Discussion Paper Series

IZA DP No. 18375

February 2026

Directedness in Search

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IZA NETWORK @ LISER
ISSN: 2365-9793

IZA DP No. 18375 February 2026

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Abstract

We present a tractable, hybrid framework that nests random and perfectly directed search, in which workers are more likely to direct their search toward submarkets with higher returns, while still searching in inferior submarkets with positive probability. The choice of submarket is governed by a logit choice model with noise parameter $\mu \in [0,\infty)$. In the respective limits, search becomes either completely random or perfectly directed. We characterize the model equilibrium and show that even the perfectly directed search limit is inefficient, in contrast to its otherwise close cousin, competitive search.

We proceed to quantify the extent of directedness on Danish matched employer–employee data. Identification relies on the insight that the two benchmark models differ qualitatively in their implications for job-to-job worker reallocation. We find evidence of substantial directedness in search. Finally, we study the implications for underinvestment due to holdup problems and show that the observed degree of directedness substantially reduces underinvestment relative to a setting with random search.

JEL classification

J62, J63, D83, D4

Keywords

random search, directed search, partly directed search, structural estimation, efficiency

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* This paper has benefited from numerous questions and comments from participants at several seminars, conferences and workshops. Among them, Hector Chade, Philipp Kircher, Guido Menzio, and Laura Pilossoph (discussant). A previous version of this paper has circulated under the title "Competitive or Random Search?". We are grateful to the ECONAU project database (Department of Economics and Business Economics, Aarhus University) for making their data available.

1 Introduction

In the search model literature, there is little consensus on how to model the search technology. One approach is to model search as random, where agents of different types on the same side of the market search in a common pool, thereby generating search externalities for one another. Another option is to model search as directed, in which case agents of different types tend to be separated into distinct submarkets. An example of random search is the canonical DMP model (Diamond 1982, Pissarides 1985, Mortensen 1982), but the Burdett–Mortensen (Burdett and Mortensen 1998, Christensen et al. 2005) and Postel–Vinay and Robin (Postel-Vinay and Robin 2002) types of search models also fall into this category. In Competitive Search Equilibrium, by contrast, firms divide the market into submarkets through their wage advertisement policies and hence endogenously create directed search. However, competitive search is not a prerequisite for directed search, as search may also be directed if wages are determined *ex post* (after agents meet), for instance through bargaining, although the definition of directed search in these cases is, in general, not as clearly defined.

The extent to which search is directed has consequences for the welfare properties of search equilibria. Acemoglu and Shimer (1999) show that if search is random and wages are determined by bargaining, firms invest too little in capital, even if the rent-sharing rule satisfies the Hosios condition. If search is directed, by contrast, so that firms that invest more in physical capital search in separate submarkets, efficiency prevails. Similarly, Acemoglu and Pischke (1999) show that with random on-the-job search, matches under-invest in general human capital even under efficient bargaining, while Moen and Rosén (2004) show that under competitive search, efficiency prevails.[1]

The extent to which search is directed also has consequences for the empirical predictions for wages before and after a job-to-job transition. Garibaldi et al. (2016) point out that if search is perfectly random, and a worker changes jobs whenever a new employer offers a higher wage than the current one, then the distribution of the wage in the destination job w_d is independent of the origin job wage w_o, conditional on being above it, $w_d > w_o$. In contrast, directed search implies a positive dependence between the origin wage w_o and the destination wage w_d, and if perfectly directed, the destination wage distribution is degenerate at a point. However, Godøy and Moen (2013) document that this independence property between ante and post wages (or productivities) cannot be found in Norwegian data.

The identification strategy in this paper exploits this difference in predictions from the random and the directed search model to arrive at a search technology estimate that is neither perfectly random nor perfectly directed. We develop a model of partly directed search. In our model,

[1] More broadly, while the random search model equilibrium can be socially efficient when the appropriate version of the Hosios condition is satisfied, in the event that Hosios is not satisfied, optimal policy prescriptions are subject to second-best considerations that depend on the estimates of matching and bargaining technology parameters. Optimal policy design analyses often make the sensible assumption of disregarding the second-best considerations through the imposition of the Hosios condition, but the mechanisms by which the economy would itself arrive at the Hosios condition are murky at best.

firms differ in productivity, and workers search both on and off the job, gradually climbing a job ladder. The contract space is sufficiently rich that an employee's search behavior maximizes joint (firm–worker) income.

Our starting point is a model in which search is directed but not competitive. Workers decide which firm type to apply to. All firms of a given type and the workers searching for jobs in those firms form a submarket, and workers choose among these different submarkets. When a match forms, wages are determined through bargaining, with the net present value of joint income in the previous job as the worker's outside option. If the worker was previously unemployed, the outside option is the value of unemployment. We assume that workers' bargaining power satisfies the Hosios condition, which ensures efficiency in a search market in which workers and firms are identical. Note that employment contracts are not advertised publicly by firms but negotiated *ex post* after a match has taken place, as in the standard DMP model, but with the modification that the outside option is as in Dey and Flinn (2005) and Cahuc et al. (2006). Furthermore, the division into submarkets is not determined by firms' wage advertisement policies, but rather by firm types and workers' choices of which firms to approach. As a result, the equilibrium in our model has a much simpler structure than in a Competitive Search Equilibrium and, in our view, captures the core elements of directed search with bargaining.

Although workers direct their search, we allow the technology for doing so to be imperfect. As a result, the submarket in which a worker ends up is partly determined by random factors. We use a logit model to describe this noisy choice process.[2] Hence, a given worker may search in any of the existing submarkets, but the probability of entering a particular submarket depends on the expected return to search there relative to other submarkets. The more sensitive these probabilities are to relative returns, the more directed the search process.[3] This sensitivity is governed by a logit parameter μ, ranging from zero (perfectly sensitive) to infinity (completely insensitive). In the latter case, search is completely undirected, and workers are assigned to submarkets in accordance with the relative size (relative number of job openings) in that submarket. In the former case, search is perfectly directed, in the sense that workers only search in their preferred submarkets.

Maybe surprisingly, we find that the perfectly directed search equilibrium is not equal to the competitive search equilibrium; a firm that is allowed to advertise a wage would advertise a lower wage than the wage it pays in equilibrium in our model. Relatedly, although welfare improves as directedness increases, the equilibrium allocation with perfectly directed search is not efficient, even though the Hosios condition is satisfied. The reason is that workers, although equally productive, create different match surpluses when matched with a firm of a given productivity. When a worker enters a submarket, this generates a positive externality for the firms in the market, and this externality is stronger the more productive the firms in that submarket are. Furthermore, it creates

[2] Our analysis does not require us to take a stand on the particular microfoundation, but one interpretation is that of costly precision in decisions used in Matějka and McKay (2015) and Cheremukhin et al. (2020).

[3] An alternative interpretation of the noise process, which we do not pursue, is that workers have preferences over more aspects of a job than wages, such as amenities, which are unobservable to the econometrician. The choice of interpretation matters for our welfare analysis, but not for the predictive properties of the model.

a negative externality for the workers already searching in that market, and this externality is lower the higher the submarket is, since the workers in the higher submarket are currently employed in higher-productivity firms, so that their gain from search is lower. Since workers do not internalize these effects, they join submarkets with too low productivity compared to what is socially optimal. Hence, even with perfectly directed search, submarkets with high-productivity firms attract too few applicants. In Competitive Search Equilibrium, by contrast, firms govern workers' choices through their wage-posting policies, and these externalities are internalized.

Given our model, we identify the degree of directedness in the Danish labor market for the period 1995-2005. Specifically, we use the administrative matched employer–employee spell data that allow us to credibly observe job-to-job transitions and the productivities of both the destination and origin jobs associated with them. As argued above, directedness manifests directly in these observed mobility patterns.

To this end, we first derive theoretical predictions for how transition patterns associated with job-to-job transitions differ between directed and random search. With random search, all firms meet workers at the same rate. If there is some direction in search, submarkets with high-productivity firms attract more workers, which reduces the job-finding rates in these markets. In isolation, this reduces their attractiveness, and more so for workers at the low end of the job ladder with low current incomes. Let firm productivity be increasing in firm type, and consider a searching worker employed in a firm of type i. Denote by $\hat{h}_{i,k}$ the probability that this worker searches in submarket $k > i$, conditional on searching in a submarket k or higher. A fundamental property of directedness is that this probability is *higher* the lower the firm type i that the worker is currently in. The reason is that as long as there is some directedness in search, jobs in submarket k are easier to get but pay less than in higher submarkets, and this makes submarket k relatively more attractive the lower the current income of the worker. Therefore, $\hat{h}_{i,k}$ is an important moment to target when estimating the model.

As the submarkets form a ladder, we can identify them following the revealed preference argument used in both Sorkin (2018) and Bagger and Lentz (2019). Similarly to Bagger and Lentz (2019), we use the poaching rank index. In a higher submarket, a greater fraction of its inflow comes from other firms relative to unemployment. With this, we can then immediately measure the $\hat{h}_{i,k}$ statistics, which provide the core of the identification strategy. Combined with other statistics, we estimate the model using a standard simulated minimum distance estimator. A particular virtue of the estimation is its implicit recognition that the poaching rank submarkets identify the submarkets with some classification error, which affects the inference the model draws from the $\hat{h}_{i,k}$ mobility patterns. This corrects a bias that would otherwise have been present had we adopted an alternative such as the commonly used two-step strategy where submarkets or firm types are classified in a first step and then treated as data in the second step.

The magnitude of the model parameter that governs the degree of directedness, μ, is subject to the measurement unit choices in the model. As such, the particular estimated value is, by itself, not all that informative about the degree of directedness in the estimated model. We proceed to provide three different perspectives on its magnitude.

First, as an interpretable measure of the degree of directedness, we construct a welfare-based index of directedness. Welfare is measured as the net present value (NPV) of utility for an unemployed worker; as shown by Pissarides (2000), this provides an appropriate welfare metric in search models. Using our estimated parameter values (except for μ), let welfare under perfectly directed search ($\mu = 0$) be denoted by S^P, and welfare under random search by S^R. Finally, let S^O denote welfare evaluated at our estimated value of μ. The degree of directedness, d_r, is then defined as

$$d_r = \frac{S^O - S^R}{S^P - S^R}.$$

Our estimation indicates that d_r is approximately 0.41. That is, the estimated economy realizes about 41 percent of the welfare difference between an economy with a perfectly directed search technology and one in which search is entirely random.

Second, we provide an elasticity measure of how sensitive submarket tightnesses are to changes in match values. Here, we are contrasting the two extremes, perfectly random and perfectly directed search. The random search model is an ordinal model, where only firm type ranks matter for mobility patterns, whereas the directed search model is cardinal, so that productivity differences between firms also matter. Therefore, if search is entirely random, submarket tightness elasticities are zero. In the perfectly directed search model, they are infinite. For a receiving submarket with average tightness, a one percent increase in the search flow value relative to top-firm productivity increases tightness by 2.5 times the relative size of the sending market.

Third, we adopt another welfare-related perspective; in this case, we consider the interaction between directedness and investment incentives. Specifically, we consider the possible hold-up problem associated with ex ante capital investment, as studied in, for example, Acemoglu and Shimer (1999) and Card et al. (2014). We evaluate how much ex ante underinvestment in capital we see in a simplified version of the estimated model in which only unemployed workers search, in which case the perfectly directed search model is efficient. We compare the perfectly random and perfectly directed economies. Again, we are leveraging a sharp contrast between the two models: In the perfectly directed search model, ex ante capital investments are efficient. As search gets less directed, the capital underinvestment problem becomes worse. Similar to above, we state the degree of directedness in terms of the degree to which the estimated economy realizes the efficiency gains of the perfectly directed search model relative to the perfectly random model. In this case, we obtain a measure of 85 percent.

The paper proceeds as follows: In Section 2 we set up our model. In Section 3 we discuss the equilibrium properties of our model in the two extremes (completely random and perfectly directed search). In Section 4 we discuss the general efficiency properties of the directed search equilibrium in our model. We further show that the version with perfectly directed search is not efficient. In Section 5 we set up our empirical exercise and define an extended version of our model (adding bells and whistles). We also discuss estimates and model fit. In Section 6 we discuss the overall implications of our estimates regarding directedness in search and consequences for underinvestment. Finally, we conclude.

2 Model

In this section, we present the basic model. Some extensions are added later for estimation purposes.

The model is set in continuous time. The economy is populated by a continuum of risk-neutral workers and firms with a common discount rate r. All workers are identical, infinitely lived, and search both on and off the job. The measure of workers in the economy is exogenous and normalized to one. The measure of firms is endogenously determined by entry.

Unemployed workers receive an exogenous net income of y_0 that also includes the monetary equivalent of the disutility of that state. Entrepreneurs incur a cost K to open a firm. After K is sunk, but before production starts, the "type" j of the firm is drawn, $j = 1, ...J$. The probability of drawing type j is τ_j, with $\sum_{j=1}^{J} \tau_j = 1$. The type determines the productivity y_j of the firm, $y_0 < y_1, ... < y_{J-1} < y_J$. All firms have a constant-returns-to-scale technology in labor.

Each firm is equipped with a search technology. We adopt the "fishing line" hiring interpretation where each firm has one vacancy, which is reposted immediately every time a worker is hired. All matches dissolve with a Poisson rate δ. Workers have one unit of search capacity they can exert at no cost.

The search market is divided into J submarkets. All firms in submarket j are of type j. In each submarket j, the flow of new matches is given by a concave and constant returns to scale matching function $x(\tilde{n}_j, v_j)$, where $\tilde{n}_j$ and v_j denote the aggregate measure of searching workers and vacancies in the submarket, respectively. The submarket-specific job finding rates and worker finding rates are denoted $p_j = p(\theta_j)$ and $q_j = q(\theta_j)$, respectively, where $\theta_j = v_j/\tilde{n}_j$.

As is common in the competitive and directed search literature, we assume efficient contracting between the worker and the firm.[4] This implies that (1) the worker searches to maximize the joint income of the worker–firm pair, (2) only jobs that yield a higher joint net present value (NPV) are accepted, (3) meetings with inferior firms do not lead to a wage increase, and (4) the previous firm provides full rent extraction as a bargaining position for the worker in the new job. Several contractual arrangements can support this outcome, the simplest being that the worker buys the job up front. Note that (3) eliminates rent extraction from the current employer as a motivation for on-the-job search, consistent with the assumption that the worker searches to maximize joint surplus.

Wages are determined by Nash bargaining. The firm's outside option in the bargaining game is zero.[5] The worker's outside option is the joint income in the previous job, as specified in (4) above. The worker and the firm split the surplus according to the Hosios sharing rule, implying that the worker's share of the surplus, β, satisfies $\beta = -\theta q'(\theta)/q(\theta)$.[6]

[4] Similar assumptions regarding efficient contracting are made in Lentz (2010, 2015), Bagger and Lentz (2019), Moen and Rosén (2004), Garibaldi et al. (2016), and Menzio and Shi (2011).

[5] This follows from the assumption that the firm continuously searches, so that a worker does not fill a slot that otherwise could have been filled by another worker.

[6] This is the same type of wage determination and job acceptance rule as in Dey and Flinn (2005) and Cahuc

At any point in time, the worker chooses which submarket to enter. With perfectly directed search, all workers end up searching in the submarket that contributes the most to the joint value of the current worker–firm pair. In the general case, the return to search for a worker in a given search market influences the probability that the worker searches in that market, but not fully, as noise or other random factors may disturb the search process. We denote the probability that a worker employed in a firm of type i searches in submarket j by π_{ij}.

In transitions, we will generally refer to the sending firm's type by i and the receiving firm's type by j. We will refer to a worker employed in a firm of type i as an *i-worker*. Let M_i denote the joint value of a worker–firm pair when the firm is of type i. As stated above, a worker accepts a job offer from a firm of type j if and only if $M_j > M_i$, which in turn holds if and only if $j > i$.[7] We assume that randomization over the markets in which a worker searches occurs continuously. It follows that M_i is independent of the submarket in which the worker happens to be searching at any given moment. Hence, we have

$$rM_i = y_i - \delta(M_i - M_0) + \beta \sum_j \pi_{ij} p(\theta_j) \left(M_j - M_i\right)^+ , \tag{1}$$

where $(M_j - M_i)^+ \equiv \max(M_j - M_i, 0)$. The NPV of the unemployed worker is

$$rM_0 = y_0 + \beta \sum_j \pi_{0j} p(\theta_j) \left(M_j - M_0\right)^+ .$$

Next, we determine π_{ij}. To that end, and as part of the technology underlying the workers' choice of submarket, define the *perceived value of search* for an i-worker searching in market j, m_{ij}, as

$$m_{ij} = y_i - \delta(M_i - M_0) + \beta p(\theta_j) \left(M_j - M_i\right)^+ - p(\theta_j) \left(M_i - M_j\right)^+ . \tag{2}$$

The first three terms in (2) are identical to the first three terms in (1), and together they constitute the flow value of searching in submarket j for an i-worker. The last term in (2) is absent in (1). Since workers are free to reject offers, there are no costs associated with receiving inferior offers (offers that will be rejected). As such, there is no difference in value between directing search toward different inferior submarkets. However, when building a theory of how workers direct their search, it is appealing to assume that workers also perceive an ordering among the inferior submarkets. The last term achieves this through a thought experiment in which the worker cannot refuse the offer and must bear the entire cost of doing so. A benefit of this construct is that workers in high submarkets are, to a lesser extent, crowding the lowest submarkets with jobs they would never

et al. (2006), except that we do not allow for renegotiation within the job. However, this differs from the wage-setting mechanism in the canonical on-the-job search model of Pissarides (1994), where the worker's outside option in bargaining with the new employer after a job switch is unemployment.

[7] Given the ordering of y_j, this follows directly. In a more general model with more dimensions of heterogeneity across submarkets, order the submarkets so that $M_j > M_i \Leftrightarrow j > i$.

accept.

We are now ready to define π_{ij}, the probability that an i-worker searches in submarket j. We define it by the logit expression

$$\pi_{ij} = \frac{\tau_j \exp\left(\frac{m_{ij}}{\mu}\right)}{\sum_{j=1}^{J} \tau_j \exp\left(\frac{m_{ij}}{\mu}\right)}. \tag{3}$$

If $\mu = 0$, we have

$$\begin{aligned} \pi_{ij} &= 0, \quad \text{if } m_{ij} < \max_j m_{ij}, \\ \pi_{ij} &\in [0,1], \quad \text{otherwise.} \end{aligned} \tag{4}$$

Note that the probabilities are scale-dependent. To be more specific, let $m_{ij} = \bar{m}_{ij}k$, where k is a scaling factor. It follows directly from (3) that an increase in k has the same effect as an increase in the inverse of μ and is invariant to a proportional change in k and μ.[8]

The stochastic nature of the workers' choice of submarkets may be given different interpretations; see, for example, Anderson et al. (1992). In one interpretation, it reflects genuine noise. In our setting, this may represent noise in the search process: workers aim to direct their search toward the submarkets that yield the highest return to search, but frictions imply that they are only partly successful. They may, for instance, misjudge the wage or productivity of firms, or the likelihood of obtaining the jobs they offer. A related interpretation is that of Sims (1998) and Sims (2003), where the cost of concentrating a choice distribution around particular choices is based on its divergence from uniformity. As used in Matějka and McKay (2015) and Cheremukhin et al. (2020), if the cost is the Kullback-Leibler divergence, one obtains the logit form. Thus, workers may end up in submarkets other than the optimal one, yet with a higher probability of landing in the more attractive submarkets.[9]

The notion that agents tremble while deciding on actions, and that other agents take this into account, is similar to the underlying logic of the *Quantal Response Equilibrium* (QRE); see McKelvey and Palfrey (1995). Under this interpretation, it is implicitly assumed that workers learn the true submarket type at some point prior to making their acceptance or rejection decision.

Another interpretation is that the stochastic choice of submarkets reflects unobserved heterogeneities associated with different jobs or search markets that influence their attractiveness. Examples include amenities (as in Taber and Vejlin 2020), geographic location, or changes in preferences over job characteristics that lead to lower wages, and so on. Hence, the observed wage is only a noisy measure of the attractiveness of a submarket; other aspects also play a role in determining the overall desirability of a job or market. Under this interpretation, workers may in fact target their

[8] The choice probabilities are unaltered if all values of m_{ij} increase with a constant.

[9] As an abstraction, we may introduce a "market maker" that allocates workers and firms to submarkets. Within this abstraction, our assumption is that the market maker "trembles" when allocating workers across submarkets, but not when allocating firms, possibly because firms are larger. We conjecture that allowing for two-sided trembles would qualitatively yield the same results.

search fully when choosing between submarkets, but because of these unobserved heterogeneities, their choices appear random to the econometrician. In this case, workers may accept job offers in submarkets below their current rank, provided that the amenities are associated with the jobs themselves (in contrast to our assumption regarding job switches). If the amenities are instead associated with the search process, they would not.

Which interpretation one adopts is critical for how the observed randomness in workers' search behavior should be understood. If it reflects genuine noise, it points to randomness in the search process itself, in the sense that workers are not fully able to direct their search. If it reflects unobserved heterogeneities, it merely indicates that wages are a noisy measure of a job's overall attractiveness. We adopt the "noise" interpretation. Hence, when a worker directs search toward a submarket that does not maximize her monetary payoff, her NPV utility is reduced and a welfare loss arises. We treat this randomness as an integral and exogenous component of the search technology, consistent with the interpretation of the matching function in Petrongolo and Pissarides (2001).

Even though workers do not accept inferior jobs, their presence when searching in the associated submarket crowds the market. The tightness of submarket j is therefore given by

$$\theta_j = \frac{\tau_j k}{\sum_{i=0}^{J-1} \pi_{ij} N_i}, \tag{5}$$

where N_i is the measure of i-workers and N_0 is the unemployment rate.

The value of a fishing line to a firm in submarket j is

$$rV_j = q(\theta_j)(1-\beta) \sum_i (M_j - M_i)^+ f_{ij}, \tag{6}$$

where f_{ij} is the conditional probability that a meeting is with an i-worker. By definition, $\sum_i f_{ij} = 1$. This probability is defined by

$$f_{ij} = \frac{\pi_{ij} N_i}{\sum_{i \geq 0} \pi_{ij} N_i}. \tag{7}$$

In equilibrium, the total number of firms (equal to the total number of vacancies) is given by the free entry condition,

$$E[V] = \sum_{j=1}^{J} \tau_j V_j = K. \tag{8}$$

Balanced-flow equations

Turning to the steady state, the numbers of i-workers have to satisfy a set of flow-balance equations for $i = 0, \ldots, N$. The equations are given by

$$N_0 \sum_j \pi_{0j} p(\theta_j) = \delta(1 - N_0) \tag{9}$$

$$N_i \left(\delta + \sum_{j>i} \pi_{ij} p\left(\theta_j\right) \right) = p\left(\theta_i\right) \sum_{j<i} \pi_{ji} N_j, \; i = 1, \ldots, N. \tag{10}$$

Equation (9) combines the balance of flows in and out of unemployment with the normalization of population size at unity.

We are now ready to define equilibrium:

Definition 1 (Equilibrium)**.** A steady-state equilibrium with wage bargaining is a vector of net present values M_j, $j = 0, \ldots, J$, a matrix of perceived flow values m_{ij}, $i = 0, \ldots, J$, $j = 1, \ldots, J$, a matrix of choice probabilities π_{ij}, $i = 0, \ldots, J-1$, $j = 1, \ldots, J$, a vector of labor market tightnesses $\theta = (\theta_1, \ldots, \theta_J)$, a number of firms k, and an allocation of workers over unemployment and firm types $N = (N_0, \ldots, N_J)$, such that:

1. The value functions M_i and m_{ij} satisfy equations (1) and (2).
2. The choice probabilities π_{ij} are given by (3) when $\mu > 0$, and by (4) when $\mu = 0$.
3. The vector of labor market tightnesses $(\theta_1, \ldots, \theta_J)$ satisfies (5).
4. The number of firms k is determined by (6) and the free-entry condition (8).
5. The allocation of workers across firm types and unemployment N is determined by the flow-balance equations (9)–(10).

Proposition 1. *The equilibrium exists.*

3 Equilibrium properties

In this section, we first discuss the limit properties of our equilibrium as μ approaches 0 and ∞, before we derive a fundamental property of directed search.

3.1 Limit properties

Consider first the equilibrium of the model when $\mu \to \infty$. In this case, (3) implies that the probability that a worker goes to submarket j is equal to $\frac{\tau_j}{\sum_{j=1}^{J} \tau_j} = \tau_j$. Hence, the worker has the same probability of meeting any vacancy, as in any model of random search.[10] Consider then the limit as $\mu \to 0$. In this limit, the transition probabilities are given by equation (4). Search is perfectly directed, and a worker will enter submarket j only if this submarket maximizes his return from search. It is instructive to compare this equilibrium with the Competitive Search Equilibrium (CSE) concept as presented in Garibaldi et al. (2016), and we will establish results highlighting both similarities and differences between the two equilibrium concepts.

We first point out the similarities. At a general level, workers in both models enter the submarket or submarkets that maximize their return to search. Furthermore, in both models, the surplus is shared according to the Hosios sharing rule–by assumption in the directed search model and as an equilibrium outcome of a wage-posting game in the CSE model. Finally, in both models, there is only one firm type in each submarket.

It is well known that in competitive search equilibrium, employed workers with a high current wage search for firms offering a higher wage than employed workers with a lower current wage do; see, for instance, Delacroix and Shi (2006) and Menzio and Shi (2010). Garibaldi et al. (2016) go one step further and derive a maximum separation result, showing that workers with higher-ranked current employers tend to search for higher-productivity firms. A similar result holds in the perfectly directed search equilibrium. To be more precise, let I_i denote the set of submarkets in which a worker currently employed at firm i searches. Then, for all $j \in I_i$, $M_{ij} = \max_l M_{il}$. By definition, it follows that $\pi_{ij} = 0$ if and only if $j \notin I_i$. If I_i contains a single element j^*, then $\pi_{ij^*} = 1$.

Proposition 2 (Maximum separation)**.** *Suppose $i < i'$. Then the highest firm type in I_i is smaller than or equal to the lowest firm type in $I_{i'}$.*

The proposition implies that the equilibrium leans toward a strict job ladder, and that workers in two different firm types meet in at most one submarket. The higher the current state, the higher the submarkets in which the agent searches.

In the appendix, we prove the proposition using a "revealed-preference" type of argument. Workers who currently have a high wage are more willing to accept longer waiting time for high wages than workers who currently have a low wage. To gain intuition, consider two workers who are currently employed in firms of type $i = l$ and $i = l + 1$, respectively, both searching in submarket j ($j > i$). Their flow income (rM_{ij}) while searching in market j (equal to the perceived value of search when search is perfectly directed) is

$$m(y_i, y_j) = y_i + \delta\,(M(y_i) - M_0) + \beta\, p(\theta_j)\big(M(y_j) - M(y_i)\big),$$

[10] The equilibrium does not converge to the random search equilibrium if search is competitive, as in GMS, in which case we conjecture that the Albrecht-Axell equilibrium would emerge; see Albrecht and Axell (1984). In the Albrecht-Axell equilibrium, low-productivity firms offer a wage equal to the reservation wage of unemployed workers. With sufficient dispersion in firm productivities, productive firms offer a wage exceeding the productivity of the low-productivity firms so that workers employed in these firms would also accept a job offer. The reason why our equilibrium does not converge to the Burdett-Mortensen equilibrium is that the worker's job acceptance decision in our model (in contrast with the Burdett-Mortensen model) is set so as to ensure efficient job transitions and is therefore independent of the wage the firm pays.

and the slope of their indifference curves in submarket j is given by

$$-\frac{d\theta_j}{dy_j} = \frac{p(\theta_j)}{p'(\theta_j)} \cdot \frac{M'(y_j)}{M(y_j) - M(y_i)}. \tag{11}$$

Since $M(y_i)$ is lower for $i = l$ than for $i = l + 1$, it follows that the absolute value of the slope of the indifference curve for the l worker is lower than for the $l + 1$ worker, evaluated at (θ_j, y_j) (or at any point where the two curves intersect). First, this implies single crossing: the indifference curves intersect at most once. Second, if worker l is indifferent between submarket l and submarket i, then worker $l + 1$ will strictly prefer submarket l when $i < l$, and submarket i when $i > l$.

An important difference between perfectly directed search and competitive search concerns the allocation of workers across submarkets. Under directed search, firms would prefer low-i workers to join their submarkets, because such workers generate larger match surpluses, of which the firm obtains a share through bargaining. However, since firms do not advertise wages, they have no instrument with which to influence workers' choice of submarkets. We now examine this difference more closely. To that end, consider the perfectly directed search equilibrium, and then allow a group of type-j firms (for arbitrary j) with measure zero to deviate by posting an NPV wage W (hereafter simply referred to as a wage).

To avoid discreteness problems and to make calculus applicable, we study the limit properties of the perfectly directed search equilibrium as the number of firm types converges to a continuous distribution on $[y_{\min}, y_{\max}]$. Garibaldi et al. (2016) show that a competitive search equilibrium exists with a continuum of types, and we conjecture that the same argument applies to the perfectly directed search equilibrium. It follows from the maximum separation result that each submarket, which we index by y_j, attracts workers whose current employer is of exactly one type, which we index by y_i.[11] For each submarket, we can write the endogenous variables—NPV wages, tightness, and sending-firm type—as functions of y_j: $W(y_j)$, $\theta(y_j)$, and $y_i(y_j)$ (the productivity of the sending firm). It follows that these functions are continuous and strictly monotone; a proof is available upon request. Finally, let $\mathcal{W}$ denote the set of equilibrium wages in the perfectly directed search allocation, i.e., the image of $W(\cdot)$.

If a deviating set of firms with measure 0 (hereafter, a deviating firm) advertises a wage $W \in \mathcal{W}$, workers will enter the resulting submarket up to the point at which they are indifferent between searching in the new submarket and their best alternative submarket. In the appendix, we show that this is equivalent to assuming that the deviating firm can choose which of the existing submarkets to enter and pay the equilibrium wage in that submarket, independently of its own productivity.

Because $W(\cdot)$ is strictly increasing, it admits an inverse, so that $y_j = W^{-1}(W)$ for $W \in \mathcal{W}$. It follows that the tightness facing a deviating firm advertising a wage $W' \in \mathcal{W}$ is equal to the tightness in the existing submarket that pays the equilibrium wage W', namely the submarket in

[11] Suppose workers employed in both y_i-firms and y'_i-firms, with $y'_i > y_i$, entered submarket y_j. The result that the targeted submarket is non-decreasing in the searching worker's value y_i would still apply. Hence, all sending submarkets in the interval $[y_i, y'_i]$ would enter submarket y_j, violating the maximum separation principle.

which the receiving firms have productivity $y'_j = W^{-1}(W')$, with tightness $\theta(y'_j)$.

Proposition 3. *Consider the perfectly directed search equilibrium with a continuum of firm types. Suppose that a set of measure zero of firms with productivity y'_j are allowed to advertise wages as described above, where $y_i(y'_j) > y_{\min}$. Then the deviating firms will choose a wage that is strictly lower than the NPV equilibrium income $W(y'_j)$ in the y'_j-submarket.*

The proof is given in Appendix A.2. To understand the result, note that both sides of the market have preferences over both wages and the tightness in a submarket. In a given submarket, wages are determined by bargaining under the Hosios condition, which ensures that positive and negative externalities from search cancel out. Hence, in all submarkets, the indifference curves of the workers in the market and the iso-profit curves of the firms in that market are tangent, so that trade in the θ–W space is efficient given the identities of the agents in that submarket.

However, there are no analogous mechanisms that ensure an efficient allocation of workers of different values y_i across submarkets. As already mentioned, a firm would prefer to attract low-y_i workers, who have a weaker bargaining position, but has no instrument with which to attract them. From a worker's perspective, he cannot join a higher submarket and obtain the going wage in that submarket, since his outside option in the bargaining game, and hence his wage, would be lower than that of the (high-value) workers already populating that submarket (the choice set in (θ, W) – space is worse for the low y_i worker than for the high y_i worker). As a result, the allocation of workers across submarkets is inefficient, and this inefficiency is exploited by the deviating firms.

As the deviating firm lowers its wage, it attracts workers with a lower y_i, who obtain a lower wage partly because their bargaining position is weaker, that is, they are less able to extract rents than their peers with a higher y_i. These workers are therefore cheaper, and for small reductions in W below $W(y_j)$, this manifests itself in a smaller increase in θ than if the firm were still attracting the same workers.

When the deviating y_j-firms reduce the wage, they deviate from the Hosios condition for these workers, since they pay them less than the share β of the surplus. The deviating firms effectively pay the same wage as less productive firms do in equilibrium, which in turn pay the Hosios wage. However, this deviation from the Hosios condition has only a second-order effect on firms' profits. Hence, the deviating firms gain from reducing the wage somewhat.

3.2 A Fundamental Property of Directed Search

We now turn to the general case in which $0 < \mu < \infty$. In this case, workers direct their search to some extent, yet all submarkets remain in the support in the sense that each is chosen with strictly positive probability.

Define by h_{ij} the equilibrium transition probability for a worker who is currently employed at a firm of type i and moves to a firm of type j. Furthermore, for any $i < j$, define

$$\hat{h}_{ij} = \frac{h_{ij}}{\sum_{k=j}^{J} h_{ik}}, \tag{12}$$

which is the probability of moving to firm j, conditional on moving to submarket j or above, while currently in i.

Proposition 4. *$\hat{h}_{ij}$ defined by* (12) *is strictly decreasing in i as long as search is not completely random (that is, for all $\mu < \infty$). When search is completely random, $\hat{h}_{ij}$ is constant in i.*

The proposition states that as long as search is at least to some extent directed, the probability of transitioning to a given firm $j > i$ (that is, to a firm with a higher type than the current employer), conditional on moving to j or higher, is strictly lower the higher the type of the firm the worker is currently employed in. We consider this a fundamental property of directedness in search.

To build intuition for the result, consider first the case of completely random search. In this case, the probability that a worker transitions to a firm of type $j > i$, conditional on transitioning to a firm of type j or higher, is equal to the ratio of the number of vacancies posted by firms of type j to the total number of vacancies posted by firms of type j or higher. This ratio is, of course, independent of the current employer type i for all $i < j$.

This changes fundamentally when search is directed. Workers then face a trade-off between obtaining a high job-finding rate and obtaining a high wage when a job is found. Moreover, how a worker makes this trade-off depends on her current employer's type. The better the current employer, the more willing the worker is to give up a high job-finding rate in return for a higher wage; see equation (11). Therefore, the higher the type of the current employer, the less attractive it is to apply to firm j relative to more productive firms. Since the choice of submarkets is influenced by their relative attractiveness, we therefore expect $\hat{h}_{ij}$ to be decreasing in i. Proposition 4 confirms this expectation.

Intuitively, we would also expect the dependence of $\hat{h}_{ij}$ on the current employer's type i to be stronger the more directed search is (that is, the smaller is μ). Our simulations confirm this intuition. We will use an empirical version of $\hat{h}_{ij}$ in our indirect inference setup for identification of the model.

4 Efficiency properties of directed search equilibrium

In the $\mu \to \infty$ limit, where search is random, all submarkets have the same labor market tightness. Consequently, job-filling rates are identical across firms. Efficiency would improve if more workers entered the high submarkets at the expense of the low ones, thereby increasing tightness in the high-productivity markets relative to the low-productivity ones. Directed search allows this reallocation and therefore raises welfare. It is less clear, however, whether private and social incentives to enter the various markets coincide, particularly in the case of perfectly directed search. Proposition 3 indicates that the perfectly directed equilibrium is not efficient, and below we confirm this formally.

For a given number of firms k, the welfare function can be written as

$$W = \int_0^\infty \sum_{i=0}^{I} N_i(t) y_i e^{-rt}\, dt.$$

In the socially optimal allocation, W is maximized with respect to π_{ij} for $i = 0, \ldots, J-1$ and $j = 1, \ldots, J$, subject to the dynamic constraints (9) and (10).

Perfectly directed search

In the social planner problem above, let λ_i be the adjoint variables associated with the law of motion for N_i. Let $\tilde{\lambda}_j = \sum_{l=0}^{j-1} f_{lj}\lambda_l$ denote the average value of λ in submarket j. In Appendix A.4, we show that λ_i can be written as

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \beta \sum_{j>i} \pi_{ij} p(\theta_j)(\lambda_j - \lambda_i) - (1-\beta) \sum_{j>i} p(\theta_j)\pi_{ij}(\lambda_i - \tilde{\lambda}_j). \tag{13}$$

The second term on the right-hand side is zero if $i = 0$. The adjoint variable λ_i can be interpreted as the value to the planner of an increase in N_i of one unit; see Seierstad and Sydsaeter (1986), ch. 3.5. It is the social counterpart to M_i, the joint income of a worker–firm pair with firm type i. The flow value $r\lambda_i$ consists of the flow value of output y_i, the flow loss from exogenous job separation, and the flow gains from on-the-job search. Importantly, we show in the appendix that the expression for λ_i gives the social value of employment in market i for any given (steady state) allocation $(\theta_j)_{j=1}^J$, not only at the optimal allocation.[12]

As a preliminary observation, note that if $\pi_{ij} = 1$ for some j, and $\pi_{lj} = 0$ for all $l \neq i$ (i-workers only search in submarket j while no other workers search in that market), then the last term in (13) is zero, and we have

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \beta p(\theta_j)(\lambda_j - \lambda_i),$$

which is identical to the expression for M_i under the same conditions. This is as expected: if a worker enters a search market, it reduces tightness in the market and crowds out some of the matches that would otherwise have taken place in that market. For each match the worker creates, $(1-\beta)$ matches are crowded out, where $\beta = -q'(\theta)\theta/q$. If the match surplus of the new matches and the crowded-out matches are equal, the social and private values of entering exactly balance when workers obtain a share β of the match surplus, that is, when the Hosios condition is satisfied.[13]

With this observation in mind, note that the last term in equation (13) captures the additional search externalities that arise because the match surpluses differ across searching workers depending

[12] In the last part of Appendix A.4, we derive equation (13) for any given restrictions on π_{ij}, that is, for any allocation $(\theta_j)_{j=1}^J$. For a given matrix $\bar{\pi}_{ij}$, we maximize welfare with respect to π_{ij} given the constraint that $\pi_{ij} = \bar{\pi}_{ij}$. We then obtain the marginal values of changing the π_{ij}'s.

[13] The Hosios condition implies that the bargaining power of the worker, β, is equal to $-q'(\theta)\theta/q$, which is satisfied by assumption in our model.

on their current status. We refer to this term as the net search externalities. In submarket j, the match surplus between a firm and an i-worker is $\lambda_j - \lambda_i$. Hence, the higher is i, the lower is the surplus. For each submarket $j > i$,

$$\lambda_i - \tilde{\lambda}_j \equiv \left(\lambda_j - \tilde{\lambda}_j\right) - \left(\lambda_j - \lambda_i\right)$$

is equal to the difference between the average match surplus in submarket j and the surplus generated by a match involving an i-worker. If the other workers in the submarket tend to be employed in firms with lower productivity than y_i, so that $\lambda_i > \tilde{\lambda}_j$, the net externality of the i-worker in the j-market is negative. That makes sense since the net externality is zero when the match surpluses of the new and crowded-out matches are equal. Summing over all the submarkets in which i-workers search gives the last term in (13).

Next, we consider the optimal choices of π_{ij}. In Appendix A.4, we show that the planner chooses π_{ij} to maximize the social values λ_i given by (13), analogous to the market solution in which a worker employed in firm i chooses submarkets to maximize M_i. The planner solves a linear programming problem with first-order conditions given by

$$\beta p(\theta_j)\left(\lambda_j - \lambda_i\right) - (1-\beta)\, p(\theta_j)\left(\lambda_i - \tilde{\lambda}_j\right) \leq \sigma_i N_i^{-1}. \tag{14}$$

With complementary slackness (the inequality in 14 binds unless $\pi_{ij} = 0$, in which case it is strict), and with the Lagrangian parameter $\sigma_i > 0$, the right-hand side is independent of j. Since the externality is positive in markets above those in which the worker in question searches, the externality moves the planner toward sending workers to higher submarkets than the workers choose in the market solution.

As in Proposition 3, we assume that the distribution of firms is continuous, in which case there is a one-to-one correspondence between the types of the sending and the receiving firms, as established in the discussion preceding Proposition 3. To obtain clear results, we analyze the planner's incentives along the perfectly directed search equilibrium, in which case λ and M coincide. Along this path, we can write θ_i and λ_i as functions of y_i (that is, $\theta_j = \theta(y_j)$ and $y_j = y_j(y_i) = y_i^{-1}(y_j)$, where $y_j(y_i)$ is the function used in the proof of Proposition 3). Along the equilibrium path, $\tilde{\lambda}_j(y_i) = \lambda_i$ and $d\tilde{\lambda}_j(y_i)/dy_i = \lambda'(y_i)$. The choice of submarket y_j maximizes λ_i. Differentiating λ_i with respect to y_j and evaluating at $y_j = y_i$ yields

$$\frac{d}{dy_j}\left[\beta\, p(\theta(y_j))\left(\lambda(y_j) - \lambda(y_i)\right)\right] \;+\; (1-\beta)\, p(\theta_j)\, \lambda'(y_i) \;=\; 0. \tag{15}$$

By the definition of the perfectly directed search equilibrium, the first term is zero, since the worker maximizes the private gain from search. The second term is strictly positive. Since the second-order conditions are satisfied locally, this implies that the planner chooses a higher optimal value of y_j than in the market solution.

Proposition 5. *Consider the perfectly directed search equilibrium with a continuum of types.*

Suppose the planner can decide the search behavior of a given worker. Then the planner will direct the worker to a higher submarket than the one the worker chooses in equilibrium.

In Appendix A.4 we analyze efficient entry of firms. We show that the entry decision is optimal if a weighted sum of the externalities associated with entry is equal to zero. Hence, the entry decision is optimal if the private and social values of entering, properly weighted, are equal. See the appendix for details.

Partly directed search

Let us analyze the value of a worker–firm match and the value of searching in the various submarkets for an arbitrary given matrix of search probabilities $\bar{\pi}_{ij}$.[14] Note that the differential equations governing $\dot{N}_i$ are the same as with perfectly directed search; however, θ_j now also includes searchers who search in inferior submarkets. Define

$$\tilde{\lambda}_j^{adj} = \sum_{l=0}^{I} a_l \max[\lambda_l, \lambda_j].$$

It follows that $\tilde{\lambda}_j^{adj} \leq \tilde{\lambda}_j$. In the appendix, we show that λ_i can be written as

$$\begin{aligned} r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \sum_{j>i} \pi_{ij}\beta p(\theta_j)(\lambda_j - \lambda_i) - \sum_{j>i} \pi_{ij}(1-\beta)p(\theta_j)(\tilde{\lambda}_j^{adj} - \lambda_i) \\ + \sum_{j \leq i}(1-\beta)p(\theta_j)\pi_{ij}\left[\lambda_j - \tilde{\lambda}_j^{adj}\right] \end{aligned} \tag{16}$$

If we compare this with the corresponding equation for λ_j given by (13), we see two differences. First, $\tilde{\lambda}_j$ is replaced by $\tilde{\lambda}_j^{adj}$, which adjusts for the fact that not all matches are accepted. Second, there is an additional term in (16) that captures the externalities arising from the search of i-workers in inferior submarkets.

Relative to the private counterpart M_i defined by (1), there are two differences. First, the fourth (second-to-last) term reflects the net externality discussed above. Second, the last term captures the social cost of searching in submarkets below the worker's current submarket without accepting the offers obtained there. These negative search externalities imposed on the searching agents in those markets enter the social value but not the private value when computing M_i.

Let us then analyze the social value of increasing π_{ij}, which we denote Λ_{ij}. In the appendix, we show that for $j > i$,

$$\Lambda_{ij} = N_i\beta p(\theta_j)(\lambda_j - \lambda_i) + N_i(1-\beta)p(\theta_j)(\tilde{\lambda}_j^{adj} - \lambda_i). \tag{17}$$

Suppose for an instant that $M_i = \lambda_i$. As before, the net search externalities are lower the higher is j,

[14] If $\pi_{ij} = \bar{\pi}_{ij}$ for all i, j, the restriction that $\sum_{l=0}^{J} \pi_{li} = 1$ is superfluous.

and this will tend to increase the planner's incentive to send workers to higher submarkets relative to the private incentives. In a logit setup, this translates into higher probabilities of searching in these markets in the planner's solution. For $j \leq i$ we get that

$$\Lambda_{ij} = -N_i(1-\beta)p(\theta_j)(\tilde{\lambda}_j^{adj} - \lambda_j).$$

In the counterfactual situation with $\lambda_i = M_i$ for all i, the incentives to search downward in the market solution are lower than in the planner's solution if

$$\lambda_i - \lambda_j \geq (1-\beta)(\lambda_i - \tilde{\lambda}_j).$$

The left-hand side is the private penalty of searching downward (recall our assumption that the searching agent, when calculating the perceived value of search, incurs the entire loss), while the right-hand side is the social penalty. If $i = j$, the left-hand side is zero and the right-hand side is also zero, so the private loss is certainly no greater than the social loss. However, this need not hold if λ_j is sufficiently lower than λ_i.

5 Estimation

Until now, we have analyzed the theoretical properties of our model, including its efficiency properties. We now discuss how the model parameters, and in particular the noise parameter μ, can be estimated.

5.1 Model extensions

In order to take the model to the data, we extend the model with a few added dimensions of heterogeneity. First, as will be clear below, firm size will be an important determinant of measurement errors in the classification of firm types. In order to match the size distribution of firms, we vary the number of vacancies the different firm types are in possession of, denoted ϕ_i.

In order to explain downward moves, we introduce advance notice layoff shocks. We also allow layoff rates to vary with firm type. Specifically, workers are separated into unemployment at a rate that depends both on the employer's type and the worker's tenure. All jobs start with a low, firm-specific layoff rate of $\bar{\delta}_i$. At the Poisson rate ν, this rate increases to $\bar{\delta}_i + \Delta$, where $\Delta > 0$. Hence, the layoff rate is $\delta_{\mathbf{i}} = \bar{\delta}_i + z\Delta$, where $\mathbf{i} = \{i, z\}$, $z \in \{0, 1\}$, referred to as the worker's state. Unemployment is represented by $\mathbf{i} = 0$.

Finally, we distinguish between search intensities when employed and unemployed. Employed workers have search intensity normalized at unity, and unemployed workers search with intensity $1 + \gamma$.

The joint value of a firm–worker match in state $\mathbf{i}$ can now be written as

$$rM_{\mathbf{i}}=y_i + \delta_{\mathbf{i}}(M_0 - M_{\mathbf{i}}) + \nu(M_{i1} - M_{\mathbf{i}}) + \beta(1 + \mathbf{1}_{\mathbf{i}}\gamma)\sum_j \pi_{\mathbf{i}j}p(\theta_j)\left(M_j - M_{\mathbf{i}}\right)^+, \tag{18}$$

where $\mathbf{1}_{\mathbf{i}} = 1$ if $\mathbf{i} = 0$ and zero otherwise. The perceived value of search $m_{\mathbf{i}j}$ is similarly rewritten,

$$m_{\mathbf{i}j} = y_i + \delta_{\mathbf{i}}(M_0 - M_{\mathbf{i}}) + \nu(M_{i1} - M_{\mathbf{i}}) + \beta(1 + \mathbf{1}_{\mathbf{i}}\gamma)p(\theta_j)(M_j - M_{\mathbf{i}})^+ - p(\theta_j)(M_{\mathbf{i}} - M_j)^+. \tag{19}$$

For a worker in state $\mathbf{i}$, define the *effective productivity of the match* as

$$\tilde{y}_{\mathbf{i}} = y_i + \delta_{\mathbf{i}}(M_0 - M_{\mathbf{i}}) + \nu(M_{i1} - M_{\mathbf{i}}) \tag{20}$$

In equilibrium, we can index the worker–firm matches according to $\tilde{y}_{\mathbf{i}}$. Let $\tilde{i}(\mathbf{i})$ denote the rank of a worker based on the rank of $\tilde{y}_{\mathbf{i}}$, $\tilde{i} = 0, \cdots, 2J$. Define the ranked joint values by $M_{\tilde{i}(\mathbf{i})} = M_{\mathbf{i}}$.

Lemma 1. *In equilibrium of the extended model, the following is true given sufficiently low y_0 so that $M_0 \leq M_1$.*

1. *A worker's search behavior only depends on the effective productivity of the match.*
2. *$M_{\tilde{i}}$ is increasing in $\tilde{i}$.*
3. *Propositions 2 and 4 hold when the worker–firm matches are indexed according to $\tilde{i}$.*

Lemma 1 is not completely trivial and is proven in Appendix A.5. The first result follows from the fact that for $\mathbf{i} > 0$ equation (18) depends on $\tilde{y}_{\mathbf{i}}$ only, not its various parts. Since we can then rank the jobs according to the rank of $\tilde{y}_{\mathbf{i}}$, Proposition 2 and Proposition 4 follow. Note that a higher unemployed job-finding rate, $1 + \gamma$, makes unemployed workers require a greater match value to compensate for the search efficiency loss associated with employment. Every submarket below the resulting reservation threshold is empty in equilibrium. Our data necessarily only show observations for non-empty submarkets, and the estimation will therefore necessarily pick a y_0 so as to satisfy that all markets are populated.

The structure of the equilibrium is very similar to the structure of the equilibrium of the simpler model, with the difference that we have to include flows between the different layoff states. See the appendix for details and for a proof of equilibrium existence. However, note that Proposition 4 concerns jobs, not necessarily firms. To be more specific, the proposition implies that the inflow of workers to a firm j, given that the worker transitions to a firm at rung j or above, is decreasing in the effective productivity $\tilde{y}$ of the current match. However, the outflow of workers in the low layoff state is lower in high-productivity firms than in low-productivity firms. It follows that the fraction of job switchers who are in the high layoff state is higher the higher the type of the sending firm. For example, all workers in the highest type of firms who leave for another job are in the high-layoff state. Therefore, the average effective productivity $\tilde{y}$ *among job switchers* does not have to increase in firm type j. So, it is not an anomaly if $\hat{h}_{ij}$ increases in i for some i, j.

In order to explain horizontal transitions between firms of the same type, we assume that workers switch jobs when they meet a firm of the same type as their current employer. For the same reason, we also assume that workers in the highest firm type search. This has minor implications for the aggregate economy.

The welfare analysis also extends easily; in particular, we obtain the same structure of the results. The social value of a job, $\lambda_{\mathbf{i}}$, will be an increasing function of $\tilde{y}_{\mathbf{i}}$.

5.2 Data

We use Danish matched employer–employee (MEE) data for our analysis, and our data sources are similar to those in, e.g., Bagger et al. (2013) and Bertheau and Vejlin (2025).

Our starting point is a population-wide dataset that contains the universe of employment spells and labor market transitions in the period 1985-2013.[15] A key advantage of these data is that we can distinguish employer-to-employer transitions from similar transitions that have a small period of non-employment in between, thereby avoiding time aggregation bias (see, e.g., Bertheau and Vejlin (2022)). Further, since we observe the universe of workers and transitions, including firm identifiers, we can construct firm ranks exploiting complete employment histories at the firm level (e.g., all hires and separations over our sample period).

The data on employment spells build on information about worker earnings and are very granular. We make some sample manipulations to clean the data and, for example, merge employment spells at the same employer with only small periods of non-earnings. We also discard very short employment and non-employment spells; see further information on these data cleaning choices in Appendix E. Lastly, we add non-employment spells, defined as periods in which the workers are not observed in employment, to the data so that we have complete employment and non-employment histories for the workers in our sample.

We add information on individuals (e.g., education level, age, and gender) and firms (e.g., industry and value added) using some of the other registers available at Statistics Denmark. We link across different data sets using either individual identifiers or firm identifiers, which are readily available in all the registers (see more details on merging and variables in Appendix E.2).

To arrive at our analysis sample, we employ a number of sample restrictions to ensure a somewhat harmonized sample and reduce unmodeled heterogeneity and other features of our sample that our model abstracts from. First, we focus on the time period 1995-2005 to consider a time period in which the Danish labor market and the economy as a whole are relatively stable. Second, we focus on workers in the age range 25-50 to avoid early and late career dynamics that influence the type of employment transitions workers make. Third, we drop observations where we have questionable information about labor market entry or education.

The final merged dataset contains information about the individual worker (background characteristics such as age, gender, education, and wages) as well as the current employer (value added, industry,

[15] Formally, this is known as the SPELL data set and is created and maintained by ECONAU at Aarhus University.

and firm size). The unit of observation in our final sample is a worker-year. In the final dataset, we construct our firm ranking, which we return to further below.

5.2.1 Constructing the Poaching Rank

As we return to below, we employ the poaching rank presented in Bagger and Lentz (2019) as our preferred grouping of firms into markets/types. The poaching rank is constructed for each firm by considering all transitions into the firm (hiring) and distinguishing them by whether the arriving workers come from non-employment or employment. For each firm, we calculate the share of new hires coming from employment (the poached workers). Subsequently, firms are ranked by this poaching share. Firms that are in the public sector or in agriculture, forestry/fishing, and financial/insurance activities are treated as an additional (and auxiliary) firm group. The same holds for firms that are only present in the first two years of our sample window and/or have fewer than five hires over our sample period. These restrictions are invoked as we do not believe the poaching rank is adequate for these firms, either due to inaccuracies in terms of firm boundaries, which makes it hard to differentiate and distinguish between, for example, internal and external new hires, or simply considerable noise in the rankings due to few hires in general. In total, there are approximately 76 thousand firms with a well-defined poaching rank. These firms are divided into 10 equally sized groups/bins.[16]

When we characterize aggregate and submarket-specific flow statistics (e.g. job separation rates), we use all transitions available in our dataset (including transitions from or to ranked and non-ranked firms). When we analyze the composition of job transitions, however, we focus on a selected set of transitions to focus on transitions that resemble transitions in our model and not transitions that arise from mass layoffs and the like (we discuss how we select these transitions next).

5.2.2 Building transition matrices

As outlined above, the key identifying variation in the data for direction in search concerns the composition of EE transitions (e.g., the share of EE transitions going to markets above your current market). A key moment of interest in the data is therefore the composition of moves between employers. When we sample EE transitions for this moment, we invoke three mild sample restrictions to avoid our key identifying moments being influenced by worker behavior around firm entry or exit or in relation to large firm size reductions.[17] In particular, we do not include EE transitions away from firms in the year of exit (defined as the last two years in which the sending firm is seen in the data) or transitions into firms during their first (or last) year in the data. Lastly, we do not consider EE transitions to firms in years where the number of hires is above one thousand workers/spells.

[16] Appendix E.3 provides some summary statistics across firm bins.

[17] The idea is that transitions around these events may not represent revealed preference over two firms' rankings, and this process is not something the model is set up to handle. In Appendix F we also show transition moments without these restrictions and show that while overall patterns are unchanged, there is some noise reduction gain.

This "transition" data set used to analyze the composition of EE transitions contains around 650 thousand EE transitions to and from firms for whom we have a firm (poaching) rank (1-10) in our sample period. Around 36% of the EE transitions we work with are transitions into a lower-ranked firm than the origin firm. Including transitions from non-employment gives us a total of around 1.8 million transitions into these firms.

5.3 Identification strategy

The model estimation is done by simulated minimum distance. The object of the estimation is to minimize the distance between a given set of data moments and the corresponding simulated data moments. The set of moments is chosen to characterize key relationships in the data that deliver identification of the model parameters. The model is solved for a given set of model parameters. Given the solution, a data set with identical structure to that of the data is simulated, and the simulated data moments follow. The distance between simulated and data moments is then calculated as the weighted sum of squared differences.

Identification of the model is focused around the insight established in Proposition 4 that the directed and random search benchmarks differ in the job-to-job move destination distributions conditional on current firm type. Specifically, the relative likelihood of moving to one of two higher-ranked firms is unaffected by the worker's current position in the firm hierarchy under random search. This is generally not true under directed search. Hence, the set of identifying moments will pay particular attention to the worker job-to-job flow rates between firm types.

The type of a firm and its associated submarket are not directly observed in the data. To rank firms, we follow the revealed preference argument used in both Sorkin (2018) and Bagger and Lentz (2019), where there is global agreement among workers on the ranking of submarkets. Specifically, we use the poaching index from Bagger and Lentz (2019) to rank firms relative to each other.

The poaching rank index measures the share of a firm's hires that come directly from other firms instead of from non-employment. With this, we rank firms and choose a number of equally sized, by firm count, bins, $J = 10$, which are thought of as separate submarkets and map into the separate types in the model. J is set as part of the definition of the data and simulated moments. With this, we can directly calculate empirical transition rates h_{ij}, as we observe workers make transitions from empirical submarket i to j. From equation (12), the empirical measurement of $\hat{h}_{ij}$ follows directly.

The set of identifying moments includes $\{h_{ij}\}_{(i,j)\in\{1,\dots,J\}^2}$ (that is, the transition probability for a worker who is currently employed at a firm of type i and moves to a firm of type j). We show these in Table 2. While we have so far emphasized upward mobility, we use this entire set of mobility rates to also discipline the downward job-to-job mobility rates induced by the advance notice layoff shocks (ν). In addition, we also include the current-submarket-i conditional rate at which a worker moves to submarket j relative to any market above j, $\{\hat{h}_{ij}\}$. This set of moments speaks specifically to the directedness of search (see Proposition 4).

In Table 3, we show $\hat{h}_{ij}$ for $i \in \{1, \dots, 9\}$ and $10 > j \geq i$. Immediately, we see substantial variation in $\hat{h}_{ij}$ across i for given j as well as an overall decreasing relationship between $\hat{h}_{ij}$ and submarket i rank for given destination market j. By Proposition 4, we expect such a relationship when search is at least partially directed, $\mu > 0$.

On its face, the patterns in Table 3 would suggest evidence that search is at least partially directed. However, job-to-job moves made by workers in the advance notice layoff state are a confounding factor; see the discussion in Section 5.1. A worker in the advance notice layoff state is effectively searching and moving from a lower submarket equivalent. In such a job-to-job move, we are effectively misclassifying the worker's origin submarket. This is not confounding if search is entirely random, because in this case, $\hat{h}_{ij}$ does not vary with origin market. However, if search is partially directed, it has the potential to generate noise in the empirical $\hat{h}_{ij}$ patterns. We discipline the advance notice layoff process by also including the downward part of the job-to-job mobility rates shown in Table 2.

Firm submarket misclassification is another confounding factor. In Appendix B, we demonstrate that classification error in the canonical on-the-job random search model results in variation in the measured $\hat{h}_{ij}$ across the origin market i, where, in the absence of the classification error, $\hat{h}_{ij}$ would not vary with i. Furthermore, depending on the pattern of the classification error, it is possible that it can by itself induce a broadly decreasing relationship between i and $\hat{h}_{ij}$, as seen in the data.

This highlights a virtue of our estimation strategy as opposed to the commonly used two-step alternative, where the firm classification is obtained in a first step and is subsequently treated as data without a sense of the uncertainty of the classification. In our estimation strategy, the firm classification along with its error is embodied in the simulated data moments. We understand through our structure how misclassification is a byproduct of the Bagger and Lentz (2019) poaching rank firm classification, and it is replicated in the simulated data.

The firm classification error embodied in the poaching rank is particularly sensitive to the number of hires a firm has. The more hires, the lower the variance of the firm's estimated type rank. Consequently, we include empirical moments to discipline firm size as well as the submarket-conditional labor flows. Specifically, we include submarket-conditional average firm size as well as the submarket-conditional average firm layoff rate and the submarket-conditional separation rate into other jobs. These moments are shown in Table 4. Generally, firm size is increasing in the rank of the firm and separations are decreasing.

Table 4 shows submarket-conditional layoff rates, average firm size, as well as the share of job-to-job transitions from the submarket that are downward rank. The latter moment is used to discipline the layoff notice process; see subsection 5.1, which is how our model understands job-to-job transitions that are observed to be in the direction of lower-rank submarkets. The process is akin to the exogenous job-to-job reallocation process in Jolivet et al. (2006) but is modified to have the feature that the destination rank distribution is origin-rank dependent, an empirically relevant feature also emphasized in Bagger and Lentz (2019). Down-rank mobility is common in the data, and the inclusion of the advance notice process allows the model to more flexibly fit worker flows, match distributions, and firm size distributions.

Worker reallocation is sensitive to the cardinal features of the model, more specifically, the productivity gains associated with matches with better firms. We include submarket-conditional value added per worker as a moment to discipline the estimate of y_n. These values are also included in Table 4.

One could consider controlling for worker heterogeneity in the data. One approach (however, computationally demanding) could be a fully flexible interaction with observed characteristics that would be implied by separate estimations on subsamples. In Appendix F, we provide the central identifying mobility moments (or summary measures hereof) for subsamples by education and sex. The mobility patterns do not vary substantially across education categories. There is some distinction between men and women. This is only suggestive, but at least along these dimensions of heterogeneity, we do not see an obvious sensitivity of our identifying moments to worker heterogeneity.

5.3.1 Model specification and parameterization

The discount rate is set at an annual rate of 5 percent, $r = 0.05$. We specify the matching function to be homogeneous of degree one Cobb–Douglas, where we follow Petrongolo and Pissarides and adopt a parameter of one half:

$$p(\theta) = A\theta^{1/2}, \text{ and } q(\theta) = A\theta^{-1/2}.$$

Given the Hosios condition, we have bargaining power $\beta = 1/2$. A is a basic scale parameter to be estimated.

Given the complexity of the estimation procedure, we economize on the number of parameters that are estimated, trading off model flexibility to fit the data. We have arrived at the following specifications as a compromise between these two concerns. The submarket-specific technology parameters $y(i)$, $\delta(i)$, and $\phi(i)$ are parameterized as follows. Match productivity is characterized by two parameters (y_1, y_9) by linear interpolation and normalization of productivity in the top-rank submarket:

$$y(i) = \begin{cases} y_1 + \frac{i-1}{8}(y_9 - y_1) & \text{if } 1 \leq i \leq 9 \\ 1 & \text{if } i = 10. \end{cases}$$

The submarket-conditional layoff rate is characterized by five parameters $(\delta_1, \delta_2, \delta_9, \delta_{10}, \delta_c)$ by

$$\delta(i) = \begin{cases} \delta_2 - (\delta_2 - \delta_9)\left(\frac{i-2}{7}\right)^{\delta_c} & \text{if } 2 \leq i \leq 9 \\ \delta_i & \text{if } i = 1 \text{ or } i = 10. \end{cases}$$

Finally, the recruiting technology is fully flexibly estimated, where $\phi(1)$ is normalized at unity, $\phi(1) = 1$, and $\phi(i) = \phi_i$ for $i \geq 2$ is characterized by the estimated nine parameters $(\phi_2, \ldots, \phi_{10})$. Thus, the estimation of (y, δ, ϕ) is achieved using 16 parameters. These parameters are constrained so that $y(i)$ is monotonically increasing and $\delta(i)$ is monotonically decreasing. $\phi(i)$ is fully flexible.

Table 1: Estimated model parameters

		Layoffs		Hiring		Productivity	
μ	0.405					y_0	-8.201
K	436.3	δ_1	0.501	ϕ_1	1.000	y_1	0.680
A	0.972	δ_2	0.123	ϕ_2	0.257	y_2	0.711
ν	0.227	δ_3	0.123	ϕ_3	0.551	y_3	0.742
s	0.218	δ_4	0.122	ϕ_4	0.878	y_4	0.772
$1+\gamma$	3.565	δ_5	0.118	ϕ_5	1.665	y_5	0.803
		δ_6	0.109	ϕ_6	1.610	y_6	0.834
		δ_7	0.091	ϕ_7	0.477	y_7	0.864
		δ_8	0.060	ϕ_8	1.170	y_8	0.895
		δ_9	0.012	ϕ_9	0.494	y_9	0.926
		δ_{10}	0.010	ϕ_{10}	0.093	y_{10}	1.000

The rest of the estimated model parameters are $(A, K, s, y_0, \gamma, \lambda, \mu)$. Thus, the estimation determines 23 parameters.

5.3.2 Simulated minimum distance estimator

The estimation is done by simulated minimum distance. The estimator is given by

$$\arg\min \left[\Psi(\omega_0) - \Psi^S(\omega)\right]^T \Sigma \left[\Psi(\omega_0) - \Psi^S(\omega)\right],$$

where $\Psi(\omega_0)$ is the vector of data moments based on the real data, which is a function of the true model parameters ω_0. $\Psi^S(\omega)$ is the set of data moments based on s data set simulations from the model solution for model parameters ω. Σ is a positive definite, symmetric weighting matrix. The set of moments includes the transition probability matrix moments $\{h_{ij}\}_{ij}$ as well as the upper diagonal matrix $\{\hat{h}\}_{ij}$ moments shown in Table 3. The latter are implicitly represented in the transition probability moments, but we include them as our theory speaks directly to them. Furthermore, the moments include the 44 moments in Table 4. The weighting matrix is a diagonal matrix chosen to provide a balanced fit to the data.

The estimation is done with $s = 1$. We have tried using a greater number of simulations, but the already substantial data set size results in noise reduction gains that are second order. The search for the minimum is done with a combination of global optimization using a swarm algorithm combined with Nelder–Mead.

5.4 Estimation results and fit

Table 1 presents the estimated model parameters. μ is estimated at 0.405. In the next section, we quantify and discuss the implied directedness of search. Layoff rates δ_i are strongly submarket-dependent, decreasing from a high of .501 in submarket 1 to almost zero in the highest submarket. We estimate $\nu = 0.227$, so that a match deteriorates into the high layoff state after slightly more

Table 2: Transition to submarket n conditional on move to n or above.

$\hat{h}_{mn}$	1	2	3	4	5	6	7	8	9
1	.012	.051	.097	.154	.203	.213	.368	.454	.607
	.016	.068	.094	.153	.213	.262	.360	.481	.639
2		.046	.081	.141	.189	.198	.340	.470	.618
		.035	.081	.133	.193	.233	.311	.454	.619
3			.081	.150	.195	.204	.353	.451	.619
			.071	.127	.188	.223	.298	.453	.614
4				.131	.185	.199	.335	.452	.627
				.119	.178	.214	.295	.451	.610
5					.184	.193	.360	.459	.619
					.170	.209	.289	.448	.608
6						.175	.311	.448	.602
						.201	.284	.449	.608
7							.308	.448	.617
							.249	.434	.605
8								.408	.570
								.407	.586
9									.554
									.529

Note: Data in black. Estimated model moments in red.

than four years of duration on average. The layoff rate then increases by 0.218. Table 4 shows that the model captures the decreasing EU rates well. The EE flows that constitute the remaining part of the separations are overall matched well, but the model overestimates the decreasing relationship. The hiring rates are broadly declining so as to ensure correct relative inflows and the overall increasing relationship between firm size and submarket.

The flow rate of income for unemployed workers is quite low, ensuring that workers enter into the lower rungs on the firm ladder at a sufficiently high rate so as to allow the model to generate the empirically observed flows from low-rung to high-rung matches. Unemployed search is estimated to be around 3.5 times as efficient as employed search (γ).

Firm productivity is estimated to be increasing by firm type, as expected, and the fit to the estimated submarket-conditional productivity is generally good.

Table 2 presents the empirical relative upward transition rates $\hat{h}_{ij}$ as well as the model fit. The estimated model captures the general pattern that $\hat{h}_{ij}$ is decreasing in i, although while both the layoff shocks and the misclassification mechanisms in the model do result in some non-monotonicity in the estimated $\hat{h}_{ij}$, it does not manage to match the prevalence of the phenomenon in the data moments. In fact, the estimated model's firm type misclassification contributes to the overall negative relationship between $\hat{h}_{ij}$ and origin market i, thereby reducing the need for μ to deliver the pattern. We conjecture that had we adopted a two-step estimation method that does not recognize firm misclassification, the estimation would have arrived at a greater estimate for μ in order to match the empirical negative relationship between $\hat{h}_{ij}$ and origin market i.

Table 3: Transition probabilities conditional on job-to-job move

h_{mn}	1	2	3	4	5	6	7	8	9	10
1	.013	.078	.145	.150	.143	.113	.117	.097	.079	.052
	.016	.067	.086	.127	.150	.145	.147	.126	.087	.049
2	.019	.052	.154	.155	.135	.106	.117	.102	.080	.049
	.003	.035	.078	.117	.148	.144	.148	.148	.111	.068
3	.010	.046	.031	.173	.148	.103	.108	.097	.076	.046
	.001	.029	.069	.115	.148	.143	.148	.158	.117	.073
4	.007	.038	.078	.038	.161	.108	.109	.087	.077	.047
	.001	.025	.064	.108	.143	.141	.153	.165	.122	.078
5	.005	.030	.070	.118	.040	.130	.152	.128	.108	.049
	.001	.025	.061	.103	.137	.140	.154	.170	.127	.082
6	.003	.030	.064	.112	.145	.047	.157	.138	.136	.076
	.001	.025	.059	.095	.131	.138	.156	.177	.132	.085
7	.003	.024	.052	.096	.134	.121	.061	.157	.139	.088
	.001	.024	.055	.090	.115	.124	.147	.192	.152	.099
8	.003	.022	.048	.088	.125	.117	.184	.075	.158	.113
	.001	.026	.056	.081	.099	.100	.125	.208	.178	.125
9	.001	.017	.037	.072	.102	.108	.180	.197	.093	.133
	.001	.025	.058	.085	.104	.105	.114	.169	.180	.160
10	.002	.019	.036	.071	.092	.096	.162	.195	.181	.149
	.001	.027	.059	.089	.110	.108	.112	.127	.178	.188

Note: Data in black. Estimated model moments in red.

Table 4: Submarket conditional moments.

submarket	Avg firm size	Avg value added per worker	EE rate	EU rate	Non-employment rate
Aggregate	16.9		.020	.029	.255
	18.9		.024	.041	.252
1	11.2	0.709	.025	.104	
	4.7	0.713	.030	.117	
2	12.6	0.764	.032	.079	
	8.7	0.762	.030	.054	
3	14.9	0.750	.032	.063	
	14.7	0.794	.028	.046	
4	20.9	0.734	.032	.057	
	20.7	0.816	.026	.046	
5	24.1	0.769	.030	.045	
	24.6	0.834	.026	.045	
6	19.4	0.755	.033	.041	
	23.1	0.843	.025	.045	
7	26.0	0.806	.032	.035	
	24.6	0.878	.023	.041	
8	25.1	0.949	.032	.031	
	29.4	0.927	.022	.034	
9	22.8	0.936	.035	.028	
	24.4	0.967	.020	.029	
10	22.3	1.000	.032	.022	
	17.3	1.000	.019	.024	

Note: Data in black. Estimated model moments in red. Aggregate average firm size includes unclassified firms.

Table 3 presents the empirical transition rates h_{mn} and the model fit. The fit to the transition rates is generally good while leaving some room for improvement in the transition patterns out of measured submarkets 3 and 4, which empirically have considerably more mass focused on moves to the immediate submarkets above whereas the estimated model spreads out transitions a bit more. It is also these two submarkets that represent a fit challenge for the $\hat{h}_{ij}$ measures.

6 Directedness

The particular value of the noise parameter μ is not very informative by itself regarding the degree of directedness in the labor market, and it may depend on arbitrary details in the parameterization of the choice probabilities. In the following, we provide three perspectives on its magnitude.

In Section 6.1 we provide a welfare-based measure that compares the welfare of the estimated economy to that of the two extreme benchmarks of either perfectly random or perfectly directed search.

Table 5: Classification error matrix

	1	2	3	4	5	6	7	8	9	10
1	.847	.153	.000	.000	.000	.000	.000	.000	.000	.000
2	.128	.281	.161	.107	.078	.088	.094	.046	.015	.001
3	.007	.298	.243	.157	.113	.096	.069	.016	.001	.000
4	.000	.198	.280	.203	.155	.104	.053	.006	.000	.000
5	.000	.029	.197	.292	.258	.168	.055	.001	.000	.000
6	.000	.002	.039	.135	.248	.325	.236	.015	.000	.000
7	.000	.025	.062	.085	.123	.185	.309	.178	.034	.000
8	.000	.000	.000	.000	.001	.007	.144	.619	.229	.000
9	.000	.000	.000	.000	.000	.000	.002	.029	.472	.498
10	.000	.002	.001	.003	.005	.009	.022	.086	.279	.591

Note: True type in rows and estimated type in columns.

In Section 6.2 we put the μ estimate into the context of how sensitive submarket tightnesses are to changes in match values. This perspective uses the contrast that the perfectly random search model is an ordinal model where only rank matters, and a marginal match value change will not affect market tightnesses for a given firm type. This contrasts with the directed search model, where cardinality matters. In the extreme case where search is perfectly directed, tightnesses are infinitely sensitive.

In Section 6.3 we pose another welfare-related question in a setting where we consider an extension to a model that includes an initial capital investment and a possible hold-up problem similar to Card et al. (2014). In this case, we quantify the extent of the ex ante capital underinvestment inefficiency implied by our μ estimate. Investment in the perfectly directed search model is efficient. The underinvestment problem becomes increasingly severe as search becomes more random.

6.1 Directedness measured in terms of welfare

One main implication of directedness is that it increases welfare. We therefore construct a measure of directedness that is based on welfare. Welfare is measured as the NPV utility of an unemployed worker; as demonstrated in Pissarides (2000), this is an appropriate welfare measure in search models. With our estimated parameter values (except μ), denote welfare with perfectly directed search ($\mu = 0$) by S^P, and welfare with random search by S^R. Finally, let S^O denote welfare with our estimated value of μ. The degree of directedness, d_r, is defined as

$$d_r = \frac{S^O - S^R}{S^P - S^R} \tag{21}$$

It follows that our measure of directedness is equal to the welfare gain, relative to random search, generated by our estimated value of μ, divided by the welfare gain obtained under perfectly directed search. Clearly, $d_r = 0$ for $\mu = \infty$ and $d_r = 1$ for $\mu = 0$.

For the estimated value $\mu = 0.405$, we obtain a directedness value of $d_r = 0.41$. That is, the estimated economy realizes 41 percent of the welfare difference between an economy with a perfectly directed search technology and one in which search is entirely random.

6.2 Sensitivity of θ_i to the flow search values m_{ij}

In order to give more economic meaning to our estimate, we interpret our results in terms of the responsiveness of the tightness θ_j in a market with respect to the flow payoffs m_{ij}.

First, as noted above, μ is scale dependent, i.e., it depends on the scale of the m_{ij} values. In our estimation, we normalized the productivity in the top job to 1, so the flow values m_{ij} are measured relative to the flow value in the top job.

Consider a submarket j that is small relative to the entire economy. Let $\bar{\theta} = k/\sum_{i=0}^{J-1} N_i$ denote the economywide ratio of vacancies to searching workers. Let $\tilde{N}_i = N_i/(1-N_J)$ denote the fraction of the searching workers that are in submarket i. From (3) and (5) it follows that the tightness in any market j is given by

$$\frac{\bar{\theta}}{\theta_j} = \sum_{i=0}^{J-1} \frac{\tilde{N}_i \exp(m_{ij}/\mu)}{\sum_{k=0}^{J-1} \tau_k \exp(m_{ik}/\mu)}. \tag{22}$$

Suppose τ_j is small, so that we can ignore the effect of m_{ij} on the denominator. Then we get that the derivative with respect to m_{ij} is given by

$$\frac{\bar{\theta}}{\theta_j^2}\theta_j'(m_{ij}) = -\frac{1}{\mu}\frac{\tilde{N}_i \exp(m_{ij}/\mu)}{\sum_{k=0}^{J-1} \tau_k \exp(m_{ik}/\mu)}$$

In the special case in which market ij is an "average market" with $\exp(m_{ij}/\mu) = \sum_{k=0}^{J-1} \tau_k \exp(m_{ik}/\mu)$, the latter expression simplifies to

$$\frac{\theta_j'(m_{ij})}{\theta_j} = -\frac{\tilde{N}_i}{\mu}\frac{\theta_j}{\bar{\theta}} \tag{23}$$

The left-hand side shows the semi-elasticity of θ_j with respect to m_{ij}, where m_{ij} is normalized by y_J. Thus, the absolute value of the percentage change in θ_j when m_{ij} increases by one percentage point relative to y_J is equal to the inverse of μ (2.5 with our estimated value), multiplied by two factors. The first factor is $\tilde{N}_i$, the share of searching workers that is directly influenced by the change in m_{ij}. The second factor is the initial ratio of θ_j to the average tightness $\bar{\theta}$ in the economy. If the latter is 1, the semi-elasticity is simply the inverse of μ scaled by $\tilde{N}_i$.

Note that if τ_j is strictly greater than 0, submarket j has "market power," in the sense that an increase in m_{ij} will increase the overall value of search expressed by the denominator on the right-hand side of (22). As a result, the semi-elasticity of θ_j with respect to m_{ij} will be lower.

6.3 Directedness and investment incentives

In this subsection, we discuss how our directedness estimate, μ, affects the sensitivity of labor market tightness and thereby the severity of the hold-up problem facing firms when investing in capital *ex ante*, as in Acemoglu and Shimer (1999) and also discussed in Card et al. (2014).

We collapse all firm types to one common type and assume there is no on-the-job search, only unemployment search. This is a reasonable assumption when there is only one firm type.[18] We assume that the productivity of a firm is $f(K)$, where $K > 0$ is the investment undertaken by the firm at the entry stage. For expositional reasons, we assume that the number of firms is given. Hence, in the symmetric steady state, tightness is given independently of the equilibrium value of K. We denote this equilibrium value by θ^*.

Since firms can immediately repost the vacancy (the fishing-line assumption), the outside option of the firm when bargaining with the worker is still zero. From equation (6) it follows that the value of the firm is

$$\begin{aligned} rV(K) &= q(\theta)(1-\beta)\big(M(K)-M_0\big) \\ &= q(\theta)(1-\beta)\frac{f(K)-rM_0}{r+\delta}. \end{aligned} \tag{24}$$

since $M(K) - M_0 = (f(K) - rM_0)/(r+\delta)$. Firms choose K so as to maximize $V(K) - K$, and at the optimum $V'(K) = 1$.

For later reference, we first derive the profit-maximizing K under random search, with θ independent of K. By taking the derivative of (24) we get that

$$(1-\beta)f'(K) = \frac{r+\delta}{\bar{q}}, \tag{25}$$

where $\bar{q} = q(\bar{\theta})$. It is easy to show that the socially optimal investment level is obtained for $\beta = 0$, and that there is underinvestment under the Hosios condition.[19]

Now let search be partly directed. When setting K, the firms take into account that θ may depend on K, $\theta = \theta(m(K))$, and that the responsiveness of θ to K depends on the degree to which search is directed. Taking the derivative of (24), we find that the first-order condition for K is given by

$$q'(\theta)\theta'(m)m'(K)(1-\beta)\left(M(K)-M_0\right) + q(\theta)(1-\beta)\frac{f'(K)}{r+\delta} = r. \tag{26}$$

Now, the value $m(K)$ is, from (2),

$$m(K) = y_0 + p(\theta(K))\,\beta\left(M(K)-M_0\right).$$

[18] Out of equilibrium, firms may differ and hence trigger on-the-job search since on-the-job search is costless. However, we conjecture that a small search cost will prevent on-the-job search as a response to deviations in investments.

[19] Opening up the entry margin, $\beta = 0$ is clearly suboptimal, as there will be too much entry.

Note that M_0 on the right-hand side is constant, as the workers constantly rerandomize. It follows that

$$m'(K) = p'(\theta)\,\theta'(m)m'(K)\beta\,(M(K) - M_0) + p(\theta)\,\beta\frac{f'(K)}{r+\delta}$$

which gives

$$m'(K) = \frac{1}{1 - \beta p'(\theta)\theta'(m)(M(K) - M_0)}\beta p(\theta)\frac{f'(K)}{r+\delta}. \tag{27}$$

Let K^* denote the equilibrium value of K, and let θ^* denote the corresponding equilibrium value of θ. Consider a firm (or a set of firms with measure 0) that deviates and posts a slightly different wage. From (23) it follows that for this firm, $\theta'(m) = -\theta^*/\mu$. Inserting this and $m'(K)$ into (26) gives that[20]

$$\frac{r}{q(\theta^*)} = (1 - \beta + \rho\beta)\frac{f'(K^*)}{r+\delta}. \tag{28}$$

where

$$\rho = \frac{\beta p(\theta^*)(M(K^*) - M_0)}{\beta p(\theta^*)(M(K^*) - M_0) + \mu/(1-\beta)} < 1, \tag{29}$$

Proposition 6. *The equilibrium value K^* is equal to the random search equilibrium value when $\mu \to \infty$, is increasing in μ, and is equal to the efficient level when search is perfectly directed ($\mu \to 0$).*

We see that in the limit, as $\mu \to 0$, ρ converges to 1 and we get efficiency. As $\mu \to \infty$, ρ converges to 0 and we are back in the random search solution.

We will now use the results from our estimated model to give an indication of the order of magnitude of μ. To that end, we calibrate the investment model as follows:

- The value of M_0 is set equal to the value of M_0 in our estimated economy.
- The value of $p(\theta^*)$ is set equal to the average unemployment outflow rate in our estimated economy.
- $M(K)$ is set equal to the average value of obtaining a job in our estimated economy, which we calculate as

$$M(K^*) = \frac{rM_0 - y_0}{\beta p(\theta^*)}$$

[20] From (26) we get that

$$-q'(\theta)\theta\mu^{-1}\frac{(1-\beta)(M(K^*) - M_0)}{1 + \beta p'(\theta)\theta\mu^{-1}[M - M_0]}\beta p(\theta^*)\frac{f'(K^*)}{r+\delta} + q(\theta)(1-\beta)\frac{f'(K^*)}{r+\delta} = r$$

This can be written as in (28), with

$$\rho = \frac{p(\theta)\mu^{-1}\beta(1-\beta)(M(K^*) - M_0)}{1 + \beta p'(\theta)\theta\mu^{-1}[M - M_0]} = \frac{p(\theta)\mu^{-1}\beta(1-\beta)(M(K^*) - M_0)}{1 + \beta(1-\beta)p(\theta)\mu^{-1}[M - M_0]} = \frac{\beta p(\theta^*)(M(K^*) - M_0)}{\beta p(\theta)[M - M_0] + \mu/(1-\beta)}$$

Inserting these values into the model gives $\rho = 0.85$. Hence, the estimated degree of directedness places us 85 percent of the way from random-search investment incentives toward directed-search investment incentives.

One may ask why we get closer to efficiency in the simplified model with identical firms and investments than in our estimated model with heterogeneous firms.

Note first, however, that to the best of our knowledge, there is no theorem or result implying that, for a given μ, the fraction of the efficiency potential realized when moving from random to directed search should be the same across different environments.

Furthermore, the estimated model is far more complex than the investment model. The private gains from entry into the different submarkets may be higher or lower than the social value, depending on the submarket in question. Although increased responsiveness to search incentives increases welfare overall, the effect is not positive in all submarkets. This contrasts with the investment model, where increased responsiveness to search incentives always increases welfare. We conjecture that this may explain why the investment model, for a given μ, yields outcomes closer to efficiency.

That being said, we find the differences in the efficiency response to μ across environments intriguing, and obtaining a better understanding of these differences is on our agenda for future research.

7 Conclusion

This paper develops and estimates a tractable model of partly directed search in the labor market. The model nests random search and perfectly directed search and allows workers to sort across submarkets in a probabilistic manner, with the degree of directedness governed by a single parameter. Workers are more likely to search in submarkets that offer higher expected returns, but may still search in inferior submarkets with positive probability. Wages are determined through bargaining after a match is formed, as in standard search models, but workers internalize joint surplus when directing their search.

The model delivers a number of sharp theoretical implications. We show that even with perfectly directed search, equilibrium deviates from competitive search equilibrium and does not give rise to an efficient allocation of searching workers across submarkets (and searching firms). Furthermore, as long as search is not completely random, the composition of job-to-job transitions depends systematically on the worker's current position in the job ladder. Conditional on moving to a given or higher-ranked submarket, workers employed at higher-ranked firms are less likely to move to any fixed destination submarket than workers employed at lower-ranked firms. This property provides a natural and robust source of identification of the degree of directedness in search.

We take the model to matched employer–employee data from Denmark and estimate it using simulated minimum distance. The estimation exploits detailed information on job-to-job transitions, wages, firm characteristics, and worker histories. Firms are grouped into submarkets using a poaching rank, and the estimation strategy explicitly accounts for classification error in firm ranks as well as for downward mobility induced by advance notice layoffs. The estimated model provides a good fit to key features of worker flows, firm size differences, and transition patterns across submarkets.

Our estimates indicate that search in the Danish labor market is neither fully random nor fully directed. Using a welfare-based measure, we find that the estimated economy realizes about 41 percent of the welfare gain that separates perfectly directed search from random search. Alternative interpretations based on the sensitivity of market tightness to match values and on investment incentives also indicate a substantial degree of directedness. In particular, the estimated degree of directedness substantially mitigates, but does not eliminate, the inefficiencies associated with random search.

Overall, the results suggest that allowing for imperfectly directed search provides a useful and empirically relevant middle ground between the standard random search framework and models with fully directed or competitive search. The framework developed here can be applied to study a range of questions related to worker reallocation, wage dynamics, and efficiency in frictional labor markets, and it offers a promising basis for further empirical work on search technologies.

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Appendix to Directness in Search

A Mathematical appendix

A.1 Proof of Proposition 1

We prove existence for $\mu > 0$ and for $\mu = 0$ separately. We start with $\mu > 0$.

The proof is based on a fixed-point argument. Define the compact convex set

$$\mathcal{X} = \Big\{(M, \pi, k, N) : \; M_i \in [\underline{M}, \overline{M}], \; \pi_{ij} \in [\underline{\pi}, 1], \; \sum_{j=1}^{J} \pi_{ij} = 1 \; \forall i, \; k \in [\underline{k}, \overline{k}], \; N \in \Delta^{J+1} \Big\},$$

with $\Delta^{J+1} = \{N_i \geq 0, \; \sum_{i=0}^{J} N_i = 1\}$, where bounds $(\underline{M}, \overline{M})$ and $\underline{\pi} > 0$ are provided below. Given $(M, \pi, k, N) \in \mathcal{X}$, define $\Gamma(M, \pi, k, N) = (M', \pi', k', N')$ as follows:

1. Define tightness. From (M, π, k, N), calculate

$$\theta_j = \frac{\tau_j k}{\sum_{i=0}^{J} \pi_{ij} N_i}$$

   Given the bounds, θ_j is strictly greater than zero and bounded above. Denote the bounds on θ by $\underline{\theta} > 0$ and $\overline{\theta} < \infty$.

2. Compute m_{ij}^p from equation (2).

3. Given θ, M, and π_{ij}, compute f_{ij} from (7), then V_j from equation (6). Let $EV = \sum_{j=1}^{J} \tau_j V_j$. Since $q(\theta_j) < q(\overline{\theta})$, V_j is finite for all j.

4. Update π from (3)

$$\pi'_{ij} = \frac{\phi_j \tau_j \exp\left(\frac{m_{ij}^p}{\mu}\right)}{\sum_{j=1}^{N} \phi_j \tau_j \exp\left(\frac{m_{ij}^p}{\mu}\right)}$$

5. Update M. For $i \geq 0$,

$$rM'_i \quad = \quad y_i - \delta(M_i - M_0) + \beta \sum_j \pi'_{ij} p(\theta_j) \left(M_j - M_i\right)^+ , \tag{30}$$

6. Update k:

$$k' = kEV/K$$

.

7. Update N:

$$N_0' \sum_{j=1}^{J} \pi_{0j}' p(\theta_j) = \sum_{j=1}^{J} N_j \delta \tag{31}$$

$$N_i' \left(\delta + \sum_{j=i+1}^{N} \pi_{ij}' p(\theta_j) \right) = p(\theta_i) \sum_{j=1}^{i-1} \pi_{ij}' N_j \tag{32}$$

If any of the values (M', π', k', N') defined above exceeds (falls below) the respective upper (lower) bound, the updated value is instead set equal to the corresponding upper (lower) bound.

The mapping is continuous, and it follows from Brouwer's fix-point theorem that the mapping has a fixed point. We denote this by (M^*, π^*, k^*, N^*). By construction, the fix-point satisfies all the equilibrium requirements provided that bounds do not bind.

Hence the proof is complete if we can construct bounds that don't bind at the fixed point. We will now derive such bounds.

1. An upper bound for M_j is y_J/r, and a lower bound is y_0/r.

2. Lower bound for k. As $k \to 0$, we have $N_0 \to 1$ and $M_0 \to y_0/r$. Then there exists $j > 0$ with $\theta_j \to 0$. Since $M_j - M_0 \to (y_j - y_0)/(r + \delta) > 0$ and $q(\theta_j) \to \infty$ under the standard Inada condition on the matching function, it follows that $V_j \to \infty$ and hence $EV \to \infty$. Thus there exists $\underline{k} > 0$ such that $EV > K$ for all $k \leq \underline{k}$. This lower bound cannot bind at the fixed point.

3. Upper bound for k. Total worker mass is 1. The mass of type-J firms is $\tau_J k$. Thus the average number of workers per type-J firm is at most $1/(\tau_J k)$. Since wages are at least y_0, flow profit per type-J firm is bounded by $(y_J - y_0)/(\tau_J k)$. Discounting at rate r,

$$V_J \ \leq \ \frac{y_J - y_0}{r \, \tau_J \, k}.$$

Because $y_j \leq y_J$ for all j, $V_j \leq V_J$. Hence

$$EV = \sum_{j=1}^{J} \tau_j V_j \ \leq \ V_J \ \leq \ \frac{y_J - y_0}{r \, \tau_J \, k}.$$

As a consequence, for $k \geq \overline{k} := (y_J - y_0)/(rK\tau_J)$, we have $EV \leq K$, so no fixed point can lie above $\overline{k}$. Thus $k^* \leq \overline{k}$ in equilibrium.

4. Lower bound for π_{ij}. From the logit expression

$$\pi_{ij} = \frac{\tau_j \exp(m_{ij}^p/\mu)}{\sum_{\ell=1}^{J} \tau_\ell \exp(m_{i\ell}^p/\mu)},$$

with $m^p_{ij} \in [y_0, y_J]$, the worst case for j is $m^p_{ij} = y_0$ and all $m^p_{i\ell} = y_J$ for $\ell \neq j$. Hence

$$\pi_{ij} \;\geq\; \frac{\tau_j e^{y_0/\mu}}{\sum_{\ell=1}^{J} \tau_\ell e^{y_J/\mu}} \;\geq\; \tau_{\min}\, e^{-(y_J - y_0)/\mu} \;=:\; \underline{\pi} \;>\; 0.$$

Suppose then that $\mu = 0$. We define the fixed point as above; however, we change the update of π_{ij}. For a given i, let $\tilde{j}$ denote a value of j that maximizes m^p_{ij} defined in step 2, and let $m^{\max}_i$ denote the corresponding maximum value. Let $\bar{m}_i = \sum_j \pi_{ij} m^p_{ij}$.

$$\pi'_{ij} = \pi_{ij}\left(1 + \frac{m^p_{ij} - \bar{m}_i}{m^{\max}_i}(1 - \pi_{i\tilde{j}})\right)$$

It follows that π'_{ij} is bounded by 0 and 1, sums over j to 1, and is continuous in π_{ij} and m^p_{ij}. Furthermore, at a fixed point, $\pi_{ij} > 0$ only in the submarkets with $m^p_{ij} = m^{\max}_i$.

We use the same bounds as in the proof with $\mu \neq 0$, and show that the bounds do not bind in the same way except for the lower bound $\underline{\pi}$ for π_{ij}, which is no longer defined by (3). Instead, we assume a lower bound on a combination of variables, $\tilde{n}_j = \sum_{i<j} N_i \pi_{ij}$, the measure of workers searching in submarket j, and show that it does not bind for sufficiently low values of the bound for any $j > 0$. This is sufficient to ensure an upper bound on θ_j, which is what we need for the mapping Γ to be well defined. To this end, construct a decreasing sequence of lower bounds on $\tilde{n}$, ϵ_z, which converges to zero. For each z, there exists a fixed point. We want to show that for sufficiently high z, the bound $\tilde{n}_j$ does not bind for any j. Suppose it does. For each of these fixed points, θ_j is non-increasing in j. Since all workers at $j < J$ search, there must exist a lowest market l at which the sequence $\tilde{n}^z_l$ is bounded below by a strictly positive number, and denote the associated upper bound on the tightness by $\overline{\theta}$. Some unemployed workers must search in market l (due to maximum separation, see 2), so that $rM_0 \leq y_0 + \beta p(\overline{\theta})(M_l - M_0)$. Furthermore, $M_1 - M_0 \geq \frac{y_1 - y_0}{r + \delta + p(\overline{\theta})}$, which is a fixed number strictly greater than zero. However, as $\tilde{n}_1 \to 0$, and since $k \geq \underline{k}$, $p(\theta_1) \to \infty$, and an unemployed who is searching in the $j = 1$ market obtains an NPV income that converges to M_1, a contradiction. Hence, for some $z < \infty$ the bound does not bind for $j = 1$, and since θ_j is non-increasing in j, for none of the markets. This completes the proof.

A.2 Proof of Proposition 2

Recall that, in the limit economy,

$$rM_i = y_i + \delta(M_0 - M_i) + \beta \max_j \left[p(\theta_j)(M_j - M_i)\right]. \tag{33}$$

Suppose $i' > i$. Further, suppose there exists a market j that workers in i' search in, and another market $j' > j$ that workers in i search in. We will show that this leads to a contradiction.

Assume the claim is true. For the worker in i', it follows that:

$$y_{i'} + \delta(M_0 - M_{i'}) + \beta p(\theta_j)(M_j - M_{i'}) \geq y_{i'} + \delta(M_0 - M_{i'}) + \beta p(\theta_{j'})(M_{j'} - M_{i'}).$$

This simplifies to:

$$p(\theta_j)(M_j - M_{i'}) \geq p(\theta_{j'})(M_{j'} - M_{i'}). \tag{34}$$

Similarly, for the worker in i, it follows that:

$$p(\theta_j)(M_j - M_i) \leq p(\theta_{j'})(M_{j'} - M_i). \tag{35}$$

Combining (34) and (35), we obtain:

$$(p(\theta_{j'}) - p(\theta_j))(M_{i'} - M_i) \geq 0. \tag{36}$$

We know that $p(\theta_{j'}) < p(\theta_j)$ and that $M_{i'} > M_i$. Therefore, the left-hand side is strictly negative, contradicting the inequality. This completes the proof.

Proof of Proposition 3

First, we want to show that it does not matter for the profitability of the deviating firms whether they form their own submarket or join the existing submarket with the same wage, since in both cases they will face the same tightness (and will also attract workers with the same employer type).

Suppose the deviating firms' submarket with wage W' did obtain a strictly higher tightness than the market with the corresponding productivity $y'_j = W^{-1}(W')$, which in equilibrium has tightness $\theta(y'_j)$. Then workers currently employed in firms with productivity $y'_i = y_i(y'_j)$ would strictly prefer to join the new market, which is inconsistent with equilibrium. If, on the other hand, the resulting tightness were strictly lower than $\theta(y'_j)$, the new market would be strictly dominated by the y'_j-market, and we could not be in equilibrium. Hence it follows that the tightness must be equal to $\theta(y'_j)$. Given this tightness, the deviating market would be dominated by another market for all workers except those currently employed in firms of type y'_i, who will therefore be the workers who populate the new market.

Given this equivalence, we now let the deviating firm choose which submarket to enter. When choosing between submarkets, the firm chooses between a menu of wage–tightness combinations $(W(y_j), \theta(y_j))$. It follows that we can write θ as a function of W, $\theta = \theta^e(W) \equiv \theta(W^{-1}(W))$ for all $W \in \mathcal{W}$. Hence

$$\frac{d\theta^e(W)}{dW} = \frac{\theta'(y_j)}{W'(y_j)}, \tag{37}$$

where $y_j = W^{-1}(W)$, $W \in \mathcal{W}$.

We now investigate the derivative of $\theta^e(W)$ further. Let $g(y_i)$ denote the value of search in the perfectly directed search equilibrium, defined as

$$g(y_i) = \max_{y_j} p(\theta(y_j))\, \beta\, (M(y_j) - M(y_i))\,. \tag{38}$$

By the envelope theorem it follows that

$$g'(y_i) = -\beta\, p(\theta(y_j))\, M'(y_i), \tag{39}$$

where y_j is the productivity of the firms in the optimal submarket, given by $y_j = y_i^{-1}(y_i)$. Note that $g'(y_i) < 0$.

Next, we take the derivative of (38) with respect to y_j, taking into account that $y_i = y_i(y_j)$. We obtain

$$\begin{aligned} g'(y_i)\, y_i'(y_j) = {} & p'(\theta)\, \theta'(y_j)\, \beta\, \big(M(y_j) - M\big(y_i(y_j)\big)\big) \\ & + p\big(\theta(y_j)\big)\, \beta\, \big(M'(y_j) - M'(y_i)\, y_i'(y_j)\big)\,. \end{aligned} \tag{40}$$

Recall that

$$W(y_j) = M\big(y_i(y_j)\big) + \beta\, \big(M(y_j) - M\big(y_i(y_j)\big)\big)\,,$$

hence

$$W'(y_j) = M'(y_i)\, y_i'(y_j) + \beta\, \big(M'(y_j) - M'(y_i)\, y_i'(y_j)\big)\,.$$

Substituting these expressions into (40) gives

$$g'(y_i)\, y_i'(y_j) = p'(\theta)\, \theta'(y_j)\, \big(W(y_j) - M(y_i)\big) + p\big(\theta(y_j)\big)\Big(W'(y_j) - M'(y_i)\, y_i'(y_j)\Big).$$

Using (39) and rearranging yields

$$p'(\theta)\, \theta'(y_j)\, \big(W(y_j) - M(y_i)\big) + p\big(\theta(y_j)\big)\, W'(y_j) = -(1-\beta)\, g'(y_i)\, y_i'(y_j) > 0, \tag{41}$$

since $g'(y_i) < 0$ and, by monotonicity, $y_i'(y_j) > 0$. It follows that

$$\frac{d\theta^e(W)}{dW} = -\frac{\theta'(y_j)}{W'(y_j)} > -\frac{p(\theta(y_j))}{p'(\theta(y_j))\big(W(y_j) - M(y_i)\big)}. \tag{42}$$

The deviating firm of productivity y_j seeks to maximize profit

$$rV = q(\theta^e(W))\, (M_j - W).$$

The first-order condition is

$$q'(\theta^e(W))\, \frac{d\theta^e(W)}{dW}\, (M_j - W) - q(\theta^e(W)) = 0.$$

It is sufficient to show that the left-hand side of this condition, evaluated at $(W(y_j), \theta(y_j))$, is strictly negative. In that case the firm finds it profitable to reduce the wage. This is equivalent to showing that

$$\frac{d\theta^e(W(y_j))}{dW} > \frac{q(\theta(y_j))}{q'(\theta(y_j))\,(M_j - W(y_j))}.$$

Now, using the Cobb–Douglas relations

$$\beta = \left|\frac{\theta\, q'(\theta)}{q(\theta)}\right|, \qquad 1-\beta = \frac{p'(\theta)}{q(\theta)},$$

we obtain (evaluated at y_j, and suppressing the dependence on y_j)

$$\begin{aligned} \frac{q(\theta)}{q'(\theta)\,(M_j - W)} &= \frac{q(\theta)}{q'(\theta)}\,\frac{1-\beta}{\beta}\,\frac{1}{W - M_i} \\ &= -\frac{\theta}{(1-\beta)(W-M_i)} \\ &= -\frac{p(\theta)}{p'(\theta)\,(W - M_i)}. \end{aligned} \tag{43}$$

From (42) we know that

$$\frac{d\theta^e(W(y_j))}{dW} > -\frac{p(\theta(y_j))}{p'(\theta(y_j))\,(W(y_j) - M(y_i))},$$

which proves that a small reduction in W increases profit, and therefore the deviating firm sets a strictly lower wage than $W(y_j)$.

Finally, note that the indifference curve of the worker of type $y_i(y_j)$ who searches in the y_j-market is given by

$$p(\theta(W))\,(W - M_i) = g(y_i),$$

with derivative exactly equal to

$$-\frac{p(\theta)}{p'(\theta)\,(W - M_i)}.$$

Hence, if the deviating firm were always attracting workers of type $y_i(y_j)$, it follows from (43) that it would not be profitable to deviate. However, when it reduces the wage, the deviating firm attracts workers of strictly lower type. The relevant θ–W relationship is therefore $\theta^e(W)$, which is strictly less steep than the indifference curve of the $y_i(y_j)$ worker.

A.3 Proof of Proposition 4

Let j and z be two receiving markets, with $z > j$. Let $i < j$ be arbitrary. Then we have that

$$\begin{aligned} m_{iz} - m_{ij} &= \beta p(\theta_z)(M_z - M_i) - \beta p(\theta_j)(M_j - M_i) \\ &= \beta\big(p(\theta_z)M_z - p(\theta_j)M_j + (p(\theta_j) - p(\theta_z))\,M_i\big) \end{aligned} \tag{44}$$

Since $p(\theta_j) > p(\theta_z)$, it follows that $m_{iz} - m_{ij}$ is strictly increasing in i. Recalling that $\pi_{ij} = \frac{e^{m_{ij}/\mu}}{\sum_x e^{m_{ix}/\mu}}$ we get

$$\begin{aligned}
\hat{h}_{ij} = & \quad \frac{p_j \pi_{ij}}{\sum_{z=j}^{J} p_z \pi_{iz}} \\
= & \quad \frac{p_j \frac{\tau_j e^{m_{ij}/\mu}}{\sum_x \tau_x e^{m_{ix}/\mu}}}{\sum_{z=j}^{J} p_z \frac{\tau_z e^{m_{iz}/\mu}}{\sum_x \tau_x e^{m_{ix}/\mu}}} \\
= & \quad \frac{p_j \tau_j e^{m_{ij}/\mu}}{\sum_{z=j}^{J} p_z \tau_z e^{m_{iz}/\mu}} \\
= & \quad \frac{p_j \tau_j}{\sum_{z=j}^{J} p_z \tau_z e^{(m_{iz}-m_{ij})/\mu}}
\end{aligned}$$

Suppose that $i' > i$. It follows that

$$m_{i'z} - m_{i'j} > m_{iz} - m_{ij} \tag{45}$$

(since $z > j$). It follows that $e^{(m_{i'z}-m_{i'j})/\mu} > e^{(m_{iz}-m_{ij})/\mu}$, and hence that

$$\begin{aligned}
\hat{h}_{ij} = & \frac{p_j \tau_j}{\sum_{z=j}^{J} p_z \tau_z e^{(m_{iz}-m_{ij})/\mu}} \\
> & \frac{p_j \tau_j}{\sum_{z=j}^{J} p_z \tau_z e^{(m_{i'z}-m_{i'j})/\mu}} \\
= & \hat{h}_{i'j}
\end{aligned}$$

which shows Proposition 4

A.4 Derivations related to efficiency considerations

Hence, the planner's maximization problem can be written as

$$\max_{\pi_{ij}(t)} \int_0^\infty \sum_{i=0}^{I} N_i(t) y_i e^{-rt}\, dt$$

Given the following constraints:

$$\dot{N}_i(t) = -N_i(t)\left(\delta \mathbf{1}_{i>0} + \sum_{j=i+1}^{I} \pi_{ij}(t)\theta_j(t)^{1-\beta}\right) + \sum_{l=0}^{i-1} N_l(t)\pi_{li}(t)\theta_i(t)^{1-\beta} + (1-\mathbf{1}_{i>0})\delta \sum_{i=1}^{J} N_i, \quad i = 1, \ldots, I$$

where $\mathbf{1}_{i>0} = 1$ if $i > 0$ and 0 if $i = 0$, where

$$\theta_j(t) = \frac{\tau_j k}{\sum_{l=0}^{I} \pi_{lj}(t) N_l(t)}, \quad p(\theta_j(t)) = \theta_j(t)^{1-\beta}, \quad 0 < \beta < 1,$$

and where we require that

$$\sum_{i=0}^{I} N_i(0) = 1, \quad \pi_{ij}(t) \geq 0,$$

$$\sum_{j=1}^{J} \pi_{ij} = 1 \quad i = 1, \ldots, I$$

We will first derive the solution to this maximization problem in the noiseless limit with $\mu = 0$. Then we will use the set-up (the Hamiltonian and the associated adjoint variables) to shed light on the social versus private value of entering the different submarkets when $\mu > 0$.

Perfectly directed search, $\mu = 0$

Let us first derive the optimal solution in the noiseless limit with $\mu = 0$. In this case, the planner can freely choose the probabilities π_{ij}. Furthermore, neither the planner nor the agents in the decentralized solution would ever choose $\pi_{ij} > 0$ for $j \leq i$. The current-value Hamiltonian thus reads

$$\begin{aligned} H &= \sum_{i=0}^{I} N_i y_i + \sum_{i=0}^{I} \lambda_i \left[-N_i \left(\delta \mathbf{1}_{i>0} + \sum_{j>i} \pi_{ij} p(\theta_j) \right) + \sum_{l=0}^{i-1} N_l \pi_{li} p(\theta_i) + (1 - \mathbf{1}_{i>0}) \delta \sum_{l=1}^{J} N_l \right] \\ &+ \sum_i \sigma_i \left(\sum_{j=i+1}^{J} \pi_{ij} - 1 \right) + \sum_{i=0}^{J} \sum_{j=0}^{J} \kappa_{ij} \pi_{ij} \end{aligned} \tag{46}$$

We require that $\sigma_i, \kappa_{ij} \geq 0$ and that $\pi_{ij}\kappa_{ij} = 0$ (complementary slackness).

Using that $r\lambda_i = \frac{\partial H}{\partial N_i}$ in steady state we get that

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \frac{\partial}{\partial N_i} \left(\sum_{j>i} p(\theta_j) \sum_{l<j} N_l \pi_{lj} (\lambda_j - \lambda_l) \right) \tag{47}$$

(since $\delta_i(\lambda_0 - \lambda_0) = 0$). Now $\frac{\partial \theta_j}{\partial N_i} = -\theta_j \frac{\pi_{ij}}{\tilde{n}_j}$, where $\tilde{n}_j$ is the total number of applicants in market j ($\tilde{n}_j = \sum_{l=0}^{j-1} \pi_{lj}(t) N_l(t)$). The next expression follows. For $i > 0$ we have that

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \sum_{j>i} \pi_{ij} p(\theta_j)(\lambda_j - \lambda_i) - \sum_{j>i} (1-\beta) p(\theta_j) \pi_{ij} \sum_{l<j} f_{lj} (\lambda_j - \lambda_l) \tag{48}$$

The first term is the productivity flow, the second the direct effect of on-the-job search (not including search externalities) and the third term captures search externalities. Note that if $\pi_{ik} = 1$ for some

k, and that $\pi_{lk} = 0$ for all $l \neq i$, then the equation writes

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \beta p(\theta_k)(\lambda_k - \lambda_i)$$

which is well known in competitive search. More generally, let $\tilde{\lambda}_j = \sum_0^{j-1} f_{lj}\lambda_l$. Then

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \sum_{j>i} \pi_{ij}\beta p(\theta_j)(\lambda_j - \lambda_i) - \sum_{j>i} (1-\beta)\, p(\theta_j)\pi_{ij}(\lambda_i - \tilde{\lambda}_j) \tag{49}$$

equal to (13).

Note that $\frac{\partial \theta_j}{\partial \pi_{ij}} = -\theta_j \frac{N_i}{\tilde{n}_j}$.

$$\pi_{ij} \left[\beta p(\theta_j)(\lambda_j - \lambda_i) - (1-\beta)p(\theta_j) \sum_{l<j} (\tilde{\lambda}_j - \lambda_i) = (\sigma_i + \kappa_{ij}) N_i^{-1} \right] \tag{50}$$

which is equivalent to maximizing λ given by (49). This is a linear programming problem, and using the complementary slackness condition for κ_{ij} and π_i gives first order conditions (14).

Finally, we consider entry decisions. Firms can enter at cost K. Subtracting kK from the Hamiltonian (46) and taking derivatives wrt k gives

$$\sum_{j=1}^{J} \tau_j q(\theta_j)(1-\beta)\left(\lambda_j - \tilde{\lambda}_j\right) = K \tag{51}$$

which has the same format as the equilibrium entry condition, as expected. Hence, any inefficiencies along the entry dimension will be due to differences between the expected social and private values, λ_i and M_i, of worker-firm matches.

General case

Let us then consider a situation in which the π_{ij} are constrained to be equal to $\bar{\pi}_{ij}$ for some probability matrix $\bar{\pi}$. We do not require $\bar{\pi}_{ij} = 0$ for $i \geq j$. The Hamiltonian then read (the constraint that $\sum_j \pi_{ij} = 1$ and that the probabilites are non-negative are now automatically satisfied and therefore ignored).

$$\begin{aligned} H &= \sum_{i=0}^{I} N_i y_i + \sum_{i=0}^{I} \lambda_i \left[-N_i \left(\delta \mathbf{1}_{i>0} + \sum_{j>i} \pi_{ij} p(\theta_j) \right) + \sum_{l=0}^{i-1} N_l \pi_{li} p(\theta_i) \right] \\ &+ \sum_{i=0}^{J} \sum_{j=0}^{J} \kappa_{ij} \left(\pi_{ij} - \bar{\pi}_{ij} \right), \quad \theta_j(t) = \frac{\tau_j k}{\sum_{l=0}^{I} \pi_{lj}(t) N_l(t)} \end{aligned} \tag{52}$$

Using that $r\lambda_i = \frac{\partial H}{\partial N_i}$ gives

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \sum_{j>i} \pi_{ij} p(\theta_j)(\lambda_j - \lambda_i)$$
$$- \sum_{j>i}(1-\beta)p(\theta_j)\pi_{ij} \sum_l \frac{N_l \pi_{lj}}{\tilde{n}_j}(\lambda_j - \lambda_l) - \sum_{j\le i}(1-\beta)p(\theta_j)\pi_{ij} \sum_l \frac{N_l \pi_{lj}}{\tilde{n}_j} \max\left[\lambda_j - \lambda_l, 0\right] \quad (53)$$

Now define $\tilde{\lambda}_j^{adj} = \sum_{l=0}^{I} f_{lj} \max[\lambda_l, \lambda_j]$. It follows that we can write

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \sum_{j>i} \pi_{ij} p(\theta_j)(\lambda_j - \lambda_i)$$
$$- \sum_{j>i}(1-\beta)p(\theta_j)(\lambda_j - \tilde{\lambda}_j^{adj}) + \sum_{j\le i}(1-\beta)p(\theta_j)\pi_{ij}\left[\lambda_j - \tilde{\lambda}_j^{adj}\right] \quad (54)$$

Or

$$r\lambda_i = y_i - \delta(\lambda_i - \lambda_0) + \sum_{j>i} \pi_{ij}\beta p(\theta_j)(\lambda_j - \lambda_i) + \sum_{j>i} \pi_{ij}(1-\beta)p(\theta_j)(\tilde{\lambda}_j^{adj} - \lambda_i)$$
$$- \sum_{j\le i}(1-\beta)p(\theta_j)\pi_{ij}\left[\tilde{\lambda}_j^{adj} - \lambda_j\right] \quad (55)$$

as stated in the text.

We will calculate the social value of increasing π_{ij}, which we denote Λ_{ij}. To that end we take the derivative of H given by (52) to get that, for $j > i$,

$$\Lambda_{ij} = N_i\beta p(\theta_j)(\lambda_j - \lambda_i) + N_i(1-\beta)p(\theta_j)(\tilde{\lambda}_j^{adj} - \lambda_j) \quad (56)$$

For $j \le 0$ we get that

$$\Lambda_{ij} = -N_i(1-\beta)p(\theta_j)(\tilde{\lambda}_j^{adj} - \lambda_j)$$

Finally, consider optimal entry. The optimal number of firms maximises $H - kK$ where H is given by (52). It follows that the conditions for optimality is given by (51) with $\tilde{\lambda}$ replaced by $\tilde{\lambda}^{adj}$.

A.5 Definition of extended equilibrium

Definition 2. A steady state equilibrium with wage bargaining for $\mu > 0$ is a vector of net present values, M_j, $j = 0, ..., J$, a matrix of perceived flow values, $m_{\mathbf{i}j}$, $\mathbf{i} = (i, z)$, $i = 0, ..., J$, $z = 0, 1$, and $j = 1, ..., J$, a matrix of choice probabilities, $\pi_{\mathbf{i}j}$, a vector of labour market tightnesses, $\theta = (\theta_1, ..., \theta_J)$, a number of firms, k, and an allocation of workers on firms and states, $N =$

$(N_{z1}, ..., N_{zJ})$, $z = 0, 1$, such that

1. $M_{\mathbf{i}}$ and $m_{\mathbf{i}j}$ solve equations (18) and (19).
2. The matrix of choice probabilities, $\pi_{\mathbf{i}j}$, is given by (3), with i replaced by $\mathbf{i}$.
3. The vector of labour market tightnesses $(\theta_1, ..., \theta_J)$ is given by equation (5), where the summation is over $\mathbf{i}$ instead of i.
4. The number of firms, k, is implicitly defined by equations (6) and (8).
5. The allocation of workers on firms, N, solves the following equations (replacing 9 and 10)

$$\gamma N_0 \sum_j \pi_{0j} p(\theta_j) = \delta(1 - N_0) \tag{57}$$

$$N_{\mathbf{i}} \left(\delta + \sum_j \pi_{\mathbf{i}j} p\left(\theta_j\right) \mathbf{1}\left(M_j - M_{\mathbf{i}} > 0\right) \right) = p\left(\theta_i\right) \sum_{\mathbf{l}} \pi_{\mathbf{l}i} N_{\mathbf{l}} \mathbf{1}\left(M_{i0} - M_{\mathbf{l}} > 0\right) \tag{58}$$

   where $\mathbf{1}()$ is an indicator function that takes value 1 if the expression in () is true and 0 otherwise.

The proof of existence of equilibrium is completely analogous to the proof of existence of equilibrium in the simpler model and is therefore omitted.

Proof of Lemma 1

Claim 1. The choice probabilities depend on $\tilde{y}_{\mathbf{i}j}$.

For given equilibrium values of $\theta = (\theta_1, \cdots \theta_J)$, $m^p_{\mathbf{i}j}$ defined by (2) depends on $\tilde{y}_{\mathbf{i}j}$ only. Since the probabilities $\pi_{\mathbf{i}j}$, defined by (3), only depend on values of $m^p_{\mathbf{i}j}$, the claim follows.

Claim 2. $M_{\tilde{i}}$ is increasing in y_i.

Let $M(\tilde{y})$ denote the NPV income as a continuous function of $\tilde{y}$ for all $\tilde{y} \geq y_0$, given by (18), where $\tilde{y}$ is defined generically by (20). Furthermore, define $m^p_j(\tilde{y})$ by (19). Similarly, define $\pi_j(\tilde{y})$ by (3), with m^p_{ijk} replaced by $m^p_j(\tilde{y})$.

For sufficiently high values of $\tilde{y}$, no job will be accepted (recall that the vector $y_0, \ldots, y_J$ is fixed). Then

$$rM'(\tilde{y}) = 1 > 0.$$

Suppose $M'(\tilde{y}) < 0$ for some $\tilde{y}$. Since $M'(\tilde{y})$ is continuous in $\tilde{y}$, there must then exist $\tilde{y}'$ such that $M'(\tilde{y}') = 0$. From (2) we then have that at $\tilde{y}'$

$$\frac{dm^p_j(\tilde{y}')}{d\tilde{y}} = 1 \quad \text{for all } j \in \{0, \ldots, J\}.$$

It thus follows from (3) that

$$\frac{d\pi_j(\tilde{y}')}{d\tilde{y}} = 0.$$

But by differentiating (1) with respect to $\tilde{y}$ it then follows that $M'(\tilde{y}) > 0$, contradicting the assumption that $M'(\tilde{y}') = 0$.

Claim 3. The propositions 2 and 4 hold when the worker-firm matches are indexed according to $\tilde{i}$.

These claims follow from the same arguments as those presented in the proofs of proposition 2 and 4, and the proofs are therefore omitted.

B Random Search w/ classification error

Consider a canonical random search model. Workers receive offers at Poisson rate λ whether currently employed or unemployed, and jobs are destroyed at rate δ. Denote by $f_{x,y}$ the joint offer distribution of a true type x offer and its observed type y. Worker movement is as in the regular model in that an offer (weakly) better than current offer is accepted. Denote by f_x the marginal distribution, $\sum_y f_{x,y} = f_x$, and its CDF $F_x = \sum_{n=1}^{x} f_n$. Let unemployment be given by g_0. Denote by $g_{x,y}$ the steady state match distribution over true type x and observed type y. By similar convention denote the marginal distribution by, $g_x = \sum_y g_{x,y}$ and its CDF $G_x = \sum_{n=0}^{x} g_n$.

Denote by h_{mn} the worker transition flow rate between measured type m and measured type n. A worker knows true type and accepts any true type offer that is better than current true type. An observed flow from m to n happens at rate:

$$h_{mn} = \lambda \sum_x g_{xm} \sum_{y \geq x} f_{yn}.$$

With this,

$$\hat{h}_{mn} = \frac{\sum_x g_{xm} \sum_{y \geq x} f_{yn}}{\sum_x g_{xm} \sum_{z \geq n} \sum_{y \geq x} f_{yz}}.$$

The steady state condition on g_{xy} is,

$$\left(\delta + \lambda \sum_{z \geq x} f_z - \lambda f_{xy}\right) g_{xy} = \lambda f_{xy} \left(\sum_{z=0}^{x} g_z - g_{xy}\right)$$

$$\Updownarrow$$

$$\left(\delta + \lambda \sum_{z \geq x} f_z\right) g_{xy} = \lambda f_{xy} \left(\sum_{z=0}^{x} g_z\right)$$

The reasonably straightforward way to solve this is to solve for G_x and then with that solve for

g_{xy}. The solution on the marginals is

$$\lambda\left[1-F_x\right]G_x = \delta\left[1-G_x\right]$$
$$\Updownarrow$$
$$G_x = \frac{\delta}{\delta+\lambda\left[1-F_x\right]}.$$

With that we have,

$$g_{xy} = \frac{\lambda f_{xy} G_x}{\delta+\lambda\left(1-F_{x-1}\right)} = \phi_x f_{xy},$$

where $\phi_x = \frac{\lambda G_x}{\delta+\lambda(1-F_{x-1})}$ is monotonically increasing in x and $0<\phi_x<\lambda/\delta$. With this, in steady state it follows that,

$$\hat{h}_{mn} = \frac{\sum_x \phi_x f_{xm} \sum_{y\geq x} f_{yn}}{\sum_x \phi_x f_{xm} \sum_{z\geq n}\sum_{y\geq x} f_{yz}}.$$

It is possible to design classification error examples where $\hat{h}_{mn}$ is at least locally increasing in m. However, the (far) more common pattern is that of a negative dependence in m. To illustrate, consider a symmetric classification error where a type can only be misclassified as a neighboring type. For some $\varepsilon>0$, assume,

$$f_{xy} = \begin{cases} \varepsilon & \text{if } |x-y|=1 \\ f_x-2\varepsilon & \text{if } y=x \text{ and } N>x>1 \\ f_x-\varepsilon & \text{if } y=x \text{ and } x=1 \text{ or } x=N \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, restrict ε so that $f_{xx}\geq f_{xy}$ for all y and x. With this we have for $1<n<N$,

$$\hat{h}_{mn} = \begin{cases} \frac{f_n}{\hat{F}_{n-1}} & \text{for } m\leq n-2 \\ \frac{f_n-C}{\hat{F}_{n-1}-C} & \text{for } m=n-1 \\ \frac{f_n-D}{\hat{F}_{n-1}-D-\phi_{n+1}\varepsilon^2} & \text{for } m=n \end{cases}$$

where,

$$C = \frac{\phi_n\varepsilon^2}{\sum_{x=n-2}^{n}\phi_x f_{x,n-1}}$$
$$D = \frac{\phi_n\varepsilon\left[\left(f_n-2\varepsilon\right)+\frac{\phi_{n+1}}{\phi_n}\left(f_n-\varepsilon\right)\right]}{\sum_{x=n-1}^{n+1}\phi_x f_{xn}}.$$

Hence, it immediately follows that $\hat{h}_{n-1,n} < h_{mn}$ for $m \leq n-2$. Furthermore, $\hat{h}_{n,n} \leq \hat{h}_{n-1,n}$ implies,

$$\hat{h}_{n,n} \leq \hat{h}_{n-1,n}$$
$$\Updownarrow$$
$$\frac{f_n - D}{\hat{F}_{n-1} - D - \phi_{n+1}\varepsilon^2} \leq \frac{f_n - C}{\hat{F}_{n-1} - C}$$
$$\Updownarrow$$
$$[C - D]\left[\hat{F}_{n-1} - f_n\right] \leq [f_n - C]\,\phi_{n+1}\varepsilon^2$$
$$\Updownarrow$$
$$[C - D]\,\hat{F}_n \leq [f_n - C]\,\phi_{n+1}\varepsilon^2.$$

A sufficient condition for $\hat{h}_{n,n} \leq \hat{h}_{n-1,n}$ is then that,

$$D > C$$
$$\Updownarrow$$
$$(f_n - \varepsilon)\left(1 + \frac{\phi_{n+1}}{\phi_n}\right) > \varepsilon \frac{\sum_{x=n-1}^{n+1} \phi_x f_{xn} + \sum_{x=n-2}^{n} \phi_x f_{x,n-1}}{\sum_{x=n-2}^{n} \phi_x f_{x,n-1}},$$

which is satisfied for suffiently low ε. This is a sufficient condition. In our experience, $D > C$ is the rule rather than the exception even for high ε, but $\hat{h}_{mn}$ has so far resisted broader characterization, and as mentioned, it is possible to provide examples where $\hat{h}_{mn}$ monotonically decreasing in m is violated.

C Specification and solution algorithm

Specification

Let the matching function in a submarket be Cobb-Douglas, $M(s, v) = As^{\alpha}v^{1-\alpha}$, where s is the mass of efficiency units of worker search in the market. The employed worker meets a vacancy at rate $p(\theta) = M(s, v)/s = m(\theta) = A\theta^{1-\alpha}$. The unemployed worker, by virtue of the possibly different search efficiency, meets a vacancy at rate, $p_0(\theta) = \gamma m(\theta) = \gamma A\theta^{1-\alpha}$. Vacancies meet workers at rate $q(\theta) = A\theta^{-\alpha}$.

Solution algorithm

1. The solution algorithm looks to find a fixed point in a mapping of (θ, k) into itself, $(\theta^{(m+1)}, k^{(m+1)}) = T_{\theta k}(\theta^{(m)}, k^{(m)})$.

2. The mapping $T_{\theta k}$ is determined as,

3. Take input $(\theta^{(m)}, k^{(m)})$.

4. Solve for $\left(m^{(m)}, M^{(m)}, \pi^{(m)}\right)$ for given $\theta^{(m)}$ using equation systems (2), (1), and (3).

   (a) This equation system is solved as fixed point search $M^* = T_M M^*$.

      i. Make a guess, $M^{(s,m)}$.

      ii. Solve for $m^{(s,m)}$ using equation (2) - uses $M^{(s,m)}$ and $\theta^{(m)}$.

      iii. Solve for $\pi^{(s,m)}$ using equation (3) - uses $m^{(s,m)}$.

      iv. Solve for $M^{(s+1,m)}$ using equations (1) - uses $\theta^{(m)}$ and $\pi^{(s,m)}$.

         A. This step uses that M can be written as a contraction mapping,

$$M_{ik} = \frac{y_i + (\delta_i + s_k) M_0 + \lambda M_{i1} + \sum_{j=1} \pi_{ijk} p(\theta_j) \beta \max[M_{j0}, M_{ik}]}{r + \delta_i + s_k + \lambda + \sum_{j=1} \pi_{ijk} p(\theta_j) \beta}$$

$$M_0 = \frac{y_0 + \sum_{j=1} \gamma \pi_{0jk} p(\theta_j) \beta \max[M_{j0}, M_0]}{r + \sum_{j=1} \gamma \pi_{0j} p(\theta_j) \beta}$$

         Thus, simply iterate until convergence to obtain solution. This happens to be quite fast.

      v. loop back to step i with $M^{(s+1,m)}$ as input. Continue to iterate until convergence... no convergence guaranteed.

   (b) In practice, it is faster if the algorithm does not iterate for a full solution of this system in each (m) iteration.

5. Solve for $n^{(m)}$ as the solution to the linear equation system in equations (9)-(**??**) taking as given $\pi^{(m)}$, $M^{(m)}$ and $\theta^{(m)}$.

6. Solve for $\left(\theta^{(m+1)}, k^{(m+1)}\right)$ as the solution to equations (5) and (8).

   (a) This step uses a speed up from the Cobb-Douglas formulation. We have that $q(\theta) = \theta^{-\alpha}$. Define,

$$\tilde{\theta}_j^{(m)} = \frac{\tau_j}{\sum_{k=0}^{1}\sum_{i=1}^{N}\pi_{ijk}^{(m)}N_{ik}^{(m)} + \gamma\pi_{0j}^{(m)}N_0^{(m)}}.$$

   With this, the free entry condition can then be written as,

$$k^{-\alpha}\sum_{j=1}^{N}\frac{\tau_j\tilde{\theta}_j^{-\alpha}(1-\beta)\left[\sum_{i=1}^{N}\sum_{k=0}^{1}(M_{j0}-M_{ik})^{+}f_{ijk} + (M_{j0}-M_0)\,f_{0j}\right]}{r+\delta} = K.$$

   Hence,

$$k^{(m+1)} = \left[\sum_{j=1}^{N}\frac{\tau_j\left(\tilde{\theta}_j^{(m)}\right)^{-\alpha}(1-\beta)\left[\sum_{i=1}^{N}\sum_{k=0}^{1}(M_{j0}^{(m)}-M_{ik}^{(m)})^{+}f_{ijk} + \left(M_{j0}^{(m)}-M_0^{(m)}\right)f_{0j}\right]}{K\,(r+\delta)}\right]^{1/\alpha}.$$

   And with this, it immediately follows that,

$$\theta_j^{(m+1)} = \tilde{\theta}_j^{(m)}k^{(m+1)}.$$

7. With $\left(\theta^{(m+1)}, k^{(m+1)}\right)$ loop back to step 3. Repeat to convergence.

D Simulation

A worker i's history consists of a sequence of spells $\{\gamma_{il}, w_{il}, j_{il}, k_{il}\}_{l=1}^{L_i}$, where γ is the duration, w is the wage, j_{il} is the firm type of the spell and k_{il} is the layoff rate state. Denote by h_{jk} the hazard of spell with a j type firm and layoff rate state k. Conditional on an employment spell, $j > 0$, the spell hazard is given by

$$h_{jk} = \lambda\,(1-k) + \delta_j + s_k + \sum_{j'=1}^{N}\pi_{jj'k}p(\theta_{j'})\mathbf{1}\left[M_{jk} < M_{j'0}\right].$$

If the spell is an unemployment spell, $j = 0$, the spell hazard is (where the layoff state notation is suppressed),

$$h_0 = \sum_{j'=1}^{N}\pi_{0j'}p_0(\theta_{j'}).$$

Denote by $\beta_{jj'kk'}$ the transition conditional probability that the state of the next spell is (j', k') given that the current spell is state (j, k). It follows directly that,

$$
\begin{aligned}
\beta_{jj01} &= \frac{\lambda}{h_{j0}}, \ \forall j > 0 \\
\beta_{j0k} &= \frac{\delta_j + s_k}{h_{jk}}, \ \forall j > 0, \ k \in \{0, 1\} \\
\beta_{jj'kk} &= \frac{\pi_{jj'k} p(\theta_{j'}) \mathbf{1}\left[M_{jk} < M_{j'0}\right]}{h_{jk}}, \ \forall j, j' > 0, \ k \in \{0, 1\} \\
\beta_{0j} &= \frac{\pi_{0j'} p_0(\theta_{j'})}{h_0}, \ \forall j > 0,
\end{aligned}
$$

and $\beta_{jj'kk'} = 0$ for all other (j, j', k, k') combinations.

The simulation initializes by steady state N_{jk}. Thus, from this distribution draw an initial state for worker i. Then simulate the duration $\gamma_{il} \sim \text{Exp}(h_{j_{il}k_{il}})$. The state of the next spell is then drawn according to $\beta_{jj'kk'}$. Let the length of the simulated panel of workers be T periods. Stop simulating spells for worker i when $\sum_{l=1}^{L_i} \gamma_{il} > T$. Let there be I workers in the panel.

E Data Construction Appendix

E.1 Sample selection and cleaning of the spell data

We focus on individuals which we are also able to find in the IDA registers so that we can observe age, gender, education and the like. Focusing on the set of employment spells for a given individual we we now do the following:

- We merge two employment spells into one if they take place in the same firm with less than 94 days in between spells
- We merge two employment spells in different firms with less than 21 days between spells in the sense that we change the startdate of the second spell to start immediately after the first
- We drop employment spells with a duration of below 21 days
- We drop employment spells with accumulated earnings less than 20.000 (in 2000 Dkk)
- We drop employment spells with weekly earnings less than 3000 (in 2000 Dkk) kr
- We drop employment spells where we have no hours recorded
- We drop employment spells with an average number of weekly hours below 20 hours (full time work: 37 hours * 40 weeks / 52 weeks)
- We drop employment spells with less than 150 accumulated hours

E.2 Linking across datasets and firm data

Individuals are linked across registers by their personal identifiers pnr. For individuals we use the IDA registers at Statistics Denmark: IDAP (from which we get information about age), UDDA (information on highest completed education), IDAN (from which we get information about wages).

Firms are linked across registers by their firm id (cvrnr) (firm level identifier). This firm id allows us to identify worker moves between firms and collect workers employed at the same firms. In addition we add information on industry affiliation and value added. Data on Value Added comes from the FIRM register at Statistics Denmark. FIRM contains firm level accounting data and a measure of value added.[21] Note that FIRM is only available for a selected subset of Danish firms (higher coverage in larger firms)[22], but since we focus on firms with more than 5 hires over the sample period and only need one period of non-missing value added to construct ranks and since we are not ranking firms in the public sector (where value added is not available) and we do not loose a lot of coverage using this VA measure.[23]

E.3 Summary statistics within firm ranks

In Table 6 we show various summary statistics of our sample within each firm rank bin. Average age across firm bins is similar. There are fewer women working in the highest ranked firms and slightly higher educated workers. If we instead rank firms based on wages that wage ranking is broadly increasing across firm bins. Similarly ranking firms using only women or men, or low or highly educated workers when constructing poaching ranks would also imply firm rankins which are increasing across our firm bins.

[21] See an example of the calculation here: http://www.dst.dk/extranet/staticsites/TIMES3/html/63c1f70e-7933-40fd-89de-90f8ab191b0c.htm

[22] The FIRM database we use is a collection of the registers: FIRM (Generel Firmastatistik), FIGF, FIGT, and FIRE from Statistics Denmark. The degree of coverage in the FIRM database varies across industries for historical reasons. Statistics Denmark started to collect firm accounting data in the Manufacturing sector in the early 90's and then slowly expanded the coverage until it covered all of the private sector except the financial sector.

[23] (We discard less than 3 % of observations when restricting to non-missing value added.

Table 6: Summary stats by firm rank

Firm rank	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Age	36.931	36.533	36.613	36.387	36.257	36.651	36.710	36.644	36.630	37.198
	(7.334)	(7.282)	(7.222)	(7.229)	(7.265)	(7.182)	(7.190)	(7.088)	(7.069)	(6.992)
Female (indicator)	0.425	0.434	0.388	0.363	0.337	0.307	0.287	0.296	0.271	0.290
	(0.494)	(0.496)	(0.487)	(0.481)	(0.473)	(0.459)	(0.453)	(0.457)	(0.444)	(0.454)
Education (3 levels)	2.056	2.119	2.114	2.104	2.173	2.210	2.270	2.335	2.347	2.362
	(0.802)	(0.800)	(0.811)	(0.807)	(0.803)	(0.809)	(0.778)	(0.758)	(0.744)	(0.721)
Wage Rank	3.835	4.909	5.659	6.101	5.813	6.631	6.904	7.242	7.660	7.723
	(2.260)	(2.493)	(2.412)	(2.258)	(2.215)	(2.254)	(2.162)	(2.257)	(2.128)	(2.184)
PoachWomenOnly	1.288	2.211	3.019	3.991	4.645	5.431	6.348	7.204	8.022	9.269
	(0.746)	(1.147)	(1.247)	(1.375)	(1.353)	(1.589)	(1.458)	(1.510)	(1.447)	(1.089)
PoachMenOnly	1.883	2.670	3.404	4.303	5.116	5.906	6.480	7.474	8.309	9.415
	(1.056)	(1.363)	(1.212)	(1.213)	(1.205)	(1.168)	(1.078)	(1.079)	(0.892)	(0.694)
PoachLowEducated	1.240	2.136	2.929	3.837	4.652	5.543	6.464	7.459	8.349	9.579
	(0.738)	(1.0413)	(1.033)	(1.055)	(1.151)	(1.358)	(1.277)	(1.177)	(1.084)	(0.764)
PoachHighEducated	1.160	2.100	2.890	3.734	4.553	5.517	6.160	7.113	8.016	9.255
	(0.590)	(1.065)	(1.002)	(1.251)	(1.293)	(1.349)	(1.247)	(1.197)	(1.122)	(0.865)
Maximum observations	248929	383545	715295	961878	1112074	1252607	1301965	1393169	1213494	1006872

Notes: this table provides summary statistics (mean and sd in parenthesis) within each firm rank bin 1-10 (as defined by the poaching rank). The education variable is categorical with three categories based on educational lenght in years (less than high school or missing, high school or vocational, or bachelor level and above). WageRank is the average wage ranking (1-10) within a bin (the wage ranking ranks firms by the average wage and assigns into 10 equally sized groups (firm weighted)). PoachWomenOnly (PoachMenOnly) is the average rank obtained when using only women (men) to construct poaching ranks. PoachLowEducated (PoachHighEducated) is the average rank obtained when using only low educated, defined as education below bachelor level, to construct poaching ranks. Maximum observations counts the number of worker-years (november cross sections) observed in each firm bin.

F Data appendix with robustness checks

As argued above, central for our identification of directedness is the tendency for $\hat{h}_{ij}$ to be decreasing in i. To ease interpretability and comparability across samples we measure this tendency with two measures which we report and discuss these across different samples.

The first measure is a weighted average of the j-specfic variances of $\hat{h}'_{ij}s$ (each column in the matrix of $\hat{h}_{ij}$). In our version with 10 submarkets this is calculated as:

$$\beta_{SD} = \sqrt{\frac{1}{n}\sum_{j=2}^{9}\sum_{i\leq j}^{9}\left(\hat{h}_{ij} - \hat{\bar{h}}_j\right)^2} \tag{59}$$

where $\hat{\bar{h}}_j = \frac{1}{j}\sum_{i\leq j}\hat{h}_{ij}$. β_{SD} is thus a measure of the overall variability of $\hat{h}_{ij}$ in a given sample (keep in mind that $\hat{h}_{ij} \in [0,1]$).

The second measure is a regression estimate from a regressions of $\hat{h}_{ij}$ on a linear i-trend allowing for $j-$specific intercepts:

$$\hat{h}_{ij} = \alpha_j + \beta_{REG} \cdot i + \epsilon_{ij} \tag{60}$$

in this specification β_{REG} can be thought of a an estimate of how much the $\hat{h}_{ij}$ are decreasing. [24]

F.1 Sensitivity wrt to sample restricitons

In Table 7we report our summary statistics β_{REG} and β_{SD} in our baseline sample (row 1), that is the $\hat{h}_{ij}$ moments we use in estimation. Our baseline estimate of β_{REG} is -0.006 suggesting that $\hat{h}_{ij}$'s are moderately decreasing as we move down the columns (change the i for a fixed j). Moving from row 1 to 6 suggests that $\hat{h}_{ij}$ falls by 0.03 on average (average $\hat{h}_{ij}$ is 0.33 in the sample).

In row 2 we then report the same statistics based on our full sample when we do not invoke the additional sample restrictions (where we discard certain type of EE transitions such as transitions to or from entering or exiting firms in the years of entry or exit) explained in SubSection 6.1.2. The regression based estimate of how decreasing $\hat{h}_{ij}$ is (β_{REG}) is very similar to the baseline sample however we see that β_{SD} is around 30 % higher (0.019 relative to 0.014) consistent with there being more noise in the $\hat{h}_{ij}$'s in the extended sample.

In row 3 of Table 7we report statistics in a sample where we only rank firms if they have more than 10 hires over the sample period. In row 4 we report results when we only construct $\hat{h}_{ij}$ using the first X hires of the firm.

F.2 Subsample heterogeneity

In row 2-5 in Table 10 we consider $\hat{h}_{ij}$ estimates obtained across other subsamples. In each subsample we both recalculate poaching ranks and reconstruct $\hat{h}_{ij}$ within the specific subsample.

[24] With 10 markets the regression then has 45 observations.

Table 7: Summary measures for $\hat{h}_{ij}$ across different subsamples

	β_{REG}	β_{SD}
Baseline	-0.006 (0.001)	0.014
Extended sample (all transitions)	-0.006 (0.003)	0.019
Hires > 10 sample	-0.006 (0.001)	0.014
HiresCap 20	-0.005 (0.002)	0.019
Drop smaller Empl spells	-0.007 (0.002)	0.014
Drop smaller Empl spells and merge E spells	-0.007 (0.003)	0.017
Include non-business sector firms in ranks	-0.004 (0.002)	0.017

Notes: This table reports summary statistics (β_{REG} (equation 60) and β_{SD} (equation 59)) for different samples. Row 1 displays statistics for the baseline sample (used in estimation). Row 2 reports statistics for a sample where we do not . In row 3 we report statistics in a sample where we only rank firms if they have more than 10 hires over the sample period. In row 4 we report results when we only construct $\hat{h}_{ij}$ using the first 20 hires of the firm when calculating poaching ranks and constructing $\hat{h}_{ij}$. In row 5 we report results for a sample where we require that employment spells are longer than 94 days to be included in the sample. In row 6 we only include employment spells longer than 188 days and "merge" (i.e. ignore the time in between) employment spells with less than 188 days in between them. Standard errors are robust to heteroskedasticity. All estimates of β_{REG} are significant at a 5% significance level.

In row 1 we reproduce our baseline estimates. Row 2 (3) focus on low (high) educated workers only (defined as education levels below (above) a bachelor). Row 4 focus on women and row 5 on men.

Table 8: $\hat{h}_{ij}$ baseline sample

$\hat{h}_{ij}$									
1	.012	.051	.097	.154	.203	.213	.368	.454	.607
2	.	.045	.081	.141	.190	.198	.340	.470	.618
3	.	.	.081	.150	.195	.204	.353	.451	.619
4	.	.	.	.131	.185	.199	.335	.452	.627
5	.	.	.	.	.184	.193	.360	.459	.619
6	.	.	.	.	.	.175	.311	.448	.602
7	.	.	.	.	.	.	.308	.448	.617
8	.	.	.	.	.	.	.	.408	.570
9	.	.	.	.	.	.	.	.	.554

Table 9: $\hat{h}_{ij}$ extended sample

$\hat{h}_{ij}$									
1	.016	.054	.087	.141	.187	.183	.312	.349	.470
2	.	.045	.069	.118	.163	.203	.278	.373	.581
3	.	.	.077	.141	.187	.183	.321	.405	.538
4	.	.	.	.124	.178	.180	.292	.421	.559
5	.	.	.	.	.161	.164	.286	.354	.543
6	.	.	.	.	.	.154	.291	.387	.534
7	.	.	.	.	.	.	.270	.374	.461
8	.	.	.	.	.	.	.	.322	.436
9	.	.	.	.	.	.	.	.	.483

Table 10: Summary measures for $\hat{h}_{ij}$ across different subsamples

	β_{REG}	β_{SD}
Baseline	-0.006 (0.001)	0.014
Low educated sample	-0.004 (0.002)	0.012
High educated sample	-0.007 (0.002)	0.016
Women only sample	-0.010 (0.002)	0.024
Women only (on baseline ranking)	-0.010 (0.002)	0.023
Men only sample	-0.005 (0.002)	0.016
Men only (on baseline ranking)	-0.005 (0.002)	0.012

Notes: This table reports summary statistics (β_{REG} (equation 60) and β_{SD} (equation 59)) for different samples. Row 1 displays statistics for the baseline sample (used in estimation). In row 2-5 we report statistics across different sub samples of workers. In each sample we both recalculate poaching ranks and reconstruct $\hat{h}_{ij}$ within the specific subsample. Row 5 (6) focus on low (high) educated workers only (defined as education levels below (above) a bachelor). Row 7 focus on women and row 8 on men. Standard errors are robust to heteroskedasticity. All estimates are significant at a 5% significance level.

Table 11: $\hat{h}_{ij}$ low educated sample

$\hat{h}_{ij}$									
1	.011	.060	.100	.184	.203	.201	.369	.442	.554
2	.	.050	.084	.187	.207	.180	.367	.424	.521
3	.	.	.081	.187	.206	.193	.348	.485	.590
4	.	.	.	.183	.194	.180	.333	.433	.562
5	.	.	.	.	.193	.181	.352	.560	.576
6	.	.	.	.	.	.166	.301	.425	.532
7	.	.	.	.	.	.	.299	.419	.562
8	.	.	.	.	.	.	.	.380	.531
9	.	.	.	.	.	.	.	.	.486

Table 12: $\hat{h}_{ij}$ high educated sample

$\hat{h}_{ij}$									
1	.016	.046	.114	.140	.170	.277	.331	.414	.623
2	.	.042	.105	.129	.137	.273	.333	.410	.674
3	.	.	.104	.137	.138	.288	.329	.423	.676
4	.	.	.	.122	.131	.286	.326	.416	.681
5	.	.	.	.	.129	.269	.332	.422	.648
6	.	.	.	.	.	.260	.314	.437	.654
7	.	.	.	.	.	.	.303	.407	.636
8	.	.	.	.	.	.	.	.390	.653
9	.	.	.	.	.	.	.	.	.606