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An Exploratory Foray into the Inclination to Cooperate in Spite of Cooperation-Induced Stress

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Abstract

We establish a new approach to the modeling of cooperation, and we formulate a new solution concept for cooperative games. We do this by constructing a game of cooperation between individuals who exhibit distaste for relative deprivation, *RD*, in the sense that they experience stress when their income is lower than that of their comparators. In such a game, the sharing out of the jointly earned income between these individuals when they cooperate, as prescribed by standard solutions of cooperative games, might not be acceptable to the individuals. The stress from *RD* may have the upper hand. Measuring stress by RD, we thus model a setting in which two individuals who are concerned with being relatively deprived need to decide whether or not to cooperate. We term this setting an *RD cooperative game*, and we design a rule, the *RD solution*, for the distribution of the income yielded in this game. The *RD solution* prescribes cooperation in spite of cooperation-induced stress and preserves the spirit of standardness (an equal sharing of the gain that accrues from cooperation) for two-player games (a property shared by the main solution concepts for cooperative games).

JEL classification

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Keywords

inclination to cooperate, cooperative games, social preferences, cooperation-induced stress, relative deprivation (RD), RD cooperative games, RD solution

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"Cooperation is willing collaboration by free individuals in a collective effort that creates more value than it expends."
JAMES RAYMOND LUCAS
Broaden the Vision

"Envy is ever joined with the comparing of a man's self; and where there is no comparison, no envy."
FRANCIS BACON
Essays: *Of Envy*

Preliminaries: A rationale for generalizing cooperative games, and the nature of the stress that arises from participating in cooperative games

A rationale for generalizing cooperative games

We analyze a setting in which individuals can produce jointly more than the sum of what they can produce when operating separately. Because typically individuals differ in their productivities, there will also be a difference in their contributions to their joint output / joint income, which gives rise to the issue of how to allocate the surplus that arises from the individuals' cooperation in a manner that the individuals will find acceptable.

A natural tool kit for modeling situations in which individuals can generate jointly more than the sum of what they can generate individually and need to agree on how to divide between them the resulting surplus is delivered by cooperative game theory, which yields a rich menu of applications, spanning legislative power measurement (Shapley and Shubik, 1954), environmental allocation problems (Parrachino et al., 2016; Ciardiello et al., 2018; Skovsgaard and Jensen, 2018), and credit assignment in machine learning (Yan and Procaccia, 2021). The solution concepts of cooperative game theory offer principled allocations of the resulting surplus that satisfy desirable properties.

Joint production that generates surplus is not, in and of itself, sufficient to incentivize individuals to cooperate. A related consideration concerns the notion of interpersonal comparisons, in particular the role of relative deprivation, *RD*, meaning the dismay that individuals experience when those with whom they compare themselves obtain more than they have. Cooperation between individuals alters the individuals' comparison environment:

whereas individuals operating alone may not have other individuals share their social space, joint production changes that, as it merges previously separate social spaces into one, thereby subjecting individuals to interpersonal comparisons. (By social space we mean the set of individuals with whose incomes an individual compares his own income. Social space, comparison group, and reference group can be used interchangeably.) A substantial body of research in economics, sociology, psychology, political science, and medicine highlights the role of *RD* in behavior and wellbeing (consult, for example, Stark, 2013; Stark, 2021).[1] Elevated stress stemming from *RD* can discourage cooperation and, instead, advocates lone production: the concern of an individual about being positioned below others in his comparison group when such a group forms upon joint production may outweigh the lure of the material benefits to be gained from joint production. Once aversion to *RD* is taken into account, allocation of the jointly produced income as prescribed by standard solutions of cooperative games may fall short of providing individuals with sufficient incentives to cooperate. This discussion leads us to ask: can we then design an allocation rule of the resulting surplus that will invite collaborative production while acknowledging the involved individuals' social preferences / concerns about not being subjected to *RD*?

Several modifications were made to standard cooperative-game solutions, aiming at mitigating inequality concerns. Coming to mind are the egalitarian Shapley values (Joosten, 1996; Malawski, 2004; van den Brink et al., 2013; Casajus and Huettner, 2014), the weighted Shapley values (Shapley, 1953a; Myerson, 1980; Kalai and Samet, 1987; Hart and Mas-Colell, 1989), the proportional Shapley value (Beal et al., 2018; Besner, 2019), the egalitarian core (Arin and Inarra, 2001; Arin et al., 2008), and the egalitarian bargaining solution (Kalai, 1997). While these modifications generally prescribe allocations that are more equal than the allocations prescribed by standard solutions, the notion of equality in these writings is imposed exogenously by the designer of the rule rather than being discerned from the individuals' preferences as exhibited by the individuals' utility functions.

Instead of relying on exogenously imposed equality criteria, we present in this paper a framework in which the allocation rule is derived directly from individuals' preferences, where individuals trade off the utility from additional income against the disutility from *RD*.

[1] Readers interested in reviewing papers in economics, sociology, psychology, political science, and medicine on how the experience of *RD* affects incentives and behavior can consult circa 30 papers in "Publications" at the website https://ostark.wne.uw.edu.pl/ that in the past decade alone modeled that behavior.

Formally (as discussed in the next subsection), *RD* is evaluated with respect to a specified metric (such as income or wealth). We compute the *RD* of an individual by aggregating the differences between the incomes of members of his reference group that are higher than his income, and we then divide this sum by the size of the reference group (so that *RD* is the average of the income excesses in the individual's reference group).

In the case of cooperative games involving two individuals, many of the standard solutions of cooperative games (as noted further in Section 1) coincide with each other (a coincidence that is referred to as standardness for two-player games). This feature also leads us to concentrate on the two-player setting, enabling us to "monitor" the impact of *RD* on multiple solutions of cooperative games closely and transparently. Specifically, we consider two participants in a cooperative game, each of whom is concerned both with his absolute income and with how that income compares to the income of the other participant. We formalize this setting by proposing the notion of *RD* cooperative games. We design a rule for the allocation of the joint income among the two individuals, which we call the *RD solution*. This rule accounts for the involved individuals' utility - given that we acknowledge the individuals' aversion to *RD* - and retains the spirit of standard solutions of cooperative games.

The nature of the stress that arises from participating in cooperative games

We quantify the stress that arises from having less than others by an index of *RD*. This index follows and is in line with a large body of literature on the subject of *RD* and reference (comparison) groups, spanning from the pioneering two-volume study of Stouffer et al. (1949), *Studies in Social Psychology in World War II: The American Soldier*, through Akerlof (1997) and up to recent writings, for example, those of Stark et al. (2017) and Stark (2020).[2] By definition and construction, the concept of *RD* is complementary to the concept of reference group or comparison group. The work of Stouffer et al., which opened the door to research on *RD* and reference groups, documented the distress caused not by a low military rank and weak prospects of promotion (military police) but rather by the faster pace of promotion of others (air force). It also documented the lesser dissatisfaction of Black soldiers

[2] Rich evidence from field and laboratory studies in economics, social psychology, and neuroscience confirms that individuals routinely engage in, and are affected by, interpersonal comparisons. In particular, people are dissatisfied when their income levels are lower than those of others who constitute their comparison group. Frey and Stutzer (2002), Walker and Smith (2002), and Smith et al. (2012) review a large body of evidence that lends support to the "upward comparison" hypothesis.

stationed in the South who compared themselves with Black civilians in the South compared with the dissatisfaction of their counterparts stationed in the North who compared themselves with Black civilians in the North. Soldiers' pay was much closer to the incomes of Black civilians in the South and much lower than the incomes of Black civilians in the North. While not explicitly using the term *RD*, Sen (1973) refers to the stress that arises from having low relative income. In explaining the construction of the Gini coefficient, Sen (1973, p. 33) writes: "the man with the lower income can be thought to be suffering from some depression on finding his income to be lower. Let this depression be proportional to the difference in income. The sum total of all such depressions . . . " takes us to a measure of *RD*. It is intriguing that when Sen (1973 and 1997), Sen (1976), and Sen (1982) developed a measure of wellbeing, he did so by assigning a positive weight to income divided equally, as measured by income per capita, and a negative weight to income inequality, as measured by the Gini coefficient. Both the writings of Sen and the work of Stouffer et al. from 75 years ago long preceded the often sited study by Fehr and Schmidt (1999). A derivation of the *RD* representation of an individual's social stress is presented in Stark (2013) and in an online appendix "Construction of a relative deprivation index," in Stark and Budzinski (2024).

We assume that individual i, $i \in A$, where A is the reference group of individual i, compares what he gets with what members of A get. An individual experiences *RD* when he gets less than what members of A get. Then the *RD* index of individual i is defined as

$$RD(i,x) \equiv \frac{1}{|A|}\sum_{j \in A} \max\{x_j - x_i, 0\},$$

where for any individual $j \in A$ we denote what he gets by x_j (in particular, x_i is what individual i gets), and the distribution of the aggregate payoff is denoted by $x = (x_j)_{j \in A}$.

1. Introduction: Cooperative games and *RD cooperative games*

Let there be two individuals, 1 and 2, faced with the following decision: either each of them operates alone, thereby earning, respectively, incomes v_1 and v_2, or they operate together to earn a joint income of v_{12}. Throughout this paper, by "joint income" we mean the overall income yielded upon cooperating. Let the cooperation of the individuals yield at least as high an income as the sum of the individuals' incomes had they operated separately, that is, let $v_{12} \geq v_1 + v_2$. (This condition is referred to as a superadditivity assumption.) In addition, for simplicity, and without loss of generality, we assume that $v_2 \geq v_1$. In the vocabulary of game theory, this setting is a two-player cooperative game with transferable utility and a superadditive characteristic function (Serrano, 2009; Barron, 2013). The characteristic function of a game assigns to each group of individuals the maximal joint income that this group can obtain. A central question in cooperative game theory is how to allocate joint income among the participating individuals. Candidates for such allocations are called imputations: a vector $x = (x_1, x_2)$ (where x_i, $i \in \{1,2\}$, is the part of v_{12} received by individual i) is referred to as an imputation if $x_1 + x_2 = v_{12}$ (group rationality, meaning that the entire joint income is distributed) and if $x_i \geq v_i$ for every $i \in \{1,2\}$ (individual rationality, meaning that when cooperating, no individual ends up with an income that is lower than the income he would have earned when operating alone). Typically, there are many possible imputations, and there are several criteria for choosing one of them as a solution. The notable rules of choice in general cooperative game theory are the Shapley value (Shapley, 1953), the nucleolus (Schmeidler, 1969), the equal allocation of nonseparable contribution value (Moulin, 1985), the equal surplus division value (Driessen and Funaki, 1991), and the solidarity value (Nowak and Radzik, 1994). In the case of two individuals, these rules coincide both with each other and with the Nash bargaining solution (Nash, 1950) and the Kalai-Smorodinsky bargaining solution (Kalai and Smorodinsky, 1975).[3] The reason for this concurrence is that these rules share the standardness for two-player games (Hart and Mas-Colell, 1989), according to which each individual should receive what he would have had when operating alone plus half of the gain that accrues from cooperation. In our setting, standardness is expressed as $x_i = v_i + \frac{v_{12} - v_1 - v_2}{2}$ for $i \in \{1,2\}$. Henceforth, we will refer to a division of income prescribed by the standardness for two-player games as the standard solution, and we denote this division by $S = (S_1, S_2) = \left(\frac{v_{12} + v_1 - v_2}{2}, \frac{v_{12} + v_2 - v_1}{2} \right)$, where S_i is the share of the joint income received by individual $i \in \{1,2\}$.

We modify the setting of cooperative games by assuming that when individuals cooperate, each of them compares his share of the joint income with the share of the other

[3] A systematic review of the equivalence of solution concepts for two-player games can be found in Kalai and Kalai (2013).

individual, and that if one individual receives less than the other individual, he experiences dismay or stress. We quantify this stress by the index of *RD*, defined as the aggregate of income excesses in an individual's comparison group divided by the size of the comparison group. In the case of two individuals, *i* and *j*, the *RD* index is

$$RD(i,x) \equiv \frac{1}{2}\max\{x_j - x_i, 0\} \ , \tag{1}$$

where $RD(i,x)$ is the relative deprivation of individual i when he cooperates and $x = (x_i, x_j)$.

We assume that the individuals belong to each other's comparison group if and only if they cooperate; when they operate alone, the individuals do not engage in interpersonal comparisons. Seen this way, cooperation is a technology that converts separate social spaces into one.[4]

The individuals need to decide whether or not to cooperate, subject to a specific division of their jointly acquired income. To model the individuals' choice, we measure the intensity of dissatisfaction inflicted by *RD*. We introduce a taste parameter $\beta \in [0,\infty)$. We assume that the taste parameters of the two individuals are the same. (In a given population, the preferences of its members can reasonably be assumed to be quite similar.[5]) Thus, when the individuals cooperate and the joint income is divided between them according to the distribution $x = (x_1, x_2)$, the utility of individual $i \in \{1,2\}$ is defined as

$$u(i,x) \equiv x_i - \beta RD(i,x). \tag{2}$$

If the individuals do not cooperate, then no individual belongs to the comparison group of the other individual, the *RD* of each individual is nil, and the utility of each individual is equal to his income when operating alone. Thus, if (in terms of utility) at least one of the individuals is worse off when cooperating than when operating on his own, then the individuals do not cooperate; otherwise, they do. Formally, the utility, $U(i,x)$, of individual i is

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[4] A vivid example of exposure to *RD* upon joint production versus no such exposure upon operating alone is provided by migration: individual 1 in region A has no social or production-related contact with individual 2 in region B, but the opposite holds when individual 1 migrates to region B. In this case, social contact is interwoven with production contact; the social comparison with individual 2 arises because of the collaborative production with him.

[5] However, as reasoned in Section 3, in the case of two individuals, this assumption may not be needed.

$$U(i,x) \equiv \begin{cases} u(i,x), \text{ if } u(j,x) \geq v_j \text{ for } j \in \{1,2\} \\ v_i, \text{ if there exists } j \in \{1,2\} \text{ such that } u(j,x) < v_j. \end{cases} \quad (3)$$

The definition in (3) spans the choices of the individuals whether to cooperate or not, given the distribution of their joint income. The first line in (3) refers to the case in which the individuals cooperate, and the second line in (3) refers to the case in which the individuals do not cooperate. In line with the usual formulation of cooperative games, we assume that an individual chooses to cooperate when cooperation and operating on his own are equally rewarding.[6]

We refer to the setting presented above as an *RD cooperative game*. Analogously to the usual formulation of cooperative games, we refer to *RD imputations* as divisions of the joint income for *RD cooperative games* under which individuals will cooperate. That is, an *RD imputation* is a vector $x = (x_1, x_2)$ if $x_1 + x_2 = v_{12}$ (group rationality) and $u(i,x) \geq v_i$ (individual rationality) for every $i \in \{1,2\}$. From the set of *RD imputations* we will want to select an imputation as the *RD solution,* ensuring that the selected imputation will retain the spirit of the standardness for two-player games, whereas the "discarded" imputations do not.

The setting of *RD cooperative games* establishes a new approach to the modeling of cooperation. In standard cooperative games, an individual needs to know only what his share of the joint payoff will be in order to determine whether cooperation will be beneficial to him. In *RD cooperative games*, however, an individual may decline to cooperate when the stress that he experiences from comparing his share of the joint payoff with the share of the joint payoff of a coplayer is more painful to him than the elation he feels as a result of his gain from cooperation. In cooperative games, the aggregate wellbeing (as measured by the joint payoff from cooperation) of each cooperating group is constant, it is given ex ante as a component of the game setting (that is, as a value of a characteristic function of the game), and it is typically used as an indicator for choosing an imputation as a solution concept. In *RD cooperative games*, the wellbeing of a member of a cooperating group cannot be calculated prior to establishing the rules for the distribution of the payoff from cooperation between the members of the cooperating group. That is to say, even if we assume that the utility levels of

[6] A utility function that gives a positive weight to absolute income and a negative weight to low relative income can be used to obtain qualitative results that are akin to the core results reported in this paper. Thus, there need not be a loss of generalization in drawing on the additively separable utility function exhibited in equation (2).

different individuals are comparable and additive, we cannot calculate the aggregate wellbeing of a cooperating group prior to knowing the distribution between the individuals of the group's joint payoff.

To illustrate the concept of *RD cooperative games* we present a motivating example.

Example 1. The effect of concern about relative deprivation on the viability of the standard solution of a cooperative game.

Let there be two individuals, 1 and 2. When they operate alone, the incomes of these individuals are, respectively, $v_1 = 6$ and $v_2 = 12$. When the individuals cooperate, they generate a joint income $v_{12} > 18$. We consider two cases: (a) the jointly produced income is $v_{12} = 19$ and (b) the jointly produced income is $v_{12} = 22$. How should the individuals divide the gain from cooperation so that each of them will prefer cooperating to not cooperating? The standard solution prescribes that from the gain conferred by cooperation each individual will receive half, so that $S_i = v_i + \frac{v_{12} - (v_1 + v_2)}{2}$ for $i \in \{1,2\}$. Thus, in case (a) the incomes of the cooperating individuals 1 and 2 are, respectively, 6.5 and 12.5, and in case (b) the incomes of the cooperating individuals 1 and 2 are, respectively, 8 and 14.

Given that $v_{12} > v_1 + v_2$, in both cases (a) and (b), the standard solution prescribes for each individual an income that is higher than what the individual could have obtained without cooperating. Therefore, prima facie, it seems that the individuals will be inclined to cooperate.

Suppose that an individual who cooperates with another individual experiences stress when the other individual's income is higher than his, that is, we consider the setting of *RD cooperative games*, and we let $\beta = 1$. When the individuals operate on their own, then each of them occupies a separate social space, no income comparisons occur, and no comparison-derived stress arises. Because of distaste for relative deprivation, the individuals need to decide whether it is worth their while to cooperate, given that cooperation merges their social spaces. When the two individuals cooperate and the joint income is divided between them according to the vector $x = (x_1, x_2)$, then the utility of individual $i \in \{1,2\}$ is as per equation (2), whereas, recall, when the individuals operate alone, the utility of individual $i \in \{1,2\}$ is v_i. Obviously, cooperation occurs if and only if $u(i,x) \geq v_i$ for $i = 1,2$, that is, if and only if neither of the individuals loses from cooperating. To incorporate the individuals' concern about experiencing *RD*, we relabel cases (a) and (b) cases (a') and (b'), respectively. We ask whether, in the relabeled setting, there can be ways of distributing the joint income such that the individuals will be incentivized to cooperate, that is, whether the set of *RD imputations* is nonempty.

In cases (a') and (b'), distributions according to the standard solution (6.5 and 12.5 in case (a') and 8 and 14 in case (b')) do not increase the utility of individual 1 enough to induce him to cooperate with individual 2. To see this, we perform some calculations.

In case (a'), if the individuals were to cooperate and divide their gain according to the standard solution (that is, $S(a)=(6.5,12.5)$), then, by equation (2), the utility of individual 1 would be $u(i,S(a))=6.5-\frac{12.5-6.5}{2}=3.5<6=u_1$, which is lower than his utility without cooperation. In this case, then, the individuals will not cooperate; the utility of individual 1 will remain at $u_1=v_1=6$, and the utility of individual 2 will remain at $u_2=v_2=12$. Moreover, in case (a'), "no cooperation" is the only possible outcome because the set of *RD imputations* is empty: individual 2 is unable to offer individual 1 a distribution of the joint income that will compensate individual 1 for the relative deprivation experienced by him, without individual 2 sacrificing his own utility. Indeed, for all distributions $x=(x_1,x_2)$ that do not lower the utility of individual 2 below $v_2=12$, the maximal amount that individual 2 can offer individual 1 is the entire gain from cooperation, that is, income 1, so the income of individual 1 becomes 7, in which case his utility will be

$$u(1,(7,12))=7-\frac{12-7}{2}=4.5<6=u_1,$$

and this utility will still be lower than his utility when operating alone. Apparently, there is not enough in the "common pot" to relieve individual 1 of the stress of *RD*.

In case (b'), if the individuals were to cooperate and divide their gain according to the standard solution (so that $S(b)=(8,14)$), then the utility of individual 1 would be $u(1,S(b))=8-\frac{14-8}{2}=5<6=u_1$, that is, a utility that is lower than his utility when operating alone. Thus, the individuals will not cooperate, and their utilities will remain at 6 for individual 1 and at 12 for individual 2.

It is worth noting that trying to eliminate the effect of relative deprivation by dividing equally the jointly produced income following cooperation will not work either. In case (b') with $v_{12} = 22$, an equal division is $x = (11,11)$. But then, because $u(2, x) = x_2 = 11 < 12 = v_2$, individual 2 will end up worse off cooperating than not cooperating.

However, in case (b'), it is possible to identify an *RD imputation*, that is, a distribution of the joint income 22 obtained by cooperation, which is sensitive to the concern for *RD* and is more satisfactory to both individuals than had their utilities remained at the levels of 6 and 12. Yet this distribution is different from the distribution prescribed by the standard solution. To see this, we assume, for example, that income 22 is distributed as $x = (x_1, x_2) = (9,13)$. Then $u(1, x) = 9 - \frac{13-9}{2} = 7 > 6 = u_1$, and $u(2, x) = 13 > 12 = u_2$; both individuals are better off cooperating than not cooperating. In fact, it is possible to identify multiple *RD imputations*: any distribution of the joint income $(x_1, 22 - x_1)$ such that $8.5 \le x_1 \le 10$, ensures that the utilities of both individuals will be (weakly) higher when cooperating than when operating alone, and such a distribution constitutes an *RD imputation*. What renders it possible to achieve this result is that there is enough in the "common pot" to compensate individual 1 for pursuing cooperation that entails exposure to relative deprivation, while still enabling individual 2 to enjoy a gain.

Because in case (b') invoking the standard solution is not viable, even though the set of *RD imputations* is nonempty, we draw on the preceding considerations and example to construct a new solution concept - the *RD solution*. When we do this, we account both for the impact of cooperation on the jointly produced income and for the individuals' concern about *RD*.

In the case of two individuals, *RD cooperative games* are essentially an extension or a revision of the concept of cooperative games. In particular, when $\beta = 0$, an *RD cooperative game* is equivalent to a cooperative game with the same parameters v_1, v_2, and v_{12}. Given the superadditivity assumption $v_{12} \ge v_1 + v_2$, the individuals will cooperate when offered any imputation of the *RD cooperative game* with $\beta = 0$ or, equivalently, any imputation of the cooperative game. In the more general setting of *RD cooperative games* with any $\beta \ge 0$, the set of *RD imputations* might be empty (as in case (a') of Example 1), and even when it is not empty, the standard solution does not always present an individual who is relatively deprived with an outcome that will tempt him to cooperate (as in case (b') of Example 1). In the next section, we establish when the set of *RD imputations* is nonempty (Claim 1), and we present a solution concept that preserves the spirit of standardness for two-player games (Claim 2). Our prescribed solution is acceptable to both individuals as a rule for the distribution of their joint income in an *RD cooperative game*.

2. The *RD solution* to *RD cooperative games*

In Example 1 we saw that there existed values of v_1, v_2, and v_{12} such that the joint income is insufficient to compensate individual 1 for the *RD* that he experiences when cooperating, that is, the set of *RD imputations* is empty. Thus, prior to discussing which of several possible *RD imputations* should be chosen as a solution, we provide a necessary and sufficient condition for the nonemptiness of the set of *RD imputations.*

Claim 1. **A criterion for nonemptiness of the set of *RD imputations***.

For an *RD cooperative game*, the set of *RD imputations* is nonempty if and only if [7]

$$v_{12} \geq \frac{2v_1 + (2+2\beta)v_2}{2+\beta}. \quad (4)$$

Proof. In the Appendix.

Condition (4) can be considered a generalization of the standard superadditivity assumption of cooperative games $v_{12} \geq v_1 + v_2$. In the same way in which superadditivity guarantees that the set of imputations of a cooperative game will be nonempty, condition (4) guarantees that the set of *RD imputations* of a cooperative game will be nonempty. When $\beta = 0$, that is, the case in which the *RD* cooperative game reduces to a cooperative game, the reduced condition $v_{12} \geq v_1 + v_2$ of equation (4) is just the standard superadditivity.

In case (b') of Example 1, we see that even though the "common pot" of joint income is sufficiently large to compensate for relative deprivation, the standard solution may not be acceptable to both individuals. We therefore construct a new solution, which we do by

[7] Inequality (4) can alternatively be interpreted as an upper bound on β : $\beta \leq \dfrac{2(v_{12} - v_1 - v_2)}{2v_2 - v_{12}}$.

selecting a particular *RD imputation* that retains the spirit of the standardness for two-player games as the distribution of the individuals' joint income. To preserve the spirit of the standard solution, we now modify the property of standardness for two-player games. Given that the standardness for two-player cooperative games builds on the idea that the increase in joint income yielded by cooperation should be divided equally between the participating individuals, an appealing generalization to the case of two-player *RD cooperative games* is a property of utilitarian standardness: the increase in utility accruing from cooperation should be the same for the participating individuals.

Definition 1. **Utilitarian standardness for two-player games**.

The utilitarian standardness for two-player *RD cooperative games* is a property of an *RD imputation* $x=(x_1,x_2)$ if the utilitarian gains from cooperation for both individuals are the same when the joint income is divided according to the *RD imputation* x, that is, when

$$u(1,x)-v_1=u(2,x)-v_2. \tag{5}$$

For $\beta=0$, $u(i,x)=x_i$ for $i\in\{1,2\}$, so condition (5) is reduced to $x_1-v_1=x_2-v_2$, which, in conjunction with the fact that x is an imputation, implies that $x=S$, that is, that utilitarian standardness for two-player games is equivalent to standardness for two-player games in the case of cooperative games. (This holds because $x_1-v_1=x_2-v_2=\frac{v_{12}-(v_1+v_2)}{2}$ for $x_i=S_i$.)

Because solution concepts of two-player cooperative games are fully characterized by the standardness for two-player games, we likewise require that the solution for two-individuals *RD cooperative games* be fully characterized by the utilitarian standardness for two-player games.

Definition 2. **The *RD solution***.

Assuming that the *RD cooperative game* is such that its set of *RD imputations* is nonempty, the *RD imputation* $RDS=(RDS_1,RDS_2)$ is called the *RD solution* if it satisfies the utilitarian standardness for two-player *RD cooperative games*.

Two questions arise. The first question has to do with whether the *RD imputation* satisfying equation (5) always exists, assuming that the set of *RD imputations* is nonempty, and the second with whether such an *RD imputation* is unique. As Claim 2 reveals, the answers to both these questions are positive.

Claim 2. Existence and uniqueness of the *RD solution*, and a formula for the *RD solution*.

We assume that Claim 1 holds: the set of *RD imputations* is nonempty. Then, the *RD solution* exists, it is unique, and it takes the form

$$RDS = \left(\frac{v_{12}}{2} - \frac{v_2 - v_1}{2+\beta}, \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} \right).$$

Proof. In the Appendix.

Claim 2 asserts that if it is at all possible to induce both individuals to cooperate, then they will accept the *RD solution* as the rule of the distribution of their joint income.

Example 2. The *RD solution* for the two cases of Example 1.

We draw on Claims 1 and 2 to revisit cases (a') and (b') from Example 1 in which $\beta = 1$. Then in both these cases, it holds that

$$\frac{2v_1 + (2+2\beta)v_2}{2+\beta} = \frac{2 \cdot 6 + (2 + 2 \cdot 1) \cdot 12}{2+1} = 20.$$

From Claim 1, because in case (a') $v_{12} = 19 < 20$, then case (a') with $\beta = 1$ does not admit any distribution of the joint income, which is more satisfactory to both individuals than their respective incomes from operating alone. In case (b') $v_{12} = 22 > 20$. Therefore, by Claim 1, the set of *RD imputations* is nonempty, and by Claim 2, applying the *RD solution* leads both individuals to choose to cooperate. The related computations yield

$$RDS_1(b') = \frac{v_{12}}{2} - \frac{v_2 - v_1}{2+\beta} = \frac{22}{2} - \frac{12-6}{3} = 11 - 2 = 9,$$

$$RDS_2(b') = \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} = \frac{22}{2} + \frac{12-6}{3} = 11 + 2 = 13.$$

Thus, $RDS(b') = (9,13)$ is the *RD solution* in case (b'). As already noted in Example 1, both individuals are better off cooperating than not cooperating when $RDS(b') = (9,13)$ is a distribution of their jointly produced income upon cooperation.

It is nice to see here that when $\beta = 0$, the *RD solution* reduces to the "standard" standardness solution for two-player games where each individual receives what he would have had when operating alone plus half of the gain that accrues from cooperation: $6 + \frac{1}{2} \cdot 4 = 8$ and $12 + \frac{1}{2} \cdot 4 = 14$, a distribution that is likewise yielded by the application of $RDS = \left(\frac{v_{12}}{2} - \frac{v_2 - v_1}{2+\beta}, \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} \right)$ when $\beta = 0$: $\frac{v_{12}}{2} - \frac{v_2 - v_1}{2} = \frac{22}{2} - \frac{12-6}{2} = 11 - 3 = 8$ and $\frac{v_{12}}{2} + \frac{v_2 - v_1}{2} = \frac{22}{2} + \frac{12-6}{2} = 11 + 3 = 14$.

Remark. Noticeably, when the basic incomes are $v_1 = 6$, $v_2 = 12$, and $v_{12} = 19$, if we modify case (a') in such a way that β is still strictly positive but considerably less than one, that is, if the individuals are not too concerned about experiencing *RD*, then it is possible to compute an *RD solution* that allocates shares of the joint income that are satisfactory to both individuals. For example, suppose that we set $\beta = \frac{2}{11}$. Then from equation (4) we get

$$v_{12} = 19 \geq \frac{2v_1 + (2+2\beta)v_2}{2+\beta} = \frac{2 \cdot 6 + \left(2 + 2 \cdot \frac{2}{11}\right) \cdot 12}{2 + \frac{2}{11}} = 18.5,$$

so that by Claims 1 and 2, the *RD solution* is a rule of the distribution of joint income that induces the individuals to cooperate and

$$RDS_1 = \frac{v_{12}}{2} - \frac{v_2 - v_1}{2+\beta} = \frac{19}{2} - \frac{6}{\frac{24}{11}} = 9.5 - 2.75 = 6.75,$$

$$RDS_2 = \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} = \frac{19}{2} + \frac{6}{\frac{24}{11}} = 9.5 + 2.75 = 12.25.$$

Hence, $RDS = (6.75, 12.25)$ is the *RD solution* for the modified case of (a') in which $\beta = \frac{2}{11}$. We can easily verify that in this circumstance, cooperation is more beneficial to each of the two individuals than operating alone. By equation (2)

$$u(1, RDS) = 6.75 - \frac{2}{11} \cdot \frac{12.25 - 6.75}{2} = 6.25 > 6 = v_1,$$

and

$$u(2, RDS) = 12.25 > 12 = v_2.$$

3. Discussion and conclusions

When individuals who join forces to produce income contribute unevenly to the jointly produced income, standard solution concepts prescribe a method of distribution which, in particular, guarantees that each of the individuals will obtain income that is higher than the income that he would have been able to obtain if operating on his own. This method induces cooperation. However, when the individuals' preferences are social, a larger absolute income conferred by cooperation does not necessarily translate into higher utility; the receipt of a larger income may not be a sufficient inducement to cooperate. To accommodate social preferences and design a rule of distribution that will make cooperation worthwhile, we introduce the concept of relative deprivation, *RD*, and subsequently, the concept of *RD cooperative games*. The outcome of such games is not always cooperation between the individuals involved. The set of satisfactory distributions of the outcomes of cooperation (the set of *RD imputations*) shrinks when the parameter β, which measures the strength of distaste for experiencing *RD*, increases. For sufficiently large β, the set of satisfactory cooperation solutions is reduced either to a singleton of equal division or to an empty set.[8]

However, when the set of *RD imputations* is nonempty, it contains a particularly appealing element which we referred to as the *RD solution*. Just as *RD cooperative games* constitute generalizations of cooperative games, the *RD solution* was similarly shown to be a generalization of a standard solution of two-player games. Establishing the *RD solution* as a rule of distribution provides the individuals with an incentive to contribute more because they stand to be rewarded for increasing the aggregate utility of the group that they join and, thereby, their individual take. This is to say that the payoff of individual 1 allocated to him by the *RD solution* is increasing as a function of v_1 and as a function of v_{12} and that the payoff of individual 2 allocated to him by the *RD solution* is increasing as a function of v_2 and as a

[8] A proof of this property as well as of the attributes of the *RD solution* listed in the next paragraph are available on request.

function of v_{12}. In addition, we can view the *RD solution* as a compromise between the standard solution and the equal division of income or, more precisely, as a convex combination of these two. Seen this way, the *RD solution* is one of the egalitarian Shapley values defined by Joosten (1996).

Our inquiry can serve as a basis for follow-up research. A logical direction is an extension of the setting studied in this paper to a setting of more than two individuals. Then, the well-known solutions for distributing the payoff of cooperation (such as the Shapley value, the nucleolus, and so on) no longer coincide. The incorporation of *RD* can also take various forms. As an example, relaxation of the assumption that the cooperating individuals are characterized by the same taste parameter β. In the case of two individuals, because both the income of individual 1 when operating alone and the income of individual 1 when cooperating do not exceed the corresponding incomes of individual 2, then all along individual 2 does not experience *RD*. Therefore, in applying the *RD solution*, the taste parameter of individual 2 is immaterial. However, in the case of more than two individuals, differentiation of the taste parameter might take us to a fundamentally different game environment wherein individuals may seek to signal or conceal their taste parameter in order to entice other individuals to cooperate with them or, upon cooperation, to induce other individuals to transfer more to them.

Appendix: **Proofs of Claims 1 and 2**

Proof of Claim 1.

First, we show the "if:" if there exists an *RD imputation*, then (4) holds.

We assume that $x=(x_1,x_2)$ is an *RD imputation*. Hence, $x_2 \ge u(2,x) \ge v_2$ and $x_1 \ge u(1,x) \ge v_1$. Therefore,

$$x_1 - \beta\frac{v_2 - x_1}{2} \ge x_1 - \beta\frac{x_2 - x_1}{2} \ge u(1,x) \ge v_1 \,. \tag{A1}$$

By adding $\frac{\beta}{2}v_2$ to both sides of the inequality $x_1 - \beta\frac{v_2 - x_1}{2} \ge v_1$, we get from (A1)

$$\frac{2+\beta}{2}x_1 \ge v_1 + \frac{\beta}{2}v_2 \,,$$

that is,

$$x_1 \geq \frac{2v_1 + \beta v_2}{2+\beta}.$$

Thus, if $x = (x_1, x_2)$ is an *RD imputation*, then

$$v_{12} = x_1 + x_2 \geq \frac{2v_1 + \beta v_2}{2+\beta} + v_2 = \frac{2v_1 + (2+2\beta)v_2}{2+\beta}.$$

Therefore, when the set of *RD imputations* is nonempty, equation (4) holds.

Second, we show the "only if:" if equation (4) holds, then there exists an *RD imputation*. We consider two cases:

(i). We assume that $v_{12} \geq 2v_2$. Then, $x = \left(\frac{v_{12}}{2}, \frac{v_{12}}{2}\right)$ is an *RD imputation* because $u(1,x) = u(2,x) = \frac{v_{12}}{2} \geq v_2 \geq v_1$.

(ii). We assume that $2v_2 > v_{12} \geq \frac{2v_1 + (2+2\beta)v_2}{2+\beta}$. Then $x = (v_{12} - v_2, v_2)$ is an *RD imputation* because $v_2 > v_{12} - v_2$. Thus, $u(2,x) = v_2$, and

$$u(1,x) = v_{12} - v_2 - \frac{\beta}{2}(2v_2 - v_{12}) = \left(\frac{2+\beta}{2}\right)v_{12} - (1+\beta)v_2 \geq \frac{2v_1 + (2+2\beta)v_2}{2} - (1+\beta)v_2 = v_1.$$

That is, $u(1,x) \geq v_1$. Therefore, if equation (4) holds, then the set of *RD imputations* is nonempty. Q.E.D.

Proof of Claim 2.

Proof. To begin with, we verify that the vector *RDS* is an *RD imputation*.

Clearly, $RDS_1 + RDS_2 = v_{12}$, and $RDS_1 = \frac{v_{12}}{2} - \frac{v_2 - v_1}{2+\beta} \leq \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} = RDS_2$. Because the set of *RD imputations* is nonempty, equation (4) holds and, therefore,

$$u(1,RDS) = RDS_1 - \beta RD(1,RDS) = \frac{v_{12}}{2} - \frac{v_2 - v_1}{2+\beta} - \frac{\beta}{2}\left(\frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} - \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta}\right)$$
$$= \frac{v_{12}}{2} - \frac{(v_2 - v_1)(1+\beta)}{2+\beta} \geq \frac{v_1 + (1+\beta)v_2}{2+\beta} - \frac{(v_2 - v_1)(1+\beta)}{2+\beta} = v_1,$$

and

$$u(2,RDS) = RDS_2 = \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} \geq \frac{v_1 + (1+\beta)v_2}{2+\beta} + \frac{v_2 - v_1}{2+\beta} = v_2.$$

We conclude that the vector *RDS* is an *RD imputation* if the set of *RD imputations* is nonempty.

Next, we show that the vector *RDS* satisfies the utilitarian standardness for two-player games, that is, that equation (5) is satisfied for $x = RDS$.

We know that

$$u(1,RDS) - v_1 = RDS_1 - \beta RD(1,RDS) - v_1 = \frac{v_{12}}{2} - \frac{v_2(1+\beta)+v_1}{2+\beta}$$

and that

$$u(2,RDS) - v_2 = \frac{v_{12}}{2} + \frac{v_2 - v_1}{2+\beta} - v_2 = \frac{v_{12}}{2} - \frac{v_2(1+\beta)+v_1}{2+\beta}.$$

Thus, $u(1,RDS) - v_1 = u(2,RDS) - v_2$, and *RDS* satisfies the utilitarian standardness for two-player games. Therefore, the vector *RDS* is an *RD solution*.

To complete the proof, what remains to be shown is that *RDS* is the only *RD imputation* satisfying the utilitarian standardness for two-player games.

By contradiction, we assume that there exists another *RD imputation* $y = (y_1, y_2) \neq RDS$ satisfying equation (5). From the inequality $v_2 \geq v_1$ and the utilitarian standardness for two-player games, we infer that $y_2 \geq y_1$. Without loss of generality, we assume that $y_2 > RDS_2$. Thus, $y_1 < RDS_1$ (because $y_1 + y_2 = RDS_1 + RDS_2 = v_{12}$). Therefore,

$$u(1,y) - v_1 < u(1,RDS) - v_1 = u(2,RDS) - v_2 < u(2,y) - v_2.$$

Hence, y does not satisfy equation (5) - a contradiction. Q.E.D.

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