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Peer vs. Network Effects: Microfoundations, Identification, and Beyond

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Abstract

This paper reviews the theoretical and empirical foundations of peer and network effects, aiming to bridge insights from both literatures. We first analyze the microfoundations of peer effects through linear–quadratic network games, linking equilibrium behavior to network centrality and highlighting the role of key players. Then, we examine the main identification challenges in linear-in-means models — reflection, correlated effects, and sorting — and show how introducing explicit network structures can help address them. We also review reduced-form strategies based on within-school cohort composition, exposure to peers' shocks, random assignment, and exogenous variation in network links. Finally, we discuss how structural models of network formation and individual effort choices can resolve endogeneity concerns. The paper concludes with recent advances on nonlinear and multiplex interactions, where individuals respond to specific peers and operate across multiple, interdependent layers.

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Keywords

social interactions, identification, network games, centrality, multiplex networks, non-linearities

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1 Introduction

Understanding social interactions has been a central pursuit across the social and natural sciences for decades, engaging researchers in disciplines as diverse as graph theory and applied mathematics (Newman, 2010; Bollobás, 2011), game theory and economics (Jackson, 2008; Jackson and Zenou, 2015; Jackson et al., 2017), sociology (Coleman, 1988; Granovetter, 1973; Christakis and Fowler, 2009), and psychology (Cialdini, 2007). The enduring fascination stems from a simple but powerful insight: individuals do not make decisions in isolation. Their behaviors, beliefs, and opportunities are shaped by the actions and expectations of those around them. Ignoring these interdependencies can lead to fundamentally flawed predictions and misguided policies. For instance, education programs that neglect peer effects may underestimate how classroom spillovers amplify or dampen treatment effects (Sacerdote, 2011, 2014); vaccination or deworming campaigns that ignore social diffusion achieve low coverage (Miguel and Kremer, 2004); health campaigns that treat individuals as independent may fail to achieve herd immunity thresholds or even backfire (Acemoglu et al., 2021; Lazić et al., 2021); and financial regulations that ignore network exposures can severely misjudge systemic risk (Degryse and Nguyen, 2007; Elliott et al., 2014).

Conversely, a growing body of research shows that harnessing social networks can dramatically increase the effectiveness of public policies. Targeting the most central or influential individuals—the key players—can magnify the reach of interventions at a fraction of the cost, whether the goal is to accelerate technology diffusion or the adoption of a microfinance program (Banerjee et al., 2013), increase R&D investments (König et al., 2019), curb the spread of epidemics (Bassolas et al., 2022), reduce criminality (Ballester et al., 2006; Lee et al., 2021; Giulietti et al., 2025) or prevent contagion in financial systems (Denbee et al., 2021). By explicitly modeling and measuring real social ties, rather than assuming anonymous interactions, network economics provides a framework for designing interventions that leverage rather than fight against the structure of social influence.

These advances highlight the importance of understanding precisely how influence operates within groups and networks. To do so, the literature draws a key distinction between *peer* and *network* effects, which formalize different ways in which individuals' behaviors are shaped by others. Traditional models, such as the linear-in-means (LIM) specification, describe how an individual's outcome depends on the average behavior of a reference group—for example, the average classroom performance, neighborhood crime rate, or grade-level achievement. In these frameworks, all individuals within a group are exposed to a common peer environment, implying that group boundaries are exogenously and often arbitrarily defined, and that behavioral effects are homogeneous across members. This aggregate approach captures a uniform intra-group externality that acts equally on everyone in the group.[1]

By contrast, models of social networks allow the *structure* of interactions to matter. Agents are connected through specific and potentially asymmetric links, formalized by an adjacency matrix that captures who interacts with whom. Each individual's behavior depends not on a group average but on the weighted actions and characteristics of directly connected peers. Network models therefore generalize peer-effect models by recognizing that social influence operates at the dyadic level—through specific interpersonal ties—and can propagate across multiple degrees of separation. Building on this micro-founded perspective, *games on networks* provide a natural framework for studying how equilibrium behavior depends on the pattern of social connections and, ultimately, for interpreting estimated peer effects as equilibrium outcomes of strategic interactions.

This paper provides a comprehensive overview of the theoretical and empirical foundations of peer and network effects, aiming to unify insights from both literatures. It proceeds in four steps.

First, we develop the microfoundations of peer effects (LIM model) using linear–quadratic network games under an exogenous network structure.[2] This framework links equilibrium behavior to network centrality, illustrating how an individual's position within a graph shapes both influence and welfare. It introduces the

[1]We acknowledge that the term "peer effects" is used broadly in the literature to describe both group-average and network-based models of social interactions; see, e.g., Bramoullé et al. (2020) or De Giorgi et al. (2020), who use "peer effects" in a network context. Our use of the peer/network distinction is a *modeling* choice, not a claim that two fundamentally different causal mechanisms are at work. As we clarify in Section 2, group-based LIM models are formally a special case of network models (a block-diagonal adjacency matrix), and we use the label "peer effects" to refer specifically to that group-average special case throughout the paper.

[2]The game-theoretic foundation of the LIM model is a network framework in which each agent's best-reply function is linear in the mean action of their peers; see Patacchini and Zenou (2012), Blume et al. (2015), Boucher (2016), Ushchev and Zenou (2020), and Boucher et al. (2024). For comprehensive overviews of identification in social interaction models, see Blume et al. (2011) and Kline and Tamer (2020).

concepts of *intercentrality* and the *key-player policy*, and it connects local-average models to behavioral foundations for the linear-in-means specification.[3]

Second, we review the econometric identification challenges in the LIM framework—including the reflection problem, correlated effects, and sorting—and show how embedding an explicit network structure helps mitigate these issues. We then discuss reduced-form identification strategies based on within-school cohort composition, exposure to peers' shocks, random assignment, and exogenous variation in network links.

Third, by unifying the theoretical and the reduced-form approaches, we outline three complementary approaches to structural network formation: (i) a model jointly determining effort choices and link formation through anonymous meetings; (ii) a sequential model of link formation first and then effort choices in which agents take as given the expected effort of others when investing in their social connections; (iii) a control-function approach correcting for endogenous network formation.[4]

Fourth, we move beyond the standard linear framework by reviewing recent advances on *nonlinear* peer effects, where individuals respond selectively to particular peers—for example, to high achievers or low performers—rather than to the group mean. These models help rationalize heterogeneous social responses and motivate targeted network interventions.

Finally, we extend the analysis to *multiplex networks*, in which individuals interact across several interdependent layers (e.g., social, advice, and financial ties). We show how cross-layer complementarities or crowding-out effects can arise when agents allocate effort across activities subject to a common resource constraint, and we discuss empirical strategies to identify such interdependencies using multilayer data.

This paper is deliberately methodological. Its main goal is to synthesize and formalize the conceptual and econometric distinctions between peer and network effects, and to highlight how recent theoretical advances provide the structural foundations necessary for credible identification and policy analysis. Rather than presenting new empirical results, the contribution lies in bridging reduced-form approaches with structural network models, showing how the latter generate testable implica-

[3] For an overview of the games-on-network literature, see Jackson and Zenou (2015), Bramoullé and Kranton (2016), and Zenou (2026).

[4] There are many overviews on network formation. See, in particular, Graham (2015), Chandrasekhar (2016), and De Paula (2020).

tions and counterfactual predictions. In this sense, the paper complements rather than competes with empirical studies, offering a unifying analytical framework and a roadmap for how theory and structure can inform empirical work.

Building on this perspective, the paper also distinguishes itself from existing surveys in the literature on social interactions. Blume et al. (2015) provide a thorough treatment of identification in LIM models and discuss microfoundations, but predate the structural network-formation, nonlinear-norm, and multiplex literatures that we offer here. Boucher and Fortin (2016) similarly focus on structural estimation of network-game models, but the identification discussion is more limited and does not cover the recent quasi-experimental designs of Section 4. Bramoullé et al. (2020) explicitly frame peer effects as a special case of network models and provide an authoritative identification overview; our contribution relative to theirs lies in the explicit microfounded bridge between reduced-form estimands and structural primitives, and in the coverage of nonlinear peer effects and multiplex networks that postdate their survey. Taken together, the nonlinear and multiplex sections (Sections 6–7) are among the most novel contributions of this survey: they extend the standard linear framework in two directions—heterogeneous social norms and multilayer interactions—and show how the microfoundational approach of earlier sections scales to these richer environments.

Throughout, we emphasize that reduced-form estimands should be interpreted as functions of equilibrium objects in structural network models, thereby bridging empirical strategies and theoretical frameworks.

Guide for readers. This paper is written to be accessible to readers with different backgrounds. Sections 2 and 4 require familiarity with standard econometric identification arguments (instrumental variables, fixed effects) but no prior exposure to game theory or network analysis. Section 3 introduces network concepts—walks, adjacency matrices, centrality—from first principles; readers already comfortable with these may proceed directly to the equilibrium results. Sections 5–6 are more technical and draw on both structural econometrics and game theory. Section 7 requires familiarity with the linear-quadratic game framework of Section 3 but is otherwise self-contained.

What this survey adds. Relative to existing reviews of peer effects and networks, this article develops a *microfounded bridge* between reduced-form peer-effect specifications and *games on networks*, showing how standard linear-in-means estimands map to equilibrium outcomes in linear–quadratic network games and under what conditions those estimands admit a structural interpretation. It further *integrates identification with structure*: network-based instrumental-variable strategies are analyzed alongside the equilibrium system, clarifying which quasi-experimental designs recover which primitives. Finally, the survey moves beyond linear averages to *nonlinear social norms* and *multiplex environments* with cross-layer complementarities and crowding-out, offering a unified framework that connects theory, estimation, and *policy design*—from key-player targeting to norm-sensitive interventions.

2 Peer versus network effects: Empirical perspectives

Before proceeding, a terminological clarification is warranted. The distinction we draw between "peer effects" and "network effects" is one of *modeling granularity*, not a claim that two fundamentally different causal mechanisms are at work. As Bramoullé et al. (2020) note, group-based peer-effect models can formally be viewed as a special case of network models: the LIM specification corresponds to a fully connected, equally weighted graph within each group, with the adjacency matrix taking a block-diagonal structure in which all within-block entries equal $1/(n_r - 1)$ for group r of size n_r. Network models therefore *generalize* traditional peer-effect models by allowing richer, sparser, and potentially asymmetric definitions of peer exposure. The label "peer effects" in this survey refers to this group-average special case; "network effects" refers to the more granular dyadic approach. This framing emphasizes a continuum rather than a dichotomy, and it implies that the identification and structural results of this paper nest the classical LIM results as limiting cases.

We begin this section with the linear-in-means (LIM) framework and discuss identification challenges such as the reflection problem, correlated and contextual effects, and sorting. We then show how embedding an explicit social network structure resolves some identification difficulties and connects the econometrics of peer effects to the graph-theoretic measurement of exposure to others' outcomes and characteristics.

2.1 The Linear-in-Means (LIM) model

In the standard LIM specification, individuals are partitioned into groups r (e.g., classrooms, schools, neighborhoods), and each individual's outcome is affected by the *average* outcome and characteristics of the group. For individual i in group r, the LIM model is given by

$$y_{i,r} = \alpha + \phi\,\mathbb{E}(y_r) + \delta\,\mathbb{E}(x_r) + \gamma\,x_{i,r} + \varepsilon_{i,r}, \tag{1}$$

where $y_{i,r}$ is the outcome of interest of individual i belonging to group r (e.g., a measure of educational achievement, crime, or mental health); $x_{i,r}$ are individual covariates (e.g., parental education, gender, race); $\mathbb{E}(y_r)$ denotes the average outcome in group r; and $\mathbb{E}(x_r)$ the average of group characteristics ("contextual effects"). In the conventional interpretation, $\phi > 0$ indicates *endogenous peer effects*—the impact of peers' outcomes on one's own outcome—while $\delta > 0$ captures *exogenous/contextual effects*—the impact of peers' characteristics on one's outcome.

Applications highlight how (1) is implemented in practice. In education, $y_{i,r}$ may be test-scores or study effort for a student i in classroom r, with $\mathbb{E}(y_r)$ the classroom mean grade and $\mathbb{E}(x_r)$ the mean background characteristics. In crime, $y_{i,r}$ could be an index of criminal activity for a resident in neighborhood r, with $\mathbb{E}(y_r)$ the local crime rate and $\mathbb{E}(x_r)$ the neighborhood socioeconomic profile. In mental health, $y_{i,r}$ could be depression (binary or continuous) among adolescents in schools or grades, and $\mathbb{E}(y_r)$ the share of depressed peers.[5]

The Reflection Problem A central challenge in LIM models is Manski's *reflection problem* (Manski, 1993). Taking group means in (1) and assuming $\mathbb{E}(\varepsilon_{i,r}) = 0$—where

[5] In the LIM model, it is difficult to disentangle the underlying mechanisms driving the observed relationships. An exception is Bursztyn et al. (2014), who separately identify two channels of peer effects in financial decisions: social learning and social utility, the latter of which can be interpreted as a network effect.

$\mathbb{E}(\cdot)$ denotes the group average—yields[6]

$$\mathbb{E}(y_r) = \alpha + \phi\,\mathbb{E}(y_r) + (\delta + \gamma)\,\mathbb{E}(x_r). \tag{2}$$

Solving gives

$$\mathbb{E}(y_r) = \frac{\alpha}{1-\phi} + \frac{\delta+\gamma}{1-\phi}\,\mathbb{E}(x_r). \tag{3}$$

Substituting back into (1) produces

$$y_{i,r} = \frac{\alpha}{1-\phi} + \left[\frac{\gamma\phi+\delta}{1-\phi}\right]\mathbb{E}(x_r) + \gamma x_{i,r} + \varepsilon_{i,r}, \tag{4}$$

which makes clear that endogenous (ϕ) and contextual (δ) effects are not separately identified: three reduced-form coefficients map to four structural parameters. Identification fails because in the linear-in-means model (1), individuals simultaneously determine their behavior in response to that of their peers. This simultaneity generates *perfect collinearity* between the group's mean outcome and its mean characteristics, making it difficult to disentangle *endogenous effects*—the influence of peers' behavior—from *contextual effects*—the influence of peers' exogenous attributes (Manski, 1993), as shown in equation (4). Consequently, observed correlations in behavior may reflect either mutual influence or shared characteristics.

2.2 From peer to network effects

The peer-effect formulation treats all group members symmetrically. A network approach, by contrast, models *who influences whom*. Let $\mathbf{g}$ denote a (possibly directed) graph on n nodes with adjacency matrix $\mathbf{G} = [g_{ij}]$, and let $d_i = \sum_j g_{ij}$ be the degree of i. Row-normalization by degrees, $\widehat{\mathbf{G}} = [\widehat{g}_{ij}]$ such that $\widehat{g}_{ij} := g_{ij}/d_i$ if $d_i > 0$ and $\widehat{g}_{ij} := 0$ otherwise, implements the local-average operator. Thus, a network version of (1) is[7]

$$y_i = \alpha + \phi\sum_j \widehat{g}_{ij}\,y_j + \delta\sum_j \widehat{g}_{ij}\,x_j + \gamma x_i + \varepsilon_i. \tag{5}$$

[6]A subtle but important point: in the LIM model, $\mathbb{E}(\cdot)$ has been defined as the group-average operator, not the statistical expectation. Conditioning on (y_r, x_r) in the assumption $\mathbb{E}(\varepsilon_{i,r} \mid y_r, x_r) = 0$ would mix these two meanings, since y_r is itself a function of all $\varepsilon_{i,r}$, making mean-independence inconsistent with the model's simultaneous structure. The standard approach in this literature (following Manski 1993) is to impose $\mathbb{E}(\varepsilon_{i,r}) = 0$ and appeal to the law of large numbers for large groups to justify treating the group average as approximating the population expectation. This blurring of average and expectation is well known (see, e.g., Blume et al. 2011) and becomes exact as group size grows. We follow this convention throughout.

[7]For the sake of the exposition, we assume that each agent i has one characteristic x_i, which is clearly not true in the empirical applications. It is straightforward to include several characteristics

where ε_i's are i.i.d. innovations with zero mean and variance σ^2 for all i.[8] Now, the size of the reference group of i corresponds to their degree d_i. This *local-average* model replaces group means with averages over i's neighbors. Directed networks encode asymmetric influence, while undirected networks impose $g_{ij} = g_{ji}$.

2.3 Identification issues in network models

Empirically identifying peer effects within social networks poses three well-documented challenges. The first issue is the *reflection problem* (Section 2.1). In network models, however, this problem can be mitigated because each individual's reference group is determined by their specific network neighborhood rather than by a group-wide mean. This structure naturally generates exclusion restrictions—for instance, through intransitive triads—that help resolve the reflection problem. Therefore, unless the network is complete or highly regular, instrumenting peers' outcomes with the characteristics of friends-of-friends breaks the perfect collinearity between peers' mean outcomes and their characteristics.

A second challenge arises from *correlated effects*. Even if simultaneity is addressed, estimation may still be confounded by unobserved factors that jointly affect all members of a peer group. Local shocks, environmental factors, or shared institutional contexts may influence both an individual's outcome and those of her peers. One way to address this problem is to exploit the *architecture of network connections* to construct valid instrumental variables (IVs) for the endogenous peer effect. Because peer groups are individual-specific, characteristics of *indirect friends*—for example, the attributes of one's friends-of-friends—can serve as natural instruments, provided they are correlated with peers' behavior but uncorrelated with one's own unobservables. This would work assuming that indirect friends are not subjected to

of i by replacing δx_i by $\mathbf{x}_i^T\boldsymbol{\delta}$, where $\mathbf{x}_i^T$ is a $(1 \times k)$ vector of k observable characteristics ($\mathbf{x}_i^T$ is the transpose of $\mathbf{x}_i$) and $\boldsymbol{\delta}$ is a $(k \times 1)$ vector of parameters. Denote $\bar{x}_{-i} = \sum_j \widehat{g}_{ij} x_j$ and the corresponding $(1 \times k)$ vector by $\bar{\mathbf{x}}_{-i}$, which is the vector of k average exogenous peer characteristics of i's neighbors (contextual variables), such as the average age and the share of girls within peers. Then, for the contextual effect term, we can replace $\delta \sum_j \widehat{g}_{ij} x_j$ by $\bar{\mathbf{x}}_{-i}^T \boldsymbol{\delta}$.

[8] The i.i.d. and homoscedasticity assumptions on ε_i are imposed here for expositional simplicity and are not required for the identification results that follow. Bramoullé et al. (2009) establish identification under the weaker assumption $\mathbb{E}[\boldsymbol{\varepsilon} \mid \widehat{\mathbf{G}}, \mathbf{x}] = \mathbf{0}$, allowing for arbitrary heteroscedasticity and cross-sectional dependence conditional on the network and covariates. Allowing for network fixed effects, as discussed below, further relaxes this to controlling for group-level unobservables.

the same unobserved shock (e.g., the same school but not the same classroom).

To illustrate, consider equation (5). Stacking observations within each network yields the matrix form:

$$\mathbf{y} = \alpha\mathbf{1} + \phi\,\widehat{\mathbf{G}}\mathbf{y} + \delta\,\widehat{\mathbf{G}}\mathbf{x} + \gamma\,\mathbf{x} + \boldsymbol{\varepsilon}, \tag{6}$$

with $\mathbb{E}\left[\varepsilon \mid \widehat{\mathrm{G}}, \mathrm{x}\right] = 0$. Equation (6) resembles a spatial autoregressive (SAR) model, which is identified if and only if $\mathbb{E}\left[\widehat{\mathrm{G}}\mathrm{y} \mid \mathrm{x}\right]$ is not perfectly collinear with the regressors $(\mathrm{x}, \widehat{\mathrm{G}}\mathrm{x})$, allowing instruments to be constructed for the endogenous term $\widehat{\mathrm{G}}\mathrm{y}$.

As shown by Bramoullé et al. (2009), a sufficient condition for identification of the endogenous peer effect ϕ in (5) is that I_n, $\widehat{\mathrm{G}}$, and $\widehat{\mathrm{G}}^2$ be linearly independent, which holds generically in *partially overlapping* networks—those in which some agents are not linked to their friends' friends.[9] Allowing network structure to vary across individuals thus breaks the symmetry underlying the reflection problem and enables separate identification of endogenous and contextual effects. Identification proceeds by instrumenting $\widehat{\mathrm{G}}\mathrm{y}$ with exogenous network-based functions such as x, $\widehat{\mathrm{G}}\mathrm{x}$, and $\widehat{\mathrm{G}}^2\mathrm{x}$. The variables x and $\widehat{\mathrm{G}}\mathrm{x}$ appear as regressors in (6) and are therefore included instruments; the key *excluded* instrument is $\widehat{\mathrm{G}}^2\mathrm{x}$—the characteristics of friends-of-friends—which is correlated with $\widehat{\mathrm{G}}\mathrm{y}$ through the equilibrium but excluded from the outcome equation by the assumption that indirect peers' characteristics affect i's outcome only through their effect on direct peers' behavior.

A third challenge concerns *sorting*. Because individuals often choose their peers, unobserved traits correlated with both link formation and outcomes can generate endogeneity. For instance, individuals with similar unobserved preferences or abilities may sort into the same networks, biasing estimates of peer effects. Introducing *network fixed effects*, as in Bramoullé et al. (2009), helps control for such unobserved heterogeneity by absorbing factors common to a given network or subnetwork, thereby mitigating selection bias due to assortative matching on unobservables. A

[9] See also Lee (2007)'s approach to identification in group-based interactions, where variation in group size solves the reflection problem. De Giorgi et al. (2020) exploit the partial-overlap structure of peer groups directly, using the fact that students share some but not all classmates across courses; their strategy is an early empirical application of the Bramoullé et al. (2009) identification logic and illustrates how partially overlapping peer groups can resolve the reflection problem even without an explicit network. It should be noted that Calvó-Armengol et al. (2009) study a related but distinct model—a local-aggregate game in which outcomes depend on the *sum* (not the average) of peers' efforts, without a contextual effect term.

sufficient condition for identification is that $\mathbf{I}_n$, $\widehat{\mathrm{G}}$, $\widehat{\mathrm{G}}^2$, and $\widehat{\mathrm{G}}^3$ be linearly independent (Bramoullé et al., 2009).[10]

Finally, these identification issues become even more pronounced once network formation is explicitly modeled. The standard *Local Average Model* with network fixed effects η, that is, adding η in equation (6), typically assumes that the network $\widehat{\mathrm{G}}$ is *conditionally exogenous*, i.e., exogenous given observable individual characteristics. This assumption is often untenable in practice. For example, in observational data on farmers' adoption of a new, risky technology, we might observe that more connected farmers are more likely to adopt, but this pattern need not imply causality. It may simply reflect that more risk-loving individuals—who are inherently more likely to adopt—also tend to be more sociable and, hence, more connected. In such cases, the exogeneity condition

$$\mathbb{E}\left[\boldsymbol{\varepsilon} \mid \widehat{\mathrm{G}}, \mathbf{x}\right] = 0, \quad \forall i, \tag{7}$$

is violated, and causal interpretation requires either exogenous variation in the network or structural modeling of the link-formation process.

3 Network games as a foundation for linear peer effects

We have seen that the standard peer effects (LIM) model, defined by equation (1), studies the impact of the average behavior of peers on own behavior. The corresponding network model, defined by (5), proposes a more detailed view of peer effects in which the *structure* of interactions matters: agents are connected through specific and potentially asymmetric links, formalized by an adjacency matrix $\mathbf{G} = (g_{ij})$. Each individual's behavior depends not on a group average but on the weighted actions and characteristics of directly connected peers.

[10] A fourth source of bias is *exclusion bias*, which arises when individuals are randomly grouped within selection pools (e.g., classrooms, tournaments) and pool fixed effects are included. It stems from the mechanical exclusion of an individual from their own peer average. Traditional estimators—whether based on peers-of-peers' instruments (Bramoullé et al., 2009) or variation in group size (Lee, 2007; Graham, 2008)—fail with fixed, non-overlapping groups, where no valid instruments or sufficient size variation exist. Spatial ML approaches (Anselin, 1988; Drukker et al., 2013) estimate peer effects from outcome covariances but remain biased in this case. Caeyers and Fafchamps (2025) develop a new estimator that corrects for exclusion bias, allowing consistent estimation even with fixed, non-overlapping peer groups.

What is the microfoundation of (5)? *Games on networks* provide a natural framework for studying how equilibrium behavior depends on the *structure* of social connections. Individuals interact strategically with their neighbors, choosing actions that depend on the choices of those to whom they are linked. These interactions can exhibit either *strategic complementarities*—where one's incentives to increase effort rise when peers increase theirs—or *strategic substitutes*, where the opposite holds. Examples of complementarities include education, crime, or drug use, where imitation or reinforcement amplifies behaviors within a network; substitutes arise in contexts such as public good provision or technology adoption, where others' actions reduce one's incentive to contribute or adopt.

Given that our focus is on peer effects, we will concentrate on games with strategic complementarities.

3.1 Linear–quadratic network games: Local aggregate

Consider a game with n agents, each choosing an effort level $y_i \in \mathbb{R}_+$ in some activity. Let $\mathbf{y} = (y_1, \dots, y_n)^\top$ denote the column vector of individual efforts. Agents are embedded in a *network* g, represented by its $n \times n$ adjacency matrix $\mathbf{G}$, whose element g_{ij} indicates whether i and j are connected (and possibly the strength of their link). The utility of agent i is

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha_i y_i - \frac{1}{2} y_i^2 + \phi \sum_{j=1}^{n} g_{ij} y_i y_j, \tag{8}$$

where $\phi > 0$. Agents differ in observable characteristics $\alpha_i > 0$ and in their network positions. The first two terms, $\alpha_i y_i - \frac{1}{2} y_i^2$, represent private benefits and costs of effort, independent of others. The last term, $\phi \sum_j g_{ij} y_i y_j$, captures *strategic complementarities* between connected agents. If $g_{ij} = 1$, actions are strategic complements since $\partial^2 u_i / (\partial y_i \partial y_j) = \phi > 0$; if $g_{ij} = 0$, the cross-effect vanishes. We impose $g_{ii} = 0$.

Katz–Bonacich Centrality The *Katz–Bonacich centrality*, introduced by Katz (1953) and extended by Bonacich (1987), measures a node's importance by counting *all walks* emanating from it, discounting longer walks by a factor of ϕ^k per step. Intuitively, an individual is central if she has many direct friends, and even more so if those friends are themselves well-connected, and so on recursively. In a social-interaction context, an agent with high Katz–Bonacich centrality exerts influence not only on her immediate peers but also, at diminishing strength, on peers-of-peers throughout the network.

Before defining Katz–Bonacich centrality, it is helpful to introduce the key building block: the notion of a *walk*. A walk of length k from node i to node j is a sequence of k edges connecting i to j, possibly revisiting nodes. The (i, j) entry of $\mathbf{G}^k$ counts the number of such walks of length k; in particular, $g_{ij}^{[1]} = g_{ij}$ (direct links), $g_{ij}^{[2]}$ counts common neighbors of i and j, and so on. Walks capture how influence can propagate indirectly: even if i and j are not directly connected, j's action reaches i through chains of mutual acquaintances.

Then, the weighted Katz–Bonacich centrality vector is

$$\mathbf{b}_{\boldsymbol{\alpha}}(\mathbf{G}, \phi) = \mathbf{M}(\mathbf{G}, \phi)\boldsymbol{\alpha} = (\mathbf{I} - \phi\mathbf{G})^{-1}\boldsymbol{\alpha} = \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k \boldsymbol{\alpha}, \tag{9}$$

where the ith component sums, over all walk lengths k, the total "productive capacity" α_j of all agents j reachable from i in k steps, weighted by ϕ^k. Two forces determine centrality: the structural position of i in the network (how many walks of each length start from i) and the heterogeneity in intrinsic productivity α_j of agents reachable from i.

The series converges if and only if $\phi\mu_1(\mathbf{G}) < 1$, where $\mu_1(\mathbf{G})$ is the largest eigenvalue (spectral radius) of G. This condition has a natural economic interpretation: it requires that the complementarity parameter ϕ be sufficiently small relative to the network's connectivity. If ϕ were too large, each agent's best-response effort would trigger ever-stronger responses from peers, which would feed back into yet-higher effort levels, spiralling without bound. The spectral condition rules out this divergence and guarantees that a finite equilibrium exists. In practice, $\mu_1(\mathbf{G})$ is determined by how densely and evenly connected the network is; for a k-regular graph (every node has exactly k friends), $\mu_1 = k$, so the condition reduces to $\phi < 1/k$.

Strategic complementarities vs. substitutes: scope of the analysis The utility function in equation (8) imposes $\phi > 0$, reflecting *strategic complementarities*: each agent's incentive to exert effort rises when her peers do the same. This is the relevant case for education, crime, and R&D—activities where imitation or reinforcement amplifies peer behavior—and it is where the theoretical literature is most fully developed. The framework can in principle accommodate *strategic substitutes* ($\phi < 0$), which arise when peer effort reduces one's own marginal returns: classic examples include public-good provision (Bramoullé and Kranton, 2007), information acquisition, and certain peer-tutoring settings. Carrell et al. (2013), for instance, document a case where high-ability students are randomly mixed with low-ability peers at the U.S. Air Force Academy, and contrary to the prediction of a complementarities model, the lowest-ability students *deteriorate*: high-ability students sort away from them rather than tutoring them, a finding more consistent with strategic substitutes or heterogeneous peer-matching. Such contexts fall partly outside the framework developed here, and the equilibrium characterization and policy prescriptions differ substantially from the complementarities case; we refer the reader to Bramoullé and Kranton (2016) and Jackson and Zenou (2015) for overviews of games with strategic substitutes on networks. Throughout the remainder of the paper, we maintain $\phi > 0$ and the spectral condition $\phi\mu_1(\mathbf{G}) < 1$.

Nash Equilibrium Each agent i chooses $y_i \geq 0$ to maximize $u_i(\mathbf{y}, \mathrm{g})$. The first-order condition is

$$y_i = \alpha_i + \phi \sum_{j=1}^{n} g_{ij} y_j. \tag{10}$$

Let $\mu_1(\mathbf{G})$ denote the largest eigenvalue of $\mathbf{G}$. Ballester et al. (2006) show that if $\phi\mu_1(\mathbf{G}) < 1$, the game admits a unique interior Nash equilibrium,

$$\mathbf{y}^* = \mathbf{b}_{\boldsymbol{\alpha}}(\mathbf{G}, \phi). \tag{11}$$

This equilibrium embodies *social multiplier* effects, where network interactions amplify individual efforts. Consider two symmetric agents, 1 and 2, with $\alpha_1 = \alpha_2 = \alpha$. If $g_{12} = g_{21} = 0$, equilibrium efforts are $y_1^* = y_2^* = \alpha$. When they are linked ($g_{12} = g_{21} = 1$) and $\phi < 1$, equilibrium becomes

$$y_1^* = y_2^* = \frac{\alpha}{1 - \phi}. \tag{12}$$

Strategic complementarities thus raise effort above the autarky level. The factor $(1-\phi)^{-1} > 1$ quantifies this amplification—the *social multiplier*. Estimating ϕ empirically is central to measuring peer effects (see Section 2). For instance, if $\phi = 0.5$, the multiplier equals 2. In a crime context, an individual who would commit α crimes alone will commit 2α when paired with another offender, solely due to mutual influence rather than personal traits.[11]

[11] See Glaeser et al. (1996, 2003) for theoretical and empirical analyses of the social multiplier in crime.

Eigenvector Centrality A related and widely used measure is *eigenvector centrality*, which assigns higher influence to agents connected to highly connected peers. Golub and Lever (2010) and Zenou (2025) show that eigenvector centrality emerges as the limiting case of Katz–Bonacich centrality. Let $\bar{\phi} = 1/\mu_1(\mathbf{G})$. As $\phi \to \bar{\phi}^-$,

$$\lim_{\phi \to \bar{\phi}^-} \frac{\mathbf{b}(\mathbf{G}, \phi)}{B(\mathbf{G}, \phi)} = \mathbf{e}(\mathbf{G}),$$

where $B(\mathbf{G}, \phi)$ is the sum of all entries of $\mathbf{b}(\mathbf{G}, \phi)$ and $\mathbf{e}(\mathbf{G})$ is the nonnegative right eigenvector of $\mathbf{G}$. Hence, for any i, j,

$$\lim_{\phi \to \bar{\phi}^-} \frac{y_i(\boldsymbol{\alpha}; \phi)}{y_j(\boldsymbol{\alpha}; \phi)} = \frac{e_i(\mathbf{G})}{e_j(\mathbf{G})}. \tag{13}$$

As highlighted by Zenou (2025), this has important implications for empirical tests of peer effects. As shown in Section 4, identifying ϕ in a causal manner is challenging, which makes it difficult to estimate the effect of individual Katz–Bonacich centrality on outcomes, as predicted by Ballester et al. (2006) (see equation (11)). By contrast, when spillovers are very large (i.e., $\phi \to \bar{\phi}^-$), one no longer needs to estimate ϕ: the relevant measure becomes eigenvector centrality—a parameter-free centrality—whose effect on outcomes follows directly from (13).

A caveat is warranted: as $\phi \to \bar{\phi}^-$, the *level* of aggregate equilibrium effort $Y^* = \sum_i y_i^* = \mathbf{1}^\top \mathbf{b}_{\boldsymbol{\alpha}}(\mathbf{G}, \phi)$ diverges to infinity, since the Leontief inverse $(\mathbf{I} - \phi\mathbf{G})^{-1}$ is not bounded as $\phi \to \mu_1(\mathbf{G})^{-1}$. The eigenvector-centrality result in equation (13) is therefore a statement about the *relative* effort of agents—the cross-sectional ranking converges to the eigenvector regardless of the level—not a literal equilibrium that can be observed in finite-sample data. The practical implication is that eigenvector centrality is most useful as an empirical proxy when ϕ is large but below $\bar{\phi}$, where the spectral condition is still satisfied but Katz–Bonacich centrality is approximately proportional to the eigenvector, so that the researcher need not estimate ϕ to rank agents by influence. This use is valid and empirically well-motivated (Zenou, 2025); it simply requires care not to interpret the limit literally as a feasible equilibrium.

Welfare The Nash equilibrium in network games with strategic complementarities is generally inefficient because individuals neglect the positive externalities that their efforts exert on others. As a result, equilibrium effort levels are below the social optimum. Assume for simplicity that $\alpha_i = \alpha$ for all i. Following Helsley and Zenou (2014), the equilibrium welfare is

$$\mathcal{W}(\mathbf{x}^*, \mathbf{g}) = \tfrac{1}{2}\, \mathbf{b}_1^\top(\mathbf{g}, \phi)\, \mathbf{b}_1(\mathbf{g}, \phi)\,, \qquad \mathbf{b}_1(\mathbf{g}, \phi) = (\mathbf{I} - \phi\mathbf{g})^{-1}\mathbf{1}, \tag{14}$$

while the social planner chooses x to maximize aggregate welfare, leading to the optimal effort profile

$$\mathbf{x}^O = \alpha(\mathbf{I} - 2\phi\mathbf{g})^{-1}\mathbf{1} = \alpha\,\mathbf{b_1}(\mathbf{g}, 2\phi). \tag{15}$$

Since $\mathbf{x}^O > \mathbf{x}^*$, each individual exerts too little effort at equilibrium. A Pigouvian subsidy can restore efficiency. If each individual receives a per-effort subsidy

$$s_i = \phi \sum_j g_{ij} x_j^O, \tag{16}$$

then $\mathbf{x}^O$ becomes a Nash equilibrium. The optimal subsidy is increasing in network centrality, implying that more central agents should be subsidized more, as their actions generate stronger positive spillovers throughout the network.

Targeting and key players An important policy question in networked environments with strategic complementarities is how to optimally target individuals whose removal or intervention most effectively reduces overall activity. Assuming a fixed network g, the *key player policy* aims to identify the individual whose elimination leads to the largest decrease in total equilibrium activity, defined as $Y^*(\mathbf{g}) = \sum_{i=1}^n y_i^*$, where y_i^* denotes the Nash equilibrium effort defined in (11). Under the condition $\phi\mu_1(\mathbf{G}) < 1$, the *intercentrality* or *key-player centrality* of individual i is defined as[12]

$$d_i(\mathbf{G}, \phi) = \frac{b_{\alpha_i}(\mathbf{G}, \phi)\, b_{1_i}(\mathbf{G}, \phi)}{m_{ii}}, \tag{17}$$

where $\mathbf{M}(\mathbf{G}, \phi) = (\mathbf{I} - \phi\mathbf{G})^{-1}$ and m_{ii} denotes its ith diagonal element. Ballester et al. (2006) show that the key player is the individual with the highest intercentrality, that is, $i^* = \arg\max_i d_i(\mathbf{G}, \phi)$.

Intercentrality measures both an individual's own centrality and her contribution to the centrality of others. Hence, the key player is not necessarily the most central node but rather the one whose position amplifies activity throughout the network. In empirical applications, such as criminal networks, removing or rehabilitating the key player can substantially reduce aggregate delinquent behavior (Lee et al., 2021; Lindquist et al., 2024; Giulietti et al., 2025). When the interaction strength ϕ is low, the most central agent (by Bonacich centrality) often coincides with the key player, but as ϕ increases, the two may diverge—highlighting the nonlinear relation between structural position and systemic impact. This framework provides a microfounded

[12]For an overview on key player policies, see Zenou (2016).

rationale for targeted network interventions in settings such as crime prevention, education, or epidemic control.[13]

3.2 Linear-quadratic network games: Local average

We now show that the game-theoretic foundation of the linear-in-means (LIM) model corresponds to a network setting in which each agent's best-response function is linear and depends on the average action of their peers. In particular, a variation of the first-order condition in the Ballester et al. (2006) (BCZ) model leads naturally to a LIM formulation.

Recall that the first-order condition of the BCZ model is given in equation (10). This formulation is often referred to as the *local aggregate model* (Liu et al., 2014), as each agent responds to the *sum* of efforts of their direct neighbors. The LIM model can be written as:

$$y_i = \alpha_i + \phi \sum_{j=1}^{n} \widehat{g}_{ij} y_j, \tag{18}$$

where, as above, $\widehat{g}_{ij} := g_{ij}/d_i$ and $\alpha_i = \mathbf{x}_i^T \boldsymbol{\delta} + \varepsilon_i > 0$, with $\mathbf{x}_i$ being a $(k \times 1)$ vector of observable characteristics, $\boldsymbol{\delta}$ a $(k \times 1)$ coefficient vector, and ε_i an unobservable individual-specific component. The term $\sum_{j=1}^{n} \widehat{g}_{ij} y_j$ represents the *average effort* of individual i's neighbors leaving out i.

Comparing equations (10) and (18), we see that the key distinction lies in the peer component: the BCZ model uses the *aggregate* peer effort $\sum_{j=1}^{n} g_{ij} y_j$, while the LIM model uses the *average* peer effort $\sum_{j=1}^{n} \widehat{g}_{ij} y_j$. This distinction reflects different assumptions about how individuals process information from their social environment—either by summing or averaging the actions of their peers.

3.2.1 Microfoundations of the LIM model

Boucher et al. (2024) demonstrate that two distinct models can serve as microfoundations for the linear-in-means (LIM) model.[14] The first is the *spillover model*, which closely resembles the BCZ framework but replaces the aggregate peer effort with the average. The utility function in this case is given by:

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha_i \, y_i - \frac{1}{2} y_i^2 + \phi \sum_{j=1}^{n} \widehat{g}_{ij} y_i y_j. \tag{19}$$

[13] For a theoretical analysis on targeting based on welfare and subsidies, see Galeotti et al. (2020).

[14] See also Blume et al. (2015) and Boucher and Fortin (2016) for an earlier treatment.

The second is the *conformist model*, originally introduced by Akerlof (1997) and later developed in network settings by Patacchini and Zenou (2012), Boucher (2016), and Ushchev and Zenou (2020). In this model, individuals derive disutility from deviating from the average behavior of their peers. The utility function takes the form:

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha_i y_i - \frac{1}{2} y_i^2 - \frac{\phi}{2} \left(y_i - \sum_{j=1}^{n} \widehat{g}_{ij} y_j \right)^2 . \tag{20}$$

It is straightforward to verify that, under suitable normalization or variable transformations, the first-order conditions derived from both models yield the LIM model as defined in equation (18). These microfoundations help ground the LIM specification in behavioral principles—either as the result of strategic complementarities in average peer effort (spillover) or from a preference for conformity (conformist).

We can embed the two models together to obtain the following utility function:

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha_i y_i + \phi_1 y_i \sum_{j=1}^{n} \widehat{g}_{ij} y_j - \frac{1}{2} \left(y_i^2 + \phi_2 \left(y_i - \sum_{j=1}^{n} \widehat{g}_{ij} y_j \right)^2 \right) . \tag{21}$$

Let $\lambda_1 := \frac{\phi_1}{1+\phi_2}$ and $\lambda_2 := \frac{\phi_2}{1+\phi_2}$. The best-reply function of individual i is then given by:

$$y_i = (1 - \lambda_2)\alpha_i + (\lambda_1 + \lambda_2) \sum_{j=1}^{n} \widehat{g}_{ij} y_j . \tag{22}$$

Do the three utility functions represent different preferences? The spillover model (19), the conformist model (20), and their combination (21) are *not* different functional forms of the same underlying preferences; they represent genuinely distinct behavioral motivations that happen to generate the same reduced-form best-reply function.

In the spillover model, the agent values interactions with peers *directly*: the cross-term $\phi\, y_i \sum_j \widehat{g}_{ij} y_j$ captures a positive production externality—exerting effort is more rewarding when connected peers also exert effort. Doubling all peer efforts while holding one's own effort constant raises utility by $\phi\, y_i \cdot \Delta \bar{y}_{-i}$, a pure externality benefit.

In the conformist model, the agent derives disutility from *deviation*: the term $-\frac{\phi}{2}(y_i - \bar{y}_{-i})^2$ is a social-distance cost, not a productivity gain. An agent with very high intrinsic motivation α_i will *reduce* effort toward the peer average to minimize this cost—a prediction with no analogue in the spillover model, where high α_i always leads to higher equilibrium effort. The two models therefore make different predictions about how an agent responds to a peer whose behavior is very far from the group norm, and about the welfare effects of increasing social pressure (ϕ_2).

The combination (21) nests both motives, with ϕ_1 governing the spillover intensity and ϕ_2 the taste for conformity. Since both models yield the same linear best-reply function (37), they are *observationally equivalent* with respect to the reduced-form peer-effect coefficient $\lambda = \lambda_1 + \lambda_2$. Distinguishing them empirically requires either identifying variation in ϕ_1 and ϕ_2 separately—for example, through experimental manipulations of social norms (Krupka and Weber, 2013)—or extending the framework to nonlinear norms as in Section 6, where the curvature parameter β breaks this equivalence.

Non-uniqueness of microfoundations. A linear best-response equation of the form $y_i = a_i + \lambda \sum_j \widehat{g}_{ij} y_j$ is consistent with a wide class of utility functions beyond (21). Any utility function whose cross-partial $\partial^2 u_i/(\partial y_i \partial \bar{y}_{-i})$ is constant and positive will generate a linear best-response, including asymmetric payoffs, non-separable preferences over own and peer effort, and models with higher-order peer interactions that happen to aggregate linearly. Equation (21) is a natural and tractable representative of this class—it nests the two most commonly studied motives (spillovers and conformity) and admits a clean characterization of equilibrium—but it should not be understood as the unique or canonical microfoundation for the LIM.

Identifiability of microfoundations. This non-uniqueness raises an important empirical question: are the structural parameters (ϕ_1, ϕ_2) of (21)—or equivalently the composite parameters (λ_1, λ_2)—separately identifiable from reduced-form peer-effect estimates? The answer is generally no without additional restrictions. The best-reply function (37) depends only on the composite parameter $\lambda = \lambda_1 + \lambda_2$; separately identifying the spillover intensity ϕ_1 from the conformity weight ϕ_2 requires either additional moment conditions (e.g., from the variance of outcomes across network positions) or experimental variation that independently shifts the two motives. Boucher et al. (2024) make progress on this by using the nonlinear norm parameter β (Section 6) as a diagnostic: the curvature of the social-norm function breaks the observational equivalence between the spillover and conformist models, allowing the structural parameters to be separately estimated via GMM under suitable moment conditions. Outside this nonlinear extension, the identifiability of microfoundations from LIM estimates remains an open and important question for the literature.

Incorporating contextual effects. A careful reader will notice that the best-reply functions derived from the spillover model (19), the conformist model (20), and their combination (21) all yield a LIM specification *without* contextual effects: the equilibrium effort of agent i depends on the average effort of her peers, but not on their observable characteristics x_j. Yet the empirical model of interest—equation (5)—includes a contextual term $\delta \sum_j \widehat{g}_{ij} x_j$. This gap deserves explicit acknowledgment.

Contextual effects can be incorporated into the structural framework by allowing the standalone productivity parameter α_i to depend on neighbors' characteristics. Specifically, replace α_i in equation (21) with

$$\alpha_i = \gamma x_i + \delta \sum_{j=1}^{n} \widehat{g}_{ij} x_j + \varepsilon_i, \tag{23}$$

so that individual i's private marginal benefit of effort depends on her own characteristics *and* those of her neighbors. Substituting into the best-reply function (37) then recovers the full empirical specification (5) with both endogenous peer effects (slope on $\sum_j \widehat{g}_{ij} y_j$) and contextual effects (slope on $\sum_j \widehat{g}_{ij} x_j$).

This extension is straightforward algebraically, but it carries an important interpretive caveat. When α_i absorbs contextual effects through (23), the contextual coefficient δ is no longer a structural parameter of the game's payoff function in the same sense as the complementarity ϕ; rather, it enters through the type distribution. Two implications follow. First, empirical evidence of large and significant $\hat{\delta}$ is fully consistent with the LQ game framework—it merely indicates heterogeneity in types that correlates with peers' characteristics. Second, and more substantively, *testing* whether $\delta = 0$ is feasible even when δ is not point-identified alongside ϕ: when the researcher observes multiple predetermined peer characteristics $\mathbf{x}_i = (x_i^{(1)}, \ldots, x_i^{(K)})$, the over-identifying restrictions implied by the SAR system (6) provide a standard test of the joint null $H_0 : \boldsymbol{\delta} = \mathbf{0}$ (Bramoullé et al., 2009). Rejection of this null—as is common in education applications where gender composition predicts test scores conditional on own score—indicates that the game involves richer channels than effort complementarities alone, motivating extensions of the structural framework.

We now present a series of reduced-form solutions to address the endogeneity of networks.[15] These approaches are not based on explicit theoretical models of network formation; such structural solutions are discussed in Section 5.

[15] A recent contribution by De Paula et al. (2025) addresses identification in the LIM model without observing the network. They show how social networks can be identified from observational panel data that contain no explicit information on social ties between agents.

4 Reduced-form solutions to identification issues

This section reviews reduced-form empirical strategies used to estimate peer effects and clarifies the connection between these approaches and the structural model in Section 5. A key message is that each empirical specification implicitly targets a well-defined estimand, which can be interpreted as either a reduced-form function (this section) or a structural parameter (Section 5) of the underlying network model.

We now present four complementary reduced-form strategies for identifying peer effects when network structure, treatment assignment, or exogenous shocks provide quasi-experimental variation.

4.1 Instrumental variable approach (cohorts)

A common empirical approach to solve the identification issues of network formation for which the exogeneity condition (7) does not hold is the cohort instrumental-variables strategy (e.g., Hoxby, 2000; Bifulco et al., 2011; Lavy and Schlosser, 2011; Patacchini and Zenou, 2016; Olivetti et al., 2020; Giulietti et al., 2022; Merlino et al., 2019, 2024), which estimates *contextual (composition) effects* by exploiting quasi-random variation in peers' *pre-determined traits* across cohorts within the same school.

Let i index individuals in school s, grade g, and cohort/year t. Let x_i be individual covariates measured pre-treatment and let z_j be a peer trait measured pre-treatment (e.g., language proficiency at entry). Define the pre-treatment, leave-one-out peer composition for individual i as (assuming $g_{ii}^{(0)} = 0$)

$$m_{it} = \sum_j \hat{g}_{ij}^{(0)} z_j. \tag{24}$$

One can estimate the contextual effect of peer composition via

$$y_{it} = \alpha + \beta m_{it} + \gamma' x_i + \eta_{sg} + \eta_t + \varepsilon_{igt}, \tag{25}$$

where η_{sg} are school-by-grade fixed effects and η_t are cohort fixed effects.

Identification requires that, conditional on η_{sg} and η_t, the within-school cohort composition is as-good-as-random—i.e., residual differences in m_{it} across cohorts reflect quasi-random cohort mix rather than sorting on unobservables. Under this assumption, β captures the causal effect of exposure to peers' *traits*. This estimand is a contextual effect, it does not by itself identify the endogenous peer effect in the linear-in-means model. The key assumption is that parents and students sort across schools based on average school traits, not on the precise demographic mix of an entering cohort, which is typically unknown at choice time; thus, within-school differences in cohort shares (e.g., gender, race, ability) shift peers' characteristics exogenously.

If the object of interest is the endogenous peer effect—the impact of peers' outcomes—then cohort-mix measures constructed from pre-treatment variables can instrument the contemporaneous peer mean outcome. In practice, the cohort-share measure m_{it} can be used as an instrument for the contemporaneous peer mean outcome $\bar{y}_{it} = \sum_j \widehat{g}_{ij}^{(0)} y_{jt}$ in a two-stage least squares specification with school×grade and year fixed effects.

Structural interpretation. What does $\hat{\beta}$ (or the IV estimate of ϕ obtained by using m_{it} as an instrument) recover within the structural Linear-Quadratic (LQ hereafter) network game of Section 3? At a Nash equilibrium of the local-average game, individual effort satisfies $y_i = \alpha_i + \phi \sum_j \widehat{g}_{ij} y_j$ (equation (18)). Solving for the equilibrium vector yields $\mathbf{y}^* = (\mathbf{I} - \phi\widehat{\mathbf{G}})^{-1}\boldsymbol{\alpha}$, so the reduced-form coefficient on average peer characteristics, β, captures the product of ϕ and the (i, j) element of the Leontief inverse $(\mathbf{I} - \phi\widehat{\mathbf{G}})^{-1}$, summed over i's cohort peers. In other words, $\hat{\beta}$ is a *reduced-form convolution* of the structural complementarity ϕ and the network-weighted exposure to pre-treatment peer traits; it does *not* separately identify ϕ or any single entry of G. To recover ϕ alone—the structural parameter of direct policy relevance for computing the social multiplier or the key-player centrality—one needs either the full network matrix $\widehat{\mathrm{G}}$ (as in Section 2) or a structural approach (Section 5) that imposes the equilibrium restrictions of the LQ game.

Relation to intent-to-treat estimation. It is worth noting that the cohort-based peer-composition measure m_{it} can also be interpreted as an *intent-to-treat* (ITT) estimator. Rather than recovering the structural peer-effect parameter ϕ, the reduced-form regression of y_{it} on m_{it} (without a second stage) estimates the reduced-form effect of being randomly assigned to a cohort with a given peer composition. ITT estimates are often preferred by applied researchers because they are robust to violations of the exclusion restriction and do not require the monotonicity assumption needed for a LATE interpretation. In settings where the first stage is weak or the instrument only imperfectly shifts the peer environment, ITT may be the most credible object to report. The tradeoff is that ITT estimates are harder to interpret structurally: they blend the direct effect of peer characteristics (contextual effect δ) with any indirect effect operating through changes in peer outcomes (endogenous effect ϕ), without separating the two. The structural framework of Section 3.1 clarifies this blend: the ITT coefficient on m_{it} equals $(\delta + \phi\hat{\pi})/(1-\phi)$, where $\hat{\pi}$ is the first-stage coefficient of peer outcomes on peer characteristics.

4.2 Random assignments

In this section, we first discuss research in which network formation itself is randomized. In the second part, we examine field experiments based on random assignments (such as RCTs) where the network is fixed, but omitted-variable bias may still arise and require appropriate correction.

4.2.1 Randomization that affects network formation

A way to address the issue of endogenous network formation (or sorting) is through field experiments based on random assignments. For example, Mas and Moretti (2009) exploit the quasi-random exposure of cashiers to highly productive co-workers arising from shift-based register placement. They show that individual productivity increases when workers are observed by top performers. Similarly, Carrell et al. (2009) estimate peer effects in college achievement using a data set from the U.S. Air Force Academy in which individuals are exogenously assigned to peer groups of about 30 students with whom they are required to spend the majority of their time interacting. They find long lasting peer effects on academic achievement.[16]

Following Algan et al. (2026), let y_{ij} denote the absolute pairwise difference in an outcome (e.g., grades or political opinions) between individuals i and j, and let $g_{ij} \in \{0,1\}$ indicate (undirected) friendship. A baseline dyadic regression

$$y_{ij} = \alpha_1 + \phi_1 g_{ij} + \delta_1 x_{ij} + \varepsilon_{ij},$$

with x_{ij} capturing common and differential predetermined traits (e.g., same gender, parental education/income, residential proximity), targets the average causal effect of friendship on outcome convergence, $\phi_1 \equiv \mathbb{E}[y_{ij} \mid g_{ij} = 1, x_{ij}] - \mathbb{E}[y_{ij} \mid g_{ij} = 0, x_{ij}]$. However, homophily renders g_{ij} endogenous, risking attribution of similarity to influence rather than selection.

[16]Numerous experiments explicitly manipulate the network structure to assess its impact on individual and collective behavior. See, for instance, Rand et al. (2011) on inducing cooperation, Rand et al. (2014) on inducing public goods production, and Shirado et al. (2019) on affecting collective welfare.

Concretely, $\mathbf{x}_{ij}$ is a vector of dyadic covariates that typically includes both indicator variables for shared characteristics (e.g., $\mathbf{1}$[same gender], $\mathbf{1}$[same nationality]) and continuous measures of difference (e.g., the absolute gap in parental income or distance between home addresses). This distinction matters: shared-characteristic indicators capture homophily along discrete dimensions, while difference measures capture distance along continuous ones, and the two enter the link-formation and outcome equations in potentially different ways.

To address endogeneity, we can instrument friendship with *exogenous treatment* or random assignment. Specifically, let $T_{ij} = 1$ if i and j are assigned to the same treatment. In Algan et al. (2026), the treatment is that students i and j are randomly allocated (based on alphabetical order) to the same integration group (IG_{ij}) during a pre-term "integration week" before starting university (Sciences Po). The first stage,

$$g_{ij} = \alpha_2 + \phi_2 T_{ij} + \delta_2 x_{ij} + \epsilon_{ij},$$

exploits that T_{ij} shifts the probability of becoming friends but—by design—does not directly affect later outcomes, satisfying the exclusion restriction. Empirically, we implement a dyadic parametric specification

$$y_{ij} = \alpha + \phi\, IG_{ij} + \delta\, \mathbf{x}_{ij} + \varepsilon_{ij},$$

where IG_{ij} indicates same-IG membership and $\mathbf{x}_{ij}$ includes rich pre-treatment controls (baseline opinions, common gender, nationality, admission type, high-school honors/district, parents' profession, residence ZIP, and tuition-fee differences). This design isolates the causal effect of friendship on convergence (negative ϕ) or divergence (positive ϕ) in outcomes, without relying on potentially endogenous realized networks alone.

4.2.2 Exposure to peers' shocks when the network is fixed

In the previous section, we considered a treatment that affected network formation: students assigned to the same integration group were 17% more likely to form friendship links than those not assigned to the same group (Algan et al., 2026). In this section, we instead study a randomized treatment across individuals within a *fixed social network*. This provides a natural way to estimate spillovers, as it compares individuals whose peers happened to receive the treatment with those whose peers did not. The framework of Borusyak and Hull (2023) formalizes this intuition and shows how to construct valid instruments from treatment assignments that respect the experimental design.

The setting. Consider a fixed, pre-existing network with weights $\widehat{g}_{ij}$ representing connections between individuals (e.g., co-workers, friends, classmates, or neighbors), where $\widehat{g}_{ii} = 0$ and weights are row-normalized, i.e., $\sum_j \widehat{g}_{ij} = 1$. Treatment $T_i \in \{0, 1\}$ is randomized at the individual level, with design probabilities $\pi_i = \mathbb{E}[T_i]$ that may vary across individuals (e.g., by risk stratum or geography). Unlike designs that randomize network formation itself (Section 4.2.1), here the network is fixed and only treatment varies.

Why naive instruments fail. For contextual effects of peer treatment (e.g., neighborhood spillovers in early-childhood programs as in List et al., 2023), define i's exposure as

$$\mathcal{E}_i = \sum_j \widehat{g}_{ij} T_j.$$

Treating $\mathcal{E}_i$ as exogenous because T_j is randomized can be misleading under stratified designs. If high-risk peers have $\pi_j = 0.7$ and low-risk peers $\pi_j = 0.4$, then

$$\mathbb{E}[\mathcal{E}_i \mid \widehat{\mathbf{g}}, \boldsymbol{\pi}] = \sum_j \widehat{g}_{ij} \pi_j,$$

which varies mechanically with peer composition. Individuals linked to higher-π peers have larger *expected* exposure even before treatment realization; if peer composition correlates with unobservables (e.g., family background or neighborhood quality), $\mathcal{E}_i$ violates the exclusion restriction.[17]

The recentering solution. Recentering each peer's realized assignment by its design mean yields

$$\widetilde{\mathcal{E}}_i = \sum_j \widehat{g}_{ij} \,(T_j - \pi_j), \tag{26}$$

so that $\mathbb{E}[\widetilde{\mathcal{E}}_i \mid \widehat{\mathbf{g}}, \boldsymbol{\pi}] = 0$ by construction. This removes mechanical correlation with peer composition and isolates idiosyncratic variation in treatment assignments around their design expectations.

[17]The failure of instrumental variables in this framework was already recognized in earlier work; for instance, Aronow (2012) note that "randomization of treatment to individuals does not imply simple randomization of proximity to treated units." In other words, random assignment does not guarantee econometric exogeneity.

Implementation and extension. To estimate the effect of *peer treatment exposure* $\mathcal{E}_i$ on outcomes y_i, one can instrument $\mathcal{E}_i$ with $\widetilde{\mathcal{E}}_i$, controlling for own treatment T_i and design strata, and including appropriate fixed effects with clustered standard errors. The same instrument also identifies *endogenous peer effects* when instrumenting $m_i = \sum_j \widehat{g}_{ij} y_j$, provided that T_j affects y_j and peer outcomes influence y_i.

Illustration. To fix ideas, consider how the recentering framework of Borusyak and Hull (2023) applies in a stylized version of a deworming experiment. Suppose treatment $T_i \in \{0, 1\}$ is randomized across students in a school network, but with *heterogeneous* design probabilities π_i (e.g., higher treatment probability for students in high-risk areas). Define student i's peer exposure as $\mathcal{E}_i = \sum_j \widehat{g}_{ij} T_j$. Even though treatment is randomized, $\mathcal{E}_i$ need not be mean-zero conditional on peer composition: if high-π peers are also more likely to be in certain neighborhoods, their expected exposure varies systematically with background characteristics, potentially biasing a naive OLS regression of outcomes on $\mathcal{E}_i$. Recentering by design probabilities, $\widetilde{\mathcal{E}}_i = \sum_j \widehat{g}_{ij}(T_j - \pi_j)$, removes this mechanical variation and restores exogeneity.

We emphasize that this is a *hypothetical* illustration. The seminal deworming paper by Miguel and Kremer (2004) does not face this problem as stated: their main specification controls for the *number* of treated peers (not their proportion), and they carefully control for school size (the total number of pupils in a school), which is the relevant measure of degree in their setting (see their equation (1) and the discussion on pages 175–176). Under uniform treatment probabilities, expected exposure is proportional to degree, and controlling for degree as they do removes the potential bias. The recentering approach adds value primarily in stratified or two-tier designs where design probabilities vary across individuals, extending the validity of spillover estimates to more complex experimental designs.

Scope: what the recentered instrument identifies. It should also be noted that the claim "the same instrument also identifies endogenous peer effects" requires qualification in the presence of *contextual* peer effects induced by the treatment. If treated peers differ from untreated peers in characteristics that directly affect i's outcome (i.e., $\delta \neq 0$ in equation (5)), then using $\widetilde{\mathcal{E}}_i$ as an instrument for peer outcomes $\sum_j \widehat{g}_{ij} y_j$ conflates the endogenous effect ϕ with the contextual channel δ. Separating the two requires either: (i) multiple pre-treatment peer characteristics to generate over-identifying restrictions, as discussed in Section 2; or (ii) an instrument that shifts peer *outcomes* without shifting peer *characteristics*, which the treatment assignment $\widetilde{\mathcal{E}}_i$ generally cannot guarantee if T_j affects both y_j and x_j.

Structural interpretation and scope. The recentered instrument $\widetilde{\mathcal{E}}_i$ identifies the *contextual spillover* of peer treatment on own outcome. In the structural framework, this corresponds to the parameter δ in equation (5): the direct effect of neighbors' exogenous characteristics (here, treatment status) on i's outcome, holding peers' endogenous effort fixed. It does *not* identify the endogenous peer effect ϕ, which requires peer *outcomes* (not treatment assignments) to vary. To see this, note that in the equilibrium of the LQ game, $\mathbf{y}^* = (\mathbf{I} - \phi\widehat{\mathbf{G}})^{-1}(\boldsymbol{\alpha} + \delta\widehat{\mathbf{G}}\mathbf{T})$, so using $\widetilde{\mathcal{E}}_i$ as an instrument for peer exposure $\mathcal{E}_i$ yields a reduced-form estimate that conflates δ and the network-multiplier $(\mathbf{I} - \phi\widehat{\mathbf{G}})^{-1}$ unless ϕ is separately pinned down. One can use the same instrument to identify ϕ via two-stage least squares if one instruments the endogenous peer outcome $\sum_j \widehat{g}_{ij} y_j$ with $\widetilde{\mathcal{E}}_i$ and the network is known—provided that T_j affects y_j (the relevance condition) and that the exclusion restriction $\widetilde{\mathcal{E}}_i \perp \varepsilon_i \mid \widehat{\mathbf{G}}, \boldsymbol{\pi}$ holds. Even then, the resulting IV estimate of ϕ is a local average treatment effect that depends on the specific network and treatment design; it is not the same as the structural ϕ_0 estimated by the full-information methods of Section 5.

Relation to the network-interference literature. It is worth noting that the network-interference literature offers several complementary approaches to the problem addressed by Borusyak and Hull (2023). These include inverse-probability weighting by π_i (which down-weights high-degree nodes with larger expected exposure), randomisation-based estimators that condition on the observed network (see Aronow and Samii 2014), and Horvitz–Thompson estimators of direct and spillover effects under partial interference (Hudgens and Halloran, 2008). We focus on recentered instruments here because they fit naturally into the IV framework adopted throughout Sections 2 and 4, and because their explicit use of design probabilities π_i maps directly onto the adjacency matrix $\widehat{\mathbf{G}}$ of our structural model. Readers interested in the broader reduced-form literature on spillover estimation are referred to Sävje et al. (2021) and Leung (2022).

4.3 Exogeneous changes in the network structure

A complementary approach to address endogeneity in networks exploits *exogenous variation in network structure* arising from node or edge "failures." When particular nodes or links are removed for reasons orthogonal to agents' choices, the resulting shocks can serve as natural experiments for identification. Examples include interlocking directorates in India where links between firms are severed by the *death of a shared board member*, plausibly an unpredictable event at hiring time (Helmers et al., 2017), and the *forced removal of academics* from German universities during the Nazi regime, which generated abrupt, externally imposed changes in departmental networks (Waldinger, 2010, 2012). In both settings, the research strategy is to compare outcomes before and after the network shock, tracing out how the disappearance of a node (or its incident edges) propagates through immediate neighbors and, potentially, through neighbors-of-neighbors.

Lindquist et al. (2024) apply this logic to Swedish co-offender networks constructed from the Suspects Register (2010–2012), covering 29,369 networks and 108,018 individuals, and documenting 679 *exogenous co-offender deaths* over the period. Treating a death as a node removal in the co-offending graph, they study how outcomes for surviving offenders change with the *graph-theoretic distance* to the deceased (one-, two-, and three-step neighbors). Conceptually, if peer spillovers operate primarily along direct ties, effects should be strongest for one-step neighbors and attenuate with distance; non-trivial responses at two or three steps would indicate broader diffusion mechanisms (e.g., displacement of opportunities, reputation, or enforcement spillovers). As with all node-failure designs, validity hinges on agents not anticipating or systematically responding to the failure in ways that violate exogeneity (e.g., recruiting replacements based on observables correlated with outcomes) and on accurate measurement of the network itself; misclassifying a missing node due to data error rather than true exit would bias estimates toward zero.

Structural interpretation. Node-removal shocks have a precise interpretation in the LQ network game of Section 3.1. When node k is removed from the network, aggregate equilibrium effort changes by $\Delta Y^* = Y^*(\mathbf{G}) - Y^*(\mathbf{G}_{-k})$, where $\mathbf{G}_{-k}$ denotes the subnetwork excluding k and all her links. Ballester et al. (2006) show that this quantity equals $d_k(\mathbf{G}, \phi)$, which is the *intercentrality* (key-player centrality) of node k defined in equation (17) of Section 3.1. Hence, the causal effect of an exogenous co-offender death estimated by Lindquist et al. (2024) is, within the structural model, an estimate of ϕ times the deceased's intercentrality. This provides a direct link between the node-failure identification strategy and the key-player policy framework: the individual whose removal reduces aggregate activity the most is precisely the one with the highest intercentrality, and the attenuation of spillover effects with graph-theoretic distance (one-, two-, and three-step neighbors) maps onto the decay of off-diagonal elements of the Leontief inverse $(\mathbf{I}-\phi\widehat{\mathbf{G}})^{-1}$ at increasing powers of $\widehat{\mathbf{G}}$.[18] This structural reading also clarifies what such designs cannot recover: because only a single node is removed, the researcher observes one row of the Leontief inverse, not the full matrix; separately identifying ϕ and $\widehat{\mathbf{G}}$ requires either additional variation in network composition or the structural estimation strategy of Section 5.

4.4 Summary: what reduced-form strategies recover from the structural model

A key take-away of this section is that no single quasi-experimental design simultaneously recovers both the structural complementarity ϕ and the network matrix $\widehat{\mathbf{G}}$. Reduced-form strategies exploit exogenous variation to identify *scalar* functionals of the equilibrium—contextual effects, total spillovers, or changes in aggregate activity—while remaining agnostic about the full structural parameterisation. Recovering the complete parameter vector $(\phi, \delta, \gamma, \widehat{\mathbf{G}})$ and using it for counterfactual policy analysis requires the structural approaches surveyed in the next section. The two strategies are therefore complementary: reduced-form designs provide credible causal benchmarks against which structural estimates can be validated, while structural models provide the equilibrium framework needed to extrapolate beyond the quasi-experiment.

5 Structural models of network formation

From Reduced-Form Estimands to Structural Network Models A central objective of this survey is to clarify how reduced-form empirical strategies map into structural models of peer effects and networks. To fix ideas, consider the canonical linear-in-means (LIM) model (equation (5)):

$$y_i = \alpha + \phi \sum_j \widehat{g}_{ij} y_j + \delta \sum_j \widehat{g}_{ij} x_j + \gamma x_i + \varepsilon_i, \tag{27}$$

[18] In fact, Lindquist et al. (2024) use eigenvector centrality rather than intercentrality. This is because, as shown in equation (13), the ratio of activities between two agents is equal to the ratio of eigenvector centralities when spillovers are very large.

or, in matrix form,

$$\mathbf{y} = (\mathbf{I} - \phi\widehat{\mathbf{G}})^{-1}(\delta\widehat{\mathbf{G}}\mathbf{x} + \gamma\mathbf{x} + \boldsymbol{\varepsilon}). \tag{28}$$

Equation (27) highlights three key objects of interest: (i) the endogenous peer effect ϕ, capturing how peers' outcomes affect individual outcomes; (ii) the contextual effect δ, capturing the influence of peers' characteristics; and (iii) the network structure $\widehat{\mathrm{G}}$.

Most empirical strategies reviewed in this paper can be interpreted as identifying either these structural parameters or reduced-form objects that are functions of $(\phi, \delta, \gamma, \widehat{\mathrm{G}})$. This observation provides a unifying lens through which to organize the literature.

Before surveying the three structural approaches, it is useful to establish a common framework that clarifies what each model takes as observed, what it treats as latent, and whether it jointly models network formation and outcome choices or focuses on network formation alone.

In all three models, the *observed* variables are: individual outcomes or effort levels y_i (or their proxies), individual characteristics x_i, and—in most cases—the realized network G (or a summary of it such as degree sequence or link indicators). What differs across models is the treatment of *unobservables* and the scope of joint modelling.

- **Section 5.1 (Anonymous network formation, Boucher et al. 2021):** jointly models effort choices and network formation. The latent variables are the preference biases γ_{ij} and socialization efforts y_i (which are recovered in the first stage via GMM before being used to estimate link probabilities). The network $\widehat{g}_{ij}$ is treated as endogenous and is part of the equilibrium outcome.
- **Section 5.2 (Network Competitive Equilibrium, Battaglini et al. 2022):** also jointly models link formation and effort. The latent variable is each agent's *effectiveness* E_i, which is unobserved and must be inferred from the system of equilibrium equations. Because the equilibrium system is nonlinear, no closed-form likelihood exists; estimation proceeds via Approximate Bayesian Computation (ABC), which simulates the model at candidate parameter values and accepts draws that match observed moments.
- **Section 5.3 (Control function approach, Hsieh and Lee 2016):** jointly models network formation and outcomes, but via an econometric (rather than fully structural) approach. The latent variable is an individual-level unobserved

type z_i, which affects both link formation and the outcome disturbance ε_i. The joint distribution of (z_i, ε_i) is parametrized as bivariate normal; estimation uses MCMC to sample over the latent types and structural parameters simultaneously.

5.1 Anonymous network formation

We follow Boucher et al. (2021), who extend the utility function in equation (19) to endogenize the process of link formation. Building on Cabrales et al. (2011), they argue that a framework inspired by random networks provides a useful perspective on network formation—one in which links emerge endogenously rather than through a predetermined socialization process. In this setting, socializing does not correspond to drawing up a nominal list of intended relationships, as in Jackson and Wolinsky (1996).[19] This modeling choice, which treats network formation as the outcome of random meetings without earmarked socialization, greatly enhances the tractability of the analysis. Unlike richer models of link formation, it allows for standard Nash equilibrium analysis without the severe (combinatorial) multiplicity problems that often plague more structural approaches.[20]

In this model, the utility of each agent i is still given by equation (19), but y_i is now interpreted as a *socialization effort*. The corresponding first-order condition is

$$y_i = \alpha_i + \phi \sum_{j=1}^{n} \widehat{g}_{ij} y_j. \tag{29}$$

The key innovation lies in the specification of the link-formation probability. The probability that agent i befriends agent j is defined as

$$\widehat{g}_{ij} = \frac{\gamma_{ij} y_i y_j}{\sum_k \mathbf{1}\{k \in C_i(j)\}}, \tag{30}$$

where γ_{ij} denotes the preference bias of i toward j, and $C_i(j)$ captures *congestion effects*. Specifically, $C_i(j)$ is the set of agents (excluding i but including j) that are

[19] See Sheng (2020) who provides a partial identification solution to the severe multiplicity problems in network formation using pairwise stability as a solution concept (Jackson and Wolinsky, 1996) and De Paula et al. (2018) who develop a framework for identifying preference parameters in pairwise-stable network formation models by linking observed local network structures to underlying preferences, deriving necessary and sufficient conditions for identification under bounded degree, and proposing a quadratic programming algorithm to characterize the identified set.

[20] See also Canen et al. (2023) who structurally estimate a similar model in political economy.

comparable with j from i's perspective—for example, individuals of the same ethnicity or gender as j. The indicator function $\mathbf{1}\{k \in C_i(j)\}$ equals 1 if agent k belongs to $C_i(j)$ and 0 otherwise. Equation (30) implies that the probability of a link between i and j increases with i's socialization effort y_i, reflecting the fact that more sociable individuals are more likely to form connections. The probability also depends on the preference bias γ_{ij}—capturing homophily in link formation—and is inversely related to congestion, which reduces the likelihood of connection as the number of comparable individuals increases.

Empirically, Boucher et al. (2021) estimate this system in two stages. Using a GMM approach, they first estimate equation (29) to recover socialization efforts $\{y_i\}$ and then substitute these estimated values into equation (30) to obtain estimates of the link probabilities $\widehat{g}_{ij}$.

5.2 Endogenous social interactions under a network competitive equilibrium

Battaglini et al. (2022) develop a model of endogenous network formation using a new equilibrium concept: the Network Competitive Equilibrium (NCE). The model unfolds in two stages. In the first stage, agents endogenously form links (*network formation*); in the second, they engage in a network game of efforts following the local aggregate specification introduced in Section 3.1. Each agent's effectiveness, denoted by $E_i \in [0, 1)$, is defined as:

$$E_i = \rho \left(\sum_j g_{ij} E_j \right)^{\alpha} (y_i)^{1-\alpha} + \varepsilon_i, \tag{31}$$

where ρ captures the intensity of spillover effects, and ε_i represents an idiosyncratic component that affects agent i's efficacy independently of her connections or effort.

5.2.1 Second stage: Effort choices

Each agent i chooses effort y_i to maximize her utility:

$$u_i(\mathbf{y}, \mathbf{g}) = E_i - cy_i = \rho \left(\sum_j g_{ij} E_j \right)^{\alpha} (y_i)^{1-\alpha} + \varepsilon_i - cy_i. \tag{32}$$

Solving for y_i and substituting into (31) yields:

$$E_i(\boldsymbol{\varepsilon}, \mathbf{g}) = \varepsilon_i + \phi \sum_{j=1}^{n} g_{ij} E_j(\boldsymbol{\varepsilon}, \mathbf{g}), \tag{33}$$

where $\phi := \rho \left(\frac{(1-\alpha)\rho}{c}\right)^{\frac{1-\alpha}{\alpha}}$. This expression is equivalent to (10), with E_i and ε_i replacing y_i and α_i, respectively.

5.2.2 First stage: Link formation

By substituting the equilibrium expression for E_i from (33) into (32), the equilibrium utility becomes:

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha\phi \sum_j g_{ij} E_j(\boldsymbol{\varepsilon}, \mathbf{g}) + \varepsilon_i - \sum_j \frac{\lambda}{1+\lambda} \left(\frac{g_{ij}}{x_{ij}}\right)^{\frac{1+\lambda}{\lambda}}, \tag{34}$$

where the cost of establishing a link of intensity g_{ij} is given by:

$$C(g_{ij}, x_{ij}) = \frac{\lambda}{1+\lambda} \left(\frac{g_{ij}}{x_{ij}}\right)^{1+\frac{1}{\lambda}}.$$

Here, x_{ij} captures *homophily*, i.e., the degree of compatibility between agents i and j: the more similar they are, the lower the cost of forming a link. Each agent i then chooses $\mathrm{g}^i = (g_{i1}, \ldots, g_{in})$ to maximize $u_i(\mathrm{y}, \mathrm{g})$. Since any link proposal $g_{ij} > 0$ is always reciprocated, we have $g_{ji} > 0$. Solving this maximization problem, Battaglini et al. (2022) obtain the following result.

Proposition 1. *Consider an interior solution for g^*_{ij}. A Network Competitive Equilibrium (NCE)*[21] *exists and is characterized by a vector $\mathbf{E}^*$ and a matrix $\mathbf{G}^*$ that jointly solve:*

$$\begin{cases} E^*_i = \phi \sum_j g^*_{ij} E^*_j + \varepsilon_i, \\ g^*_{ij} = (x_{ij})^{1+\lambda} (\alpha\phi)^\lambda (E^*_j)^\lambda, \end{cases}$$

for all $i, j \in \mathcal{N}$.

In this framework, agents' effectivenesses cannot be represented by a *linear system of equations*, unlike the familiar Katz–Bonacich formulation in (11). When g_{ij} is endogenous, agent i optimally chooses g_{ij} to be proportional to $(E_j)^\lambda$. As $\lambda \to 0$, endogenous links become completely inelastic with respect to effectiveness, and $g^*_{ij} \to (x_{ij})^{1+\lambda}$. In that case, we recover the standard Katz–Bonacich representation of effectiveness: if $\phi\mu_1(\mathbf{G}) < 1$, then $\mathbf{E} = (\mathbf{I} - \phi\mathbf{G})^{-1}\boldsymbol{\varepsilon}$.[22]

[21] A NCE $(\mathbf{y}^*, \mathbf{E}^*, \mathbf{G}^*)$ satisfies: (i) network connections are optimal for each agent i at $t = 1$ given $\mathbf{E}$; (ii) effort levels are optimal for each agent i at $t = 2$ given $\mathbf{E}$ and $\mathbf{G}$; (iii) the vector of effectiveness levels satisfies the production function (31) given $\mathbf{y}$ and $\mathbf{G}$.

[22] Battaglini et al. (2022) provide conditions for the uniqueness of the interior NCE using the Contraction Mapping Theorem.

Modeling both network formation and actions is inherently difficult due to the combinatorial nature of link formation. As discussed in Section 5.1, Boucher et al. (2021) address this by assuming *anonymous networks*, where agents choose socialization efforts rather than targeting specific partners. In contrast, Battaglini et al. (2022) overcome this challenge by introducing the concept of a Network Competitive Equilibrium (NCE), analogous to general equilibrium analysis in economics. Agents select their socialization efforts while taking others' equilibrium effectiveness as given—akin to "price-taking" behavior in competitive markets. These equilibrium effectiveness levels must be jointly consistent with individual choices, leading to a system of nonlinear equations that characterizes the NCE. Thus, when forming links, agents disregard the indirect effects on others' effectiveness.

This assumption allows the authors to exploit the analytical characterization of equilibrium conditions for estimation. Because the structure of the model precludes an explicit likelihood function, they employ an *Approximate Bayesian Computation* approach to estimate parameters. Applying this methodology to data from the 109th–113th U.S. Congresses, they find strong evidence that social connections significantly affect legislative effectiveness.

5.3 Control function approach

Consider the model of Hsieh and Lee (2016), which, as above, add a network formation model to the outcome equation (5). However, the mechanism of network formation is very different. Following a control–function strategy in the spirit of Goldsmith-Pinkham and Imbens (2013), they posit a latent trait z_i that affects links and is correlated with the outcome disturbance ε_i in the local–average model (5) according to a bivariate normal distribution

$$(z_i, \epsilon_i) \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_z^2 & \sigma_{\varepsilon z} \\ \sigma_{\varepsilon z} & \sigma_\varepsilon^2 \end{pmatrix} \right),$$

where σ_z^2 captures the variance of z and $\sigma_{\varepsilon z}$ the covariance between ε and z.

Each agent i chooses to be friends with j according to a vector of observed and unobserved characteristics in a standard link formation probabilistic model:

$$\Pr(g_{ij} = 1 \mid x_{ij}, z_i, z_j, \gamma, \upsilon) = \Lambda\left(\gamma_0 + \sum_k |x_i^{(k)} - x_j^{(k)}| \, \gamma_k + |z_i - z_j| \, \upsilon \right),$$

with $\Lambda(\cdot)$ logistic and $\upsilon < 0$ (similarly $\gamma_k < 0$) encoding assortative matching. If $\sigma_{\varepsilon z} \neq 0$ and $\upsilon \neq 0$, then g_{ij} is endogenous in the outcome equation; joint normality implies $E[\varepsilon_i \mid z_i] = (\sigma_{\varepsilon z}/\sigma_z^2)\, z_i$, yielding the augmented outcome equation of (5), given by (assuming $\alpha = 0$ for simplicity)

$$y_i = \phi \sum_j \widehat{g}_{ij} y_j + \delta \sum_j \widehat{g}_{ij} x_j + \gamma x_i + \eta + \frac{\sigma_{\varepsilon z}}{\sigma_z^2}\, z_i + v_i,$$

where $v_i \sim N\Big(0, \sigma_\varepsilon^2 - \frac{\sigma_{\varepsilon z}^2}{\sigma_z^2}\Big)$ and η is a network fixed effect. Identification in the baseline case (with an exogenous network) relies on the exogeneity of x and the intransitivities in $\widehat{\mathrm{G}}$, such that $\widehat{\mathrm{G}}^2$x provides a valid exclusion restriction. In the extended model that jointly considers network formation and outcomes, the dyadic regressors $|x_i - x_j|$ enter the link formation equation but are excluded from the outcome equation, thereby generating additional (nonlinear) exclusion restrictions. Because link decisions are interdependent—reflecting friends-of-friends connections, clustering, and popularity—and because the network state space grows exponentially with n ($2^{n(n-1)}$ in directed binary graphs), likelihood-based estimation quickly becomes infeasible for even moderately sized networks. Consequently, Hsieh and Lee (2016) estimate the joint link–outcome system using Bayesian methods, where the latent variables z_i and the structural parameters are sampled via a Markov Chain Monte Carlo procedure.[23]

Table 1 summarizes the identification issues and how we can address them.

5.4 A practical guide: when to use network data

A natural question for applied researchers is: given the additional cost and complexity of collecting and analyzing network data, when does moving beyond the group-average LIM model pay off? The answer depends on the available data, the research question, and the degree of heterogeneity in peer influence.

5.4.1 Available data and the state of the empirical evidence

Most empirical work on peer and network effects draws on one of three types of network data, each with distinct strengths and limitations.

[23] See also König et al. (2019), who estimate the adjacency matrix using a homophily-based network formation model before incorporating it into the outcome equation; Battaglini et al. (2020), who address network endogeneity through a Heckman correction that controls for individual-level unobserved heterogeneity; and Hsieh et al. (2020), who develop a unified framework in which individuals anticipate how the network structure influences the utility of their interactions when forming links.

Table 1: Identification Challenges and Solutions in Peer/Network Settings

Challenge	**Problem in brief**	**Representative solutions (what they identify)**	**Model link**
Reflection	Group mean $\mathbb{E}(y_r)$ collinear with group mean covariates $\mathbb{E}(x_r)$; endogenous vs. contextual not separable	(i) Network heterogeneity: use $\widehat{\mathbf{G}}\mathrm{x}$, $\widehat{\mathbf{G}}^2\mathrm{x}$ as IV; (ii) experimental/cohort variation.	LIM $\leftrightarrow$ local-average network game; ϕ identified when $\mathbf{I}, \widehat{\mathbf{G}}, \widehat{\mathbf{G}}^2$ are linearly independent
Correlated effects	Common shocks or environments bias peer coefficients even without simultaneity	Network fixed effects; instruments using friends-of-friends' x.	LIM $\leftrightarrow$ local-average network game; ϕ identified when $\mathbf{I}, \widehat{\mathbf{G}}, \widehat{\mathbf{G}}^2, \widehat{\mathbf{G}}^3$ are linearly independent
Sorting / endogenous links	Unobserved traits drive both links and outcomes	(i) Within-school cohort composition; (ii) Exposure to peers' shocks; (iii) Random assignment; (iv) node/edge "failures" (exogenous exits); (v) structural link formation (anonymous networks, network competitive equilibrium, control-function)	Endogenous $\widehat{\mathbf{G}}$: joint model for y and g

Friendship nominations. The most widely used network data in economics come from surveys in which respondents nominate a fixed number of friends or acquaintances. The most influential example is the National Longitudinal Study of Adolescent to Adult Health (Add Health), a stratified sample of over 90,000 U.S. adolescents in grades 7–12 who were asked to nominate up to five male and five female friends from a school roster. These data underpin a large body of work on peer effects in education (Calvó-Armengol et al., 2009), crime (Patacchini and Zenou, 2012), and health (Giulietti et al., 2022). The friendship-nomination design has the advantage of capturing perceived, active social ties rather than mere proximity, but it introduces several well-documented measurement problems. Nominations are *truncated* at the maximum allowed number, so high-degree nodes may appear to have fewer links than they actually do. Nominations are also *asymmetric*: if i nominates j but j does not nominate i, the researcher must choose whether to treat the link as directed, undirected, or absent, with different choices leading to different estimates (Hardy et al., 2022).

Administrative and digital trace data. An increasingly common alternative constructs networks from administrative records: co-offending registers (Lindquist et al., 2024), patents and R&D networks (König et al., 2019; Bhaskarabhatla et al., 2021), mobile phone call records (Barwick et al., 2023), or social media connections (Chetty et al., 2022). These data have the advantage of large sample sizes and near-complete network observation within their domain, but they capture a specific type of interaction that may not always correspond to the economically relevant peer relationship. Co-offenders are connected by criminal activity, which may be endogenous to the very outcome (crime) being studied. Social media connections reflect online engagement that may differ substantially from real-world influence.

5.4.2 Advantages and issues of using network data

Group-based LIM is typically sufficient when:

- The research question concerns the *average* spillover within well-defined, relatively homogeneous groups (classrooms, military units), and heterogeneity in peer influence across individuals within those groups is not the object of interest.
- Network data are unavailable or too costly to collect reliably, and cohort-based or random-assignment variation provides a valid instrument for group-average

peer behavior.

- The policy question does not require targeting specific individuals but rather concerns group-level interventions (e.g., changing classroom composition).

Network data add substantial value when:

- The research question concerns *who influences whom*: for example, targeting the most influential individuals (key players), designing vaccination or information-seeding campaigns, or estimating heterogeneous treatment effects that depend on network position.
- Peer groups are poorly defined, overlapping, or potentially endogenous (e.g., friendship networks in which group boundaries are themselves shaped by the outcome of interest).
- The reflection problem cannot be resolved by cohort variation, but intransitive triads or friends-of-friends instruments—which require observing the network—provide valid exclusion restrictions (Section 2).
- The researcher wishes to conduct counterfactual equilibrium analysis—for example, computing the aggregate crime reduction from removing the key player—which requires knowledge of the full network structure and the structural parameter ϕ (Sections 3 and 5).

5.4.3 Practical limitations of network-based approaches.

Network data come with non-trivial costs and risks that applied researchers should bear in mind. First, *measurement error* in self-reported networks is well-documented: respondents in surveys such as Add Health systematically under-report links, and the resulting attenuation bias can be severe when the network is used as a regressor (Chandrasekhar, 2016). Second, *data requirements* are demanding: obtaining reliable network data typically requires surveying all members of a population (to avoid missing-link bias) or using administrative records that capture actual interactions. Partial network observation introduces its own identification problems (Boucher and Houndetoungan, 2026). Third, *modeling complexity* increases substantially with network data: structural estimation of the LQ game requires specifying and estimating the network-formation process (Section 5) to address endogeneity, and the resulting estimators are computationally intensive and sensitive to the assumed functional forms. These limitations do not undermine the case for network analysis, but they underscore that the approach is most valuable when the research question genuinely requires it and when sufficiently rich network data are available.

6 Non-linear peer effects

Most empirical models of social interactions discussed above rely on strong simplifying assumptions, typically assuming that each agent's outcome is a linear function of the average behavior of her peers. This linear-in-means (LIM) specification conveniently summarizes aggregate peer influence but imposes restrictive behavioral and structural assumptions. In particular, it rules out heterogeneity in the strength or direction of peer effects and abstracts from the underlying microfoundations that shape how individuals actually respond to their social environment. A key question, therefore, is how to model social interactions when peer influence is neither linear nor solely driven by mean behavior.

Recent studies have explored alternative mechanisms in which individuals are affected by specific members of their group rather than by the average. Some highlight the influence of high achievers or "leaders" (Carrell et al., 2010; Tao and Lee, 2014; Díaz et al., 2021; Jones and Christakis, 2024), while others emphasize the negative impact of low performers or "bad apples" (Bietenbeck, 2020), and a few consider both types of effects (Hoxby and Weingarth, 2005; Tatsi, 2015). However, until Boucher et al. (2024), a general theoretical framework capable of unifying these different cases had been lacking. From a policy perspective, identifying the relevant *social norm*—whether it is anchored in high or low performers—is essential for designing interventions that effectively target the most influential or disruptive individuals within social networks.

Boucher et al. (2024) develop a unified framework that extends the linear-in-means (LIM) model by incorporating both spillover and conformity motives, while allowing for flexible definitions of social norms. The utility function, which generalizes (21) to accommodate an arbitrary social norm, is specified as:

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha_i y_i + \phi_1 y_i \widetilde{y}_{-i}(\beta) - \frac{1}{2}\Big[y_i^2 + \phi_2\big(y_i - \widetilde{y}_{-i}(\beta)\big)^2\Big], \tag{35}$$

where $0 \leq \phi_1 < 1$ captures the strength of spillovers, $\phi_2 \geq 0$ measures the taste for conformity, and $\widetilde{y}_{-i}(\beta)$ denotes agent i's perceived *social norm*, defined as:

$$\widetilde{y}_{-i}(\beta) = \left(\sum_{j=1}^{n} \widehat{g}_{ij} y_j^{\beta} \right)^{1/\beta}. \tag{36}$$

This specification nests the LIM model as a special case with $\beta = 1$, where $\widetilde{y}_{-i}(\beta)$ reduces to the average peer effort $\sum_{j=1}^{n} \widehat{g}_{ij} y_j$. By varying $\beta \in [-\infty, +\infty]$, it accommodates a continuum of social norm definitions, ranging from focus on the lowest to the highest peer actions.

To express best responses compactly, define $\lambda_1 = \frac{\phi_1}{1+\phi_2}$ and $\lambda_2 = \frac{\phi_2}{1+\phi_2}$. The individual best-reply function becomes:

$$y_i = (1 - \lambda_2)\alpha_i + (\lambda_1 + \lambda_2) \left(\sum_{j=1}^{n} \widehat{g}_{ij} y_j^{\beta} \right)^{1/\beta}. \tag{37}$$

This expression shows how heterogeneity and social norms jointly shape behavior. Specific parameter restrictions yield familiar models: with $\lambda_1 = 0$, only conformity matters; with $\lambda_2 = 0$, only spillovers matter. The framework thus unifies several canonical peer-effect formulations and allows for nonlinear behavioral responses driven by endogenous norms.

Using Generalized Method of Moments (GMM), the authors structurally estimate this model on U.S. adolescent data (AddHealth). They find that, across various activities, students respond not to average but to different peers from least- to top-performing peers.

In the educational domain (measured by GPA),[24] the estimated norm parameter is $\beta = 372$, implying that students benchmark themselves against the best in their network peers rather than the mean. To explore policy implications of this result, Boucher et al. (2024) contrast optimal interventions under two regimes: the LIM case ($\beta = 1$) and the general nonlinear norm model. The resulting policy prescriptions differ sharply. Under LIM, optimal subsidies are nearly uniform, reflecting homogeneous marginal externalities. In contrast, with $\beta = 372$, many individuals receive no transfer because they generate negligible spillovers, while a small group of influential students receive substantial subsidies. When social norms are driven by low performers, *targeted* support to a few *key agents* can significantly shift the benchmark and magnify aggregate welfare effects.

[24] To the extent that GPA can be directly mapped into effort choices.

This comparison underscores the importance of accounting for the structure of peer preferences when designing network-based policies.[25] Table 2 summarizes the policy implications of the non-linear peer model.

Table 2: Nonlinear Social Norms: Which Peers Matter?

Norm parameter	**Behavioral meaning**	**Estimation / policy implication**
$\beta = 1$ (mean)	Respond to average neighbor action	Uniform targeting; LIM-like subsidies
$\beta \to +\infty$ (max/leaders)	Benchmark top performers ("shining lights")	Target few high-influence nodes; big returns on limited transfers
$\beta \to -\infty$ (min/bad apples)	Benchmark worst performers	Remedial targeting to lowest performers; large welfare gains from correcting low tail
Mixed / estimated β	Heterogeneous responses across domains	Design norm-sensitive interventions

7 Multiplex networks: Multiple dimensions of peers

The analysis so far has focused on a single network $\mathbf{G}$. In many applications, however, interactions occur across multiple layers, such as friendship, geographic proximity, and online connections. Multiplex network models extend the LIM framework by replacing $\mathbf{G}$ with a collection of matrices $\{\mathbf{G}^{(s)}\}_{s=1}^{S}$, each capturing a distinct type of interaction.

From an empirical perspective, this extension raises new identification challenges, as reduced-form estimands now reflect a combination of effects across layers. Understanding how these estimands map into structural parameters requires generalizing the equilibrium mapping in (28) to a multi-layer setting.

[25] See also Houndetoungan (2025), who proposes a flexible structural framework to estimate nonlinear peer effects across different quantiles of the peer outcome distribution.

Thus, multiplex networks can be viewed as a natural extension of the framework developed in Section 3, rather than a separate literature.

Another important but largely unexplored aspect of peer effects—at least within economics[26]—is that individuals are simultaneously embedded in multiple networks and engage in multiple types of interactions. In many real-world contexts, people participate in multiplex networks, where the same set of actors are connected through several distinct types of relationships. For instance, in a workplace, employees may be linked through a professional layer (collaboration on tasks), a friendship layer (socializing outside work), and an advice layer (seeking or providing guidance). Behavior in one layer often spills over to others: a highly productive worker who is also well liked can simultaneously raise colleagues' effort through both professional imitation and social motivation. Similarly, in rural villages, households are connected through credit, information, and kinship networks. Access to credit in the financial layer may depend on trust formed in the social layer, while information about new agricultural technologies diffuses more effectively when these layers overlap. In schools, students are connected through academic, friendship, and extracurricular layers, so a motivated student may influence peers not only by sharing study habits but also by shaping social norms about effort and achievement. Across these examples, the multiplex structure amplifies or dampens peer effects depending on whether the layers reinforce or counteract one another, thus shaping aggregate outcomes in complex ways.

7.1 Multiplex networks: Theory

Zenou and Zhou (2026) were among the first to formalize *multiplex* interactions within network games. Specifically, they extend the framework of Ballester et al. (2006) (hereafter BCZ), whose utility function is given by (8). They introduce a slight modification to this function to obtain

$$u_i(\mathbf{y}, \mathbf{g}) = \alpha_i y_i - \frac{1}{2} y_i^2 + \phi \sum_{j=1}^{n} g_{ij} y_i y_j - \frac{c}{2} (y_i)^2. \tag{38}$$

The only difference with (8) is the inclusion of an additional effort cost term. The first-order conditions are then given by

$$y_i = \frac{1}{1+c} \alpha_i + \frac{\phi}{1+c} \sum_{j=1}^{n} g_{ij} y_j. \tag{39}$$

[26] A notable exception is Chandrasekhar et al. (2024).

As in the BCZ model, the Nash equilibrium can be solved in closed form. When $\phi\lambda_{\max}(\mathbf{G}) < 1 + c$, the unique interior equilibrium is

$$\mathbf{y} = \left[\mathbf{I}_n - \frac{\phi}{1+c}\mathbf{G}\right]^{-1} \frac{1}{1+c}\boldsymbol{\alpha}. \tag{40}$$

Let $\mu_i := c\, y_i$ denote the marginal cost of total effort. Then,

$$\boldsymbol{\mu} = c\,\mathbf{y} = c\left[\mathbf{I}_n - \frac{\phi}{1+c}\mathbf{G}\right]^{-1} \frac{1}{1+c}\boldsymbol{\alpha}. \tag{41}$$

We can now extend the BCZ framework to a *multiplex* setting. For each layer $s \in \mathcal{S}$, let $\mathbf{G}^s = (g_{ij}^s)_{1\leq i,j\leq n}$ denote the adjacency matrix. The utility function of agent i is then given by

$$u_i^M(\mathbf{y}, \mathbf{g}) = \sum_{s\in\mathcal{S}} v^s \left(\alpha_i^s y_i^s - \frac{1}{2}(y_i^s)^2 + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s y_i^s y_j^s\right) - \frac{c}{2}\left(\sum_{s\in\mathcal{S}} y_i^s\right)^2, \tag{42}$$

where $v^s > 0$ represents the preference weight associated with layer s, $\phi^s > 0$ is the within-layer spillover parameter, and g_{ij}^s denotes the social tie between i and j in layer s. Zenou and Zhou (2026) show that a unique equilibrium exists if $1 - \lambda_{\max}(\mathbf{G}^s)\phi^s > 0$ holds for each layer $s \in \mathcal{S}$. At an interior equilibrium, we have

$$y_i^s = \alpha_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s y_j^s - \frac{c\sum_{s\in\mathcal{S}} y_i^s}{v^s}. \tag{43}$$

Let $\mu_i := c\sum_{s\in\mathcal{S}} y_i^s$ denote the marginal cost of total effort. Then,

$$\mathbf{y}^s = [\mathbf{I}_n - \phi^s\mathbf{G}^s]^{-1}\left(\boldsymbol{\alpha}^s - \frac{1}{v^s}\boldsymbol{\mu}^*\right) = \mathbf{M}^s\left(\boldsymbol{\alpha}^s - \frac{1}{v^s}\boldsymbol{\mu}^*\right), \tag{44}$$

where $\mathbf{M}^s := [\mathbf{I}_n - \phi^s\mathbf{G}^s]^{-1}$. However, $\boldsymbol{\mu}$ is now endogenous and satisfies the following *linear* system of equations:

$$\boldsymbol{\mu} = c\sum_{s\in\mathcal{S}} \mathbf{y} = c\sum_{s\in\mathcal{S}} \mathbf{M}^s\left(\boldsymbol{\alpha}^s - \frac{1}{v^s}\boldsymbol{\mu}\right). \tag{45}$$

The key new equation is (45), which endogenously links the different layers to one another through the common marginal cost of total effort, $\boldsymbol{\mu}$. In the monolayer model, each network (or layer) is analyzed *independently*, as if the agent operated in a single environment. Agents in each network respond only to local complementarities *within* that layer, and there is no strategic interaction across layers. The cost parameter c

affects total effort, but it is treated separately in each layer, without any cross-layer interdependence.

By contrast, in the multilayer model, agents choose efforts across all layers *simultaneously*, facing a total cost that depends on their *aggregate* effort across layers. In this setting, the cost of effort becomes *interdependent* across layers: exerting effort in one layer increases the marginal cost of exerting effort in another. Agents internalize this interdependence and strategically allocate their efforts across layers, balancing network complementarities against rising marginal costs of total effort.[27]

It is straightforward to modify this model to incorporate the *local-average* specification instead of the *local-aggregate* one. Indeed, note that we can rewrite the utility function in (42) by redefining the interaction parameter as $\phi'^s = \phi^s/d_i^s$. All subsequent derivations remain valid,[28] and the equilibrium effort is still given by (43), but now with $\widehat{g}_{ij} = g_{ij}/d_i$ instead of g_{ij}.

7.2 Multiplex networks: Empirical considerations

The theoretical predictions derived from monolayer and multilayer network models differ substantially. But what are their empirical implications? In this section, we show that estimating each model yields fundamentally distinct econometric specifications, reflecting the structural differences inherent in the monolayer and multilayer frameworks.

Consider first the *monolayer* model from the previous section, but with a row-normalized network $\widehat{\mathrm{g}}$ instead of g and a parameter $\widetilde{\phi}$ in place of ϕ. Then:

$$y_i = \frac{1}{1+c}\alpha_i + \frac{\widetilde{\phi}}{1+c}\sum_{j=1}^{n}\widehat{g}_{ij}y_j. \tag{46}$$

Let $c = 1$, define $\frac{\widetilde{\phi}}{1+c} := \phi$, and set $\alpha_i := 2\left(\mathbf{x}_i^T\boldsymbol{\delta}_i + \epsilon_i\right)$, where $\mathbf{x}_i$ is a $(k \times 1)$ vector of observable characteristics (a vector with superscript T indicates the transpose of this vecxtor) and $\boldsymbol{\delta}_i$ a $(k \times 1)$ coefficient vector. Equation (46) can then be rewritten as:

$$y_i = \mathbf{x}_i^T\boldsymbol{\delta}_i + \phi\sum_{j}\widehat{g}_{ij}y_j + \varepsilon_i. \tag{47}$$

[27] An extension of this framework to non-linear peer effects in multiplex networks can be derived using the approach in Section 6; see Zenou and Zhou (2024) for a detailed exposition.

[28] In the row-normalized network, $\lambda_{\max} = 1$, and thus, the spectral condition for existence and uniqueness is equal to $\phi^s < 1$, for each layer s.

In contrast, the *multilayer* model features endogenous marginal costs that rise with total effort. Consequently, the econometric specification must capture cross-layer interactions—such as the total effort across layers ($\sum_s y_i^s$)—as determinants of marginal cost or behavior.

Starting from the first-order conditions in equation (43), and assuming $v^s = 1$, a row-normalized network $\widehat{\mathbf{g}}^s$ instead of $\mathbf{g}^s$, and $\widetilde{\phi}^s$ instead of ϕ^s, we obtain:

$$y_i^s = \frac{\alpha_i^s}{1+c} + \frac{\widetilde{\phi}^s}{1+c} \sum_{j \in \mathcal{N}} \widehat{g}_{ij}^s y_j^s - \frac{c}{1+c} \sum_{s' \neq s} y_i^{s'}. \tag{48}$$

Let $c = 1$, define $\frac{\widetilde{\phi}^s}{1+c} := \phi^s$, set $\beta = -\frac{1}{2}$, and specify $\alpha_i^s := 2\left((\mathbf{x}_i^s)^T \boldsymbol{\delta}_i^s + \epsilon_i^s\right)$. We allow the observable and unobservable characteristics affecting a decision in one layer to differ from those influencing decisions in another. The resulting econometric equation is:

$$y_i^s = (\mathbf{x}_i^s)^T \boldsymbol{\delta}_i^s + \phi^s \sum_j \widehat{g}_{ij}^s y_j^s + \beta \sum_{s' \neq s} y_i^{s'} + \epsilon_i^s. \tag{49}$$

We can generalize this specification by assuming that, in the multiplex model, the cost function takes the form $\frac{1}{2}\left(\sum_s c^s y_i^s\right)^2$. Under the same assumptions as above and with $\beta^{s'} = -\frac{c^{s'}}{2}$, the equation to be estimated becomes:

$$y_i^s = (\mathbf{x}_i^s)^T \boldsymbol{\delta}_i^s + \phi^s \sum_j \widehat{g}_{ij}^s y_j^s + \sum_{s' \neq s} \beta^{s'} y_i^{s'} + \epsilon_i^s. \tag{50}$$

Since the actions $y_i^{s'}$ in (50) are themselves determined by analogous equations, one must estimate S equations if there are S layers. Estimating this system provides a direct test for cross-layer crowding-out effects. A statistically significant coefficient $\beta^{s'}$ indicates such effects, highlighting the role of multilayer networks in shaping individual behavior.

Illustration. Consider the dataset on multiplexing patterns in Indian villages from Chandrasekhar et al. (2024), based on Wave II data from 75 villages (Banerjee et al., 2013, 2024). Suppose we focus on two layers: the *social* layer ($s = 1$)[29] and the *advice* layer ($s' = 2$).[30] Let y_i^1 denote whether individual i adopts a microfinance program (Banerjee et al., 2013), and y_i^2 whether the same individual engages in informal borrowing and risk sharing within the village (Banerjee et al., 2024). Then equation (50)

[29] "To whose home does the respondent go and who comes to their home, as well as which close relatives live outside their household."

[30] "To whom does the respondent give information or advice."

specializes to:

$$\begin{cases} y_i^1 = (\mathbf{x}_i^1)^T \boldsymbol{\delta}_i^1 + \phi^1 \sum_j \widehat{g}_{ij}^1 y_j^1 + \beta^2 y_i^2 + \epsilon_i^1, \\ y_i^2 = (\mathbf{x}_i^2)^T \boldsymbol{\delta}_i^2 + \phi^2 \sum_j \widehat{g}_{ij}^2 y_j^2 + \beta^1 y_i^1 + \epsilon_i^2. \end{cases} \tag{51}$$

Clearly, $\widehat{g}_{ij}^1$ need not equal $\widehat{g}_{ij}^2$, since the individuals connected to i in layer 1 (close relationships involving home visits) may differ from those in layer 2 (individuals from whom i receives advice).

By jointly estimating the system in (51), we can test whether β^1 and β^2 are statistically significant, and thus assess whether the adoption of microfinance is driven solely by peers' behavior in the social network (layer 1) or also by informal borrowing and risk-sharing decisions in the advice layer (layer 2). For identification, the simplest approach is to apply an exclusion restriction. For instance, one may use an observable characteristic in $\mathbf{x}_i^1$ that is excluded from $\mathbf{x}_i^2$. When contextual effects are included, such exclusion restrictions arise naturally, as $\mathbf{G}^s\mathbf{x}_i^s$ for layer s is excluded from layer s', given that $\mathbf{G}^s$ and $\mathbf{G}^{s'}$ are, by definition, distinct.[31]

Table 3 summarizes the identification issues arising in multiplex network models and how they can be (partially) addressed.

Table 3: Empirical Tests of Multiplex Peer Effects

Specification	**Empirical Test**	**Interpretation**
Equation (47)	Within-layer spillovers	Baseline monolayer peer effects
System of equations (50)	Cross-layer interactions (crowding or complementarity)	$\beta^{s'} < 0$: shared-cost crowding; $\beta^{s'} > 0$: cross-layer complementarities
Instruments: $\widehat{\mathbf{G}}^s\mathbf{x}^s$, $(\widehat{\mathbf{G}}^s)^2\mathbf{x}^s$, and layer-specific excluded covariates	Exogeneity of peer exposure across layers	Identification strategy consistent with the theoretical first-order conditions

[31]The identification strategy based on exclusion restrictions is closely related to that of Cohen-Cole et al. (2018), who test a multi-activity network model (Chen et al., 2018). Their framework can be viewed as a special case of our multiplexing model with two layers, where $\beta^1 = \beta^2 = \beta$ and $\mathbf{G}^1 = \mathbf{G}^2$.

8 Discussion

The empirical distinction between peer and network effects is more than semantic. Group-average models are simple and intuitive but face fundamental identification challenges without additional structural assumptions. Network-based models introduce this structure by exploiting heterogeneity in connection patterns. This overview shows how formal network frameworks deepen our understanding of social interactions and clarify the conditions under which causal peer effects can be credibly identified. Embedding empirical strategies in theoretically grounded models not only disentangles social influence from correlated or contextual effects but also provides a foundation for policy evaluation through counterfactual analysis. By linking behavioral mechanisms to observable network structures, these models guide the design of targeted interventions—such as identifying key players or influential nodes—that can magnify policy effectiveness.

By treating peer effects as equilibrium interactions within explicitly modeled networks, this survey transforms the traditional divide between reduced-form and structural approaches into a continuum. Researchers can begin with network-aided identification to separate endogenous and contextual effects, embed those estimates within a *network-game backbone* to interpret magnitudes structurally, and use the same framework to design *targeted interventions*. Extending this logic to nonlinear norms and multiplex environments uncovers new dimensions of social influence: who shapes prevailing norms, how incentives propagate across layers, and where policy leverage is greatest. The resulting synthesis offers a coherent toolbox for analyzing, interpreting, and manipulating social interactions in networks.

Beyond the linear-in-means paradigm, individuals often respond to *nonlinear* and *context-dependent* peer interactions rather than to group averages. Multiplex and multilayer settings—where individuals engage simultaneously across social, professional, and financial domains—further expand the notion of "peers" and introduce new identification challenges. Together, these insights offer a unified analytical framework that connects theory, empirics, and policy, and point to promising directions for future research on how social influence propagates within and across networks.

8.1 Future Directions

Nonlinear norms at scale. New administrative and digital-network data allow for the estimation of which peers anchor norms (leaders versus laggards) and how salience shifts after interventions. Embedding estimated β into counterfactual network games can alter both policy targeting and welfare evaluations.

Multiplex policy design. When individuals allocate effort across multiple layers with shared costs, single-layer interventions can backfire through cross-layer crowding. Designing and empirically testing coordinated, cross-layer policies remain an open challenge.

Endogenous network responses. Most identification strategies treat the network as exogenous. Yet policies—information shocks, subsidies, or sanctions—may themselves rewire the network. Estimable joint models of outcomes and link dynamics are essential for credible counterfactuals.

Targeting beyond centrality. Key-player policies should be extended to include *norm-aware* targeting, focusing on who most effectively shifts the social reference norm rather than only on who is most central.

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