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Life-Cycle Wage Growth and Firm Productivity in Developed and Developing Economies

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Life-Cycle Wage Growth and Firm Productivity in Developed and Developing Economies^{*}

Abstract

Life-cycle wage growth varies substantially across countries. We examine the role of the firm productivity distribution by introducing a random-search model that disentangles firm heterogeneity, on-the-job learning, and labor-market frictions. Estimates for Brazil, Colombia, and the United States reveal two distinct disadvantages: a long left tail of low-paying firms in Brazil and a highly compressed firm ladder in Colombia. Counterfactuals show that equating each country's firm distribution to that of the United States closes most of the cross-country gap in life-cycle wage growth: Brazil's wage growth rises from 73 to 103 percent of the U.S. value, and Colombia's from 52 to 115 percent. These results are robust to alternative distributional assumptions, education residualization, and external discipline on the productivity tail.

JEL classification

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Keywords

labor markets, income profiles, developing countries

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1 Introduction

Life-cycle wage growth varies substantially across countries, with developed economies experiencing much steeper wage trajectories ([Lagakos et al., 2018](#)). Differences in on-the-job learning and job mobility have been shown to contribute to these disparities.¹ The steepness of life-cycle wage profiles, however, also depends on workers' access to high-paying firms. Evidence from less-developed economies suggests that a larger share of employment is concentrated in small, low-productivity firms, which may limit the scope for wage growth through movement up the job ladder ([Hsieh and Klenow, 2009](#); [Eslava et al., 2022](#)). In this paper, we show that this firm-side disadvantage is an important driver of cross-country differences in life-cycle wage growth, complementing the contributions of human-capital accumulation and labor-market mobility.

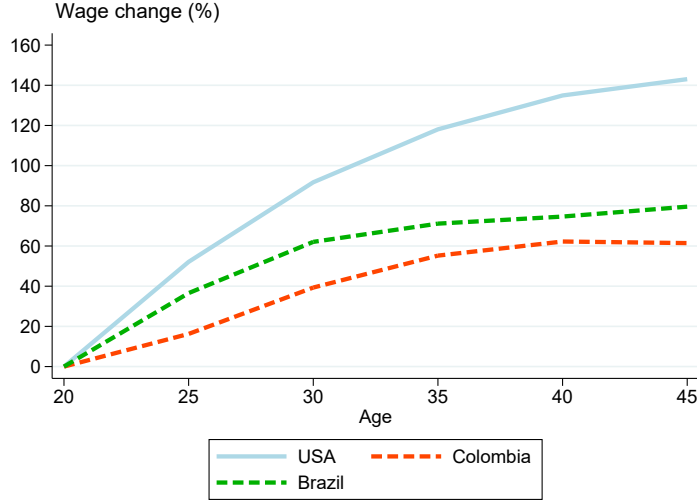
We develop a random-search model that disentangles the contributions of the firm productivity distribution, on-the-job learning, and labor-market frictions, building on [Burdett and Mortensen \(1998\)](#) and [Bagger et al. \(2014\)](#). We estimate two specifications: a baseline with a single learning rate and a life-cycle extension that allows learning to vary with age. The latter is our preferred specification because it avoids extrapolating early-career learning into later ages. The model shows that the firm productivity distribution interacts with on-the-job learning and labor-market frictions through two equilibrium channels. The shape of the wage schedule depends jointly on the friction and tail parameters. The reservation wage, which sets the bottom of the job ladder, depends on the learning rate and on the asymmetry between job-finding rates while employed and unemployed. The firm distribution therefore matters for wage growth not in isolation but through its interaction with the other primitives.

We focus on Colombia and Brazil because recent research has documented a predominance of small, older establishments in these countries, in sharp contrast to the firm-type

¹For the role of wage differences, see [Lagakos et al. \(2018\)](#). For on-the-job training, see [Ma et al. \(2024\)](#) and [Jedwab et al. \(2023\)](#). For job mobility, see [Donovan et al. \(2023\)](#) and [Engbom \(2022\)](#).

distribution in the United States (Eslava et al., 2022, 2021). Figure 1 shows that life-cycle wage growth is substantially flatter in these two economies than in the United States, consistent with prior findings (Lagakos et al., 2018).²

Figure 1: Age Profile of Wages



Notes: Wage change corresponds to the growth rate of average monthly wages for workers in each 5-year age bin relative to the average wages of workers between the ages of 20 and 24. Data for the United States come from PSID, for Brazil from PNADC, and for Colombia from the social security records. Figure A.1a in the Appendix shows the same figure for additional data sets.

Before estimating the model, we document a positive correlation between firm size composition and life-cycle wage growth across countries, across regions within country, and across workers. Following the model, we proxy the firm productivity distribution by the employment share at firms above a given size threshold, an approach supported by previous empirical studies (Hsieh and Klenow, 2014; Eslava et al., 2023). Countries and regions with a higher share of workers at larger firms exhibit steeper life-cycle wage growth, and the correlation remains positive after controlling for institutional and compositional factors. Worker-level estimates additionally show that the same worker earns substantially

²Figure 1 employs data from the PSID for the United States, from the PNADC for Brazil, and from the social security records for Colombia (also known as the “Planilla Integrada de Liquidación de Aportes,” or PILA). Very similar patterns are observed when using alternative data sources, when constructing experience-income profiles, when imposing alternative sample restrictions, and when adopting alternative identifying assumptions on the age–time–cohort decomposition, as shown in Appendix A.

higher wages when employed at a larger firm.

We estimate the model by simulated method of moments (SMM) using data commonly available in longitudinal labor-force surveys. Labor-market frictions are identified from worker transition rates and the unemployment rate. On-the-job learning is identified from the level and age profile of stayer wage growth. The remaining curvature of life-cycle wages disciplines the firm-side distribution, represented in the model by a Pareto distribution of firm productivity. The sequential design uses non-overlapping moments, so each parameter block is identified from a distinct set of moments.

Three results emerge from the estimation. First, the firm-side estimates suggest two distinct disadvantages relative to the United States: Brazil has a long lower tail of low-paying firms, whereas the Colombian wage profile pushes the model toward a highly compressed firm ladder with limited scope for advancement to substantially higher-paying firms, consistent with [Eslava et al. \(2022\)](#). Second, job-finding rates while employed are higher in Brazil and Colombia than in the United States, while job-finding rates while unemployed are lower. This reversal of the standard ranking found in developed economies is, to our knowledge, undocumented, and we validate it in cross-country data from [Donovan et al. \(2023\)](#). Third, cross-country differences in on-the-job learning operate on two margins, the level of early-career learning and the persistence of learning into older ages. Early-career learning is much lower in Colombia than in the other two countries, while persistence is higher in the United States than in Brazil. The standard finding that learning is stronger in more developed economies ([Ma et al., 2024](#)) therefore operates through the level margin in Colombia comparisons and through the persistence margin in the U.S.–Brazil comparison.

Using the model estimates, we then decompose wage growth into worker and firm components. At ages 40–44, the firm component accounts for roughly half of U.S. life-cycle wage growth, about a third in Brazil, and two-fifths in Colombia. Worker contributions vary modestly across countries, but firm contributions vary sharply, making firm heterogeneity

the dominant driver of cross-country differences in life-cycle wage growth.

A counterfactual analysis confirms the central role of the firm productivity distribution. Equalizing the firm-productivity distribution to the U.S. benchmark closes most of the life-cycle wage-growth gap: Brazil moves close to the U.S. profile, while Colombia even overshoots it. By contrast, equalizing learning or labor-market frictions has smaller and less uniform effects. These conclusions are robust to replacing the Pareto distribution with a shifted Weibull and to residualizing wage profiles for educational composition. We also use establishment-productivity data from the manufacturing sector to discipline the Pareto shape parameter. These data imply less compression of productivity in Colombia relative to the United States than the model estimates. Differences in sectoral coverage and aggregation, however, limit direct comparability. The qualitative counterfactual conclusion remains unchanged in this sensitivity exercise.

Our benchmark attributes wage growth among stayers to worker learning and measures the firm contribution primarily through workers’ movement along the job ladder. If firm heterogeneity also generates wage growth within continuing jobs, for example through firm-specific learning, promotions, or wage renegotiation, our decomposition omits these additional firm channels and may therefore understate firms’ contribution.

This paper builds on three areas of literature. The first studies the relationship between life-cycle wage growth and economic development. [Lagakos et al. \(2018\)](#) document that wage growth in developed economies is, on average, twice as steep as in developing economies, a pattern that [Fang and Qiu \(2023\)](#) confirms for China and [Jedwab et al. \(2023\)](#) extend to 145 countries. Several papers attribute these differences to human capital accumulation, whether through formal training ([Ma et al., 2024](#)), managerial learning ([Guner et al., 2018](#)), or access to high-learning locations ([Rivera-Padilla, 2025](#)). Others emphasize labor-market mobility, with [Engbom \(2022\)](#) finding that job-to-job mobility explains roughly half of the cross-country gap and [Donovan et al. \(2023\)](#) showing that job-finding and employment-exit rates are negatively correlated with development. We

contribute to this literature by emphasizing the role of the firm productivity distribution as a fundamental determinant of life-cycle wage growth.

The second is the literature on random-search models of wage growth within and across countries. Earlier work explains cross-country differences in mobility and labor market outcomes (Jolivet et al., 2006), as well as wage growth on the job (Rubinstein and Weiss, 2006; Barlevy, 2008; Yamaguchi, 2010; Gregory, 2020), but few studies decompose wage growth patterns. Bagger et al. (2014) and Menzio et al. (2016) are notable exceptions, decomposing wage growth into human capital accumulation and job search using two distinct frameworks. Building on the former, Ozkan et al. (2023) show that human capital growth patterns are important determinants of income differences within the United States. Burdett et al. (2011) propose a model related to Burdett and Mortensen (1998) that incorporates human capital accumulation but without firm heterogeneity, with a different aim from ours. In addition, Flinn et al. (2025) incorporate human-capital accumulation and wage growth into a random-search framework and clarify the relationship between the approaches of Burdett et al. (2011) and Bagger et al. (2014). Our paper adds a heterogeneous firm side and uses that structure to study how the distribution of wage opportunities shapes cross-country differences in life-cycle wage growth.

The third is the literature on the firm productivity distribution and development. Several papers document that the variance of firm productivity within narrowly defined industries is higher in less-developed countries, and that the growth rate of manufacturing plants over the life cycle is lower (Hsieh and Klenow, 2009, 2014; Hsieh and Olken, 2014; Poschke, 2018). In Latin America, Eslava et al. (2021) and Eslava et al. (2023) show that the firm size distribution is dominated by small businesses, a pattern that Eslava et al. (2022) attributes to the slower exit rate of smaller firms. Chen (2026) provides a theoretical foundation, showing that in a growth model with idea search, the right tail of the firm size distribution endogenously thickens with economic development. We contribute to this literature by linking a compressed firm productivity distribution to flatter life-cycle wage

growth for workers in developing countries.

The mechanism also has potential policy relevance. Policies that affect the creation and growth of productive firms, workers’ access to those firms, or the pass-through of firm productivity into wages may alter life-cycle wage growth by changing the job ladder workers face. Our counterfactuals quantify the importance of that ladder but do not identify the effects of any particular policy. Evaluating specific interventions requires an extension that models the relevant policy and firm responses explicitly.

The paper proceeds as follows. Section 2 presents the theoretical framework. Section 3 summarizes our data sources, and Section 4 provides the empirical motivation. Section 5 describes the identification and estimation strategy. Section 6 reports the model estimates and the counterfactual analysis. Finally, Section 7 draws the conclusion.

2 Framework

We present a random-search model to quantify the role of the firm-productivity distribution in shaping life-cycle wage profiles. The model builds on [Burdett and Mortensen \(1998\)](#) and incorporates worker heterogeneity, on-the-job learning, and an additive piece-rate wage structure, with event-timing conventions following [Bagger et al. \(2014\)](#).

2.1 Setting

Environment. The economy is populated by a continuum of workers of size M and a mass of firms of size L . Time is discrete and workers discount the future at rate $\tilde{\beta}$. Each period, a fraction ν of workers dies and is replaced by an equal mass of newborn workers. The aggregate effective discount rate is $\beta = (1 - \nu)\tilde{\beta}$.³ Workers and firms have heterogeneous productivity, indexed by h for workers and p for firms. Employed and unemployed workers meet a new firm with probability λ_1 and λ_0 , respectively, and employed workers face an

³Job acceptance decisions in our framework do not depend on life-cycle motives, following [Bagger et al. \(2014\)](#). We introduce age variation in learning in Section 2.5.

exogenous match-dissolution probability δ . We assume $\delta + \lambda_1 \leq 1$ so that at most one labor-market event occurs per period. In steady state, the flow balance $\delta(M - \tilde{U}) = \lambda_0 \tilde{U}$ pins down the unemployment mass $\tilde{U} = \delta M / (\delta + \lambda_0)$.

Workers are endowed with an initial human-capital level $h_0 \geq 0$ that increases by μ each period when employed, $h' = h + \mu$. Unemployed workers receive benefits $b(h) = b_0 + h$.⁴ Firm productivity is distributed according to $\Gamma(p)$ on $[\underline{p}, \infty)$, with anti-cdf $\bar{\Gamma}(p) = 1 - \Gamma(p)$, and density $\gamma(p)$.

Output of a worker-firm match is additive in the two productivities, $y = p + h$. Firms post a firm-specific wage component $w_0(p)$, so that the worker's total wage is

$$w(p, h) = w_0(p) + h. \quad (1)$$

Throughout, w denotes the log wage, h log human capital, and w_0 the log firm wage component. The law of motion $h' = h + \mu$ corresponds to a constant log-growth rate of human capital while employed, and (1) is a log-wage equation.⁵ The endogenous distribution of posted wage components is denoted $F(w_0)$, with $w_0 \in [\underline{w}_0, \bar{w}_0]$ and anti-cdf $\bar{F}(w_0) = 1 - F(w_0)$.

Timing. Within each period, an employed worker first separates exogenously with probability δ . If the match survives, human capital increases deterministically by μ , after which the worker receives an outside offer with probability λ_1 , observes the posted component w_0 of the offer, and moves if and only if the offer dominates the current job. Unemployed workers receive an offer with probability λ_0 and accept if and only if it exceeds the reservation

⁴The affine specification $b(h) = b_0 + h$ follows [Bagger et al. \(2014\)](#) and ensures that job-acceptance decisions are independent of h ; it is a tractability normalization rather than a literal description of real-world UI systems. Age-dependent reservation behavior, which would arise under proportional or capped UI, is captured in our framework through the aging effect in the life-cycle extension (equation 6) rather than through skill-varying benefits.

⁵The additive structure—total wage equal to a firm component plus the worker's human capital, with the firm component constant across worker skills—is the analogue of the piece-rate policy used in [Bagger et al. \(2014\)](#).

component.

2.2 Value Functions and Firm Problem

Let $U(h)$ denote the value of an unemployed worker with human capital h , and let $W(w_0, h)$ denote the value of an employed worker with human capital h at a firm offering w_0 . An unemployed worker receives the flow benefit $b(h)$ and, with probability λ_0 , meets a firm whose offer w_0 is accepted whenever it exceeds the reservation component θ^R (defined formally below). The lifetime value in unemployment is therefore

$$U(h) = b(h) + \beta U(h) + \beta \lambda_0 \int_{\theta^R}^{\bar{w}_0} [W(w_0, h) - U(h)] dF(w_0).$$

An employed worker earns $w(w_0, h)$ and faces three possibilities. With probability δ , the match dissolves and the worker returns to unemployment with value $U(h)$. If the match survives, human capital advances to $h' = h + \mu$; with probability λ_1 the worker draws an outside offer and chooses whether to switch, and otherwise stays with the incumbent. The employed worker's value is

$$\begin{aligned} W(w_0, h) = & w(w_0, h) + \beta \delta U(h) + \beta \lambda_1 \int \max\{W(w_0, h'), W(x, h')\} dF(x) \\ & + \beta(1 - \delta - \lambda_1)W(w_0, h'). \end{aligned}$$

Firms maximize profits,⁶ which under the piece-rate policy are constant per worker and equal to per-worker rents $p - w_0$ times firm size $l(w_0)$:

$$\pi(p) = \max_{w_0} (p - w_0) l(w_0).$$

⁶In a steady-state wage-posting equilibrium, the firm's discounted profit stream is constant per period at $(p - w_0)l(w_0)$, so maximizing the discounted value is equivalent to maximizing per-period profit (Burdett and Mortensen, 1998; Bagger et al., 2014). The firm's discount rate, therefore, plays no role in the wage-posting decision.

In steady state, firm size is

$$l(w_0) = \frac{A}{[\delta + \lambda_1 \bar{F}(w_0)]^2}, \quad (2)$$

where A is a constant of proportionality that depends on the population mass M , the unemployment mass $\tilde{U}$, and the firm mass L .⁷ The equilibrium-consistency condition $F(w_0(p)) = \Gamma(p)$ links posted wages to the firm-productivity distribution.

2.3 Equilibrium

Definition 2.1 (Equilibrium). An equilibrium consists of: (i) a reservation component θ^R , (ii) a strictly increasing posting function $w_0(p)$, (iii) a distribution of posted components F , and (iv) a firm size function $l(\cdot)$ such that: (a) workers optimize given (θ^R, F, w_0) ; (b) firms choose w_0 to maximize profits $(p - w_0)l(w_0)$; (c) steady-state worker flows imply $l(w_0)$; and (d) the equilibrium mapping holds, $F(w_0(p)) = \Gamma(p)$.

Reservation wages. The reservation wage component, θ^R , is the value at which an unemployed worker is indifferent between accepting and remaining unemployed, $W(\theta^R, h) = U(h)$. We guess and verify an equilibrium in which θ^R is identical for all workers (Appendix B.2, Lemmas B.1 and B.2), and obtain

$$\theta^R = \underbrace{b_0}_{\text{Unemployment Benefit}} - \underbrace{\beta(1 - \delta) \frac{\mu}{1 - \beta}}_{\text{Rosen Effect}} + \underbrace{\beta(\lambda_0 - \lambda_1) \int_{\theta^R}^{\bar{w}_0} W_x(x) \bar{F}(x) dx}_{\text{Entry Effect}}, \quad (3)$$

where $W_x(x) \equiv \partial W(x, h)/\partial x$ is the marginal value of the firm wage component at offered wage x (which under the affine h -structure is independent of h). Without learning ($\mu = 0$) and without asymmetric job-finding rates ($\lambda_0 = \lambda_1$), the reservation wage component equals the *Unemployment-Benefit* intercept, b_0 . Learning and friction asymmetries generate the two other terms.

The *Rosen effect* is the human-capital-investment channel first emphasized by [Rosen](#)

⁷See Appendix B.1 for the derivation. Appendix Proposition B.1 provides the formal expression for A .

(1972). With $\mu > 0$, employed workers accumulate human capital, which raises their continuation value and so makes them willing to accept lower wage components today. The reservation wage is therefore below the unemployment benefit: a higher μ lowers θ^R , and—under the zero-profit normalization $\underline{p} = \theta^R$ adopted below—extends the lower support of active firms, lengthening the bottom of the job ladder.

The *Entry effect* captures the asymmetry between on- and off-the-job search. When unemployed workers find new jobs more easily than employed workers ($\lambda_0 > \lambda_1$), they are more selective in unemployment because outside options will be easier to obtain in that state, which raises θ^R . When the asymmetry reverses ($\lambda_1 > \lambda_0$), workers accept low-quality jobs as stepping stones to better matches, lowering θ^R .

Wage equation. The firm problem implies (Appendix B.3, Lemma B.3) that

$$\frac{p - w_0(p)}{[\delta + \lambda_1 \bar{\Gamma}(p)]^2} - \tilde{\pi}(\underline{p}) = \int_{\underline{p}}^p \frac{1}{[\delta + \lambda_1 \bar{\Gamma}(x)]^2} dx, \quad (4)$$

where $\tilde{\pi}(\underline{p}) \equiv \pi(\underline{p})/A$ is the (scaled) profit level of the lowest-productivity active firm. Equation (4) determines the wage schedule up to the integration constant $\tilde{\pi}(\underline{p})$. No firm posts below the reservation component θ^R , so the lowest active firm satisfies $w_0(\underline{p}) = \theta^R$, and more generally

$$\pi(\underline{p}) = (\underline{p} - \theta^R) l(\theta^R) = A \tilde{\pi}(\underline{p}).$$

We adopt the standard zero-profit normalization $\underline{p} = \theta^R$ in the quantitative implementation, under which $\pi(\underline{p}) = 0$.

Rearranging (4) yields the wage equation

$$w(p, h) = h + p - \underbrace{(1 + k_1 \bar{\Gamma}(p))^2 \left[\delta^2 \tilde{\pi}(\underline{p}) + \int_{\underline{p}}^p \frac{1}{(1 + k_1 \bar{\Gamma}(x))^2} dx \right]}_{w_0(p)}, \quad (5)$$

with $k_1 \equiv \lambda_1/\delta$. Using $w_0(\underline{p}) = \theta^R$ pins down the integration constant:

$$\tilde{\pi}(\underline{p}) = \frac{\underline{p} - \theta^R}{(\delta + \lambda_1)^2} = \frac{\underline{p} - \theta^R}{\delta^2(1 + k_1)^2}.$$

The wage equation decomposes into a worker-specific component h and a firm-specific component $w_0(p)$. The firm component is itself the sum of two terms: the first increases one-for-one with firm productivity p , capturing the direct contribution of productivity to wages; the second reflects equilibrium rents and the competitive pressure exerted by higher-productivity firms.

The sensitivity of $w_0(p)$ to p is governed by the rent-normalized slope $\psi(p)$ (Lemma B.4). More productive firms retain a larger share of match output as rents because of weaker competition from other firms (Bontemps et al., 2000). The magnitude of this effect depends on the firm-productivity distribution $\Gamma(p)$ and the friction parameter k_1 .

2.4 Comparative Statics

Model parameters affect wage growth through two channels. The *direct* channel holds the wage schedule fixed. A higher learning rate μ mechanically raises wages at any age, a higher on-the-job offer rate λ_1 and a lower separation rate δ raise the contribution of job-to-job mobility to wage growth, and the off-the-job offer rate λ_0 affects unemployment but not wage growth conditional on employment. The *indirect* channel works through the equilibrium firm-specific wage component $w_0(p)$.⁸ Under the Pareto specification adopted in Section 5, the effect of the tail index α on $\psi(p)$ is non-monotonic, positive near the bottom of the support and negative sufficiently far in the right tail. These comparative statics are summarized in Appendix Table B.1.

⁸In closed form, $\psi(p) \equiv w'_0(p)/(p - w_0(p)) = 2k_1\gamma(p)/(1 + k_1\bar{\Gamma}(p))$. This implies that a stronger job ladder (higher λ_1 , lower δ) steepens $\psi(p)$ everywhere (Appendix B.4).

2.5 Life-Cycle Extension

The baseline model abstracts from life-cycle variation in learning. While this keeps the analysis tractable, a constant learning rate μ implies that human capital contributes to wage growth at the same pace at all ages, leaving the firm-side job-ladder mechanism to absorb any curvature in life-cycle wage profiles. Because this has direct implications for identification, we extend the framework to allow learning to vary with age.⁹

Setup. We partition workers into two age groups, $a \in \{Y, O\}$, where Y denotes young and O old. A young worker ages with probability η per period; an old worker dies with probability ν and is replaced by a newborn young worker. The effective discount rate for old workers, who face only the death hazard, is $\beta_O \equiv (1 - \nu)\tilde{\beta}$. Learning rates satisfy $\mu_Y \geq \mu_O \geq 0$, nesting the baseline when $\mu_Y = \mu_O = \mu$. All remaining primitives—the job-finding rates (λ_0, λ_1) , the separation rate δ , the unemployment benefit $b(h) = b_0 + h$, and the firm productivity distribution $\Gamma(p)$ —are common across age groups; we keep mobility hazards age-invariant to preserve a single firm-side problem and identify them from aggregate flow rates, leaving age-dependent hazards (which would require age-segmented wage posting) to future work. We focus on *common-support* equilibria in which firms post a single offer distribution F on $[\theta_O^R, \bar{w}_0]$ and both age groups face it; this requires $\theta_Y^R \leq \theta_O^R$, so young workers accept every posted offer and the firm’s problem remains one-dimensional in w_0 .

Relative to the baseline, the equilibrium changes in three ways: the young-worker reservation wage gains an *aging effect*, the Rosen and Entry effects become age-weighted averages, and firm size aggregates across age groups. All other equilibrium objects—the wage schedule, the firm’s envelope condition, and the rent-normalized slope $\psi(p)$ —retain the structural form derived in Section 2.3.

⁹Appendix B.5 provides further details on the life-cycle extension.

Reservation wage component. The value functions retain an affine structure in worker productivity h (Lemma B.5). Denote by $\tilde{U}_a$ and $\tilde{W}_a(w_0)$ the rescaled values net of the affine h -component for age group a , so that $U_a(h) = \tilde{U}_a + c_a h$ and $W_a(w_0, h) = \tilde{W}_a(w_0) + c_a h$, where c_a is the age-specific slope. The reservation wage component for old workers continues to satisfy equation (3) with (μ, β) replaced by (μ_O, β_O) .

For young workers, imposing $W_Y(\theta_Y^R, h) = U_Y(h)$ and using the common-support restriction yields

$$\begin{aligned} \theta_Y^R = b_0 - & \underbrace{\left[(1 - \eta)\tilde{\beta}(1 - \delta)c_Y\mu_Y + \eta\tilde{\beta}(1 - \delta)\frac{\mu_O}{1 - \beta_O} \right]}_{\text{age-weighted Rosen effect}} \\ & + \underbrace{(1 - \eta)\tilde{\beta}(\lambda_0 - \lambda_1)J_Y(\theta_Y^R) + \eta\tilde{\beta}(\lambda_0 - \lambda_1)J_O(\theta_O^R)}_{\text{age-weighted entry effect}} \\ & + \underbrace{\eta\tilde{\beta}(1 - \delta - \lambda_1)[\tilde{U}_O - \tilde{W}_O(\theta_Y^R)]}_{\text{aging effect}}, \end{aligned} \quad (6)$$

where $\tilde{W}'_a(x) \equiv \partial \tilde{W}_a(x) / \partial x$ denotes the marginal value of the firm wage component, and $J_a(w_0) \equiv \int_{w_0}^{\bar{w}_0} \tilde{W}'_a(x) \bar{F}(x) dx$ is the option value of on-the-job search for age group a .

Relative to the baseline reservation wage in (3), equation (6) (derived in Appendix B.5.3) adds an *aging effect*: young workers anticipate the decline in learning—and the associated loss in continuation value—upon aging, which raises their reservation wage because future wage growth from learning is weaker. The Rosen and Entry effects also become age-weighted averages of their young- and old-worker counterparts.

Firm problem. Because per-worker rents $p - w_0$ do not depend on worker age, the firm's envelope condition is unchanged: $\pi'(p) = l(w_0(p))$. Firm size now aggregates across age groups, $l(p) = l_Y(p) + l_O(p)$, where the young component retains the closed form $l_Y(p) = \tilde{A}_Y / [\tilde{\delta} + \tilde{\lambda}_1 \bar{F}(w_0(p))]^2$ with effective rates $\tilde{\delta} \equiv \eta + (1 - \eta)\delta$ and $\tilde{\lambda}_1 \equiv (1 - \eta)\lambda_1$ (Appendix B.5.4). The old component $l_O(p)$ does not reduce to a closed form because of inflows from aging young workers, and we compute it numerically. The wage

equation retains the structure of (5), with $l(p)$ in place of the baseline size expression (Appendix B.5.5). Setting $\eta = \nu = 0$ recovers the baseline model exactly.

3 Data

Our analysis combines two types of data from the United States, Brazil, and Colombia. On the worker side, we use household and labor force surveys that allow us to construct individual wage profiles over the life cycle. On the firm side, we use establishment-level data on productivity where available. The worker-side data are broadly comparable across the three countries, while the firm-side data vary in coverage and detail, which shapes how we use them in the analysis.

The three countries are selected for complementary reasons. The United States is the standard developed-economy benchmark, with well-documented life-cycle wage profiles (Lagakos et al., 2018). Brazil offers a high-quality nationally representative longitudinal survey and is among the most extensively studied middle-income labor markets. Colombia is the natural counterpart for our identifying variation in firm productivity. Prior work has documented its compressed firm-size distribution (Eslava et al., 2022, 2023), a feature that is central to the model’s firm-side mechanism.

3.1 Worker-Level Data

We use four datasets across the three countries to construct individual wage profiles over the life cycle.¹⁰ For the United States, we use the outgoing rotation group of the *Current Population Survey* (CPS) from 2003 to 2013, matching the sample years in Lagakos et al. (2018).¹¹ For Brazil, we use the *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC) from 2012 to 2019, a nationally representative quarterly panel that records

¹⁰Additional details on wage data construction and harmonization are provided in Appendix C.1.

¹¹We additionally use the *Panel Study of Income Dynamics* (PSID, 1975–2013) and the *National Longitudinal Survey of Youth* (NLSY, 2000–2019) to validate the CPS life-cycle wage profile in Appendix A.

employment status, wages, and demographics, in which selected households are interviewed for five consecutive quarters. For Colombia, we combine two sources. The *Gran Encuesta Integrada de Hogares* (GEIH) is a monthly household survey that supplies job-mobility measures through retrospective questions. Because the GEIH is not longitudinal and cannot identify individual wage growth, we supplement it with the *Planilla Integrada de Liquidación de Aportes* (PILA), an administrative database covering the universe of formal workers from 2009 to 2016.¹²

We harmonize the four worker-side datasets along four dimensions. We measure wages monthly, deflated to real U.S. dollars and winsorized at the 1st and 99th percentiles. We measure wage growth at the highest frequency each panel allows, which is quarterly for the CPS and PNADC and annual for the PILA, and we harmonize all moments to a common quarterly frequency for the structural estimation.¹³ We classify a worker as a job stayer if the employer is unchanged across consecutive observations, and as a job mover otherwise.¹⁴ We measure firm size as the size of the employer reported by the worker. The PNADC records this information in a longitudinal panel, so we use it at the individual level for Brazil. The CPS and GEIH report firm size cross-sectionally, so for the United States and Colombia we aggregate to the state level.

The analysis sample covers workers between ages 20 and 60. Sample years do not fully overlap across the three countries. Therefore, we use within-country averages over all available years to mitigate cyclical contamination.¹⁵

Table 1 summarizes the worker-level analytic samples. The samples are predominantly

¹²Formal workers are defined as those who contribute to health and pension, approximately 60 percent of jobs.

¹³The Colombian wage-growth measure uses the PILA and is therefore restricted to formal workers, since the PILA is the only Colombian source with the longitudinal structure required to measure individual wage growth. We discuss the implications of this restriction for cross-country comparability in Appendix C.1, which also provides additional details on harmonization.

¹⁴For Colombia, we identify job stayers retrospectively from the GEIH, following Lasso (2013).

¹⁵The cross-sectional dispersion of firm productivity is stable year-over-year in our sample windows for both the United States and Colombia (see Appendix Table C.3), supporting our interpretation of life-cycle wage profiles as steady-state objects rather than reflecting different productivity environments at different ages.

male (approximately 60 percent), with mean age 41 in the United States and 38 in Brazil and Colombia.¹⁶ Wage growth is higher among job movers than job stayers in all three countries, consistent with the job-to-job channel being a primary driver of wage growth.

3.2 Firm-Level Data

We use firm-level data to construct firm productivity measures using three complementary sources. For Colombia, we use the Annual Manufacturing Survey (EAM), a census of formal manufacturing establishments with at least ten employees observed from 1992 to 2016. For the United States, we use the BLS Dispersion in Productivity Statistics. As a common cross-country benchmark, we use the World Bank Enterprise Survey, a stratified establishment-level survey administered with a common questionnaire across countries, pooling the 2006, 2010, 2017, and 2023 waves for Colombia, the 2009 wave for Brazil, and the 2024 wave for the United States.¹⁷

4 Empirical Motivation

Life-cycle wage profiles are positively correlated with the share of employment at large firms—across countries, across regions within country, and across firm sizes for the same worker. We document these three correlations in this section after linking the empirical measures to the model’s primitives. We measure the life-cycle wage profile as the ratio of wages at ages 40–44 relative to ages 20–24, and proxy the upper tail of firm productivity by the share of workers in firms with more than 10 and more than 50 employees.

¹⁶Wage growth statistics are reported at the frequency of each dataset, quarterly for the CPS and PNADC and annual for the PILA.

¹⁷Details about the construction of firm productivity measures are provided in Appendix C.2.

Table 1: Summary Statistics

	Obs. (1)	Mean (2)	SD (3)	Median (4)	Min. (5)	Max. (6)
A. United States (CPS)						
Age	947,362	40.853	11.00	41.00	20.00	60.00
Female	947,362	0.464	0.50	0.00	0.00	1.00
Log(monthly wages)	826,668	7.928	0.70	7.95	5.69	9.35
1(Stayer)	947,362	0.928	0.26	1.00	0.00	1.00
Job-to-unemployment	947,362	0.020	0.14	0.00	0.00	1.00
Job-to-job	947,362	0.052	0.22	0.00	0.00	1.00
Unemployment-to-job	51,130	0.456	0.50	0.00	0.00	1.00
Wage growth, stayers	202,015	0.019	0.49	0.01	-3.66	3.66
Wage growth, movers	8,962	0.039	0.54	0.02	-3.66	3.20
B. Brazil (PNADC)						
Age	4,497,588	37.967	10.73	37.00	20.00	60.00
Female	4,497,588	0.421	0.49	0.00	0.00	1.00
Log(monthly wages)	4,136,189	6.289	0.83	6.23	3.37	8.48
1(Stayer)	4,497,588	0.891	0.31	1.00	0.00	1.00
Job-to-unemployment	4,497,588	0.033	0.18	0.00	0.00	1.00
Job-to-job	4,497,588	0.075	0.26	0.00	0.00	1.00
Unemployment-to-job	325,752	0.440	0.50	0.00	0.00	1.00
Wage growth, stayers	3,749,814	0.005	0.44	0.00	-5.11	5.11
Wage growth, movers	280,053	0.013	0.64	0.00	-4.62	4.09
C. Colombia						
Age	2,508,256	37.814	10.99	37.00	20.00	60.00
Female	2,508,256	0.415	0.49	0.00	0.00	1.00
Log(monthly wages)	2,047,820	5.622	0.90	5.67	2.76	7.74
1(Stayer)	2,508,256	0.636	0.48	1.00	0.00	1.00
Job-to-unemployment	2,508,256	0.080	0.27	0.00	0.00	1.00
Job-to-job	2,508,256	0.284	0.45	0.00	0.00	1.00
Unemployment-to-job	123,191	0.838	0.37	1.00	0.00	1.00
Wage growth, stayers*	31,999,316	0.023	0.36	0.01	-17.13	16.89
Wage growth, movers*	17,306,998	0.042	0.54	0.02	-18.76	17.06

Notes: All wages are expressed in 2010 USD and winsorized at the 1st and 99th percentiles. Brazilian (PNADC) data have quarterly frequency and CPS data are collected monthly for four consecutive months, so job mobility measures and wage growth rates in the CPS and PNADC are computed quarterly. Colombian measures are computed annually. Wage growth rates correspond to per-period log-wage change. All samples are restricted to workers between the ages of 20 and 60. Rates across countries are therefore not directly comparable at the frequencies reported in this table; for the common-frequency moments used in the structural estimation, see Table 3. *Computed using the Colombian social security records.

4.1 Empirical Proxies for Model Primitives

Two empirical counterparts allow us to map the model's primitives to the data. First, following (2), firm size is strictly increasing in productivity p in the model, so employment shares in size bins ($> 10, > 50$) proxy for concentration of employment in the upper tail of $\Gamma(p)$.¹⁸ In the baseline Burdett–Mortensen structure, the *unconditional* cross-firm size distribution is a rank object and does not identify the shape of Γ (Lemma E.1). The relevant object is instead the employment share at firms above a given productivity cutoff. As shown in Lemma E.2, for $p_H > \underline{p}$,

$$S(p_H) \equiv \frac{\int_{p_H}^{\infty} l(p) d\Gamma(p)}{\int_{\underline{p}}^{\infty} l(p) d\Gamma(p)} = \frac{(1 + k_1) z_H}{1 + k_1 z_H}, \quad z_H \equiv \bar{\Gamma}(p_H). \quad (7)$$

A higher employment share at large firms can therefore reflect either a stronger job ladder (higher k_1 , that is, higher λ_1 or lower δ) or greater mass in the upper tail of the firm-type distribution (higher z_H). Under the Pareto specification used in the quantitative analysis, a thicker right tail (lower α) raises z_H for any fixed $p_H > \underline{p}$ (Lemma E.3).

The second counterpart concerns wage growth. Conditional on being employed at a firm with productivity p , expected per-period wage growth is

$$E[\Delta w \mid p] = \mu + \lambda_1 \int_p^{\infty} (w_0(x) - w_0(p)) d\Gamma(x), \quad (8)$$

where the second term is the expected gain from on-the-job search. This term rises directly with λ_1 and, holding the wage schedule fixed, is amplified when the right tail of Γ places more mass on high- p draws (Lemma E.4). By contrast, higher separations δ weaken the

¹⁸The firm-size/firm-wage rank correlation is positive but imperfect: small fast-growing entrants poach from larger incumbents, and firm age, industry, and product-market position can shift the wage premium attached to a given size. Our use of firm size as a proxy is not a claim that within-country wage and size rankings coincide perfectly, but a claim about cross-country comparability: countries with thicker right tails of the firm-size distribution also have thicker right tails of the firm-wage distribution, which is the object relevant for our model. Appendix D provides direct evidence on the size–productivity link from Colombian firm-level data (across five productivity measures) and from cross-country WBES data.

job ladder and reduce the time workers spend in high- p firms. Our life-cycle wage-growth measure aggregates these per-period gains over employment histories, so differences across regions reflect variation both in μ and in the mobility-driven component of (8).

Both the thickness of the firm-productivity tail (α) and the effectiveness of the job ladder (λ_1, δ) therefore shape $S(p_H)$ and the mobility-driven part of life-cycle wage growth:

Prediction. *Areas with a higher employment share at large firms exhibit steeper life-cycle wage profiles.*

We test this prediction in three ways: across countries, across regions within country, and across firm sizes for the same worker.¹⁹

4.2 Empirical Results

Cross-country evidence. First, we consider cross-country correlations, presented in Figure 2. We complement our three countries of interest using the *European Union Statistics on Income and Living Conditions* (EU-SILC) data from 2005 to 2019.²⁰ Panels 2a and 2b correlate wage growth with the share of employment in firms with more than 10 and 50 employees, respectively. The correlation is positive in both panels (slopes and standard errors reported on each figure), indicating that countries with greater large-firm employment exhibit steeper life-cycle wage profiles.²¹

Part of this relationship could reflect institutional factors such as minimum wages and collective bargaining. We residualize against the country-level minimum wage relative to the average wage and the share of employees covered by collective bargaining agreements, and report the results in Panels 2c and 2d. The positive relationship between firm-size

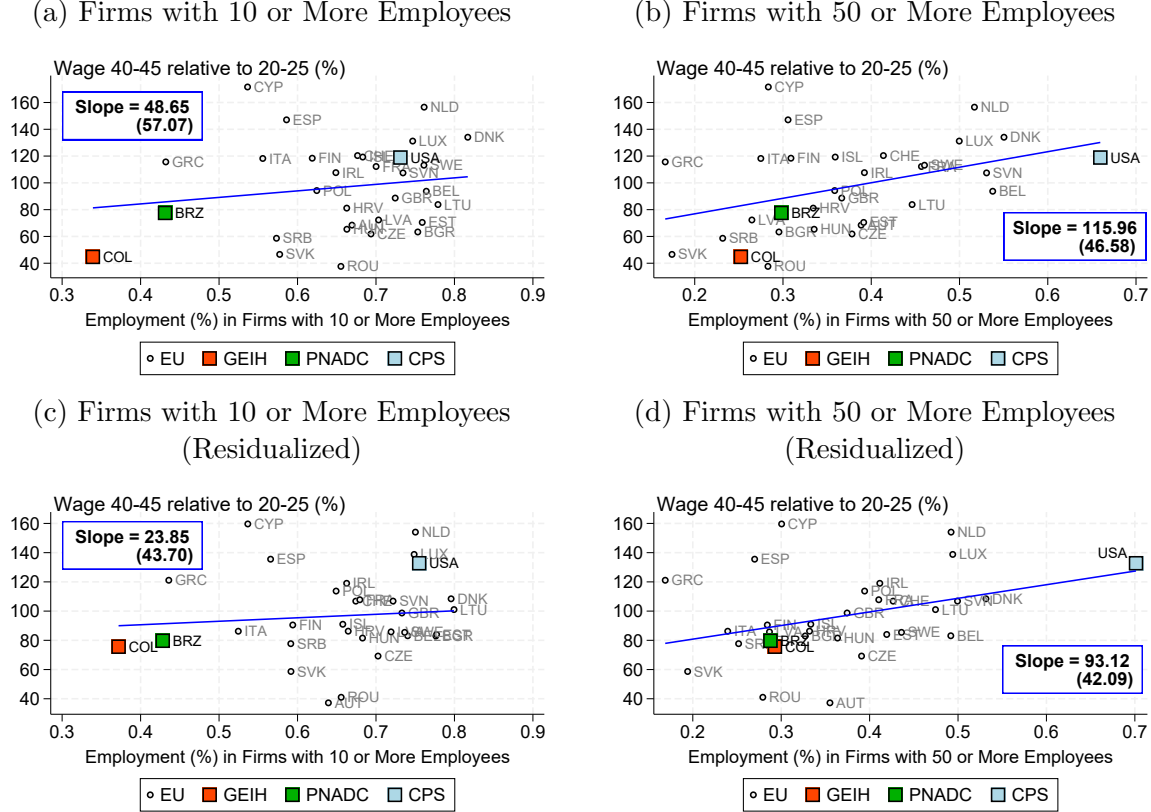
¹⁹We additionally assess the empirical relationship between the firm productivity distribution and firm size in Appendix D.

²⁰Data from EU-SILC cover a large number of European countries and are collected as a rotating yearly panel that surveys individuals over several periods. We use these data to construct cross-country correlations in our measures of interest.

²¹With only 31 observations, the relationship is precisely estimated only for the 50-employee threshold. Appendix F reports the regression analogues of all four panels of Figure 2.

composition and life-cycle wage growth remains, consistent with the view that the firm productivity distribution is a relevant driver of cross-country differences in life-cycle wage profiles.

Figure 2: Life-Cycle Wage Profiles and Firm Size Across Countries



Notes: These figures combine data from EU-SILC (gray markers), CPS (light blue markers), PNADC (green markers), and GEIH (red markers). The y-axis reports the ratio of monthly wages of workers aged 40–44 relative to those aged 20–24. The x-axis reports the share of employment in firms with more than 10 employees (left panels) and more than 50 employees (right panels). The top panels report raw values; the bottom panels report values residualized with respect to the minimum wage relative to the average wage and the share of employees covered by collective bargaining agreements. For countries without a statutory minimum wage (Austria, Denmark, Finland, Iceland, Italy, Sweden, Switzerland, Cyprus), the minimum-wage variable is set to zero. The institutional dimension is captured by collective bargaining coverage, which is high in this group. Each point corresponds to a time-invariant value computed by first collapsing individual-level data at the country-year level and then averaging across years. We exclude Portugal, Germany, and Norway from the analysis. Portuguese data differ in how incomes were recorded, Norwegian data record job transitions differently, and German data were available for only two periods with a very small sample.

Cross-regional evidence. Second, we zoom in on our three countries of interest—Brazil, the United States, and Colombia—and compute cross-regional correlations within each country. Figures 3a and 3b present state-level correlations.²² Between countries, life-cycle wage profiles are steeper in the United States, where employment is more concentrated in larger firms. States in Brazil and Colombia show a less steep gradient and a noticeably smaller share of employment in larger firms. Within each country, we observe a positive correlation between life-cycle wage profiles and the share of employment in larger firms across states.

As in the cross-country case, this relationship could be confounded by institutional and compositional factors. We residualize the within-country measures against country fixed effects, state-level formality rates, the state-level share of workers with more than a high-school degree, and the state-level minimum wage.²³ Results are presented in Figures 3c and 3d. Controlling for these factors reduces the slope, but the coefficients remain positive and statistically significant.

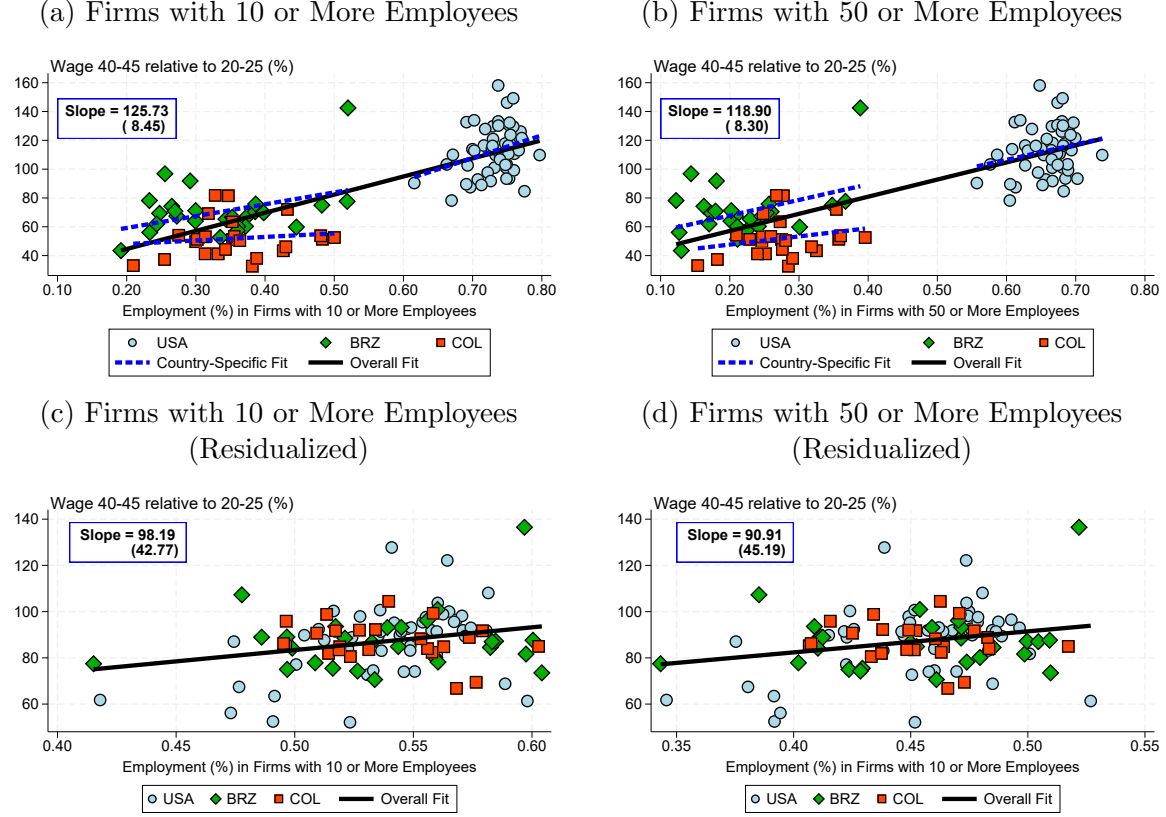
Individual-level evidence. Third, cross-country and cross-regional correlations may be susceptible to biases from worker composition—more skilled workers tend to sort into larger firms. To strip out this channel, we estimate individual-level Mincer regressions on worker-level panel data. PNADC is the only worker-level panel in our sample with both wage and firm-size information. The CPS reports firm size only cross-sectionally, and the GEIH is not longitudinal. We therefore restrict this exercise to Brazil and treat its results as suggestive for the other countries. Formally, we estimate

$$\ln w_{it} = \sum_{k \in \mathcal{K}} \beta_k \mathbf{1}(FS_{it} = k) + \gamma_1 a_{it} + \gamma_2 a_{it}^2 + \alpha_i + \nu_s + \tau_t + \varepsilon_{it}, \quad (9)$$

²²Appendix F reports the regression analogues of all four panels in Figure 3 and additional estimates varying the set of controls.

²³The minimum-wage coefficient is identified almost entirely from state-level variation within the United States, since Brazil and Colombia set the minimum wage nationally.

Figure 3: Life-Cycle Wage Profiles and Firm Size Within Countries



Notes: These figures combine data from the CPS (light blue markers), PNADC (green markers), and GEIH (red markers). The y-axis reports the ratio of monthly wages of workers aged 40–44 relative to those aged 20–24. The x-axis reports the share of employment in firms with more than 10 employees (left panels) and more than 50 employees (right panels). The top panels report raw values; the bottom panels report values residualized with respect to country fixed effects, the state-level formality rate, the state-level share of workers with more than a high-school degree, and the state-level log minimum wage. Each point corresponds to a time-invariant value computed by first collapsing individual-level data at the country-state-year level and then averaging across years. The slope and robust standard error reported in the blue panels correspond to the coefficient on firm-size share from a regression of the wage ratio on firm-size share, formality, the share of workers with more than a high-school degree, the log minimum wage, and country fixed effects.

where $\ln w_{it}$ denotes log wages for individual i in year t , a_{it} is age, and α_i , ν_s , τ_t are individual, state, and year fixed effects, respectively. The indicators $\mathbf{1}(FS_{it} = k)$ equal one if individual i is employed in a firm of size category $k \in \mathcal{K} \equiv \{[6, 10], [11, 50], > 50\}$ at period t , with firms of $[1, 5]$ employees as the omitted reference. Standard errors are clustered at the state level.

Appendix Table F.3 reports the estimates. Columns (1)–(3) progressively add controls:

column (1) includes only state and year fixed effects; column (2) adds observable individual controls (gender and years of education) alongside the age polynomial; column (3) adds individual fixed effects. All three columns show a monotonic positive association between firm size and wages. The cross-sectional gradient (column 1) is large: workers at firms with more than 50 employees earn 42 log points more than workers at firms with one to five employees. Adding observable controls (column 2) attenuates the gradient modestly to 32 log points, consistent with sorting on observables. Adding individual fixed effects (column 3) shrinks the gradient to 15 log points and the within-individual estimates remain statistically significant, indicating that the same worker earns substantially higher wages when employed at a larger firm.

To examine the job-ladder mechanism directly, we re-estimate (9) separately for *stayers* and *movers* (columns (4) and (5)). The wage–firm-size gradient is positive in both groups but markedly steeper among movers. A worker who moves to a firm with more than 50 employees gains 27 log points relative to a job in a 1–5-employee firm, compared with only 8 log points for stayers. The steeper mover gradient is consistent with the model’s job-ladder mechanism: workers gain wages by moving up the firm-size distribution, not just by happening to be employed at larger firms. The distribution of firm types is therefore a quantitatively important determinant of life-cycle wage growth, which we structurally quantify in Section 6.

5 Identification and Estimation

5.1 Identification

We estimate three blocks of parameters: labor market frictions ($\lambda_0, \lambda_1, \delta$), on-the-job learning (μ in the baseline, (μ_Y, μ_O) in the extension), and the firm productivity distribution $(\alpha, \underline{p})$.²⁴ Estimation proceeds sequentially, with non-overlapping moments for each block

²⁴The aggregate discount rate β is exogenously set to align with an annual capital return of 10 percent.

and some pinned down in closed form. A similar strategy is used in Moscarini and Postel-Vinay (2023)—who apply the same closed-form identification within the sequential-auction bargaining framework of Postel-Vinay and Robin (2002)—building on a long tradition of simulated method of moments (SMM) estimation of random-search models with firm heterogeneity (Bontemps et al., 2000; Bagger et al., 2014; Engbom, 2022). An important by-product of this sequential design is that our friction estimates are invariant to the wage-setting protocol: any model preserving the closed-form mapping between worker flows and $(\lambda_0, \lambda_1, \delta)$ generates the same friction estimates, as the sequential-auction application in Moscarini and Postel-Vinay (2023) illustrates.

Labor market frictions. Labor-market frictions are pinned down in closed form from the unemployment rate and worker transition rates. The steady-state flow equation for unemployment balances inflow at rate $(1 - u)\delta$ against outflow at rate $\lambda_0 u$, yielding

$$\frac{1 - u}{u} = \frac{\lambda_0}{\delta} = k_0.$$

We estimate $\hat{\lambda}_0 = E[UE]$ directly from the data as the share of unemployed workers who find a job within a quarter, and recover $\hat{\delta} = \hat{\lambda}_0 / \hat{k}_0$ from the empirical ratio $\hat{k}_0 = (1 - u)/u$. The expected separation rate—the sum of the job-to-job rate EE and the displacement rate δ —admits a closed-form expression in $k_1 \equiv \lambda_1/\delta$:

$$EE + \delta = \delta + \lambda_1 \int (1 - F(w))g(w) dw = \delta \ln(1 + k_1) \frac{1 + k_1}{k_1},$$

where $g(w_0) := g(w)$ is the distribution of wage components across workers (derived in Appendix B.6). We solve numerically for k_1 given the empirical $(EE + \delta)/\delta$ ratio and recover $\hat{\lambda}_1 = \hat{k}_1 \hat{\delta}$.

Learning on the job. The learning parameters are then identified from stayer wage growth, which under wage posting reflects human capital accumulation only. In the baseline, μ is disciplined by the average wage growth of stayers, $\Delta w_{\text{stay}} = \mu$, where Δw_{stay} denotes the average per-period log-wage change among job stayers. Any life-cycle curvature in wages is therefore absorbed by the firm-side mechanism.

In the life-cycle extension of Section 2.5, (μ_Y, μ_O) are disciplined by average stayer wage growth at each age.²⁵ Conditional on the aging hazard η , the probability of still being young in age bin $a = [t_0(a), t_1(a))$ is

$$\Omega_a = \frac{(1 - \eta)^{t_0(a)+1} [1 - (1 - \eta)^{t_1(a)-t_0(a)}]}{(t_1(a) - t_0(a)) \eta}, \quad (10)$$

so model-implied stayer growth in bin a is $\Omega_a \mu_Y + (1 - \Omega_a) \mu_O$ (Appendix B.5).²⁶ We recover (μ_Y, μ_O) by matching the empirical age profile of stayer growth under $\mu_Y \geq \mu_O \geq 0$. In the extension, learning absorbs the age profile of stayer wage growth, leaving the remaining life-cycle curvature in wages to discipline the firm distribution through the model’s job-ladder channel.

Firm productivity distribution. Conditional on the friction and learning estimates, the only remaining source of life-cycle wage growth in the model is the wage gains workers obtain through job-to-job mobility, which depends on the shape and scale of $\Gamma(p)$. We assume $\Gamma(p)$ is Pareto with shape parameter α and lower support $\underline{p}$.²⁷ We use SMM to pin down $(\alpha, \underline{p})$, targeting the average wage growth of workers at ages 25–29 and 30–34 relative to ages 20–24, and minimizing the squared percentage distance between empirical and simulated moments at quarterly frequency.²⁸ We focus on the first two age bins

²⁵The labor-market and firm-distribution arguments remain unchanged.

²⁶Appendix G.1 provides details about the calibration of the age-specific learning rates.

²⁷We assess sensitivity to this assumption in Section 6.3 by re-estimating the model with a shifted Weibull distribution.

²⁸Operationally, the SMM optimizes over α and the unemployment-benefit intercept b_0 ; the lower support is then recovered from the equilibrium reservation wage as $\underline{p} = \theta^R(b_0)$. We report results in terms of $(\alpha, \underline{p})$.

because they correspond to the steepest part of the life-cycle profile and so have the highest potential impact on lifetime earnings. The later bins are left untargeted and used as an out-of-sample fit check. The lower support satisfies $\underline{p} = \theta^R$ by the standard zero-profit normalization, under which no active firm posts below the unemployed worker’s reservation component.²⁹ Table 2 summarizes the full mapping.

Table 2: Mapping Between Parameters and Identifying Moments

Parameter	Moment	Identifying equation
λ_0	UE transition rate	$\hat{\lambda}_0 = E[UE]$
δ	Unemployment rate (given λ_0)	$\hat{\delta} = \hat{\lambda}_0 u / (1 - u)$ $EE + \delta = \delta \ln(1 + k_1) \times$
$\lambda_1 = k_1 \delta$	EE transition rate (given δ)	$(1 + k_1) / k_1$
μ (baseline)	Stayer wage growth rate	$\Delta w_{\text{stay}} = \mu$ $\Delta w_{\text{stay},a} = \Omega_a \mu_Y$
μ_Y, μ_O (extension)	Age profile of stayer growth	$+(1 - \Omega_a) \mu_O$
$\alpha, \underline{p}$	Wage growth at ages 25–29, 30–34	SMM, given $(\lambda_0, \lambda_1, \delta, \mu)$

Notes: Each row lists a model parameter, the empirical moment that disciplines it, and the identifying equation. Moments are disjoint across rows: no moment enters the identification of more than one parameter block. Ω_a denotes the share of quarters spent as young in age bin a (see text).

5.2 Estimation Targets

Table 3 summarizes the empirical moments used in estimation.³⁰ We do not need to pin down the initial worker-skill distribution. Because skills enter wages additively and learning is common across workers, the initial distribution affects wage levels but not changes in average wages across age bins, so we rely on relative moments alone.³¹

²⁹The wage equation (5) determines the wage schedule up to the integration constant $\tilde{\pi}(\underline{p}) = \pi(\underline{p})/A$. Setting $\underline{p} = \theta^R$ implies $\tilde{\pi}(\underline{p}) = 0$ and yields a unique wage schedule. Any $\underline{p} > \theta^R$ would generate rents at the bottom of the ladder without altering the rank ordering of firms or the qualitative predictions of the model; the normalization is standard in the Burdett–Mortensen literature.

³⁰As opposed to Table 1, the targeted wage-growth moments in Table 3 are harmonized to a common quarterly frequency.

³¹Our identification targets the age profile of mean wages, not the variance contributions of worker and firm components. The first- and second-moment identification problems are distinct: the wage variance depends on the initial worker-skill distribution, which is not pinned down by our targeted moments, and on the recovery of worker-firm variance components, which is known to be sensitive to sorting and firm-ID measurement (Bonhomme et al., 2019, 2023; Borovičková and Shimer, 2017; Ozkan et al., 2023; Huggett et al., 2011; Guvenen et al., 2021). A variance-decomposition analysis is therefore a separate research

Table 3: Estimation Targets

	Rates			Wage Growth (rel. to $w_{0/5}$)		Stayer Wage Growth		
	Unemp. (u)	UE	EE	$w_{5/10}$	$w_{10/15}$	Avg. (μ)	Young (μ_Y)	Old (μ_O)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
United States	0.068	0.472	0.056	60.08	88.15	0.0095	0.018	0.003
Brazil	0.097	0.437	0.083	37.59	64.52	0.0088	0.018	0.001
Colombia	0.105	0.370	0.089	33.79	46.04	0.0053	0.006	0.006

Notes: This table presents the estimation targets. u denotes the unemployment rate, UE the unemployment-to-employment transition rate, and EE the job-to-job transition rate. The columns $w_{5/10}$ and $w_{10/15}$ correspond to the average wage in age bins 25–29 and 30–34 relative to the reference bin 20–24, respectively. Column (6) reports average quarterly stayer log-wage growth, which identifies the baseline learning rate μ . Columns (7) and (8) report the calibrated learning rates μ_Y and μ_O , obtained by fitting the age profile of stayer wage growth under the model’s stochastic aging process in the life-cycle extension. Data for the United States come from the CPS and data for Brazil from the PNADC; for Colombia, unemployment and transition moments come from GEIH, while wage-growth moments use PILA social security records. All rates are expressed in quarterly terms.

Beyond the cross-country differences in life-cycle wage growth, two other patterns in Table 3 anticipate the estimation results. First, the job-to-job transition rate is markedly higher in Brazil and Colombia than in the United States, inverting the standard ranking of λ_1 and λ_0 found in studies of developed economies, where job-finding from unemployment typically exceeds job-finding while employed. Second, the average wage growth rate of stayers is similar across countries when measured for young workers, but differs more markedly when comparing young and old stayers within country. Cross-country differences in life-cycle wage growth therefore do not arise primarily from differences in stayer learning at young ages, pointing toward firm-side mechanisms as the natural source of variation.

question rather than a validation of our first-moment identification.

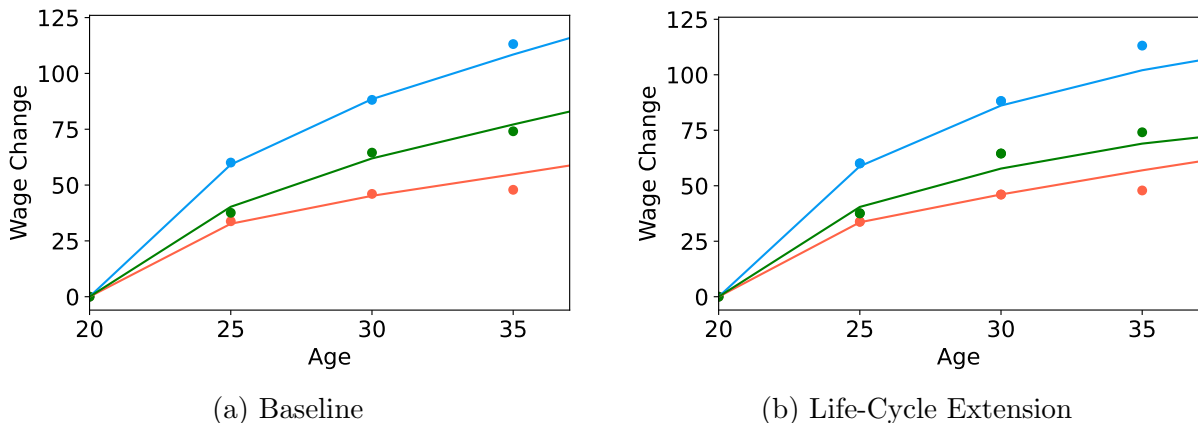
6 Quantitative Results

6.1 Model Estimates

We report estimates separate for the baseline model and the life-cycle extension. The comparison between the two serves two purposes. First, it tests whether the cross-country differences in the firm productivity distribution are robust to allowing age-specific learning. Second, it characterizes how learning varies over the life cycle in each country.

Fit and Validation. The model is estimated to match early-career wage growth and provides mixed out-of-sample fit, with good performance for the U.S. and Brazil but weaker fit for Colombia at older ages. The targeted moments are the wage-growth ratios at ages 25–29 and 30–34 relative to ages 20–24. Figure 4 compares the empirical and simulated targeted moments for all three countries.

Figure 4: Model Fit: Empirical and Simulated Life-Cycle Wage Profiles



Notes: The figures show the average wage in each 5-year age bin relative to the wage at ages 20–24, for empirical (dots) and simulated (line) data in the three countries. Data for the United States come from CPS, for Brazil from PNADC, and for Colombia from household surveys and social security records. The United States is shown in blue, Brazil in green, and Colombia in orange. Bins 25–29 and 30–34 are targeted in the estimation; later bins (35–39, 40–44) are not. The left panel shows the baseline; the right panel, the life-cycle extension.

As out-of-sample non-targeted validation, we provide two alternative sets of tests. First, we use the wage path at ages 35–39 and 40–44 to validate the model’s life-cycle

implications. Table 4 reports the results. Out-of-sample fit is good for the U.S. and Brazil, but slightly weaker for Colombia. The baseline extrapolates a single learning rate linearly into later ages, whereas the extension lets learning flatten with age. At ages 40–44, the baseline overshoots the U.S. wage path by about 10 percentage points, whereas the extension undershoots it by about 4 points. The corresponding absolute error for Brazil falls from about 14 points to less than 2. These gains come from allowing $\mu_O < \mu_Y$, which prevents the model from extrapolating early-career learning into later ages. Colombia is less well fit at older ages under either specification, consistent with selection into formal PILA employment driven by informality, a margin we abstract from.³²

Table 4: Out-of-Sample Fit on Non-Targeted Moments

Country	Moment	Data	Baseline		Extension	
			Model	Diff.	Model	Diff.
United States	Wage path, 35–39	113.14	108.46	−4.68	102.05	−11.09
	Wage path, 40–44	117.32	126.90	+9.58	112.98	−4.34
Brazil	Wage path, 35–39	74.09	77.13	+3.04	68.98	−5.11
	Wage path, 40–44	77.51	91.60	+14.09	75.98	−1.52
Colombia	Wage path, 35–39	47.88	54.80	+6.93	56.94	+9.07
	Wage path, 40–44	44.35	64.67	+20.31	66.83	+22.48

Notes: This table reports model fit on non-targeted life-cycle wage moments. The targeted moments (wage growth at ages 25–29 and 30–34 relative to 20–24) are not reported here; see Figure 4. “Wage path” refers to average wages in the indicated age bin relative to ages 20–24 (expressed as a percentage). “Data” is the empirical value; “Model” is the model-implied counterpart under each specification; “Diff.” reports Model – Data.

Second, we verify that the estimated firm-distribution parameters reproduce the cross-regional firm-size/wage-growth gradient documented in Section 4. Holding all other parameters at their country-specific values and varying $(\alpha, \underline{p})$, the model traces a locus relating the employment share in large firms to the life-cycle wage ratio. Appendix Figure H.3 reports this relationship for the three countries. The locus is upward-sloping in each country, replicating the cross-regional pattern of Figure 3. The slope is not targeted,

³²Using Chilean panel data, Hsu and Leyton (2026) document that wage growth among formally employed workers in a developing economy can be comparable to that in developed economies, and that informality is a quantitatively important margin in the cross-country gap.

so its positive sign is an emergent property of the job-ladder mechanism.

Parameter estimates. Table 5 reports estimates for both model specifications, with bootstrap standard errors in parentheses.³³ Across specifications, the firm-distribution estimates are stable for Brazil but shift for the United States and Colombia. The shift reflects how the two specifications attribute late-career wage growth: in the baseline, the single rate μ extrapolates early-career learning into older ages and matches part of the late-career profile through the learning channel. Once μ_O falls below μ_Y , this channel weakens in late career and the firm-distribution parameters adjust to compensate. Age-varying learning therefore helps separate the learning and firm-distribution channels rather than forcing the firm distribution to absorb life-cycle variation better captured by learning. Across countries, the three parameter blocks differ in distinctive ways, which we unpack block by block below.

³³Appendix G.2 provides details about the computation of the standard errors.

Table 5: Estimation Results: Firm Distribution, Frictions, and Learning

	Firm Prod. Distr.				Frictions			Learning		
	Baseline		Extension		δ	λ_1	λ_0	Baseline	Extension	
	Shape (α)	Scale ($\underline{p}$)	Shape (α)	Scale ($\underline{p}$)				μ	μ_Y	μ_O
United States	1.59 (0.91)	0.516 (0.821)	0.853 (0.099)	0.044 (0.047)	0.035 (0.000)	0.34 (0.005)	0.47 (0.003)	0.0095	0.018	0.003
Brazil	0.77 (0.00)	0.010 (0.000)	0.783 (0.006)	0.010 (0.000)	0.047 (0.000)	0.55 (0.005)	0.44 (0.001)	0.0088	0.018	0.001
Colombia	5.22 (3.39)	1.733 (1.576)	10.00 (0.535)	4.105 (0.264)	0.043 (0.000)	0.73 (0.007)	0.37 (0.002)	0.0053	0.006	0.006

Notes: This table presents estimation results for the two model specifications. U.S. denotes the United States. Bootstrap standard errors are reported in parentheses and are based on 200 bootstrap replications. α is the shape parameter of the Pareto firm productivity distribution $\Gamma(p)$. $\underline{p}$ is its scale parameter. Both are estimated separately for the baseline model and the life-cycle extension of Section 2.5. The admissible ranges are $\alpha \in [0.01, 10]$ and $\underline{p} \in [0.01, 10]$, with both parameters normalized affinely to the unit interval inside the optimizer. Colombia’s life-cycle estimate of $\alpha = 10$ lies at the upper endpoint, whereas its baseline estimate is $\alpha = 5.22$. The upper bound is reached in 34 of 200 Colombian baseline bootstrap replications and 199 of 200 life-cycle replications. The life-cycle bootstrap distribution is therefore concentrated at the boundary. Labor market friction parameters (δ , λ_1 , λ_0) are identical across specifications because they are identified in closed form from worker flows and the unemployment rate, independent of the learning specification (see Section 5); their small standard errors reflect the precision with which these flow moments are estimated. The U.S. firm-distribution parameters have smaller bootstrap standard errors in the extension than in the baseline, although uncertainty about the extension’s scale parameter remains. For Colombia, standard errors condition on the auxiliary stayer wage-growth moments from the administrative source. The baseline learning rate μ equals average stayer wage growth. The extension rates (μ_Y , μ_O) are calibrated from its age profile. These learning rates are also reported in Table 3. Standard errors are not reported separately.

Firm productivity distribution. The firm-side estimates imply two distinct disadvantages relative to the United States: a thin upper tail of model-implied firm productivity in Colombia and a long lower tail of low-productivity firms in Brazil. For Colombia, the wage moments push the Pareto shape parameter to its set upper bound, $\alpha = 10$, in the life-cycle extension. Comparing model fit across alternative values of α indicates that this endpoint should not be interpreted as a precisely estimated value.³⁴ We therefore emphasize the

³⁴Appendix G.3 profiles the Colombian life-cycle objective over α , re-optimizing $\underline{p}$ at each value. The original estimation imposes upper bounds of 10 on both parameters. For the objective profile, we evaluate larger values of α and re-optimize $\underline{p}$. The objective declines from 0.000299 at $\alpha = 5$ to 0.000165 at 10, 0.000114 at 20, and 0.000106 at 25, while the profiled $\underline{p}$ rises from 1.679 to 4.105, 9.036, and 11.511, respectively.

model-implied compression of the Colombian firm ladder rather than the exact value of α . Section 6.3 shows that the main counterfactual results remain stable across the tested values of α between 10 and 25, with $\underline{p}$ re-estimated at each value.

Brazil and the United States exhibit thicker and more dispersed distributions, with comparable shape parameters ($\alpha \approx 0.78$ – 0.85 in the extension) but sharply different scale parameters ($\underline{p} = 0.010$ in Brazil versus $\underline{p} = 0.044$ in the United States). Brazil has a comparable upper tail to the United States but a much longer lower tail of low-productivity firms, which lengthens the job ladder and exposes workers to extended periods in low-paying positions early in their careers.

Labor market frictions. Two patterns emerge. First, separation rates (δ) are higher in Brazil (0.047) and Colombia (0.043) than in the United States (0.035), indicating greater downward churn in the job ladder in less developed economies.

Second, job-finding rates display an asymmetric pattern across countries. Job-finding from unemployment (λ_0) is lower in Brazil (0.44) and Colombia (0.37) than in the United States (0.47), consistent with the cross-country pattern in [Donovan et al. \(2023\)](#). Job-finding while employed (λ_1), however, is higher in the two developing economies (0.55 in Brazil, 0.73 in Colombia) than in the United States (0.34). The U.S. ranking $\lambda_1 < \lambda_0$ is consistent with the standard search-literature finding for developed economies ([Bontemps et al., 2000](#); [Bagger et al., 2014](#)). The reverse ranking $\lambda_1 > \lambda_0$ in Brazil and Colombia is, to our knowledge, undocumented. Appendix Figure H.4 validates the pattern in aggregated cross-country data from [Donovan et al. \(2023\)](#): more developed countries tend to exhibit $\lambda_1 < \lambda_0$, less developed countries the reverse.³⁵

This pattern has a direct implication for the reservation wage. In countries where $\lambda_0 > \lambda_1$, workers are more selective when unemployed because outside offers are easier to obtain in that state, raising the reservation wage; in countries where $\lambda_1 > \lambda_0$, workers

³⁵The data in [Donovan et al. \(2023\)](#) include unemployment-to-employment and job-to-job rates, along with GDP per capita. We supplement this with unemployment rates from the International Labor Organization for the same periods.

accept lower-quality jobs as stepping stones, lowering the reservation wage and lengthening the bottom of the job ladder. Our results qualify [Donovan et al. \(2023\)](#). We confirm their negative correlation between λ_0 and economic development, but find an opposite pattern for λ_1 .

Learning on the job. Cross-country differences in learning operate on two distinct margins. The first is the *level* of early-career learning, which is much lower in Colombia than in the other two countries. The second is the *persistence* of learning into older ages, which is higher in the United States than in Brazil. Young workers in the United States and Brazil learn at essentially identical rates. The gap between the two countries appears entirely in the old-worker rate. Colombia, in contrast, exhibits uniformly low and flat learning across the life cycle. The baseline obscures these patterns by absorbing all life-cycle variation into a single rate, making the United States look only modestly ahead of Brazil. The standard finding that more developed economies exhibit higher on-the-job learning ([Ma et al., 2024](#)) therefore appears, in our sample, to operate through the persistence margin rather than the level margin, at least for the U.S.–Brazil comparison.³⁶

6.2 Decomposition of life-cycle wage growth.

Variation in the firm component dominates the worker component in explaining the cross-country wage-growth gap. We decompose model-implied wage growth at each age into a worker component, h , and a firm-specific component, $w_0(p)$, and present the results in Table 6. By age 40–44, the firm component accounts for 54 log points of U.S. life-cycle wage growth, versus 25 in Brazil and 27 in Colombia. The worker component contributes a comparable 55, 48, and 40 points, respectively. The model attributes most of the cross-country wage-growth gap to the job-ladder channel operating through the

³⁶The constraint $\mu_Y \geq \mu_O \geq 0$ binds for Colombia, where the unconstrained estimate would imply $\mu_Y < \mu_O$. We interpret this as evidence that learning is essentially flat over the life cycle in Colombia rather than as evidence of model misspecification. The implied common rate of $\mu = 0.006$ is consistent with the baseline estimate of 0.0053.

firm distribution. The relative similarity of the worker components is consistent with the comparable young-stayer learning rates documented above.

Table 6: Wage Decomposition Over the Life Cycle

Age bin	United States		Brazil		Colombia	
	Worker	Firm	Worker	Firm	Worker	Firm
20–24	0.00	0.00	0.00	0.00	0.00	0.00
25–29	21.76	35.66	20.83	17.89	10.03	23.19
30–34	36.61	47.41	33.83	22.92	19.91	25.93
35–39	47.22	52.37	42.32	24.60	29.84	26.87
40–44	55.48	54.22	48.20	24.77	39.77	26.78

Notes: This table decomposes model-implied wage growth relative to ages 20–24 into its worker and firm components under the life-cycle extension. Wage growth is measured as the change in mean log wages relative to ages 20–24, multiplied by 100; the worker and firm contributions are expressed in the same units. “Worker” denotes the contribution of worker productivity (human capital h) to the wage path; “Firm” denotes the contribution of the firm-specific wage component $w_0(p)$. The two components sum to the full life-cycle wage path reported in Figure 4. Appendix Table H.2 further decomposes the firm component into firm productivity p and rents $w_0(p) - p$.

Within each country, the worker component at age 40–44 contributes a share comparable to or larger than the firm component, in line with single-country decompositions that emphasize human capital (Bagger et al., 2014; Ozkan et al., 2023). What distinguishes the cross-country setting is the source of the *gap*: worker-productivity growth varies modestly across countries while the firm component varies sharply, making firm heterogeneity the dominant driver of cross-country differences in life-cycle wage growth, consistent with Engbom (2022).

6.3 Counterfactual Analysis

The decomposition in Table 6 attributes a large share of U.S. life-cycle wage growth to the firm component. The counterfactual exercises in this section show that this accounting decomposition has economic content: replacing the firm distribution in Brazil and Colombia with the U.S. distribution closes most of the cross-country gap in wage growth, while equalizing learning rates or labor-market frictions has substantially smaller effects.

We use the life-cycle estimates to construct counterfactual wage profiles for Brazil and Colombia, setting one parameter block at a time to its U.S. value while holding the remainder fixed. Table 7 reports the resulting wage-growth ratios at ages 30–34 relative to the U.S. benchmark. Equalizing labor-market frictions does not steepen either profile. If anything, the ratio falls slightly, from 0.73 to 0.65 in Brazil and from 0.52 to 0.50 in Colombia, because job-finding rates while employed are already higher in the two developing economies than in the United States, so the U.S. friction structure does not improve access to the upper rungs of the job ladder. Equalizing learning lowers Brazil’s ratio from 0.73 to 0.68 while raising Colombia’s from 0.52 to 0.70, bringing Colombia close to Brazil’s factual profile.³⁷ Differences in learning therefore help explain Colombia’s flatter profile but do not account for the Brazil–United States gap.

Table 7: Counterfactual Simulation

	Parameters changed to U.S. Level (1)	Relative Wage Growth	
		Brazil (2)	Colombia (3)
Share with Respect to U.S.		0.73	0.52
Counterfactual:			
Firm-Type Distribution	$\alpha, \underline{p}$	1.03	1.15
Learning on the Job	μ_Y, μ_O	0.68	0.70
Labor Market Frictions	$\lambda_1, \lambda_0, \delta$	0.65	0.50

Notes: This table presents the empirical value (first row) and three counterfactual estimation results (subsequent rows), expressed as the share of the wage growth at ages 30–34 relative to the value for the U.S. economy. Each counterfactual changes the parameters in column (1) to match the U.S. level. Columns (2) and (3) report the relative wage growth of counterfactual Brazil and counterfactual Colombia, respectively, with respect to the U.S. benchmark.

Equalizing the firm productivity distribution has the largest effect. Relative wage

³⁷This non-monotonic response reflects an indirect equilibrium effect of (μ_Y, μ_O) on the young-worker reservation wage θ_Y^R . As shown in equation (6), higher learning rates lower θ_Y^R through the Rosen effect: workers anticipating faster human capital accumulation are willing to accept lower-paying entry-level jobs to gain access to the job ladder. The counterfactual changes a different margin in each country. Brazil and the United States share μ_Y , so the counterfactual only adjusts μ_O ($0.001 \rightarrow 0.003$); the direct contribution to late-career h -growth is small, and the Rosen-effect drop in θ_Y^R dominates, lengthening the lower tail of the job ladder and slightly flattening the wage profile. In Colombia, by contrast, μ_Y rises sharply ($0.006 \rightarrow 0.018$); the direct contribution to young-worker h -accumulation is large and dominates the Rosen channel, steepening the wage profile.

growth rises to 1.03 in Brazil and 1.15 in Colombia, closing the gap with the United States and slightly overshooting it for Colombia. We conclude that the main barrier to U.S.-style life-cycle wage growth in Brazil and Colombia is not lower mobility or uniformly weaker on-the-job learning, but the distribution of firms over which workers climb. Appendix Table H.3 reports the counterfactual results for the baseline estimates and yields qualitatively identical conclusions, with the firm-distribution counterfactual quantitatively close as well (1.00 for Brazil and 1.07 for Colombia under the baseline).

Robustness. We test the sensitivity of the counterfactual exercise to three modeling choices: the Pareto assumption, education residualization of wage profiles, and external discipline on the firm-productivity tail. Results are summarized in Appendix Table H.4.³⁸

First, we re-estimate the life-cycle model replacing the Pareto firm-productivity distribution with a shifted Weibull distribution,

$$\bar{\Gamma}(p) = \exp \left[- \left(\frac{p - \underline{p}}{\underline{p}} \right)^\kappa \right], \quad p \geq \underline{p},$$

where $\underline{p}$ serves jointly as the lower support and as the scale parameter (so the distribution has the same number of free parameters as the Pareto), and κ controls the shape of the upper tail. Unlike the Pareto distribution, the Weibull has a thinner upper tail when $\kappa \geq 1$, while values $\kappa < 1$ allow a stretched-Weibull tail.³⁹ This exercise tests whether our counterfactual conclusions rely on imposing a Pareto tail, and find they do not. Equalizing the firm-productivity distribution to the U.S. level remains the dominant counterfactual force, raising relative wage growth to 1.07 for Brazil and 1.30 for Colombia, compared with empirical ratios of 0.73 and 0.52, respectively.

Second, differences in educational composition could account for part of the cross-country wage-growth gap. If the share of high-education workers rises with age more steeply

³⁸Country coverage varies because the auxiliary data sources are not available with comparable quality for all three economies.

³⁹Appendix Table H.5 reports the estimates using the Weibull distribution.

in one country than another, unconditional life-cycle profiles will diverge mechanically, independent of the model’s mechanisms. To strip out this channel, we re-estimate the life-cycle model using education-residualized wage profiles constructed following [Lagakos et al. \(2018\)](#).⁴⁰ We focus this exercise on the U.S. and Brazil, where the underlying data support the richest residualization.

The residualization requires an age-period-cohort decomposition. Therefore, we consider four alternative ways of assigning aggregate growth: to cohort effects, equally to cohort and time effects, to time effects, and to the restriction of no wage growth at the end of the life cycle. For each specification, we keep the stayer wage-growth moments fixed at the residualized moments controlling for education, year, and cohort effects. Across these alternatives, Brazil’s residualized wage growth is between 39 and 48 percent of the U.S. value in the data. Equalizing the firm-productivity distribution raises this ratio to 85–87 percent, while equalizing learning raises it to 58–68 percent and equalizing labor-market frictions raises it to 50–59 percent. Thus, even after removing differences in observed education composition, the firm-productivity distribution remains the quantitatively most important component for closing the Brazil–U.S. life-cycle wage-growth gap.

Third, because the firm-side distribution is identified from life-cycle wage growth, the SMM estimates may reflect residual wage variation in addition to physical productivity differences. As a sensitivity analysis, we therefore discipline the Pareto shape parameter α externally using directly measured establishment-productivity dispersion. We focus on the United States and Colombia, for which productivity-dispersion moments can be constructed within four-digit industry cells. Both sources are establishment-level and sectorally selective, so the exercise is not a validation of the complete model-implied firm distribution. Instead, it evaluates whether the counterfactual conclusion depends on the unrestricted SMM estimate of α .

We use the hours-weighted 90–10 log output-per-hour gap computed from the BLS

⁴⁰Details about the estimation of the residualized wage profiles are presented in [Appendix A](#).

and EAM as an empirical measure of dispersion.⁴¹ Because this moment is a difference in log productivity quantiles, it is invariant to multiplicative rescalings of productivity units across sources. In a Pareto distribution, $\log Q(0.90) - \log Q(0.10) = \log(9)/\alpha$, so the lower support $\underline{p}$ cancels and the gap identifies the shape parameter α but not the scale parameter $\underline{p}$.⁴² We therefore use the productivity data to infer α separately by country, and then re-estimate only $\underline{p}$ using wage growth at ages 25–29 relative to ages 20–24.^{43,44}

Using this externally disciplined sensitivity specification does not alter the main results. As observed in the last line of Appendix Table H.4, Colombia’s wage growth equals 52 percent of the U.S. benchmark in the data, while replacing Colombia’s re-estimated firm distribution with the U.S. distribution raises the relative wage-growth ratio to 114 percent, which is very close to the 115 percent computed using the SMM estimates.⁴⁵ Equalizing learning raises the ratio to 74 percent, whereas equalizing labor-market frictions lowers it to 47 percent. This shows that the importance of firm-side wage opportunities does not depend exclusively on the unrestricted Colombian estimate. However, due to the data limitations, this exercise should be interpreted as a sensitivity result rather than external validation of the SMM distribution.

⁴¹Appendix Section C.2 details the data construction. Quantiles are weighted by establishment hours within four-digit industry-year cells and then averaged across cells and the common years 2008–2016.

⁴²For a Pareto distribution with lower support $\underline{p}$ and shape parameter α , the q th quantile is $Q(q) = \underline{p}(1 - q)^{-1/\alpha}$. Hence the log 90–10 gap satisfies $\log Q(0.90) - \log Q(0.10) = \log(9)/\alpha$. We therefore set $\alpha = \log(9)/[\log Q(0.90) - \log Q(0.10)]$, using the weighted 90–10 log productivity gap in the productivity data.

⁴³Disciplining $\underline{p}$ externally would require productivity units and coverage to be comparable across the BLS and EAM, which is less credible than using the unit-free log gap.

⁴⁴The resulting parameters and gaps are presented in Appendix Table H.6, showing 1.6867 in the United States and 1.3193 in Colombia, which imply $\alpha = 1.3027$ and $\alpha = 1.6654$, respectively. The externally disciplined Colombian shape parameter is substantially lower than the life-cycle SMM boundary estimate. Appendix Table H.7 further evaluates the fit of the model using the externally disciplined Pareto shapes, and shows that the specification reproduces wage growth quite closely at early ages but deteriorates in the later untargeted bins, consistent with the fit of the SMM boundary estimates in Table 4.

⁴⁵We also assess in Appendix Table H.8 whether the results are robust to values of α between 10 and 25. We fix $\alpha \in \{10, 12.5, 15, 20, 25\}$, re-estimate $\underline{p}$ at each value, and rerun the counterfactuals. Modifying α leaves the factual fit and headline counterfactual ratios essentially unchanged. The fitted Colombia/U.S. wage-growth ratio at ages 30–34 remains between 0.534 and 0.535; the learning counterfactual remains between 0.706 and 0.707, and the frictions counterfactual between 0.505 and 0.511. The firm-distribution counterfactual remains 1.159 because it replaces both Colombian distribution parameters with their U.S. values; relative to each alternative fitted factual economy, its increase is 0.624–0.625.

Across all three exercises, equalizing the firm productivity distribution remains the dominant counterfactual force. The conclusion that firm heterogeneity is the central driver of cross-country wage-growth differences is robust to the functional form of $\Gamma(p)$, to education composition, and to externally disciplining the upper tail.

7 Conclusion

This paper examines how the distribution of firms shapes life-cycle wage profiles across the United States, Brazil, and Colombia. We introduce a random-search model that disentangles the contributions of the firm-side distribution, labor-market frictions, and on-the-job learning, and extend it to allow learning to vary with age. The estimates reveal two distinct firm-side disadvantages: Brazil has a long lower tail of low-paying firms, while Colombia’s wage profile implies a highly compressed firm ladder. Counterfactual simulations show that equating each country’s firm-side distribution to that of the United States closes most of the Brazil gap and more than closes the Colombia gap. The main results are robust to alternative functional forms, education residualization, and an externally disciplined sensitivity analysis using measured productivity dispersion.

Beyond the firm-side mechanism, we document three further patterns. First, the life-cycle extension reveals that learning differences operate on two distinct margins: the level of early-career learning is essentially identical in the United States and Brazil but much lower in Colombia, while the persistence of learning into later ages is higher in the United States than in Brazil. Second, job-finding rates while employed exceed those for the unemployed in the two developing economies, reversing the standard ranking documented in developed labor markets. Third, decomposing wages into worker- and firm-specific components reveals that within each country the worker component contributes a share comparable to or larger than the firm component, consistent with single-country decompositions emphasizing human capital ([Bagger et al., 2014](#); [Ozkan et al., 2023](#)).

Across countries, however, worker contributions vary modestly while firm contributions vary sharply, making firm heterogeneity the dominant driver of the cross-country gap.

Two modeling choices imply that our benchmark may understate the contribution of firms under particular omitted mechanisms. First, the estimation disciplines each parameter block with distinct moments: labor-market frictions by worker flows and unemployment; learning by stayer wage growth; and the firm-type distribution only by the residual life-cycle wage growth that these channels cannot explain. Because the firm-distribution parameters are identified residually—after netting out the contribution of learning to stayer wage growth—their estimated importance declines as the estimated learning rate rises. If some stayer wage growth attributed to learning instead reflects firm heterogeneity, our benchmark may understate firms’ contribution. The wage-posting benchmark is also empirically plausible: posted wages are common in Austria, Germany, and the United States ([Holzheu and Robin, 2024](#); [Mullins and Flinn, 2026](#)) and particularly prevalent in lower-skill jobs ([Brenčić, 2012](#); [Brenzel et al., 2014](#)). Second, we allow learning to vary over the life cycle but not across firms. If more productive firms also generate faster human-capital accumulation ([Arellano-Bover and Saltiel, 2026](#); [Gregory, 2020](#); [Arellano-Bover, 2024](#)), firm productivity would shape wage growth through learning as well as through the job ladder, and our estimates may understate the total contribution of firms.

Our findings indicate that steeper life-cycle wage profiles in developed economies reflect not only differences in human capital or mobility, but also the underlying distribution of firms over which workers climb. Policies affecting the creation of productive firms, workers’ access to them, or productivity-to-wage pass-through may shape life-cycle wage growth through the job ladder. The current counterfactuals establish the quantitative importance of that ladder but do not evaluate a specific intervention. Doing so requires modeling the relevant policy and firms’ equilibrium responses.

References

- Alviarez, V., Busso, M., Keefer, P., Santos, C., and Stucchi, R. (2025). Markets for development: Improving lives through competition.
- Arellano-Bover, J. (2024). Career consequences of firm heterogeneity for young workers: First job and firm size. *Journal of Labor Economics*, 42(2):549–589.
- Arellano-Bover, J. and Saltiel, F. (2026). Differences in on-the-job learning across firms. *Journal of Labor Economics*, 44(1):149–188.
- Bagger, J., Fontaine, F., Postel-Vinay, F., and Robin, J.-M. (2014). Tenure, experience, human capital, and wages: A tractable equilibrium search model of wage dynamics. *American Economic Review*, 104(6):1551–96.
- Barlevy, G. (2008). Identification of Search Models using Record Statistics. *The Review of Economic Studies*, 75(1):29–64.
- Bonhomme, S., Holzheu, K., Lamadon, T., Manresa, E., Mogstad, M., and Setzler, B. (2023). How much should we trust estimates of firm effects and worker sorting? *Journal of Labor Economics*, 41(2):291–322.
- Bonhomme, S., Lamadon, T., and Manresa, E. (2019). A distributional framework for matched employer employee data. *Econometrica*, 87(3):699–739.
- Bontemps, C., Robin, J.-M., and van den Berg, G. J. (2000). Equilibrium search with continuous productivity dispersion: Theory and nonparametric estimation. *International Economic Review*, 41(2):305–358.
- Borovičková, K. and Shimer, R. (2017). High wage workers work for high wage firms. NBER Working Paper 24074, National Bureau of Economic Research.
- Brenčič, V. (2012). Wage posting: evidence from job ads. *The Canadian Journal of Economics / Revue canadienne d’Economie*, 45(4):1529–1559.

- Brenzel, H., Gartner, H., and Schnabel, C. (2014). Wage bargaining or wage posting? evidence from the employers’ side. *Labour Economics*, 29:41–48.
- Burdett, K., Carrillo-Tudela, C., and Coles, M. G. (2011). Human capital accumulation and labor market equilibrium. *International Economic Review*, 52(3):657–677.
- Burdett, K. and Mortensen, D. T. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, 39(2):257–273.
- Busso, M., Madrigal, L., and Pagés, C. (2013). Productivity and resource misallocation in latin america. *The B.E. Journal of Macroeconomics*, 13(1):903–932.
- Chen, Z. (2026). Economic growth and the rise of large firms. *Econometrica*. Forthcoming.
- Deaton, A. (1997). *The Analysis of Household Surveys: A Microeconometric Approach to Development Policy*. World Bank, Washington, DC.
- Donovan, K., Lu, W. J., and Schoellman, T. (2023). Labor Market Dynamics and Development. *The Quarterly Journal of Economics*, 138(4):2287–2325.
- Engbom, N. (2022). Labor market fluidity and human capital accumulation. Working Paper 29698, National Bureau of Economic Research.
- Eslava, M., Haltiwanger, J., and Pinzón, Á. (2022). Job creation in colombia versus the usa: “up-or-out dynamics” meet “the life cycle of plants”. *Economica*, 89(355):511–539.
- Eslava, M., Haltiwanger, J., and Urdaneta, N. (2023). The size and life-cycle growth of plants: The role of productivity, demand, and wedges. *The Review of Economic Studies*, 91(1):259–300.
- Eslava, M., Meléndez, M., and Urdaneta, N. (2021). Market concentration, market fragmentation and inequality in latin america. Technical report, UNDP LAC Working Paper Series.

- Fang, H. and Qiu, X. (2023). Golden Ages: A tale of the labor markets in china and the united states. *Journal of Political Economy Macroeconomics*, 1(4):665–706.
- Flinn, C. J., Todd, P. E., and Zhang, W. (2025). Labor market returns to personality: A job search approach to understanding gender gaps. *Journal of Political Economy*, 133(4):1169–1234.
- Gregory, V. (2020). Firms as Learning Environments: Implications for Earnings Dynamics and Job Search. Working Papers 2020-036, Federal Reserve Bank of St. Louis.
- Guner, N., Parkhomenko, A., and Ventura, G. (2018). Managers and productivity differences. *Review of Economic Dynamics*, 29:256–282.
- Guvenen, F., Karahan, F., Ozkan, S., and Song, J. (2021). What Do Data on Millions of U.S. Workers Reveal About Lifecycle Earnings Dynamics? *Econometrica*, 89(5):2303–2339.
- Heckman, J. J., Lochner, L., and Taber, C. (1998). Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents. *Review of Economic Dynamics*, 1(1):1–58.
- Holzheu, K. and Robin, J.-M. (2024). Wage Bargaining and Wage Posting Firms. Working paper.
- Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing tfp in china and india. *The Quarterly Journal of Economics*, 124(4):1403–1448.
- Hsieh, C.-T. and Klenow, P. J. (2014). The Life Cycle of Plants in India and Mexico *. *The Quarterly Journal of Economics*, 129(3):1035–1084.
- Hsieh, C.-T. and Olken, B. A. (2014). The missing "missing middle". *Journal of Economic Perspectives*, 28(3):89–108.

- Hsu, M. and Leyton, S. (2026). Life cycle wage growth in a developing economy: Employment formality and sector-specific human capital accumulation. GRIPS Discussion Papers 25-14, National Graduate Institute for Policy Studies. R&R JDE.
- Huggett, M., Ventura, G., and Yaron, A. (2011). Sources of lifetime inequality. *The American Economic Review*, 101(7):2923–2954.
- Jedwab, R., Romer, P., Islam, A. M., and Samaniego, R. (2023). Human capital accumulation at work: Estimates for the world and implications for development. *American Economic Journal: Macroeconomics*, 15(3):191–223.
- Jolivet, G., Postel-Vinay, F., and Robin, J.-M. (2006). The empirical content of the job search model: Labor mobility and wage distributions in europe and the us. *European Economic Review*, 50(4):877–907.
- Lagakos, D., Moll, B., Porzio, T., Qian, N., and Schoellman, T. (2018). Life cycle wage growth across countries. *Journal of Political Economy*, 126(2):797–849.
- Lasso, F. J. (2013). La dinámica del desempleo urbano en Colombia. Borradores de Economía 667, Banco de la Republica de Colombia.
- Ma, X., Nakab, A., and Vidart, D. (2024). Human capital investment and development: The role of on-the-job training. *Journal of Political Economy Macroeconomics*, 2(1):107–148.
- Ma, Y., Zhang, M., and Zimmermann, K. (2026). Business concentration around the world: 1900-2020. Working Paper 34711, National Bureau of Economic Research.
- Manning, A. (2021). The elusive employment effect of the minimum wage. *Journal of Economic Perspectives*, 35(1):3–26.
- Menzio, G., Telyukova, I. A., and Visschers, L. (2016). Directed search over the life cycle. *Review of Economic Dynamics*, 19:38–62.

- Moscarini, G. and Postel-Vinay, F. (2023). The Job Ladder: Inflation vs. Reallocation. NBER Working Paper 31466, National Bureau of Economic Research.
- Mullins, J. and Flinn, C. (2026). Firms' Choices of Wage-Setting Protocols. Unpublished manuscript, March 6. Available at SSRN: <https://ssrn.com/abstract=6353938>.
- Ozkan, S., Song, J., and Karahan, F. (2023). Anatomy of lifetime earnings inequality: Heterogeneity in job-ladder risk versus human capital. *Journal of Political Economy Macroeconomics*, 1(3):506–550.
- Poschke, M. (2018). The firm size distribution across countries and skill-biased change in entrepreneurial technology. *American Economic Journal: Macroeconomics*, 10(3):1–41.
- Postel-Vinay, F. and Robin, J.-m. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6):2295–2350.
- Rivera-Padilla, A. (2025). Life-cycle wage growth and internal migration. *International Economic Review*, 66(4):1505–1523.
- Rosen, S. (1972). Learning and experience in the labor market. *The Journal of Human Resources*, 7(3):326–342.
- Rubinstein, Y. and Weiss, Y. (2006). Chapter 1 post schooling wage growth: Investment, search and learning. In Hanushek, E. and Welch, F., editors, *Handbook of the Economics of Education*, volume 1 of *Handbook of the Economics of Education*, pages 1–67. Elsevier.
- Vaghul, K. and Zipperer, B. (2022). Historical state and sub-state minimum wages. <https://github.com/benzipperer/historicalminwage>. Accessed: 05/06/2026.
- Yamaguchi, S. (2010). Job search, bargaining, and wage dynamics. *Journal of Labor Economics*, 28(3):595–631.

Online Appendix to:

Life-Cycle Wage Growth and Firm Productivity in Developed and Developing Economies

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September 25, 2026

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A Age-Earnings Profiles

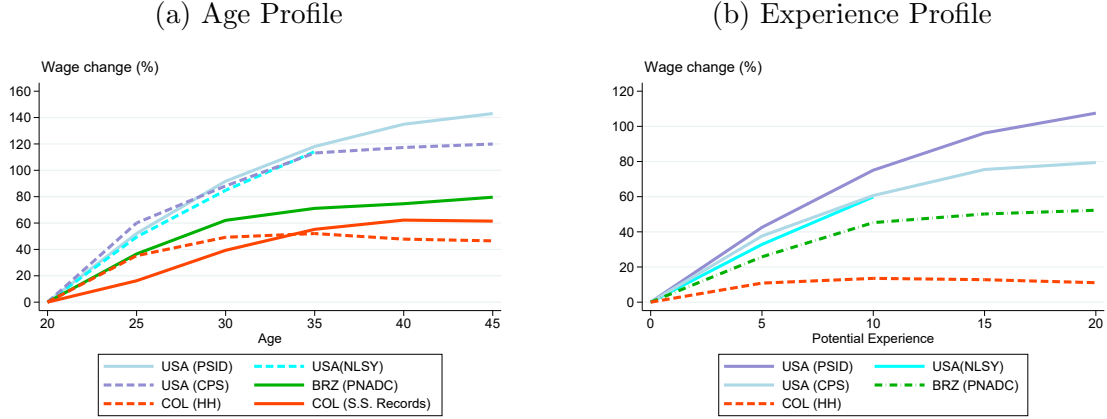
This appendix expands the descriptive evidence on life-cycle wage profiles introduced in the main text. We present age profiles across multiple data sets for the United States (PSID, CPS, and NLSY97), Brazil (PNADC), and Colombia (GEIH and Social Security Records), and document the robustness of the cross-country differences in life-cycle wage growth to alternative sample restrictions and to alternative identifying assumptions on the age–time–cohort decomposition.

Age and experience profiles across data sets. Figure A.1 shows that the cross-country differences in life-cycle wage growth highlighted in Figure 1 are not an artifact of the particular data set used as benchmark. The left panel plots age profiles using the PSID, the CPS, and the NLSY for the United States, together with the PNADC for Brazil, and household-survey data and social security records for Colombia. The right panel plots the corresponding profiles by potential experience, defined as the lesser of the time since age 18 and the time since graduating from the highest level of education. Across all U.S. data sets, life-cycle wage growth is substantially steeper than in Brazil and Colombia, both in age and in experience.

Sample restrictions. We next examine the sensitivity of the age profiles to the sample of workers, comparing all employed workers to the subset of full-time male employees. Figure A.2 reports the two profiles side by side. The cross-country ranking is unchanged: U.S. profiles are systematically steeper than Brazilian and Colombian profiles, regardless of the sample restriction. Restricting to full-time male employees attenuates the cross-country differences slightly, consistent with female workers and part-time workers contributing less wage growth on average, but the qualitative pattern is preserved.

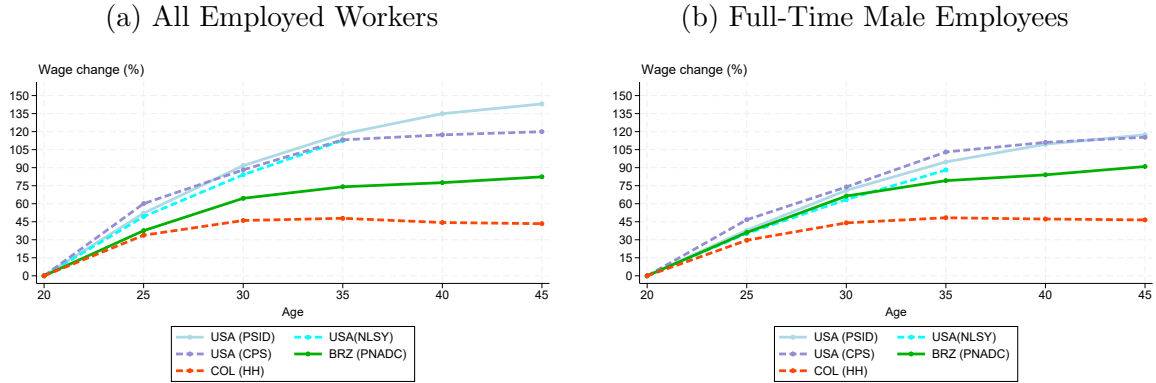
Age, time, and cohort effects. Cross-sectional age profiles confound the effects of age, time, and birth cohort, since age equals year minus cohort by construction. Following

Appendix Figure A.1: Age and Experience Profiles of Wages Across Multiple Data Sets



Notes: Wage change corresponds to the growth rate of average monthly wages relative to a baseline group. In the left panel, wages are measured for workers in each five-year age bin relative to workers aged 20–24. In the right panel, wages are measured for workers in each five-year experience bin relative to workers with 0–4 years of experience. Potential experience is defined as the lesser of two duration measures: the time since reaching age 18 and the time since graduating from the highest level of education.

Appendix Figure A.2: Age–Wage Profiles Across Samples



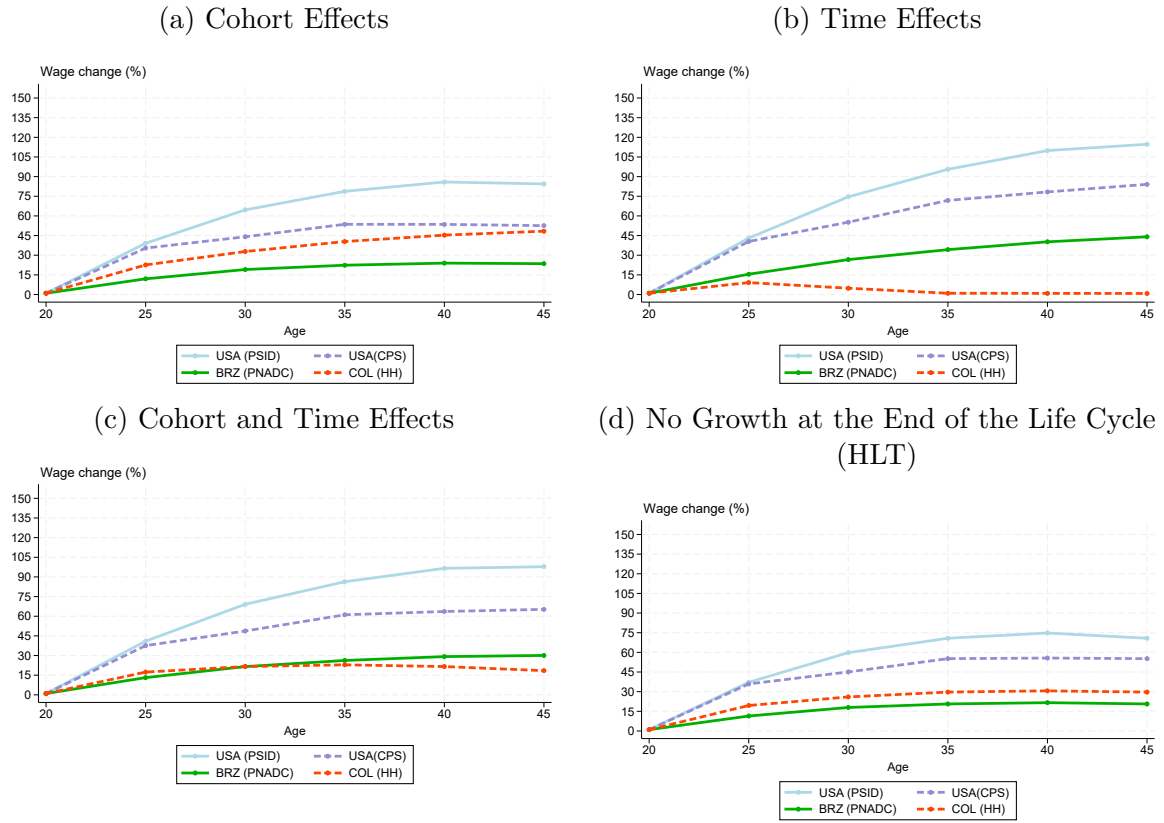
Notes: The figures plot the growth rate of average monthly wages by five-year age bin, relative to workers aged 20–24. Panel (a) includes all employed workers in each data set; panel (b) restricts to full-time male employees.

Lagakos et al. (2018), we report profiles under four alternative identifying assumptions that resolve this collinearity. We estimate a Mincer regression of log wages on years of schooling, five-year experience-group dummies, and either cohort dummies, year dummies, or a restriction implied by economic theory; under each restriction, the experience-group coefficients trace out the life-cycle wage profile.

The four panels in Figure A.3 correspond to the four restrictions discussed in Lagakos

et al. (2018). Panel (a) attributes all aggregate wage growth to birth-cohort effects and uses year dummies only to capture cyclical fluctuations, following Deaton (1997). Panel (b) attributes all aggregate growth to time effects, with cohort dummies absorbing only level differences across cohorts. Panel (c) averages the two assumptions, splitting aggregate growth equally between time and cohort. Panel (d) implements the Heckman et al. (1998) (HLT) restriction adopted by Lagakos et al. (2018), which uses the theoretical prediction that wage growth is approximately zero in the final years of the working life to disentangle the contributions of time and cohort effects.

Appendix Figure A.3: Age–Wage Profiles Following Lagakos et al. (2018)



Notes: Each panel reports estimates from a Mincer regression of log wages on years of schooling and five-year experience-group dummies under a different identifying assumption on time and cohort effects, following Lagakos et al. (2018). Panel (a): aggregate wage growth attributed to birth-cohort effects (Deaton, 1997). Panel (b): aggregate wage growth attributed to time effects. Panel (c): aggregate growth split equally between time and cohort. Panel (d): the Heckman et al. (1998) (HLT) restriction that the experience profile is flat in the final working years.

Table A.1 summarizes the wage-growth ratios at ages 40–44 (relative to ages 20–24)

under each of the four identifying restrictions. The U.S. PSID profile is the steepest across all four panels, exceeding both Brazilian and Colombian profiles at every age. The cross-country gap is largest under the time-effects restriction in panel (b) and smallest under the cohort-only restriction in panel (a). Under the HLT restriction in panel (d)—the specification [Lagakos et al. \(2018\)](#) consider most defensible—U.S. wage growth still exceeds the Brazilian and Colombian counterparts by a factor of three to four. The robustness of this ordering across all four identifying restrictions reinforces the conclusion that the cross-country differences in life-cycle wage growth documented in the main text are not an artifact of how aggregate productivity growth is attributed to time and cohort effects.

Appendix Table A.1: Life-Cycle Wage Growth at Ages 40–44 Across Identifying Restrictions

Restriction	United States	Brazil	Colombia
(a) Cohort effects only	$\approx 85\%$	—	—
(b) Time effects only	$\approx 115\%$	$\approx 44\%$	$\approx 0\%$
(c) Cohort and time effects (equal)	$\approx 100\%$	—	—
(d) HLT (Heckman et al., 1998)	$\approx 75\%$	$\approx 21\%$	$\approx 30\%$

Notes: Approximate values read off Figure [A.3](#). Each cell reports the wage at ages 40–44 relative to ages 20–24, expressed as a growth percentage. The U.S. source is the PSID, Brazil the PNADC, and Colombia the GEIH/PILA. Entries marked — indicate that the corresponding panel does not report a comparable number in the figure.

B Derivations and Implementation Details

B.1 Firm Size

Let $l(w_0)$ denote average employment per firm among firms posting wage component w_0 . Let F denote the cross-firm CDF of posted wage components with density $f(w_0) \equiv F'(w_0)$, and let G denote the cross-worker CDF of posted wage components among employed workers with density $g(w_0) \equiv G'(w_0)$. The per-worker exit rate from a firm posting w_0 is

$$s(w_0) = \delta + \lambda_1 \bar{F}(w_0), \quad \bar{F}(w_0) \equiv 1 - F(w_0). \quad (\text{B.1})$$

Proposition B.1 (Firm size). *In steady state, the size of firms posting wage component w_0 satisfies*

$$l(w_0) = \frac{A}{(\delta + \lambda_1 \bar{F}(w_0))^2}, \quad (\text{B.2})$$

for some constant $A > 0$.

Proof sketch. Consider the set of employed workers at firms posting wage components at most w_0 . In steady state, inflows into this set from unemployment equal outflows from this set due to separations and job-to-job moves to firms posting above w_0 . Hence,

$$u\lambda_0 F(w_0) = (1 - u)G(w_0)(\delta + \lambda_1 \bar{F}(w_0)). \quad (\text{B.3})$$

Using the unemployment steady state $u = \delta/(\delta + \lambda_0)$, equation (B.3) implies

$$G(w_0) = \frac{\delta F(w_0)}{\delta + \lambda_1 \bar{F}(w_0)}. \quad (\text{B.4})$$

Differentiating (B.4) yields the worker density

$$g(w_0) = G'(w_0) = \frac{\delta(\delta + \lambda_1)}{(\delta + \lambda_1 \bar{F}(w_0))^2} f(w_0).$$

Since $(1 - u)Mg(w_0)$ is the density of employed workers across wage components and $Lf(w_0)$ is the density of firms across wage components, average employment per firm at w_0 is

$$l(w_0) = \frac{(1 - u)Mg(w_0)}{Lf(w_0)} = \frac{(1 - u)M\delta(\delta + \lambda_1)}{L(\delta + \lambda_1\bar{F}(w_0))^2}.$$

Renaming the constant $A \equiv (1 - u)M\delta(\delta + \lambda_1)/L$ yields (B.2).

B.2 Reservation Wage Component

This subsection derives the reservation wage component stated in (3). We take as given the worker Bellman equations after imposing the reservation strategy and applying integration by parts:

$$U(h) = b(h) + \beta U(h) + \beta \lambda_0 \int_{\theta^R}^{\bar{w}_0} W_x(x, h) \bar{F}(x) dx, \quad (\text{B.5})$$

$$W(w_0, h) = w(w_0, h) + \beta \delta U(h) + \beta(1 - \delta)W(w_0, h') + \beta \lambda_1 \int_{w_0}^{\bar{w}_0} W_x(x, h') \bar{F}(x) dx, \quad (\text{B.6})$$

where $b(h) = b_0 + h$, $w(w_0, h) = w_0 + h$, and $h' = h + \mu$ when employed.

Lemma B.1 (Affine structure in worker productivity). *Under $b(h) = b_0 + h$ and $w(w_0, h) = w_0 + h$, there exist functions $\tilde{U} \in \mathbb{R}$ and $\tilde{W}(\cdot)$ such that*

$$U(h) = \tilde{U} + \frac{h}{1 - \beta}, \quad W(w_0, h) = \tilde{W}(w_0) + \frac{h}{1 - \beta}.$$

In particular, $W_x(x, h) = \tilde{W}'(x)$ does not depend on h .

Proof sketch. Guess $U(h) = \tilde{U} + h/(1 - \beta)$ and $W(w_0, h) = \tilde{W}(w_0) + h/(1 - \beta)$. Since $W_x(x, h) = \tilde{W}'(x)$, the integral terms in (B.5)–(B.6) do not depend on h . Substituting into (B.5), the coefficients on h match identically because $\frac{h}{1 - \beta} = h + \beta \frac{h}{1 - \beta}$. The remaining terms pin down $\tilde{U}$. Substituting into (B.6) and using $h' = h + \mu$ yields the same identity for the h terms and pins down $\tilde{W}(\cdot)$. Hence the affine representation holds and W_x is

independent of h .

Lemma B.2 (Reservation wage component). *Let θ^R be the reservation wage component.*

Define

$$J(\theta^R) \equiv \int_{\theta^R}^{\bar{w}_0} W_x(x, \cdot) \bar{F}(x) dx,$$

(where W_x is independent of h by Lemma B.1). Then θ^R satisfies

$$\theta^R = b_0 - \beta(1 - \delta) \frac{\mu}{1 - \beta} + \beta(\lambda_0 - \lambda_1) J(\theta^R), \quad (\text{B.7})$$

which is (3).

Proof sketch. Using Lemma B.1, write $U(h) = \tilde{U} + h/(1 - \beta)$ and $W(w_0, h) = \tilde{W}(w_0) + h/(1 - \beta)$. Then (B.5) becomes, after canceling the h terms,

$$(1 - \beta)\tilde{U} = b_0 + \beta\lambda_0 J(\theta^R). \quad (\text{B.8})$$

Similarly, (B.6) becomes

$$\tilde{W}(w_0) = w_0 + \beta\delta\tilde{U} + \beta(1 - \delta)\tilde{W}(w_0) + \beta(1 - \delta) \frac{\mu}{1 - \beta} + \beta\lambda_1 J(w_0),$$

where $J(w_0) \equiv \int_{w_0}^{\bar{w}_0} \tilde{W}'(x) \bar{F}(x) dx$.

At the reservation component, $W(\theta^R, h) = U(h)$ for all h , hence $\tilde{W}(\theta^R) = \tilde{U}$ and $J(\theta^R)$ is the integral appearing in (B.5). Evaluating the previous display at $w_0 = \theta^R$ and substituting $\tilde{W}(\theta^R) = \tilde{U}$ gives

$$(1 - \beta)\tilde{U} = \theta^R + \beta\lambda_1 J(\theta^R) + \beta(1 - \delta) \frac{\mu}{1 - \beta}. \quad (\text{B.9})$$

Equating (B.8) and (B.9) yields

$$b_0 + \beta\lambda_0 J(\theta^R) = \theta^R + \beta\lambda_1 J(\theta^R) + \beta(1 - \delta) \frac{\mu}{1 - \beta},$$

which rearranges to (B.7).

B.3 Wage Equation

Lemma B.3 (Wage schedule up to a boundary condition). *In any equilibrium with a strictly increasing posting function $w_0(\cdot)$, the profit function satisfies the envelope condition*

$$\pi'(p) = l(w_0(p)) = \frac{A}{(\delta + \lambda_1 \bar{\Gamma}(p))^2}. \quad (\text{B.10})$$

Integrating from $\underline{p}$ to p gives

$$\pi(p) - \pi(\underline{p}) = A \int_{\underline{p}}^p \frac{1}{(\delta + \lambda_1 \bar{\Gamma}(x))^2} dx. \quad (\text{B.11})$$

Equivalently, dividing by A and using $\pi(p)/A = (p - w_0(p))/(\delta + \lambda_1 \bar{\Gamma}(p))^2$ yields

$$\frac{p - w_0(p)}{(\delta + \lambda_1 \bar{\Gamma}(p))^2} - \frac{\pi(\underline{p})}{A} = \int_{\underline{p}}^p \frac{1}{(\delta + \lambda_1 \bar{\Gamma}(x))^2} dx. \quad (\text{B.12})$$

Proof sketch. The envelope theorem gives $\pi'(p) = \partial[(p - w_0)p]/\partial p = l$ because $w_0(p)$ is chosen optimally. Substituting the equilibrium size expression yields (B.10). Integrating gives (B.11). The final expression follows from rewriting $\pi(p)/A$ using $l = A/(\delta + \lambda_1 \bar{\Gamma}(p))^2$.

B.4 Comparative Statics

Lemma B.4 (Comparative statics of the rent-normalized slope). *Let $m(p) \equiv p - w_0(p)$ denote the per-worker rent (markup) at firm productivity p , with $w_0(\cdot)$ differentiable and strictly increasing. Define the rent-normalized slope*

$$\psi(p) \equiv \frac{w'_0(p)}{m(p)}.$$

$\psi(p)$ satisfies

$$\psi(p) = \frac{2k_1 \gamma(p)}{1 + k_1 \bar{\Gamma}(p)}, \quad k_1 \equiv \frac{\lambda_1}{\delta}. \quad (\text{B.13})$$

(i) Holding (Γ, γ) fixed, $\psi(p)$ is strictly increasing in k_1 :

$$\frac{\partial \psi(p)}{\partial k_1} = \frac{2 \gamma(p)}{(1 + k_1 \bar{\Gamma}(p))^2} > 0. \quad (\text{B.14})$$

(ii) Under Pareto $\bar{\Gamma}_\alpha(p) = (\underline{p}/p)^\alpha$ (with density $\gamma_\alpha(p) = \alpha(\underline{p}/p)^\alpha p^{-1}$), write $z_\alpha(p) \equiv (\underline{p}/p)^\alpha$. Then

$$\psi(p; \alpha, k_1) = \frac{2\alpha}{p} \cdot \frac{k_1 z_\alpha(p)}{1 + k_1 z_\alpha(p)}, \quad (\text{B.15})$$

and its derivative w.r.t. the Pareto shape parameter is

$$\frac{\partial \psi(p; \alpha, k_1)}{\partial \alpha} = \frac{2}{p} \cdot \frac{k_1 z_\alpha(p)}{(1 + k_1 z_\alpha(p))^2} \left(1 + k_1 z_\alpha(p) - \alpha \ln(\underline{p}/p) \right). \quad (\text{B.16})$$

In particular, $\partial \psi / \partial \alpha$ is positive for p close to $\underline{p}$ and becomes negative for sufficiently large p (i.e. far in the right tail).

(iii) Under Pareto, $\psi(p; \alpha, k_1)$ is strictly increasing in the scale parameter $\underline{p}$:

$$\frac{\partial \psi(p; \alpha, k_1)}{\partial \underline{p}} = \frac{2\alpha}{p} \cdot \frac{k_1}{(1 + k_1 z_\alpha(p))^2} \cdot \frac{\partial z_\alpha(p)}{\partial \underline{p}} > 0, \quad (\text{B.17})$$

where $\frac{\partial z_\alpha(p)}{\partial \underline{p}} = \frac{\alpha}{\underline{p}} z_\alpha(p)$. Equivalently,

$$\frac{\partial \psi(p; \alpha, k_1)}{\partial \underline{p}} = \frac{2\alpha^2}{p \underline{p}} \cdot \frac{k_1 z_\alpha(p)}{(1 + k_1 z_\alpha(p))^2} > 0. \quad (\text{B.18})$$

Proof sketch. By definition $m(p) = p - w_0(p)$ and hence $m'(p) = 1 - w'_0(p)$. The posting FOC in the BM model implies the ODE

$$m'(p) = 1 - \frac{2k_1 \gamma(p)}{1 + k_1 \bar{\Gamma}(p)} m(p),$$

which is equivalent to (B.13) after rearranging, i.e. $w'_0(p) = \psi(p)m(p)$. Differentiating (B.13) w.r.t. k_1 holding (Γ, γ) fixed yields (B.14).

Under Pareto, substitute $\bar{\Gamma}_\alpha(p) = z_\alpha(p)$ and $\gamma_\alpha(p) = \alpha z_\alpha(p)/p$ into (B.13) to obtain (B.15). For (B.16), note that $z_\alpha(p) = \exp\{-\alpha \ln(p/\underline{p})\}$ so $\partial z_\alpha(p)/\partial \alpha = -z_\alpha(p) \ln(p/\underline{p})$, and apply the product rule to $\psi(p; \alpha, k_1) = (2/p) \alpha [k_1 z_\alpha / (1 + k_1 z_\alpha)]$, which yields (B.16). The final sign statement follows because $\ln(p/\underline{p}) \downarrow 0$ as $p \downarrow \underline{p}$ and $\ln(p/\underline{p}) \rightarrow \infty$ as $p \rightarrow \infty$, while $1 + k_1 z_\alpha(p) \in (1, 1 + k_1]$.

For (iii), note $\partial z_\alpha(p)/\partial \underline{p} = (\alpha/\underline{p}) z_\alpha(p) > 0$ and differentiate (B.15) in $\underline{p}$ using the chain rule; this gives (B.17)–(B.18) and the strict positivity.

These comparative statics are summarized in Appendix Table B.1.

Appendix Table B.1: Effects on the Rent-Normalized Slope

Labor market frictions		
Job finding rate (employed)	λ_1	$\partial \psi(p)/\partial \lambda_1 > 0$
Job finding rate (unemployed)	λ_0	$\partial \psi(p)/\partial \lambda_0 = 0$
Separation rate	δ	$\partial \psi(p)/\partial \delta < 0$
Firm productivity		
Scale (left tail)	$\underline{p}$	$\partial \psi(p)/\partial \underline{p} > 0$
Shape (right tail)	α	$\partial \psi(p)/\partial \alpha \gtrless 0$

Notes: Comparative statics of the rent-normalized slope $\psi(p)$ with respect to labor-market frictions and the parameters of the firm productivity distribution $\Gamma(p)$. A positive (negative) sign indicates that an increase in the parameter steepens (flattens) the rent-normalized slope; $\gtrless 0$ indicates that the sign depends on parameter values. The derivative with respect to λ_0 is zero for $\psi(p)$ holding the firm distribution and k_1 fixed; λ_0 still affects reservation wages, unemployment, and the lower support through θ^R . Derivations are in Lemma B.4.

B.5 Life-Cycle Extension

B.5.1 Setting

We extend the baseline model to allow for age-dependent learning. Workers belong to one of two age groups $a \in \{Y, O\}$, where Y denotes young and O old. Learning rates satisfy $\mu_Y \geq \mu_O \geq 0$. A young worker becomes old with probability η each period; an old worker dies with probability ν . An equal measure of newborn young workers replaces

those who die. All other primitives are common across age groups: job-finding rates (λ_0, λ_1) , separation rate δ , unemployment benefits $b(h) = b_0 + h$, and the firm-productivity distribution $\Gamma(p)$.

Workers collect flow payoffs at the beginning of the period. At the end of the period, age transition is realized first. Next, labor-market shocks occur: an employed worker separates with probability δ and, if the match survives, receives an outside offer with probability λ_1 ; an unemployed worker receives an offer with probability λ_0 . Finally, workers who remain employed learn on the job. We define

$$\beta_O \equiv (1 - \nu)\tilde{\beta}, \quad \beta_Y \equiv (1 - \eta)\tilde{\beta}.$$

To preserve a single common posting distribution across age groups, we restrict attention to common-support equilibria in which old workers are weakly more selective at the bottom of the ladder:

$$\theta_Y^R \leq \theta_O^R,$$

and firms do not post wage components below θ_O^R . Equivalently, the lower bound of active firm types satisfies

$$w_0(\underline{p}) = \theta_O^R.$$

This rules out age-segmented low-wage firms and keeps the firm-side problem one-dimensional.

B.5.2 Value Functions

Old workers face the baseline model with parameters (μ_O, β_O) . Hence the affine representation from the baseline model applies:

$$U_O(h) = \tilde{U}_O + \frac{h}{1 - \beta_O}, \quad W_O(w_0, h) = \tilde{W}_O(w_0) + \frac{h}{1 - \beta_O}.$$

For young workers, unemployment value mixes over future age states:

$$\begin{aligned}
U_Y(h) = & b(h) + (1 - \eta)\tilde{\beta} \left[U_Y(h) + \lambda_0 \int_{\theta_Y^R}^{\bar{w}_0} W_{Y,x}(x, h) \bar{F}(x) dx \right] \\
& + \eta\tilde{\beta} \left[U_O(h) + \lambda_0 \int_{\theta_O^R}^{\bar{w}_0} W_{O,x}(x, h) \bar{F}(x) dx \right], \tag{B.19}
\end{aligned}$$

and the employment value is

$$\begin{aligned}
W_Y(w_0, h) = & w(w_0, h) \\
& + (1 - \eta)\tilde{\beta} \left[\delta U_Y(h) + (1 - \delta)W_Y(w_0, h + \mu_Y) \right. \\
& \quad \left. + \lambda_1 \int_{w_0}^{\bar{w}_0} W_{Y,x}(x, h + \mu_Y) \bar{F}(x) dx \right] \\
& + \eta\tilde{\beta} \left[\delta U_O(h) + (1 - \delta)W_O(w_0, h + \mu_O) \right. \\
& \quad \left. + \lambda_1 \int_{w_0}^{\bar{w}_0} W_{O,x}(x, h + \mu_O) \bar{F}(x) dx \right]. \tag{B.20}
\end{aligned}$$

Because both $W_Y(\cdot, h)$ and $W_O(\cdot, h)$ are increasing in w_0 , the job-to-job decision remains threshold-based: an employed worker moves if and only if the outside offer posts a higher wage component.

Lemma B.5 (Affine structure for young workers). *Under $b(h) = b_0 + h$ and $w(w_0, h) = w_0 + h$, there exist a scalar $\tilde{U}_Y$ and a function $\tilde{W}_Y(\cdot)$ such that*

$$U_Y(h) = \tilde{U}_Y + c_Y h, \quad W_Y(w_0, h) = \tilde{W}_Y(w_0) + c_Y h,$$

where

$$c_Y = \frac{1 + \frac{\eta\tilde{\beta}}{1 - \beta_O}}{1 - (1 - \eta)\tilde{\beta}}. \tag{B.21}$$

In particular, $W_{Y,x}(x, h) = \tilde{W}_Y'(x)$ is independent of h .

Proof sketch. Guess $U_Y(h) = \tilde{U}_Y + c_Y h$ and $W_Y(w_0, h) = \tilde{W}_Y(w_0) + c_Y h$. Substituting

into (B.19)–(B.20) and collecting coefficients on h gives

$$c_Y = 1 + (1 - \eta)\tilde{\beta} c_Y + \eta\tilde{\beta} \frac{1}{1 - \beta_O},$$

which yields (B.21). The integral terms depend on w_0 but not on h , hence $W_{Y,x}$ is independent of h .

B.5.3 Reservation Wage Components

Define, for $a \in \{Y, O\}$,

$$J_a(w_0) \equiv \int_{w_0}^{\bar{w}_0} \tilde{W}'_a(x) \bar{F}(x) dx.$$

For old workers, the reservation wage component is the baseline expression with (μ_O, β_O) :

$$\theta_O^R = b_0 - \beta_O(1 - \delta) \frac{\mu_O}{1 - \beta_O} + \beta_O(\lambda_0 - \lambda_1) J_O(\theta_O^R). \quad (\text{B.22})$$

For young workers, the reservation identity is:

$$\begin{aligned} \theta_Y^R = & b_0 - (1 - \eta)\tilde{\beta}(1 - \delta)c_Y\mu_Y - \eta\tilde{\beta}(1 - \delta) \frac{\mu_O}{1 - \beta_O} + (1 - \eta)\tilde{\beta}(\lambda_0 - \lambda_1)J_Y(\theta_Y^R) \\ & + \eta\tilde{\beta} \left[\lambda_0 J_O(\theta_O^R) - \lambda_1 J_O(\theta_Y^R) + (1 - \delta)(\tilde{U}_O - \tilde{W}_O(\theta_Y^R)) \right]. \end{aligned} \quad (\text{B.23})$$

Under the common-support restriction $\theta_Y^R \leq \theta_O^R$ and $w_0(\underline{p}) = \theta_O^R$, this simplifies to

$$\begin{aligned} \theta_Y^R = & b_0 - \underbrace{\left[(1 - \eta)\tilde{\beta}(1 - \delta)c_Y\mu_Y + \eta\tilde{\beta}(1 - \delta) \frac{\mu_O}{1 - \beta_O} \right]}_{\text{Age-weighted Rosen effect}} \\ & + \underbrace{\left[(1 - \eta)\tilde{\beta}(\lambda_0 - \lambda_1)J_Y(\theta_Y^R) + \eta\tilde{\beta}(\lambda_0 - \lambda_1)J_O(\theta_O^R) \right]}_{\text{Age-weighted entry effect}} \\ & + \underbrace{\eta\tilde{\beta}(1 - \delta - \lambda_1)[\tilde{U}_O - \tilde{W}_O(\theta_Y^R)]}_{\text{Aging effect}}. \end{aligned} \quad (\text{B.24})$$

The last term is non-negative because $\theta_Y^R \leq \theta_O^R$ implies $\tilde{W}_O(\theta_Y^R) \leq \tilde{W}_O(\theta_O^R) = \tilde{U}_O$.

Proof sketch. Using Lemma B.5, write

$$U_Y(h) = \tilde{U}_Y + c_Y h, \quad W_Y(w_0, h) = \tilde{W}_Y(w_0) + c_Y h,$$

and use the old-worker affine representation. After canceling the h terms in (B.19),

$$(1 - (1 - \eta)\tilde{\beta})\tilde{U}_Y = b_0 + (1 - \eta)\tilde{\beta}\lambda_0 J_Y(\theta_Y^R) + \eta\tilde{\beta}[\tilde{U}_O + \lambda_0 J_O(\theta_O^R)]. \quad (\text{B.25})$$

Evaluating (B.20) at $w_0 = \theta_Y^R$ and using $\tilde{W}_Y(\theta_Y^R) = \tilde{U}_Y$ gives

$$\begin{aligned} (1 - (1 - \eta)\tilde{\beta})\tilde{U}_Y &= \theta_Y^R + (1 - \eta)\tilde{\beta} [(1 - \delta)c_Y \mu_Y + \lambda_1 J_Y(\theta_Y^R)] \\ &\quad + \eta\tilde{\beta} \left[\delta\tilde{U}_O + (1 - \delta)\tilde{W}_O(\theta_Y^R) + (1 - \delta)\frac{\mu_O}{1 - \beta_O} + \lambda_1 J_O(\theta_Y^R) \right]. \end{aligned} \quad (\text{B.26})$$

Equating (B.25) and (B.26) yields (B.23). Under $\theta_Y^R \leq \theta_O^R$ and the common-support restriction, one has

$$J_O(\theta_Y^R) = [\tilde{U}_O - \tilde{W}_O(\theta_Y^R)] + J_O(\theta_O^R),$$

which turns (B.23) into (B.24).

B.5.4 Steady State and Firm Size

The age-specific population masses satisfy

$$M_Y = \frac{\nu}{\eta + \nu} M, \quad M_O = \frac{\eta}{\eta + \nu} M. \quad (\text{B.27})$$

Define the effective rates

$$\tilde{\delta} \equiv \eta + (1 - \eta)\delta, \quad \tilde{\lambda}_1 \equiv (1 - \eta)\lambda_1, \quad \hat{\delta} \equiv \nu + (1 - \nu)\delta, \quad \hat{\lambda}_1 \equiv (1 - \nu)\lambda_1. \quad (\text{B.28})$$

The steady-state unemployment rates are

$$u_Y = \frac{\eta(1 - \lambda_0) + (1 - \eta)\delta}{\eta + (1 - \eta)(\lambda_0 + \delta)}, \quad (\text{B.29})$$

and

$$u_O = \frac{\delta + \nu u_Y(1 - \lambda_0 - \delta)}{\nu + (1 - \nu)(\lambda_0 + \delta)}. \quad (\text{B.30})$$

Let $G_a(w_0)$ denote the cross-worker CDF of posted wage components among employed workers of age a , and let g_a denote the corresponding density. For young employed workers, the same stock-balance logic as in the baseline Burdett–Mortensen model yields

$$G_Y(w_0) = \frac{\tilde{\delta} F(w_0)}{\tilde{\delta} + \tilde{\lambda}_1 \bar{F}(w_0)}. \quad (\text{B.31})$$

For old employed workers, aging young workers generate an additional inflow. The resulting distribution is

$$G_O(w_0) = \frac{\hat{\phi}_1 F(w_0) + \hat{\phi}_2 G_Y(w_0) [1 - \delta - \lambda_1 \bar{F}(w_0)]}{\hat{\delta} + \hat{\lambda}_1 \bar{F}(w_0)}, \quad (\text{B.32})$$

where

$$\hat{\phi}_1 \equiv \frac{[(1 - \nu)u_O + \nu u_Y]\lambda_0}{1 - u_O}, \quad \hat{\phi}_2 \equiv \frac{\nu(1 - u_Y)}{1 - u_O}.$$

Firm size is the sum of age-specific employment densities per firm:

$$l(w_0) = l_Y(w_0) + l_O(w_0), \quad l_a(w_0) = \frac{(1 - u_a)M_a g_a(w_0)}{L f(w_0)}.$$

Since (B.31) has the standard BM form,

$$l_Y(w_0) = \frac{\tilde{A}_Y}{[\tilde{\delta} + \tilde{\lambda}_1 \bar{F}(w_0)]^2}, \quad \tilde{A}_Y \equiv \frac{(1 - u_Y)M_Y \tilde{\delta}(\tilde{\delta} + \tilde{\lambda}_1)}{L}. \quad (\text{B.33})$$

For old workers,

$$l_O(w_0) = \frac{(1 - u_O)M_O g_O(w_0)}{L f(w_0)}, \quad (\text{B.34})$$

where g_O is obtained by differentiating (B.32). In contrast to the young block, $l_O(w_0)$ does not reduce to a simple squared-reciprocal expression and is therefore evaluated numerically.

Using the equilibrium mapping $F(w_0(p)) = \Gamma(p)$, one can write

$$l(p) = l_Y(p) + l_O(p).$$

Setting $\eta = \nu = 0$ recovers the baseline model:

$$\tilde{\delta} \rightarrow \delta, \quad \tilde{\lambda}_1 \rightarrow \lambda_1, \quad l_O(\cdot) \rightarrow 0, \quad l(p) \rightarrow \frac{A}{[\delta + \lambda_1 \bar{\Gamma}(p)]^2}.$$

B.5.5 Wage Equation

The firm chooses w_0 to maximize

$$\pi(p) = \max_{w_0} (p - w_0) l(w_0).$$

Since per-worker profit $p - w_0$ is independent of worker age, the envelope condition is unchanged:

$$\pi'(p) = l(w_0(p)) = l(p). \quad (\text{B.35})$$

With boundary condition $\pi(\underline{p}) = 0$, integration gives

$$\pi(p) = \int_{\underline{p}}^p l(x) dx. \quad (\text{B.36})$$

Hence the wage schedule is

$$w_0(p) = p - \frac{\int_{\underline{p}}^p l(x) dx}{l(p)}, \quad w(p, h) = w_0(p) + h. \quad (\text{B.37})$$

Thus the wage equation has exactly the same structure as in the baseline model; the only difference is that firm size now equals the sum of a young component available in closed form and an old component computed numerically.

Finally, define the rent-normalized slope

$$\psi(p) \equiv \frac{w'_0(p)}{p - w_0(p)}.$$

Differentiating $\pi(p) = (p - w_0(p))l(p)$ and using (B.35) yields

$$\psi(p) = \frac{l'(p)}{l(p)}. \quad (\text{B.38})$$

When $\eta = \nu = 0$ and $l(p) = A/[\delta + \lambda_1 \bar{\Gamma}(p)]^2$, this collapses to the baseline expression

$$\psi(p) = \frac{2\lambda_1\gamma(p)}{\delta + \lambda_1\bar{\Gamma}(p)} = \frac{2k_1\gamma(p)}{1 + k_1\bar{\Gamma}(p)}, \quad k_1 \equiv \frac{\lambda_1}{\delta}.$$

B.6 Exit Rate

Lemma B.6 (Average exit rate among employed workers). *Let F denote the cross-firm CDF of posted wage components $w_0 \in [\underline{w}_0, \bar{w}_0]$, with density f , and let G denote the cross-worker CDF of posted wage components among employed workers, with density g . Suppose an employed worker at a firm posting w_0 exits at rate*

$$qr(w_0) = \delta + \lambda_1 \bar{F}(w_0), \quad \bar{F}(w_0) \equiv 1 - F(w_0). \quad (\text{B.39})$$

In the steady-state equilibrium, the average exit rate among employed workers is

$$\mathbb{E}[qr] = \int_{\underline{w}_0}^{\bar{w}_0} qr(w_0) dG(w_0) = \delta \frac{1 + k_1}{k_1} \ln(1 + k_1), \quad k_1 \equiv \frac{\lambda_1}{\delta}. \quad (\text{B.40})$$

In particular, $\mathbb{E}[qr]$ depends on the firm-type distribution only through the equilibrium mapping between F and G and is otherwise distribution-free.

Proof sketch. By (B.39),

$$\mathbb{E}[qr] = \delta + \lambda_1 \int_{\underline{w}_0}^{\bar{w}_0} \bar{F}(w_0) dG(w_0) = \delta + \lambda_1 \int_{\underline{w}_0}^{\bar{w}_0} \bar{F}(w_0) g(w_0) dw_0. \quad (\text{B.41})$$

F and G satisfy the standard mapping

$$G(w_0) = \frac{\delta F(w_0)}{\delta + \lambda_1 \bar{F}(w_0)} = \frac{F(w_0)}{1 + k_1 \bar{F}(w_0)}, \quad k_1 = \frac{\lambda_1}{\delta}. \quad (\text{B.42})$$

Differentiating (B.42) yields

$$g(w_0) = \frac{\delta(\delta + \lambda_1)}{(\delta + \lambda_1 \bar{F}(w_0))^2} f(w_0) = \frac{(1 + k_1)}{(1 + k_1 \bar{F}(w_0))^2} f(w_0). \quad (\text{B.43})$$

Substituting (B.43) into (B.41) and using $\lambda_1 = \delta k_1$,

$$\mathbb{E}[qr] = \delta + \lambda_1(1 + k_1) \int_{\underline{w}_0}^{\bar{w}_0} \frac{\bar{F}(w_0)}{(1 + k_1 \bar{F}(w_0))^2} f(w_0) dw_0. \quad (\text{B.44})$$

Let $s = \bar{F}(w_0)$. Then $ds = -f(w_0) dw_0$, and the bounds are $w_0 = \underline{w} \Rightarrow s = 1$ and $w_0 = \bar{w} \Rightarrow s = 0$. Hence

$$\mathbb{E}[qr] = \delta + \lambda_1(1 + k_1) \int_0^1 \frac{s}{(1 + k_1 s)^2} ds. \quad (\text{B.45})$$

Compute the integral by setting $u = 1 + k_1 s$, so that $du = k_1 ds$ and u runs from 1 to $1 + k_1$:

$$\int_0^1 \frac{s}{(1 + k_1 s)^2} ds = \frac{1}{k_1^2} \int_1^{1+k_1} \left(\frac{1}{u} - \frac{1}{u^2} \right) du = \frac{1}{k_1^2} \left(\ln(1 + k_1) - \frac{k_1}{1 + k_1} \right). \quad (\text{B.46})$$

Plugging (B.46) into (B.45) gives

$$\mathbb{E}[qr] = \delta + \lambda_1(1 + k_1) \frac{1}{k_1^2} \left(\ln(1 + k_1) - \frac{k_1}{1 + k_1} \right) = \delta \frac{1 + k_1}{k_1} \ln(1 + k_1),$$

where the final equality uses $\lambda_1 = \delta k_1$ and cancels the $-\delta$ term. This proves (B.40).

C Data Harmonization

C.1 Wage Harmonization

Our earnings analysis combines six data sources across three countries, summarized in Table C.1.

Appendix Table C.1: Data Sources

Country	Source	Type	Years	Role
United States	CPS	Household panel (rotating)	2003–2013	Main
United States	PSID	Long longitudinal panel	1975–2013	Validation
United States	NLSY97	Cohort panel (born 1980–84)	2000–2019	Validation
Brazil	PNADC	Household panel (5 quarters)	2012–2019	Main
Colombia	GEIH	Cross-sectional household survey	2009–2016	Main
Colombia	PILA	Administrative register (formal sector)	2009–2016	Main

Notes: Data sources used in the main empirical analysis (Main) and in validation exercises (Validation). The CPS, PSID, and NLSY97 are described in Section 3. Details on harmonization across sources are given in this appendix.

For the United States, our main source is the outgoing rotation group of the *Current Population Survey* (CPS), a household survey in which households are interviewed for four consecutive months, rotated out for eight, and surveyed again for four months one year later. We complement the CPS with two additional U.S. sources used only to validate the life-cycle wage profiles: the *Panel Study of Income Dynamics* (PSID), a long-running longitudinal household survey,⁴⁶ and the 1997 cohort of the *National Longitudinal Survey of Youth* (NLSY97), a panel of individuals born between 1980 and 1984.⁴⁷

For Brazil, we use the *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC), a nationally representative panel in which households are interviewed for five consecutive quarters.

⁴⁶The PSID was collected annually from 1968 to 1998 and biennially after 1999. We restrict attention to household heads and use the waves from 1975 to 2013, the same window used by Lagakos et al. (2018). The PSID does not contain information on job mobility, which is the main reason it is not used in the structural estimation.

⁴⁷We use the NLSY97 waves from 2000 to 2019, covering cohorts approximately aged 20 to 40 over this window. The NLSY97 is too narrow in age coverage to support life-cycle moments past age 40, which is why it complements rather than replaces the CPS.

For Colombia, we combine two complementary sources: the *Gran Encuesta Integrada de Hogares* (GEIH), a monthly cross-sectional household survey with detailed retrospective questions on employment histories, and the *Planilla Integrada de Liquidación de Aportes* (PILA), an administrative register covering the universe of workers contributing to the health and pension systems. The PILA is access-restricted: the data can only be processed on-site at the Banco de la República, and our analysis was carried out under such an arrangement.

None of these sources was designed for cross-country comparison, so harmonization is a sequence of deliberate choices. This appendix documents those choices, following the structure of Section 3 and organized around four groups of variables: wage levels, wage growth, job mobility, and firm size.

Throughout, we restrict each sample to workers between the ages of 20 and 60 and exclude individuals out of the labor force. We deflate all monetary variables to constant 2010 U.S. dollars using country-specific consumer price indices and the 2010 average exchange rate. The same age and labor force restrictions apply to all countries. Any further sample selection is dictated by the structure of the source data and is described below.

Wage levels. We focus on monthly earnings as our wage measure because it is the only concept that can be constructed comparably in all data sources. Hourly wages would be a natural alternative, and for the United States and Brazil we observe usual weekly hours and can compute them, but the household surveys in Colombia record hours unevenly and the PILA does not record hours at all.⁴⁸ To preserve a common definition across countries we therefore take monthly earnings as the baseline.⁴⁹

We winsorize wages at the first and 99th percentiles within each country–year cell

⁴⁸The PILA records contributed days within each month, which we use as a coarse full-time indicator but not as a continuous hours measure.

⁴⁹Appendix Figures A.2a and A.2b report life-cycle profiles using hourly wages for the data sources where they can be computed, and the resulting cross-country ordering is the same as in our baseline.

to limit the influence of outliers. The CPS sample runs from 2003 to 2013, the years that align with [Lagakos et al. \(2018\)](#) and that allow us to compute wage changes at a monthly frequency. The PNADC sample runs from 2012 to 2019, the period for which the survey has been operating in its current form and for which longitudinal identifiers are reliable. The GEIH and PILA samples cover 2009 to 2016, so that the household survey and the administrative register overlap in time and any composition differences cannot be attributed to different sample windows. The PSID and NLSY97 samples are used only to compute life-cycle wage profiles for the validation exercises in Appendix Figures [A.1a](#) and [A.1b](#), and we apply the same age, labor force, and winsorization restrictions to them.

Wage growth. Wage growth rates are constructed at the worker level, but the underlying frequency at which we can observe a given worker twice differs across data sources. In the CPS, the rotation structure lets us compute monthly wage changes within the rotation group and, by aggregation, quarterly and annual changes. In the PNADC, households are interviewed for five consecutive quarters, so wage growth is naturally measured at a quarterly frequency. The GEIH does not follow the same household over time and therefore cannot be used to compute individual wage changes. Therefore, we rely on the PILA, which links workers to their employers month by month and from which we compute annual wage growth.⁵⁰

This unequal frequency is unavoidable and we treat it transparently rather than smooth it away. Table [1](#) reports the raw moments at the frequency native to each data source (quarterly for the CPS and PNADC, annual for the PILA). These moments are not directly comparable across countries. For the structural estimation, we map all moments to a common quarterly frequency before targeting them. The converted moments are reported in Table [3](#).

⁵⁰The PSID and NLSY97 also support individual wage growth measures and are used in Appendix Figure [A.1a](#) to validate the U.S. profile. Their native frequencies are annual (PSID through 1998; NLSY97 through 2011) and biennial thereafter, which we adjust by halving the implied growth rates over biennial intervals.

The PILA covers only formal workers, defined as those contributing to the health and pension systems, which represents roughly 60 percent of Colombian employment. This restriction is a real limitation: because formal workers in Colombia are positively selected on schooling and earnings, our wage-growth moments for Colombia speak to the formal segment of the labor market rather than to the labor market as a whole. We discuss the implications for the structural estimates in Section 6 and return to selection into formality in the model-fit discussion. We adopt the restriction because it is the only way to obtain individual-level wage trajectories in Colombia, and applying the same age and labor-force restrictions to a comparable formal subsample preserves consistency with the wage-growth concept used in the CPS and the PNADC.

Job mobility. We classify employed workers as either job stayers or job movers and combine the resulting transition rates with movements into and out of unemployment to characterize labor market flows. The underlying data structures force three different operationalizations of the same concept. In the CPS, the dependent interviewing question on whether the respondent still works for the same employer reported in the previous month delivers a direct stayer indicator. We define a stayer as a worker who reports the same employer for four consecutive months and a mover as anyone who changes employer within that window. In the PNADC, the analogous indicator is built from employer tenure across consecutive interviews, and a stayer is a worker who has held the same job for at least one full quarter. In Colombia, the GEIH does not track workers but asks retrospective questions about previous employment and unemployment durations. Following [Lasso \(2013\)](#), we combine current tenure with reported time since last job to identify movers, stayers, and transitions in and out of unemployment.⁵¹

These three procedures are not identical, and we do not claim they are. They are unified by a common conceptual definition: a stayer is a worker observed in the same

⁵¹The PSID does not separately identify employer changes, so we follow [Lagakos et al. \(2018\)](#) in defining stayers as workers who do not change occupation or industry across consecutive observations. The NLSY97 contains employer identifiers and supports an exact stayer/mover definition.

employment relationship across two reference periods. However, they differ in the recall horizon underlying that definition: four consecutive months in the CPS, one quarter in the PNADC, and a year in the GEIH. As with wage growth, we report the raw transition rates at their native frequency in Table 1 and convert them to a common quarterly frequency for the structural estimation in Table 3. The conversion treats the underlying transition probabilities as Markov and aggregates or disaggregates them accordingly, and necessarily relies on a stationarity assumption within each native window, which is reasonable for the short horizons of the CPS and PNADC and somewhat more demanding for the GEIH.

Firm size. Firm size is the variable for which our data sources differ most. The PNADC asks employed workers about the size of their employer in every interview, so we observe firm size at the individual level over time. The GEIH asks the same question but is not longitudinal, which means we observe a worker’s firm size in the cross-section but cannot follow that worker across firm-size transitions. The CPS Outgoing Rotation Group does not include firm size, but cross-sectional supplements to the CPS do, and we use these supplements to construct state-level employment shares by firm size class for the United States.⁵²

This difference in the level of observation has direct consequences for what we can do empirically. For Brazil, where firm size is observed at the individual level in a panel, we estimate the within-worker wage–firm-size relationship using individual fixed-effects regressions, as reported in Section 4. For the United States and Colombia, the same exercise is not feasible. We instead compute employment shares by firm size class at the state–year level and exploit cross-state variation in firm size composition to relate the steepness of life-cycle wage profiles to the prevalence of large firms. This is the source of the cross-country and within-country evidence in Figures 2 and 3.

A direct consequence of operating at different levels of aggregation is that the firm-

⁵²The PILA contains firm identifiers but not firm size directly. In principle, one could construct firm size by collapsing the register at the employer level. We do not exploit this margin, since the household-side firm-size variable in the GEIH is sufficient for the cross-state moments used in the empirical analysis.

size moments used in the structural estimation cannot be the same individual-level objects across countries. We use country-level employment shares in firms above two size thresholds—10 and 50 employees—as our targets, since these are the moments that can be constructed consistently in all three countries. The thresholds are chosen to align with reporting conventions in EU-SILC, which we use to embed our three economies in a broader cross-country picture in Figure 2.

Building Country Moments. The harmonization choices above leave us with three samples that are comparable in concept but unequal in coverage. The CPS sample tracks employed workers at a monthly frequency, the PNADC sample tracks the full employed population at a quarterly frequency, and the Colombian sample combines the GEIH for cross-sectional moments and labor market transitions with the PILA for wage growth in the formal sector. We use the available data, and the alternative—restricting the analysis to a single common data source across countries—would either eliminate the United States and Colombia from the comparison or eliminate wage growth from Colombia. The harmonization we adopt instead is to make every concept as similar as possible across countries, to be explicit about where it is not, and to convert all targeted moments to a common frequency at the point where the model requires it. Appendix Figures A.1a and A.1b show that the resulting life-cycle profiles are robust to using alternative U.S. data sources (PSID and NLSY97), and Appendix Figure A.2b shows that they are robust to conditioning on full-time male workers and to residualizing on schooling and cohort.

C.2 Productivity Measures

The productivity evidence in Section 4 draws on a different set of sources than the wage evidence since cross-country comparisons require firm-level data, and these are only available in establishment surveys and administrative registers that have no household-side counterpart. The two sources we use are summarized in Table C.2. The Colombian source

is the *Encuesta Anual Manufacturera* (EAM), an annual establishment-level census of formal manufacturing firms with at least ten employees. For the United States we use the BLS–Census *Dispersion Statistics on Productivity* (BLS), which reports establishment-level dispersion of output per hour and total factor productivity at the four-digit NAICS by year level. The two sources are different by design. One delivers establishment-level microdata, the other delivers pre-computed dispersion statistics. However, both speak to the same underlying object, and combining them is the only way to discipline cross-country differences in firm productivity dispersion within the constraints of what each statistical system makes available.

Appendix Table C.2: Productivity Data Sources

Country	Source	Level of Observation	Years
Colombia	EAM	Establishment (manufacturing, 10+ employees)	1992–2016
United States	BLS	4-digit NAICS $\times$ year dispersion stats	1987–2021

Notes: Firm-level productivity sources. EAM: *Encuesta Anual Manufacturera*, the Colombian establishment-level manufacturing census. BLS: U.S. Bureau of Labor Statistics–Census *Dispersion Statistics on Productivity*, reporting establishment-level dispersion of output per hour and total factor productivity at the 4-digit NAICS by year level.

As with the wage harmonization, the goal is to construct measures of firm-level productivity that are as comparable as possible in both countries while being honest about the differences that remain. For this, we take the BLS data and try to replicate the same moments using EAM.

Sample and deflation. We restrict the EAM to manufacturing establishments with at least ten employees, which is the lower coverage bound of the survey and is also the natural cutoff for comparability across countries. All monetary variables are deflated to constant 2010 U.S. dollars using the Colombian CPI and the 2010 average exchange rate. Non-positive values for sales, value added, gross output, and the wage-bill components are set to missing rather than dropped, since establishments may legitimately report zero or missing values for individual revenue components in years of low activity. The BLS

reports dispersion statistics at the four-digit NAICS by year level.

Productivity measures. For Colombia, we construct four labor-productivity measures at the establishment level: value added per worker, sales per worker, gross output per worker, and gross output per implied hour. The first three are standard measures of revenue productivity at different points along the production process and at the worker level. The fourth introduces an hours adjustment that is necessary because the EAM does not record hours worked, and corresponds to the most comparable to the BLS data. For the United States, we take the dispersion statistics directly from the BLS at the comparable level of aggregation.

Recovering hours from the wage bill. The EAM does not record hours worked, which is a limitation that does not affect the BLS data. To make the Colombian establishment data comparable, we recover implied hours at each establishment in two steps, using sector-by-year hourly wages from the GEIH as a bridge. First, in the GEIH we compute hourly wages at the worker level as monthly earnings divided by usual monthly hours, restricting the sample to employed workers with positive labor earnings, positive usual weekly hours, and employed at firms with more than ten employees, so that the GEIH sample matches the EAM coverage threshold. We then collapse to the sector-by-year level, averaging across workers. Second, we merge these sector-year hourly wages into the EAM and recover implied hours at each establishment by dividing the establishment-level wage bill by the corresponding sector-year hourly wage. This procedure restricts the EAM data to 2008 to 2016 in order match the GEIH.⁵³ We then use these implied hours as the denominator to compute a measure of gross output per hour, which is comparable to the BLS. This procedure assumes that the average hourly wage in the household survey is a reasonable approximation to the average hourly wage actually paid by establishments in

⁵³The GEIH was launched in 2007 and provides reliable sector-by-year hourly wage estimates from 2008 onward, which is the binding constraint on the start year for our hours-based productivity measure. Earlier EAM years are still used for productivity measures based on workers (rather than hours).

the same sector and year. The assumption is imperfect—the GEIH wage concept reflects worker-side take-home earnings while the EAM wage bill includes employer-side non-wage labor costs—but it is the only way to construct an hours-based productivity measure for Colombia.⁵⁴ The resulting per-hour and per-worker productivity rankings across firm size classes deliver the same qualitative ordering, which we view as evidence that the procedure is not driving the cross-country comparisons.

Harmonization. We then combine the Colombian productivity measure with the output-per-hour measure from the BLS, restricting both countries to the 2008–2016 window over which the two data sets overlap. Appendix Table C.3 reports the resulting distribution of firm productivity. Column (1) reports the standard deviation, column (2) the difference between the 90th and 10th percentiles, and column (3) the interquartile range. All three statistics are computed on the within-industry distribution of log productivity. Because the percentile gaps are differences in log productivity quantiles, they are invariant to multiplicative changes in productivity units across sources; they therefore put the Colombian EAM and U.S. BLS measures on a common unit-free scale.

Appendix Table C.3 above reports unweighted descriptive productivity gaps. The external-discipline exercise in Section 6.3 instead uses gaps in which establishment observations are weighted by hours when computing the within-cell 90th and 10th percentiles, before averaging across four-digit industry-year cells and the common years 2008–2016. We present such results in Appendix Table H.6 and report both sets of gaps side by side, together with the implied Pareto shape parameters.

⁵⁴Implied hours per worker have a median of approximately 3,200 in our sample, around 40 percent above external benchmarks for Colombian annual hours per worker. The likely source is the wedge between the EAM wage-bill concept (which includes social security contributions, payroll taxes, severance accruals, and mandated bonuses) and the GEIH earnings concept. The wedge is multiplicative and roughly uniform within sector-year cells, leaving within-cell dispersion statistics invariant to it.

Appendix Table C.3: Firm Productivity Distributions in Colombia and the United States

		Standard Dev. (1)	$\ln p_{90} - \ln p_{10}$ (2)	$\ln p_{75} - \ln p_{25}$ (3)
A. Overall				
Colombia		0.742	1.763	0.865
United States		0.708	1.837	0.932
B. Yearly				
Colombia	2008	0.729	1.752	0.829
	2009	0.736	1.744	0.873
	2010	0.718	1.705	0.842
	2011	0.747	1.795	0.860
	2012	0.746	1.802	0.835
	2013	0.737	1.740	0.908
	2014	0.751	1.770	0.879
	2015	0.756	1.783	0.872
	2016	0.757	1.775	0.891
United States	2008	0.696	1.798	0.892
	2009	0.690	1.783	0.877
	2010	0.693	1.791	0.906
	2011	0.711	1.858	0.923
	2012	0.723	1.871	0.972
	2013	0.730	1.881	0.972
	2014	0.709	1.841	0.957
	2015	0.714	1.867	0.943
	2016	0.705	1.846	0.951

Notes: This table presents the productivity distributions for Colombia and the United States using the EAM and the BLS, respectively. Productivity is computed as output per hour at the establishment level. Dispersion statistics are computed within four-digit industry by year cells and reported as unweighted averages across cells. For Colombia, output per hour is constructed using the implied-hours procedure described in Section C.2. For the United States, the statistics are taken directly from the BLS, which computes within-industry-year dispersion of establishment-level output per hour at the four-digit NAICS level following the same definitions.

C.3 World Bank Enterprise Survey

As a complementary cross-country measure of firm productivity, we use the *World Bank Enterprise Survey* (WBES), a stratified establishment-level survey administered using a common questionnaire across countries. Because the WBES asks the same questions of formal firms in each country, it is designed for cross-country comparison from the start. Its main limitation is coverage. The survey samples on the order of a thousand establishments

per country per wave, so we use it as a robustness check on our main productivity evidence rather than as a replacement for it.

We pool the available WBES waves for the three countries in our analysis. The Colombian sample uses the 2006, 2010, 2017, and 2023 waves. The Brazilian sample uses the 2009 wave, and the U.S. sample uses the 2024 wave, the first WBES round in which the country was included. The sample is restricted to firms reporting positive employment.⁵⁵

The WBES does not record hours worked, so we cannot construct an output-per-hour measure comparable to the EAM and BLS comparison. We instead use log sales per worker, the only firm-productivity measure available consistently across waves and countries. Sales are deflated to constant 2010 U.S. dollars using country-specific CPI factors and the 2010 average exchange rate, and winsorized at the 99th percentile within each country-year cell to limit the influence of extreme outliers. Total employment is the sum of permanent full-time and temporary workers reported by the establishment. Firm size classes are defined as one to nineteen, twenty to ninety-nine, and one hundred or more workers, which match the size bins reported consistently across waves and countries.

⁵⁵We use the WBES median sampling weights so that descriptive statistics are representative of the underlying business population in each country.

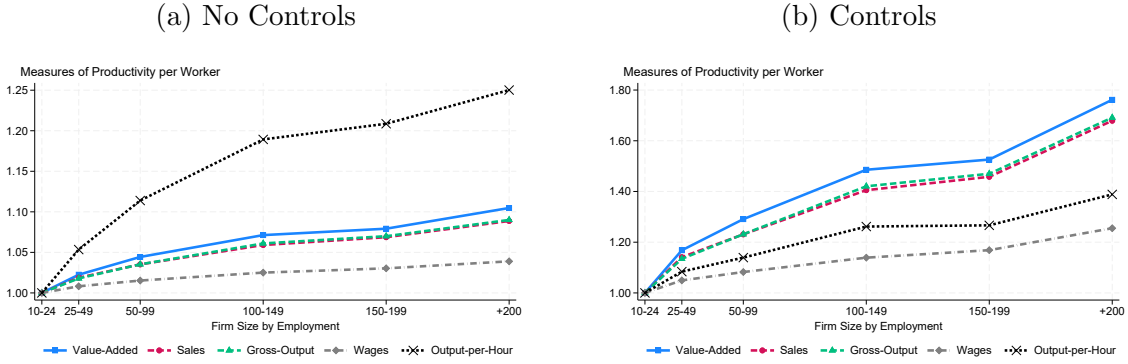
D Firm Productivity and Size

The empirical analysis in Section 4 uses firm size as a proxy for firm productivity, since firm size is the only variable that can be measured consistently across the household and labor force surveys we use in all three countries. This appendix shows that the proxy is justified: across the firm-level data sources where productivity is directly observable, larger firms are systematically more productive than smaller ones, both within Colombia and across the three countries of interest. Construction of the productivity measures, deflation, and the implied-hours procedure for the EAM are described in Appendix C.2.

Within-Colombian evidence. We begin with Colombia, where the EAM provides the richest firm-level data—a full census of formal manufacturing establishments with ten or more employees, observed annually from 1992 to 2016, with multiple revenue and cost variables that can be turned into alternative productivity measures. Figure D.1 plots five labor-productivity measures by firm size class, expressed relative to the smallest size class. Panel (a) reports unconditional means; Panel (b) reports values residualized against industry-by-year-by-department cells, so that the gradient reflects within-cell comparisons.

Three observations follow from Figure D.1. First, all five productivity measures increase monotonically with firm size in both panels, supporting the use of firm size as a proxy for productivity. Second, the gradient is substantially steeper once we condition on industry, year, and department. The unconditional gradient in Panel (a) is attenuated by sectoral composition—industries dominated by small firms differ in average productivity from industries dominated by large firms, and the two effects partially offset. Comparing establishments *within* the same industry-year-department cell, the productivity gradient is steeply increasing in firm size across all four revenue-based measures, with the slope being most pronounced for value added and gross output per worker. Third, the wage-bill gradient is the flattest of the five measures, consistent with our reduced-form evidence in Section 4.2: the wage premium associated with firm size is real but smaller than the

Appendix Figure D.1: Productivity and Firm Size in Colombia



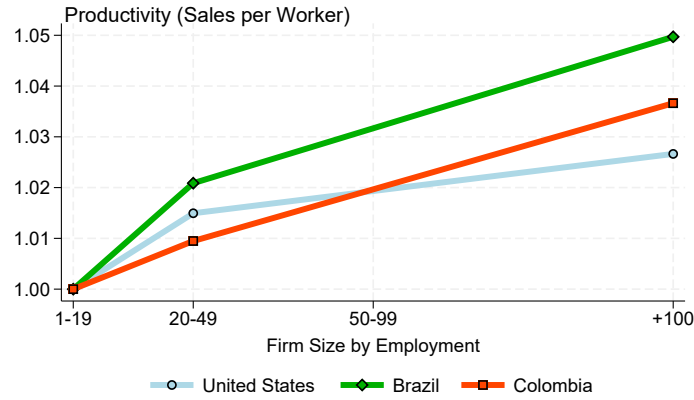
Notes: These figures plot the relationship between five different measures of productivity and firm size using the Colombian manufacturing census of firms (EAM). Value-added, sales, gross-output, and wages are computed as per-worker measures. Output per hour is computed using hours-worked measures from the GEIH, as explained in Appendix C.2, which replicates the BLS measures for the United States. The sample includes manufacturing firms with more than ten employees from 2008 to 2016. Each measure is expressed relative to its value in the smallest size class. Panel (a) reports unconditional averages within each firm size class; panel (b) reports averages after residualizing against industry-by-year-by-department cells.

underlying productivity premium, leaving room for the job-ladder and on-the-job-learning mechanisms emphasized in the model. The same pattern is documented in the broader Latin American firm-level literature (Busso et al., 2013; Alviarez et al., 2025).

Cross-country evidence. We next ask whether the firm-size/productivity relationship holds across countries using the World Bank Enterprise Survey (WBES), which administers a common questionnaire to formal establishments in each country and is therefore designed for cross-country comparison. The cost of comparability is coverage: the WBES samples roughly one thousand firms per country per wave, and only records sales per worker as a productivity measure. We use it as a complementary cross-country check rather than as a replacement for the EAM.

Figure D.2 shows that the firm-size/productivity gradient is positive in all three countries. The slopes are flatter than in the EAM panel-(b) figure for Colombia because the WBES does not condition on industry and the sample is small; what matters for our purposes is that the *ordering* of firms by size in each country lines up with their

Appendix Figure D.2: Firm Productivity and Size Across Countries



Notes: This figure plots sales per worker by firm size class for the United States, Brazil, and Colombia using the World Bank Enterprise Survey. Sales per worker is expressed relative to its value in the smallest size class within each country. Sample restrictions and the construction of size classes are described in Appendix C.2.

productivity ordering. The broader cross-country evidence on this regularity, including the 1900–2020 panel evidence in [Ma et al. \(2026\)](#), is consistent with this pattern.

Taken together, the within-country evidence from the EAM and the cross-country evidence from the WBES support the use of firm size as a tractable proxy for firm productivity when worker-side data preclude a direct productivity measurement. The relationship is monotonic, robust to within-cell residualization, and present in all three countries we study.

E Top-Share Identity and Its Comparative Statics

Define the tightness-type ratio

$$k_1 \equiv \frac{\lambda_1}{\delta}, \quad z \equiv \bar{\Gamma}(p) = 1 - \Gamma(p), \quad z_H \equiv \bar{\Gamma}(p_H) \in (0, 1).$$

Then

$$l(p) = A[\delta + \lambda_1 \bar{\Gamma}(p)]^{-2} = \frac{A}{\delta^2} [1 + k_1 z]^{-2}. \quad (\text{E.1})$$

Lemma E.1 (Unconditional firm size is a rank object). *Because $p \sim \Gamma$ across firms, the variable $z = \bar{\Gamma}(p)$ is uniformly distributed on $[0, 1]$. Hence, by (E.1), the unconditional cross-firm size distribution depends on Γ only through rank and therefore does not identify its shape.*

Proof sketch. By (E.1), firm size depends on productivity p only through the tail rank

$$z = \bar{\Gamma}(p).$$

Across firms, productivity is distributed according to Γ , so z is uniformly distributed on $[0, 1]$. It follows that the unconditional distribution of firm size is generated by the mapping from rank z into size, rather than by the particular shape of Γ . In that sense, the unconditional cross-firm size distribution is a rank object and does not identify the shape of Γ .

Lemma E.2 (Closed form for $S(p_H)$). *Let $l(p) = \frac{A}{\delta^2} [1 + k_1 \bar{\Gamma}(p)]^{-2}$ and define*

$$S(p_H) \equiv \frac{\int_{p_H}^{\infty} l(p) d\Gamma(p)}{\int_p^{\infty} l(p) d\Gamma(p)}.$$

Then

$$S(p_H) = \frac{(1 + k_1) z_H}{1 + k_1 z_H}. \quad (\text{E.2})$$

Proof sketch. Using $z = \bar{\Gamma}(p)$ implies $d\Gamma(p) = -dz$ and the bounds are $p = p_H \Rightarrow z = z_H$, $p \rightarrow \infty \Rightarrow z \rightarrow 0$, and $p = \underline{p} \Rightarrow z = 1$. Hence

$$\int_{p_H}^{\infty} l(p) d\Gamma(p) = \int_0^{z_H} \frac{A/\delta^2}{(1+k_1z)^2} dz, \quad \int_{\underline{p}}^{\infty} l(p) d\Gamma(p) = \int_0^1 \frac{A/\delta^2}{(1+k_1z)^2} dz.$$

Since $\int (1+k_1z)^{-2} dz = -\frac{1}{k_1(1+k_1z)}$, we obtain

$$\int_0^{z_H} \frac{A/\delta^2}{(1+k_1z)^2} dz = \frac{A}{\delta^2 k_1} \left(1 - \frac{1}{1+k_1 z_H}\right), \quad \int_0^1 \frac{A/\delta^2}{(1+k_1z)^2} dz = \frac{A}{\delta^2 k_1} \left(1 - \frac{1}{1+k_1}\right).$$

Taking the ratio yields (E.2).

Lemma E.3 (Comparative statics of $S(p_H)$). *Holding z_H fixed,*

$$\frac{\partial S}{\partial k_1} = \frac{z_H(1-z_H)}{(1+k_1 z_H)^2} > 0, \quad \frac{\partial S}{\partial z_H} = \frac{1+k_1}{(1+k_1 z_H)^2} > 0. \quad (\text{E.3})$$

Let $\Gamma(p) = 1 - (\underline{p}/p)^\alpha$ for $p \geq \underline{p}$, $\alpha > 0$. Then

$$z_H = \bar{\Gamma}(p_H) = \left(\frac{\underline{p}}{p_H}\right)^\alpha, \quad \frac{\partial z_H}{\partial \alpha} = z_H \ln\left(\frac{\underline{p}}{p_H}\right) < 0 \quad \text{for } p_H > \underline{p},$$

and therefore

$$\frac{\partial S}{\partial \alpha} = \frac{1+k_1}{(1+k_1 z_H)^2} z_H \ln\left(\frac{\underline{p}}{p_H}\right) < 0. \quad (\text{E.4})$$

Lemma E.4 (Right-tail mass amplifies direct OJS gain holding w_0 fixed). *Fix p and w_0 .*

Let Γ and $\tilde{\Gamma}$ be two productivity distributions such that $\tilde{\Gamma}$ has (weakly) more right-tail mass above p in the sense that $\tilde{\Gamma}(x) \geq \bar{\Gamma}(x)$ for all $x \geq p$. Equivalently, $\tilde{\Gamma}$ first-order stochastically dominates Γ on $[p, \infty)$. With w_0 being absolutely continuous and weakly increasing

$$\int_p^\infty (w_0(x) - w_0(p)) d\tilde{\Gamma}(x) \geq \int_p^\infty (w_0(x) - w_0(p)) d\Gamma(x). \quad (\text{E.5})$$

Hence, holding (w_0, λ_1) fixed, more right-tail mass increases the expected OJS gain.

Proof sketch. Using integration by parts,

$$\int_p^\infty (w_0(x) - w_0(p)) d\Gamma(x) = \int_p^\infty w'_0(x) \bar{\Gamma}(x) dx,$$

where the boundary term vanishes since $\bar{\Gamma}(x) \rightarrow 0$ as $x \rightarrow \infty$. Because $w'_0(x) \geq 0$ and $\widetilde{\bar{\Gamma}}(x) \geq \bar{\Gamma}(x)$ for all $x \geq p$, we obtain

$$\int_p^\infty w'_0(x) \widetilde{\bar{\Gamma}}(x) dx \geq \int_p^\infty w'_0(x) \bar{\Gamma}(x) dx,$$

which is (E.5).

F Details of the Reduced Form Results

This appendix presents the regression analogues of the residualized panels in Figures 2 and 3. Across all specifications, the coefficient on firm size remains positive and significant, supporting our interpretation that the cross-country and cross-regional firm-size/wage-growth gradients are not driven by institutional or compositional differences alone.

F.1 Cross-Country Evidence

We estimate

$$R_c = \gamma L_c + \beta_1 \text{CB}_c + \beta_2 \log \left(\frac{w^{\min}}{\bar{w}} \right)_c + \varepsilon_c, \quad (\text{F.1})$$

where R_c denotes the ratio of mean wages of workers aged 40–44 to those aged 20–24 in country c , L_c is the share of country employment in firms with more than 10 (or 50) employees, CB_c is the share of employees covered by collective bargaining agreements, and $\log(w^{\min}/\bar{w})_c$ is the log of the country’s minimum wage relative to its average wage.⁵⁶ Table F.1 reports estimates of equation (F.1), sequentially adding the institutional controls.

F.2 Cross-State Evidence

We estimate

$$R_{sc} = \alpha_c + \gamma L_{sc} + \beta_1 \text{Formality}_{sc} + \beta_2 \text{Skilled}_{sc} + \beta_3 \log w_{sc}^{\min} + \varepsilon_{sc}, \quad (\text{F.2})$$

where R_{sc} denotes the ratio of mean wages of workers aged 40–44 to those aged 20–24 in state s of country c , L_{sc} is the share of state employment in firms with more than

⁵⁶Collective bargaining coverage and minimum-wage data come from the OECD/AIAS ICTWSS database. We complement the ICTWSS minimum-wage series with Eurostat data for Serbia, which is not covered in ICTWSS. For Austria, Denmark, Finland, Iceland, Italy, Sweden, Switzerland, and Cyprus, no statutory minimum wage exists during our period of analysis; in these countries, sectoral collective bargaining sets the binding wage floor, and we set the minimum wage to zero. The institutional dimension is then absorbed by the collective bargaining coverage variable, which is high in this group (above 80% in seven of the eight countries).

Appendix Table F.1: Cross-Country Wage Growth and Firm Size

	(1)	(2)	(3)
A. Firms with 10 or More Employees			
Emp. share in firms with 10 or more employees	48.650 (57.067)	36.065 (48.081)	23.853 (43.702)
Minimum relative to average wage		−0.775** (0.302)	−0.527 (0.381)
Collective bargaining coverage (%)			0.356* (0.197)
Observations	31	31	31
B. Firms with 50 or More Employees			
Emp. share in firms with 50 or more employees	115.957** (46.584)	107.826** (44.964)	93.122** (42.085)
Minimum relative to average wage		−0.750** (0.293)	−0.556 (0.371)
Collective bargaining coverage (%)			0.280 (0.187)
Observations	31	31	31

Notes: This table reports estimates from regressing the ratio of monthly wages of workers aged 40–44 to those aged 20–24 on the country-level share of employment in firms with more than 10 employees (Panel A) or more than 50 employees (Panel B). The unit of observation is the country. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

10 (or 50) employees, Formality_{sc} is the share of formally employed workers, Skilled_{sc} is the share of workers with more than a high-school degree, and $\log w_{sc}^{\min}$ is the log of the binding state-level minimum wage.⁵⁷ The country fixed effect α_c absorbs all between-country variation. The coefficient of interest is γ , which captures the within-country, cross-state association between firm-size composition and life-cycle wage growth, conditional on differences in formality, education, and minimum wages. Standard errors are heteroskedasticity-consistent.

Table F.2 reports estimates of equation (F.2) sequentially adding controls. Column (1) reports the unconditional cross-state correlation without country fixed effects.

⁵⁷Minimum-wage data for the United States are from Vaghul and Zipperer (2022) and defined as the log of the maximum of the state and federal minimum, following Manning (2021). In Brazil and Colombia, the minimum wage is set at the national level with limited regional deviation; the coefficient on the minimum wage is therefore identified almost entirely from across-state variation within the United States.

Appendix Table F.2: Cross-State Wage Growth and Firm Size

	(1)	(2)	(3)	(4)	(5)
A. Firms with 10 or More Employees					
Emp. share in firms with 10+ employees	125.725*** (8.452)	74.581** (35.271)	124.218** (48.910)	130.356*** (48.657)	98.185** (43.879)
Formality rate			-53.307 (32.412)	-57.540* (32.337)	-76.677** (30.564)
Log(state minimum wage)				92.028*** (22.228)	55.971** (26.062)
Emp. share with more than high school					137.771*** (28.536)
B. Firms with 50 or More Employees					
Emp. share in firms with 50+ employees	118.904*** (8.296)	93.221** (44.190)	118.747** (51.768)	128.097** (51.468)	90.910* (46.366)
Formality rate			-26.557 (25.985)	-31.195 (25.983)	-53.770** (23.965)
Log(state minimum wage)				93.669*** (23.280)	57.246** (27.055)
Emp. share with more than high school					136.777*** (28.230)
Observations	102	102	102	102	102
Country FE		Yes	Yes	Yes	Yes

Notes: This table reports estimates from regressing the ratio of monthly wages of workers aged 40–44 to those aged 20–24 on the state-level share of employment in firms with more than 10 employees (Panel A) or more than 50 employees (Panel B). The unit of observation is the country–state. Specifications from column (2) onward include country fixed effects. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

F.3 Individual-Level Evidence

We estimate individual-level Mincer regressions on worker-level panel data. The PNADC is the only worker-level panel in our sample with both wage and firm-size information. The CPS reports firm size only in its cross-sectional sample, and the GEIH is not longitudinal, hindering us to include individual fixed effects. Therefore, we restrict this exercise to Brazil. Formally, we estimate

$$\ln w_{it} = \sum_{k \in \mathcal{K}} \beta_k \mathbf{1}(FS_{it} = k) + \gamma_1 a_{it} + \gamma_2 a_{it}^2 + \alpha_i + \nu_s + \tau_t + \varepsilon_{it}, \quad (\text{F.3})$$

where $\ln w_{it}$ denotes log wages for individual i in year t , a_{it} is age, and α_i , ν_s , τ_t are individual, state, and year fixed effects, respectively. The indicators $\mathbf{1}(FS_{it} = k)$ equal one if individual i is employed in a firm of size category $k \in \mathcal{K} \equiv \{[6, 10], [11, 50], > 50\}$ at period t , with firms of $[1, 5]$ employees as the omitted reference. Standard errors are clustered at the state level. Table F.3 reports the estimates: the wage–firm-size gradient is positive and monotonic, attenuates with controls, and is steeper for movers than for stayers, consistent with the job-ladder mechanism.

Appendix Table F.3: Individual Wages and Firm Size

	Overall			Stayers	Movers
	(1)	(2)	(3)	(4)	(5)
6–10 Emp.	0.245*** (0.043)	0.227*** (0.030)	0.085*** (0.007)	0.048*** (0.004)	0.171*** (0.019)
11–50 Emp.	0.326*** (0.051)	0.278*** (0.035)	0.126*** (0.011)	0.074*** (0.006)	0.226*** (0.029)
50 or More	0.422*** (0.049)	0.323*** (0.032)	0.147*** (0.012)	0.084*** (0.006)	0.271*** (0.028)
Observations	4,571,895	4,571,895	4,019,253	2,286,272	81,835
State FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Controls		Yes	Yes	Yes	Yes
Individual FE			Yes	Yes	Yes

Notes: This table reports estimates of equation (9) using log wages as the dependent variable, on the longitudinal module of the Brazilian PNADC. The omitted firm-size category is firms with 1–5 employees. “Controls” include age, age-squared, a gender indicator, and years of education; columns (3)–(5) include individual fixed effects, which absorb the gender indicator and years of education, while still controlling for age and age-squared. Column (4) restricts to job stayers; column (5) to job movers. Estimates are weighted by survey weights. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

G Details about Model's Estimation

G.1 Calibration of Age-Specific Learning Rates

Young workers learn at rate μ_Y and old workers at rate μ_O , with $\mu_Y \geq \mu_O$. Since aging is exogenous and labor-market hazards are common across age groups, conditional on being a stayer the probability of being young is the same as in the underlying cohort. If all workers start young and age with probability η each period, then the probability of still being young at the learning step in quarter t is

$$\omega_t = (1 - \eta)^{t+1}.$$

Hence, the model-implied expected wage growth of a stayer in quarter t is exactly

$$\mathbb{E}[\Delta w_t \mid \text{stayer}] = \omega_t \mu_Y + (1 - \omega_t) \mu_O.$$

Let age bin a contain quarters $t = t_0(a), \dots, t_1(a) - 1$. Then the model-implied average stayer growth in that bin is

$$E_{\text{stay},a} = \Omega_a \mu_Y + (1 - \Omega_a) \mu_O,$$

where

$$\Omega_a \equiv \frac{1}{t_1(a) - t_0(a)} \sum_{t=t_0(a)}^{t_1(a)-1} (1 - \eta)^{t+1}.$$

Using the geometric-sum formula,

$$\Omega_a = \frac{(1 - \eta)^{t_0(a)+1} [1 - (1 - \eta)^{t_1(a)-t_0(a)}]}{(t_1(a) - t_0(a)) \eta}.$$

Given η , we estimate (μ_Y, μ_O) by matching the observed profile of age-specific stayer wage growth to this expression, imposing $\mu_Y \geq \mu_O \geq 0$.

G.2 Details on Standard Error Computation

Standard errors are computed by a nonparametric bootstrap of the empirical moments used in the estimation. In each bootstrap replication, we resample individuals with replacement within country, recompute the wage-growth, stayer-growth, unemployment, and transition-rate moments, and then re-estimate the model using the same SMM criterion as for the corresponding full-sample specification. Each replication is initialized at the corresponding full-sample parameter estimates, independently of previous bootstrap replications. The reported standard errors are the standard deviations of the bootstrap distribution of the estimated parameters. This procedure accounts for sampling uncertainty in both the targeted wage-profile moments and the auxiliary moments used to discipline learning and labor-market frictions.

For Colombia, the stayer wage-growth moments are obtained from a separate administrative source for which we do not observe the underlying individual-level records. We therefore hold these moments fixed across bootstrap replications and bootstrap the remaining moments from the available household-survey micro data.

G.3 Colombian Objective Profile over the Pareto Shape

Because Colombia's life-cycle estimate $\alpha = 10$ lies at the upper bound of the admissible range $[0.01, 10]$ (see the main text), we present in Appendix Table [G.1](#) the life-cycle SMM objective over α , re-optimizing $\underline{p}$ at each value, to characterize identification near and beyond that bound. This traces a high- α , high- $\underline{p}$ ridge rather than a well-identified interior optimum, showing that the $\alpha = 10$ estimate is bound-dependent rather than a sharply pinned-down interior value.

Appendix Table G.1: Colombia: Life-Cycle SMM Objective Profile over the Pareto Shape

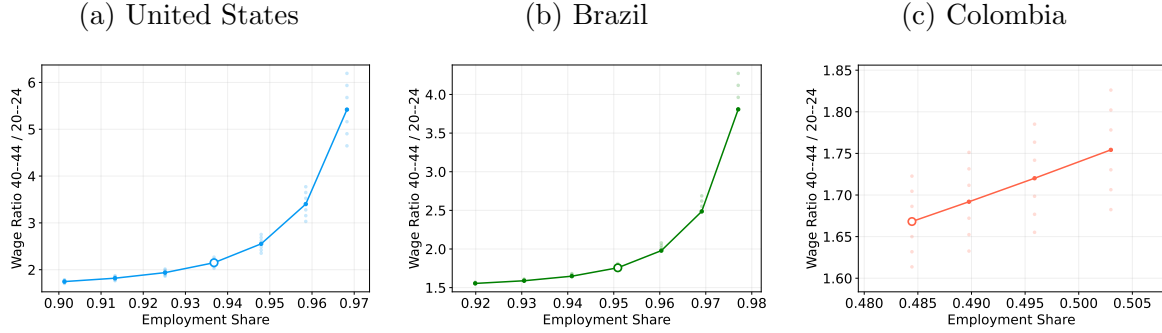
α	Profiled $\underline{p}$	Two-target objective
0.5	0.010	25,640.285
1.0	0.040	0.003905
1.5	0.168	0.001756
2.0	0.343	0.001017
3.0	0.758	0.000550
5.0	1.679	0.000299
7.5	2.883	0.000204
9.0	3.615	0.000177
10.0	4.105	0.000165
12.5	5.333	0.000143
15.0	6.566	0.000130
20.0	9.036	0.000114
25.0	11.511	0.000106
30.0	13.987	0.000100

Notes: At each displayed value of α , the lower-support parameter $\underline{p}$ is re-optimized over $[0.01, 20]$. In the baseline, the upper bound is 10 for both α and $\underline{p}$; larger values are joint relaxed-bound diagnostics. The objective uses the two life-cycle wage-growth targets, ages 25–29 and 30–34 relative to ages 20–24, for Colombia’s life-cycle specification. The profile uses $N = 10,000$ simulation draws and common random numbers.

H Results Appendix

H.1 Identification Check: Firm-Size Locus

Appendix Figure H.3: Model-Implied Locus: Firm Size and Life-Cycle Wage Growth



Notes: Each panel fixes the country's life-cycle-extension estimates and varies (α, p) . The horizontal axis reports the model-implied employment share at firms in the top 0.1% of the productivity distribution; the vertical axis, the model-implied wage ratio at ages 40–44 relative to 20–24. The open circle marks each country's benchmark estimate. The solid line varies α at fixed p ; dots represent additional draws. The locus is not targeted in estimation; its upward slope is a model-implied property of the job-ladder mechanism.

H.2 Detailed Wage Decomposition

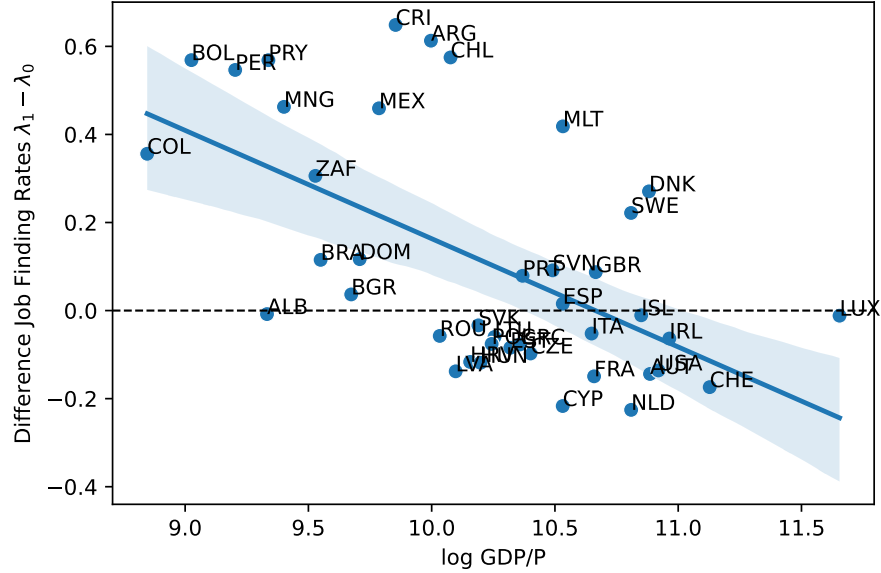
Appendix Table H.2: Detailed Wage Decomposition Over the Life Cycle

	Bin	Wage	Worker	Firm		
		path	prod.	Comp.	Prod.	Rents
U.S.	20–24	0.00	0.00	0.00	0.00	0.00
	25–29	57.42	21.76	35.66	133.52	−97.87
	30–34	84.02	36.61	47.41	185.30	−137.89
	35–39	99.60	47.22	52.37	203.95	−151.57
	40–44	109.70	55.48	54.22	216.25	−162.02
Brazil	20–24	0.00	0.00	0.00	0.00	0.00
	25–29	38.72	20.83	17.89	67.84	−49.95
	30–34	56.75	33.83	22.92	87.40	−64.48
	35–39	66.92	42.32	24.60	99.07	−74.47
	40–44	72.97	48.20	24.77	99.70	−74.93
Colombia	20–24	0.00	0.00	0.00	0.00	0.00
	25–29	33.22	10.03	23.19	36.19	−12.99
	30–34	45.84	19.91	25.93	41.66	−15.73
	35–39	56.71	29.84	26.87	43.73	−16.86
	40–44	66.55	39.77	26.78	43.88	−17.10

Notes: This table decomposes model-implied wage growth relative to ages 20–24 into worker and firm contributions under the life-cycle extension. Wage growth is measured as the change in mean log wages relative to ages 20–24, multiplied by 100; the worker and firm contributions are expressed in the same units. “Bin” is the 5-year age bin. “Wage path” equals the sum of “Worker prod.” (h) and the firm component “Comp.” ($w_0(p)$). The firm component is further decomposed into “Prod.” (p) and “Rents” ($w_0(p) - p$), which are negative because firms capture part of the match surplus through piece-rate wage posting. U.S. denotes the United States.

H.3 Cross-Country Validation of Job-Finding Rates

Appendix Figure H.4: Difference in Job Finding Rates and Economic Development



Notes: The figure shows the relationship between the job-finding rate for the employed (λ_1) and the job-finding rate for the unemployed (λ_0) across countries. Data are derived from [Donovan et al. \(2023\)](#) on UE and EE rates and GDP per capita, supplemented with unemployment rates from the International Labor Organization. We use our identification procedure to back out λ_1 and λ_0 .

H.4 Counterfactual and Robustness

Appendix Table H.3: Counterfactual Simulation – Baseline Estimates

	Parameters changed to U.S. Level (1)	Relative Wage Growth	
		Brazil (2)	Colombia (3)
Share with Respect to U.S.		0.73	0.52
Counterfactual:			
Firm-Type Distribution	$\alpha, \underline{p}$	1.00	1.07
Learning on the Job	μ	0.72	0.67
Labor Market Frictions	$\lambda_1, \lambda_0, \delta$	0.65	0.47

Notes: Baseline-specification counterpart to Table 7. The first row presents the empirical value; subsequent rows present three counterfactual estimation results, expressed as the share of wage growth at ages 30–34 relative to the value for the U.S. economy. Each counterfactual changes the parameters in column (1) to match the U.S. level. Columns (2) and (3) report the relative wage growth of counterfactual Brazil and counterfactual Colombia, respectively, with respect to the U.S. benchmark.

Appendix Table H.4: Robustness: Counterfactual Simulations

Specification	Country	Data	Firms	Learning	Frictions
A. Shifted Weibull firm-productivity distribution					
Baseline wage moments	Brazil	0.73	1.07	0.67	0.62
Baseline wage moments	Colombia	0.52	1.30	0.70	0.53
B. Education-residualized wage profiles					
Cohort effects	Brazil	0.43	0.85	0.68	0.59
Cohort and time effects	Brazil	0.44	0.86	0.63	0.55
Time effects	Brazil	0.48	0.87	0.58	0.50
No late-life-cycle growth	Brazil	0.39	0.85	0.67	0.58
C. External productivity dispersion					
External Pareto shape	Colombia	0.52	1.14	0.74	0.47

Notes: Entries report wage growth at ages 30–34 relative to the corresponding U.S. value. “Data” reports the empirical relative wage-growth ratio. “Firms”, “Learning”, and “Frictions” report counterfactual ratios obtained by setting the corresponding parameter block to the U.S. level, holding all other country-specific parameters fixed. In panel A, the firm-productivity distribution is shifted Weibull rather than Pareto. In panel B, wage profiles are residualized following [Lagakos et al. \(2018\)](#) under the four identifying assumptions illustrated in Figure [A.3](#). In panel C, the Pareto shape parameter is disciplined using the weighted 90–10 log productivity gap, and the scale parameter is re-estimated using wage growth at ages 25–29 relative to ages 20–24.

Appendix Table H.5: Weibull Robustness: Estimation Results

	Firm Prod. Distr.		Frictions			Learning	
	Shape (κ)	Scale ($\underline{p}$)	δ	λ_1	λ_0	μ_Y	μ_O
U.S.	0.239	0.025	0.035	0.34	0.47	0.018	0.003
Brazil	0.240	0.010	0.047	0.55	0.44	0.018	0.001
Colombia	1.506	0.815	0.043	0.73	0.37	0.006	0.006

Notes: This table reports estimates from the life-cycle specification using a shifted Weibull firm-productivity distribution. The survival function is $\bar{\Gamma}(p) = \exp[-((p - \underline{p})/\underline{p})^\kappa]$ for $p \geq \underline{p}$. The parameter κ governs the shape and $\underline{p}$ denotes the lower support. Labor-market frictions ($\delta, \lambda_1, \lambda_0$) and learning parameters (μ_Y, μ_O) are identified from the same moments as in the baseline life-cycle specification. U.S. denotes the United States.

Appendix Table H.6: External Productivity Dispersion and Implied Pareto Shapes

Within-cell quantile weights	Colombia		United States	
	Gap	Implied α	Gap	Implied α
Establishment hours	1.3193	1.6654	1.6867	1.3027
Unweighted	1.7629	1.2464	1.8374	1.1958

Notes: The gap is the 90–10 difference in log establishment output per hour. Quantiles are computed within four-digit industry-year cells. In the first row, establishments receive weights proportional to hours when the within-cell quantiles are calculated. Colombian establishment hours are implied by dividing the EAM wage bill by the corresponding sector-year mean hourly wage in the GEIH; the U.S. values use the BLS establishment-hours weights. Cell statistics are averaged without additional weights, and the reported entries are means over the common years 2008–2016. The external exercise uses the hours-weighted row; the unweighted row (identical to Appendix Table C.3) is descriptive. Implied shapes use $\alpha = \log(9)/[\log Q(0.90) - \log Q(0.10)]$.

Appendix Table H.7: Wage-Profile Fit with Externally Disciplined Pareto Shapes

Age	United States		Colombia		Colombia/U.S.	
	Data	Model	Data	Model	Data	Model
25–29	60.08	60.05	33.79	33.80	0.5625	0.5628
30–34	88.15	85.42	46.04	48.36	0.5223	0.5661
35–39	113.14	99.84	47.88	59.47	0.4232	0.5957
40–44	117.32	110.28	44.35	69.33	0.3780	0.6287

Notes: Entries for each age group are cumulative wage growth relative to ages 20–24; country entries are in percentage points and the last two columns are ratios. The external specification fixes α at 1.3027 for the United States and 1.6654 for Colombia. It then re-estimates $\underline{p}$ separately by country using only the 25–29 row. The 30–34 row is the left-out second original SMM target; the 35–39 and 40–44 rows are untargeted later-age moments. Model moments use $N = 10,000$ simulation draws.

Appendix Table H.8: Counterfactual Robustness along Colombia’s Pareto-Parameter Ridge

α	Profiled $\underline{p}$	Objective	Factual	Firms	Δ Firms	Learning	Frictions
10.0	4.105	0.000165	0.535	1.159	0.624	0.707	0.505
12.5	5.333	0.000143	0.535	1.159	0.625	0.707	0.507
15.0	6.566	0.000130	0.534	1.159	0.625	0.707	0.508
20.0	9.036	0.000114	0.534	1.159	0.625	0.707	0.510
25.0	11.511	0.000106	0.534	1.159	0.625	0.706	0.511

Notes: Each row fixes Colombia’s Pareto shape and re-optimizes its lower support over $[0.01, 20]$ using the two original wage-growth targets. The upper bound is 10 for both parameters. Factual and counterfactual entries report model-implied wage growth at ages 30–34 relative to the U.S. model value. The corresponding data ratio is 0.522. Firms replaces both Colombian firm-distribution parameters with their U.S. values; Δ Firms is Firms minus Factual. Learning replaces (μ_Y, μ_O) , and Frictions replaces $(\lambda_1, \lambda_0, \delta)$, with U.S. values. Because Firms replaces the entire firm block, its level is identical across rows by construction; robustness of its effect is assessed relative to the alternative fitted factual profiles. Profiling and counterfactual simulations both use $N = 10,000$.