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IZA DP No. 18068

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ABSTRACT

Output Fluctuations and Firm Recruitment Effort*

This paper examines the relationship between output fluctuations and firms' recruitment efforts using Danish data that link online job ads with high-frequency firm-level revenue and value-added. While overall output growth is weakly correlated with advertisement rates, decomposing output into permanent and transitory components reveals a strong link between persistent shocks and recruitment effort. A one standard deviation permanent shock raises advertisement rates by 10-16% of a standard deviation, whereas transitory shocks show no significant effect. These results highlight the importance of shock persistence in labor demand and offer empirical support for dynamic search-and-matching models of the labor market.

JEL Classification: J23, J63

Keywords: vacancies, online job advertisements, output growth, revenue growth, value-added growth, permanent and transitory shocks

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1 Introduction

The continuous, large-scale reallocation of workers across firms is a persistent and well-documented feature of market economies (Davis and Haltiwanger, 1999; Davis, Faberman, and Haltiwanger, 2012). Understanding the sources and implications of this reallocation process is essential, particularly due to its central role in driving productivity growth (Bartelsman and Doms, 2000; Foster, Haltiwanger, and Krizan, 2006) and wage growth (Topel and Ward, 1992; Haltiwanger, Hyatt, Kahn, and McEntarfer, 2018). A substantial body of theoretical work has emerged to interpret this phenomenon, especially through models of firm dynamics. These models typically posit that firms are subject to frequent idiosyncratic shocks—such as changes in demand, productivity, or input costs—which affect their marginal revenue product and lead to adjustments in their desired labor input (Hopenhayn, 1992).

The literature has been further enriched by incorporating search and matching frictions, which provide a more realistic depiction of labor market dynamics (Mortensen and Pissarides, 1994). A central insight from these models is that hiring is a costly activity for firms and it requires taking particular recruitment actions. Consequently, a key empirical implication is that a firm's recruitment effort should respond positively to shocks to its marginal revenue product. Despite its theoretical prominence, this prediction has not been empirically tested, primarily due to the lack of suitable data.

This paper addresses this gap by leveraging a novel dataset that combines high-frequency firm-level data on output (revenue and value-added) and online job advertisements, a key measures of recruitment effort. We empirically examine the relationship between firms' output growth and advertisement rate. Our findings qualitatively confirm core predictions of canonical firm-dynamics and search-and-matching models and provide empirical targets for their quantitative calibration.[1]

Our empirical analysis is based on data from Denmark. We construct a novel dataset by merging online job advertisement data with two rich administrative sources: value-added tax (VAT) records and a matched employer-employee dataset. While VAT and matched employer-employee datasets have been widely used in previous research, the integration with high-frequency data on

[1] We should note, however, that posting job advertisements is not the only form of recruitment effort for firms. Bagger and Galenianos (2025) show that a significant share of hires occurs through recall and that recall hiring is negatively correlated with the advertisement rate; this suggests that recruitment effort through the recall recruitment channel is negatively correlated with the posting of online job advertisements.

job advertisements is unique.[2] This merger enables us to explore previously unaddressed questions regarding firm-level recruitment behavior. Specifically, we obtain granular, high-frequency measures of firm recruitment effort—proxied by online job postings—alongside detailed indicators of firm performance, such as growth in revenue and in value-added.

We study the relationship between output growth and the advertisement rate using two complementary empirical approaches. In the first approach, output fluctuations are treated as realizations of a generic shock process, and the advertisement rate is assumed to respond only to the magnitude of these fluctuations. In the second approach, output growth is decomposed into permanent and transitory components, allowing us to explore whether the persistence of shocks influences their relationship with the advertisement rate. These two approaches yield markedly different results.

In the first empirical approach, we find very small average partial effects of output shocks on job advertisement activity. A one standard deviation increase in revenue is associated with, at most, an increase in the advertisement rate of 2% of a standard deviation, while value-added growth is essentially uncorrelated with the advertisement rate. This result is robust across several estimation strategies, including fixed effects and random effects models with time-invariant firm and firm-quarter heterogeneity. It also holds when we apply a Tobit estimator to account for the fact that the dependent variable (the advertisement rate) is non-negative by definition and exhibits a mass point at zero.

Motivated by the weak associations uncovered through the first empirical approach and the documented high volatility of output growth, we proceed to examine the relationship between the advertisement rate and output shocks of differing persistence. To do so, we estimate a process for firm output that distinguishes between permanent and transitory shocks, using a combination of random walk and moving average components. A notable, and somewhat surprising, finding from our exploration of the output data is the presence of strong firm-specific seasonal patterns. If not properly addressed, this feature induces long autocovariances and oscillatory dynamics in output growth. We mitigate this issue by focusing on year-on-year growth rates, which effectively

[2] There is an extensive literature linking firm productivity to wages both empirically and theoretically, see Card, Cardoso, Heining, and Kline (2018) for a review. Roys (2016) and Maibom and Vejlin (2021) consider the joint response of wages and employment to profitability shocks. Recent papers that merge administrative firm-level data with information on vacancies have detailed employment, worker flows and wage series but lack measures of firm output, see e.g. Carrillo-Tudela, Gartner, and Kaas (2021) and Mueller, Osterwalder, Zweimüller, and Kettemann (2020).

control for firm-quarter time-invariant heterogeneity. The empirical model of the output process fits the data well and reveals that the transitory component is much more volatile than the permanent component: in the revenue data the standard deviation of the transitory component is 60% larger than that of the permanent component, while in the value-added data it is 2.5-3 times larger, depending on the specification.

To estimate the relationship between permanent output shocks and advertisement-posting, we employ two approaches. First, we use a long-difference regression approach similar to Juhn, McCue, Monti, and Pierce (2018). The estimates from this approach are much larger in magnitude than the single-shock models and, unlike in the single-shock models, they are of similar magnitude for revenue and value-added. Specifically, a positive permanent one standard deviation shock in revenue or value-added is associated with an increase in the advertisement rate of 5-6% of a standard deviation.

Second, we employ an instrumental variables strategy similar to Guiso, Pistaferri, and Schivardi (2005) which identifies even stronger associations: a positive permanent one standard deviation shock to revenue is associated with an increase of 10% of a standard deviation in the advertisement rate and an equivalent shock to value-added is associated with a rise of 16% of a standard deviation in the advertisement rate. The long-difference regression approach is likely to suffer from attenuation bias due to measurement error and, therefore, it is unsurprising that it leads to lower estimates. Finally, we use the instrumental variables approach to estimate the effect of transitory shocks on the advertisement rate which yields quantitatively trivial estimates.

We draw two main conclusions from this analysis. First, high-frequency measures of output are extremely volatile and, so, the relationship between raw output growth and our measure of recruitment effort is economically negligible. Second, focusing on the persistent components of output growth provides a measure of shocks that are economically meaningful to the firm and that correlate strongly with our measure of recruitment effort. The estimates of the effect of permanent output shocks vary somewhat between estimation methods and the choice of output measure (revenue or value-added) but are quantitatively meaningful. A theoretically coherent interpretation of these findings is immediate: recruitment effort is a form of investment and, hence, firms only exert recruiting effort in response to output shocks that they understand to be persistent, while they do not respond to output shocks that are transitory.

Related literature. Our paper contributes to the empirical literature on vacancy posting and the recruitment behavior of firms. Davis, Faberman, and Haltiwanger (2013) use the Job Openings and Labor Turnover Survey (JOLTS) data in the US and show that the job-filling rate rises steeply with employer growth and with gross hires, suggesting that employers rely on other instruments to hire workers. However, they do not have direct measures of firm output and productivity and cannot identify the exact drivers of firm recruitment.[3] A parallel strand of literature uses online job advertisement data from job boards (see, for example, Marinescu and Wolthoff (2020), Davis and Samaniego de la Parra (2017), Modestino, Shoag, and Ballance (2019), Hershbein and Kahn (2018) and Banfi and Villena-Roldan (2019) among others). While these studies take advantage of more detailed data about job postings— such as skill and education requirements and geographical location of the job—they have limited information about firms and thus cannot speak to what drives the advertisement-posting actions. More recent papers link the vacancy data with the administrative matched employer-employee datasets (see for example Mueller, Osterwalder, Zweimüller, and Kettemann (2020) for Austria and Carrillo-Tudela, Gartner, and Kaas (2021) for Germany), making it possible to examine the relationship between vacancies and a wide range of outcomes. However, similarly to Davis, Faberman, and Haltiwanger (2013), these papers lack information on firm output.

Methodologically, our paper relates to the literature on the pass-through of firm-level shocks to workers' income. We adapt the estimation procedure from Juhn, McCue, Monti, and Pierce (2018) and Guiso, Pistaferri, and Schivardi (2005) to distinguish the effects of permanent and transitory shocks on the advertisement rate. For a review of papers that use a similar approach to examine the effect of firm-level shock on wages, see Card, Cardoso, Heining, and Kline (2018). Finally, for papers that estimate the effects of productivity shocks on employment outcomes see Roys (2016), Kline, Petkova, Williams, and Zidar (2019), and Maibom and Vejlin (2021).

In a companion paper, Bagger, Fontaine, Galenianos, and Trapeznikova (2022), we use the same data sources to document the relationship between advertisements (vacancies), hires and separations, and the growth of output at the firm-level. Here, we extend the analysis by formalizing the effect of idiosyncratic shocks to firm output on the recruitment actions of firm.

[3]Other studies that use firm-level survey data include Barron and Bishop (1985), Burdett and Cunningham (1998) and Faberman and Menzio (2018) for the US, van Ours and Ridder (1991), van Ours and Ridder (1993) and van Ommeren and Russo (2014) for the Netherlands, Carrillo-Tudela, Gartner, and Kaas (2021) for Germany.

2 Data

This section presents the data sources, describes how we build the analysis data, and reports a set of descriptive statistics that will guide our analysis.

2.1 Data sources

We use firm-level information on online job advertisements, output (revenue and value-added), employment, and industry of activity.

Job Advertisements. The job advertisement data originates with Jobindex A/S, a private firm that operates Denmark's largest online job board. A job advertisement is included in the Job index data either because a firm has paid Jobindex to feature the advertisement or because Jobindex scraped the job advertisement elsewhere on the internet (e.g., from public job center websites, firm websites, online newspapers, other job boards, etc.).[4] Jobindex thus provides a comprehensive database of online job advertisements and claims to capture more than 90% of online job advertisements in Denmark. The unit of observation is a job advertisement which includes the date of posting, the occupation of the posted job opening (recorded according to the job board's own detailed occupation classification), and the firm's unique identifier in the business registry (the CVR-number) for approximately two-thirds of the job postings. The CVR-number is a firm's main administrative identifier and routinely appears on invoices and company websites. The CVR number was either provided directly by the firm in the event that it paid Jobindex to post an advertisement or it was included in the information scraped by Jobindex. Some advertisements might refer to multiple available positions, but we do not have this information. Jobindex provided the job advertisement data from January 1st, 2002 to August 23rd, 2009. The dataset contains approximately two million online job advertisements.

Output. Firm output is measured by revenue and value-added, as recorded in administrative VAT accounts data. In Denmark, firms with expected annual revenue that exceeds 50,000 DKK (approximately USD 8,000) must register a VAT account with the tax authorities, which is linked to the firms' CVR-number, and are required to settle their VAT on a monthly, quarterly,

[4] Jobindex operates an algorithm that identifies *new* job advertisements and removes identical entries. Brodersen, Dimova, and Rosholm (2016) report the sources of job advertisements posted on Jobindex in July 2014 as 35% direct posting on Jobindex, 35% other job boards, 25% public job centers, and 5% firm websites.

or semi-annual frequency, depending on the level of revenues.[5] When settling their VAT accounts, firms report their revenues from sales and expenses from purchases.[6] The unit of observation in the VAT account data is a firm-month and an observation includes firm revenues, purchases, value-added (the difference between revenues and purchases), and an indicator of whether the firm settles VAT monthly, quarterly, or semi-annually. For firms that report quarterly and semi-annually, the monthly data is the result of imputations. Our analysis is conducted at a quarterly frequency and we will drop firms that report semi-annually, so we will not use any imputed data. The monthly VAT accounts data is available from 2001M1 to 2013M8. We use Denmark's consumer price index to deflate revenues, purchases, and value-added.

Employment. Firm employment is measured using person-level job spells data. The job spells data records the start- and end-dates for all jobs of all legal residents in Denmark aged 15-70 and it is constructed from income tax reports and a host of other administrative data sources.[7] Workers are identified by (an anonymized version of) their social security number. Employers are identified at the firm-level via their CVR-number and at the establishment-level via an establishment identification number (`LBNR`, identifying a physical workplace). The unit of observation in the job spells data set is a person-spell-year combination. The job spells data spans the period from January 1st, 1985 to December 31st, 2012.

Industries. Firms' industry codes stem from IDA-S, the employer component of IDA (Integreret Database for Arbejdsmarkedforskning), an annual matched employer-employee panel that covers the entire Danish population and all firms with economic activity and is constructed and maintained by Statistics Denmark. IDA-S contains annual information on all physical workplaces

[5] As of 2021, firms report VAT monthly if annual revenues exceed 50 million DKK (≈USD 8 million), quarterly if revenues fall between 5 and 50 million DKK (≈USD 800,000-8 million) or during their first six quarters, and semi-annually if revenues are below 5 million DKK. VAT must be reported on fixed dates. For instance, semi-annual filers report January–June VAT on September 1st and July–December on March 1st. Quarterly filers report on June 1st, September 1st, December 1st, and March 1st. Monthly filers report on the 25th–27th of the following month, except for June, which is due August 17th.

[6] Transactions between Danish counterparties (firm-to-firm or firm-to-consumer) are subject to VAT and thus automatically recorded. For cross-border transactions, VAT is generally levied in the buyer's country, so purchases by Danish firms from abroad are recorded (consumer imports are not relevant here). Sales by Danish firms within the EU are not subject to Danish VAT but are tracked by EU authorities and settled through national tax agencies, and thus appear in our data. Exports to non-EU countries must be declared via the EORI system and to Danish tax authorities, so firm-to-firm goods exports are recorded. However, direct sales to non-EU consumers and sales of services outside the EU are generally not subject to Danish VAT and are likely missing from our data.

[7] Henning Bunzel (Aarhus University) has been instrumental in developing the labor market spells dataset. Hejlesen (2016) provides a technical description of the construction of the labor market spell data from administrative records.

(establishments) in Denmark with industry classification recorded according to Nomenclature statistique des Activités économiques dans la Communauté Européenne (NACE).[8] The unit of observation in IDA-S is an establishment-year and the IDA-S data set covers 1980-2019.

2.2 Merging the data sources and creating the analysis panel

We describe in detail the steps we take to merge the different datasets, clean them and create a quarterly firm-level panel.

First, we merge the job spells data with the IDA-S data set using the shared establishment ID. We retain all job spells data observations, whether matched or not to an IDA-S observation and discard unmatched IDA-S observations. The unit of observation in this merged matched employer-employee panel is a worker-spell-year and it covers 1985-2012 with industry information.[9] From the merged MEE panel we create a monthly firm employment panel for 1998M1-2010M12: for each firm ID and each month, we record the number of workers on the firm's payroll and the industry code of the firm (including a "missing industry" category).

Second, we turn to the advertisement data. We discard observations with missing firm-ID (one-third of the total) and then we aggregate from the original advertisement-by-date structure to a monthly 2002M1-2009M8 firm panel that records the number of advertisements (total and by occupation) for each firm in each month. We merge this with the firm employment panel using the shared firm-ID. We match 80 percent of firm-months in the advertisement data to a firm-month in the employment panel. We retain all firm-months in the firm employment panel and discard unmatched advertisement panel observations. Employment panel firm-months that are not matched to a firm-month in the advertisement data are coded as firm-months with zero advertisement posting events.

Third, we merge the monthly firm output panel with the firm employment panel by the firm ID. We are able to assign output data to 85 percent of the firm-months in the 2001M1-2010M12 employment panel (conversely, we match 75 percent of the 2001M1-2010M12 output panel observations to the employment panel); again, we retain all firm-months in the original

[8] NACE classifications changed in 2003 (NACE 1.1 replacing 1.0) and in 2007 (NACE 2.0 replacing 1.1). Statistics Denmark provides both old and new codes in those years, allowing us to build empirical correspondence tables. We use these to assign NACE 1.1 codes to pre-2003 observations and then to assign NACE 2.0 codes to pre-2007 observations. Hence, industry is classified using NACE 2.0 throughout the observation period.

[9] Industry codes stem from IDA-S, where they are recorded at the establishment level. A firm in the merged MEE panel may comprise multiple establishments in any given year in which case the firm is assigned the industry code of the establishment with the highest employment in that year.

firm employment panel, whether matched or not to an output panel observation, and discard unmatched output panel observations. The discarded output panel observations are mostly from very small firms. We have now built a monthly panel of all Danish firms with information on employment (1998M1-2010M12), advertisement posting (2002M1-2009M8), and output (2001M1-2010M12) where firms are classified by industry.

Fourth, we aggregate the monthly firm panel to a quarterly frequency: quarterly advertisements, revenues, and purchases are the sum of the three monthly observations and quarterly employment is measured as the employment on the first day of the quarter, e.g. on January 1st for Q1 observations. We restrict attention to the 26-quarter period 2003Q1 to 2009Q2 for which we have quarterly advertisement data. We keep firms that belong to the NACE 2.0 business sector and discard the firms that are not assigned to an industry and those whose industry is outside the business sector.[10]

Finally, we discard firms with missing or negative values for revenues or purchases, we discard firms that are ever observed to report VAT semi-annually or with missing information on the reporting frequency.[11] In the last step, we discard firms that are never observed to post an online job advertisement during the 26-quarter observation period, which leaves us with 21,132 firms. Table 1 provides a comparison between retained firms (observed with at least one online job advertisement) and discarded firms (never observed with an online job advertisement). Although numerous, the discarded firms account for only 27 percent of total employment, and 23 percent of total revenue (21 percent of value-added). Some of the discarded firms engaged in active recruiting over the observation period (they account for 31 percent of total hires), but we either do not observe their online advertisement, do not have their CVR-numbers, or they made use of recruiting channels other than online job advertisement. Overall, the retained firms account

[10]Using NACE 2.0 section labels, the business sector consists of the following industries: Mining and quarrying; Manufacturing; Electricity, gas, steam and airconditioning supply; Water supply, sewerage contractors, waste management, and remediation activities; Construction; Wholesale and retail trade, repair of motor vehicles and motorcycles; Transport and storage; Accommodation and food service activities; Information and communication; Financial and insurance activities; Real estate activities; Professional, scientific, and technical activities; and Administrative and support service activities. The discarded non-business sector firms consists of the following NACE 2.0 industries: Agriculture, forestry and fishing; Public administration and Defence and compulsory social security; Education; Human health and social work activities; Arts, entertainment and recreation; Other service activities; Activities of households as employers; Activities of extraterritorial organizations and bodies.

[11]Only a few firms are discarded because they have missing information on the reporting frequency. We discard about 48,000 firms that are ever observed to settle VAT accounts at a semi-annual frequency. Firms that always report semi-annually have average employment of 1.5 workers, average annual revenue of DKK 660,000 (USD 100,000), and average annual value-added of DKK 264,000 (USD 40,000) so they are very small and account for a very small share of the economy.

Table 1: Characteristics of firms with and without an online job advertisement

	W/ ONLINE JOB ADVERT.	W/O ONLINE JOB ADVERT.
NUMBER OF FIRMS	21,132	69,230
AVERAGE NUMBER OF EMPLOYEES	42	8
AVERAGE QUARTERLY REVENUE (IN DKK 1,000)	21,918	3,417
AVERAGE QUARTERLY VALUE ADDED (IN DKK 1,000)	7,389	1,030
SHARE OF TOTAL EMPLOYMENT	0.73	0.27
SHARE OF TOTAL REVENUE	0.77	0.23
SHARE OF TOTAL VALUE ADDED	0.79	0.21
SHARE OF TOTAL HIRES	0.69	0.31

Notes: In May, 2021 the exchange rate of the Danish Krone to the US Dollar was approximately 1 USD = 6.2 DKK.

for almost 80% of the economic activity, employment and hires in the Danish business sector.

The firm panel used for analysis contains 21,132 firms and comprises 477,092 firm-quarters spanning the 26-quarter period 2003Q1-2009Q2 with firm-level information on advertisement-posting and output.

2.3 Key data features

We now define the central variables that we will use in our analysis of a firm's output growth and its recruitment effort, and present some features of their distribution that will guide our empirical analysis.

Notation and variable definitions. Let j index firms and let t index quarters, let x_{jt} be a generic panel data variable and $\Delta x_{jt} = x_{jt} - x_{jt-1}$ be the first difference operator. Output for firm j in quarter t is measured by revenue and value-added, denoted by R_{jt} and Y_{jt}, respectively. The natural logarithms of these variables are denoted by $r_{jt} = \ln R_{jt}$ and $y_{jt} = \ln Y_{jt}$, so Δr_{jt} and Δy_{jt} are the firm's quarter-on-quarter growth rates in revenue and value-added, respectively.

Recruitment effort for firm j in quarter t is measured via online job advertisements. Let A_{jt} denote the number of online job advertisements posted by firm j in quarter t. Let N_{jt} denote the employment of firm j in quarter t and define the advertisement rate of firm j in quarter t by $a_{jt} = A_{jt}/(N_{jt} + A_{jt})$.

Cross-sectional variation. Table 2 presents summary statistics of the quarterly advertisement rate a_{jt}, growth rates of revenue, Δr_{jt} and value-added, Δy_{jt}. Table 2 documents that 22

Table 2: Quarterly advertisement rate and output growth

	Ad rate		Output growth	
	a	$a > 0$	Δr	Δy
Average	0.028	0.223	0.007	0.011
Standard deviation	0.094		0.495	0.946
10th percentile	0.000		−0.469	−0.941
25th percentile	0.000		−0.190	−0.374
50th percentile	0.000		0.007	0.012
75th percentile	0.000		0.211	0.400
90th percentile	0.083		0.484	0.963
Number of firms	21,132	21,132	21,046	20,820
Number of firm-quarters	83,850	83,850	83,220	80,708
Number of observations	479,578	479,578	454,915	398,648

Notes: a is the quarterly advertisement rate, Δr is the quarterly revenue growth rate, and Δy is the quarterly value added growth rate. To comply with disclosure rules, the kth percentile P_k is computed as the average value in the $(P_{k-2.5}, P_{k+2.5}]$-interval.

percent of firm-quarters involve the posting of online job advertisements. The average quarterly advertisement rate is 0.028 and the 90th percentile of the ad rate distribution is 0.083 (the 95th percentile, not shown, is 0.167). The average quarterly growth rate is 0.7 percent for revenue and 1.1 percent for value added. Both revenue and value-added growth are highly volatile, with standard deviations of 0.495 for revenue and 0.946 for value added.

3 Recruitment effort and output growth

We want to evaluate the theoretical prediction that firms experiencing positive shocks would like to increase their labor input and, hence, will exert more recruitment effort. This section estimates the relationship between growth in revenue and value-added, our measures of firm output shocks, and the posting of online job advertisements, our measure of recruitment effort.

The key aspect of this section's analysis is that the relationship between output growth and the advertisement rate is fully captured by the magnitude of output fluctuations; in other words, output fluctuations are interpreted as the outcome of a single-shock process whose only relevant characteristic for advertisement-posting is the magnitude of the shock. In sections 4 and 5, we consider a different model where output growth is also characterized by its persistence, and the relationship of an output shock with the advertisement rate might depend on the shock's persistence.

An important dimension of the job advertisement data is that the advertisement rate is non-negative and has mass at zero (as shown in table 2, only 22 percent of the firm-quarters feature positive advertisement rate). For this reason, we estimate two empirical models: a linear regression model and a corner-solution regression (Tobit) model that explicitly accounts for this non-negativity.

3.1 Linear regression model

We estimate the *average partial effect* (*APE*) of observed output growth on the advertisement rate. The linear regression model is

$$a_{jt} = \gamma \Delta z_{jt} + \mathbf{g}'_{jt}\boldsymbol{\kappa} + \rho_{j\mathrm{Q}(t)} + \xi_{jt}, \tag{1}$$

where a_{jt} is firm j's advertisement rate at time t; Δz_{jt} is output growth between time $t-1$ and t and z_{jt} denotes revenue or value-added ($z \in \{r, y\}$); $\mathbf{g}_{jt}$ is a column-vector containing a constant term, time dummies, industry dummies, and industry-time interaction dummies; $\mathrm{Q} : \{1, 2, ..., T\} \to \{1, 2, 3, 4\}$ maps time period t to a calendar quarter; $\rho_{jQ(t)}$ is an unobserved time-invariant firm-quarter heterogeneity component; and ξ_{jt} is the error term.[12] Strictly speaking, all variables should indicate whether they refer to the relationship between the advertisement rate and revenue or value-added, e.g. γ^z, $\boldsymbol{\kappa}^z$ etc. but we will avoid this to keep our notation manageable. For future reference, let $\Delta \mathbf{z}_j = (\Delta z_{j1}, \Delta z_{j2}, \ldots, \Delta z_{jT})'$, $\mathbf{G}_j = (\mathbf{g}'_{j1}, \mathbf{g}'_{j2}, \ldots, \mathbf{g}'_{jT})'$, $\boldsymbol{\rho}_j = (\rho_{j1}, \rho_{j2}, \rho_{j3}, \rho_{j4})$, and $\boldsymbol{\rho}_{j,-\mathrm{Q}(t)}$ be the vector obtained by removing $\rho_{j\mathrm{Q}(t)}$ from $\boldsymbol{\rho}_j$; for example, if $Q(t) = 1$ then we have $\boldsymbol{\rho}_{j,-Q(t)} = (\rho_{j2}, \rho_{j3}, \rho_{j4})$.[13]

The linearity of (1) means that the average partial effect of output growth on the advertisement rate conditional on covariates and time-invariant heterogeneity is given by the estimate of γ. Table 3 reports estimates of equation (1) for revenue and value-added under different assumptions about the term $\rho_{jQ(t)}$ and the error ξ_{jt}, which correspond to different ways of modeling

[12] We have repeated this analysis including lagged output growth as an additional explanatory variable. The estimates of the coefficient on lagged output growth are quantitatively insignificant in all specifications and, therefore, we omit them here.

[13] Each estimator of (1) invoke assumptions about the relationship between the error term ξ_{jt} and the regressors Δz_{jt} and $\mathbf{g}'_{jt}$. Some, for example, assume strict exogeneity conditional on the heterogeneity component, i.e., $\mathbb{E}(\xi_{jt}|\mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_j) = 0$. Although our notation is for a balanced panel, the data is in fact an unbalanced panel. Let $s_{jt} = 1$ if firm-j is observed in quarter-t, and 0 otherwise, and define $\mathbf{s}_j \equiv (s_{j1}, s_{j2}, \ldots, s_{jT})$. In this context, strict exogeneity becomes $\mathbb{E}(\xi_{jt}|\mathbf{z}_j, \mathbf{G}_j, \mathbf{s}_j, \boldsymbol{\rho}_j) = 0$, which allows for selection into the panel on output growth, observed covariates, and time-invariant firm heterogeneity, but not on the error term. Throughout, we omit $\mathbf{s}_j$ in notation for brevity.

unobserved time-invariant heterogeneity.

Pooled OLS estimator. Column (1) in table 3 reports the estimates of equation (1) for revenue and value-added from a pooled OLS estimator, i.e. where we impose $\rho_{j\mathrm{Q}(t)} = 0$ for all j and all t. This estimator provides a consistent estimate of γ under the assumption that Δz_{jt} and $\mathbf{g}'_{jt}$ are contemporaneously exogenous: $\mathbb{E}(\xi_{jt}|\Delta z_{jt}, \mathbf{g}'_{jt}) = 0$. Identification of γ stems from both between-firm and within-firm variation in output growth, conditional on $\mathbf{g}'_{jt}$. We cluster the standard errors at the firm-level, so inference is robust to arbitrary heteroskedasticity and serial correlation in the errors. The estimate of γ for revenue is 0.003, which is statistically significant but quantitatively insignificant: a revenue growth rate event equal to one standard deviation (0.495, see Table 2) corresponds to an increase in a firm's advertisement rate of 0.0015, which is less than 2 percent of a standard deviation of the advertisement rate (0.094, see Table 2). The estimate of γ for value-added is 0.000.

Fixed effect estimator. Column (2) in table 3 reports estimates of equation (1) from a fixed effect estimator that allows for time-invariant firm heterogeneity ρ_j, i.e. we impose $\rho_{j\mathrm{Q}(t)} = \rho_j$ for all j and all t. The estimator provides a consistent estimate of γ under the assumption of strict exogeneity of Δz_{jt} and $\mathbf{g}'_{jt}$ conditional on time-invariant heterogeneity: $\mathbb{E}(\xi_{jt}|\Delta \mathbf{z}_j, \mathbf{G}_j, \rho_j) = 0$. This estimator does not impose restrictions on $\mathbb{E}(\rho_j|\Delta \mathbf{z}_j, \mathbf{G}_j)$, i.e. the dependence between ρ_j, $\Delta \mathbf{z}_j$ and $\mathbf{G}_j$ is unrestricted. The identification of γ stems from within-firm variation in output growth, conditional on $\mathbf{g}'_{jt}$. We cluster the standard errors at the firm level. The estimate of γ is 0.002 for revenue and, hence, remains quantitatively insignificant. The estimate of γ for value-added is 0.000. The standard deviation of time-invariant firm heterogeneity is 0.061 for revenue and 0.060 for value-added and they are comparable to the standard deviation of the error at 0.071 and 0.69. The fixed effects estimators thus identifies significant time-invariant heterogeneity in the advertisement rate intercept across firms.

Column (3) in table 3 reports estimates of equation (1) from a fixed effect estimator that allows for time-invariant firm-quarter heterogeneity $\rho_{jQ(t)}$, which provides a consistent estimate of γ under the assumption of strict exogeneity of Δz_{jt} and $\mathbf{g}'_{jt}$ conditional on time-invariant heterogeneity: $\mathbb{E}(\xi_{jt}|\Delta \mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_j) = 0$. This estimator does not impose restrictions on $\mathbb{E}(\rho_{j\mathrm{Q}(t)}|\Delta \mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_{j,-\mathrm{Q}(t)})$, i.e. the dependence between $\boldsymbol{\rho}_j$, $\Delta \mathbf{z}_j$ and $\mathbf{G}_j$ is unrestricted. The identification of γ stems from within-firm year-on-year variation in output growth rates, conditional on $\mathbf{g}'_{jt}$. We cluster the standard errors at the firm level. The estimate of γ is 0.003 for revenue and 0.000 for value-added. The standard deviation of the firm-quarter fixed effects is 0.071 for both output series, which is 16-18 percent larger than the standard deviation of the coarser firm fixed effect in column (2), and slightly larger than the standard deviation of the error.

Random effect estimator. Columns (4) and (5) in table 3 report estimates of equation (1) from a random effect estimator and allow for time-invariant firm heterogeneity and time-invariant firm-quarter heterogeneity, respectively. For the consistent estimation of γ, we impose strict exogeneity of Δz_{jt} and $\mathbf{g}'_{jt}$ conditional on time-invariant heterogeneity, i.e. $\mathbb{E}(\xi_{jt}|\Delta\mathbf{z}_j, \mathbf{G}_j, \rho_j) = 0$ for column (4) and $\mathbb{E}(\xi_{jt}|\Delta\mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_j) = 0$ for column (5). Furthermore, for column (4), we assume that the time-invariant firm heterogeneity component ρ_j is mean-independent of the observed covariates, i.e. $\mathbb{E}(\rho_j|\Delta\mathbf{z}_j, \mathbf{G}_j) = 0$; and for column (5) we assume that the time-invariant firm-quarter heterogeneity component $\rho_{j\mathrm{Q}(t)}$ is mean-independent of the observed covariates and the other three firm-quarter heterogeneity components $\boldsymbol{\rho}_{j,-\mathrm{Q}(t)}$, i.e. $\mathbb{E}(\rho_{j\mathrm{Q}(t)}|\Delta\mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_{j,-\mathrm{Q}(t)}) = 0$.

For revenue, the estimate of γ is 0.003 under time-invariant firm heterogeneity, see column (4), and 0.004 under time-invariant firm-quarter heterogeneity, see column (5), and hence, the estimates remain quantitatively insignificant. The estimates of γ for value-added is 0.000 for both heterogeneity specifications. The standard deviation of time-invariant firm heterogeneity is 0.070 for revenue and 0.071 for value-added, while the standard deviation of time-invariant firm-quarter heterogeneity is 0.060 for revenue and 0.058 for value-added. These estimates point to significant time-invariant heterogeneity in advertisement rates among firms and firm-quarters.

In summary, our estimates of the linear regression model suggest that the relationship between output growth and the advertisement rate is extremely weak, regardless of the way that we model time-invariant heterogeneity, which is estimated to be substantial.

Table 3: Advertisement rate and output growth

	Linear model					Corner solution model		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Revenue growth ($\Delta z_{jt} = \Delta r_{jt}$)							
γ	0.003***	0.002***	0.003***	0.003***	0.004***	0.014***	0.015***	0.012***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
APE	0.003***	0.002***	0.003***	0.003***	0.004***	0.003***	0.003***	0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$APE \times \sigma_{\Delta r}$	0.002	0.001	0.002	0.001	0.001	0.001	0.002	0.001
σ_ρ		0.061	0.071	0.070	0.060		0.122	0.127
σ_ξ	0.084	0.071	0.070	0.071	0.070		0.195	0.193
$\sqrt{\sigma_\rho^2 + \sigma_\xi^2}$	0.084	0.093	0.100	0.089	0.092	0.233	0.230	0.231
Firms	21,046	21,046	21,046	21,046	21,046	21,046	21,046	21,046
Observations	454,915	454,915	454,915	454,915	454,915	454,915	454,915	454,915
	Value added growth ($\Delta z_{jt} = \Delta y_{jt}$)							
γ	0.000***	0.000**	0.000**	0.000***	0.000***	0.002***	0.002***	0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
APE	0.000***	0.000**	0.000**	0.000***	0.000***	0.000***	0.000***	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$APE \times \sigma_{\Delta y}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
σ_ρ		0.060	0.071	0.071	0.058		0.123	0.124
σ_ξ	0.082	0.069	0.068	0.069	0.068		0.197	0.194
$\sqrt{\sigma_\rho^2 + \sigma_\xi^2}$	0.082	0.091	0.098	0.087	0.089	0.230	0.232	0.230
Firms	20,820	20,820	20,820	20,820	20,820	20,820	20,820	20,820
Observations	398,648	398,648	398,648	398,648	398,648	398,648	398,648	398,648
Covariates	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	No	Yes	No	No	No	No	No	No
Firm-qrt FE	No	No	Yes	No	No	No	No	No
Firm RE	No	No	No	Yes	No	No	No	No
Firm-qrt RE	No	No	No	No	Yes	No	No	No
Firm CRE	No	No	No	No	No	No	Yes	No
Firm-qrt CRE	No	No	No	No	No	No	No	Yes

Notes: Asymptotic standard errors in parentheses. The standard errors are clustered at the firm level for specifications (1) through (6); for specifications (7) and (8), we assume i.i.d. errors. The covariates in specifications (1) through (5) include time dummies, whereas specifications (6), (7) and (8) the covariates include a quadratic polynomial trend. ***, ** and * indicates statistical significance at the 1, 5, and 10 percent level, respectively.

3.2 Corner solution regression model

For the *corner solution regression model*, we define a latent advertisement rate variable a_{jt}^* and specify the measured advertisement rate to be the non-negative part of a_{jt}^*:

$$\begin{aligned} a_{jt}^* &= \gamma \Delta z_{jt} + \mathbf{g}_{jt}' \boldsymbol{\kappa} + \rho_{jQ(t)} + \xi_{jt}, &(2)\\ a_{jt} &= \max(0, a_{jt}^*). &(3) \end{aligned}$$

The set of explanatory variables is identical to that in equation (1), with the exception that, for computational reasons, the time trend is specified as a quadratic polynomial.[14] In this empirical model, the relationship between the advertisement rate and output growth is non-linear: the partial effect of Δz_{jt} on a_{jt} depends on whether a_{jt}^* is positive or negative which, in turn, depends not only on Δz_{jt}, but also on $\mathbf{g}_{jt}$, $\rho_{jQ(t)}$, and ξ_{jt}.

Let $\mathbb{E}(a_{jt}|\Delta z_{jt}, \mathbf{g}_{jt}', \rho_{jQ(t)})$ denote the conditional mean function of the advertisement rate a_{jt}, given output growth Δz_{jt}, the vector of observed covariates $\mathbf{g}_{jt}$, and the unobserved time-invariant heterogeneity component $\rho_{jQ(t)}$. We will estimate the *average partial effect* (*APE*) of observed output growth Δz_{jt} on $\mathbb{E}(a_{jt}|\Delta z_{jt}, \mathbf{g}_{jt}', \rho_{jQ(t)})$. The APE is given by:

$$APE = \int \frac{\partial \mathbb{E}(a_{jt}|\Delta z_{jt}, \mathbf{g}_{jt}', \rho_{jQ(t)})}{\partial \Delta z_{jt}} \mathrm{d}D(\Delta \mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_j) \qquad (4)$$

where $D(\Delta \mathbf{z}_{jt}, \mathbf{G}_j, \boldsymbol{\rho}_j)$ is the distribution of characteristics for firm j.

The variables $\Delta \mathbf{z}_j$ and $\mathbf{G}_j$ are observed while $\boldsymbol{\rho}_j$ is unobserved and, hence, it is useful to separate them so that we can be explicit about the assumptions we make about $\boldsymbol{\rho}_j$. Using the fact that $D(\Delta \mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_j) = D(\boldsymbol{\rho}_j|\Delta \mathbf{z}_j, \mathbf{G}_j) D(\Delta \mathbf{z}_j, \mathbf{G}_j)$ we rewrite equation (4) as follows:

$$APE = \int \left\{ \int \frac{\partial \mathbb{E}(a_{jt}|\Delta z_{jt}, \mathbf{g}_{jt}', \rho_{jQ(t)})}{\partial \Delta z_{jt}} \mathrm{d}D(\boldsymbol{\rho}_j|\Delta \mathbf{z}_j, \mathbf{G}_j) \right\} \mathrm{d}D(\Delta \mathbf{z}_j, \mathbf{G}_j),$$

We proceed to estimate APE using a Tobit model with two different assumptions about the distribution of time-invariant heterogeneity $D(\boldsymbol{\rho}_j|\Delta \mathbf{z}_j, \mathbf{G}_j)$.

[14] In the linear regression model and the Pooled Tobit estimator, the estimates of the coefficient of interest, γ, are nearly identical whether we use the fully flexible time dummies or the quadratic time trend.

Pooled Tobit estimator. Column (6) of table 3 reports the estimates from a pooled Tobit estimator where we impose that $\rho_{jQ(t)} = 0$ for all j and t. This estimator provides a consistent estimate of APE under the assumptions that Δz_{jt} and $\mathbf{g}'_{jt}$ are contemporaneously exogenous (i.e. $\mathbb{E}(\xi_{jt}|\Delta z_{jt}, \mathbf{g}'_{jt}) = 0$) and ξ_{jt} is conditionally normal:

$$\xi_{jt}|\Delta z_{jt}, \mathbf{g}_{jt} \sim \mathcal{N}(0, \sigma_\xi^2).$$

The parameters of the model are estimated by Partial Maximum Likelihood. With these estimates, we can compute the APE using the partial effect of Δz_{jt} on $\mathbb{E}[a_{jt}|\Delta z_{jt}, \mathbf{g}_{jt}]$ which, following standard calculations (Wooldridge, 2010), is given by:

$$\frac{\partial \mathbb{E}\left(a_{jt}|\Delta z_{jt}, \mathbf{g}_{jt}\right)}{\partial \Delta z_{jt}} = \gamma \Phi\left(\frac{\gamma \Delta z_{jt} + \mathbf{g}'_{jt}\boldsymbol{\kappa}}{\sigma_\xi}\right).$$

The estimate of APE is 0.003 for revenue and, while it is highly statistically significant, the estimate remains quantitatively insignificant. A one-standard deviation revenue shock is associated with a rise in the advertisement rate by 0.002, which is 2% of the standard deviation of a_{jt}. The estimate of APE for value-added is 0.000.

Correlated random effects Tobit estimator. Columns (7) and (8) of table 3 report the estimates from a correlated random effects Tobit estimator with time-invariant firm heterogeneity and time-invariant firm-quarter heterogeneity, respectively. This estimator provides a consistent estimate of APE under the assumptions of strict exogeneity of Δz_{jt} and $\mathbf{g}'_{jt}$ conditional on $\rho_{jQ(t)}$ (i.e. $\mathbb{E}(\xi_{jt}|\Delta \mathbf{z}_j, \mathbf{G}_j, \rho_{jQ(t)}) = 0$) and conditional normality of ξ_{jt} and $\boldsymbol{\rho}_j$.

The distribution of ξ_{jt} is given by:

$$\xi_{jt}|\Delta \mathbf{z}_j, \mathbf{G}_j, \boldsymbol{\rho}_j \sim \mathcal{N}(0, \sigma_\xi^2).$$

For $\rho_{jQ(t)}$ we take a correlated random effects approach to parameterize the dependence between unobserved and observed heterogeneity and we do not impose independence between $\boldsymbol{\rho}_j$ and $\Delta \mathbf{z}_j, \mathbf{G}_j$. Specifically we assume:

$$\rho_{jQ(t)} = \lambda \overline{\Delta z}_j + \overline{\mathbf{g}}'_j \boldsymbol{\varkappa} + \varrho_{jQ(t)}; \quad \boldsymbol{\varrho}_j|\Delta \mathbf{z}_j, \mathbf{G}_j \sim \mathcal{N}(\mathbf{0}, \sigma_\rho^2 \mathbf{I}_4).$$

where $\overline{\Delta z}_j$ and $\overline{\mathbf{g}}_j$ are the time-averaged values of the variables. The parameters of the model are estimated by Maximum Likelihood. As before, standard calculations imply that the partial effect of Δz_{jt} on $\mathbb{E}[a_{jt}|\Delta \mathbf{z}_j, \mathbf{G}_j]$ is given by

$$\frac{\partial \mathbb{E}\left(a_{jt}|\Delta \mathbf{z}_j, \mathbf{G}_j\right)}{\partial \Delta z_{jt}} = \gamma \Phi \left(\frac{\gamma \Delta z_{jt} + \mathbf{g}_{jt}' \boldsymbol{\kappa} + \lambda \overline{\Delta z}_j + \overline{\mathbf{g}}_j' \boldsymbol{\varkappa}}{\sqrt{\sigma_\rho^2 + \sigma_\xi^2}} \right).$$

The estimate of APE remains at 0.003 for revenue and at 0.000 for value-added. A one-standard deviation revenue shock is associated with a rise in the advertisement rate by 0.002, which is 2% of the standard deviation of a_{jt}.

3.3 Summary and next steps

We have examined the relationship between advertising rates and firm-level output fluctuations using a series of regression models. Our findings show that the advertising rate is only weakly correlated with revenue growth and is uncorrelated with value-added growth. These results are obtained controlling for unobserved time-invariant firm traits and addressing the advertising rate's non-negativity and its mass point at zero via Tobit models. Notably, the APE estimates from the Tobit models are nearly identical to those from the linear regressions, indicating that accounting for specific distributional features of the advertising rate offers little empirical advantage in our context.

Taken at face value, this might appear to be a negative result for theoretical models where firms adjust their recruitment effort in response to shocks in demand or productivity, e.g. search and matching models. However, an important aspect of this analysis is highly restrictive: this section's empirical models assume that the only dimension of output fluctuations that matters for recruitment effort is the magnitude of the output shock. In particular, there is no heterogeneity in the persistence of the underlying output shock. Since output growth is extremely volatile (see table 2) it is reasonable to inquire whether some of this output volatility is transitory and correlates differently with the advertisement rate than persistent output volatility. We turn to this issue in sections 4 and 5, where we consider richer models of the output process that distinguish between persistent and transitory output shocks, and we estimate the relationship between the advertisement rate and output shocks with different persistence.

4 The output process

This section aims to identify and estimate firm-level output processes that allow shocks with different levels of persistence. We analyze the empirical autocovariance structure of revenue and value-added growth to develop a parsimonious representation of the firm-level output processes, which we then estimate. The results both motivate and inform our further analysis of the relationship between the advertisement rate and output growth in section 5.

4.1 Autocovariance analysis

We start by documenting the autocovariance structure of output growth. We first residualize output growth with respect to $\mathbf{g}'_{jt}$, i.e. with respect to time effects, industry effects, and their interactions, which will prove useful in the analysis of section 5.[15] Residualized output growth $\Delta\tilde{z}_{jt}$ is defined as:

$$\Delta\tilde{z}_{jt} = \Delta z_{jt} - \mathbf{g}'_{jt}\hat{\boldsymbol{\delta}}_{\Delta z}, \tag{5}$$

where $\hat{\boldsymbol{\delta}}_{\Delta z}$ contains the estimated coefficients in a regression of Δz_{jt} onto $\mathbf{g}_{jt}$.

Let Δ_k denote the kth difference operator, so that $\Delta_k x_{jt} = x_{jt} - x_{jt-k}$ for some generic variable x_{jt}. The kth difference of residualized output growth is computed as follows:

$$\Delta_k\tilde{z}_{jt} = \sum_{i=0}^{k-1} \Delta\tilde{z}_{jt-i}.$$

To document the autocovariance structure of output growth, we estimate the autocovariance functions of residualized quarter-on-quarter output growth and year-on-year (4th difference) output growth as a function of lag-length s, i.e. $\mathbb{C}ov(\Delta\tilde{z}_{jt}, \Delta\tilde{z}_{jt-s})$ and $\mathbb{C}ov(\Delta_4\tilde{z}_{jt}, \Delta_4\tilde{z}_{jt-s})$. Our estimation follows Abowd and Card (1989). Table 4 shows the results for the residualized series.

Columns (1) and (3) in Table 4 report the autocovariances of residualized quarter-on-quarter revenue and value-added growth, respectively. Both series exhibit economically and statistically significant autocovariances at very long lags, even after 16 quarters, and oscillatory dynamics

[15] Residualizing output growth in this way does not affect the estimates of γ in Table 3: regressing the advertisement rate on residualized output growth yields identical γ as in (1) since the removed variation is orthogonal to the variation that identifies γ. We will also residualize the series for the advertisement rate in section 5 in order to estimate the relationship between residualized output growth and residualized advertisement rate.

Table 4: Autocovariances of residualised revenue and value-added growth

	(1)	(2)	(3)	(4)
	Revenue growth		Value added growth	
	$\Delta\tilde{r}$	$\Delta_4\tilde{r}$	$\Delta\tilde{y}$	$\Delta_4\tilde{y}$
Lag 0 quarters	0.223*** (0.003)	0.189*** (0.003)	0.852*** (0.009)	0.559*** (0.007)
Lag 1 quarters	−0.091*** (0.002)	0.059*** (0.001)	−0.412*** (0.005)	0.029*** (0.002)
Lag 2 quarters	−0.005** (0.002)	0.037*** (0.001)	0.036*** (0.003)	0.031*** (0.001)
Lag 3 quarters	−0.041*** (0.002)	0.018*** (0.001)	−0.106*** (0.003)	0.027*** (0.001)
Lag 4 quarters	0.089*** (0.003)	−0.042*** (0.001)	0.193*** (0.003)	−0.197*** (0.004)
Lag 5 quarters	−0.041*** (0.002)	0.002** (0.001)	−0.109*** (0.003)	0.010*** (0.001)
Lag 6 quarters	−0.006*** (0.002)	0.002*** (0.001)	0.027*** (0.003)	0.003** (0.001)
Lag 7 quarters	−0.039*** (0.002)	0.003*** (0.001)	−0.099*** (0.003)	0.001 (0.001)
Lag 8 quarters	0.084*** (0.003)	0.002*** (0.001)	0.176*** (0.003)	−0.003** (0.002)
Lag 12 quarters	0.082*** (0.003)	0.000 (0.001)	0.168*** (0.003)	0.001 (0.002)
Lag 16 quarters	0.078*** (0.003)	0.000 (0.001)	0.156*** (0.003)	−0.002 (0.002)
Lag 20 quarters	0.077*** (0.003)	−0.002 (0.002)	0.147*** (0.004)	−0.003 (0.004)
Number of firms	21,046	20,514	20,820	19,274
Number of observations	454,915	385,579	398,648	301,258

Notes: $\Delta\tilde{r}$ and $\Delta\tilde{y}$ denote log revenue growth and log value-added growth, respectively, net of quarter effects, industry effects, and quarter–industry interaction effects. Asymptotic standard errors in parentheses (Abowd and Card, 1989).

(some autocovariances are positive, others are negative). These features indicate the presence of seasonality at the quarterly frequency in the growth series which, furthermore, is firm-specific as we have residualized with respect to aggregate quarter effects.

Columns (2) and (4) in Table 4 report the autocovariances of residualized year-on-year revenue and value-added growth, respectively. By construction, year-on-year growth removes firm-specific time-invariant quarterly seasonality. The autocovariances of both series are quantitatively significant for the first four quarters and are quantitatively very small thereafter (they drop by roughly an order of magnitude). Furthermore, the only quantitatively significant negative term that remains is the four-quarter lag; this indicates the presence of some stochastic firm-specific seasonality which, however, we will not further pursue here. In sum, the autocovariance estimates in Table 4 suggest that output is well-represented by a stochastic process that includes stochastic trends and firm-specific seasonality.

4.2 A model of the output process

We postulate a stochastic process for output that is consistent with the autocovariance structure documented in Table 4. The key features that we want to capture are the persistent autocovariances of quarter-on-quarter growth, the shorter autocovariances for year-on-year growth, and the oscillatory dynamics of quarter-on-quarter growth.

Consider the following output process:

$$\tilde{z}_{jt} = \mu_{jQ(t)} + u_{jt} + w_{jt}, \tag{6}$$

where $\mu_{jQ(t)}$ is a firm-quarter effect representing deterministic firm-specific seasonal output variation, u_{jt} is a Random Walk process, and w_{jt} is a Moving Average (MA) process of order q. Specifically,

$$u_{jt} = u_{jt-1} + \eta_{jt}, \tag{7}$$

$$w_{jt} = \zeta_{jt} + \sum_{s=1}^{q} \theta_s \zeta_{jt-s}, \tag{8}$$

where $\{\eta_{jt}\}$ and $\{\zeta_{jt}\}$ are i.i.d. innovation processes with $\mathbb{E}[\eta_{jt}] = \mathbb{E}[\zeta_{jt}] = \mathbb{E}[\eta_{jt}\eta_{jt-s}] = \mathbb{E}[\zeta_{jt}\zeta_{jt-s}] = \mathbb{E}[\eta_{jt}\zeta_{jt'}] = 0$ for all t, t' and any $s \neq 0$; furthermore, let $\sigma_\eta^2 = \mathbb{E}[\eta_{jt}^2] < \infty$ and $\sigma_\zeta^2 = \mathbb{E}[\zeta_{jt}^2] < \infty$. The η_{jt} innovations are *permanent* shocks and the ζ_{jt} innovations are *transitory* shocks.[16]

The first-differences and fourth-differences of process (6) are, respectively, given by:

$$\Delta\tilde{z}_{jt} = \mu_{jQ(t)} - \mu_{jQ(t-1)} + \eta_{jt} + \zeta_{jt} - \zeta_{jt-1} + \sum_{s=1}^{q} \theta_s(\zeta_{jt-s} - \zeta_{jt-1-s}) \tag{9}$$

$$\Delta_4\tilde{z}_{jt} = \eta_{jt} + \eta_{jt-1} + \eta_{jt-2} + \eta_{jt-3} + \zeta_{jt} - \zeta_{jt-4} + \sum_{s=1}^{q} \theta_s(\zeta_{jt-s} - \zeta_{jt-4-s}) \tag{10}$$

The process (6) is consistent with the observations that the quarter-on-quarter growth rate features persistent and oscillatory autocovariances, while the year-on-year growth rate does not. To see this, note that $\mu_{jQ(t)} = \mu_{jQ(t+4)}$ implies $\mathbb{E}[\Delta\mu_{jQ(t)}\Delta\mu_{jQ(t-s)}] = \mathbb{E}[\Delta\mu_{jQ(t)}\Delta\mu_{jQ(t-s-4)}]$, which, on the one hand, creates a persistent and oscillatory component in the autocovariance

[16] The ζ_{jt} innovations could comprise measurement errors as well. We shall not attempt to disentangle transitory shocks and measurement error and will refer to ζ_{jt} as a transitory shock.

Table 5: Output processes: Optimally Weighted Minimum Distance estimates

	(1)	(2)	(3)	(4)	(5)	(6)
			The revenue process			
SD random walk-innovation, σ_η	0.135*** (0.001)	0.136*** (0.001)	0.136*** (0.001)	0.140*** (0.002)	0.137*** (0.002)	0.139*** (0.002)
SD MA-innovation, σ_ζ	0.221*** (0.002)	0.220*** (0.002)	0.218*** (0.002)	0.211*** (0.003)	0.227*** (0.003)	0.221*** (0.004)
Order-1 MA-coefficient, θ_1		−0.010 (0.007)	−0.019** (0.009)	−0.097*** (0.019)	−0.046*** (0.014)	−0.105*** (0.025)
Order-2 MA-coefficient, θ_2			−0.023** (0.009)	−0.079*** (0.015)	−0.039*** (0.011)	−0.069*** (0.017)
Order-3 MA-coefficient, θ_3				−0.057*** (0.010)	−0.024*** (0.008)	−0.062*** (0.014)
Order-4 MA-coefficient, θ_4					0.080*** (0.008)	0.060*** (0.011)
Order-5 MA-coefficient, θ_5						−0.035*** (0.009)
OWMD objective function	782.826	780.993	774.367	736.970	665.556	646.407
			The value-added process			
SD random walk-innovation, σ_η	0.128*** (0.001)	0.135*** (0.001)	0.138*** (0.001)	0.140*** (0.001)	0.137*** (0.001)	0.139*** (0.001)
SD MA-innovation, σ_ζ	0.508*** (0.003)	0.511*** (0.003)	0.506*** (0.003)	0.501*** (0.003)	0.526*** (0.003)	0.521*** (0.004)
Order-1 MA-coefficient, θ_1		−0.055*** (0.003)	−0.063*** (0.003)	−0.083*** (0.004)	−0.063*** (0.004)	−0.079*** (0.005)
Order-2 MA-coefficient, θ_2			−0.026*** (0.003)	−0.036*** (0.004)	−0.019*** (0.003)	−0.026*** (0.004)
Order-3 MA-coefficient, θ_3				−0.022*** (0.003)	−0.010*** (0.002)	−0.021*** (0.003)
Order-4 MA-coefficient, θ_4					0.066*** (0.004)	0.061*** (0.004)
Order-5 MA-coefficient, θ_5						−0.016*** (0.003)
OWMD objective function	1,940.082	1.525.224	1,468.358	1,402.821	1,106.659	1,073.787

Notes: Asymptotic standard errors in parentheses. ***, ** and * indicates statistical significance at the 1, 5, and 10 percent level, respectively. Coefficient estimates from the over-identified OWMD-estimator fitting the first 13 autocovariances of $\Delta_4\tilde{r}_{jt}$ (top panel) and $\Delta_4\tilde{y}_{jt}$ (bottom panel), including the order-0 autocovariance (see Table 4). The OWMD objective function is asymptotically distributed as $\chi^2(k)$, where k is the number of over-identifying restrictions. For example, the column (1) specification has two free parameters and the OWMD objective function is distributed as $\chi^2(11)$.

function of the first-differenced output series (9), and, on the other hand, eliminates the firm-quarter fixed effects from the year-on-year growth rate process (10).

4.3 The estimated output process

Given a choice of the order of the MA-process, q, the output process (6) has $q+2$ parameters: the standard deviation of the permanent shock σ_η, the standard deviation of the transitory shock σ_ζ, and the q moving average parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_q)'$. To estimate these parameters we use a Minimum Distance estimation procedure and we match the theoretical autocovariances of year-on-year residualized growth $\Delta_4\tilde{z}_{jt}$ from equation (10) with the first 13 empirical autocovariances of revenue and value-added (including the order-0 autocovariance) that are reported, respectively, in columns (2) and (4) of Table 4.

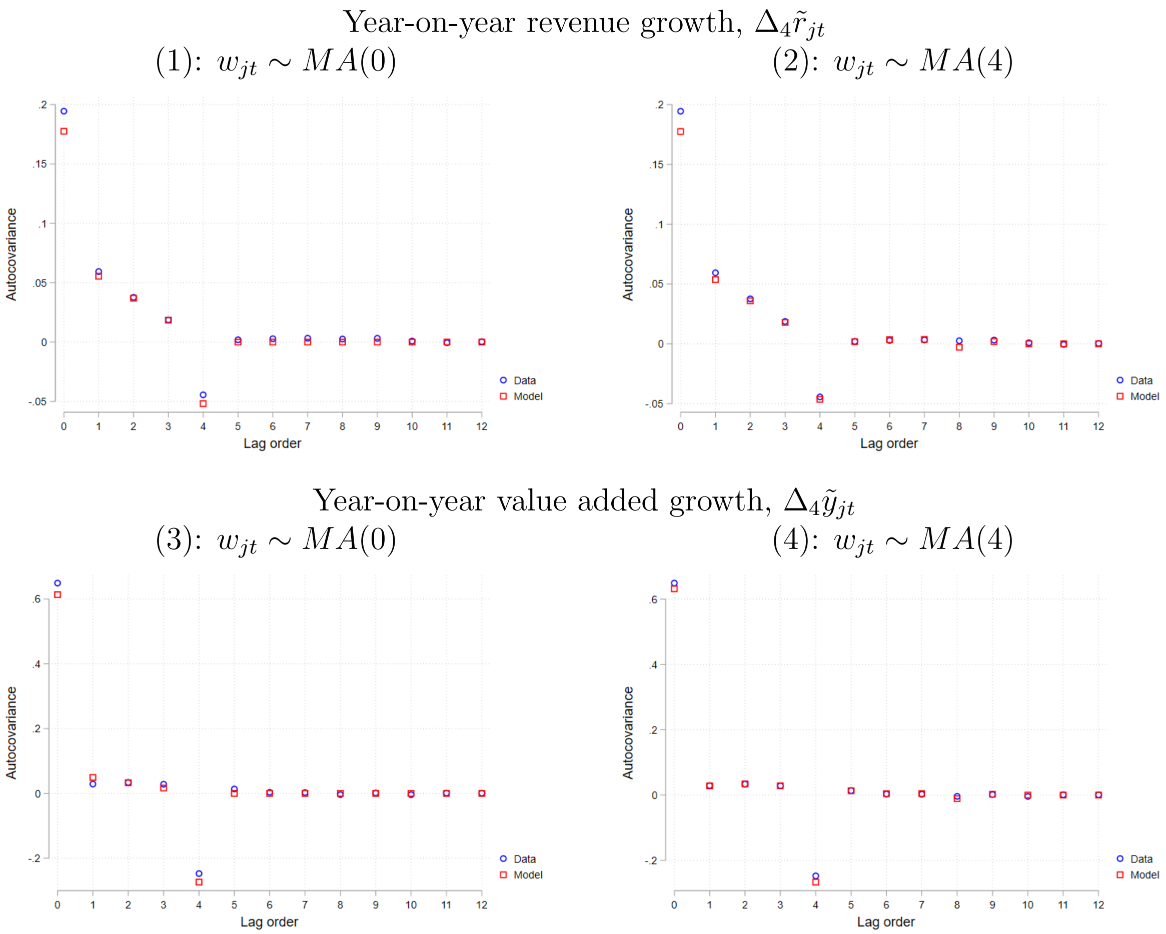
We estimate the output process (6) for $q = 0, 1, \ldots, 5$ which renders the Minimum Distance estimators over-identified. Our preferred estimates obtain from the asymptotically efficient Optimally Weighted Minimum Distance (OWMD) estimator that weights the distance between the theoretical and empirical autocovariances by the inverse variance-covariance matrix of the empirical autocovariances. The estimates from just- and over-identified Equally Weighted Minimum Distance (EWMD) estimators are very similar and are reported in Appendix B.

Table 5 reports the estimates of the revenue and value-added processes, for $q = 0, 1, 2, 3, 4, 5$. For both output series, the estimates of the standard deviations of the permanent and transitory shocks are precise, economically meaningful, and highly stable across the different specifications of the moving average process. In both output series, the transitory shock are estimated to be more volatile than the permanent shocks: in the revenue process the standard deviation of transitory shocks is 60% greater than that of permanent shocks and in the value-added process it is 2.5-3 times larger, depending on the specification. The coefficients of the moving average process (the θ_q's) are also precisely estimated throughout. In both output series, the estimates of the coefficients are small and negative with the exception of the $MA(4)$ and $MA(5)$ specifications where the estimate for θ_4 is positive and larger in magnitude. This feature possibly reflects some stochastic seasonality that is not accounted for by the firm-quarter fixed effects.

The most parsimonious specification, $w_{jt} \sim MA(0)$, in Table 5 fits the empirical autocovariances almost as well as the richest specification, $w_{jt} \sim MA(5)$. As we will show in section 5, the pass-through of permanent and transitory output shocks to the advertisement rate are only modestly affected by the order of the moving average specification for the transitory component.

The objective function values exceed conventional χ^2 critical values, thereby formally rejecting the over-identifying restrictions in all cases. However, this is a result of highly precise autocovariance estimates rather than poor model fit. This can be seen from figure 1 which plots the empirical and model-predicted autocovariances for the revenue and value-added processes and demonstrates that they are very close to each other even in the most parsimonious specification, namely $MA(0)$.

Figure 1: Empirical and model-predicted autocovariances for year-on-year output growth



Notes: The empirical autocovariances (circles) are tabulated in Table 4. The predicted autocovariances (squares) obtain from the estimated revenue processes reported in Table 5, columns (1) and (5).

5 Recruitment effort and permanent and transitory output shocks

Section 3 documents that firms' output growth and advertisement rate correlate very weakly. Section 4 shows that the output process can be decomposed into a very volatile transitory component and a less volatile permanent component. In this section, we investigate the extent to which the advertisement rate is differentially associated with permanent and transitory output shocks.

5.1 An empirical advertisement-posting model

We model output as a random walk plus a moving average process, as in equation (6). This implies that quarterly output growth is

$$\Delta \tilde{z}_{jt} = \Delta \mu_{jQ(t)} + \Delta u_{jt} + \Delta w_{jt}, \tag{11}$$

where (7) implies that $\Delta u_{jt} = \eta_{jt}$ is the innovation to the permanent component of output, (8) implies that $\Delta w_{jt} = \zeta_t - \zeta_{t-1} + \sum_{s=1}^{q} \theta_s (\zeta_{jt-s} - \zeta_{jt-1-s})$ is a moving average process of order $q+1$ obtained from the transitory component of output, and $\Delta \mu_{jQ(t)}$ is the difference in the firm-quarter fixed effects.

We posit a linear relationship between the advertisement rate and the three components of residualized output growth. To ensure a consistent definition of the error variance in this regression and to obtain valid standard errors for our estimates, we residualize the advertisement rate a_{jt} with respect to $\mathbf{g}_{jt}$, mirroring the procedure to obtain residualized output growth $\Delta \tilde{z}_{jt}$. Let $\tilde{a}_{jt}$ be the residualized advertisement rate.[17] Our empirical model for the residualized advertisement rate is

$$\tilde{a}_{jt} = \psi \Delta \mu_{jQ(t)} + \alpha \Delta u_{jt} + \beta \Delta w_{jt} + \xi_{jt}, \tag{12}$$

where ψ, α, and β are the loading coefficients that capture the relationship between the advertisement rate and growth in the three output components, and ξ_{jt} is an error term. We assume throughout that Δu_{jt} and Δw_{jt} in (12) are strictly exogenous regressors conditional on

[17] Formally, $\tilde{a}_{jt} = a_{jt} - \mathbf{g}'_{jt} \hat{\boldsymbol{\delta}}_a$ where $\hat{\boldsymbol{\delta}}_a$ contains the estimated coefficients in a regression of a_{jt} onto $\mathbf{g}_{jt}$.

$\Delta\mu_{j\mathrm{Q}(t)}$, i.e. $\mathbb{E}(\xi_{jt}|\Delta\mathbf{u}_j, \Delta\mathbf{w}_j, \Delta\boldsymbol{\mu}_j) = 0$ for all j and t, where $\Delta\mathbf{u}_j \equiv (\Delta u_{j1}, \Delta u_{j2}, \ldots, \Delta u_{jT})'$, $\Delta\mathbf{w}_j \equiv (\Delta w_{j1}, \Delta w_{j2}, \ldots, \Delta w_{jT})'$ and $\Delta\boldsymbol{\mu}_j \equiv (\Delta\mu_{j1}, \Delta\mu_{j2}, \Delta\mu_{j3}, \Delta\mu_{j4})'$.

In order to remove the firm-quarter fixed effects in (12), our analysis focuses on the relationship between the advertisement rate and year-on-year output growth. Specifically, let Σ_k denote a k-period cumulative sum operator so that for any variable x_{jt} we have $\Sigma_k x_{jt} = x_{jt} + x_{jt-1} + \ldots + x_{jt-k+1}$ and $\Sigma_k \Delta x_{jt} = x_{jt} - x_{jt-k} = \Delta_k x_{jt}$ where we note that $\Sigma_{4k}\Delta\mu_{j\mathrm{Q}(t)} = 0$ for $k = 1, 2, \ldots$ because $Q(t) = Q(t-4k)$. Hence, cumulating (12) over any multiple of four quarters eliminates the firm-quarter fixed effects and yields the following advertisement rate regression equation:

$$\Sigma_{4k}\tilde{a}_{jt} = \alpha\Delta_{4k}u_{jt} + \beta\Delta_{4k}w_{jt} + \Sigma_{4k}\xi_{jt}, \tag{13}$$

for $k = 1, 2, \ldots$, where the cumulated error term is $\Sigma_{4k}\xi_{jt} = \sum_{s=0}^{4k-1}\xi_{jt-s}$. Equations (7) and (8) mean that $\Delta_{4k}u_{jt}$ and $\Delta_{4k}w_{jt}$ take the forms

$$\Delta_{4k}u_{jt} = \sum_{s=0}^{4k-1}\eta_{t-s}, \tag{14}$$

$$\Delta_{4k}w_{jt} = \zeta_t - \zeta_{t-4k} + \sum_{s=1}^{q}\theta_s(\zeta_{jt-s} - \zeta_{jt-4k-s}), \tag{15}$$

Our interest is in the identification and estimation of α and β in equation (13). The key econometric challenge is that we do not separately observe the permanent and transitory components of output growth, Δu_{jt} and Δw_{jt}. Our proposed permanent/transitory shocks framework bears many similarities to those use in e.g. Guiso, Pistaferri, and Schivardi (2005) and Juhn, McCue, Monti, and Pierce (2018) who are concerned with the pass-through of permanent and transitory firm-level productivity shocks to the wages of individual workers, and our identification strategy borrows heavily from these papers.

5.2 Identification and estimation of α

In this subsection, we examine the relationship between the advertisement rate and permanent shocks to output. We describe two distinct identification strategies due to Juhn, McCue, Monti, and Pierce (2018) and Guiso, Pistaferri, and Schivardi (2005) which leverage in different ways the same feature of the shock structure: that permanent shocks to output affect the advertisement rate over longer periods than do transitory shocks to output. We provide estimates from both strategies.

Long-difference regressions. We adapt the identification strategy of Juhn, McCue, Monti, and Pierce (2018) to our setting. Consider the projection of the advertisement rate cumulated over $4k$ quarters, $\Sigma_{4k}\tilde{a}_{jt}$, onto the output growth rate cumulated over $4k$ quarters, where $\sum_{4k} \Delta\tilde{z}_{jt} = \Delta_{4k}\tilde{z}_{jt}$:[18]

$$\Sigma_{4k}\tilde{a}_{jt} = \tau^k + \gamma^k \Delta_{4k}\tilde{z}_{jt} + \epsilon^k_{jt}, \tag{16}$$

for $k = 1, 2, \ldots$, where k indexes the horizon of the differencing.

The coefficients τ^k and γ^k are such that $\mathbb{E}(\epsilon^k_{jt}) = 0$ and $\mathbb{C}ov(\Delta_{4k}\tilde{z}_{jt}, \epsilon^k_{jt}) = 0$ which implies

$$\gamma^k = \frac{\mathbb{V}ar(\Delta_{4k}u_{jt})}{\mathbb{V}ar(\Delta_{4k}u_{jt}) + \mathbb{V}ar(\Delta_{4k}w_{jt})}\alpha + \frac{\mathbb{V}ar(\Delta_{4k}w_{jt})}{\mathbb{V}ar(\Delta_{4k}u_{jt}) + \mathbb{V}ar(\Delta_{4k}w_{jt})}\beta, \tag{17}$$

for $k = 1, 2, \ldots$ Therefore, the projection coefficient γ^k is a weighted average of α and β and the weights are given by the relative contributions of the permanent and transitory output components to the variance of $\Delta_{4k}\tilde{z}_{jt}$. Let $\hat{\gamma}^k$ denote the OLS estimator of γ^k in (16) and note that it is a consistent estimator of γ^k but it is not in general a consistent estimator of α or of β.[19] This inconsistency has two sources. First, without controls for $\Delta_{4k}w_{jt}$ or for $\Delta_{4k}u_{jt}$, the variation in $\Delta_{4k}\tilde{z}_{jt}$ that identifies γ^k confounds permanent and transitory shocks, and thus, does not identify α or β. Second, even if $\beta = 0$, the transitory shocks lead to attenuation bias.

Juhn, McCue, Monti, and Pierce (2018) observe that as the differencing horizon k increases, the sequence of $\hat{\gamma}^k$ becomes informative about α. We demonstrate their insight under the assumption $w_{jt} \sim MA(0)$ for expositional simplicity but the result holds generally. When $q = 0$, we have $\Delta_{4k}w_{jt} = \zeta_t - \zeta_{t-4k}$, $\mathbb{V}ar(\Delta_{4k}u_{jt}) = 4k\sigma^2_\eta$, and $\mathbb{V}ar(\Delta_{4k}w_{jt}) = 2\sigma^2_\zeta$ and (17) becomes

$$\gamma^k = \frac{2k\sigma^2_\eta}{2k\sigma^2_\eta + \sigma^2_\zeta}\alpha + \frac{\sigma^2_\zeta}{2k\sigma^2_\eta + \sigma^2_\zeta}\beta,$$

[18]The cumulation needs to be a multiple of four quarters to remove the firm-quarter fixed effects from (12).

[19]There are three cases where the OLS estimator $\hat{\gamma}^k$ is a consistent estimator of α or β or both: if there are no permanent shocks to output, then $\mathbb{V}ar(\Delta_{4k}u_{jt}) = 0$, $\gamma^k = \beta$, and $\hat{\gamma}^k$ is a consistent estimator of β; if there are no transitory shocks to output, then $\mathbb{V}ar(\Delta_{4k}w_{jt}) = 0$, $\gamma^k = \alpha$, and $\hat{\gamma}^k$ is a consistent estimator of α; finally, if $\alpha = \beta$, then $\gamma^k = \alpha = \beta$, and $\hat{\gamma}^k$ is a consistent estimator of α and β.

for $k = 1, 2, \ldots$ As the differencing horizon k increases, the projection coefficient γ^k converges to α. Effectively, as k increases, the variation in $\Delta_{4k}\tilde{z}_{jt}$ becomes a progressively more accurate signal of the permanent component of output growth, with the transitory component contributing progressively less to overall variation. As the OLS estimator $\hat{\gamma}^k$ consistently estimates γ^k, this enables using these long-difference projections for approximate inference about α. We can also learn about the relative size of α and β: If $\hat{\gamma}^k$ is increasing in k, we can conclude that $\alpha > \beta$, and vice versa. These lines of argument hold for any finite-order $MA(q)$ process, though longer differences may be required for precision when q is larger.

Table 6 presents estimates of the projection coefficient γ^k from equation (16) for a range of differencing horizons k for the growth of revenue and value-added. The estimates are highly precise. For both output processes, the estimates of γ^k are low for small k and increase as k rises, by a factor of more than two for revenue and more than seven for value-added. Hence, we conclude that α exceeds β in both output processes. For the revenue process, the γ^k estimates stabilize at somewhat more than 0.04 for difference horizons of 12 to 20 quarters. This implies that a 1SD permanent revenue shock is associated with an increase in a firm's advertisement rate between 0.005 and 0.006, or 5 to 6 percent of a standard deviation of the advertisement rate. For the value-added process, the γ^k estimates stabilize at 0.035-0.040 for difference horizons of 16 to 20 quarters, which implies that a 1SD permanent value-added shock is associated with an increase of about 0.005 in a firm's advertisement rate, or 5 percent of a standard deviation of the advertisement rate.

In contrast to the results of section 3, the response to a 1SD permanent output shock are almost identical for revenue and value-added shocks. Furthermore, the estimates of the response to a 1SD permanent output shock in table 6 are 3 times larger than the estimates of a 1SD output shock for revenue and more than 10 times larger for value-added in section 3.

Instrumental variable regressions. We adapt the methodology of Guiso, Pistaferri, and Schivardi (2005) to our setting and employ appropriately-centered output differences as instrumental variables for the permanent shocks, in order to point identity and estimate α.

We illustrate this approach for the case where $w_{jt} \sim MA(0)$ for simplicity. Setting $k = 1$, (13) becomes $\Sigma_4\tilde{a}_{jt} = \tau^1 + \gamma^1\Delta_4\tilde{z}_{jt} + \epsilon_{jt}$ where $\Delta_4\tilde{z}_{jt} = \Delta_4 u_{jt} + \Delta_4 w_{jt}$. The instrumental variable

Table 6: Long-difference projections by difference horizon k

	Revenue				Value added			
Diff. horizon	γ^k	$\gamma^k \times \sigma_\eta$	Firms	Obs.	γ^k	$\gamma^k \times \sigma_\eta$	Firms	Obs.
4 quarters ($k=1$)	0.019*** (0.002)	0.003	20,514	385,579	0.005*** (0.001)	0.001	19,274	301,258
8 quarters ($k=2$)	0.030*** (0.003)	0.004	19,499	300,757	0.013*** (0.002)	0.002	16,813	213,246
12 quarters ($k=3$)	0.040*** (0.006)	0.005	18,247	223,496	0.025*** (0.004)	0.003	14,223	146,916
16 quarters ($k=4$)	0.046*** (0.007)	0.006	16,719	152,887	0.035*** (0.006)	0.005	11,530	94,441
20 quarters ($k=5$)	0.043*** (0.009)	0.006	15,431	88,058	0.039*** (0.009)	0.005	9,585	51,528

Notes: Asymptotic standard errors clustered at the firm-level in parentheses. ***, ** and * indicate statistical significance at the 1, 5, and 10 percent level, respectively. The column labeled $\gamma^k \times \sigma_\eta$ reports the advertisement rate response to a permanent output shock equal to 1SD σ_η, as estimated in the $MA(4)$ specification of section 4. For the revenue process it is equal to 0.137 and for the value-added process it is, coincidentally, also equal to 0.137 (see Table 5).

for $\Delta_4 u_{jt}$ is one-period leaded output, differenced over 8 quarters, $\Delta_8 \tilde{z}_{jt+1}$:

$$\Delta_8 \tilde{z}_{jt+1} = \eta_{jt+1} + \underbrace{\eta_{jt} + \eta_{jt-1} + \eta_{jt-2} + \eta_{jt-3}}_{\Delta_4 u_{jt}} + \eta_{jt-4} + \eta_{jt-5} + \eta_{jt-6} + \underbrace{\zeta_{t+1} - \zeta_{t-7}}_{\text{Uncorr. w}/\Delta_4 w_{jt}} . \tag{18}$$

Plainly, $\Delta_8 \tilde{z}_{jt+1}$ correlates with the quarter-t year-on-year growth in the permanent component $\Delta_4 u_{jt}$, but has no covariance with the quarter-t year-on-year growth in the transitory component $\Delta_4 w_{jt}(= \zeta_{jt} - \zeta_{jt-4}$ when $q = 0)$. Hence, a regression of $\Sigma_4 \tilde{a}_{jt}$ on $\Delta_4 \tilde{z}_{jt}$ using $\Delta_8 \tilde{z}_{jt+1}$ as an instrumental variable for $\Delta_4 u_{jt}$ yields a consistent estimate of α under an $MA(0)$ process for the transitory output component w_{jt}. Similar arguments yield instrumental variables for any order of the moving average process (see Guiso, Pistaferri, and Schivardi (2005)).

Table 7 presents the estimates of α from the instrumental variable approach for $MA(0)$- and $MA(4)$-specifications of the transitory output component. Appendix C reports the estimates for the full set of orders for the moving average process ($q = 0, 1, 2, 3, 4, 5$), which are very similar to those reported here. The first stage F-statistic is substantial for both specifications of w_{jt} and for both output measures, which means that the proposed instrumental variables are strong. In each case, the loading coefficient on permanent output shocks α is precisely estimated. For the revenue process, the estimate of α is 0.056 when $w_{jt} \sim MA(0)$ and 0.065 when $w_{jt} \sim MA(4)$. This translates into advertisement rate responses to a 1SD permanent revenue shock of 0.008 and 0.009, respectively, or 9-10% of a standard deviation of the advertisement rate. For the value-added process, the estimate of α is 0.067 when $w_{jt} \sim MA(0)$ and 0.109 when $w_{jt} \sim MA(4)$,

Table 7: Instrumental variables estimates of advertisement rate responses to permanent and transitory revenue and value added shocks

	Revenue				Value added			
	$w \sim MA(0)$		$w \sim MA(4)$		$w \sim MA(0)$		$w \sim MA(4)$	
	α	β	α	β	α	β	α	β
Estimate	0.056***	−0.003	0.065***	−0.003	0.067***	−0.004**	0.109***	−0.004**
	(0.007)	(0.007)	(0.014)	(0.007)	(0.011)	(0.002)	(0.023)	(0.002)
1SD shock	0.135	0.312	0.137	0.330	0.128	0.718	0.137	0.769
Est. × 1SD	0.008	−0.001	0.009	−0.001	0.009	−0.003	0.015	−0.003
IVs	$\Delta_8\tilde{r}_{jt+1}$	$\Delta_4\tilde{r}_{jt+4}$	$\Delta_{16}\tilde{r}_{jt+5}$	$\Delta_4\tilde{r}_{jt+4}$	$\Delta_8\tilde{y}_{jt+1}$	$\Delta_4\tilde{y}_{jt+4}$	$\Delta_{16}\tilde{y}_{jt+5}$	$\Delta_4\tilde{y}_{jt+4}$
1st stage F	3,634	2,327	1,584	2,327	816	7,807	511	7,807
Firms	19,499	19,499	16,719	19,499	16,813	16,813	11,530	16,813
Firm-qrts	76,216	76,216	64,829	76,216	63,485	63,485	42,913	63,485
Observations	300,757	300,757	152,887	300,757	213,246	213,246	94,441	213,246

Notes: Asymptotic standard errors clustered at the firm-level in parentheses. ***, ** and * indicates statistical significance at the 1, 5, and 10 percent level, respectively.

which implies that advertisement rate responses to a 1SD permanent value added shock of 0.009 and 0.015, respectively, or 10-16% of a standard deviation of the advertisement rate.

Across the w_{jt}-specifications and output processes, the instrumental variables estimates of α are larger than those obtained from the long-difference regressions approach. Recall that the long-difference regressions identifies α in the limit for very long difference horizons k. In practice, with the difference horizons that are feasible, the α estimates from the long-difference regressions likely suffer from attenuation bias due to the highly volatile transitory output component.

5.3 Identification and estimation of β

In this subsection, we adapt the methodology of Guiso, Pistaferri, and Schivardi (2005) to our setting and use an instrumental variable approach to point-identify β.

As before, we illustrate the approach for the case where $w_{jt} \sim MA(0)$. The appropriate instrumental variable is the 4-quarter leaded year-on-year output growth $\Delta_4\tilde{z}_{jt+4}$. Noting that

$$\Delta_4\tilde{z}_{jt+4} = \underbrace{\eta_{jt+4} + \eta_{jt+3} + \eta_{jt+2} + \eta_{jt+1}}_{\text{Uncorr. w/ } \Delta_4 u_{jt}} + \zeta_{jt+4} - \underbrace{\zeta_{jt}}_{\text{Corr.w/}\Delta_4 w_{jt}}, \tag{19}$$

it is evident that $\Delta_4\tilde{z}_{jt+4}$ correlates with $\Delta_4 w_{jt} = \zeta_{jt} - \zeta_{jt-4}$ through the shared component ζ_{jt}; moreover, $\Delta_4\tilde{z}_{jt+4}$ does not correlate with the innovations to the permanent output component in $\Delta_4 u_{jt} = \eta_{jt} + \eta_{jt-1} + \eta_{jt-2} + \eta_{jt-3}$. Hence, $\Delta_4\tilde{z}_{jt+4}$ is instrumental for $\Delta_4 w_{jt}$ in (13) and identifies β. In fact, this instrumental variable identifies β for any finite order q of the moving average process. We implement the estimator by GMM.

Table 7 reports the estimates of β when output is measured by revenue and value-added.[20] In each of the reported cases in Table 7, the first stage F-statistic is substantial, which suggests that the proposed instrument is strong. The point estimate of β when output is measured by revenue is negative, but statistically and economically insignificant: firms do not adjust their advertisement rate in response to growth in the transitory revenue component. The point estimate of β when output is measured by value added is also negative and is statistically significant at the 5 percent significance level; however, the estimated value of -0.004 is economically insignificant and implies that a firm's advertisement rate decreases by 0.003 in response to a 1SD growth event to the transitory value added component.

5.4 Summary

This section shows that the persistence of output shocks is a very important determinant in the relationship between output growth and the advertisement rate. The relationship between the advertisement rate and permanent shocks to output is estimated to be statistically and economically significant, in both estimation methods and all the specifications that we have tried. The relationship between the advertisement rate transitory output shocks is estimated to be quantitatively insignificant.

Our preferred estimates obtain from the instrumental variable estimator where the moving average process is of order 4. This specification does not suffer from attenuation bias, as the long-difference regressions might do, and it controls for the stochastic firm-specific seasonality that remains after we account for deterministic firm-specific seasonality. According to this specification, a one standard deviation output shock is associated with a rise in the advertisement rate that equals 10-16% of the standard deviation of a_{jt}, depending on whether we use revenue or value-added as our measure of output. By contrast, transitory shocks to either measure of output have, essentially, no effect on the advertisement rate. This feature explains why we did not find a quantitatively significant relationship between output growth and the advertisement rate in section 3: when we do not distinguish the persistence of the shock, the transitory shocks attenuate the estimates to quantitative insignificance.

[20]The volatility of growth in the transitory output components is computed as $\mathbb{V}ar(\Delta w_{jt})$; hence, for $w_{jt} \sim MA(0)$, $\mathbb{V}ar(\Delta w_{jt}) = 2\sigma_\zeta^2$ and for $w_{jt} \sim MA(4)$, $\mathbb{V}ar(\Delta w_{jt}) = \sigma_\zeta^2(1+[\theta_1-1]^2+[\theta_2-\theta_1]^2+[\theta_3-\theta_2]^2+[\theta_4-\theta_3]^2+\theta_4^2)$ with population parameters replace by their estimated values.

6 Conclusion

The literatures on firm dynamics and search and matching typically interpret firms' employment adjustments as responses to firm-level output shocks, emphasizing that such adjustments are key channels through which shocks propagate to the broader labor market and economy. Yet, direct empirical evidence on the magnitude and nature of these responses has been limited.

To address this gap, we combine firm-level job advertisement data from an online job board with quarterly output data from administrative VAT records, providing a unique opportunity to examine the relationship between firms' advertisement-posting behavior and output growth. Our findings qualitatively confirm core predictions of canonical firm-dynamics and search-and-matching models and provide empirical targets for their quantitative calibration.

We first document that correlations between output growth and job advertisement posting are weak, which might seem at odds with theoretical predictions. Further analysis, however, reveals that the relationship between output growth and the advertisement rate masks important differences that relate to the persistence of output growth shocks. Decomposing output growth into permanent and transitory components, modeled respectively as a random walk and a low-order moving average process, we find that only permanent shocks exhibit quantitatively significant correlations with the advertisement rate. Specifically, a one standard deviation positive permanent shock is associated with an increase in the advertisement rate equal to 10–16% of the standard deviation of postings, depending on whether output is measured as revenue or value-added. In contrast, transitory shocks are essentially unrelated to the advertisement rate. These results are robust to alternative specifications and estimation methods.

From a theoretical perspective, our findings are consistent with an environment in which firms observe the persistence of their firm-level shocks and adjust recruitment only when shocks are permanent or sufficiently persistent. This selective adjustment behavior aligns with theoretical models emphasizing costly vacancy creation and search frictions, and highlights the importance of shock decomposition for understanding firm hiring dynamics.

Several empirical extensions are worth pursuing. One direction is to explore potential nonlinearities in both output processes and their link to recruitment. Recent advances in nonlinear panel data methods (e.g., Arellano, Blundell, and Bonhomme, 2017) provide tools for such analyses, though these are beyond the scope of the present paper.

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Appendices

A Data

A.1 Data sources

The analysis data is constructed from eight raw data sources. The raw data sources were accessed on ECONAU-servers in Statistics Denmark under project 707275 in 2024.

1. **The job spells data set.** Start- and end-dates, earnings and (an estimate of) hours worked for all jobs of all legal residents in Denmark. Persons are identified by `PNR`, an anonymized social security number. Employers are identified at the firm-level via the business registry identifier (the CVR-number) and at the establishment-level via an establishment identification number (`LBNR`, identifying a physical workplace). The unit of observation is a person-spell-year combination (i.e. a job spell for a particular worker that is ongoing in, say, three calendar years is represented by three observations, one for each calendar year). The job spells data covers the period from January 1st, 1985 to December 31st, 2012.

2. **The PERSONER data set.** Socio-economic and demographic information on the population of legal residents in Denmark from administrative registers. Persons are identified by `PNR`, an anonymized social security number. The unit of observation is a person-year combination. The PERSONER data set covers 1979-2021.

3. **The IDA-P data set.** The person-component of the Integreret Database for Arbejdsmarkedforskning (IDA), an annual comprehensive matched employer-employee panel that is constructed, updated, and maintained by Statistics Denmark and which covers the entire Danish population and all firms with economic activity. Persons are identified by `PNR`, an anonymized social security number. The unit of observation is a person-year combination. The UDDA data set covers 1980-2019.

4. **The IDA-S data set.** The establishment-component of the Integreret Database for Arbejdsmarkedforskning (IDA), an annual comprehensive matched employer-employee panel that is constructed, updated, and maintained by Statistics Denmark and which covers the

entire Danish population and all firms with economic activity. Establishments are identified by an establishment identification number (`LBNR`, identifying a physical workplace) and each establishment is associated with a firm that is identified by its business registry number (the CVR-number). The unit of observation is an establishment-year combination. The IDA-S data set covers 1980-2019.

5. **The job advertisement data set.** Online job advertisements from Jobindex A/S, Denmark's largest online job board. The job advertisement data set records the date the job opening was posted and information on the occupation of the posted job opening, and often the posting firm's business registry identifier (the CVR-number). The dataset covers job advertisements posted from January 1st, 2003 to August 31st, 2009.

6. **The Value Added Tax (VAT) accounts data set.** Revenues and purchases from firms' administrative VAT accounts. Firms are identified Employers are identified at the firm-level via the business registry identifier (the CVR-number). The unit of observation in the VAT account data is a firm-month. The monthly VAT accounts data set covers January 2001 to August 2013.

A.2 Merging the data sources

Merging the different data sources is implemented in SAS in a series of steps outlined below.

We first merge the job spells data set with worker-level information in the PERSONER, IDA-P and IDA-S data sets and establishment-level information IDA-S in five steps (recall that `PNR` is the person identifier, while `LBNR` is the establishment identifier).

1. **Merging job spells and PERSONER data sets.** We merge the 1985-2012 PERSONER data set with the 1985-2012 job spells data set by `PNR` and year. We use the gender and age information in the PERSONER data set to select labor market histories for persons with non-missing gender and age information and who are aged 15-70 in the job spells data set.

2. **Merging job spells and IDA-P data sets.** We merge the 1985-2012 IDA-P data set with the 1985-2012 job spells data set by `PNR` and year. We keep all the observations in

the job spells data, whether or not they are merged to an observation from the IDA-P data set, but discard all unmatched IDA-P data set observations.

3. **Merging job spells and IDA-S data sets.** We merge the 1985-2012 IDA-S data set with the 1985-2012 job spells data set by `LBNR`-year. We retain establishment-level information industry and keep all the observations in the job spells data, whether or not they are merged to an observation from the IDA-S data set, but discard all unmatched IDA-S data set observations.

We next use the job spells data, enriched with worker-level information, to construct a monthly 1998M1-2010M12 firm employment panel, including hiring and separations, and merge this firm employment panel with firm-level information in the Job Advertisement and the VAT Accounts data sets. The construction of the firm-level employment panel entails aggregating establishment-level (i.e. `LBNR`-level) information to the firm-level as described in the main text (recall that firms are identified by their business registry identifiers `CVNR`). The Job Advertisement and the VAT Accounts data sets are subsequently merged with the firm employment panel in two steps.

1. **Merging the firm employment panel with the Job Advertisement data set.** We first aggregate the 2002M7-2009M8 job advertisement data from the advertisement-level to the `CVRNR`-year-month-level, retaining simply the number of advertisements per firm per month, also by occupation. We merge this firm-level monthly job advertisement panel with the 2002M1-2009M12 firm employment panel by `CVRNR`-year-month. We keep all the observations in the firm employment panel, whether or not they are merged to an observation from job advertisement panel, but discard all unmatched job advertisement panel observations.

2. **Merging the firm employment panel with the VAT Account data set.** We merge the 2001M1-2010M12 VAT Accounts data with the 2001M1-2010M12 firm employment panel by `CVRNR`-year-month. We keep all the observations in the firm employment panel, whether or not they are merged to an observation from VAT Account data, but discard all unmatched VAT Account data observations.

B Additional results on the output process

The tables in appendix B report estimates of the revenue and value-added process from the just-identified and over-identified Equally Weighted Minimum Distance estimators. These are very similar to the estimates reported in section 4.

Table B.1: Output processes: Just-identified Equally Weighted Minimum Distance estimates

	(1)	(2)	(3)	(4)	(5)	(6)
			The revenue process			
SD random walk-innovation, σ_η	0.140***	0.136***	0.138***	0.153***	0.142***	0.140***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
SD MA-innovation, σ_ζ	0.235***	0.240***	0.238***	0.184	0.251***	0.257***
	(0.002)	(0.002)	(0.002)	(3.869)	(0.003)	(0.003)
Order-1 MA-coefficient, θ_1		0.026***	0.015*	−0.472	−0.029**	0.003
		(0.007)	(0.008)	(41.947)	(0.012)	(0.013)
Order-2 MA-coefficient, θ_2			−0.018***	−0.344***	−0.048***	−0.028***
			(0.008)	(22.150)	(0.009)	(0.009)
Order-3 MA-coefficient, θ_3				−0.183	−0.026***	−0.008
				(7.709)	(0.006)	(0.008)
Order-4 MA-coefficient, θ_4					0.184***	0.199***
					(0.008)	(0.008)
Order-5 MA-coefficient, θ_5						0.015**
						(0.007)
EWMD objective function	0.000	0.000	0.000	0.000	0.000	0.000
			The value-added process			
SD random walk-innovation, σ_η	0.097***	0.124***	0.128***	0.183***	0.129***	0.130***
	(0.003)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
SD MA-innovation, σ_ζ	0.510***	0.498***	0.496***	0.387***	0.536***	0.534***
	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)
Order-1 MA-coefficient, θ_1		−0.035***	−0.042*	−0.526	−0.038**	−0.042
		(0.004)	(0.004)		(0.004)	(0.006)
Order-2 MA-coefficient, θ_2			−0.008***	−0.307***	−0.006***	−0.009***
			(0.004)	(0.004)	(0.003)	(0.004)
Order-3 MA-coefficient, θ_3				−0.167***	0.006**	0.004
				(0.003)	(0.003)	(0.004)
Order-4 MA-coefficient, θ_4					0.173***	0.171***
					(0.005)	(0.005)
Order-5 MA-coefficient, θ_5						−0.003
						(0.004)
EWMD objective function	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Asymptotic standard errors in parentheses. ***, ** and * indicates statistical significance at the 1, 5, and 10 percent level, respectively. Coefficient estimates from the just-identified EWMD-estimator fitting the first $2 + q$ autocovariances of $\Delta_4 \tilde{r}_{jt}$ (upper panel) and $\Delta_4 \tilde{y}_{jt}$ (lower panel), including the order-0 autocovariance (see Table 4), where q is the order of the MA-component in (8).

Table B.2: Output processes: Over-identified Equally Weighted Minimum Distance estimates

	(1)	(2)	(3)	(4)	(5)	(6)
			The revenue process			
SD random walk-innovation, σ_η	0.143***	0.144***	0.146***	0.149***	0.147***	0.149***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
SD MA-innovation, σ_ζ	0.227***	0.226***	0.223***	0.217***	0.227***	0.220***
	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)	(0.005)
Order-1 MA-coefficient, θ_1		−0.020***	−0.034**	−0.118***	−0.086***	−0.181***
		(0.008)	(0.010)	(0.022)	(0.018)	(0.040)
Order-2 MA-coefficient, θ_2			−0.081***	−0.110***	−0.084***	−0.120***
			(0.012)	(0.017)	(0.014)	(0.025)
Order-3 MA-coefficient, θ_3				−0.076***	−0.059***	−0.103***
				(0.012)	(0.011)	(0.021)
Order-4 MA-coefficient, θ_4					0.052***	0.036**
					(0.011)	(0.014)
Order-5 MA-coefficient, θ_5						−0.057***
						(0.012)
EWMD objective function	0.000	0.000	0.000	0.000	0.000	0.000
			The value-added process			
SD random walk-innovation, σ_η	0.134***	0.149***	0.154***	0.158***	0.148***	0.151***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
SD MA-innovation, σ_ζ	0.484***	0.476***	0.472***	0.468***	0.497***	0.493***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Order-1 MA-coefficient, θ_1		−0.067***	−0.077**	−0.107***	−0.073***	−0.103***
		(0.004)	(0.004)	(0.006)	(0.005)	(0.006)
Order-2 MA-coefficient, θ_2			−0.044***	−0.054***	−0.031***	−0.038***
			(0.005)	(0.005)	(0.004)	(0.005)
Order-3 MA-coefficient, θ_3				−0.029***	−0.011***	−0.026***
				(0.004)	(0.003)	(0.004)
Order-4 MA-coefficient, θ_4					0.076***	0.072***
					(0.004)	(0.004)
Order-5 MA-coefficient, θ_5						−0.023***
						(0.004)
EWMD objective function	0.003	0.002	0.001	0.001	0.001	0.001

Notes: Asymptotic standard errors in parentheses. ***, ** and * indicates statistical significance at the 1, 5, and 10 percent level, respectively. Coefficient estimates from the over-identified EWMD-estimator fitting the first 13 autocovariances of $\Delta_4\tilde{r}_{jt}$ (upper panel) and $\Delta_4\tilde{y}_{jt}$ (lower panel), including the order-0 autocovariance (see Table 4).

C Additional results on IV regressions

The table reports estimates from the instrumental variable estimation of α and β for moving average processes of order $q = 0, 1, 2, 3, 4, 5$.

Table C.1: Instrumental variables estimates of advertisement rate responses to permanent and transitory revenue and value added shocks

	$w \sim MA(0)$		$w \sim MA(1)$		$w \sim MA(2)$		$w \sim MA(3)$		$w \sim MA(4)$		$w \sim MA(5)$	
	α	β	α	β	α	β	α	β	α	β	α	β
Revenue ($z_{jt} = r_{jt}$)												
Estimate	0.056***	−0.003	0.057***	−0.003	0.064***	−0.003	0.058***	−0.003	0.065***	−0.003	0.060***	−0.003
	(0.007)	(0.007)	(0.007)	(0.007)	(0.010)	(0.007)	(0.010)	(0.007)	(0.014)	(0.007)	(0.014)	(0.007)
1SD shock	0.135	0.312	0.136	0.313	0.136	0.311	0.140	0.313	0.137	0.330	0.139	0.331
Ad rate response	0.008	−0.001	0.008	−0.001	0.009	−0.001	0.008	−0.001	0.009	−0.001	0.008	−0.001
1st stage F	3,634	2,327	3,914	2,327	2,639	2,327	3033	2,327	1,584	2,327	1,769	2,327
Firms	19,499	19,499	19,499	19,499	18,247	19,499	18,247	19,499	16,719	19,499	16,719	19,499
Firm-qrts	76,216	76,216	76,216	76,216	70,609	76,216	70,609	76,216	64,829	76,216	64,829	76,216
Observations	300,757	300,757	300,757	300,757	223,496	300,757	223,496	300,757	152,887	300,757	152,887	300,757
Value-added ($z_{jt} = y_{jt}$)												
Estimate	0.067***	−0.004**	0.062***	−0.004**	0.094***	−0.004**	0.072***	−0.004**	0.109***	−0.004**	0.100***	−0.004**
	(0.011)	(0.002)	(0.010)	(0.002)	(0.016)	(0.002)	(0.012)	(0.002)	(0.023)	(0.002)	(0.021)	(0.002)
1SD shock	0.128	0.718	0.135	0.744	0.138	0.740	0.140	0.740	0.137	0.769	0.139	0.770
Ad rate response	0.009	−0.003	0.008	−0.003	0.013	−0.003	0.010	−0.003	0.015	−0.003	0.014	−0.003
1st stage F	816	7,807	845	7,807	862	7,807	1,008	7,807	511	7,807	467	7,807
Firms	16,813	16,813	16,813	16,813	14,223	16,813	14,223	16,813	11,530	16,813	11,530	16,813
Firm-qrts	63,485	63,485	63,485	63,485	52,475	63,485	52,475	63,485	42,913	63,485	42,913	63,485
Observations	213,246	213,246	213,246	213,246	146,916	213,246	146,916	213,246	94,441	213,246	94,441	213,246
IVs	$\Delta_8 \tilde{z}_{jt+1}$	$\Delta_4 \tilde{z}_{jt+4}$	$\Delta_8 \tilde{z}_{jt+2}$	$\Delta_4 \tilde{z}_{jt+4}$	$\Delta_{12} \tilde{z}_{jt+3}$	$\Delta_4 \tilde{z}_{jt+4}$	$\Delta_{12} \tilde{z}_{jt+4}$	$\Delta_4 \tilde{z}_{jt+4}$	$\Delta_{16} \tilde{z}_{jt+5}$	$\Delta_4 \tilde{z}_{jt+4}$	$\Delta_{16} \tilde{z}_{jt+4}$	$\Delta_4 \tilde{z}_{jt+4}$

Notes: Asymptotic standard errors clustered at the firm-level in parentheses. ***, ** and * indicates statistical significance at the 1, 5, and 10 percent level, respectively.