along the worker's optimal asset path,

$$\dot{W}_a(a,x,y) = W_{aa}(a,x,y)\dot{a}^e.$$

Using the first-order conditions $W_a(a,x,y) = \mathfrak{u}'(c^e)$ and $U_a(a,x) = \mathfrak{u}'(c^u)$, we also have

$$\dot{W}_a = \mathfrak{u}''(c^e)\dot{c}^e.$$

Substituting these expressions into the envelope equation gives

$$\mathfrak{u}''(c^e)\,\dot{c}^e = (\rho - r - \omega_a)\,\mathfrak{u}'(c^e) - \sigma\left[\mathfrak{u}'(c^u) - \mathfrak{u}'(c^e)\right]$$

Therefore,

$$\underbrace{-\frac{\mathfrak{u}''(c^e)\,c^e}{\mathfrak{u}'(c^e)}}_{\text{relative risk aversion}} \cdot \underbrace{\frac{\dot{c}^e}{c^e}}_{\text{consumption growth}} = r - \rho + \omega_a + \sigma\left[\frac{\mathfrak{u}'(c^u)}{\mathfrak{u}'(c^e)} - 1\right]$$

Unemployed workers. Applying the envelope theorem to unemployed workers' HJB with respect to assets a,

$$(\rho+\delta)U_a(a,x) = p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[W_a(a,x,y) - U_a(a,x)\right]\mathrm{d}y + (r+\delta)U_a(a,x) + \dot{a}U_{aa}(a,x)$$

where $B(a,x) \coloneqq \{y : W(a,x,y) \geq U(a,x)\}$. Canceling the δU_a terms yields

$$\rho U_a(a,x) = p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[W_a(a,x,y) - U_a(a,x)\right]\mathrm{d}y + rU_a(a,x) + \dot{a}U_{aa}(a,x)$$

Along the unemployed worker's optimal asset path,

$$\dot{U}_a(a,x) = U_{aa}(a,x)\dot{a}^u.$$

Using the first-order conditions $W_a(a,x,y) = \mathfrak{u}'(c^e)$ and $U_a(a,x) = \mathfrak{u}'(c^u)$, we also have

$$\dot{U}_a = \mathfrak{u}''(c^u)\dot{c}^u.$$

Substituting these expressions into the envelope equation gives

$$\mathfrak{u}''(c^u)\,\dot{c}^u = (\rho - r)\,\mathfrak{u}'(c^u) - p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[\mathfrak{u}'(c^e) - \mathfrak{u}'(c^u)\right] \mathrm{d}y$$

Therefore,

$$-\frac{\mathfrak{u}''(c^u)\,c^u}{\mathfrak{u}'(c^u)} \cdot \frac{\dot{c}^u}{c^u} = r - \rho + p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v}\left[\frac{\mathfrak{u}'(c^e)}{\mathfrak{u}'(c^u)} - 1\right] \mathrm{d}y,$$

where $-c\mathfrak{u}''(c)/\mathfrak{u}'(c)$ is the coefficient of relative risk aversion. Under CRRA preferences, this coefficient is constant and equal to γ.

I.3 On-the-Job Search

We extend the baseline model by allowing employed workers to search on the job with relative search intensity λ. The measure of effective searchers is $s = u + \lambda e$, so labor market tightness is $\theta = v/(u + \lambda e)$. An unemployed worker receives job offers at rate $p(\theta)$, an employed worker receives outside offers at rate $\lambda p(\theta)$, and a vacancy meets an effective searcher at rate $q(\theta)$.

Worker values. The HJB equation for an unemployed worker retains the same form as in the baseline model:

$$\begin{aligned}(\rho+\delta)U(a,x) &= \max_c \mathfrak{u}(c) + p(\theta)\int \frac{d_v(y)}{v}\left[W(a,x,y) - U(a,x)\right]^+ \mathrm{d}y + \dot{a}U_a(a,x),\\ \text{s.t. } \dot{a} &= (r+\delta)a + b(x) - c,\\ a &\geq \underline{a}.\end{aligned}$$

For an employed worker at job y, on-the-job search creates an additional option value from receiving and accepting better outside offers. Define

$$\Psi(a,x,y,y') \coloneqq \mathbb{1}\left\{W(a,x,y') \geq W(a,x,y)\right\}\Phi(a,x,y').$$

as the job-to-job transition rule. An employed worker at job y moves to a new job y' if the new job yields a higher value than her current job and the new worker–job pair forms an acceptable match. The first condition captures the worker's willingness to move, while $\Phi(a,x,y')$ ensures that the new match is mutually acceptable to the worker and the firm.

The HJB equation for an employed worker is therefore

$$\begin{aligned}(\rho+\delta)W(a,x,y) &= \max_{c}\mathfrak{u}(c)+\sigma\left[U(a,x)-W(a,x,y)\right]+\tilde{\sigma}\left[U(a,x)-W(a,x,y)\right]^{+}\\ &\quad+\lambda p(\theta)\int\frac{d_v(y')}{v}\Psi(a,x,y,y')\left[W(a,x,y')-W(a,x,y)\right]\mathrm{d}y'+\dot{a}W_a(a,x,y),\\ \text{s.t.}\quad &\dot{a}=(r+\delta)a+\omega(a,x,y)-c,\\ &a\geq\underline{a}.\end{aligned}$$

Firm values. If an employed worker moves from her current job y to a new job y', the previous job becomes vacant. Hence the HJB equation for a producing job is

$$\begin{aligned}rJ(a,x,y) &= f(x,y)-\omega(a,x,y)+(\sigma+\delta)\left[V(y)-J(a,x,y)\right]+\tilde{\sigma}\left[V(y)-J(a,x,y)\right]^{+}\\ &\quad+\lambda p(\theta)\int\frac{d_v(y')}{v}\Psi(a,x,y,y')\left[V(y)-J(a,x,y)\right]\mathrm{d}y'+\dot{a}^{e}(a,x,y)J_a(a,x,y),\end{aligned}$$

where $\dot{a}^{e}(a,x,y)\coloneqq(r+\delta)a+\omega(a,x,y)-c^{e}(a,x,y)$ is the worker's optimal asset-accumulation policy while employed.

A vacancy meets an effective searcher at rate $q(\theta)$. Conditional on a meeting, the searcher is unemployed with probability $u/(u+\lambda e)$ and employed with probability $\lambda e/(u+\lambda e)$. The value of a vacant job therefore satisfies

$$\begin{aligned}rV(y) &= q(\theta)\frac{u}{u+\lambda e}\iint\frac{d_u(a,x)}{u}\left[J(a,x,y)-V(y)\right]^{+}\mathrm{d}a\,\mathrm{d}x\\ &\quad+q(\theta)\frac{\lambda e}{u+\lambda e}\iiint\frac{d_m(a,x,y')}{e}\Psi(a,x,y',y)\left[J(a,x,y)-V(y)\right]\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y'.\end{aligned}$$

The first term captures meetings with unemployed workers. The second captures meetings with employed workers who leave their current job y' for the vacancy of type y. In the latter case, $\Psi(a,x,y',y)=1$ implies that the new match is acceptable, so the surplus $J(a,x,y)-V(y)$ is nonnegative under Nash bargaining.

Kolmogorov forward equations. The stationary density of employed workers satisfies

$$\begin{aligned}0 &= -\frac{\partial}{\partial a}\left[\dot{a}^{e}(a,x,y)\,d_m(a,x,y)\right]\\ &\quad-\left\{\sigma+\delta+\tilde{\sigma}\left[1-\Phi(a,x,y)\right]+\lambda p(\theta)\int\frac{d_v(y')}{v}\Psi(a,x,y,y')\,\mathrm{d}y'\right\}d_m(a,x,y)\\ &\quad+d_u(a,x)\,p(\theta)\frac{d_v(y)}{v}\Phi(a,x,y)+\int d_m(a,x,y')\,\lambda p(\theta)\frac{d_v(y)}{v}\Psi(a,x,y',y)\,\mathrm{d}y',\quad\forall\ a,x,y.\end{aligned}$$

The first term captures asset drift. The second collects outflows from the state (a, x, y) due to exogenous separation, exit, endogenous separation, and job-to-job transitions. The third term captures inflows from unemployment, while the final term captures workers arriving from other jobs through on-the-job search.

The stationary density of unemployed workers has the same form as in the baseline model, with $p(\theta)$ now evaluated at $\theta = v/(u + \lambda e)$:

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}^u(a,x)\, d_u(a,x)\right] - p(\theta)\int \frac{d_v(y)}{v}\Phi(a,x,y)\, d_u(a,x)\,\mathrm{d}y - \delta d_u(a,x)$$
$$+ \sigma\int d_m(a,x,y)\,\mathrm{d}y + \tilde{\sigma}\int \left[1-\Phi(a,x,y)\right] d_m(a,x,y)\,\mathrm{d}y + \delta d_w(x)h(a), \quad \forall\ a, x.$$

Job-to-job transitions do not appear in the unemployment KF equation because they move workers directly between employed states without an intervening unemployment spell.

II Algorithmic Appendix

II.1 Overview of the Algorithm

The algorithm we develop to solve for the stationary equilibrium extends Achdou et al. (2022) to frictional sorting with wages bargained over the entire state space. We use grids $\{a_1, \ldots, a_{N_a}\}$ for assets, $\{x_1, \ldots, x_{N_x}\}$ for worker types, and $\{y_1, \ldots, y_{N_y}\}$ for job types, with step lengths $\Delta_a, \Delta_x, \Delta_y$ (the non-uniform asset grid is described in the next subsection). In the discretized problem, we let $d_i^{jk,W}$, $d_i^{j,U}$, and d_v^k denote the masses associated with the corresponding grid cells. Thus,

$$e = \sum_{i,j,k} d_i^{jk,W}, \qquad u = \sum_{i,j} d_i^{j,U}, \qquad v = \sum_k d_v^k.$$

1. Guess the vacancy masses d_v^k and the match masses $d_i^{jk,W}$. The remaining masses follow from the add-up conditions. A convenient initialization is $v = 1 - e = 0.1$, $d_v^k/v = 1/N_y$, and $d_i^{jk,W} = e/(N_a N_x N_y)$.

2. Initialize the value functions $\{W(a,x,y), U(a,x), J(a,x,y), V(y)\}$. For numerical implementation, we use as the initial guess the corresponding risk-neutral equilibrium, which can be solved from the frictional sorting model of Shimer and Smith (2000).

3. Given the current value functions, compute the consumption policy from $c^e(a,x,y) = (\mathfrak{u}')^{-1}(W_a(a,x,y))$, and compute the wage schedule directly from equation (A2) in Appendix I.2.2.

4. Given the resulting wage schedule, update the workers' and firms' value functions by solving their HJB equations using the implicit finite-difference method (Achdou et al., 2022) described below.

5. Iterate Steps 3–4 until the value functions and the implied wage schedule converge.

6. Given the converged value and policy functions, solve the Kolmogorov forward (KF) equations for the stationary distribution using the upwind scheme (Achdou et al., 2022) and normalize the cell masses to sum to one.

7. Update d_v and d_m. In practice, we use damping in the density update to facilitate convergence.

8. Check convergence of the equilibrium densities. If they have not converged, return to Step 3 using the current value functions as the initialization for the new density iteration.

Each iteration solves three systems of $N_a \times N_x \times N_y$ equations: the workers' and firms' HJBs, which are nonlinear (the drift and the option terms depend on the value functions) and are themselves solved by inner iterations, and the linear KF system, solved in one step up to normalization.

II.2 Details of the Algorithm

II.2.1 HJB Equations

Let $W(a,x,y)$ and $U(a,x)$ denote the employed and unemployed values. Mortality arrives at rate δ and delivers a continuation value of zero, so it enters the HJBs only through the effective discount rate $\rho+\delta$. The reallocation of the deceased to $a=0$ affects the KF equations, not the HJBs. The value functions solve

$$\begin{aligned}(\rho+\delta)W(a,x,y) &= \max_c \mathfrak{u}(c) + \sigma\left[U(a,x) - W(a,x,y)\right] + \tilde{\sigma}\left[U(a,x) - W(a,x,y)\right]^+ \\ &\quad + \left(\tilde{r}a + \omega(a,x,y) - c\right) W_a(a,x,y), \\ (\rho+\delta)U(a,x) &= \max_c \mathfrak{u}(c) + p(\theta)\int \frac{d_v(y)}{v}\left[W(a,x,y) - U(a,x)\right]^+ \mathrm{d}y \\ &\quad + \left(\tilde{r}a + b(x) - c\right) U_a(a,x),\end{aligned}$$

with the first-order conditions $\mathfrak{u}'(c) = W_a(a,x,y)$ and $\mathfrak{u}'(c) = U_a(a,x)$. Because the bargained wage satisfies $\operatorname{sign}(J-V) = \operatorname{sign}(W-U)$, the worker's and firm's endogenous-separation decisions coincide. We encode both through the single indicator $\mathbb{1}_i^{jk} = \mathbb{1}\{W(a_i,x_j,y_k) > U(a_i,x_j)\}$

and write $\sigma_i^{jk} = \sigma$ when $\mathbb{1}_i^{jk} = 1$ and $\sigma_i^{jk} = \sigma + \tilde{\sigma}$ otherwise. The finite-difference approximations are

$$\begin{aligned}(\rho+\delta) W(a_i, x_j, y_k) &= \mathfrak{u}(c_{i,j,k}) + \sigma_i^{jk} [U(a_i, x_j) - W(a_i, x_j, y_k)] \\ &\quad + (\tilde{r} a_i + \omega(a_i, x_j, y_k) - c_{i,j,k}) W_a(a_i, x_j, y_k), \qquad \text{(A3)} \\ (\rho+\delta) U(a_i, x_j) &= \mathfrak{u}(c_{i,j}) + p(\theta) \sum_k \frac{d_v^k}{v} [W(a_i, x_j, y_k) - U(a_i, x_j)]^+ \\ &\quad + (\tilde{r} a_i + b(x_j) - c_{i,j}) U_a(a_i, x_j). \qquad \text{(A4)}\end{aligned}$$

The producing and vacant firms' HJBs are

$$\begin{aligned}r J(a_i, x_j, y_k) &= f(x_j, y_k) - \omega(a_i, x_j, y_k) + \left(\sigma_i^{jk} + \delta\right) [V(y_k) - J(a_i, x_j, y_k)] \\ &\quad + (\tilde{r} a_i + \omega(a_i, x_j, y_k) - c_{i,j,k}) J_a(a_i, x_j, y_k), \qquad \text{(A5)} \\ r V(y_k) &= q(\theta) \sum_i \sum_j \frac{d_i^{j,U}}{u} [J(a_i, x_j, y_k) - V(y_k)]^+. \qquad \text{(A6)}\end{aligned}$$

Note that σ_i^{jk} already folds in endogenous separation. The producing firm's asset state a drifts at its employee's saving rate $\tilde{r} a_i + \omega_{i,j,k} - c_{i,j,k}$, so the discretization of J_a reuses the worker's upwind coefficients.

II.2.2 Upwind Scheme for the HJBs

Following Achdou et al. (2022), we use forward differences where saving is positive and backward differences where it is negative. Define

$$W_{a,F}(a_i, x_j, y_k) = \frac{W(a_{i+1}, x_j, y_k) - W(a_i, x_j, y_k)}{\Delta_a},$$

$$W_{a,B}(a_i, x_j, y_k) = \frac{W(a_i, x_j, y_k) - W(a_{i-1}, x_j, y_k)}{\Delta_a},$$

and let $\bar{W}_a(a_i, x_j, y_k) = \mathfrak{u}'(\tilde{r} a_i + \omega(a_i, x_j, y_k))$ be the marginal value at zero saving. With $c = (\mathfrak{u}')^{-1}(W_a)$, the implied forward and backward saving rates are

$$s_{i,j,k,F}^W = \tilde{r} a_i + \omega(a_i, x_j, y_k) - (\mathfrak{u}')^{-1}(W_{a,F}(a_i, x_j, y_k)),$$

$$s_{i,j,k,B}^W = \tilde{r} a_i + \omega(a_i, x_j, y_k) - (\mathfrak{u}')^{-1}(W_{a,B}(a_i, x_j, y_k)),$$

and the upwind derivative is

$$W_a\left(a_i, x_j, y_k\right) = W_{a,B}\,\mathbb{1}\left\{s^W_{i,j,k,B} < 0\right\} + W_{a,F}\,\mathbb{1}\left\{s^W_{i,j,k,F} > 0\right\} + \bar{W}_a\,\mathbb{1}\left\{s^W_{i,j,k,F} < 0 < s^W_{i,j,k,B}\right\}. \tag{A7}$$

Concavity of W implies $s^W_{i,j,k,F} < s^W_{i,j,k,B}$, so exactly one branch applies. If $s^W_{i,j,k,F} < 0 < s^W_{i,j,k,B}$, saving is set to zero. The firm derivative J_a uses the same saving rates s^W. Writing $s^+ = \max\{s, 0\}$ and $s^- = \min\{s, 0\}$ and substituting (A7) into (A3),

$$\begin{aligned}\left(\rho + \delta\right) W_i^{jk} = \mathfrak{u}\left(c_i^{jk}\right) + \sigma_i^{jk}\left[U_i^j - W_i^{jk}\right] \\ + \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+} \end{array}\right]\left[\begin{array}{c} W_{i-1}^{jk} \\ W_i^{jk} \\ W_{i+1}^{jk} \end{array}\right],\end{aligned} \tag{A8}$$

and analogously for U, replacing the separation term with the job-finding term, and for J, using $f^{jk} - \omega_i^{jk}$ as the flow payoff, $\sigma_i^{jk} + \delta$ as the separation-plus-mortality rate, and the same upwind discretization for asset drift.

Non-uniform grid. It is often efficient to use a non-uniform asset grid, since the wealth distribution is right-skewed with a mode at low wealth. One choice is $a_i = a_{\min} + (a_{\max} - a_{\min})\frac{q^{\xi_i} - 1}{q - 1}$ with $q > 1$ and ξ_i uniform on $[0, 1]$. Defining $\Delta^F_{a,i} = a_{i+1} - a_i$ and $\Delta^B_{a,i} = a_i - a_{i-1}$, replace Δ_a by $\Delta^F_{a,i}$ in forward differences and by $\Delta^B_{a,i}$ in backward differences.

II.2.3 Implicit Method

Stack all worker value functions in $\mathbf{W}$, ordered by assets (innermost), then worker type, then firm type (outermost), with the unemployed block appended last. Collecting the transition terms of (A8) into a generator $\mathbf{A}\left(\mathbf{W}^n\right)$, the implicit method updates the value functions in the following way:

$$\frac{1}{\Delta}\left(\mathbf{W}^{n+1} - \mathbf{W}^n\right) + (\rho + \delta)\mathbf{W}^{n+1} = \tilde{\mathbf{u}}\left(\mathbf{W}^n\right) + \mathbf{A}\left(\mathbf{W}^n\right)\mathbf{W}^{n+1}$$

so that

$$\mathbf{W}^{n+1} = \left(\left(\rho + \delta + \frac{1}{\Delta}\right)\mathbf{I} - \mathbf{A}\left(\mathbf{W}^n\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^n\right) + \frac{1}{\Delta}\mathbf{W}^n\right)$$

where $\tilde{\mathbf{u}}$ collects the flow utilities using $\mathfrak{u}(c_i^{jk}) = \mathfrak{u}((\mathfrak{u}')^{-1}[W_a(a_i, x_j, y_k)])$.

The generator decomposes as $\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3$:

- $\mathbf{A}_1 = \text{blockdiag}\left(\mathbf{A}_{1e}, \mathbf{A}_{1u}\right)$ is the asset-drift block. Each diagonal sub-block is tridiagonal in the asset index, with subdiagonal, diagonal, and superdiagonal entries

$$\alpha_i^{jk,W} = \frac{-s_{i,B}^{jk,W-}}{\Delta_a}, \qquad \beta_i^{jk,W} = \frac{s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}}{\Delta_a}, \qquad \gamma_i^{jk,W} = \frac{s_{i,F}^{jk,W+}}{\Delta_a},$$

  and analogously for the unemployed. Each row sums to zero.

- $\mathbf{A}_2$ is the separation block. For each employed state (i, j, k) it places $-\sigma_i^{jk}$ on the diagonal and $+\sigma_i^{jk}$ in the column of the unemployed state (i, j); unemployed rows are zero.

- $\mathbf{A}_3$ is the job-finding block. For each unemployed state (i, j) it places $+p\left(\theta\right)\frac{d_v^k}{v}\mathbb{1}_i^{jk}$ in the column of each employed state (i, j, k) and $-p\left(\theta\right)\sum_k \frac{d_v^k}{v}\mathbb{1}_i^{jk}$ on the diagonal; employed rows are zero.

Firms are handled similarly. Stacking $\mathbf{J}$ and letting $\mathbf{\Pi}$ collect flow profits $f^{jk} - \omega_i^{jk}$, the implicit method updates firms' value functions in the following way

$$\mathbf{J}^{n+1} = \left[\left(r + \tfrac{1}{\Delta}\right)\mathbf{I} - \mathbf{B}\left(\mathbf{W}\right)\right]^{-1}\left(\mathbf{\Pi} + \tfrac{1}{\Delta}\mathbf{J}^n\right),$$

where $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3$. The asset-drift block $\mathbf{B}_1$ equals $\mathbf{A}_{1e}$, obtained for free from the workers' problem. The separation block $\mathbf{B}_2$ places $-\left(\sigma_i^{jk} + \delta\right)$ on the producing-firm diagonal and $+\left(\sigma_i^{jk} + \delta\right)$ in the vacancy column $V\left(y_k\right)$. The meeting block $\mathbf{B}_3$ places $q\left(\theta\right)\frac{d_i^{j,U}}{u}\left[\,\cdot\,\right]^+$ terms mapping $V\left(y_k\right)$ to $J\left(a_i, x_j, y_k\right)$.

II.2.4 Stationary Density

The Kolmogorov forward equations for the match density d_m and the unemployed density d_u are

$$\begin{aligned}
0 = &-\frac{\partial}{\partial a}\left[s^W\left(a,x,y\right)d_m\left(a,x,y\right)\right] - \{\sigma + \delta + \tilde{\sigma}[1 - \Phi(a,x,y)]\}d_m\left(a,x,y\right) \\
&+ p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u\left(a,x\right) \\
0 = &-\frac{\partial}{\partial a}\left[s^U\left(a,x\right)d_u\left(a,x\right)\right] - \int p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u(a,x)\,\mathrm{d}y - \delta d_u\left(a,x\right) \\
&+ \sigma\int d_m\left(a,x,y\right)\mathrm{d}y + \tilde{\sigma}\int\left[1 - \Phi\left(a,x,y\right)\right]d_m\left(a,x,y\right)\mathrm{d}y + \delta d_w(x)\cdot h(a)
\end{aligned}$$

where $\Phi\left(a,x,y\right) = \mathbb{1}\left\{W(a,x,y) \geq U(a,x)\right\}$. Mortality removes mass from every state at rate δ and returns it, for each type x, as unemployed newborns with asset distribution $h(a)$ and skill

distribution $d_w(x)$. These densities satisfy

$$1 = \iiint_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x\,\mathrm{d}y + \iint_{\underline{a}}^{\infty} d_u(a,x)\,\mathrm{d}a\,\mathrm{d}x,$$

as well as

$$\begin{aligned} d_w(x) &= \iint_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}y + \int_{\underline{a}}^{\infty} d_u(a,x)\,\mathrm{d}a, \\ d_j(y) &= \iint_{\underline{a}}^{\infty} d_m(a,x,y)\,\mathrm{d}a\,\mathrm{d}x + d_v(y). \end{aligned}$$

Discretization. Let $d_i^{jk,W}$ and $d_i^{j,U}$ denote the masses in the corresponding employed and unemployed grid cells. Likewise, let d_v^k denote the vacancy mass at job type y_k. We discretize the KF equations using the same upwind scheme as for the HJB equations. The KF transition matrix is the transpose of the corresponding HJB transition matrix. The discretized equations are

$$\begin{aligned} 0 = &\frac{s_{i-1,F}^{jk,W+}}{\Delta_a} d_{i-1}^{jk,W} + \frac{s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}}{\Delta_a} d_i^{jk,W} - \frac{s_{i+1,B}^{jk,W-}}{\Delta_a} d_{i+1}^{jk,W} \\ &- \left(\sigma_i^{jk} + \delta\right) d_i^{jk,W} + p(\theta)\,\frac{d_v^k}{v}\mathbb{1}_i^{jk}\, d_i^{j,U}, \\ 0 = &\frac{s_{i-1,F}^{j,U+}}{\Delta_a} d_{i-1}^{j,U} + \frac{s_{i,B}^{j,U-} - s_{i,F}^{j,U+}}{\Delta_a} d_i^{j,U} - \frac{s_{i+1,B}^{j,U-}}{\Delta_a} d_{i+1}^{j,U} \\ &- \left[\delta + p(\theta) \sum_k \frac{d_v^k}{v}\mathbb{1}_i^{jk}\right] d_i^{j,U} + \sum_k \sigma_i^{jk} d_i^{jk,W} + \delta\,\mathbb{1}\{i=1\}\left[\sum_{i'} d_{i'}^{j,U} + \sum_{i'}\sum_k d_{i'}^{jk,W}\right]. \end{aligned}$$

Relative to a naive transpose, two adjustments matter. First, both equations carry a mortality outflow $-\delta d$ on the diagonal. Second, the separation outflow $-\sigma_i^{jk} d_i^{jk,W}$ in the employed equation is the source of the inflow $\sum_k \sigma_i^{jk} d_i^{jk,W}$ in the unemployed equation; both use σ_i^{jk}, which already includes endogenous separation. The newborn term reinjects the deceased of each type at $i = 1$.

In matrix form the system is

$$\tilde{\mathbf{A}}\,\mathbf{d} = 0, \tag{A9}$$

where $\tilde{\mathbf{A}} = (\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3)^\top + \mathbf{D}$, with $\mathbf{d}$ the stacked vector of cell masses in the same order as $\mathbf{W}$. Unlike Achdou et al. (2022), $\tilde{\mathbf{A}}$ is not simply the transpose of the HJB generator, because mortality enters the value functions as a discount but the flow equations as a physical reallocation. The correction $\mathbf{D}$ implements this: it subtracts δ from every diagonal entry (mortality outflow) and adds δ from every type-j state—employed (i,j,k) and unemployed

(i, j)—into the newborn state $(1, j, U)$. By construction every column of $\tilde{\mathbf{A}}$ sums to zero, so mass is conserved.

Normalization. The system (A9) is homogeneous, so we fix one component, e.g., $d_1^{1,U} = 0.1$, solve for the remaining $\tilde{d}$, and rescale by the total mass:

$$d_i^{jk,W} = \frac{\tilde{d}_i^{jk,W}}{M}, \quad d_i^{j,U} = \frac{\tilde{d}_i^{j,U}}{M}, \qquad M = \sum_{i,j,k} \tilde{d}_i^{jk,W} + \sum_{i,j} \tilde{d}_i^{j,U}.$$

The other masses then follow from the add-up conditions.

III Empirical Appendix

This appendix reports further empirical results on the relationships between liquid wealth, match quality, and labor-market outcomes that motivate the mechanism studied in the paper. We use the NLSY79 sample and the measure of liquid wealth defined in Section 3.

Specification. For worker i in period t we estimate

$$o_{it} = \beta \, \text{wealth}_{it} + \gamma' X_{it} + \varepsilon_{it}, \tag{A10}$$

where o_{it} is an outcome and wealth_{it} is liquid wealth in thousands of dollars. Match quality is summarized by the gap between the worker's productivity type x and the matched job's type y: the mismatch magnitude is $|y - x|$; under-match restricts the sample to matches with $y < x$, and over-match to $y > x$. Labor-market outcomes are job tenure (months), log wages, and months spent unemployed. The controls X_{it} enter cumulatively across columns in three blocks: demographic controls (race, gender, education, and a quadratic in age); experience and skill controls (quadratics in cumulative labor-market experience and occupational tenure, and ASVAB score); and economic controls (region fixed effects and the national unemployment rate). To absorb time-invariant worker heterogeneity, we also report worker fixed-effects estimates of

$$o_{it} = \beta \, \text{wealth}_{it} + \gamma' Z_{it} + \alpha_i + \varepsilon_{it}, \tag{A11}$$

where α_i is a worker fixed effect and Z_{it} collects the time-varying controls (the quadratic in age, the experience controls, and the economic controls); the time-invariant demographic controls are absorbed by α_i.

Pooled results. Table A-1 shows that higher liquid wealth is associated with significantly less under-match and a smaller overall mismatch magnitude, consistent with wealthier workers accepting matches closer to their own productivity. The over-match estimates are imprecise, reflecting the small subsample of over-matched workers. Table A-2 shows the complementary labor-market pattern: wealth predicts longer job tenure, higher wages, and more months unemployed, consistent with wealthier workers searching longer and forming better matches.

Fixed-effects results. Tables A-3 and A-4 re-estimate the specifications with worker fixed effects. The signs are preserved throughout: wealth continues to reduce under-match and mismatch magnitude and to raise tenure and wages.

Table A-1: Effect of Wealth on Mismatch

	Under-match			Over-match			Mismatch magnitude		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Liquid wealth ($1,000s)	-0.829***	-0.803***	-0.792***	1.374	1.382	1.470	-0.864***	-0.832***	-0.826***
	(0.112)	(0.113)	(0.113)	(1.609)	(1.618)	(1.650)	(0.116)	(0.117)	(0.117)
Demographic controls	✓	✓	✓	✓	✓	✓	✓	✓	✓
Experience controls		✓	✓		✓	✓		✓	✓
Economic controls			✓			✓			✓
Observations	5042	5042	5042	385	385	385	5427	5427	5427
Adjusted R^2	0.488	0.489	0.491	0.045	0.040	0.038	0.492	0.493	0.494

Notes: Pooled OLS estimates of equation (A10). All coefficients and standard errors are multiplied by 100. Under-match restricts the sample to matches with $y < x$, over-match to $y > x$, and mismatch magnitude uses $|y - x|$ on the full sample. Control blocks are defined in the text. Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A-2: Effect of Wealth on Labor Market Outcomes

	Job tenure (months)			Log wages			Months unemployed		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Liquid wealth ($1,000s)	0.421***	0.402***	0.365***	0.011***	0.010***	0.010***	0.092***	0.097***	0.087***
	(0.052)	(0.053)	(0.052)	(0.001)	(0.001)	(0.001)	(0.023)	(0.023)	(0.023)
Demographic controls	✓	✓	✓	✓	✓	✓	✓	✓	✓
Experience controls		✓	✓		✓	✓		✓	✓
Economic controls			✓			✓			✓
Observations	11727	11727	11727	5927	5927	5927	6510	6510	6510
Adjusted R^2	0.032	0.040	0.061	0.144	0.164	0.170	0.032	0.032	0.049

Notes: Pooled OLS estimates of equation (A10). Control blocks are defined in the text. Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A-3: Effect of Wealth on Mismatch (Worker Fixed Effects)

	Under-match		Over-match		Mismatch magnitude	
	(1)	(2)	(3)	(4)	(5)	(6)
Liquid wealth ($1,000s)	-0.637***	-0.490***	-2.228	-2.225	-0.671***	-0.520***
	(0.156)	(0.158)	(2.867)	(3.114)	(0.163)	(0.164)
Time-varying controls		✓		✓		✓
Worker fixed effects	✓	✓	✓	✓	✓	✓
Observations	5042	5042	385	385	5427	5427
Within R^2	0.005	0.019	0.005	0.061	0.005	0.017

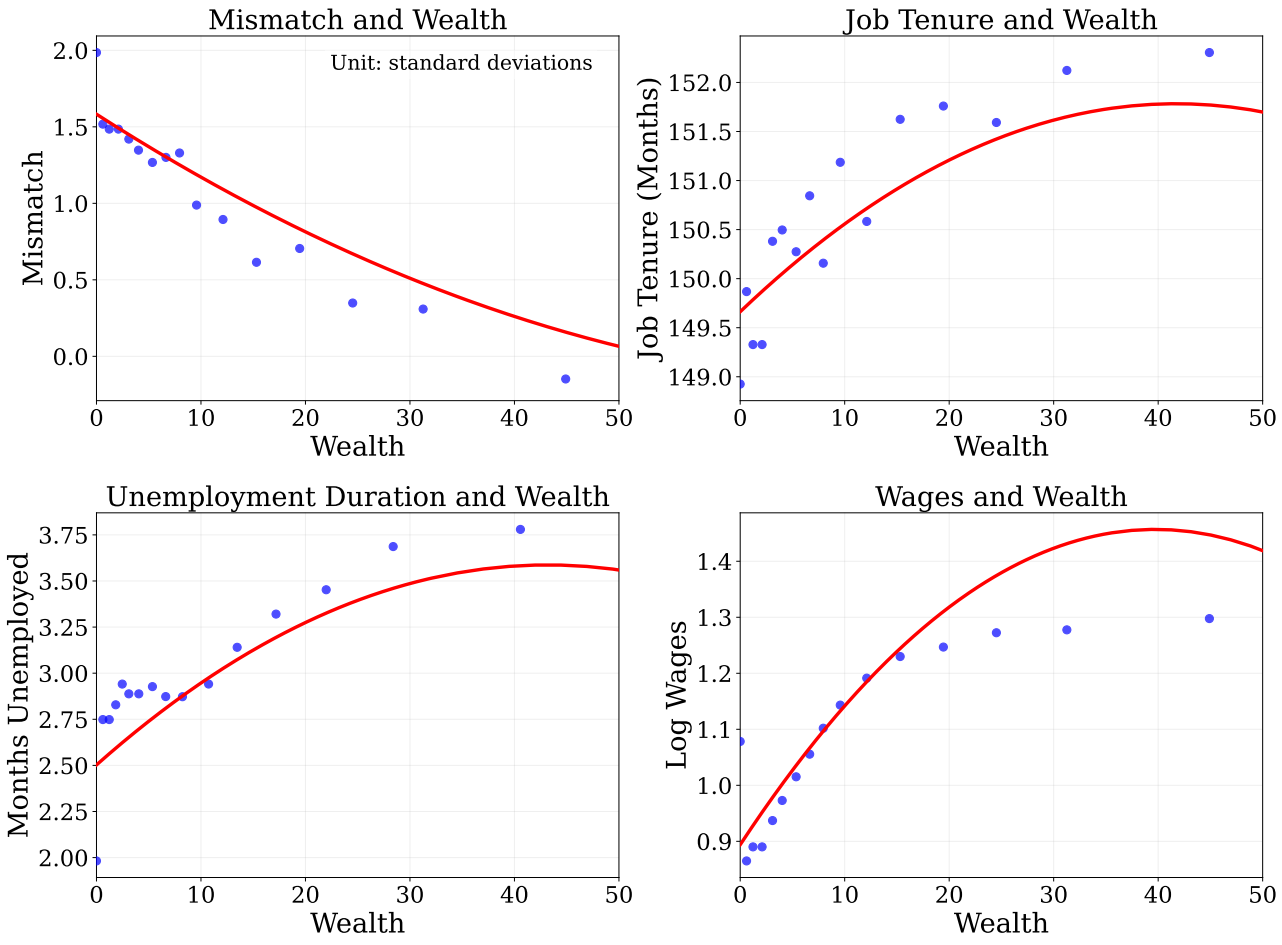
Notes: Worker fixed-effects estimates of equation (A11). All coefficients and standard errors are multiplied by 100. Sample definitions are as in Table A-1. Time-varying controls comprise the quadratic in age, the experience controls, and the economic controls; time-invariant controls (race, gender, education, ASVAB score) are absorbed by the worker fixed effects. Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A-4: Effect of Wealth on Labor Market Outcomes (Worker Fixed Effects)

	Job tenure (months)		Log wages		Months unemployed	
	(1)	(2)	(3)	(4)	(5)	(6)
Liquid wealth ($1,000s)	0.537***	0.147**	0.004***	0.003**	0.066*	0.025
	(0.066)	(0.067)	(0.001)	(0.001)	(0.036)	(0.036)
Time-varying controls		✓		✓		✓
Worker fixed effects	✓	✓	✓	✓	✓	✓
Observations	12133	11727	6139	5927	6732	6510
Within R^2	0.007	0.086	0.003	0.021	0.001	0.083

Notes: Worker fixed-effects estimates of equation (A11). Time-varying controls comprise the quadratic in age, the experience controls, and the economic controls; time-invariant controls (race, gender, education, ASVAB score) are absorbed by the worker fixed effects. Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Figure A-3: Labor Market Impacts of Wealth in the Model



Notes: This figure plots the model-implied counterparts to Figure 2 using a large simulated sample from the stationary equilibrium. Because the model imposes a borrowing constraint at zero, the simulated wealth distribution has no support for negative wealth.