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ABSTRACT

Prudent Job Search and Consumption Sensitivity

This paper connects classical preference theory to quantitative job search and savings models. We treat job acceptance as a choice between stochastic income lotteries. By integrating Decreasing Absolute Risk Aversion (DARA) and Prudence (DAP), we derive five contributions. First, we prove the standard positive wealth effect on reservation wages is driven by the gap between the risk premium (equating total utilities) and the prudence premium (equating marginal utilities), while resolving value function concavity. Second, a unified theorem shows the wealth effect depends on the stochastic dominance of on-the-job search (OJS) versus unemployed search. We uncover a novel "Investment Fund" regime: when OJS is superior, reservation wages decrease with wealth as agents purchase access to high-growth states. Third, this explains consumption puzzles, showing the high MPC of the unemployed is an endogenous response to background risk. Fourth, we demonstrate an isomorphism between labor and savings: the reservation wage exhibits the same prudence-driven diminishing sensitivity as the consumption function. Fifth, borrowing constraints amplify wealth sensitivity without altering qualitative risk rankings.

JEL Classification: E21, H55, J64

Keywords: job search, wealth, consumption, risk-aversion, prudence

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1 Introduction

Understanding how individuals jointly manage consumption, savings, and job search decisions in the face of labor market uncertainty is central to modern macroeconomics. Workers engage in self-insurance by accumulating assets to buffer against the risk of job loss and to facilitate upward mobility through job-to-job transitions. Consequently, during joblessness, they draw down savings to smooth consumption and adjust their job acceptance criteria, lowering reservation wages to expedite reemployment. This paper studies the joint determination of savings and search behavior in a frictional labor market featuring both unemployment and on-the-job search (OJS), demonstrating that consumption smoothing and job acceptance are intrinsically linked decisions, jointly shaped by asset holdings, labor market prospects, and risk preferences.

Despite mature, parallel literatures on precautionary savings (Leland 1968, Kimball 1990, Carroll 1997) and foundational job search theory (McCall 1970, Mortensen 1986, Burdett and Mortensen 1998), the analytical connection between fundamental risk preferences and behavior in dynamic search environments remains incomplete. Initial attempts to bridge these fields, notably Danforth (1979), were limited by a static absorbing state assumption and lacked a formal proof for the wealth effect, relying instead on an analogy to earlier work on generalized lotteries (Danforth 1977). While later work also integrated savings (Lentz and Tranaes 2005, Rendon 2006, Lentz 2009), including in directed search models without OJS (Eeckhout and Sepahsalari 2023), this literature often focused on binding borrowing constraints relied heavily on numerical results, or gained tractability by abstracting from wealth effects on the reservation wage (Acemoglu and Shimer 1999, Shimer and Werning 2008).

This paper provides the analytical micro-foundations connecting classic preference-based arguments to modern quantitative OJS models with savings and inequality (Krusell, Mukoyama and Sahin 2010, Lise 2013, Golosov, Maziero and Menzio 2013, Chaumont and Shi 2022). We make the implicit mechanisms linking preferences to outcomes explicit by recasting the agent's problem in the spirit of the background risk literature (Gollier 2014, Eeckhoudt, Gollier and Schlesinger 1996). Our key insight is that the agent is actively choosing between two distinct background risk lotteries, "the "unemployed" lottery and the "employed" lottery. Following the foundational work of Hadar and Russell (1969) and Hanoch and Levy (1969), we treat the job acceptance decision as a ranking of these stochastic income streams. We show that while the level of the reservation wage is determined by First Order Stochastic Dominance (FOSD), ranking the value of the employed versus unemployed lottery, the sensitivity of this choice to wealth is governed by higher-order risk preferences, specifically Decreasing Absolute Risk Aversion (DARA) and Decreasing Absolute Prudence (DAP).

We first develop a rigorous baseline by formalizing the classic Danforth (1979) model. We provide a direct, formal proof that the reservation wage strictly increases with wealth under DARA. This is driven by the endogenous gap between the risk premium, the reservation wage, and the prudence premium (Kimball 1990), which we call the "consumption reservation wage". We formally link this to the Euler equation, showing that while the Bellman equation equates total utilities at the risk premium, the Euler equation equates marginal utilities at the prudence premium. We further resolve the long-standing value function concavity issue by establishing a new, sufficient analytical condition.

Building on this clarified baseline, we develop our primary contribution: an extended search-consumption model incorporating OJS and job separations, capturing the volatility and mobility inherent in modern labor markets. Unlike standard consumption models where income, even if stochastic, is treated as exogenous, here the income process is partially controlled by the agent's search behavior. The reservation wage is a risk premium for this background labor market risk. This yields our central finding: a unified theorem that proves that the sign of the wealth effect and several qualitative properties are regime-dependent, governed by the relative productivity, and thus dominance, of unemployed versus on-the-job search.

Consistent with empirical findings on the liquidity-dependence of search behavior (Krueger and Mueller 2010), we prove that reservation wages increasing in wealth is a special case that holds only when unemployed search is the more productive channel. We find the novel result that when OJS is more productive, wealth acts as an investment fund, lowering the reservation wage. By contrast, when OJS is more productive, we uncover a novel "Investment Fund" mechanism: wealth allows agents to lower their reservation wage to purchase access to superior future job ladders. This extends the logic of Low (2005), who established that flexible labor supply acts as a substitute for precautionary savings; just as his agents work harder when young to build a buffer against shocks, our agents accept lower initial wages to buffer against future career stagnation. In the language of Hadar (1971), agents diversify intertemporally accepting a deterministic short-term loss to access a future income distribution that stochastically dominates the current one. This mechanism offers a theoretical counterpart to recent evidence from the COVID-19 pandemic, which suggests that high liquidity allowed workers to 'purchase' access to the labor market's future career ladder even when the immediate wage fell below the benefit floor (Ganong, Greig, Noel, Sullivan and Vavra 2024).

This regime-dependence provides a powerful, preference-based explanation for consumption sensitivity. Standard consumption theory (based on quadratic utility (Hall 1978), logarithmic preferences (Samuelson 1969), or CRRA (Barro 1976, Deaton 1991)) often yields a constant MPC (Sargent 1979), empirical evidence consistently shows that unemployed individuals exhibit a significantly higher MPC. This high MPC is typically attributed to binding liquidity constraints (Carroll, Hall and Zeldes 1992, Jappelli and Pistaferri 2010, Herkenhoff 2019), benefit exhaustion (Ganong and Noel 2019), and hand-to-mouth consumption (Kaplan and Violante 2014), though recent evidence suggests low liquidity is not the only explanation (Ganong et al. 2024). We show that higher MPCs arise naturally as a consequence of DAP interacting with labor market frictions. Unemployment acts as a state of heightened background risk, an increase in risk in the sense of Rothschild and Stiglitz (1970), that amplifies precautionary motives for saving. Our theorem demonstrates that the higher MPC, effective risk aversion, and effective prudence of the unemployed endogenously depend on unemployment being the more productive lottery, thus demonstrating that consumption sensitivity is governed by the same underlying preferences that determine reservation wages. The phenomenon is not merely that lower income forces the unemployed onto a steeper portion of a common concave consumption function, regardless of proximity to the borrowing constraint (Carroll and Kimball 1996). Instead, the consumption function itself is specific to the employment status, where unemployment implies a layered uncertainty that structurally amplifies precautionary motives.

We fully characterize the consumption and value function curvature. As established by Carroll and Kimball (1996) and Huggett and Vidon (2002), prudence alone is not sufficient for a concave consumption function, which is essential for the precautionary motive to translate into wealth accumulation. Furthermore, we show that the unemployed are endogenously more risk-averse and more prudent than the employed evaluated at the reservation wage, linking labor market status directly to effective risk attitudes. We also establish conditions under which the DARA and DAP properties of the underlying utility function are preserved in the value functions despite the complexities introduced by search frictions.

We further demonstrate that both the risk premium (the reservation wage) and the prudence premium (the consumption reservation wage) share the fundamental geometry of the consumption function: diminishing marginal sensitivity to wealth. Whether the reservation wage is increasing (under productive unemployment) or decreasing (under productive employment), its responsiveness to assets attenuates as wealth rises, driven by the same higher-order preference structures—DARA and DAP—that smooth consumption.

Finally, we show that borrowing constraints introduce a critical level effect by interrupting the intertemporal attenuation of risk aversion; this locks agents into regions of high marginal utility, amplifying effective risk aversion and prudence while preserving their regime-dependent ranking across employment states. Furthermore, by preventing consumption smoothing, borrowing constraints steepen the wealth gradient of the reservation wage, making job acceptance decisions significantly more responsive to marginal changes in asset holdings.

The paper proceeds as follows. Section 2 presents the baseline model, formalizing the proof that reservation wages are increasing in wealth via the prudence/risk premium gap and resolving the concavity issue. Section 3 introduces the extended model with OJS, derives the unified theorem on wealth-contingent reservation wages and analyzes the implications for precautionary saving, consumption sensitivity, and quits. Section 4 concludes.

2 The Baseline Search-Savings Model

In this section, we formalize the classic job search model where employment is an absorbing state, as analyzed by Danforth (1979). We first establish properties for an agent with a fixed income, which will serve as a building block for the search model.

2.1 Consumption with Income Certainty

Consider an individual with a period-by-period utility function $U(\cdot)$ over consumption, characterized by $U_1 > 0$, $U_2 < 0$, $\lim_{C\to 0} U_1(C) = \infty$, $r_C^U < 0$ (DARA), $p_C^U < 0$ (DAP), and Decreasing Absolute Temperance (DAT, $t_C^U < 0$).[1] These properties are fulfilled by constant relative risk aversion (CRRA) utility functions. The rate of return ρ is constant, and the subjective discount factor is $\beta \in (0, 1)$ with $\beta(1 + \rho) \leq 1$.

[1] Throughout this paper, subscripts are used to denote partial derivatives.

The value function for an individual employed at a constant wage w is:

$$V^e(A,w) = \max_{A^e}\left\{U\left(A+w-\frac{A^e}{1+\rho}\right)+\beta V^e(A^e,w)\right\}.$$

The agent cannot run down wealth below the present discounted value of his future income stream, given by $B^e(w) = -Rw$, $R = \frac{1+\rho}{\rho}$. The properties of this deterministic value function are well established. As reiterated in the Companion Appendix, the value function V^e preserves the DARA and DAP properties of the underlying utility function U. That is, for $r^{V^e} = -\frac{V^e_{AA}}{V^e_A}$, $r^{V^e}_A < 0$, and for $p^{V^e} = -\frac{V^e_{AAA}}{V^e_{AA}}$, $p^{V^e}_A < 0$. Moreover, value and consumption are increasing in wealth and in wages $V^e_w = RV^e_A > 0$, $C^e_w = RC^e_A > 0$, while the marginal value of wealth is decreasing in wealth and wages, $V^e_{Aw} = RV^e_{AA} < 0$. This deterministic setup is characterized by wealth decumulation: $A^e < A$.

2.2 The Reservation Wage as a Risk Premium

We now consider an unemployed individual who receives non-labor income b. Each period, with probability λ, this agent draws a wage offer x from a known distribution $F(\cdot)$. If the offer is accepted, the agent is employed at that wage forever. The value function when unemployed is

$$V^u(A) = \max_{A^u}\left\{U\left(A+b-\frac{A^u}{1+\rho}\right)+\beta EV^u(A)\right\},$$

where

$$EV^u(A) = (1-\lambda)V^u(A) + \lambda\int_0^\infty \max\left[V^e(A,x),V^u(A)\right]dF(x).$$

The agent's wealth is bounded below by a natural borrowing limit, determined by the present discounted value of their lowest possible no-labor income stream, given by $B = -Rb$. Since $V^e(A,w)$ is monotonically increasing in w, i.e. $V^e_w > 0$, the reservation wage property exists. We define this as the risk premium $w^*(A)$, which equates total utility across states: $w^*(A) = \{w \mid V^e(A,w) = V^u(A)\}$.

We can write the expected continuation value of unemployment as

$$EV^u(A) = V^u(A) + \lambda\int_{w^*(A)}^\infty \left[V^e(A,x) - V^u(A)\right]dF(x).$$

This reservation wage must be greater than the unemployment benefit, that is, there are returns to job search.

Theorem 2.2.1 *The Reservation Wage Exceeds Benefits:* $w^*(A) > b$, $\forall$ A.
Proof: Since $V^u(A) > V^e(A,b)$, $\forall A$, *the value of searching is higher than the value of being employed at wage* b *forever, the definition* $V^e(A,w^*) = V^u(A)$ *requires* $V^e(A,w^*) > V^e(A,b)$. *Given* $V^e_w > 0$, *it must be that* $w^* > b$ ■

The solution for wealth next period A^u is defined by the first order necessary condition or the Euler equations:

$$U_1\left(A + b - \frac{A^u}{1+\rho}\right) - \beta(1+\rho)\,EV^u_A(A^u) = 0, \tag{1}$$

where

$$EV^u_A(A) = V^u_A(A) + \lambda \int_{w^*(A)}^{\infty} \left[V^e_A(A,x) - V^u_A(A)\right] dF(x). \tag{2}$$

We now define a second, related threshold: the prudence premium, $\hat{w}(A)$, or "consumption reservation wage." This is the wage that equates the marginal utility of wealth across states: $\hat{w}(A) = \{w \mid V^e_A(A,w) = V^u_A(A)\}$. By the Envelope Theorem, this condition implies $U_1(C^e(A,\hat{w})) = U_1(C^u(A))$, meaning current consumption levels are equalized. Thus, $\hat{w}$ is the wage required for indifference at the margin (a prudence-based concept), while w^* is the wage required for indifference *in total* (a risk-aversion-based concept). This threshold also exceeds unemployment transfers.

Theorem 2.2.2 *The Prudence Premium Exceeds Benefits:* $\hat{w}(A) > b$, $\forall$ A.
Proof: Since $V^u_A(A) < V^e_A(A,b)$, $\forall A$, *the marginal value of searching is lower than the value of being employed at wage* b *forever, the definition* $V^e_A(A,\hat{w}) = V^u_A(A)$ *requires* $V^e_A(A,\hat{w}) < V^e_A(A,b)$. *Given* $V^e_{Aw} < 0$, *it must be that* $\hat{w} > b$ ■

A central finding of this paper is that these the risk-premium and the prudence premium are not equal, and the gap between them is equivalent to the wealth effect.

Theorem 2.2.3 *Under DARA for* U, *the following properties hold jointly:*

1. *The consumption reservation wage is strictly less than the reservation wage:* $\hat{w}(A) < w^*(A)$.

2. *The reservation wage is strictly increasing in wealth,* $w^*_A(A) > 0$.

3. *Consumption when employed is strictly greater than consumption when unemployed:* $C^e(A,w) > C^u(A)$, *for all* $w \geq w^*(A)$.

Proof. In Appendix A

This theorem formalizes the intuition that wealthier individuals are more selective. It also establishes a key equivalence: this wealth effect (Property 3) is equivalent to consumption being higher when employed at the reservation wage than when unemployed (Property 2). This demonstrates that labor market selectivity and consumption smoothing are jointly determined by the same underlying risk preferences.

Property 1 provides the core economic intuition, highlighting the distinction between prudence and risk aversion. The theorem proves that the consumption reservation wage $\hat{w}$, or prudence premium, is strictly lower than the full reservation wage w^*, a risk premium. While the Bellman equation equates total utilities at the risk premium w^*, the Euler equation equates marginal utilities at the prudence premium $\hat{w}$. Thus, the gap between the risk premium and the prudence premium is the structural wedge driving the wealth effect and the direct cause of the consumption gap in Property 2. Because $w^* > \hat{w}$ and consumption $C^e(A,w)$ is increasing in wages, an agent at the reservation wage w^* must consume more than an agent at $\hat{w}$, who, by definition, consumes the same as the unemployed, C^u. This shows that the lower consumption of the unemployed is not merely a mechanistic result of lower income, but an optimal response to the state of being unemployed itself. Notably, the proof's structure reveals that these results are driven by the DARA property of the *employed* deterministic value function V^e. The theorem does not require DARA to hold for the unemployed value function V^u, whose curvature may be more complex due to the search option.

2.3 On the Concavity of the Value Function

A critical requirement for the model to produce well-behaved comparative statics is ensuring the expected value function of the unemployed EV^u is concave. To see why, consider the marginal propensity to consume (MPC) out of wealth for the unemployed:

$$C_A^u = \frac{\beta(1+\rho)^2 EV_{AA}^u}{U_2^u + \beta(1+\rho)^2 EV_{AA}^u}.$$

For $C^u_A \in (0,1)$, the numerator and denominator must have the same sign, which requires the numerator to be negative, since $U^u_2 < 0$.[2] Hence, $EV^u_{AA} < 0$ is a necessary and sufficient condition for a positive MPC, to preserve concavity of the value function itself ($V^u_{AA} = U^u_2 C^u_A < 0$), and to maintain positive effective risk aversion ($r^{V^u} = r^{U^u} C^u_A > 0$). Were EV^u_{AA} instead positive, the MPC and the effective risk-aversion coefficient would be both negative, implying counter-intuitive results such as locally risk-loving behavior.

This is a known technical issue in job search models with savings: $V^u(A)$ may not be globally concave, because EV^u_{AA} is not guaranteed to be negative. The `max` operator in the Bellman equation, combined with a wealth-dependent reservation wage $w^*(A)$, may introduce regions of local convexity.

The literature has addressed this potential complication along three main lines. A first approach, exemplified by Danforth (1979), accepts the presence of nonconcavities as an inherent feature of the model, interpreting them as a source of multiple solutions and asserted that "*the unemployed individual will be a risk preferrer for some levels of wealth holdings.*" However, he contended that "*no fundamental analytical difficulties result from such nonuniqueness.*" A second strand, such as Lentz and Tranaes (2005), Lentz (2009), and Gomes, Greenwood and Rebelo (2001), also acknowledges the threat to concavity and introduces mechanisms—such as wealth lotteries or productivity shocks—aimed to restore concavity by effectively smoothing the expected value function. A third line of work, illustrated by Hopenhayn and Nicolini (1997), Pissarides (2002), Chetty (2008), and Lise (2013), argues that for empirically plausible parameters, nonconcavities do not arise and proceeds under the maintained assumption of concavity.

Our approach is to analyze the potential lack of concavity explicitly and establish a sufficient analytical condition to guarantee it. To find the source of the problem, we apply the Leibniz rule to $EV^u_A(A)$:

$$\begin{aligned} EV^u_{AA}(A) &= (1-\lambda\left[1-F(w^*(A))\right])V^u_{AA}(A) + \lambda\int_{w^*(A)}^{\infty} V^e_{AA}(A,x)\,dF(x) \\ &\quad + \lambda f(w^*(A)) w^*_A(A)\left[V^u_A(A) - V^e_A(A,w^*(A))\right]. \end{aligned} \tag{3}$$

While the first two terms are negative and tend to preserve concavity, the third

[2] Notice that the denominator must be negative, even if EV_{AA} were positive, because it is the Second-Order Sufficient Condition (SOSC) for a maximum.

term is unambiguously positive[3] precisely because DARA implies $w_A^* > 0$, which is equivalent to $V_A^u > V_A^w$, as established in Theorem 2.2.3. This positive term, directly linked to wealth effects under DARA, is the sole source of potential convexity.[4]

[Figure 1 here]

Figure 1 provides a graphical intuition for the technical issue. The wealth-contingent reservation wage introduces a discontinuity in the marginal value function EV_A^u, which translates into the positive term in EV_{AA}^u shown in Equation (3). Our approach relies on direct verification that this positive term is not large enough to make the whole second derivative positive.

The following theorem provides a simple, verifiable sufficient condition that guarantees $EV_{AA}^u < 0$ by ensuring the negative terms dominate this positive term.

Theorem 2.3.1 (Sufficient Condition for Concavity of the Expected Value Function) *$EV_{AA}^u(A) < 0$, if $r_{V^u} \geq h_A$, where the "wealth hazard rate" h_A is defined as*

$$h_A \equiv \frac{\lambda f\left(w^*\left(A\right)\right) w_A^*\left(A\right)}{1 - \lambda\left[1 - F\left(w^*\left(A\right)\right)\right]}.$$

Proof. In Appendix B.

This theorem provides a simple, verifiable condition for concavity by comparing two opposing forces. It states that $EV_{AA}^u < 0$ is guaranteed if the unemployed's effective risk aversion r_{V^u}, "their desire to smooth consumption" is greater than the wealth hazard rate h_A. This h_A term represents the marginal decrease in the job-finding probability driven by $w_A^* > 0$ as a ratio of the probability of remaining unemployed, capturing the behavioral effect of wealth-driven selectivity. Concavity holds, therefore, if the preference for smoothing dominates this behavioral selectivity effect. This condition is sufficient, not necessary; as noted in the proof, the concavity of V^e (the second term in Equation 3) also contributes to preserving concavity, so it can hold even if $r_{V^u} < h_A$.

[3] A similar third term arises in the Euler equation, but its value is zero at the optimum because $V^u = V^{w^*}$.

[4] Under Constant Absolute Risk Aversion (CARA) or its special case with zero risk-aversion, quadratic utility ($r_C^U = 0$), $w_A^* = 0$ and this term vanishes, eliminating the concavity concern (Acemoglu and Shimer 1999).

Having established a sufficient condition under which $EV^u_{AA} < 0$, we can confirm the Second-Order Sufficient Condition (SOSC) for the agent's optimization problem:

$$U_2\left(C^u\right) + \beta\left(1+\rho\right)^2 EV^u_{AA}\left(A^u\right) < 0, \tag{4}$$

which confirms the validity of the maximum.

This paper proceeds under the condition that $EV^u_{AA} < 0$ holds and thus $V^u_{AA} < 0$. By extension, for the preservation of higher-order properties (DARA, DAP), we maintain that the analogous terms in V^u_{AAA} and V^u_{AAAA} arising from the $w^*_A > 0$ mechanism are also sufficiently small as to not alter the sign of the overall derivatives, allowing for a full characterization of the model's comparative statics.

2.4 Dissavings

Assuming the expected value function is concave, $EV^u_{AA} < 0$, we can derive a series of sharp comparative statics results. These demonstrate how employment status endogenously shapes risk preferences and consumption behavior.

Theorem 2.4.1 *Danforth (1979) The unemployed decumulate:* $A^u < A$.
Proof. In Appendix C.

This result captures the intuition that, absent labor income, unemployed individuals finance consumption by running down their assets.

Corollary 2.4.1 *The reservation wage declines during unemployment. Since* $A > A^u$, *then* $w^*(A) > w^*(A^u)$.
Proof. Follows directly from Theorem 2.4.1 and Theorem 2.2.3.

This corollary follows directly: as assets diminish, individuals become more willing to accept lower wages, consistent with the declining reservation wage over time.

Theorem 2.4.2 *The employed decumulate less than the unemployed:* $A^e(A, w) > A^u(A)$ *for all* $w \geq b$.
Proof. In Appendix C.

This theorem implies that conditional on the same starting wealth, employed individuals will hold higher assets in the next period than the unemployed, due to labor income inflows. Notice this theorem applies to wages even below the reservation wage $w^* \geq b$.

2.5 Preservation of DARA and DAP in the Baseline Model

We examine the intertemporal transmission of risk attitudes. Following earlier research (Neave 1971, Nachman 1979, Nachman 1982), we establish that the fundamental properties of the instantaneous utility function U are generally preserved in the dynamic value function V^u. As detailed below, the value function inherits these higher-order preference structures under specific analytical conditions.

Theorem 2.5.1 *If U satisfies DARA, that is $r_C^U < 0$, and DARA is preserved by integration, $r_A^{EV^u} < 0$, then the value function of the unemployed also satisfies DARA, $r_A^{V^u} < 0$.*
Proof. In Appendix C.

This result ensures that decreasing absolute risk aversion at the level of utility translates into the same property at the level of the value function. It is crucial for the internal consistency of risk-related behavior in the dynamic model. DARA implies prudence, as the inequality $r_A^{V^u} = r^{V^u}(r^{V^u} - p^{V^u}) < 0$ holds only if $p^{V^u} > r^{V^u} > 0$, implying a sufficiently convex marginal utility of consumption.

Theorem 2.5.2 *If U satisfies DAP, that is, $p_C^U < 0$, and DAP is preserved by integration, $p_A^{EV^u} < 0$, then unemployed prudence is decreasing in wealth, $p_A^{V^u} < 0$, under the condition*

$$-p_C^U (C_A^u)^3 - p_A^{EV^u}(1+\rho)^2(1-C_A^u)^3 > 2(p^U C_A^u - p^{EV^u} A_A^u)^2 C_A^u (1-C_A^u).$$

Proof. In Appendix C.

2.6 The unemployed are more prudent

This section examines how consumption behavior and value functions differ systematically across employment states. These comparative statics highlight how risk preferences, consumption smoothing, and marginal valuation of wealth evolve between unemployed and employed individuals (Carroll and Kimball 1996, Low 2005, Lentz and Mortensen 2008).

Theorem 2.6.1 *Carroll and Kimball (1996) If the utility function is CRRA/homogeneous, then $C_{AA}^u < 0$.*
Proof. In Appendix C.

This theorem establishes the concavity of the consumption function for the unemployed, showing that increases in wealth lead to diminishing increases in consumption. By contrast, the MPC of the employed for a CRRA/homogenous utility function is constant, so that $C^e_{AA} = 0$.

Theorem 2.6.2 *If U satisfies DAP, that is $p^U_C < 0$, then for all $w \geq w^*$:*

1. *the marginal propensity to consume is higher for the unemplo2yed than for the employed: $C^u_A > C^w_A$.*
2. *the curvature of the value function is greater for the unemployed than for the employed: $-V^u_{AA} > -V^w_{AA}$.*
3. *the unemployed are more risk averse than the employed: $r^{V^u} > r^{V^w}$.*
4. *the unemployed are more prudent than the employed: $p^{V^u} > p^{V^w}$, if U is a CRRA/homogeneous utility function.*
   Proof. In Appendix C.

This unified result captures a core implication of precautionary saving theory: the agent's employment status acts as a powerful determinant of their effective risk attitudes and saving behavior. The theorem confirms that the unemployed have a higher MPC compared to the employed. This aligns with standard buffer-stock behavior: individuals facing greater income uncertainty have stronger incentives to consume a larger fraction of their wealth, effectively drawing down assets to smooth consumption. Conversely, employed individuals benefit from current income and display a more tempered consumption response to wealth changes. This behavior is supported by the finding that the consumption function for the unemployed is concave, meaning the increases in consumption diminish as wealth increases. Analytically, the higher MPC of the unemployed structurally implies their value function is more concave, translating directly into the conclusion that the unemployed agent is structurally more risk-averse and more prudent than their employed counterpart. Prudence, defined as the strength of the motive to save against risk, is amplified when income is uncertain. Since the unemployed face both income volatility and low current earnings, their marginal valuation of additional savings is steeper; each unit of wealth plays a more protective role for them. The employed agent's greater income certainty and stability allow them to tolerate future consumption variation more easily, which mathematically reduces their effective aversion to risk relative to the unemployed state.

2.7 Concavity of the Reservation Wage

This section characterizes the higher-order properties of the reservation wage functions, relating their curvature to the fundamental risk attitudes defined by the value functions.

Theorem 2.7.1 *The reservation wage function $w^*(A)$ is concave with respect to assets, $w^*_{AA}(A) < 0$, if $\overline{r}^{V^{Aw*}} < r^{V^u}$, where $\overline{r}^{V^{Aw*}} \equiv r^{V^{w*}}(1 + Rw^*_A)$.*
Proof. In Appendix C.

The concavity depends on a core comparison: the total effective risk aversion along the reservation wage locus must be less than the coefficient of absolute risk aversion for the unemployed. The total effective risk aversion $\overline{r}^{V^{Aw*}}$ includes an additional wealth effect $w^*_A\, r^{V^{Aw*}} = w^*_A\, Rr^{V^{w*}}$ arising from the endogeneity of w^*. This effect, which distinguishes the analysis from standard consumption-saving problems with exogenous income streams, means that the necessary condition arising from DARA and DAP $r^{V^{w^*}} < r^{V^u}$, established in Theorem 2.6.2, is not sufficient for concavity. Concavity is only guaranteed if this indirect effect is relatively small.

Theorem 2.7.2 *The reservation wage function satisfies:*

1. $\lim_{A\to B} w^*_A(A) = \infty$;
2. $\lim_{A\to\infty} w^*(A) = w^c$, *provided* $\lim_{A\to\infty} r^U(C) = 0$, *where w^c is the reservation wage under risk neutrality;*
3. $\lim_{A\to\infty} w^*_A(A) = 0$.

Proof. In Appendix C.

These asymptotic results confirm the model's structural consistency across wealth levels. As wealth approaches the borrowing limit ($A \to B$), the marginal utility of wealth for the unemployed explodes; consequently, the marginal reservation wage w^*_A also diverges to infinity. Conversely, as $A \to \infty$, abundant wealth provides perfect self-insurance against the risk of continued unemployment and behavior converges to risk-neutrality, so that w^* converges to the constant w^c, causing its derivative w^*_A to approach zero. For a CRRA utility function, $r^U = \gamma C^{-1}$, and hence $\lim_{A\to\infty} r(C) = 0$. The consumption reservation wage exhibits similar properties of being increasing and concave with respect to wealth, subject to a sufficiency condition related to prudence when employed.

Theorem 2.7.3 *If U satisfies DAP, that is $p_C^U < 0$, the consumption reservation wage is*

1. *increasing in wealth $\widehat{w}_A(A) > 0$ for all $w \geq \widehat{w}$.*

2. *concave with respect to assets, $\widehat{w}_{AA} < 0$, if $\overline{p}^{V^{A\widehat{w}}} < p^{V^u}$, where $\overline{p}^{V^{A\widehat{w}}} \equiv p^{V^{\widehat{w}}}(1 + R\widehat{w}_A)$.*

Proof: In Appendix C.

As it occurs with the reservation wage, the wage prudence premium is increasing in wealth at diminishing returns. The additional effect $R\widehat{w}_A$ arising from the endogeneity of $\widehat{w}$ is also critical in determining the concavity of the prudence reservation wage. Accordingly, the condition $p^{V^{\widehat{w}}} < p^{V^u}$ established in Theorem 2.6.2 is necessary but not sufficient for the concavity of the consumption reservation wage function.

These results reveal a fundamental isomorphism: both the reservation wage and the consumption function behave as increasing, concave functions of wealth. However, we note that the drivers are distinct. The concavity of the consumption function is a direct consequence of DAP, the desire to smooth marginal utility. The concavity of the reservation wage, by contrast, is driven by the convergence of lotteries: as wealth increases, the value gap between employment and unemployment shrinks at a diminishing rate. Yet, despite these distinct mechanical origins, they share the same prudence-driven geometry, unifying the agent's policy rules under a single structural framework.

3 Job Turnover, OJS, and Precautionary Savings

We now extend the baseline framework to incorporate job separations and on-the-job search (OJS), key features of modern labor markets central to quantitative models studying the joint dynamics of earnings and wealth (Lise 2013, Chaumont and Shi 2022). Our contribution is to provide complementary analytical characterizations, demonstrating how fundamental preference properties like DARA and DAP shape optimal consumption and job search decisions. This logic, analyzing how prudence and risk aversion shape responses to layered uncertainties, aligns with results in Kimball (1990) and Carroll and Kimball (1996) and clarifies behavior in this richer environment with endogenous employment risk.

In contrast to the baseline where employment was an absorbing state, the extended model introduces risk even while employed. Employment is no longer a safe benchmark; rather, both labor market states represent dynamic lotteries over future income streams. This valuation highlights the importance of background risk in intertemporal decision-making. The relative search productivity of the employed versus the unemployed is central here. Recent empirical work finds that arrival rates are higher while employed than when unemployed, consistent with employed individuals having superior networks (Faberman, Mueller, Sahin and Topa 2022). The alternative hypothesis, posits that the unemployed have a larger time endowment for search, permitting a higher probability or receiving a wage offer (Burdett and Mortensen 1977, Flinn and Heckman 1983). This is a relevant extension that involves some technical subtleties: comparing two risky processes (employment and unemployment lotteries) is more analytically challenging than comparing a risky process to a risk-free, absorbing state.

3.1 Reservation Wages as Background Risk Insurance

The value function for an employed individual earning wage w now solves:

$$V^e(A,w) = \max_{A^e} \left\{ U\left(A + w - \frac{A^e}{1+\rho}\right) + \beta E V^e(A^e, w) \right\}. \tag{5}$$

The natural lower bound on wealth is still given by $B = -Rb$. The expected continuation value incorporates the risk of job loss θ and the possibility of upward wage mobility π:

$$\begin{aligned} EV^e(A,w) &= (1-\theta-\pi)\max\left[V^e(A,w), V^u(A)\right] \\ &\quad + \pi \int_0^\infty \max\left[V^e(A,x), V^e(A,w), V^u(A)\right] dF(x) + \theta V^u(A). \end{aligned}$$

The value function $V^u(A)$ is defined as in the baseline model. The embedded max operator expresses that the employed individual retains the option to quit to unemployment at any time if their current job's value falls below $V^u(A)$, ensuring their continuation value never falls below the value of unemployment. The value function $V^e(A,w)$ is strictly increasing in w under standard assumptions. This yields a well-defined reservation wage, $w^*(A) = \{w \mid V^e(A,w) = V^u(A)\}$, which ceases to be a simple risk premium to become a background risk premium. It is the wage that equates the value of two distinct stochastic processes: the unemployment lottery and the employment lottery.

Under these conditions, the expected value function $EV^e(A,w)$ can be written piecewise, distinguishing between acceptable ($w \geq w^*$) and unacceptable ($w < w^*$) wages:

$$EV^e(A,w) = \begin{cases} V^e(A,w) + \pi \int_w^\infty \left[V^e(A,x) - V^e(A,w)\right] dF(x) \\ +\theta \left[V^u(A) - V^e(A,w)\right], & \text{if } w \geq w^*(A); \\ \\ V^u(A) + \pi \int_{w^*(A)}^\infty \left[V^e(A,x) - V^u(A)\right] dF(x), & \text{if } w < w^*(A). \end{cases}$$

If the worker's current wage is greater than or equal to their reservation wage, they optimally remain employed. In this state, the worker's employment spell ends only upon an exogenous layoff or upon receiving and accepting a superior job offer. Conversely, if the current wage is strictly less than the reservation wage, the individual will optimally quit their current job to become voluntarily unemployed, unless he receives a better wage offer. This quitting option fundamentally ensures that the employed agent's continuation value never falls below the value of unemployment.

The worker's optimal behavior is formalized by the Euler equation for asset choice A^e:

$$U_1\left(A + w - \frac{A^e}{1+\rho}\right) - \beta(1+\rho)\, EV_A^e(A^e, w) = 0, \tag{6}$$

where the marginal continuation value isolates the competing motives:

$$EV_A^e(A,w) = V_A^e(A,w) + \pi \int_w^\infty \left[V_A^e(A,x) - V_A^e(A,w)\right] dF(x) + \theta \left[V_A^u(A) - V_A^e(A,w)\right].$$

Concavity of the employed value function ($V_{AA}^e < 0$) is preserved for $w \geq w^*(A)$, provided $V^u(A)$ is concave.

As with the baseline model, we define a prudence premium or "consumption reservation wage," as the wage that equates the marginal utility of wealth across states: $\hat{w}(A) = \{w \mid V_A^e(A,w) = V_A^u(A)\}$.

Endogenous Search Regimes and Wealth Effects

The valuation of the distinct unemployment and employment lotteries is governed by their relative productivity of search (λ versus π), establishing three distinct search regimes. These lotteries' relative desirability is governed by Stochastic Dominance. Following Hadar and Russell (1969), the agent ranks the unemployment lottery' against the employment lottery'. The direction of the wealth effect depends entirely on which lottery stochastically dominates the other in terms of option value. Wealth acts as a tool to secure the dominant lottery: if unemployment dominates (Productive Unemployment), wealth is used to prolong search (positive wealth effect); if employment dominates (Productive Employment), wealth is used to pay for entry (negative wealth effect) (see Appendix D).

Theorem 3.1.1 *(Reservation Wage Level and Slope by Search Regime) If U satisfies DARA, $r_C^U < 0$:*

Productive Unemployment: $\lambda > \pi$**.** $w^*(A) > \hat{w}(A) > b \iff w_A^*(A) > 0 \iff V_A^u(A) > V_A^e(A, w^*) \iff C^e(A, w^*) > C^u(A)$, $\forall A$.

Neutral Search: $\lambda = \pi$**.** $w^*(A) = \hat{w}(A) = b \iff w_A^*(A) = 0 \iff V_A^u(A) = V_A^e(A, w^*) \iff C(A, w^*) = C^u(A)$, $\forall A$.

Productive Employment. $\pi > \lambda$**.** $w^*(A) < \widehat{w}(A) < b, \iff w_A^*(A) < 0 \iff V_A^e(A, w^*) > V_A^u(A) \iff C^e(A, w^*) < C^u(A)$, $\forall A$.

Proof. In Appendix E.

The theorem provides the central analytical results for the extended model, revealing a rich, state-dependent relationship between wealth and search selectivity across three distinct regimes. It demonstrates that the wealth effect w_A^*, the gap between risk-aversion and prudence (w^* vs $\widehat{w}$), the marginal value gap (V_A^u vs $V_A^{w^*}$), and the consumption gap (C^{w^*} vs C^u) are all jointly and endogenously determined by the single sign of $(\lambda - \pi)$.

Productive Unemployment: $\lambda > \pi$**.** This is the contextualized classic Danforth (1979) result. When unemployment offers the more productive search channel, wealth acts as a "search subsidy." The agent's DARA preference manifests as a willingness to prolong this profitable search, requiring the risk premium to be strictly positive, $w_A^*(A) > 0$. This is analytically equivalent to the marginal value of unemployment being higher, $V_A^u > V_A^{w^*}$, and consumption while employed being higher than while unemployed, $C^{w^*} > C^u$.

Neutral Search: $\lambda = \pi$**.** The lotteries are exactly the same and the marginal wealth effect disappears. With no instrumental value of wealth in driving job acceptance, the marginal values and consumption levels are equalized across states at the point of indifference, yielding the knife-edge, risk-neutral-like outcome.

Productive Employment. $\pi > \lambda$**.** This the most striking finding; it provides a novel and apparently counter-intuitive prediction: when OJS is the superior search technology, the reservation wage decreases with wealth, $w_A^*(A) < 0$. This result is a new manifestation of DARA. In this regime, accepting a job at $w^* < b$ is a form of investment. In the spirit of Hadar (1971), the agent engages in 'intertemporal diversification' sacrificing a deterministic short-term current income $(b - w^*)$ to access a future wage distribution that stochastically dominates the current one. A wealthier DARA agent is more willing to 'purchase' this exposure to the superior π-lottery. This implies $V_A^{w^*} > V_A^u$ and, as the theorem states, $C^u > C^{w^*}$ at the point of indifference.

This unified theorem also clarifies the role of background risk. In the baseline model, employment was a safe state. Here, both V^u and V^e are distinct stochastic lotteries, and w^* is the price that equates them. The agent's preference between them depends on which state is riskier and which is more productive. In the Productive Unemployment regime, $\lambda > \pi$, the employment state with its low π can be viewed as the stagnant background risk, and unemployment state offers the productive upside. Wealth, via DARA, insures the agent's ability to hold onto this "good" risk, maintaining the positive wealth effect. If $\pi > \lambda$, the roles are reversed: the unemployment state is the background risk (the risk of missing the π-channel), and the V^e lottery is the high-upside gamble. DARA makes the agent more willing to pay the investment cost $(b - w^*)$ to enter this better lottery, hence $w_A^*(A) < 0$. This theorem also states that, when values are equal, consumption is greater at the employment status with the higher arrival rate.

3.2 Job Search driven Asset Accumulation

The extended framework introduces job turnover, fundamentally altering the baseline model's dynamics by creating a powerful precautionary saving motive for employed agents. We first confirm the core dissaving dynamics: the unemployed are expected to decumulate assets, $A^u < A$, causing their reservation wage to decline during unemployment, $w^*(A) > w^*(A^u)$, as they become more willing to accept lower wages (Theorem 2.4.1 and Corollary 2.4.1). Conversely, employed agents may accumulate assets to hedge against future unemployment risk, particularly when current wages are high.

This dynamics of asset accumulation and job search suggest a cycle where agents aim at achieving a level of employment stability and compensation high enough to render the voluntary quitting option irrelevant.

3.2.1 Precautionary Savings when Employed

As shown formally by Lise (2013), the decision to accumulate or decumulate wealth is governed by the Euler equation, which compares the marginal cost/benefit of wealth today versus the marginal expected value tomorrow. For an agent employed at $w > w^*(A^e)$,[5], the condition $A \lesseqgtr A^e$ is determined by two competing dynamic forces:

$$V_A^e(A^e, w) \lesseqgtr \frac{\beta(1+\rho)}{1-\beta(1+\rho)}\left[\pi \int_w^\infty \left[V_A^e(A^e, x) - V_A^e(A^e, w)\right] dF(x) + \theta\left[V_A^u(A^e) - V_A^e(A^e, w)\right]\right]. \tag{7}$$

This equivalence formally links the precautionary saving decision to the wealth-sensitivity of the job search strategy. Assuming $\beta(1+\rho) \leq 1$, the right-hand side of this expression isolates two competing motives: the OJS Motive (dissaving, driven by π) and the Precautionary Motive (saving against layoff loss, driven by θ):

1. **OJS Motive:** The first term is negative provided that $\pi > 0$, and encourages dissaving to smooth consumption upwards in anticipation of future wage growth.

2. **Precautionary Motive:** This term is active only if $\theta > 0$ and $\lambda > \pi$, which encourages saving against the expected marginal loss from a layoff: $V_A^u - V_A^w < 0$.

Net saving can only occur when the precautionary motive dominates the OJS dissaving motive. This is most likely when the current wage w is high, diminishing the marginal value of OJS, or the layoff risk θ is large. The precautionary motive must dominate as assets approach the borrowing limit $A \to B$, as $V_A^u \to \infty$, necessitating saving. If $\pi \geq \lambda$, per Theorem 3.1.1 $V_A^u(A^e) - V_A^e(A^e, w^*) \leq 0$ and the precautionary motive vanishes, so $A^e < A$.

Asset Accumulation by Employment Status

Asset accumulation by employment status, A^e vs A^u, is highly sensitive to the search regime, reflecting the instrumental value of wealth in each state.

[5]If $w \leq w^*(A^e)$, the individual exits employment, in which case the layoff probability becomes irrelevant to the decision problem.

In the Productive Unemployment regime, $\lambda > \pi$, the employed hold structurally higher assets in the next period than the unemployed, as formalized by the following theorem.

Theorem 3.2.1 *If U satisfies DARA, $r_C^U < 0$,*

If $\lambda > \pi$, $A^e(A, w) > A^u(A)$ for all $w \geq w^*$.

If $\lambda = \pi$, $A^e(A, w) \geq A^u(A)$ for all $w \geq w^*$.

If $\pi > \lambda$, $A^e(A, w^*) < A^u(A)$.

Proof. In Appendix F.

This theorem is similar to Theorem 2.4.2 and implies that, if $\lambda > \pi$, even when the employed decumulate they still hold more wealth next period than their unemployed counterparts due to labor income inflows.

For Neutral Search, $\lambda = \pi$, the expected value functions are identical across states, causing asset accumulation and consumption to be equal across states, conditional on the benefit wage $w^* = b$, that is, $A^e(A, b) = A^u(A)$ and $C^e(A, b) = C^u(A)$. Consequently, assets next period and consumption when employed are higher than when unemployed for all wages above the benefit level: $A^e(A, w) > A^u(A)$ and $C^e(A, w) > C^u(A)$, for all $w > b$.

For Productive Employment Search, $\pi > \lambda$, the dynamics reverse at low acceptable wages. Since the employed state is superior, consumption is higher when employed at the lowest acceptable wage, $C^e(A, w^*) < C^e(A, \widehat{w}) = C^u(A) < C^e(A, b)$, by Theorem 3.1.1. This higher consumption forces faster asset decumulation for the employed at the reservation wage level and at the consumption reservation wage level than when unemployed, $A^u(A) > A^e(A, b) > A^e(A, \widehat{w}) > A^e(A, w^*)$, to finance that consumption. Therefore, assets next period when employed at low acceptable wages are lower than when unemployed. As the wage increases, $w > b$, assets increase and eventually equalize, $A^e(A, w) = A^u(A)$, since $A_w^e > 0$.

Quit Triggers

Quits to unemployment are an optimal, endogenous search strategy, occurring if an agent's current wage falls below their reservation wage, $w^*(A^e) > w$. This dynamic depends on the worker's asset accumulation and the wealth-sensitivity of their reservation wage, as defined by the three regimes in Theorem 3.1.1.

If $\lambda > \pi$, quits are "precaution-driven." They are triggered if the agent accumulates assets, $A^e > A$, raising their wealth-sensitive reservation wage, $w^*_A > 0$, above their current wage, $w^*(A^e) > w$. A quit can even be triggered from a reservation wage job, $w = w^*(A)$, if the agent is a net saver at that point, $A^e(A, w^*) > A$.

If $\lambda = \pi$, quits to unemployment are impossible, because $w^*_A = 0$ ensures w^* cannot rise above the floor b. An agent employed at a viable wage $w \geq b$ will never see their reservation wage rise to meet or exceed their current wage.

If $\pi > \lambda$, quits are "OJS-driven." A quit can be triggered if the agent dissaves, $A^e < A$, in anticipation of a better π-offer, and this very act makes them "poorer," which in this regime makes them *more* selective, since $w^*_A < 0$. This can cause their w^* to rise above their current, $w^*(A^e) > w$, triggering a quit.

Consistent with general search literature, quitting is an endogenous strategy to reach higher wages,[6] determined by the interplay of precautionary saving and On-the-Job Search.

Long-Run Steady State

The nature of the long-run steady state for an employed agent depends critically on the relative search productivity.

If $\lambda > \pi$, conditional on a given wage w, the precautionary motive is active, supporting an interior long-run steady state, $\overline{A}(w)$. At this point, the OJS motive, encouraging dissaving, is perfectly offset by the precautionary motive, encouraging saving. If current assets are below this target, the agent accumulates assets, $A < \overline{A}(w)$; if assets are above the target, the agent decumulates, $A > \overline{A}(w)$. Wealth thus adjusts endogenously toward this wage-contingent steady state where saving and dissaving motives are perfectly offset, $A^e = A$.

Moreover, if $\lambda > \pi$, the pair $\left(\overline{A}^*, \overline{w}^*\right)$ defines a stable, interior long-run steady state where two conditions are simultaneously met: the agent is indifferent between employment states, $\overline{w}^* = w^*\left(\overline{A}^*\right)$, and there is no net incentive to save or dissave, $\overline{A}^* = \overline{A}(\overline{w}^*)$. If an employed agent has assets and wages below this steady state,

[6] The existing literature has explained in different ways that people quit to become unemployed. Workers may take jobs without knowing their true wage. Once they learn about it, they quit jobs where their true wage is revealed to be low (Jovanovic 1979). An economic boom may change wage offer distributions or arrival rates such that a worker finds profitable to quit to search while unemployed (Jovanovic 1987, Lippman and Mamer 1989). This explains the observed trend that quits are procyclical. With a finite horizon and accumulation of work experience, people quit because the value of investing in working decreases over time (Wolpin 1992).

they will experience a cycle of job search and saving toward $\overline{A}^*$, eventually increasing their reservation wage above their current wage and quitting their job. The terminal steady-state means that an agent will optimally remain at this asset and wage level, only searching on-the-job, and not quitting. He only becomes unemployed if a layoff occurs; then he decumulates ($A^u < A^*$), and the cycle of search and saving begins again.

If $\pi \geq \lambda$, the precautionary motive is absent, thus the OJS motive (dissaving) dominates at all asset levels $A > B$. No interior target asset level $\overline{A}(w)$ exists, and the only steady state for an employed agent is the borrowing limit $A^e = B$.

3.3 Employment-contingent Risk Attitudes

The value functions, $V^u(A)$ and $V^e(A, w)$, for both employed and unemployed individuals, preserve the foundational DARA and DAP properties of the underlying utility function, as in the baseline model (Theorem 2.5.1 and Theorem 2.5.2). Intuitively, individuals become less risk-averse (Arrow 1965, Pratt 1964) and less prudent (Kimball 1990) as wealth increases, even when facing endogenous search frictions. This is consistent with the consumption concavity finding of Carroll and Kimball (1996), as both unemployed and employed individuals exhibit a declining MPC as wealth increases, that is, $C^u_{AA} < 0$ and $C^e_{AA} < 0$, Theorem 2.6.1.

However, while these properties are preserved in both states, the relative magnitude of these effective risk attitudes and the resulting consumption sensitivity is not universal but depends critically on the search regime.

The following theorems, which define the comparative MPC, risk aversion r^V, and prudence p^V, formalize the structural ranking across all three regimes:

Theorem 3.3.1 *(Consumption Sensitivity by employment status) If DAP holds, the relative MPC, value function curvature, and risk aversion are determined by the search regime:*

Productive Unemployment: $\lambda > \pi$. $C^u_A > C^w_A$, $-V^u_{AA} > -V^w_{AA}$, *and* $r^{V^u} > r^{V^w}$ *for all* $w \geq w^*$.

Neutral Search: $\lambda = \pi$. $C^u_A = C^w_A$, $-V^u_{AA} = -V^w_{AA}$, *and* $r^{V^u} = r^{V^w}$ *for* $w = w^* = \widehat{w} = b$.

Productive Employment. $\pi > \lambda$. $C^{w^*}_A > C^u_A$, $-V^{w^*}_{AA} > -V^u_{AA}$, *and* $r^{V^{w^*}} > r^{V^u}$.

Proof. *In Appendix F.*

Theorem 3.3.2 *(Greater Prudence in Unemployment) If DAT holds, the relative prudence is also determined by the search regime:*

Productive Unemployment: $\lambda > \pi$. *The unemployed are more prudent, $p^{V^u} > p^{V^w}$, and exhibit greater curvature than the employed, $C^{w^*}_{AA} > C^u_{AA}$ for all $w \geq w^*$ under the condition $\frac{p^{EV^u}}{r^{EV^u}} A^u_A \geq \frac{p^{EV^{w^*}}}{r^{EV^{w^*}}} A^{w^*}_A$.*

Neutral Search: $\lambda = \pi$. *The unemployed are equally prudent, $p^{V^u} = p^{V^w}$, and exhibit the same curvature as the employed, $C^w_{AA} = C^u_{AA}$, at wage $w = w^* = b$.*

Productive Employment. $\pi > \lambda$. *The employed are more prudent, $p^{V^u} > p^{V^w}$, and exhibit greater curvature than the unemployed, $C^w_{AA} < C^u_{AA}$ at wage $w = w^*$ under the condition $\frac{p^{EV^u}}{r^{EV^u}} A^u_A \leq \frac{p^{EV^{w^*}}}{r^{EV^{w^*}}} A^{w^*}_A$.*

Proof. *In Appendix F.*

This set of results provides a powerful, preference-based rationale for consumption sensitivity, grounded in the principle of DAP, for the widely documented empirical finding that households experiencing unemployment exhibit a higher MPC. The agents with higher arrival rates at their employment status are thus more risk-averse, exhibit higher MPC, and, under an additional sufficient condition, more prudent

In the Productive Unemployment search regime, $\lambda > \pi$, the unemployed state is associated with higher effective MPC, $C^u_A > C^w_A$, because it represents the more productive background risk lottery. This higher MPC analytically implies the unemployed agent is structurally more risk-averse and more prudent than the employed agent, consistent with the buffer-stock logic of high risk exposure.

In the Neutral Search regime, the search risks are identical, the effective MPC, curvature, risk aversion, and prudence are all equalized across states at the reservation wage floor ($w = b$).

In the Productive Employment search regime, $\pi > \lambda$, the MPC and risk attitudes rankings reverse. The employed agent exhibits a higher MPC, value function curvature, and prudence, both at the reservation wage and at the consumption reservation wage, than the unemployed. This is because the employed state offers the superior investment opportunity (π-channel), leading to an amplified consumption response for the employed agent.

These findings formalize the idea that employment status endogenously shapes an individual's attitudes toward risk and prudence and their propensity to save for precautionary reasons.

3.4 Curvature of the Reservation Wage and Prudence Premium

This section characterizes the higher-order properties of the wealth-contingent reservation wage functions, relating their curvature to the fundamental risk attitudes defined by the value functions.

Theorem 3.4.1 *(Curvature of the Reservation Wage) The curvature of the reservation wage* $w^*_{AA}(A)$ *is determined by the relative difference in effective risk aversion between employment states. The condition for concavity is:*

$$sign\left(w^*_{AA}(A)\right) = sign\left(\overline{r}^{V^{Aw*}} - r^{V^u}\right),$$

where where r^{V^u} *is the effective risk aversion of the unemployed, and* $\overline{r}^{V^{Aw*}}$ *is the total risk aversion of the employed state along the* w^* *locus:* $\overline{r}^{V^{Aw*}} = a\overline{r}^{V^{Aw*}} + (1-a)\,\overline{r}^{V^{ww*}}$, *where* $\overline{r}^{V^{Aw*}} \equiv \frac{-V^{w^*}_{AA}}{V^{w^*}_{A}} + \frac{-V^{w^*}_{Aw}}{V^{w^*}_{A}} w^*_A$, $\overline{r}^{V^{ww*}} \equiv \frac{-V^{w^*}_{Aw}}{V^{w^*}_{w}} + \frac{-V^{w^*}_{ww}}{V^{w^*}_{w}} w^*_A$, *and* $a = \frac{V^{w^*}_{A}}{V^{w^*}_{A} + w^*_A V^{w^*}_{w}}$
Proof: In Appendix F.

As in the baseline model, the term $\overline{r}^{V^{Aw*}}$ includes an indirect effect arising from the endogeneity of $w^*(A)$, since $w^*(A) \neq 0$, distinguishing this analysis from problems with exogenous income streams. The three search regimes dictate the slope and the likelihood of concavity:

If $\lambda > \pi$, since $w^*_A > 0$, concavity $w^*_{AA} < 0$ is expected, provided the additional positive wealth effects in $\overline{r}^{V^{Aw*}}$ are not large enough to reverse the core inequality $\overline{r}^{V^{Aw*}} < r^{V^u}$.

If $\lambda = \pi$, the curvature is trivially zero, $w^*_{AA} = 0$, since $w^* = b$ and $w^*_A = 0$.

If $\pi > \lambda$, since $w^*_A < 0$, convexity $w^*_{AA} < 0$ is expected, provided $\overline{r}^{V^{Aw*}}$ remains greater than r^{V^u}.

The consumption reservation wage, $\widehat{w}_A$, exhibits analogous properties, driven by the preference for prudence:

Theorem 3.4.2 *(Slope and Curvature of the Consumption Reservation Wage) If U satisfies DAP, that is* $p^U_C < 0$,

1. *sign*$(\widehat{w}_A(A))$ =*sign*$(\lambda - \pi)$.
2. *sign*$(\widehat{w}_{AA})$ =*sign*$\left(\overline{p}^{V^{A\widehat{w}}} - p^{V^u}\right)$

where $\overline{p}^{V^{A\widehat{w}}} \equiv p^{V^{\widehat{w}}} + \widehat{w}_A p^{V^{A\widehat{w}}}$. $\overline{p}^{V^{A\widehat{w}}} \equiv -\frac{V^{\widehat{w}}_{AAA}}{V^{\widehat{w}}_{AA}} - \widehat{w}_A \frac{V^{\widehat{w}}_{AAw}}{V^{\widehat{w}}_{AA}}$, $\overline{p}^{V^{ww*}} \equiv -\frac{V^{\widehat{w}}_{Aww}}{V^{\widehat{w}}_{Aw}} - \widehat{w}_A \frac{V^{\widehat{w}}_{www}}{V^{\widehat{w}}_{Aw}}$, *and* $\alpha \equiv \frac{V^{\widehat{w}}_{AA}}{V^{\widehat{w}}_{AA} + \widehat{w}_A V^{\widehat{w}}_{Aw}}$.
Proof: In Appendix F.

The marginal prudence premium $\widehat{w}_A(A)$ tracks the sign of the relative search productivity $\lambda - \pi$, meaning $\widehat{w}$ increases only when unemployment search is more productive. Its concavity, $\widehat{w}_{AA} < 0$, is determined by comparing the total prudence along the $\widehat{w}$ locus $\overline{p}^{V^{A\widehat{w}}}$ against the absolute prudence of the unemployed p^{V^u}. The indirect effect arising from $\widehat{w}_A$ is also critical in determining the concavity of the prudence premium.

In this extended framework, Theorem 2.7.2 on asymptotic behavior also applies. As wealth approaches the borrowing limit, the marginal reservation wages diverge to infinity $w^*_A \to \pm\infty$, based on the sign of $\lambda - \pi$. Conversely, as wealth grows its marginal effect on wages vanish, $w^*(A) \to w^c$ (the risk-neutral wage), and its derivative $w^*_A(A) \to 0$.

These results underscore that the same underlying attitudes toward risk, specifically risk aversion (DARA) and prudence (DAP), are the fundamental drivers that jointly shape both consumption and savings decisions and wage acceptance criteria. Analytically, the higher-order curvature of the utility function directly and correspondingly determines the resulting curvature of the consumption function C_{AA} and the reservation wage functions over wealth w^*_{AA} and $\widehat{w}_{AA}$.

3.5 Risk and Prudence Under Borrowing Constraints

This section builds on the foundational literature on "excess sensitivity" (Flavin 1981) and "buffer-stock" saving (Deaton 1991, Carroll 1997) by explicitly analyzing how a binding borrowing constraint, jointly impacts savings behavior, reservation wage decision, risk-aversion, and prudence across employment states.

A constraint amplifies risk aversion: the effective risk aversion r^V is the product of intrinsic risk aversion r^U and the MPC C_A. When the constraint binds, the MPC reaches its maximum ($C_A = 1$), and the agent's effective risk aversion and prudence equal their respective intrinsic levels ($r^V = r^U$ and $p^V = p^U$).We analyze the impact of a binding constraint B, formalized as a fraction of the natural borrowing limit, $0 \leq B = -sRb$, where $s \in [0,1]$ measures tightness.

The binding status is sensitive to the search regime, as established by Theorem 3.2.1. If $\lambda > \pi$, the unemployed hit the constraint at a higher wealth level than the employed, $A^e(A, w^*) > A^u(A)$; if $\pi > \lambda$, this relationship reverses, implying $A^u(A) > A^e(A, w^*)$. We characterize the joint behavior depending on which agents are constrained:

Interior Solutions: $A^u > B$ and $A^e > B$. Both employed and unemployed agents attain an interior solutions for wealth next period. Standard comparative statics from the unconstrained model apply.

Single-State Constraint (Only One State Binds). When only one state is constrained, the resulting consumption difference structurally determines the sign of w_A^*:

Unemployed Binds ($\lambda > \pi$): The unemployed are constrained, while the employed are not, $A^{w^*} > A^u = B$. This reduces unemployed consumption further relative to the unconstrained case, which was already lower than the employed, $C^{w^*} > C^u$. The marginal value of wealth satisfies $EV_{AA}^u(B) = 0$, implying $C_A^u = 1$. This constraint creates a kink and reinforces a positive gradient of the reservation wage, $w_A^* > 0$. Consequently, $r^{V^u} > r^{V^{w^*}}$ and $p^{V^u} > p^{V^{w^*}}$.

Employed Binds ($\pi > \lambda$): Only the employed agent at w^* is constrained, $A^u > A^{w^*} = B$. This reduces the employed consumption further, $C^u > C^{w^*}$; therefore, the wealth effect is negative, $w_A^* < 0$, and effective risk aversion/prudence are amplified for the employed: $r^{V^{w^*}} > r^{V^u}$ and $p^{V^{w^*}} > p^{V^u}$, given that $C_A^{w^*} = 1$.

Both States Constrained: $A^u = A^e = B$. When both states hit the lower bound on wealth, $EV_{AA}^u(B) = EV_{AA}^{w^*}(B, w^*) = 0$, so that there are no additional savings, $C_A^u = C_A^{w^*} = 1$.

The allocation remains dependent on the search regime. If $\lambda \gtreqless \pi$, then $EV^u(B) \gtreqless EV^e(B, b)$, implying $w^* \gtreqless b$. In this case, consumption levels are given by

$$C^{w^*} = A + w^* - \frac{B}{1+\rho} \gtreqless C^u = A + b - \frac{B}{1+\rho},$$

which implies $U_1(C^u) \gtreqless U_1\left(C^{w^*}\right)$, leading to $w_A^* \gtreqless 0$.

In particular:

If $\lambda = \pi$, the risks are identical, leading to $C^{w^*} = C^u$, where effective risk-aversion, and prudence are equalized. There are no wealth effects, that is, $w^*_A = 0$.

If $\lambda > \pi$, there is a strictly positive risk-premium, $w^* > b$. In this case, consumption levels are $C^{w^*} > C^u$, and the reservation wage remains increasing, $w^*_A > 0$. Effective risk aversion and prudence are amplified for the unemployed ($r^{V^u} > r^{V^{w^*}}$ and $p^{V^u} > p^{V^{w^*}}$).

If $\pi > \lambda$, there is a strictly negative risk-premium, $w^* < b$. Then $C^u > C^{w^*}$, so that the outcome is $w^*_A < 0$, and effective risk aversion and prudence are amplified for the employed ($r^{V^{w^*}} > r^{V^u}$ and $p^{V^{w^*}} > p^{V^u}$).

As agents deplete assets, they eventually reach the constraint $B \leq 0$, where consumption is determined solely by current income and debt servicing needs,
$C = (1 - s)b > 0$.[7]

In summary, the introduction of borrowing constraints preserves the model's key relative findings concerning the wealth effect and risk ranking. Specifically, the reservation wage function remains increasing in wealth $w^*_A > 0$ when $\lambda > \pi$, and the unemployed continue to exhibit higher effective risk aversion and prudence in this regime. The constraints introduce a critical level effect by interrupting the intertemporal attenuation of risk aversion implied by DARA and DAP. Because constrained agents are locked in a region of high marginal utility, they behave with greater effective risk aversion and prudence than their unconstrained counterparts at any given level of assets.

4 Conclusions

This paper bridges a significant gap in dynamic macroeconomic theory by providing the requisite analytical structure that links the foundational job search literature with modern research on consumption and prudence. By formally integrating higher-order risk preferences, specifically decreasing absolute risk aversion and prudence, into a model featuring on-the-job search, we supply the micro-foundations necessary to understand how wealth endogenously shapes worker trajectories and amplifies inequality in dynamic labor markets.

[7] This implies that agents facing tighter borrowing constraints (lower s) consume more at the constraint due to lower future repayment obligations, despite having less flexibility ex ante.

Our analysis yields five distinct contributions that structurally revise standard intuitions regarding the optimal reservation wage.

First, we provide the rigorous proof that the classic positive wealth effect on reservation wages is driven by the analytical gap between the risk premium and the prudence premium. We explicitly link this to the fundamental equations of dynamic optimization: whereas the Bellman equation equates total utilities at the risk premium, the Euler equation equates marginal utilities at the prudence premium. We show that the reservation wage is determined by the interplay of these two distinct stochastic wedges. We further resolve the long-standing technical ambiguity regarding value function concavity by establishing a verifiable sufficient condition based on the "wealth hazard rate."

Second, and most significantly, we derive a unified theorem of search regimes that fundamentally challenges the universality of the wealth effect. We establish that the standard result—that reservation wages increase with wealth—holds only when the unemployment state offers more productive search opportunities than employment. We show that when the employment lottery stochastically dominates the unemployment lottery (via a "job ladder" structure), this logic inverts. We uncover a novel "Investment Fund" mechanism: in this regime, wealth no longer subsidizes leisure or patience, but rather subsidizes entry. Wealthier agents utilize their assets to "purchase" access to the superior employment lottery by accepting lower starting wages. This provides a theoretical base to empirical evidence showing that financial access facilitates entry into high-risk, high-growth states.

Third, we show that this regime-dependence extends critically to consumption dynamics, determining relative consumption sensitivity and effective prudence across states. Our model explains the persistent finding that unemployed households maintain high MPC, even those with high liquidity, as a feature of Prudent Job Search. Under DAP, unemployment acts as a state of heightened background risk, which endogenously induces a higher MPC independent of liquidity constraints. By linking these outcomes to the primitives of risk preference and search technology, this paper provides the analytical sign-restrictions required to ground the complex dynamics observed in frontier quantitative models.

Fourth, we demonstrate that both the risk premium (the reservation wage) and the prudence premium (the consumption reservation wage) mirror the asymptotic geometry of the consumption function. While the direction of the wealth effect depends on the search regime, the curvature is universally governed by DARA and

DAP: the sensitivity of job acceptance to wealth diminishes as assets accumulate, effectively 'smoothing' labor market transitions for the wealthy just as savings smooth consumption.

Fifth, we show that the introduction of borrowing constraints leaves the model's key qualitative predictions regarding the wealth effect and risk ranking intact, acting as a distinct amplifier. The primary impact of constraints is a critical level effect: by restricting intertemporal smoothing and locking agents into regions of high marginal utility, constraints interrupt the attenuation of risk aversion implied by DARA and DAP. This amplifies the effective risk aversion and prudence of constrained agents relative to their unconstrained counterparts, heightening the sensitivity of reservation wages to wealth beyond the baseline effects of uncertainty.

These findings carry immediate implications for policy. The identified "Investment Fund" regime suggests that public policies conditioning unemployment assistance on asset liquidation—such as strict asset tests—may be counterproductive. By depleting the private funds required to accept low-wage, high-growth entry jobs, such policies may inadvertently trap workers in low-mobility employment trajectories.

More broadly, the analytical toolkit derived here, which rigorously connects the sign of the preference-driven wealth effect to the structural properties of layered risk, is directly transferable to any dynamic programming problem involving optimal switching between two risky regimes. This methodology can be applied to model decisions across finance, banking, and general macroeconomics, such as determining the optimal default threshold for consumer credit products as a function of borrower equity, thereby providing a robust preference-based analytical foundation for decisions beyond the labor market.

Appendix

Some derivations are presented in the Companion Appendix.

A Proof: Jobs Forever - Reservation Wage

Proof of Theorem 2.2.3. Under DARA, iterative integration and maximization yields $V_A^u(A) > V_A^e(A, w^*)$. We proceed inductively, starting with $V_A^e(A, w^*) \;\; = V_A^u(A) \;\; = U_1(A + b)$. We first characterize the static risk and prudence premia, and then leverage these results to determine the corresponding dynamic values.

1. Integration and Static Premia. We first characterize the search lottery by defining two static thresholds. Let w^{**} denote the *static risk premium*, defined as the wage that equates the value of employment to the expected continuation value of unemployment: $V^e(A, w^{**}) = EV^u(A)$. Similarly, let $\widetilde{w}$ denote the *static prudence premium*, defined as the wage that equates the marginal value of employment to the marginal expected continuation value: $V^e_A(A, \widetilde{w}) = EV^u_A(A)$.

We express $EV^u(A)$ in terms of V^e

$$\begin{aligned} EV^u(A) &= (1-\lambda[1-F(w^*)])\, V^u(A) + \lambda \int_{w^*}^{\infty} V^e(A,x)\, dF(x), \\ &= (1-\lambda[1-F(w^*)])\, V^e(A,w^*) + \lambda \int_{w^*}^{\infty} V^e(A,x)\, dF(x). \end{aligned}$$

Since EV^u_A is an expected value function based on V^e, then the static risk premium w^{**} represents its certainty value $V^e(A, w^{**}) = EV^u(A)$. Then, DARA of V^e implies

$$V^e_A(A, w^{**}) < (1-\lambda[1-F(w^*)])\, V^e_A(A, w^*) + \lambda \int_{w^*}^{\infty} V^e_A(A,x)\, dF(x) \leq EV^u_A(A).$$

That is, $V^e_A(A, w^{**}) < EV^u_A(A)$. By the definition of the static prudence premium $V^e_A(A, \widetilde{w}) = EV^u_A(A)$. Therefore, $V^e_A(A, w^{**}) < V^e_A(A, \widetilde{w})$. Since $V^e_{Aw} < 0$, this inequality implies that the static risk premium is strictly greater than the static prudence premium: $w^{**} > \widetilde{w}$. Thus, DARA implies that for the wage $\widetilde{w}$ equating marginal values, the total value must be strictly lower: $V^e(A, \widetilde{w}) < EV^u(A)$. Moreover, this result is also valid for certainty values at different asset levels. That is, DARA of V^e also implies that $V^e_A(A+\Delta, \widetilde{w}) = EV^u_A(A)$ implies $V^e(A+\Delta, \widetilde{w}) < EV^u(A)$.

2. Maximization. We exploit the structural coincidence between the definition of the dynamic prudence premium and the Euler equation. The dynamic prudence premium $\widehat{w}$ is defined defined as the wage that equates the marginal value of wealth across states: $V^e_A(A, \widehat{w}) = V^u_A(A)$. By the Envelope Theorem, $V_A = U_1$, this condition implies that the Euler equations governing consumption dynamics for both employment and unemployment coincide:

$$\begin{aligned} V^u_A(A) &= U_1(C^u) = \beta(1+\rho)\, EV^u_A(A^u) \\ = V^e_A(A, \widehat{w}) &= U_1\left(C^{\widehat{w}}\right) = \beta(1+\rho)\, V^e_A\left(A^{\widehat{w}}, \widehat{w}\right). \end{aligned}$$

These expressions readily imply that $C^u = C^{\widehat{w}}$ and $U(C^u) = U\left(C^{\widehat{w}}\right)$. Since $V^e_A(\cdot,\cdot)$ is a deterministic function, these three equations are equivalent $V^e_A(A^1, w^1) = V^e_A(A^2, w^2)$, $C^e(A^1, w^1) = C^e(A^2, w^2)$, and $V^e(A^1, w^1) = V^e(A^2, w^2)$ for any combination (A, w). See Companion Appendix. Then, DARA of V^e implies that

$$\begin{aligned} EV^u_A(A^u) &= V^e_A(A^u, \widetilde{w}) = V^e_A\left(A^{\widehat{w}}, \widehat{w}\right), \text{ implies that} \\ EV^u(A^u) &> V^e(A^u, \widetilde{w}) = V^e\left(A^{\widehat{w}}, \widehat{w}\right). \end{aligned}$$

Adding $U\left(C^{u}\right)=U\left(C^{\widehat{w}}\right)$ to each respective side, we obtain

$$\begin{aligned} U\left(C^{u}\right)+\beta E V^{u}\left(A^{u}\right) &> U\left(C^{\widehat{w}}\right)+V^{e}\left(A^{\widehat{w}}, \widehat{w}\right), \\ V^{u}(A) &> V^{e}(A, \widehat{w}). \end{aligned}$$

Since $V^{e}\left(A, w^{*}\right)=V^{u}(A)>V^{e}(A, \widehat{w})$, then $w^{*}>\widehat{w}$, implying $V_{A}^{u}(A)>V_{A}^{e}\left(A, w^{*}\right)$. That is, integration and maximization preserve this property over all iterations, which establishes Property 1.

Property 2. By implicit differentiation of $V^{e}\left(A, w^{*}\right)=V^{u}(A)$:

$$w_{A}^{*}(A)=\frac{V_{A}^{u}(A)-V_{A}^{e}\left(A, w^{*}\right)}{V_{w}^{e}\left(A, w^{*}\right)}>0.$$

Property 3. By the envelope theorem, $V_{A}^{u}(A)>V_{A}^{e}\left(A, w^{*}\right)$ is equivalent to $U_{1}\left(C^{u}\right)>U_{1}\left(C^{w^{*}}\right)$. Given that $C_{w}^{e}>0$, then $C^{e}(A, w)>C^{e}\left(A, w^{*}\right)>C^{u}(A)$, for all $w>w^{*}$ ■

B Proof: Concavity

Proof of Theorem 2.3.1. From Equation (3), $E V_{AA}^{u}(A)$ is the sum of two negative terms and one positive term. A sufficient (but not necessary) condition for $E V_{AA}^{u}(A)<0$ is for the first negative term to dominate the positive term, ignoring the second negative term, which only strengthens the condition. This requires:

$$-\left(1-\lambda\left[1-F\left(w^{*}\right)\right]\right) V_{AA}^{u} \geq \lambda f(w^{*}) w_{A}^{*}\left[V_{A}^{u}-V_{A}^{w^{*}}\right],$$

Since $V_{A}^{u}>V_{A}^{w^{*}}$, by Theorem 2.2.3), a stricter condition that implies the one above is:

$$-\left(1-\lambda\left[1-F\left(w^{*}\right)\right]\right) V_{AA}^{u} \geq \lambda f(w^{*}) w_{A}^{*} V_{A}^{u}.$$

Rearranging and dividing by $V_{A}^{u}>0$ yields:

$$\frac{-V_{AA}^{u}}{V_{A}^{u}}>\frac{\lambda f\left(w^{*}(A)\right) w_{A}^{*}(A)}{1-\lambda\left[1-F\left(w^{*}(A)\right)\right]}.$$

This is equivalent to $r_{V^{u}}>h_{A}$ ■

C Proofs: Jobs Forever - Consumption

Proof of Theorem 2.4.1. From the Envelope Theorem and Euler Equation (1) for the V^{u} value function, we have

$$V_{A}^{u}(A)=\beta(1+\rho)\left[V_{A}^{u}\left(A^{u}\right)+\lambda \int_{w^{*}\left(A^{u}\right)}^{\infty}\left[V_{A}^{e}\left(A^{u}, x\right)-V_{A}^{u}\left(A^{u}\right)\right] d F(x)\right].$$

From Theorem 2.2.3, the second term of the RHS is negative. If $\beta(1+\rho) \leq 1$, then $V_{A}^{u}(A)<V_{A}^{u}\left(A_{u}\right)$. For $V_{AA}^{u}<0$, it holds that $A>A^{u}$ ■

Proof of Theorem 2.4.2. Suppose $A^b \leq A^u$, then $V_A^e\left(A^b, b\right) > EV_A^u\left(A^u\right)$, but also $A + b - \frac{A^b}{1+\rho} \geq A + b - \frac{A^u}{1+\rho}$, or $C^b \geq C^u$ and thus $U_1\left(C^b\right) \leq U_1\left(C^u\right)$, which forces $V_A^e\left(A^b, b\right) \leq EV_A^u\left(A^u\right)$, a contradiction. Therefore, $A^e\left(A, b\right) > A^u\left(A\right)$ and $C^u\left(A\right) > C^e\left(A, b\right)$. Since $A_w^e > 0$, then $A^e\left(A, w\right) > A^e\left(A, b\right)$, $\forall w \geq b$ ■

Proof of Theorem 2.5.1. Risk aversion is just preserved if the MPC is positive, $r_{V^u} = r_U C_A^u < 0$. The utility function has DARA, $r_C^U < 0$. Then, DARA is preserved for V^u, if DARA is preserved by the integration, that is $r_A^{EV'^u} < 0$:

$$\frac{r_A^{V^u}}{r^{V^u}} = \frac{r_C^U}{r^U}\left(C_A^u\right)^2 + \frac{r_A^{EV'^u}}{r^{EV'^u}}\left(1+\rho\right)\left(1 - C_A^u\right)^2 < 0 \blacksquare$$

Proof of Theorem 2.5.2: The derivative of prudence for V^u is

$$p_A^{V^u} = p_C^U\left(C_A^u\right)^3 + p_A^{EV'^u}\left(1+\rho\right)^2\left(1 - C_A^u\right)^3 + 2\left(p^U C_A - p^{EV'^u} A_A^u\right)^2 C_A^u\left(1 - C_A^u\right).$$

While the first term is always negative and the second term is negative if DAP is preserved by integration, the third term of the RHS is positive by Theorem 2.6.1. Then, DAP is preserved intertemporally if the two first terms negative values are larger than the third terms, as the Theorem states■

Proof of Theorem 2.6.1. The second derivative of consumption can be expressed in terms of prudence:

$$C_{AA}^u = r^{V^u}\left(\frac{p^U}{r^U} - \frac{p^{EV'^u}}{r^{EV'^u}}\right) C_A^u\left(1 - C_A^u\right), \tag{8}$$

then $C_{AA}^u < 0$, if $\frac{p^{EV'}}{r^{EV'}} = \frac{EV_3' EV_1'}{\left(EV_2'\right)^2} > \frac{p^U}{r^U} = \frac{U_3 U_1}{\left(U_2\right)^2}$. We know that for the CRRA utility function $U_3 U_1 = \gamma\left(\gamma+1\right) C^{-2\gamma-2}$, and $\left(U_2\right)^2 = \gamma^2 C^{-2\gamma-2}$, thus

$$\frac{p^U}{r^U} = \frac{U_3 U_1}{\left(U_2\right)^2} = \frac{\gamma+1}{\gamma}.$$

We also know that $V_1 = U_1$, $V_2 = U_2 C_1$, $V_3 = U_3\left(C_1\right)^2 + U_2 C_2$. Thus

$$\frac{V_1 V_3}{\left(V_2\right)^2} = \frac{U_1 U_3\left(C_1\right)^2}{U_2^2\left(C_1\right)^2} + \frac{U_1 U_2 C_2}{\left(U_2\right)^2\left(C_1\right)^2} = \frac{\gamma+1}{\gamma} - \frac{U_1 C_2}{U_2\left(C_1\right)^2} \geq \frac{\gamma+1}{\gamma}.$$

As shown in Equation (1) in the Companion Appendix, for

$$\begin{aligned} V_3 V_1 &\geq \frac{\gamma+1}{\gamma}\left(V_2\right)^2, \\ \text{integration implies } EV_3 EV_1 &> \frac{\gamma+1}{\gamma}\left(EV_2\right)^2. \end{aligned}$$

Then

$$\frac{p^{EV'}}{r^{EV'}} = \frac{EV_3'EV_1'}{(EV_2')^2} > \frac{\gamma+1}{\gamma} = \frac{U_3U_1}{(U_2)^2} = \frac{p^U}{r^U} \blacksquare$$

Proof of Theorem 2.6.2. As seen in Theorem 2.2.3, $V_A^e(A^e, \widehat{w}) = EV_A^u(A^u)$. Then, DAP of V_A^e implies $V_{AA}^e(A^e, \widehat{w}) > EV_{AA}^u(A^u)$, which on its turn implies Property 1:

$$C_A^u = \frac{\beta(1+\rho)^2 EV_{AA}^u}{U_2^u + \beta(1+\rho)^2 EV_{AA}^u} > \frac{\beta(1+\rho)^2 V_{AA}^e}{U_2^{\widehat{w}} + \beta(1+\rho)^2 V_{AA}^e} = C_A^{\widehat{w}}.$$

When jobs are forever, $C_{AA}^w = 0$, then C_A^w is the same for all the employed and thus $C_A^u > C_A^w$ for all w.

Since $U_2\left(C^{\widehat{w}}\right) = U_2(C^u)$, then $V_{AA}^{\widehat{w}} = U_2\left(C_A^{\widehat{w}}\right)C_A^{\widehat{w}} > V_{AA}^u = U_2(C_A^u)C_A^u$. This implies, together with $V_{AAA}^w > 0$, Property 2, $V_{AA}^w > V_{AA}^{\widehat{w}} > V_{AA}^u$ for all $w \geq w^* > \widehat{w}$.

The equivalency $V_A^{\widehat{w}} = V_A^u$ implies that $-\frac{V_{AA}^u}{V_A^u} > -\frac{V_{AA}^{\widehat{w}}}{V_A^{\widehat{w}}}$. This result, together with **DARA in wages $r_w^{V^e} < 0$,** for $w \geq w^* > \widehat{w}$, imply Property 3, that $r^{Vu} > r^{V^{\widehat{w}}} > r^{V^w}$ for all $w \geq w^* > \widehat{w}$.

By definition,

$$\begin{aligned} p^{Vu} &= p^{U^u}C_A^u - \frac{C_{AA}^u}{C_A^u}, \text{ and} \\ p^{V^{w*}} &= p^{U^{w*}}C_A^{w*}. \end{aligned}$$

By Theorem 2.2.3 $C^{w*} > C^u$, which under DAP of V^e implies $p^{U^u} > p^{U^{w*}}$. By $C_A^u > C_A^{w*}$ and Theorem 2.6.1, for a CRRA/homogeneous utility function, $C_{AA}^u \leq 0$, then, $p^{U^u}C_A^u - \frac{C_{AA}^u}{C_A^u} > p^{U^{w*}}C_A^{w*}$, which implies $p^{Vu} > p^{V^{w*}} > p^{V^w}$ for all $w \geq w^*$, Property 4 $\blacksquare$

Proof of Theorem 2.7.1. From $V_w^{w^*} = RV_A^{w^*}$ and Theorem 2.2.3:

$$w_A^* = \frac{V_A^u - V_A^{w^*}}{RV_A^{w^*}}.$$

Taking the derivative over wealth:

$$\begin{aligned} w_{AA}^* &= \left(\frac{V_{AA}^u}{V_A^u} - \frac{V_{AA}^{w^*}}{V_A^{w^*}}(1+Rw_A^*)\right)\frac{V_A^u}{RV_A^{w^*}}, \\ &= \left(\overline{r}^{V^{Aw*}} - r^{V^u}\right)\frac{V_A^u}{RV_A^{w^*}}, \end{aligned}$$

where $\overline{r}^{V^{Aw*}} = r^{V^{Aw*}}(1+Rw_A^*)$. Since $\frac{V_A^u}{RV_A^{w^*}} > 0$, the sign of w_{AA}^* equals the sign of $\left(\overline{r}^{V^{Aw*}} - r^{V^u}\right) \blacksquare$

Proof of Theorem 2.7.2.

1. Since $\lim_{A\to B} V_A^u = \infty$ and $\lim_{A\to B} V_A^{w^*} < \infty$, then

$$\lim_{A\to B} w_A^*(A) = R^{-1}\left(\lim_{A\to B}\frac{V_A^u}{V_A^{w^*}} - 1\right) = \infty.$$

2. If the utility function satisfies DARA such that $\lim_{A\to\infty} r(C) = 0$, the utility function is linear, reflecting risk neutrality.

3. Since $\lim_{A\to\infty} V_A^u = 0$ and $\lim_{A\to\infty} V_A^{w^*} = 0$, we apply L'Hôpital's rule:

$$\lim_{A\to\infty} w_A^*(A) = R^{-1}\left(\lim_{A\to\infty}\frac{V_A^u}{V_A^{w^*}} - 1\right) = R^{-1}\left(\lim_{A\to\infty}\frac{V^u}{V^{w^*}} - 1\right) = 0 \blacksquare$$

Proof of Theorem 2.7.3. Property 1. By implicit differentiation of $V_A^{\widehat{w}} = V_A^u$ and Theorem 2.6.2:

$$\widehat{w}_A(A) = \frac{V_{AA}^u - V_{AA}^{\widehat{w}}}{RV_{AA}^{\widehat{w}}} = \frac{r^{V^u} - r^{V^{\widehat{w}}}}{Rr^{V^{\widehat{w}}}} > 0$$

Property 2. Taking the derivative of $\widehat{w}_A$, we obtain $\widehat{w}_{AA}(A) = \left(\overline{p}^{V^{Aw*}} - p^{V^u}\right)\frac{V_{AA}^u}{RV_{AA}^{\widehat{w}}}$, where $\overline{p}^{V^{Aw*}} = p^{V^{Aw*}}(1 + R\widehat{w}_A)$ and $\frac{V_{AA}^u}{RV_{AA}^{\widehat{w}}} > 0$ in a similar way as in the proof of Theorem 2.7.1 $\blacksquare$

D Stochastic Dominance and Lottery Valuation

In this framework, wage dynamics and offer uncertainty define stochastic lotteries for both the employment and unemployment states. To rigorously compare these lotteries, we utilize the framework of Stochastic Dominance established by Hadar and Russell (1969) and Hanoch and Levy (1969). We define the cumulative distribution functions (CDFs) for the unemployment state F^u and the employment state F^e as follows

$$\begin{aligned} F^u(x) &= \begin{cases} 1 - \lambda\left[1 - F(w^*)\right], & \text{if } x = w^*, \\ 1 - \lambda\left[1 - F(x)\right], & \text{if } x \geq w^*, \end{cases} \\ F^e(x) &= \begin{cases} \theta, & \text{if } w > x \geq w^*, \\ 1 - \pi\left[1 - F(w)\right], & \text{if } x = w, \\ 1 - \pi\left[1 - F(x)\right], & \text{if } x \geq w. \end{cases} \end{aligned}$$

These distributions represent mixed continuous-discrete lotteries governed by the underlying wage offer distribution F, the arrival rates λ and π, and the state-dependent thresholds, the current wage w and the reservation wage w^*.

The unemployment lottery F^u. This distribution contains a single mass point at the reservation wage $x = w^*$ with probability $1 - \lambda\left[1 - F(w^*)\right]$. This mass represents the probability of remaining in the unemployment state, which is the sum of two disjoint events: receiving no job offer, probability $1 - \lambda$, and receiving an offer below the reservation wage, probability $\lambda F(w^*)$.

The employment lottery F^e. This distribution contains two distinct mass points reflecting the specific risks of the employment state:

- at $x = w^*$ (The Downside Risk): A mass point with probability θ, representing the exogenous risk of layoff where the agent falls back to the value of unemployment, evaluated at w^*.
- at $x = w$ (The Status Quo): A mass point with probability $1 - \pi [1 - F(w)]$. This probability, calculated as the jump in the CDF $F^e(w) - \lim_{x \to w^-} F^e(x) = 1 - \pi [1 - F(w)] - \theta$ represents the likelihood of retaining the current wage. It is the sum of the probability of stability, not being laid off and receiving no outside offer, $1 - \theta - \pi$, and the probability of receiving an outside offer that fails to improve upon the current wage $\pi F(w)$.

First-order stochastic dominance (FOSD) implies that an employment status is preferred to the other for any increasing utility function. For example if, employment dominates unemployment, it is defined as $F^e(x) \leq F^u(x)$ for all x This happens in the following scenarios:

- Unemployment FOSD employment, if $\lambda > \pi$ and $w = w^*$.
- Employment FOSD unemployment, if $\pi > \lambda$ and $w = w^*$ or if $\pi > \lambda$, $1 - \lambda > \theta$, and if $w > w^*$.

These scenarios are illustrated in Figure 2 and 3.

[Figure 2 and Figure 3 here]

Second-order stochastic dominance (SOSD) implies that an employment status is preferred to the other for risk-averse agents (i.e., all U with $U_1 > 0$, $U_2 < 0$). Employment SOSD unemployment, defined by $\int_0^\infty F_e(x) \leq \int_0^\infty F_u(x)$, in one scenario:

- if $\lambda > \pi$, $1 - \lambda > \theta$, and $w \geq w^s$, where the threshold w^s is defined by equating the expected values of the lotteries:

$$w^s = \{w \mid \theta (w - w^*) + \int_w^\infty [1 - \pi [1 - F(x)]] \, dx = \int_{w^*}^\infty [1 - \lambda [1 - F(x)]] \, dx\}.$$

Integrating by parts, using $\int [1 - F(x)] \, dx = x [1 - F(x)] + \int x f(x) \, dx$, and rearranging yields the following expression:

$$\begin{aligned} & (1 - \theta) w^s + \pi [1 - F(w^s)] [E[x | x \geq w^s] - w^s] \\ = \; & (1 - \theta) w^* + \lambda [1 - F(w^*)] [E[x | x \geq w^*] - w^*]. \end{aligned}$$

This threshold, necessarily higher than the reservation wage ($w^s > w^*$), identifies a sufficiently high wage for employment to be preferred by *any* risk-averse agent.

DARA, Prudence, and the Static Valuation of Lotteries

While stochastic dominance provides universal rankings for entire classes of utility functions, this section focuses on the specific valuation of the continuation lotteries EV^u and EV^e by a DARA agent. This static comparison establishes the mechanism driving the main dynamic theorem (Theorem 3.1.1). We define two static wage thresholds for the regimes where they are applicable ($\lambda \geq \pi$):

- The Static Risk Premium w^{**}, the wage that equates total expected values: $EV^e(A, w^{**}) = EV^u(A)$.
- The Static Prudence Premium $\widetilde{w}$, the wage that equates marginal expected values: $EV^e_A(A, \widetilde{w}) = EV^u_A(A)$.

Neutral Search. If $\lambda = \pi$, the search lotteries are identical. Consequently, the expected continuation values and the marginal expected values are equal at the benefit wage, $EV^e(A, b) = EV^u(A)$ and , $EV^e_A(A, b) = EV^u_A(A)$, for all assets A. This implies the static risk and prudence premia are simply the benefit level: $w^{**} = \widetilde{w} = b$.
Productive Unemployment. If $\lambda > \pi$, the unemployment lottery $EV^u(A)$ is more productive.

Using the expected value definitions of EV^e and EV^u and substituting $V^u(A) = V^e(A, w^*)$, the static risk premium w^{**} is derived from

$$
\begin{aligned}
& V^e(A, w^{**}) + \frac{\pi}{1-\theta}\int_{w^{**}}^{\infty}\left[V^e(A, x) - V^e(A, w^{**})\right] dF(x) \\
= \; & V^e(A, w^*) + \frac{\lambda}{1-\theta}\int_{w^{*}}^{\infty}\left[V^e(A, x) - V^e(A, w^{*})\right] dF(x) .
\end{aligned}
$$

This expression confirms that for $\lambda > \pi$, the static risk premium is strictly higher than the dynamic reservation wage: $w^{**} > w^*$.

By the DARA/Arrow-Pratt theorem, the lottery with the higher arrival rate EV^u must have a higher total value relative to its marginal value. Therefore, the equality of marginal values, $EV^e_A(A, \widetilde{w}) = EV^u_A(A)$, implies an inequality in total values: $EV^u(A) > EV^e(A, \widetilde{w})$. Since w^{**} is the wage premium that equates total values, this establishes the gap between the two static premia: $w^{**} > \widetilde{w}$.

This principal extends to different asset levels: for any $EV^u_A(A) = EV^e_A(A + \Delta, w)$, DARA and $\lambda > \pi$ imply $EV^u(A) > EV^e(A + \Delta, w)$, for all asset changes Δ.
Productive Employment. If $\pi > \lambda$, the employed lottery EV^e is strictly more productive, a result of First-Order Stochastic Dominance. This implies that the expected value of employment is always strictly higher than the expected value of unemployment, $EV^e(A, w) > EV^u(A)$, for any wage below the reservation wage floor, $w < w^*(A)$, as it can be seen from their definitions:

$$
\begin{aligned}
EV^e(A, w) &= V^u(A) + \pi\int_{w^*(A)}^{\infty}\left[V^e(A, x) - V^u(A)\right] dF(x)\,, \text{if } w < w^*(A)\,, \\
EV^u(A) &= V^u(A) + \lambda\int_{w^*(A)}^{\infty}\left[V^e(A, x) - V^u(A)\right] dF(x)\,.
\end{aligned}
$$

The fact that individuals retain the quitting option structurally prevents the higher expected employment value from declining to the unemployment value at the same asset level. In the absence of the standard wage premia, the values are equalized by wealth premia. Let Δ^* be the wealth premium needed to equalize total expected values, $EV^u(A + \Delta^*) = EV^e(A, w)$, and $\widehat{\Delta}$ be the premium needed to equalize marginal expected values, $EV^u_A\left(A + \widehat{\Delta}\right) = EV^e_A(A, w)$. The DARA property implies that for $\pi > \lambda$, the wealth premium needed to equate marginal values $\widehat{\Delta}$ is strictly higher than the premium needed to equate total values, that is,

$EV^u\left(A+\widehat{\Delta}\right) > EV^e(A,w)$, creating a gap between the two static wealth premia: $\widehat{\Delta} > \Delta^*$.

This static comparison of the continuation lotteries is the key logical step used in the inductive proof of Theorem 3.1.1 (see next Appendix). The *dynamic* premia (the reservation wage w^* and the dynamic prudence premium $\hat{w}$) inherit this same sign structure from their *static* continuation-value wage counterparts, w^{**} and $\widetilde{w}$, if $\lambda > \pi$, and from their *static* continuation-value wealth counterparts Δ^* and $\widehat{\Delta}$, if $\pi > \lambda$.

E Proofs: Job Turnover - Reservation wage

Proof of Theorem 3.1.1: From the analysis in Appendix D, the higher expected continuation value of employment status is the one with the higher arrival rate, that is, sign$(EV^u(A) - EV^e(A,b))$ =sign$(\lambda - \pi)$. This implies

$$U\left(A+b-\frac{A'}{1+r}\right)+\beta EV^u(A') \gtreqqless U\left(A+b-\frac{A'}{1+r}\right)+\beta EV^e(A',b)\,,\text{ if }\lambda \gtreqqless \pi.$$

This simplifies to $V^u(A) \gtreqqless V^e(A,b)$. By the definition of the reservation wage, $V^u(A) = V^e(A,w^*)$. Therefore, if $\lambda \gtreqqless \pi$, it follows that $V^e(A,w^*) \gtreqqless V^e(A,b)$. Since V^e is strictly increasing in w, this directly implies $w^*(A) \gtreqqless b, \forall A$ It is also true that $C^u(A) = C^e(A,\hat{w}) \gtreqqless C^e(A,b)$, if $\lambda \gtreqqless \pi$, implying $\hat{w}(A) \gtreqqless b$, $\forall A$.

By the envelope theorem and the Euler equations for employment and unemployment:

$$\begin{aligned} V_A^u(A) &= U_1(C^u) = \beta(1+\rho)\,EV_A^u(A^u) \\ = V_A^e(A,\hat{w}) &= U_1\left(C^{\hat{w}}\right) = \beta(1+\rho)\,EV_A^e\left(A^{\hat{w}},\hat{w}\right). \end{aligned}$$

Equating these expressions yields two critical implications:

1. The current consumption levels must be equal: $C^u = C^{\hat{w}}$, which also means $U(C^u) = U(C^{\hat{w}})$.

2. The marginal continuation values must be equal: $EV_A^u(A^u) = EV_A^e\left(A^{\hat{w}},\hat{w}\right)$.

By the DARA implication derived in Appendix D

$$\text{sign}\left(EV^u(A^u) - EV^e\left(A^{\hat{w}},\hat{w}\right)\right) = \text{sign}(\lambda-\pi)\,.$$

If $\lambda > \pi$, the equality of marginal values $EV_A^u(A^u) = EV_A^e\left(A^{\hat{w}},\hat{w}\right)$ implies an inequality of total values: $EV^u(A^u) > EV^e\left(A^{\hat{w}},\hat{w}\right)$. Adding the equal current utility $U(C^u)$ to both sides:

$$U(C^u) + \beta EV^u(A^u) > U\left(C^{\hat{w}}\right) + \beta EV^e\left(A^{\hat{w}},\hat{w}\right).$$

This is equivalent to $V^u(A) > V^e(A,\hat{w})$. Since $V^e(A,w^*) = V^u(A)$, we have $V^e(A,w^*) > V^e(A,\hat{w})$, which implies $w^* > \hat{w}$. Since V_{Aw}^e is negative in w, $w^* > \hat{w}$

implies $V_A^u(A) > V_A^e(A, w^*)$. By the Implicit Function Theorem, $w_A^* = \frac{V_A^u - V_A^{w^*}}{V_w^e} > 0$. Finally, $V_A^u > V_A^{w^*}$ implies $U_1(C^u) > U_1\left(C^{\hat{w}}\right)$, which means $C^u < C^{w^*}$.

If $\lambda = \pi$, then $V^u(A) = V^e(A, \hat{w})$, so $w^* = \hat{w} = b$. All inequalities become equalities: $w_A^* = 0$, $V_A^u = V_A^{w^*}$, and $C^u = C^{w^*}$.

If $\pi > \lambda$, then $EV^u(A^u) < EV^e\left(A^{\hat{w}}, \hat{w}\right)$, which leads to $V^u(A) < V^e(A, \hat{w})$. This implies $V^e(A, w^*) < V^e(A, \hat{w})$, meaning $w^* < \hat{w}$, further implying $V_A^u < V_A^e(A, w^*)$, $w_A^* < 0$, and $C^u > C^{w^*}$ ■

F Proofs: Job turnover - Consumption

Proof of Theorem 3.2.1. If $\lambda \gtreqqless \pi$, then $C^e(A, w^*) \gtreqqless C^e(A, \widehat{w}) = C^u(A) \gtreqqless C^e(A, b)$, by Theorem 3.1.1, which implies $A^u(A) \lesseqqgtr A^e(A, b) \lesseqqgtr A^e(A, \widehat{w}) \lesseqqgtr A^e(A, w^*)$ ■

Proof of Theorem 3.3.1. Since $\widehat{w}$ is defined such that $V_A^{\widehat{w}} = V_A^u$, this is equivalent to $U_1^{\widehat{w}} = U_1^u$, and thus $C^{\widehat{w}} = C^u$, and $U_2^{\widehat{w}} = U_2^u$. As seen in Theorem 3.1.1 and Appendix D, this also implies that $EV_A^e(A^e, \widehat{w}) = EV_A^u(A^u)$. Then, DAP of V_A^e implies $EV_{AA}^e(A^e, \widehat{w}) \gtreqqless EV_{AA}^u(A^u)$, if $\lambda \gtreqqless \pi$, which on its turn implies that

$$\begin{aligned}
\frac{\beta(1+\rho)^2 EV_{AA}^u}{U_2^u + \beta(1+\rho)^2 EV_{AA}^u} &\gtreqqless \frac{\beta(1+\rho)^2 EV_{AA}^{\widehat{w}}}{U_2^{\widehat{w}} + \beta(1+\rho)^2 EV_{AA}^{\widehat{w}}}, \text{ if } \lambda \gtreqqless \pi, \\
\text{or equivalently } C_A^u &\gtreqqless C_A^{\widehat{w}}, \text{ if } \lambda \gtreqqless \pi.
\end{aligned}$$

This implies:

$$\begin{aligned}
U_2^u C_A^u &\gtreqqless U_2^{\widehat{w}} C_A^{\widehat{w}}, \text{ if } \lambda \gtreqqless \pi, \\
\text{or equivalently } -V_{AA}^u &\gtreqqless -V_{AA}^{\widehat{w}}, \text{ if } \lambda \gtreqqless \pi.
\end{aligned}$$

and further, since by definition $V_A^{\widehat{w}} = V_A^u$, then

$$\begin{aligned}
\frac{-V_{AA}^u}{V_A^u} &\gtreqqless \frac{-V_{AA}^{\widehat{w}}}{V_A^{\widehat{w}}}, \text{ if } \lambda \gtreqqless \pi, \\
\text{or equivalently } r^{V^u} &\gtreqqless r^{V^{\widehat{w}}}, \text{ if } \lambda \gtreqqless \pi.
\end{aligned}$$

For a CRRA/homogeneous utility function, $C_{AA}^w < 0$, and given that $V_{AAA}^w > 0$ and **DARA in wages** $r_w^{V^e} < 0$.

Productive Unemployment. $\lambda > \pi$**.** Since $C_A^{\widehat{w}} > C_A^{w^*}$, then $C_A^u > C_A^w$ for all acceptable wages for $w \geq w^* > \widehat{w}$. Similarly, since $-V_{AA}^{\widehat{w}} > -V_{AA}^{w^*}$ and $r^{V^{\widehat{w}}} > r^{V^{w*}}$, then $-V_{AA}^u > -V_{AA}^w$ and $r^{V^u} > r^{V^w}$ for $w > w^* > \widehat{w}$.

Neutral Search: $\lambda = \pi$**.** Since $C_A^{\widehat{w}} = C_A^{w^*}$, then $C_A^u \geq C_A^w$ for all acceptable wages for $w \geq w^* = \widehat{w}$. Similarly, since $-V^{\widehat{w}}_{AA} = -V^{w^*}_{AA}$and $r^{V^{\widehat{w}}} = r^{V^{w*}}$, then $-V^{u}_{AA} \geq -V^{w}_{AA}$ and $r^{V^u} \geq r^{V^w}$ for $w > w^* = \widehat{w}$.

Productive Employment. $\pi > \lambda$**.** Since $\widehat{w} > w^*$, then $C^{w^*}_A > C^{\widehat{w}}_A > C^u_A$, $-V^u_{AA} < -V^{\widehat{w}}_{AA} < -V^{w^*}_{AA}$, and $r^{V^u} < r^{V^{\widehat{w}}} < r^{V^{w*}}$ ■

Proof of Theorem 3.3.2. We first decompose the difference in effective prudence:

$$\frac{p^{V^u}}{r^{V^u}} - \frac{p^{V^{w*}}}{r^{V^{w*}}} = \frac{p^{U^u}}{r^{U^u}} C^u_A - \frac{p^{U^{w^*}}}{r^{U^{w*}}} C^e_A + \frac{p^{EV'^u}}{r^{EV'^u}} \frac{A^u_A}{1+\rho} - \frac{p^{EV'^{w^*}}}{r^{EV'^{w*}}} \frac{A^{w^*}_A}{1+\rho}.$$

Since we established that $\frac{p^{U^u}}{r^{U^u}} = \frac{p^{U^{w^*}}}{r^{U^{w*}}} = \frac{\gamma+1}{\gamma}$ for the CRRA/homogeneous utility function and $C^u_A \gtreqless C^{w^*}_A$ and $r^{V^u} \gtreqless r^{V^{w*}}$, if $\lambda \gtreqless \pi$, then $\frac{p^{EV'^u}}{r^{EV'^u}} A^u_A \gtreqless \frac{p^{EV'^{w^*}}}{r^{EV'^{w*}}} A^{w^*}_A$ is a sufficient condition for Property 1, that $\frac{p^{V^u}}{r^{V^u}} \gtreqless \frac{p^{V^{w*}}}{r^{V^{w*}}}$, if $\lambda \gtreqless \pi$.

From the derivation of the consumption function's curvature, we have the identity:

$$\frac{-C^i_{AA}}{C^i_A} = r_{V^i} \left(\frac{p^{EV'^i}}{r^{EV'^i}} - \frac{p^{U^i}}{r^{Ui}} \right) \frac{A^i_A}{1+\rho}, \text{ for } i = u, e.$$

If $\lambda \gtreqless \pi$, given that $r^{V^u} \gtreqless r^{V^{\widehat{w}}}$, then $\frac{p^{EV'^u}}{r^{EV'^u}} A^u_A \gtreqless \frac{p^{EV'^{w^*}}}{r^{EV'^{w*}}} A^{w^*}_A$ is a sufficient condition for $\frac{-C^u_{AA}}{C^u_A} \gtreqless \frac{-C^{\widehat{w}}_{AA}}{C^{\widehat{w}}_A}$. Then, since $C^u_A \gtreqless C^{\widehat{w}}_A$, if $\lambda \gtreqless \pi$, it has to be true that $C^{\widehat{w}}_{AA} \gtreqless C^u_{AA}$, if $\lambda \gtreqless \pi$, which proves Property 2 ■

Proof for 3.4.1. Taking the derivatives of $w^*_A = \frac{V^u_A - V^{w^*}_A}{V^{w^*}_w}$ over wealth A:

$$\begin{aligned} w^*_{AA} &= \frac{V^u_{AA} - V^{w^*}_{AA} - 2V^{w^*}_{Aw} w^*_A - V^{w^*}_{ww} \left(w^*_A\right)^2}{V^{w^*}_w}, \\ &= \left(\frac{V^u_{AA}}{V^u_A} - \frac{V^{w^*}_{AA} + 2V^{w^*}_{Aw} w^*_A + V^{w^*}_{ww} \left(w^*_A\right)^2}{V^{w^*}_A + w^*_A V^{w^*}_w} \right) \frac{V^u_A}{V^{w^*}_w}, \\ &= \left(\overline{r}^{V^{Aw*}} - r^{V^u} \right) \frac{V^u_A}{V^{w^*}_w}. \end{aligned}$$

where $\overline{\overline{r}}^{V^{Aw*}} = \overline{r}^{V^{Aw*}} \alpha + \overline{r}^{V^{ww*}} (1-\alpha)$, $\overline{r}^{V^{Aw*}} = -\frac{V^{w^*}_{AA} + V^{w^*}_{Aw} w^*_A}{V^{w^*}_A}$, $\overline{r}^{V^{ww*}} = -\frac{V^{w^*}_{Aw} + V^{w^*}_{ww} w^*_A}{V^{w^*}_w}$, and $\alpha = \frac{V^{w^*}_A}{V^{w^*}_A + w^*_A V^{w^*}_w}$, and $\frac{V^u_A}{V^{w^*}_w} > 0$ ■

Proof of Theorem 3.4.2. Property 1. Same proof as Theorem 2.7.3, but with Theorem 3.3.1.
Property 2. Taking the derivative of $\widehat{w}_A$,,in a similar way as in the proof of Theorem

3.4.1, we obtain $\widehat{w}_{AA} = \left(\overline{\overline{p}}^{V^{A\widehat{w}}} - p^{V^u}\right)\frac{V^u_{AA}}{V^{\widehat{w}}_{Aw}}$, where $\overline{\overline{p}}^{V^{A\widehat{w}}} = \overline{p}^{V^{A\widehat{w}}}\alpha + \overline{p}^{V^{\widehat{w}\widehat{w}}}(1-\alpha)$, $\overline{p}^{V^{A\widehat{w}}} = -\frac{V^{\widehat{w}}_{AAA}+V^{\widehat{w}}_{AAw}\widehat{w}_A}{V^{\widehat{w}}_{AA}}$, $\overline{p}^{V^{\widehat{w}\widehat{w}}} = -\frac{V^{\widehat{w}}_{AAw}+V^{\widehat{w}}_{Aww}\widehat{w}_A}{V^{\widehat{w}}_{AAw}}$, and $\alpha = \frac{V^{\widehat{w}}_{AA}}{V^{\widehat{w}}_{AA}+\widehat{w}_A V^{\widehat{w}}_{Aw}}$, and $\frac{V^u_{AA}}{V^{\widehat{w}}_{Aw}} > 0$ ■

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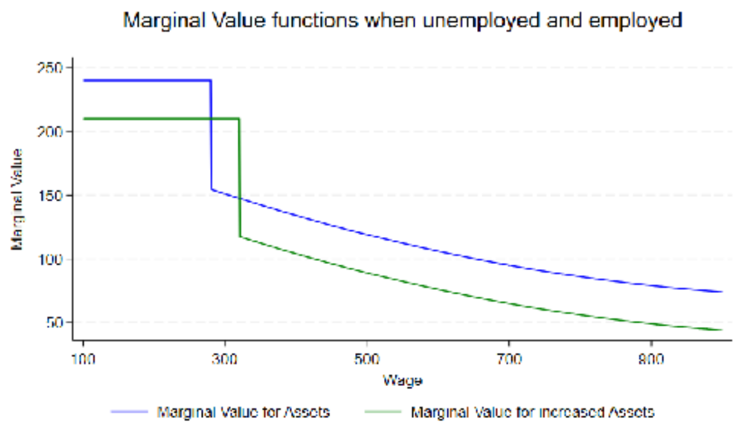

Figure 1. Marginal Value functions while unemployed and employed

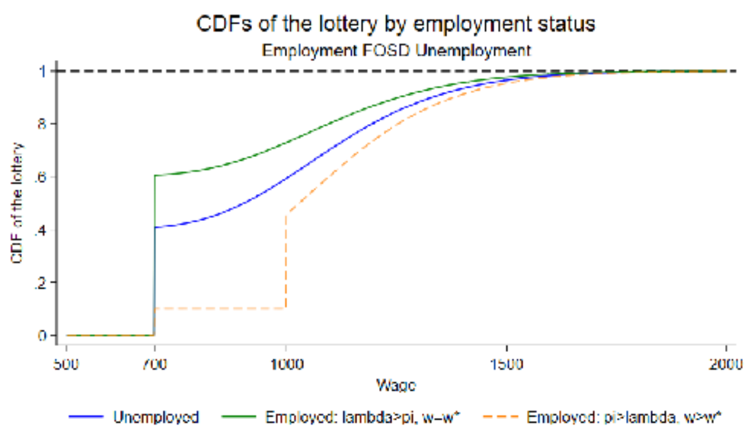

Figure 2. Two cases of First Order Stochastic Dominance: 1. unemployment dominates if $\lambda > \pi$, $w = w^*$ and 2. employment dominates if $\pi > \lambda$, $1 - \lambda > \theta$, and $w > w^*$.

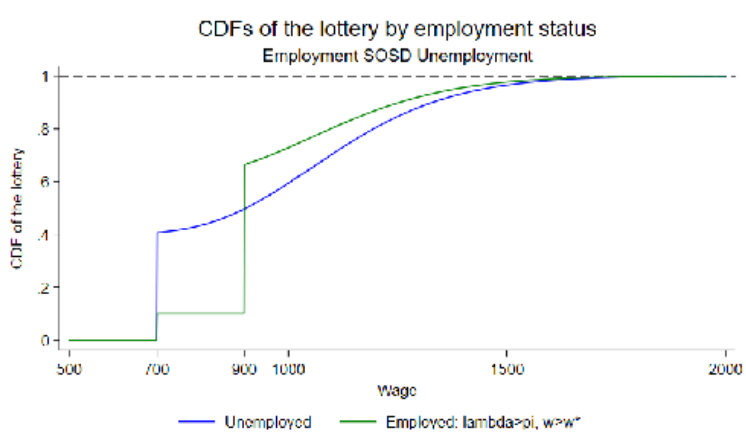

Figure 3. First Order Stochastic Dominance does not occur if $\lambda > \pi$, $w > w^*$, but Employment may Second Order Stochastically Dominate Unemployment if w is high enough.