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ABSTRACT

Economics for a Safe Operating Space: A Green-Growth-Degrowth Model

We present a model of economic growth that bridges Green Growth and Degrowth perspectives. The model demonstrates that a minimum physical per capita consumption level can be maintained without recourse to tech-optimism, and moreover with degrowth in material resource throughput - respecting planetary boundaries. We critically discuss the assumptions necessary for this result, explore relaxing these, and illustrate that eventually a full transition to renewable energy and materials will be needed to sustain consumption levels in a post-growth economy. We identify areas for future growth modelling, emphasising that these require genuine interdisciplinary cooperation.

JEL Classification: O44, Q32, Q56, Q55

Keywords: economic growth, degrowth, post-growth, green growth, sustainability

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1 Introduction

The pursuit and possibility of continued growth of the world economy has become a contentious issue, particularly so after the Limits to Growth report of the 1970s (Meadows et al., 1972) and rising concerns about the impacts of climate change in more recent times (Naudé, 2023; Susskind, 2024). It is widely recognized that the extraordinary run of exponential economic growth that the world has experienced since the early 1800s, and which accelerated since the 1950s, has brought unprecedented wealth, prosperity and wellbeing to humanity generally - even if not equally across and within countries (Syvitski et al., 2020; Steffen et al., 2015a).

It is also recognized that economic growth has come at a price in terms of causing an ecological overshoot, which is "when the consumption of bio-resources and the production of wastes exceed the regenerative and assimilative capacities respectively, of supportive ecosystems" (Rees, 2022, p.2262). Ecological overshoot threatens transgression of various so-called *Planetary Boundaries* (Steffen et al., 2015b) - an estimated six out of nine planetary boundaries have been breached by 2023 (Richardson et al., 2023). According to Bradshaw et al. (2021, p.3) the global economy consumed 170% of the planet's "regenerative capacity" already in 2016. A symptom of ecological overshoot is climate change, which according to many poses a catastrophic threat to humanity (Bradshaw et al., 2021).

Some have argued that this means that economic growth now results in more damages than benefits and that the world need to move to a post-growth economy (Jackson, 2009; Spash, 2015). In this regard some advocate for a deliberate downscaling of the size of the global economy, through degrowth (Hickel et al., 2022). Mainstream economics however - and the vast majority of policymakers in the world - are less pessimistic about the continuation of, and the need, for economic growth. This view, however, relies on Romer (1990, 1986) wherein technological progress is the fundamental determinant of economic growth. Because technological progress is based on ideas, which are non-rival thus generating increasing returns to scale, and that can be joined in an infinite number of combinations, the belief is that world will never run out of ideas, and hence never run out of economic growth. All that government needs to do, as the growth modelling of e.g. Acemoglu et al. (2012) and Acemoglu et al. (2016) conclude, is to direct technological change towards clean technologies through an optimal policy of carbon taxes, and subsidies for research and development (R&D) in clean technologies.

Rather than argue for a continuation exponential economic growth as per the mainstream tech-optimist approach, or the opposite, the planned degrowth of the world economy, some have in recent years argued the case for a different *type* of economic growth, namely growth within a "safe operating space for humanity." This is growth that respects Planetary Boundaries (Rockström et al., 2009). In this approach, Planetary Boundaries do not rule out growth, nor require degrowth: in fact it has been argued that within a safe operating space growth that promotes and ensures basic social and material needs of humanity - i.e. a sustainable and adequate *physical* consumption level - is even desirable (Raworth, 2017).

In this paper we ask whether and physical consumption and GDP growth within a safe operating space is possible? What assumptions are crucial to allow this? To answer these questions we provide a simple green-growth-degrowth model of a sustainable economy that functions within the safe operating space of humanity, which here is modelled as an economy with degrowth in material throughput. We bridge the perspectives of green growth and degrowth by breaking with traditional green growth assumptions by including finite material resources and not making the assumption that all resources can be perfectly substituted. We also break with the tech-optimism of mainstream growth modelling.

We also depart from the degrowth/ post-growth and ecological economics' rather strict view that further growth is *per se* undesirable, and that technology does not matter for economic growth which is argued to only depend on "energy, materials, and human labour" (Kallis et al., 2025, p.e65). Thus while we break with tech-optimism, we do also consider how the realisation of appropriate technological innovations can make a difference. But given the Degrowth movement's distrust that appropriate technological innovations will be made, we also depart from tech-optimism.

Furthermore, whereas the ideas proposed by Degrowth scholars have not yet been presented in an internally consistent mathematical growth model, our model incorporate some critical degrowth insights in a mathematically rigorous, and hence internally consistent, model. A further contribution of this paper is therefore an attempt to initiate the formalization of Degrowth theory.

In the process, our proposed model of (consumption) growth within the safe operating space of humanity can be described as a *green-growth-degrowth model.* It is an approach that bridges the green growth - degrowth paradigms which are typically presented as mutually exclusive.

Through the approach that we take in this paper we wish to promote interdisciplinary communication. For this we follow the broad approach of Bretschger and Karydas (2019) and stick to relatively simple descriptive modelling of the key macroeconomic dynamic processes to facilitate understanding of our economic reasoning for a broader audience also beyond economics. As Bretschger and Karydas (2019, p.561) pointed out with reference to economic modelling of climate change "contributions have become very technical and quite specialized; for a broader audience it is often difficult to get an overview." We hope that through this paper a broader audience will be able to get an overview of the key challenges in modelling economic growth and bridging different perspectives on the assumptions critical for the results emanating from such modelling.

The rest of the paper proceeds as follows. The core of the paper is section 2, where we present a simple model of economic growth with finite material resources, and use it to analyse three scenarios. In section 3 we relax the assumption of infinity divisible material resources and introduce the use of renewable resources. Section 4 contains a discussion of key questions that future interdisciplinary work must address. Section 5 concludes.

2 Economic Growth within the Safe Operating Space

In this section we present an economic growth model where material resources are finite, and use it to analyse three scenarios: (i) capital accumulation (section 2.2) that substitutes for material resource depletion, (ii) continuous technological progress (section 2.3) that improves resource efficiency and recycling, and a (iii) combination of capital accumulation and continuous technological. We show that the latter scenario is, subject to the assumptions, consistent with sustainable consumption growth and degrowth in material resource use (section 2.4). In section 3 we will discuss the critical assumptions, one of which is the infinite diminishing use of physical resources - the relaxation of which brings to the fore the need for a full transition to renewable resources.

First though, we define our notion of a safe operating space economy for purposes of this paper.

2.1 Definition of a Safe Operating Space Economy

A Safe Operating Space economy is one wherein economic growth respect planetary boundaries, as defined and described by Rockström et al. (2009). A Safe Operating Space economy is a sustainable economy in the sense that it can provide at least a certain minimum comfortable per capita consumption over an infinite time horizon. This economy can either be stationary, or growing in per capita consumption, but must not fall below the defined minimum level of comfort without exceeding previously defined requirements for the preservation of planetary systems, i.e., maintaining them in the long term. Formally, in this approach a *sustainable economy* is defined by three elements.

The first element is that for an infinite time horizon the economy must not fall below a comfortable minimum level of physical consumption, i.e.:

$$c(t) = \frac{C(t)}{L(t)} \geqq c_{\min}, \quad \text{for } t = 0, .., \infty \tag{1}$$

The second condition is that a *sustainable growth path* of an economy must be non-negative, i.e.:

$$g_c(t) = \frac{\dot{c}(t)}{c(t)} \geqq 0, \quad \text{as systematic trend} \tag{2}$$

Equations (1) and (2) define the term sustainable economy and sustainable growth path with respect to the outcome for humans. The definition requires a decent, non-declining, level of human wellbeing, proxied by physical (and not monetary) consumption, for an "infinitely" long-living human species[1].

The third condition is the *Resource Conservation* condition which relates to the state of a planetary resource. A planetary resource is a finite resource that humans affect through their (economic) activities. In a simple case, this can be a resource stock such as oil reserves or a certain material, or the basis of a regeneration process for a renewable resource. Even more generally, it can also be a global or local ecological system - a planetary boundary - that needs to be maintained at a particular state or range. This state is defined as the

[1]A note on the use of the word "infinite" in this paper is in order. We do not believe that the human species will be infinitely long-lived, at least not on planet Earth, given the finite lifetime of the Sun and neither elsewhere, given the eventual heat-death of the universe. Some authors, such as Gee (2025) in fact reckon that humans will likely be extinct in around 10,000 years. From the current vantage point though,10,000 years may perhaps be treated in our stylized model as an"infinite" period.

permanently maintained state $S^{boundary}$. It is not necessarily the initial state or natural state $S^{natural}$, but rather the state that must be maintained at a minimum to prevent triggering ecological tipping points. Therefore, all states between the natural state and the state that must be maintained are fundamentally feasible. E.g. for a natural stock of a finite material resource $S^{natural}$ this means that the available stock for human uses $\bar{S}$ is

$$\bar{S} = S^{natural} - S^{boundary},$$

and all extraction from now until infinity must not exceed this stock

$$\bar{S} = \int_{t=0}^{t=\infty} R_t^{ex} dt, \tag{3}$$

with R_t^{ex} denoting the extracted material resource flow in period t.

Having thus defined the sustainable economy in the context of planetary boundaries, we turn now to modelling three scenarios.

2.2 Scenario 1: Capital Accumulation, Recycling and Sustainability

In this sub-section we outline model of economic growth with finite material resources, where capital accumulation substitutes for material resource depletion, under the assumption that there is no technological progress or population growth. This scenario corresponds to a world where innovation and technological progress has stagnated, perhaps due to a declining and ageing in population, as for instance in Jones (2022). It also corresponds to Degrowth's belief that technology offers no solution to the ecological predicament facing humanity with only "energy, materials, and human labour" (Kallis et al., 2025, p.e65) determining economic growth. The modelling in this sub-section refers back to chapter 7 of the classic book by Dasgupta and Heal (1979).

The order in this sub-section is as follows. First we outline the model set-up, and then we find a solution to the model from which we derive six growth paths for the economy with pure capital accumulation. We also represent these growth paths graphically in Figure 1.

2.2.1 Model set-up

In terms of equations (1)-(3) a sustainable (Safe Operating Space) economy is defined as an economy that provides for a physical consumption level per capita above a defined minimum level. Without population growth this requires at least a fixed consumption per capita $\frac{\bar{C}}{L} \geqq c_{\min}$.

Production: We start off by describing how this economy produces its output. Total production, or GDP, is denoted by Q_t, and is assumed to be the result of using three production factors, in line with the Degrowth movement's belief that only "energy, materials, and human labour" (Kallis et al., 2025, p.e65) determines GDP. The three production factors in our model corresponding to these are labor L, and energy and materials represented by capital K_t and energy and other material resource input flows, denoted by R_t.

All production factors including the natural resource are essential, as $R = 0$, implies $Q = 0$. Thus, economic growth depends on some availability of material resource inputs - there is no full decoupling possible, as the Degrowth movement insists (e.g. Kallis et al. (2025); Hickel (2015)). Furthermore, it is assumed that the total population is available as labor, and that total labor is only used in production $L = L_Q$. Hence, production (GDP) is given by

$$Q_t = \theta K_t^{\alpha} R_t^{\beta} L_Q^{\gamma}, \text{ with } \alpha + \beta + \gamma = 1, \tag{4}$$

where θ is a given total factor productivity.

The production function in (4) is a Cobb-Douglas production function, which is also used by Bretschger and Karydas (2019) in their basic climate model. The Cobb Douglas production function is a special case of the the much used Constant Elasticity of Substitution (CES) production function:

$$Q = A\left(\alpha K^{-\frac{1-\sigma}{\sigma}} + \beta R^{-\frac{1-\sigma}{\sigma}} + \gamma L_Q^{-\frac{1-\sigma}{\sigma}}\right)^{-\frac{\sigma}{1-\sigma}}$$

The CES-function boils down to a Cobb-Douglas function if $\sigma = 1$.We use the Cobb-Douglas function because we wish to break with the standard Green Growth modelling assumption of perfect substitutability between inputs, which is operationalised by assuming that $\sigma > 1$.This means that the decreasing availability of a factor is not a problem. The economy can always be operated sustainably with the production factors assumed here, even if production would take place without the finite resource. The sustainability problem posed above would already be solved with this assumption. This is a fundamental support for the mainstream Green Growth belief that infinite growth is possible.

As far back as 1974 Nordhaus and Tobin (1974, p.522) stated in this regard, "reproducible capital is a near-perfect substitute for land and other exhaustible resources." And more recently Aghion et al. (2025) continued in this vein, assuming the substitution between material and service inputs in production to have a $\sigma > 1$ which allows them to show that economic growth could continue without environmental damage by just substituting the material inputs by services, arguing that this entails a shift from "quantity" to "quality" inputs in production.

By rejecting this standard assumption in mainstream growth modelling, and thus rather using a Cobb-Douglas production function we therefore move closer to the Degrowth perspective that there is no perfect substitutability between resources. However, in a strict Degrowth world, the view is that a minimum input level of resource inputs, material throughput, is required for each specific output level - there is no complete decoupling possible. If the resource becomes scarce the output level must decline. The economy cannot be sustainable. Degrowth is the only feasible outcome. In the context of the CES production function the Degrowth perspective implies the assumption that $\sigma < 1$.

Hence, by assuming that $\sigma = 1$, production in our economy takes place in the middle ground between Green Growth and Degrowth. Resources are essential, but to a limited extent some substitution is possible. Future research could clarify whether this is a reasonable compromise or not.

Since this is a descriptive model we do not need to take care of optimal choices of private consumers or firms in markets. We can just describe paths of the economy and analize the effects of certain parameter changes. However, part of this description is that we consider the price path for resource use as if private firms had to make allocative decisions as in a market economy.

In such a case we can derive the optimal factor allocations by firms as the result of minimizing costs:

$$\min_{K,R,L} : Costs = rK + p_R R + wL_Q - \lambda \left(\theta K_t^{\alpha} R_t^{\beta} L_Q^{\gamma} - \bar{Q} \right)$$

Cost minimization will lead to an relation between an optimal resource (to capital) intensity and resource input the relative (to interest rates) price of the resource

$$\frac{R_t}{K_t} = \frac{\beta}{\alpha} \frac{r}{p_{Rt}}, \quad \text{with } \frac{d\frac{R_t}{K_t}}{dp_{Rt}} = -\frac{\beta}{\alpha} \frac{r}{p_{Rt}^2} < 0. \tag{5}$$

Equation (4) implies that the price path of the material resource determines firms resource use - an increasing price of the resource will lead to a decline in material resource intensity. That is, in a market economy, prices are an important signal and control instrument for profit oriented companies regarding resource use. Price paths incentivize private companies. We therefore describe the relationship between resource price developments and the resulting resource use. Therefore, price developments can be used as a key instrument for decentralized resource consumption control in companies. It can encourage companies to adopt sustainable behaviour. The assumption of course is that the market will work adequately to accurately price material resources. In section 4 below we point out that relaxing this implicit assumption remains a challenge for future research and policy making.

Technology: As mentioned, in this scenario we assume that technological innovation has stagnated and cannot be relied on to generate sustainable growth. To make this explicit in the model, we describe technology θ as an index of total factor productivity, which, is here assumed to be constant. Formally this means that

$$g_\theta = \frac{\dot{\theta}_t}{\theta_t} = 0 \tag{6}$$

Capital: In this scenario, our economy is characterised by a financial system that enables a fixed value for savings to be perfectly channelled into investments $\bar{I}$ and capital accumulation, thus

$$\dot{K}_t = \bar{I}. \tag{7}$$

Note, that this capital accumulation is not specified in a particular way. However, even if it is unspecified, the implicit assumption is that this kind of capital can indeed substitute for resource inputs. To the extent that it substitutes for finite material resource inputs it can be seen as "green capital".

Resource use and resource stock dynamics: A key feature of our model which is also different from Green Growth models, is our incorporation of finite material resources, again reflecting the Degrowth movement's belief that we live on a finite planet.

Hence, every extracted material resource R_t^{ex} reduces the material resource stock available in future, $\dot{S}_t = -R_t^{ex}$.

Finite material resources include fossil fuel energy sources (e.g. coal. gas and oil) and non-fossil fuel material inputs such as minerals or other non-renewable inputs. In case of such material inputs, the importance of the circular economy has been recognised (Bauwens et al., 2020; Blum et al., 2020). We therefore include elements of the circular economy in our model.

We do so assuming that an already extracted and used resource can be recycled at the recycling rate ρ. If the recycling rate $\rho < 1$, the circular economy is not perfect, reflecting the physical reality that due to thermodynamics, a used resource can be recycled only at a certain percentage rate $\rho < 1$, never perfectly $\rho = 1$. Since we do not assume any technological advances in recycling technology, the recycling rate remains constant. With recycling, the finite material stock is reduced by the extent of the non-recycled loss of the resource, hence total material resource extraction is given by

$$R_t^{ex} = (1 - \rho) R_t \tag{8}$$

$$\dot{S}_t = -R_t^{ex} \tag{9}$$

So as not to move outside of the Safe Operating Space, sustainability requires that material extraction in our model must not fully exhaust the resource stock during the existence of humanity. In fact, this sustainability condition is already discussed in section 2.1. Thus, we have to ensure that this finite stock of material resources, S, will not be used up - it can only be completely exhausted (i.e. up to the $\bar{S}$) after an infinite period of extraction:

$$\bar{S} = \int_{t=0}^{t=\infty} (1-\rho) R_t dt \tag{10}$$

This condition also means that the absolute limit of an exhaustible resource does not logically imply a final boundary of the use of this material resource for economic purposes - before infinity is reached. Otherwise, while the economy will remain within the Safe Operating Space the finite planet provides, consumption levels cannot be maintained, and material wellbeing and progress will be jeopardised.

2.2.2 Solving the model

We can now solve for the model set-up in equations (3) to (9).

Sustainable consumption: The key question is now is this: Is the material resource extraction rate necessary to ensure a required minimum level of per capita consumption $c_{\min} < \bar{c} = \bar{C}/L_Q$ consistent with the stocks of material resources left after use, as per (3)?

To answer this question we determine the material resource use for a given level of production $\bar{Q}$, as

$$R_t = \bar{Q}^{\frac{1}{\beta}} K_t^{-\frac{\alpha}{\beta}} L_Q^{-\frac{\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \tag{11}$$

If output (GDP) is used only for consumption and investment $[Q = C + I]$, and the capital stock is accumulated according to (7) we can determine the material resource input or throughput that would be required for the desired per capita consumption level:

$$R_t = \left[\frac{\bar{C} + I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left[K_0 + It\right]^{-\frac{\alpha}{\beta}} \tag{12}$$

Recycling enables the economy to extract less than the resource use, such that (8,9) and (12) lead to the resource extraction R_t^{ex}. Hence the reduction of the resource stock is

$$\dot{S}_t = -R_t^{ex} = -\left(1-\rho\right)\left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left[K_0 + It\right]^{-\frac{\alpha}{\beta}} \tag{13}$$

To evaluate whether this material resource exploitation path allows (3) respectively (10) to be met, we plug (13) into (10) to determine if

$$\bar{S} = \left(1-\rho\right)\int_{t=0}^{t=\infty}\left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left[K_0 + It\right]^{-\frac{\alpha}{\beta}} dt \tag{14}$$

exists.

After integration we find that (14) only exists if $\alpha > \beta$;[2] that is, the contribution of capital to the production process is larger than the contribution of the material resource. After rearranging we can derive per capita production and per capita consumption as

$$\frac{\bar{Q}}{L_Q} = \left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta} \theta L_Q^{\beta+\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}\left[K_0\right]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta} \text{ and} \tag{15}$$

$$\frac{\bar{C}}{L_Q} = \theta\left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta} L_Q^{\beta+\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}\left[K_0\right]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta} - \frac{I}{L_Q}. \tag{16}$$

Equations (15) and (16) indicate that the economy is only sustainable if it can use capital accumulation to substitute for finite material resources, which in our modelling here is possible, but limited, and not a near-perfect substitute as assumed in mainstream modelling (see e.g. Nordhaus and Tobin (1974)).

[2]For details see the Appendix A.1.

Maximum sustainable paths of variables: In the Appendix A.1 we derive for the maximum sustainable consumption $\bar{C}^*$, the respective required investment I, GDP, $\bar{Q}^*$, and the paths for capital accumulation K_t^* and resource extraction R_t^*, as follows:

$$\begin{aligned} I^* &= \beta^{\frac{1}{1-\beta}} \theta^{\frac{1}{1-\beta}} \left(\frac{\bar{S}}{(1-\rho)} \right)^{\frac{\beta}{1-\beta}} L_Q^{\frac{\gamma}{1-\beta}} \left(\frac{\alpha-\beta}{\beta} \right)^{\frac{\beta}{1-\beta}} [K_0]^{\frac{\alpha-\beta}{1-\beta}}, \quad (17) \\ \bar{C}^* &= \theta \left(\frac{\bar{S}}{(1-\rho)} \right)^{\beta} L_Q^{\beta+\gamma} \left(\frac{\alpha-\beta}{\beta} \right)^{\beta} [K_0]^{\alpha-\beta} \left(\frac{I^*}{L_Q} \right)^{\beta} - I^*, \quad (18) \\ \bar{Q}^* &= \bar{C}^* + I^* \quad (19) \\ K_t^* &= K_0 + I^* t. \quad (20) \end{aligned}$$

From the production function and knowing $\bar{C}^*$, I^*, K_t^* and $\bar{Q}^*$ the path of sustainable resource extraction that maximises consumption is:

$$\begin{aligned} R_t^* &= \left[\bar{C}^* + I^* \right]^{\frac{1}{\beta}} L_Q^{\frac{-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} [K_0 + I^* t]^{-\frac{\alpha}{\beta}}, \quad \text{with } \lim_{t \to \infty} R_t^* = 0 \quad (21) \\ \dot{R}_t^* &= -\frac{\alpha}{\beta} \left[\bar{C}^* + I^* \right]^{\frac{1}{\beta}} L_Q^{\frac{-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} [K_0 + I^* t]^{-\frac{\alpha}{\beta}-1} I^* < 0, \end{aligned}$$

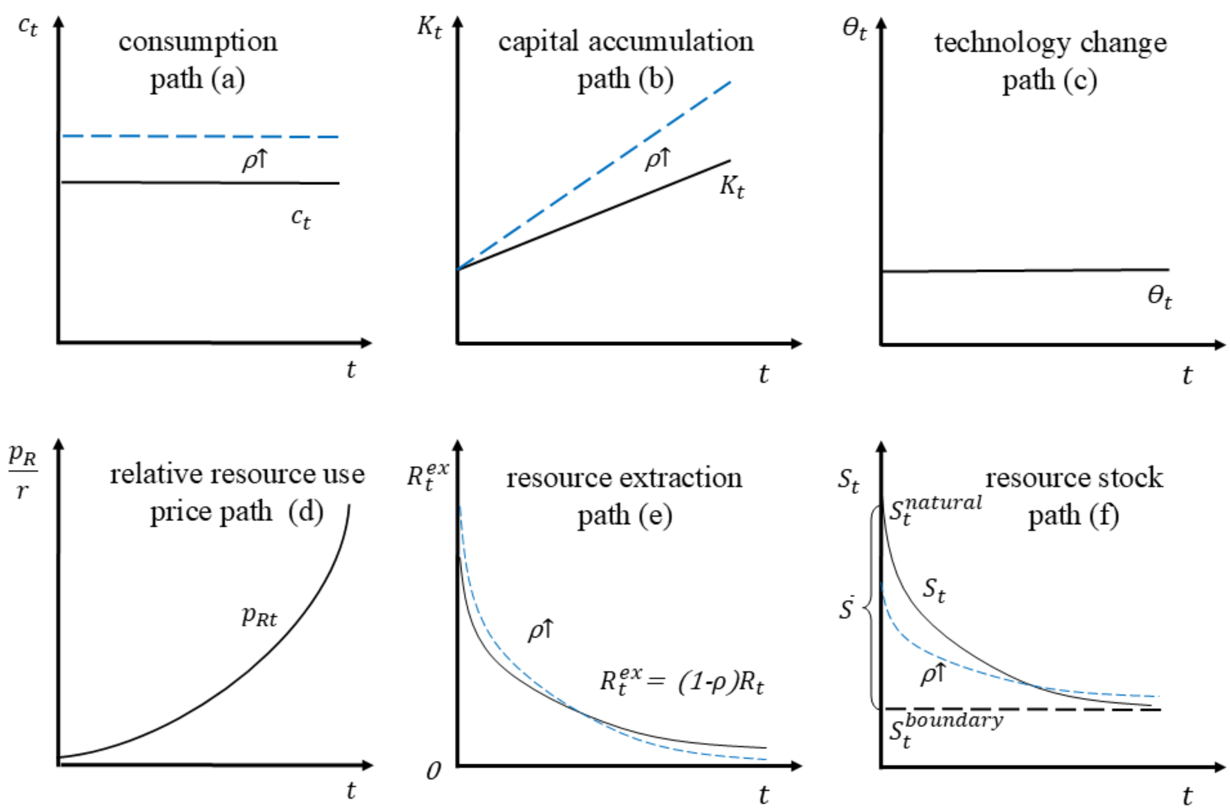
Equation (21) indicates that material resource extraction R_t^* in our economy will permanently decline - i.e. degrow. Degrowth in material resource throughput and extraction is therefore compatible with maintaining a minimum required level of consumption.

The decline in material resource extraction occurs due to the substitution of the latter by capital accumulation. For capital accumulation to achieve this in our market economy, the *finite material resource price* will need to permanently increase so as to provide a signal to producers to continuously substitute the material resource by capital and production remains sustainable. This increasing price path is given by:

$$\frac{p_{Rt}^*}{r_t} = L_Q^{\frac{\gamma}{\beta}} \theta^{\frac{1}{\beta}} \left(I^*\right)^{\alpha} \frac{\beta \left[K_0/I^* + t\right]^{\frac{\beta+\alpha}{\beta}}}{\alpha \left[\bar{C}/I^* + 1\right]^{\frac{1}{\beta}}}, \quad \lim_{t\to\infty} \frac{p_{Rt}}{r_t} = \infty \tag{22}$$

All three sustainable growth paths - in consumption, capital accumulation and material resource use - are depicted in Figure 1, panels a,b,c.

Figure 1: Paths of a sustainable economy with pure capital accumulation



Source: Authors' compilation

Figure 1 (a) shows that consumption in this scenario is flat, i.e., the minimum level of consumption required to maintain human well-being is achieved and does also not increase further. This scenario therefore represents a post-growth, stationary economy with constant consumption per capita. This is achieved through investment and capital accumulation (b), even though material throughput, resource extraction may decline in line with panel (e). Stocks will then decline continuously (f), but never fall below the level required to maintain stocks $S^{maintain}$, which was determined in the discussion as necessary for sustainability or a safe margin for action. Panel (d) additionally describes the relative resource price path that would be required to incentivize and implement this process in a decentralized market economy with profit maximizing firms.

In Appendix A.1 we also show how these paths change if we increase the recycling rate (dashed lines). Recycling (modelled here as costless) leads to a quasi factor augmenting effect of the total resource. In Figure 1 the sustainable consumption and investment path will be higher.

Maximum sustainable intertemporal welfare: In this variation of this first scenario we would like to explore issues of intertemporal valuation and allocation. To do this we start with the simplest case to our current model. Our economy still strives to provide a comfortable minimum consumption level each period $\bar{C}_t$. However, we now consider the effects of discounting future consumption at rate δ.

Now the objective of society can be written as maximizing of the total value of discounted sustainable consumption[3]

$$\begin{aligned} \max_{I} \quad &: \quad V = \int_0^\infty = e^{-\delta t}\bar{C}_t dt \\ \text{with} \quad \bar{C} \quad &= \quad \theta\left(\frac{\bar{S}}{(1-\rho)}\right)^\beta \left(\frac{\alpha-\beta}{\beta}\right)^\beta [K_0]^{\alpha-\beta}(I)^\beta - I \end{aligned}$$

[3]For simplicity we assume that $L_Q = L = 1$.

The welfare maximizing values for $\bar{C}^*$ and I^* are [4]

$$\begin{aligned} I^* &= \beta^{\frac{1}{1-\beta}} \theta^{\frac{1}{1-\beta}} \left(\frac{\bar{S}}{(1-\rho)} \right)^{\frac{\beta}{1-\beta}} \left(\frac{\alpha-\beta}{\beta} \right)^{\frac{\beta}{1-\beta}} [K_0]^{\frac{\alpha-\beta}{1-\beta}} \\ \bar{C}^* &= \theta \left(\frac{\bar{S}}{(1-\rho)} \right)^{\beta} \left(\frac{\alpha-\beta}{\beta} \right)^{\beta} [K_0]^{\alpha-\beta} (I^*)^{\beta} - I^* \end{aligned}$$

What does this mean? For this modelling of an intertemporal welfare it it trivial because by construction we forced the consumption path to a constant minimum sustainable level of consumption that we discussed in section 2.1. However, our modelling here is not the standard modelling, at least not in mainstream economic theory. In mainstream economic growth theory the dominating - and often only - approach for intertemporal modelling is

$$\max : V = \int_0^{\infty} e^{-\delta t} U(C_t) \, dt.$$

The choice of this approach is not trivial, as it implies two values that are usually not questioned. The first is the assumption of an infinite time horizon. From a sustainability perspective, this first assumption, which describes a maximum time horizon and thus appears to take all generations into account for all eternity, seems very positive. The second assumption, however, the assumption of discounting, can effectively negate this first idea. If we discount the consumption to be enjoyed by future generations the implication is that the well-being of a person in the future is less important to society today than the well-being of a person today.

Let us take, for example, a discount rate of $\delta = 5\%$ and a consumption of $\bar{C}$: Today, at time 0, we assume that this consumption generates a utility value of $V_0 = U_0 = U(\bar{C}_0) = 10$. Similarly, for a person in 100 years, this level of consumption will have a utility value of $U_{100} = U(\bar{C}_{100}) = 10$. However, according to our intertemporal valuation and discounting

[4] See the appendix for this section.

with $\delta = 5\%$, this same consumption and the same utility for future generations would only have a value of $V_{100} = e^{-0.1*100} U \left(\bar{C}_{100}\right) = 0.07$ based on today's valuation. The future consumption of future generations therefore no longer has any value in this approach.

This means that in effect, mainstream economic growth models using discounting do not take the wellbeing of future generations into account. Because such discounting of future generations wellbeing have a drastic impact on today's decisions, the time horizon to be used in modelling, and the use and extent of discounting should be seen as value judgments, which requires an interdisciplinary and intercultural discussion. In section 4 we return to this issue.

2.3 Scenario 2: Technological Innovation and Sustainability

In this sub-section, we model technological progress as the only mechanism for ensuring a sustainable economy when there are finite material resources. As in the previous case of (isolated) capital accumulation, this is a challenge, as once again a single mechanism is supposed to bring about sustainability. However, we will illustrate that there are conditions such that a sustainable economy - meeting a minimum level of consumption over time, can be possible with continuous technological change.

As before, we will first outline the model set-up, after which we will solve the model for the growth paths of the variables of concern.

2.3.1 Model set-up

Production: Production again follows a Cobb-Douglas production function, such that resources are again essential

$$Q_t = \theta_t K_t^{\alpha} R_t^{\beta} L_Q^{\gamma}.$$

Given that we are now interested in technology, as per the tenets of Green Growth models wherein green technological innovations are crucial to deliver green and clean technologies for growth, e.g. as in Acemoglu and Autor (2012), someone has to perform the innovation (e.g. through R&D). Hence labor can now be allocated either to production L_Q or to R&D to discover new ideas, L_θ.

With shares l_Q and l_θ we obtain for the allocation of labor:

$$L = L_Q + L_\theta = l_Q L + l_\theta L = 1. \tag{23}$$

Firms minimize costs by an optimal choice of input factors:

$$\min_{K,R,L_Q} : Costs = rK_0 + p_R R_t - \lambda \left(\theta_t K_0^\alpha R_t^\beta L_Q^\gamma - \bar{Q} \right).$$

Cost minimizing input allocation and an optimal resource (to capital) intensity would go along with a relative (to interest rates) price of the resource:

$$\frac{R_t}{K_0} = \frac{\beta}{\alpha} \frac{r}{p_{Rt}}, \quad \text{with } \frac{d\frac{R_t}{K_0}}{dp_{Rt}} = -\frac{\beta}{\alpha} \frac{r}{p_{Rt}^2} < 0$$

Capital: We considered capital accumulation in the previous sub-section 2.2. As we wish to isolate the technology change mechanism for sustainable growth, we assume for present purposes that there is no capital accumulation, and hence that all production is used for consumption

$$\dot{K}_t = I_t = 0. \tag{24}$$

Thus, the capital stock is fixed at a given level ($K_t = K_0$).

Technology: We introduce technological change by assuming that some labor L_θ is engaged in the innovation sector in doing R&D with productivity a, generating a fixed number of new technologies in each period

$$\dot{\theta}_t = aL_\theta \tag{25}$$

Resource use and resource stock dynamics: The finite material resource dynamics is identical to (9) when recycling is now dropped for simplicity, and hence

$$\dot{S}_t = -R_t$$

As we have already seen in (3), the economy is sustainable if material resources are not fully exhausted within a finite time period, but lasts until infinity

$$\bar{S} = \int_{t=0}^{t=\infty} R_t dt$$

Since we have already discussed the principal effects of recycling in the previous section and in order to simplify the representation of the model as much as possible, we will omit recycling from now on.

2.3.2 Solving the model

With no capital accumulation $\bar{C} = \bar{Q}$. As before we first determine the finite material resources use path

$$R_t = \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} L^{-\frac{\gamma}{\beta}} l_Q^{-\frac{\gamma}{\beta}} l_\theta^{-\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{-\frac{1}{\beta}} \tag{26}$$

The central question again is whether the finite material resource extraction described in

(26) allows for sustainable finite material resource use (3) to be achieved. To see whether this is the case we plug (26) into (3) to check if

$$\bar{S} = \int_0^{\infty} \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} L^{-\frac{\gamma}{\beta}} l_Q^{-\frac{\gamma}{\beta}} l_\theta^{-\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{-\frac{1}{\beta}} dt$$

exists.

In the Appendix A.2 we show that this integral exists only if f $\beta < 1$. If this is possible then continuous technological change will allow for a sustainable economy to exist. Assuming this we can solve again for the most interesting variables.

Per capita consumption is

$$\frac{\bar{C}}{L} = \left(\frac{1-\beta}{\beta}\bar{S}\right)^{\beta} K_0^{\alpha} L^{\gamma} l_Q^{\gamma} l_\theta \left[\theta_0/L_\theta\right]^{1-\beta} a^{\beta} \tag{27}$$

The extraction dynamics of the sustainable finite material resource path under these conditions is

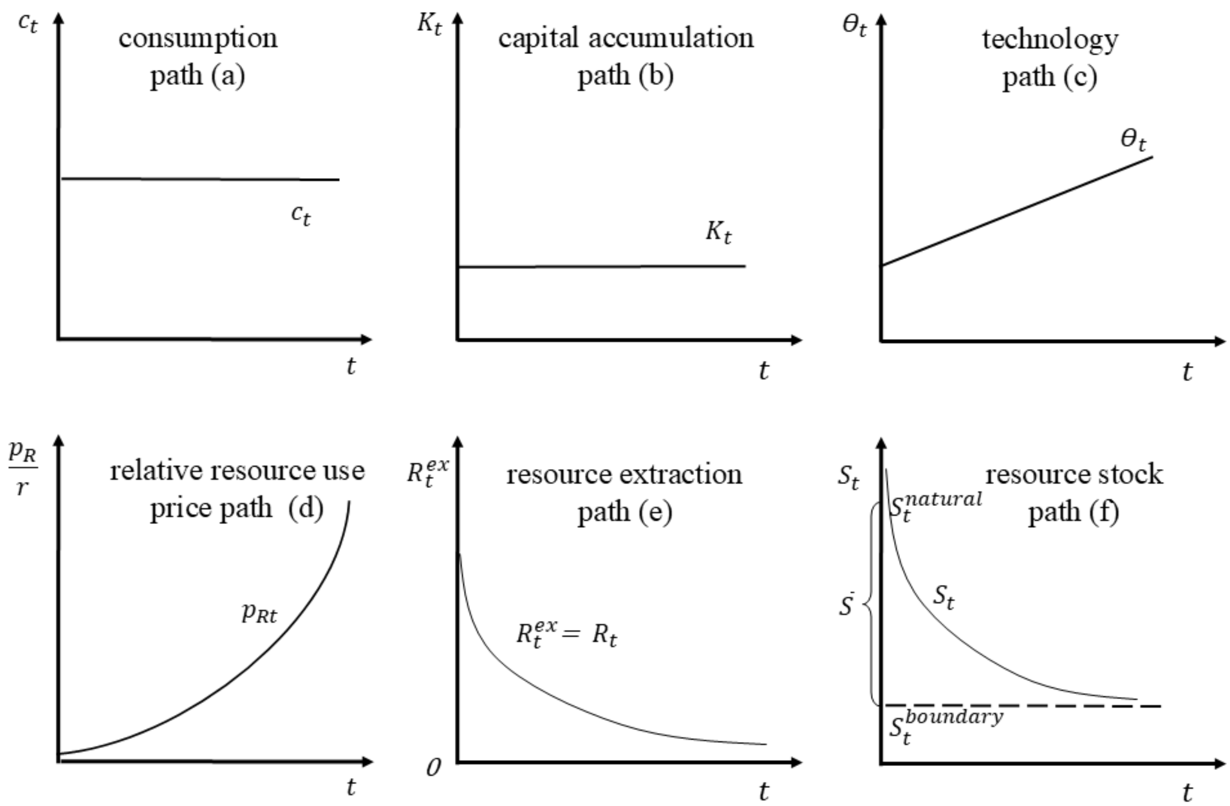
$$\begin{aligned} R_t &= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{-\frac{1}{\beta}}, \text{ with } \lim_{t\to\infty} R_t = 0, \text{ for } \beta < 1 \quad (28) \\ \dot{R}_t &= -\frac{1}{\beta}\left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} L^{-\frac{\gamma}{\beta}} l_Q^{-\frac{\gamma}{\beta}} l_\theta^{-\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{-\left(1+\frac{1}{\beta}\right)} a < 0 \end{aligned}$$

The price path consistent with the above is:

$$\frac{p_{Rt}}{r_t} = \frac{\beta}{\alpha}\left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{1+\frac{\alpha}{\beta}} L^{\frac{\gamma}{\beta}} l_Q^{\frac{\gamma}{\beta}} l_\theta^{\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{\frac{1}{\beta}} \tag{29}$$

It can be seen in Figure 2 that the consumption, material resource and price paths closely resemble the time paths in Figure 1, panels a,c and d. This means that we have again a

Figure 2: Paths of a sustainable economy with technological innovation only



Source: Authors' compilation

sustainable, but post-growth economy - with a constant level of consumption guaranteed and degrowth in material resource throughput.

2.4 Scenario 3: Consumption Growth with Degrowth in Material Inputs

In scenario's 1 and 2 in the preceding sub-sections we modelled how respectively capital accumulation and technological innovation can ensure that a minimum level of consumption can be maintained whilst achieving a degrowth in finite material resource inputs. The resulting sustainable economy, which functioned within the Safe Operating Space, was in each case a post-growth economy: growth in consumption had ceased, and use of material resource inputs was put on a declining time path, without fully exhausting stocks in finite time.

In this section we show how a green growth economy, marked by *growth* in consumption - and not a fixed level - can be consistent with degrowth in material throughputs.

As before, we will first outline the model set-up, after which we will solve the model for the growth paths of the variables of concern.

2.4.1 Model set-up

In the economy modelled here, there is both capital accumulation and continuous technological progress at the same time. Together, these two factors enable both a sustainable and growing consumption path. We are out of the post-growth economy and in the Green Growth economy.

To keep the discussion short and concise, we simply combine the above two models and mechanisms. With respect to capital accumulation we build on section 2.2. In section 2.2, capital accumulation has made it possible to replace the finite material resource so that a constant sustainable (and maximum) level of consumption per capita is possible. We can directly build upon the results in section 2.2, jump to equation (19) that describes - for the given technology level θ - the constant production for maximum sustainable consumption is

$$\bar{Q}^* = \bar{C}^* + I^* = \theta \left(K_t^*\right)^{\alpha} \left(R_t^*\right)^{\beta} L_Q^{\gamma} = const.$$

Defining $\breve{Q}^* = (K_t^*)_t^{\alpha} (R_t^*)^{\beta} L_Q^{\gamma}$ we can rewrite $\bar{Q}^*$ as

$$\bar{Q}^* = \theta \breve{Q}^*. \tag{30}$$

The only difference from the modelling in section 2.2 is that here we assume technology θ can grow over time exponentially, such that

$$\theta_t = \theta_0 e^{g_\theta t}. \tag{31}$$

Further, if we assume that initial technology in section 2.2 can be described by the index value $\theta = \theta_0 = 1$ production grows accordingly as

$$Q_t^* = \theta_t \breve{Q}^* \tag{32}$$

Thus, we can now rewrite the consumption path

$$C_t^* = Q_t^* - I = \theta_t \breve{Q}^* - I^* \tag{33}$$

As consumption level $\bar{C}_0^* = \theta_0 \breve{Q}^* - I^*$ can already be maintained solely through the accumulation of capital (see 20 in section 2.2), sustainability is already assured at level $\bar{C}_0^*$ through capital accumulation. Thus, technical change purely contributes to consumption growth.

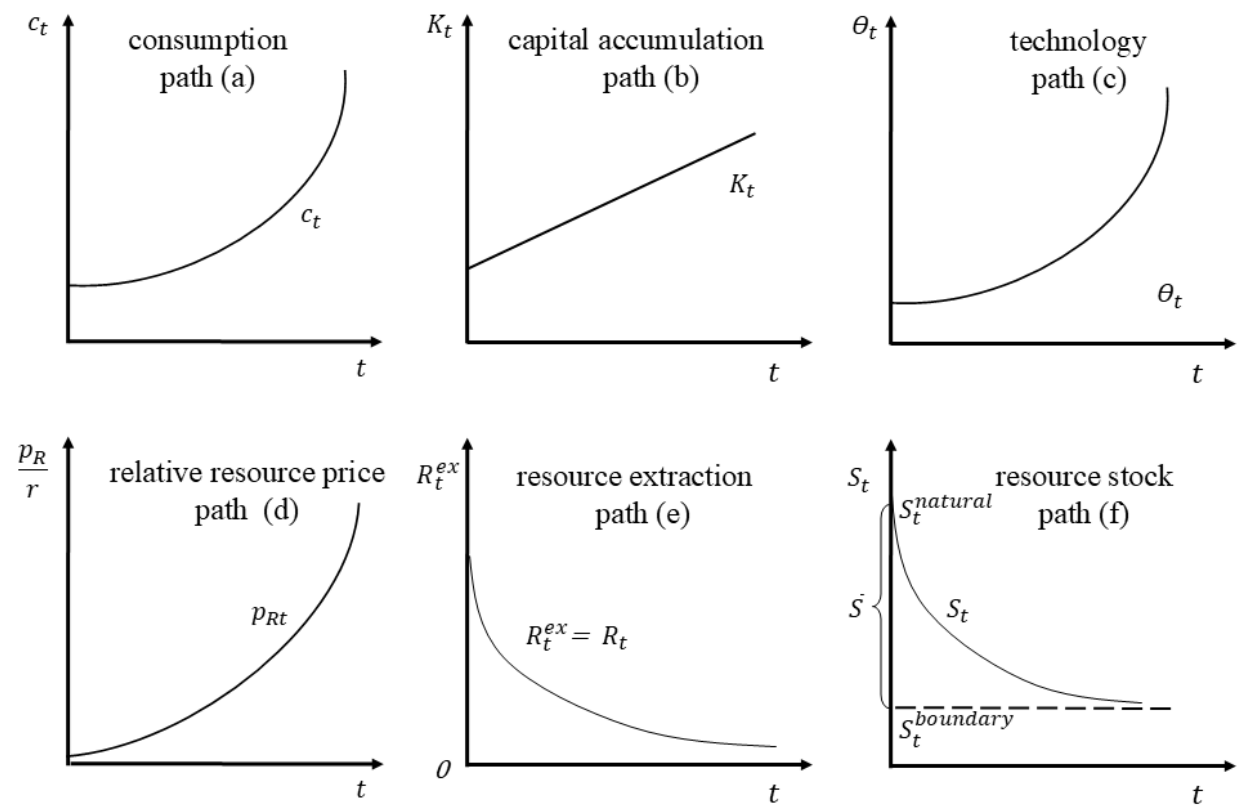
Technology: In this already sustainable economy we can now analyze a technological progress that further increases productivity. We include technological change similar as has been done in section 2.3. As in many endogenous growth approaches we specify innovations

$\dot{\theta}_t$ as being generated based on existing technology and the share of labor L_θ allocated to the technology-inventing R&D sector:

$$\dot{\theta}_t = \theta_t a L_\theta \tag{34}$$

2.4.2 Solving the model

Figure 3: Paths of a sustainable economy with consumption growth and resource degrowth



Source: Authors' compilation

To solve the model and analyse the additional effect of technological change on consumption growth over time we take the time derivative of (33) and include the generated technological progress to obtain in the long-term consumption growth path as

$$\frac{\dot{C}_t^*}{C_t^*} = \frac{\dot{\theta}_t}{\theta_t} = g_\theta = aL_\theta. \tag{35}$$

Figure 3 depicts the relevant time paths for this scenario, to be compared with those in Figures 1 and 2. It can be seen from Figure 3 that while the capital accumulation path and the finite material resource use path correspond to those of section 2.2, the technological progress path - assumed here to be driven by the allocation of labor to innovations increases total factor productivity and allow for positive consumption growth.

3 Fully Transitioning to Renewables

The three scenarios modelled in the preceding section suggest that, theoretically at least, a comfortable post-growth or even Green Growth future is possible in a world characterised by finite material resources and less than perfect substitutability between resources.

While we had removed the highly objectionable assumptions in Green Growth modelling that resources are infinite and substitutability between resource inputs is very easy ($\sigma > 1$); and while we had also modelled sustainability without reverting to pure tech-optimism, there remain various potentially problematic assumptions in our modelling.

Perhaps one of the most serious is our assumption, so far, that it is possible to infinitely divide up a finite stock of material resources so as to keep production (and consumption) going. This is a mathematical device through which we can model resource use by always for example being able to halve a specific quantity of resources an infinite number of times without fully exhausting the resource. Clearly, while this holds in a strict mathematically sense, when compared to material resource use this seems like a sleight of hand.

To move beyond this assumption we therefore refine our model to take into account that in practice material resources are finite and discrete inputs. We show that in this case that the sustainability, or not, of economic growth hinges on harnessing renewable energy and materials. As the Earth is not a fully closed system a continuous supply of materials can potentially be generated from solar energy.

3.1 Model set-up

In this model variation we take a key Degrowth assumption, namely that absolute decoupling between GDP and material resource use on a global level is unlikely, even if much dematerialised-growth can take place (Haberl et al., 2020; Kallis et al., 2025). In terms of modelling this means that a minimum physical resource input R^{min} would always need to be available for maintaining a certain physical level of production (non-reducibility). In our model, non-reducibility is justified either by the fact that the resource cannot be subdivided infinitely (resources are finite and discrete inputs) or that it becomes difficult to replace the resource with capital (low substitutability in the CES function, $\sigma < 1$) to sustain a minimum comfortable consumption level. As in section 2.2 we will show that a sustainable consumption growth is possible in the face of this key degrowth view, but however conditional on the economy mobilizing sufficient renewable resources.

Production: We split the time-path of production (GDP) into in two periods. In the first time period ($t = 0$ to τ), the economy has substituted finite material resource R_t by reproducible capital K_t - as modelled in sub-section 2.2. In the second time period beginning at time τ the economy has reached the point that $R_\tau = R^{min}$ cannot be reduced further to produce the same output. Therefore, at time τ we observe the following production

$$Q_\tau = \theta K_\tau^\alpha \bar{R}_\tau^\beta L_Q^\gamma \qquad \text{with } \; Q_t = 0 \; \text{ for } \; R_t < \bar{R} \tag{36}$$

To focus on only this one mechanism, we once more leave aside pronounced technological optimism, and freeze technology at the level θ.

Capital: In the first time period accumulation of capital took place according to the specifications in sub-section 2.2. Thus, after τ period the economy has accumulated a green capital stock K_τ. After τ capital cannot further substitute for the minimum required resource use $R_\tau = R^{min}$, such that any further investments for a sustainable consumption is useless. As sustainable consumption is the only purpose for investments in this model it will stop and the green capital stock will remain constant at the level K_τ.

Resource use and resource stock dynamics: While the material resource use is physical input R_t we can differentiate the source of this physical input. The source could be a finite, exhaustible (non-renewable) material resource R_{EXt} or it can be an renewable material resource R_{Rt} in period t. While so far the non-renewable resource was extracted and available at no costs, the renewable resource requires another input to be made useful. This input can be taken from the final good.[5] To simplify we assume a resource availability function as

$$R_t = R_{EXt} \text{ for exhaustible (non-renewable) resource inputs} \tag{37}$$

$$R_t = I_R \bar{R}_{Rt} \; \text{ for renewable resource inputs} \tag{38}$$

where $\bar{R}_{Rt}$ is the total renewable resource flow into the economy (e.g. total solar radiation)

[5]Note that we assume that the price path of the resource that generate this substitution process between resources and capital is lower or identical to the price of the non-renewable resource compared to the renewable resource, such that the reason for switching from non-renewable to renewable resource is indeed the non-reducibility of the non-renewable resource in production. A price motivated switch is another economically interesting scenario, however, it is not in the focus of our discussion here.

and I_R denotes the investments required to make the desired fraction of this resource available for production (e.g. solar panel generates electricity for production). In our current considerations the desired fraction would be $R^{min} = \bar{I}_R \bar{R}_R$. However, even if we see that more of these investments could generate a higher amount of renewable resources available, we want to keep the analysis simple and do not discuss an optimal transition or the optimal use of these renewables. For simplicity we also assume that $\bar{I}_R$ is fully depreciated each period.

3.2 Solving the model

Deriving the required resource extraction path for the first time window of using the exhaustible resource leads to

$$R_t = R_{EXt} = \left[\bar{C} + \bar{I}\right]^{\frac{1}{\beta}} L_Q^{\frac{-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} K_t^{-\frac{\alpha}{\beta}} \quad \text{for } t = 0...\tau. \tag{39}$$

As in section 2.2 sustainable consumption $\bar{C}$ is

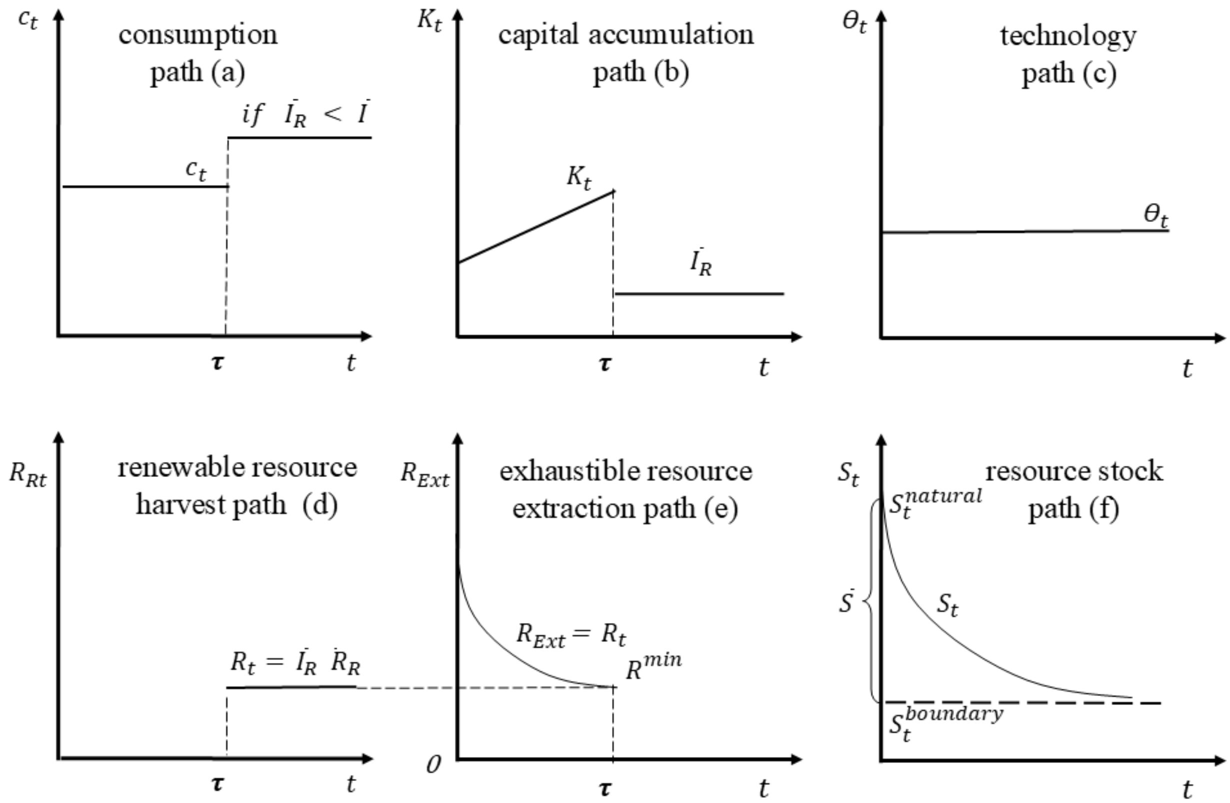
$$\bar{C} = \theta K_t^{\alpha} R_{Ext}^{\beta} L_Q^{\gamma} - \bar{I} \;\; \text{for } t = 0...\tau.$$

When reaching the critical level of resource use $R_\tau = R^{min}$ the process changes. Starting with period τ the economy produces the resource input by using the renewable flow resource which becomes available at the required level by green investments $\bar{I}_R$:

$$R_t = R^{min} = \bar{I}_R R_R \quad \text{for } t = \tau....\infty.$$

After the non-renewable resource is exploited to the extent that S_t is not sufficiently available to provide R^{min} the economy can only survive and be sustainable, if it switches to renewable resources that provide at least R^{min}.

Figure 4: Paths of a sustainable economy with renewable resources



Source: Authors' compilation

The sustainable consumption level with renewable resources then is

$$\bar{C}_{RR} = \theta K_{\tau t}^{\alpha} \left(\bar{I}_R R_R\right)^{\beta} L_Q^{\gamma} - \bar{I}_R \text{ for } t = 0...\tau. \tag{40}$$

This shows that ultimately our model economy should switch to renewable resources to maintain a sustainable level of consumption. With respect to a finite material resource such as fossil fuel energy such a switch is already empirically observable, even though small : wind and solar provides currently only around 3% of the world's total energy consumption (Rees, 2025). Progress in switching from fossil fuels to renewable energy is also widely uneven between countries - ranging in 2024 from 80% and 72% in Iceland and Norway, 40%, 24% and 18% in respectively Portugal, Germany and The Netherlands, to 0,7% in Saudi Arabia.

Renewable energy as solar and wind energy can potentially fully substitute for exhaustible fossil fuels, although formidable challenges still exist,[6] see e.g. Rees (2025), Fressoz (2024) and Berners-Lee (2025). Switching from exploiting the non-renewable fossil fuel stock to optimally using the continuous renewable energy resource flow could even make the economy better off: the sustainable consumption level could even increase if $\bar{I} < \bar{I}_R$. The time paths of the respective variables are drawn in Figure 4. It should also be noted again that we have not yet incorporated continuous technical progress in this modelling. If we were to do so, it would be conceivable, as in section 2.4, that after the switch to renewable resources, consumption levels would continue to rise in line with ongoing technical progress.

Note however, that while a switch to renewable resources is more conceivable in the case of energy, although very formidable, in the case of other finite material resources whether substitutes can always be found is much less obvious. Therefore, we should be very cautious to conclude from our modelling in this section that a sustainable economy, within the safe operating space, is at all possible. As Naudé (2025) discusses in outlining three climate technology gaps, the key uncertainty has to do with the viability, and commercialization, of the appropriate green technologies.

[6]Rees (2025) for instance calculates that to displace fossil fuels "from electricity generation alone by – let's say 2035 – would require that the world install an additional four times the entire existing 30-year cumulative stock of wind and solar generating capacity in just the next decade."

4 Discussion

Green growth models rely on the mainstream modelling approach of Romer (1990, 1986) wherein the fundamental determinant of economic growth is technological progress. Because technological progress is based on ideas, which are non-rival thus generating increasing returns to scale, and can be joined in an infinite number of combinations, the world will never run out of ideas, and hence never run out of economic growth. All that government needs to do, as the growth modelling of e.g. Acemoglu and Autor (2012) and Acemoglu et al. (2016) advocates, is to direct technological change towards clean technologies through an optimal policy of carbon taxes, and subsidies for research and development (R&D) in clean technologies. In this paper we illustrated with a simple descriptive growth model why this is neither so simple - nor sufficient.

The problem, as Degrowth scholars point out justifiably, is that the assumptions and modelling frameworks that mainstream Green Growth scholars and policy makers use to justify its tech-optimism, is just not realistic, and often at odds with empirical evidence. Adequate policy responses to the ecological overshoot crisis requires appropriate economic growth models wherein the ecological damages from economic growth is not wished away, and the relation of growth with energy and other material inputs are recognised as potentially significant constraints if not on short-run, then eventually on long-run growth (Murphy, 2022).

While economic growth theory has made important progress in the 20th century, especially in recognising the role of non-rival and combinatorial ideas driving economic growth, its remaining shortcomings as far as ecological overshoot is concerned, and which results in the Degrowth movement and others reject their tech-optimism, can briefly be summarised in the rest of this section. We restrict ourselves to four of the most problematic assumptions. Our call here is for genuine interdisciplinary cooperation to develop a growth theory moving beyond these assumptions.

A first problem is that Green Growth models do not adequately model energy. Economists do recognise the importance of energy, but because of the low share of energy in GDP, and the belief that easy substitutes will be found once energy gets scarce and its price increases, energy is not treated as a finite material resource. In sections 2 of this paper we dealt with this by explicitly incorporating finite resources (which includes energy) and modelling their finite stocks as discrete stocks. When we did this, our model showed how tenuous it is to maintain consumption levels without a full transition to renewable energy and materials (see section 3).

A second problematic assumption, as per for instance Acemoglu et al. (2012) and Acemoglu et al. (2016) is that resources are perfectly substitutable, i.e. has a substitutional elasticity in production of $\sigma = 1$ - see section 2.2 In this paper we departed from this assumption by using a the Cobb-Douglas function where the elasticity of substitution is $\sigma = 1$. Questions that needs more attention include, what are empirically realistic assumptions for the short- and long-term elasticity of substitution between green capital and resources, as well as for the continuous growth of green technologies, in order to model and discuss the above scenarios and the corresponding policy measures? Is the implicit Degrowth assumption of $\sigma < 1$ more appropriate, or not?

A third problematic assumption is that that it is possible to infinitely divide up a finite stock of material resources so as to keep production (and consumption) going. We showed in this paper - in section 3 - that this amounts to a mathematical sleigh of hand, and that by more acknowledging finite *and* discrete material resources, the implications for consumption growth is significant.

While our paper explicitly dealt with the three problematic assumptions above, and showed that using more appropriate assumptions critically change the results, there are four further problematic assumptions that future research and modelling - preferable in inter-disciplinary context- should deal with.

One is the problematic assumption of positive social discount rates used in much economic growth modelling and in social cost-benefit analysis of climate policy. For example, economic models of climate damages that are used in Integrated Assessment Models and the Intergovernmental Panel on Climate Change (IPCC) scenario's, tend to use a positive rate to discount the future. This places a higher value on people living in the present as opposed to those who will live in the future, raising issues of intergenerational equity (Lenton et al., 2023; Stern et al., 2022). As Asefi-Najafabady et al. (2021, p.1183) points out,"the climate-change denying Trump administration has used an annual discount rate of 7% for its analysis of the social cost of carbon." We think that Degrowth scholars have put forward strong arguments for zero social discount rates, see e.g. Spash (2013), Kerschner (2010) and Martinez-Alier (2009), and that the incorporation of these into economic growth models should be central in an inter-disciplinary discussion. We have mentioned this problem at the end of section 2.2.

A second further problematic assumption is the view that the commodification of nature is the way to deal with "externalities" and "market failures" and that it such a way the market will eventually solve all the negative externalities of growth, including through adequately pricing finite material resources. In section 2.2.1, we assumed that continuous price increases for a finite resource would lead to continuous decreases in its use. In standard economic modelling it is assumed that resource owners will generate this price path due to the Hotelling Rule and an infinite time horizon in an infinitely working market system. If such reliance on an assumed perfect market system with an infinite time horizon is unrealistic, the question is what economic allocation system, what kind of regulations of markets, and what kind political guidance are necessary to ensure sustainable growth within the planet's Safe Operating Space?

A third further problematic assumption is that economic growth models, including ours here (see section 2.2.1), assumes a single "representative" household and/or firm. Models that jettison this assumption shows that spiralling inequality can follow from technological innovations, and that this can undermine climate policy (see e.g. Gries and Naudé (2018)). Fix (2014) critically discusses the problematic assumption in mainstream growth models that growth is independent of income distribution and the nature and structure of the labor force. Hence, the question is how is income distribution affected by the various paths of sustainable income development?

Finally, we assumed here that a minimum physical consumption level exists that is sufficient for human objective and subjective wellbeing. This is a simple assumption - humans do need a minimum amount of physical resources to flourish, but as Sen (2000) argued, development is the *freedom* to function fully as human beings, and that this goes beyond mere physical consumption. A decrease in consumption levels, at least in wealthy countries, may not necessarily be inconsistent with maintaining or even improving wellbeing, and the way in which physical consumption is distributed among a population, certainly matters for subjective wellbeing.

5 Concluding Remarks

In this paper we addressed the contentious issue of continued economic growth in light of environmental concerns and the 1970s Limits to Growth report. Despite bringing unprecedented wealth, economic growth over the 20th century has caused ecological overshoot, where human consumption and waste exceed the Earth's regenerative capacity, leading to the breach of an estimated six out of nine Planetary Boundaries by 2023. While some advocate for a Degrowth economy, mainstream economics often assumes continuous growth - Green Growth - is possible due to technological progress. The central question the paper seeks to answer is whether economic growth is possible within a "safe operating space for humanity" that respects Planetary Boundaries, and what crucial assumptions this entails.

We developed a simple descriptive economic growth model that bridges the often mutually exclusive paradigms of Green Growth and Degrowth.

With our Green-Growth-Degrowth model we analysed three scenarios: (i) capital accumulation without technological progress, (ii) continuous technological progress without capital accumulation, and (iii) a combination of both, and introduces the importance of finite resources. A Cobb-Douglas production function is used, implying that material resources are essential and cannot be perfectly substituted.

The initial findings demonstrated that a sustainable economy is possible. First, either capital accumulation or continuous technological progress alone can ensure a minimum per capita consumption level, leading to a post-growth, stationary economy with material resource throughput consistently declining. In other words, we modelled a degrowth economy but where income and consumption (GDP) is not declining. Second, by combining capital accumulation and continuous technological progress, we showed that sustainable consumption growth with degrowth in material inputs is achievable. In other words, we modelled a sustainable Green Growth-Degrowth economy.

Hence, the main contribution of this paper was to remove several objectionable assumptions in mainstream economic growth modelling, such as that resources are infinite, that there is perfect substitutability between resource inputs, and that tech-optimism is warranted, and to provide pathways of economic growth that can be consistent with the notion of a Safe Operating Space for humanity. This illustrated that the paradigms of Green Growth and Degrowth need not be antagonistic.

A further contribution of this paper was to relax the mathematical assumption or device that it is possible to infinitely divide up a finite stock of material resources so as to keep production (and consumption) going. To move beyond this assumption we modelled material resources are finite and discrete inputs. We showed that in this case that the sustainability, or not, of economic growth hinges on harnessing renewable energy and materials. Our somewhat pessimistic conclusion in this regard was that that while a switch to renewable resources is more conceivable in the case of energy, although very formidable, in the case of other finite material resources whether substitutes can always be found is much less obvious. Therefore, we should be very cautious to conclude from our modelling in this paper that a sustainable economy, within the Safe Operating Space, is at all possible.

Of course, the final word on this has hardly been spoken, and for future research, we emphasized the need for genuine interdisciplinary cooperation around a common growth modelling framework that merges insights from both Green Growth and Degrowth scholars, and that can deal with various assumptions and desired outcomes in a mathematically consistent way.

Such interdisciplinary cooperation should try to address critical questions, including realistic assumptions on substitution elasticities in resource use, what economic and political systems would be best to achieve the desired sustainable economy paths we have outlined, what the relevance is of using infinite time horizons and how to discount the future, the role of income distribution and consumption. The challenge is how to effectively combine research across natural sciences, engineering, economics, sociology, social psychology, philosophy and political science to identify practical control mechanisms for sustainable prosperity and better integrate the important insights from Green Growth and Degrowth approaches.

A Appendix

A.1 Appendix for Section 2.2

Factor Allocation The demand for resources results from the firm's cost minimization,i.e. from:

$$\min_{K,R,L} : Costs = rK + p_R R - \lambda \left(\theta K_t^{\alpha} R_t^{\beta} L_Q^{\gamma} - Q \right)$$

The first order conditions (FOCs) are

$$\begin{aligned} r - \lambda \alpha \theta K_t^{\alpha-1} R_t^{\beta} L_Q^{\gamma} &= 0, \\ p_R - \lambda \beta \theta K_t^{\alpha} R_t^{\beta-1} L_Q^{\gamma} &= 0 \end{aligned}$$

From which resource demands depend on relative resource prices:

$$\begin{aligned} \frac{p_R}{r} &= \frac{\lambda \beta \theta K_t^{\alpha} R_t^{\beta-1} L_Q^{\gamma}}{\lambda \beta \theta K_t^{\alpha} R_t^{\beta-1} L_Q^{\gamma}} \\ \frac{R_t}{K_t} &= \frac{\beta}{\alpha} \frac{r}{p_{Rt}} \end{aligned}$$

Solving the Basic Model From the production function we can determine resource use for a given output

$$\begin{aligned} R_t &= \bar{Q}^{\frac{1}{\beta}} K_t^{-\frac{\alpha}{\beta}} L_Q^{-\frac{\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \\ R_t &= \left[\bar{C} + I \right]^{\frac{1}{\beta}} \left[K_0 + It \right]^{-\frac{\alpha}{\beta}} L_Q^{-\frac{\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \end{aligned}$$

Departing from (11), and with $L_Q = L$ and $Q = C + I$ we obtain for per capita consumption

$\frac{\bar{C}+I}{L_Q}$ the required resource input rate

$$\begin{aligned}
R_t &= \left[\bar{C}+I\right]^{\frac{1}{\beta}} [K_0+It]^{-\frac{\alpha}{\beta}} L_Q^{-\frac{\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \\
R_t &= \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} [K_0+It]^{-\frac{\alpha}{\beta}} L_Q^{\frac{1}{\beta}} L_Q^{-\frac{\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \\
R_t &= \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} [K_0+It]^{-\frac{\alpha}{\beta}}
\end{aligned}$$

The sustainable stock exploitation requires

$$\begin{aligned}
\bar{S} &= \int_0^\infty (1-\rho) R_t dt \\
&= (1-\rho) \int_0^\infty \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} [K_0+It]^{-\frac{\alpha}{\beta}} dt, \quad \frac{\alpha}{\beta} > 1
\end{aligned}$$

From integration process we can derive that the integral is finite if $\frac{\alpha}{\beta} > 1$. Integrating:

$$\begin{aligned}
\frac{\bar{S}}{(1-\rho)} &= \left| \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left(1-\frac{\alpha}{\beta}\right)^{-1} [K_0+It]^{1-\frac{\alpha}{\beta}} I^{-1} \right|_0^\infty \\
&= \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left(\frac{\beta}{\beta-\alpha}\right) [K_0+I\infty]^{1-\frac{\alpha}{\beta}} I^{-1} \\
&\quad - \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left(\frac{\beta}{\beta-\alpha}\right) [K_0]^{1-\frac{\alpha}{\beta}} I^{-1} \\
&= \left[\frac{\bar{C}+I}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left(\frac{\beta}{\alpha-\beta}\right)^{\beta} [K_0]^{1-\frac{\alpha}{\beta}} I^{-1}
\end{aligned}$$

This can be rearranged to determine the per capita consumption

$$\begin{aligned}
\left(\frac{\bar{S}}{1-\rho}\right)^{\beta} &= \left[\frac{\bar{C}+I}{L_Q}\right] L_Q^{1-\gamma}\theta^{-1}\left(\frac{\beta}{\alpha-\beta}\right)^{\beta}[K_0]^{\beta-\alpha} I^{-\beta} \\
\frac{\bar{C}+I}{L_Q} &= \left(\frac{\bar{S}}{1-\rho}\right)^{\beta}\theta L_Q^{\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta} L_Q^{\beta} \\
\frac{\bar{C}+I}{L_Q} &= \left(\frac{\bar{S}}{1-\rho}\right)^{\beta}\theta L_Q^{\beta+\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta} \\
\frac{\bar{C}}{L_Q} &= \left(\frac{\bar{S}}{1-\rho}\right)^{\beta}\theta L_Q^{\beta+\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta} - \frac{I}{L_Q}
\end{aligned}$$

Maximum Consumption and Optimal Investments For simplicity we We can explore the highest possible level of consumption consistent with finite material resources in our model economy by solving for:

$$\max_{\frac{I}{L_Q}} \frac{\bar{C}}{L_Q} = \left(\frac{\bar{S}}{1-\rho}\right)^{\beta}\theta L_Q^{\beta+\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta} - \frac{I}{L_Q}, \text{ with } L = L_Q = 1.$$

The first order conditions for an optimum is

$$\begin{aligned}
0 &= \beta\left(\frac{\bar{S}}{1-\rho}\right)^{\beta}\theta L_Q^{\beta+\gamma-1}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I}{L_Q}\right)^{\beta-1} - 1 \\
I^{1-\beta} &= \beta\left(\frac{\bar{S}}{1-\rho}\right)^{\beta}\theta L_Q^{\gamma}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}
\end{aligned}$$

From the first order conditions we obtain optimal sustainable investments, consumption and GDP, and the respective paths for capital accumulation and resource extraction.

Sustainable optimal investment and capital accumulation: Using the first order condition we can determine the optimal investments as

$$I^* = \beta^{\frac{1}{1-\beta}} \theta^{\frac{1}{1-\beta}} \left(\frac{\bar{S}}{(1-\rho)} \right)^{\frac{\beta}{1-\beta}} L_Q^{\frac{\gamma}{1-\beta}} \left(\frac{\alpha - \beta}{\beta} \right)^{\frac{\beta}{1-\beta}} [K_0]^{\frac{\alpha-\beta}{1-\beta}} ,$$

and the optimal path of the capital stock as

$$K_t^* = K_0 + I^* t.$$

Sustainable maximum consumption: Using the consumption maximizing investment I^* we can now determine all other variables such as the maximum

$$\bar{C}^* = \theta \left(\frac{\bar{S}}{(1-\rho)} \right)^{\beta} L_Q^{\gamma} \left(\frac{\alpha - \beta}{\beta} \right)^{\beta} [K_0]^{\alpha-\beta} (I^*)^{\beta} - I^*.$$

Sustainable production for maximum consumption:

$$\bar{Q}^* = \bar{C}^* + I^*$$

Sustainable resource path: From the production function and knowing $\bar{C}^*$, I^* and $\bar{Q}^*$ as well as the optimal path of capital accumulation we obtain the *optimal path of sustainable* *resource extraction* as

$$
\begin{aligned}
R_t &= \left[\bar{Q}^*\right]^{\frac{1}{\beta}} \theta^{-\frac{1}{\beta}} L_Q^{-\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}} \left[K_0 + I^* t\right]^{-\frac{\alpha}{\beta}} \\
R_t^* &= \left[\bar{C}^* + I^*\right]^{\frac{1}{\beta}} L_Q^{\frac{-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left[K_0 + I^* t\right]^{-\frac{\alpha}{\beta}}, \quad \text{with} \\
\dot{R}_t^* &= -\frac{\alpha}{\beta} \left[\bar{C}^* + I^*\right]^{\frac{1}{\beta}} L_Q^{\frac{-\gamma}{\beta}} \theta^{-\frac{1}{\beta}} \left[K_0 + I^* t\right]^{-\frac{\alpha}{\beta}-1} I^* < 0, \\
\lim_{t\to\infty} R_t^* &= \frac{\left[\bar{C}/I^* + 1\right]^{\frac{1}{\beta}}}{L_Q^{\frac{\gamma}{\beta}} \theta^{\frac{1}{\beta}} \left(I^*\right)^{\alpha} \left[K_0/I^* + t\right]^{\frac{\alpha}{\beta}}} = 0
\end{aligned}
$$

Sustainable price path

$$
\begin{aligned}
\frac{p_{Rt}}{r_t} &= \frac{\beta R_t^{-1}}{\alpha K_t^{-1}} = \frac{\beta K_t}{\alpha R_t} \\
&= L_Q^{\frac{\gamma}{\beta}} \theta^{\frac{1}{\beta}} \left(I^*\right)^{\alpha} \frac{\beta \left[K_0/I^* + t\right]^{1+\frac{\alpha}{\beta}}}{\alpha \left[\bar{C}/I^* + 1\right]^{\frac{1}{\beta}}}
\end{aligned}
$$

Investigating Recycling: increase in Recycling Rate $d\rho > 0$:

Change of investments and capital accumulation: From (17) we know that $I^* = \beta^{\frac{1}{1-\beta}} \theta^{\frac{1}{1-\beta}} \left(\bar{S}\right)^{\frac{\beta}{1-\beta}} (1-\rho)^{-\frac{\beta}{1-\beta}} L_Q^{\frac{\gamma}{1-\beta}} \left(\frac{\alpha-\beta}{\beta}\right)^{\frac{\beta}{1-\beta}} \left[K_0\right]^{\frac{\alpha-\beta}{1-\beta}}$, and we obtain

$$
\begin{aligned}
\frac{dI^*}{d\rho} &= -\frac{\beta}{1-\beta} \beta^{\frac{1}{1-\beta}} \theta^{\frac{1}{1-\beta}} \left(\bar{S}\right)^{\frac{\beta}{1-\beta}} (1-\rho)^{-\frac{\beta}{1-\beta}-1} L_Q^{\frac{\gamma}{1-\beta}} \left(\frac{\alpha-\beta}{\beta}\right)^{\frac{\beta}{1-\beta}} \left[K_0\right]^{\frac{\alpha-\beta}{1-\beta}} (-) \\
&= \frac{\beta}{1-\beta} (1-\rho)^{-1} \beta^{\frac{1}{1-\beta}} \theta^{\frac{1}{1-\beta}} \left(\bar{S}\right)^{\frac{\beta}{1-\beta}} (1-\rho)^{-\frac{\beta}{1-\beta}} L_Q^{\frac{\gamma}{1-\beta}} \left(\frac{\alpha-\beta}{\beta}\right)^{\frac{\beta}{1-\beta}} \left[K_0\right]^{\frac{\alpha-\beta}{1-\beta}} \\
\frac{dI^*}{d\rho} &= \frac{\beta}{(1-\beta)(1-\rho)} I^* > 0
\end{aligned}
$$

Thus, the path of capital accumulation is steeper.

$$\begin{aligned} K_t &= K_0 + I^* t \\ \frac{dK_t}{d\rho} &= t\frac{dI^*}{d\rho} > 0 \end{aligned}$$

Change of consumption Finally we can determine the change of consumption possible, when recycling is organized costlessly. With $\bar{C}^* = \theta\left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta} L_Q^{\beta+\gamma}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I^*}{L_Q}\right)^{\beta} - I^*$ and using the envelope theorem from "optimal investments for maximizing consumption," we obtain

$$\begin{aligned} \frac{d\bar{C}^*}{d\rho} &= -\beta\left(1-\rho\right)^{-1}\theta\left(\bar{S}\right)^{\beta}\left(\frac{1}{1-\rho}\right)^{\beta} L_Q^{\beta+\gamma}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\left(\frac{I^*}{L_Q}\right)^{\beta}(-) \\ &= \frac{\beta}{1-\rho}\left(\bar{C}^* + I^*\right) > 0 \end{aligned}$$

Change of input path of resource and extraction path: Assuming $L_Q = 1$

$$R_t^* = \left[\frac{\bar{C}^* + I^*}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}}\theta^{-\frac{1}{\beta}}\left[K_0 + I^* t\right]^{-\frac{\alpha}{\beta}}$$

$$\begin{aligned} \frac{dR_t^*}{d\rho} &= \frac{1}{\beta}\left[\frac{\bar{C}+I^*}{L_Q}\right]^{\frac{1}{\beta}-1} L_Q^{\frac{1-\gamma}{\beta}}\theta^{-\frac{1}{\beta}}\left[K_0+I^*t\right]^{-\frac{\alpha}{\beta}}\frac{1}{L_Q}\frac{d\bar{C}^*}{d\rho} \\ &\quad+\frac{1}{\beta}\left[\frac{\bar{C}+I^*}{L_Q}\right]^{\frac{1}{\beta}-1} L_Q^{\frac{1-\gamma}{\beta}}\theta^{-\frac{1}{\beta}}\left[K_0+I^*t\right]^{-\frac{\alpha}{\beta}}\frac{1}{L_Q}\frac{dI^*}{d\rho} \\ &\quad-\frac{\alpha}{\beta}\left[\frac{\bar{C}+I^*}{L_Q}\right]^{\frac{1}{\beta}} L_Q^{\frac{1-\gamma}{\beta}}\theta^{-\frac{1}{\beta}}\left[K_0+I^*t\right]^{-\frac{\alpha}{\beta}-1} t\frac{dI^*}{d\rho} \\ &= \frac{1}{\beta}R_t^*\left[\frac{\bar{C}+I^*}{L_Q}\right]^{-1}\frac{1}{L_Q}\frac{d\bar{C}^*}{d\rho} + \left[\frac{1}{\beta}R_t^*\frac{1}{L_Q}\left[\frac{\bar{C}+I^*}{L_Q}\right]^{-1} - \frac{\alpha}{\beta}R_t^*\left[K_0+I^*t\right]^{-1}t\right]\frac{dI^*}{d\rho} \\ &= \frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\frac{d\bar{C}^*}{d\rho} + \left[\frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*} - \frac{\alpha}{\beta}R_t^*\frac{1}{K_0+I^*t}t\right]\frac{dI^*}{d\rho} \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\frac{d\bar{C}^*}{d\rho}+\frac{1}{\beta}R_t^*\left[\frac{1}{\bar{C}+I^*}-\alpha\frac{1}{K_0+I^*t}t\right]\frac{dI^*}{d\rho}\\
&= \frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\frac{d\bar{C}^*}{d\rho}+\frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\left[1-\alpha\frac{\bar{C}+I^*}{K_0+I^*t}t\right]\frac{dI^*}{d\rho}\\
&= \frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\left[\frac{d\bar{C}^*}{d\rho}+\left[1-\alpha\frac{\bar{C}+I^*}{K_0+I^*t}t\right]\frac{dI^*}{d\rho}\right]\lesseqgtr 0
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\left[\frac{\beta}{1-\rho}\left(\bar{C}^*+I^*\right)+\left[1-\alpha\frac{\bar{C}+I^*}{K_0+I^*t}t\right]\left(\frac{\beta}{(1-\beta)(1-\rho)}I^*\right)\right]\\
&= \frac{\beta}{\beta}R_t^*\frac{1}{\left(\bar{C}+I^*\right)(1-\rho)}\left[\left(\bar{C}^*+I^*\right)+\left[1-\alpha\frac{\bar{C}+I^*}{K_0+I^*t}t\right]\left(\frac{1}{(1-\beta)}I^*\right)\right]\\
&= R_t^*\frac{1}{\left(\bar{C}+I^*\right)(1-\rho)(1-\beta)}\left[\left(\bar{C}^*+I^*\right)(1-\beta)+\left[1-\alpha\frac{\bar{C}+I^*}{K_0+I^*t}t\right]I^*\right]\\
&= R_t^*\frac{1}{(1-\rho)(1-\beta)}\left[(1-\beta)+\frac{I^*}{\bar{C}+I^*}-\alpha\frac{1}{K_0/I^*t+1}\right]\lesseqgtr 0
\end{aligned}
$$

Determine t for the switch of the the sign:

$$
\begin{aligned}
0 &< (1-\beta)+\frac{I^*}{\bar{C}+I^*}-\alpha\frac{1}{K_0/I^*t+1}\\
0 &< \left(\bar{C}^*+I^*\right)(1-\beta)+I^*-\alpha\frac{\bar{C}+I^*}{\left(\frac{K_0}{I^*t}+1\right)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha\frac{1}{\left(\frac{K_0}{I^*t}+1\right)} &< (1-\beta)+\frac{I^*}{\bar{C}+I^*}\\
\alpha\frac{1}{\left((1-\beta)+\frac{I^*}{\bar{C}+I^*}\right)} &< \frac{K_0}{I^*t}+1\\
\alpha\frac{I^*t}{\left((1-\beta)+\frac{I^*}{\bar{C}+I^*}\right)}-I^*t &< K_0\\
I^*t\left(\alpha-\left((1-\beta)+\frac{I^*}{\bar{C}+I^*}\right)\right) &< K_0\left((1-\beta)+\frac{I^*}{\bar{C}+I^*}\right)
\end{aligned}
$$

$$
\begin{aligned}
I^*t\frac{1}{\left(\bar{C}+I^*\right)}\left(\left(\bar{C}+I^*\right)\alpha-\left((1-\beta)\left(\bar{C}+I^*\right)+I^*\right)\right) &< K_0\left((1-\beta)\left(\bar{C}+I^*\right)+I^*\right)\frac{1}{\left(\bar{C}+I^*\right)} \\
I^*t\left(\left(\bar{C}+I^*\right)\alpha-\left(\bar{C}+I^*\right)+\beta\left(\bar{C}+I^*\right)-I^*\right) &< K_0\left((1-\beta)\left(\bar{C}+I^*\right)+I^*\right) \\
I^*t\left((\alpha+\beta-1)\left(\bar{C}+I^*\right)-I^*\right) &< K_0\left((1-\beta)\left(\bar{C}+I^*\right)+I^*\right)
\end{aligned}
$$

since we can assume that $(\alpha+\beta-1)\left(\bar{C}+I^*\right)-I^*<0$ we obtain

$$
t>\frac{K_0\left((1-\beta)\left(\bar{C}+I^*\right)+I^*\right)}{I^*\left((\alpha+\beta-1)\left(\bar{C}+I^*\right)-I^*\right)}.
$$

Therefore, the use of the corresponding resources will increase. From an economic perspective, this is directly related to the higher consumption, higher investment, and higher GDP already mentioned above.

For the extraction path, $R_t^{ex}=(1-\rho)\,R_t^*$, we obtain:

$$
\begin{aligned}
R_t^{ex} &= (1-\rho)\,R_t^*, \\
\frac{dR_t^*}{d\rho} &= \frac{1}{\beta}R_t^*\frac{1}{\bar{C}+I^*}\left[\frac{d\bar{C}^*}{d\rho}+\left[1-\alpha\frac{\bar{C}+I^*}{K_0+I^*t}t\right]\frac{dI^*}{d\rho}\right] \\
\frac{dR_t^{ex}}{d\rho} &= -R_t^*+(1-\rho)\frac{dR_t^*}{d\rho}\lesseqgtr 0 \\
&= -R_t^*+(1-\rho)\left(R_t^*\frac{1}{(1-\rho)(1-\beta)}\left[(1-\beta)+\frac{I^*}{\bar{C}+I^*}-\alpha\frac{1}{K_0/I^*t+1}\right]\right) \\
&= -R_t^*+R_t^*\frac{1}{(1-\beta)}\left[(1-\beta)+\frac{I^*}{\bar{C}+I^*}-\alpha\frac{1}{K_0/I^*t+1}\right] \\
&= R_t^*\left[-1+\frac{1}{(1-\beta)}\left[(1-\beta)+\frac{I^*}{\bar{C}+I^*}-\alpha\frac{1}{K_0/I^*t+1}\right]\right]
\end{aligned}
$$

Determine t for the switch of the the sign:

$$\begin{aligned} 0 &< -1 + \frac{1}{(1-\beta)}\left[(1-\beta) + \frac{I^*}{\bar{C}+I^*} - \alpha\frac{1}{K_0/I^*t+1}\right] \\ (1-\beta) &< (1-\beta) + \frac{I^*}{\bar{C}+I^*} - \alpha\frac{1}{K_0/I^*t+1} \\ 0 &< \frac{I^*}{\bar{C}+I^*} - \alpha\frac{1}{K_0/I^*t+1} \end{aligned}$$

$$\begin{aligned} \alpha\frac{I^*t}{K_0+I^*t} &< \frac{I^*}{\bar{C}+I^*} \\ \alpha t &< \frac{K_0+I^*t}{\bar{C}+I^*} \\ \alpha\left(\bar{C}+I^*\right)t &< K_0 + I^*t \\ \left(\alpha\left(\bar{C}+I^*\right) - I^*\right)t &< K_0 \\ t &< \frac{K_0}{\alpha\left(\bar{C}+I^*\right) - I^*} \quad for: \alpha > \frac{I^*}{\left(\bar{C}+I^*\right)} \end{aligned}$$

As t is positive we find a time at which the new path with an increasing recycling rate intersects the previous path, starting at a higher level.

Sustainable maximum intertemporal welfare:

$$V = \left|\frac{1}{-\delta}e^{-\delta t}C_t\right|_0^\infty = \frac{\bar{C}_t}{\delta} \quad \text{as} \quad \bar{C}_t = const.$$

$$\max_I : \frac{\bar{C}}{\delta} = \frac{\theta}{\delta}\left(\frac{\bar{S}}{(1-\rho)}\right)^\beta\left(\frac{\alpha-\beta}{\beta}\right)^\beta [K_0]^{\alpha-\beta}(I)^\beta - \frac{I}{\delta}$$

$$
\begin{aligned}
0 &= \beta\frac{\theta}{\delta}\left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}(I)^{\beta-1}-\frac{1}{\delta} \\
1 &= \beta\theta\left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}(I)^{\beta-1} \\
I^{-(1-\beta)} &= \left(\beta\theta\left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\right)^{-1} \\
I &= \left(\beta\theta\left(\frac{\bar{S}}{(1-\rho)}\right)^{\beta}\left(\frac{\alpha-\beta}{\beta}\right)^{\beta}[K_0]^{\alpha-\beta}\right)^{\frac{1}{(1-\beta)}}
\end{aligned}
$$

A.2 Appendix for Section 2.3

With no accumulation $\bar{C}=\bar{Q}$. We start with the resource use according to the production function

$$
\begin{aligned}
R_t &= \bar{Q}^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L_Q^{-\frac{\gamma}{\beta}}[\theta_0+aL_\theta t]^{-\frac{1}{\beta}}, \\
R_t &= \left[\bar{C}\right]^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L_Q^{-\frac{\gamma}{\beta}}[\theta_0+aL_\theta t]^{-\frac{1}{\beta}}.
\end{aligned}
$$

As we look at sustainable per capita consumption we obtain

$$
\begin{aligned}
&= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}}L^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L_Q^{-\frac{\gamma}{\beta}}[\theta_0+aL_\theta t]^{-\frac{1}{\beta}} \\
&= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}}L^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L^{-\frac{\gamma}{\beta}}l_Q^{-\frac{\gamma}{\beta}}[\theta_0+aL_\theta t]^{-\frac{1}{\beta}} \\
&= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}}L^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L^{-\frac{\gamma}{\beta}}l_Q^{-\frac{\gamma}{\beta}}[\theta_0/L_\theta+at]^{-\frac{1}{\beta}}l_\theta^{-\frac{1}{\beta}}L^{-\frac{1}{\beta}} \\
&= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}}L^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L^{-\frac{\gamma}{\beta}}L^{-\frac{1}{\beta}}l_Q^{-\frac{\gamma}{\beta}}[\theta_0/L_\theta+at]^{-\frac{1}{\beta}}l_\theta^{-\frac{1}{\beta}} \\
&= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}}K_t^{-\frac{\alpha}{\beta}}L^{-\frac{\gamma}{\beta}}l_Q^{-\frac{\gamma}{\beta}}l_\theta^{-\frac{1}{\beta}}[\theta_0/L_\theta+at]^{-\frac{1}{\beta}}
\end{aligned}
$$

For a sustainable resource use the following integral must exist:

$$\bar{S} = \int_0^\infty R_t dt = \int_0^\infty \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} [\theta_0/L_\theta + at]^{-\frac{1}{\beta}} dt$$

Applying the integration procedure results in:

$$\begin{aligned}
\bar{S} &= \left| \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \left(1 - \frac{1}{\beta}\right)^{-1} [\theta_0/L_\theta + at]^{1-\frac{1}{\beta}} a^{-1} \right|_0^\infty \\
&= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \left(1 - \frac{1}{\beta}\right)^{-1} [\theta_0/L_\theta + a\infty]^{1-\frac{1}{\beta}} a^{-1} \\
&\quad - \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \left(1 - \frac{1}{\beta}\right)^{-1} [\theta_0/L_\theta + a0]^{1-\frac{1}{\beta}} a^{-1} \\
&= - \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \left(1 - \frac{1}{\beta}\right)^{-1} [\theta_0/L_\theta]^{1-\frac{1}{\beta}} a^{-1} \\
\bar{S} &= \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \frac{\beta}{1-\beta} [\theta_0/L_\theta]^{1-\frac{1}{\beta}} a^{-1}
\end{aligned}$$

The right hand side of this expression is positive and finite if $1 > \beta$.

Per capita consumption

$$\begin{aligned}
\left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} &= \frac{1-\beta}{\beta} \bar{S} K_0^{\frac{\alpha}{\beta}} [L]^{\frac{\gamma}{\beta}} [l_Q]^{\frac{\gamma}{\beta}} [l_\theta]^{\frac{1}{\beta}} [\theta_0/L_\theta]^{\frac{1-\beta}{\beta}} a \\
\frac{\bar{C}}{L} &= \left(\frac{1-\beta}{\beta} \bar{S}\right)^{\beta} K_0^{\alpha} [L]^{\gamma} [l_Q]^{\gamma} [l_\theta] [\theta_0/L_\theta]^{1-\beta} a^{\beta}
\end{aligned}$$

Resource path:

$$R_t = \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} [\theta_0/L_\theta + at]^{-\frac{1}{\beta}}$$

with

$$\dot{R}_t = -\frac{1}{\beta}\left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_t^{-\frac{\alpha}{\beta}} L^{-\frac{\gamma}{\beta}} l_Q^{-\frac{\gamma}{\beta}} l_\theta^{-\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{-\left(1+\frac{1}{\beta}\right)} a < 0$$

and

$$\lim_{t\to\infty} R_t = \frac{\left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}}}{K_t^{\frac{\alpha}{\beta}} L^{\frac{\gamma}{\beta}} l_Q^{\frac{\gamma}{\beta}} l_\theta^{\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{\frac{1}{\beta}}} = 0, \quad \beta < 1.$$

Price path according to the first order conditions:

$$\begin{aligned}
\frac{p_R}{r} &= \frac{\beta}{\alpha}\frac{K_0}{R_t} = \frac{\beta}{\alpha}\frac{K_0}{\left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{-\frac{\alpha}{\beta}} [L]^{-\frac{\gamma}{\beta}} [l_Q]^{-\frac{\gamma}{\beta}} [l_\theta]^{-\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{-\frac{1}{\beta}}} \\
\frac{p_R}{r} &= \frac{\beta}{\alpha} K_0 \left[\frac{\bar{C}}{L}\right]^{\frac{1}{\beta}} K_0^{\frac{\alpha}{\beta}} [L]^{\frac{\gamma}{\beta}} [l_Q]^{\frac{\gamma}{\beta}} [l_\theta]^{\frac{1}{\beta}} \left[\theta_0/L_\theta + at\right]^{\frac{1}{\beta}}
\end{aligned}$$

A.3 Appendix for section 2.4

Allowing for a change in technology and productivity (33) maximum sustainable consumption can also change

$$\dot{C}_t^* = \dot{\theta}_t \breve{Q}^*.$$

Thus, with a given endogenous growth rate of technology the consumption growth rate can be determined

$$\begin{aligned}
\frac{\dot{C}_t^*}{C_t^*} &= \frac{\dot{\theta}_t \breve{Q}^*}{\theta_t \breve{Q}^* - I^*} \\
& \frac{\dot{\theta}_t \breve{Q}^*}{\left(1 - \frac{I^*}{\theta_t \breve{Q}^*}\right)\theta_t \breve{Q}^*}
\end{aligned}$$

If we include the generated technological progress the sustainable growth rate of consumption can continuously be maintained even in the long run

$$\lim_{t\to\infty}\frac{\dot{C}_t^*}{C_t^*}=\frac{\frac{\dot{\theta}}{\theta}}{1-0}=\frac{\dot{\theta}_t}{\theta_t}=aL_\theta.$$

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