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ABSTRACT

Introducing Time-to-Educate in a Job Search Model*

Transition patterns from school to work differ considerably across OECD countries. Some countries exhibit high youth unemployment rates, which can be considered an indicator of the difficulty facing young people trying to integrate into the labor market. At the same time, education is a time-consuming process, and enrolment and dropout decisions depend on expected duration of studies, as well as on job prospects with and without completed degrees. One way to model entry into the labor market is by means of job search models, where the job arrival hazard is a key parameter in capturing the ease or difficulty in finding a job. Standard models of job search and education assume that skills can be upgraded instantaneously (and mostly in the form of on-the-job training) at a fixed cost. This paper models education as a time-consuming process, a concept which we call time-to-educate, during which an individual faces the trade-off between continuing education and taking up a job.

JEL Classification: E24, J31, J41, J64

Keywords: job search, education, enrollment, dropouts

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1 Motivation

Transition patterns from school to work differ considerably across OECD countries. Column (1) of table 1 shows that the percentage of youths completing upper secondary education ranges from 68 to 100 percent in OECD countries.[1] The percentage of the population within the age bracket 25-34 and that have attained university education (tertiary-type A education), which is displayed in column (2), depends on two factors: firstly, the transition rate between upper secondary education and university, and secondly, on the survival rate in university, i.e. on the fraction of students completing their studies. The survival rate in university is displayed in column (3). Several things are noteworthy about the survival rate in university: in all countries, a number of students drop out of university without obtaining a degree. On average, 30 percent of students in OECD countries do not complete their studies. Part of the explanation for this phenomenon is probably that some students give up because they realize that they are not 'college material'. Another striking feature of column (3) is the considerable variation in survival rates across countries. While 94 percent of those starting university in Japan manage to complete their degrees, only 42 percent of students in Italy complete their studies. Taking the complement of survival rates, we arrive at the dropout rate, which ranges from 6 percent in Japan to a staggering 58 percent in Italy. This raises the question what are the driving forces between university enrolment and dropout behavior. One factor which may be relevant in understanding the transition from school to work is the youth unemployment rate, which can be considered an indicator for the difficulty of youths to integrate into the labor market.

Our conjecture is that high youth unemployment rates at the time of leaving high school may induce some youths to go to university (in particular when tuition fees are low) who would not have done so in the case of more favorable labor market conditions. These students may continue searching for a job while enroled in university and may drop out once they receive a job offer.

[1] The table contains all OECD countries with non-missing information on the four indicators.

Table 1: Education and labour market indicators

	(1)	(2)	(3)	(4)
Belgium	79	18	60[a]	15.3
Czech Republic	81	12	61	16.6
Denmark	100	23	69	8.3
Finland	85	21	75	19.9
France	82	19	59	18.7
Germany	93	13	70	8.4
Iceland	79	23	73	4.8
Ireland	77	23	85	6.2
Italy	82	12	42	27.0
Japan	92	25	94	9.7
Spain	68	25	77	20.8
Sweden	72	22	48	11.8
United States	73	31	66	10.6
Country mean	81	19	70	12.4

Source: OECD Education at a Glance 2004 and OECD Employment Outlook:
Col. 1 displays upper secondary graduation rates (2002), i.e. the percentage of upper secondary graduates to the population at the typical age of graduation in public and private institutions
Col. 2 displays percentage of the population age 25-34 that has attained tertiary-type A education (2002)
Col. 3 displays survival rates in tertiary-type A education (2000), i.e. the number of graduates divided by the number of new entrants in the typical year of entrance in all university programs
Col. 4 displays youth unemployment rates (2001), i.e. percentage of labour force aged 15-24 in unemployment

In the next section, we formalize this idea in the form of a simple job search model, in which education is explicitly modeled as a time-consuming process, which we will call *time-to-educate*. This is a novel feature in the job search literature, where education/training is usually modeled as a cost in the value functions of either the worker or the firm, depending on who is assumed to pay for the education/training (e.g. Mortensen and Pissarides, 1998, and Pissarides, 2000). In many of these models, skills can be upgraded instantaneously at some cost c (see e.g. Coles and Masters, 2000, Masters, 1998).[2] This is a reasonable assumption when the duration of training is negligible (e.g. an intensive course of a month or so). It is clearly inadequate when education/training is time-consuming and when the focus is on understanding transitions between employment, unemployment and education/training.

In the transition from high school to work, further education beyond compulsory schooling is clearly a time-consuming process. After finishing compulsory schooling, the decision to carry on with education depends on the relative job prospects in terms of wages and employment probabilities at different education levels as well as on the time expected to obtain a further degree. Time to completion varies considerably among individuals, especially in university education (see Becker, 2001).

The plan of the paper is as follows. In the following main section, we present a model of job search which introduces education as a separate labor market status, thereby capturing the idea of *time-to-educate* and endogenizing the (opportunity) cost of education. The final section concludes and discusses two empirical examples.

2 The model setup

Consider a continuous time model with two skill types, unskilled and skilled. We can think of the skilled as holding a university degree and the unskilled as being high school graduates without a university degree. The unskilled can carry on with education, and obtaining a degree, they become skilled. Unskilled workers can be either unemployed, in education, or employed, while

[2] An exception is Malcomson, Maw and McCormick, 2003, who explicitly take into account contract length of apprenticeship contracts. Their focus is, however, on firm training while our focus is on formal (classroom) education.

skilled workers can only be unemployed or employed. Denote by U_u the expected present discounted value (PDV) of income of an unskilled unemployed, and by U_s the PDV of income for a skilled unemployed. By W_u and W_s denote the PDV of being an unskilled employed and of being a skilled employed, respectively, and by E_u the PDV of being an unskilled in education. U_u, U_s, W_u, W_s, and E_u can be given asset interpretations and their relationship can be written in the form of arbitrage equations. Note that we do not model the firm side here to save on space and for the clarity of exposition.

Let b be the flow value of income while unemployed, w_u the wage rate of the unskilled and w_s the wage rate for the skilled, all of which are taken to be exogenous.[3] By r denote the rate of time preference. Assume that an unskilled unemployed has a constant probability λ_u of finding a job at any instant, and a skilled unemployed finds a job with instantaneous probability λ_s. Then, we can write the asset equations defining U_u and U_s as

$$rU_u = b + \lambda_u(W_u - U_u) \tag{1}$$

$$rU_s = b + \lambda_s(W_s - U_s) \tag{2}$$

Note that we could allow for for different flow values of income for the two skill groups ($b_u \neq b_s$). This would, however, not give any significant insights, but would come at the price of more cumbersome notation.[4] An unskilled can take further education, in which case he receives job offers with instantaneous probability η_u that he can accept or reject, and with instantaneous probability γ_i he obtains a degree and becomes skilled.[5] Note that the job arrival and degree arrival processes are assumed to be independent, i.e. they are 'competing risks'. Due to the search friction, newly graduated individuals first go through a spell of unemployment. Remark that γ is indexed by an

[3]Since the number of school-leavers entering the labor market is small compared to the total labor force, we can reasonably consider school-leavers to be price-takers, with w_u and w_s determined by the distribution of skills in the population.

[4]Actually, later on, we will even set b equal to zero, without loss of generality.

[5]Implicit in this setup is the assumption that only degrees matter and that *some* education but no degree is no better than no education at all. The assumption that only degrees matter is known as *sheepskin effects*.

individual-specific index i, allowing for heterogeneity in "degree achievement rates". This reflects the fact that the expected time for reaching a degree varies considerably by individual. γ_i can be interpreted as individual ability and the setup therefore reflects the idea that more able students obtain a degree more quickly than less able students. Let b_e be the flow value of income while in education. In the case of an explicit financial cost of education, b_e may actually be negative. We can now write the asset equation defining E_u as

$$rE_{u,i} = b_e + \gamma_i(U_s - E_{u,i}) + \eta_u \max(W_u - E_{u,i}, 0) \tag{3}$$

where we assume $\eta_u < \lambda_u$,[6] in order to rule out the unrealistic feature that no unskilled are ever observed in unemployment because always $E_{u,i} > U_u$. Modelling education as a separate labor market status captures the idea of *time-to-educate* and endogenizes the (opportunity) cost of education.

The asset equations for W_u and W_s are very simple:

$$rW_u = w_u \tag{4}$$

$$rW_s = w_s = gw_u \tag{5}$$

where $g > 1$ measures the wage gap between skilled and unskilled workers. The assumption implicit in equations (4) and (5) is that once the worker finds a job he can keep it forever, so he never again faces unemployment.[7]

2.1 Solving the model

The model features two decision thresholds. Unskilled unemployed decide whether to enroll or remain unemployed whereas enrolled individuals decide whether to accept job offers and drop out or whether to continue education. We first consider the decision to drop out of education, and then look at the enrolment decision. To keep solutions at both margins tractable and in order to focus on the most interesting effects, we will set $b = 0$, without

[6] This may be reasonable if those in education have less time for job search than those in unemployment.

[7] The model can be easily extended to allow for job destruction at the expense of more cumbersome notation. This version of the model is available upon request.

loss of generality. We will, however, let $b_e \leq 0$, thereby allowing for explicit financial costs of education. This permits us to study the effect of an increase or decrease in tuition fees on enrolment and dropping out behavior.

2.1.1 The dropout margin

When a job offer arrives, an unskilled individual in education can either accept or reject it. Given the heterogeneity in *degree achievement rates*/ability, there will be a marginal type of individual who is exactly indifferent between continuing education and dropping out. For this individual, the condition $W_u = E_{u,i}$ holds. Solving equation (1) for U_u and equation (2) for U_s and substituting in equations (4) and (5) respectively, we obtain the following expressions

$$U_u = \frac{1}{r+\lambda_u}(\lambda_u W_u) = \frac{1}{r+\lambda_u}(\frac{\lambda_u}{r} w_u) \tag{6}$$

$$U_s = \frac{1}{r+\lambda_s}(\lambda_s W_s) = \frac{1}{r+\lambda_s}(\frac{\lambda_s}{r} g w_u) \tag{7}$$

For the marginal individual, the last term in equation (3) disappears because of the condition $W_u = E_{u,i}$ and therefore equation (3) can be rewritten as

$$E_{u,i} = \frac{1}{r+\gamma_i}(b_e + \gamma_i U_s) = \frac{1}{r+\gamma_i}\left(b_e + \frac{\gamma_i}{r+\lambda_s}\frac{\lambda_s}{r} g w_u\right) \tag{8}$$

The condition $W_u = E_u$ can be expressed as

$$\frac{w_u}{r} = \frac{1}{r+\gamma_i}(b + \gamma_i U_s) = \frac{1}{r+\gamma_i}\left(b_e + \frac{\gamma_i}{r+\lambda_s}\frac{\lambda_s}{r} g w_u\right) \tag{9}$$

This expression defines a threshold value γ^d for an individual indifferent between continuing education and dropping out. For individuals with $\gamma_i > \gamma^d$ the last term in (3) disappears and they continue education until they obtain a degree. For individuals with $\gamma_i < \gamma^d$ both the second and the third term in (3) are "active" and whatever event comes first ('competing risks'), degree or job offer, they turn skilled or they drop out.

We now want to see how changes in the parameters of the model affect individuals at the margin γ^d (and thereby also individuals off the margin).

We can do so by applying the implicit function theorem to the following equation which follows directly from equation (9):[8]

$$\frac{\gamma^d}{r+\gamma^d}b_e + \frac{\gamma^d}{r+\gamma^d}\frac{\lambda_s}{r+\lambda_s}gw_u - w_u = 0 \tag{10}$$

All ceteris paribus changes have the expected impacts on the dropout margin: holding all other parameters constant, a marginal increase in the wages of the unskilled, w_u, induces more people to drop out of education. A marginal increase in the wages of the skilled, w_s, or alternatively in the wage gap g, provides an incentive for students to stay in university. As the job arrival rate for skilled unemployed, λ_s, goes up, more students tend to continue university. Notice that the job arrival rate for unskilled unemployed, λ_u, does not enter the optimum. An increase in the discount rate, r, has a negative effect on staying in university. A decrease in tuition fees, i.e. an increase in $b_e \leq 0$, reduces dropout.

We can now turn to the entry margin.

2.1.2 The entry margin

Now, we want to consider the decision to enter university, i.e. find the conditions for $E_{u,i} \geq U_u$. From the previous analysis (see equation (9)) we know that the threshold γ^d is independent from λ_u. We can therefore distinguish two cases: $\gamma_i < \gamma^d$, and hence $W_u > E_{u,i}$, and $\gamma_i > \gamma^d$, and hence $W_u < E_{u,i}$.

In the first case, the last term in (3) does not disappear and we can write

$$E_{u,i} = \frac{1}{r+\gamma_i+\eta_u}[b_e+\gamma_i U_s+\eta_u W_u] = \frac{1}{r+\gamma_i+\eta_u}[b_e+\gamma_i\{\frac{1}{r+\lambda_s}\frac{\lambda_s}{r}gw_u\}+\eta_u\frac{w_u}{r}] \tag{11}$$

From there, we can write the inequality $E_{u,i} \geq U_u$ as follows

$$\frac{b_e r}{r+\gamma_i+\eta_u} + \frac{\gamma_i}{(r+\gamma_i+\eta_u)}\frac{\lambda_s}{r+\lambda_s}gw_u + \frac{\eta_u}{(r+\gamma_i+\eta_u)}w_u \geq \frac{\lambda_u}{r+\lambda_u}w_u \tag{12}$$

This equation defines a new threshold value $\gamma^e < \gamma^d$, that determines whether an unskilled prefers to remain unemployed or to carry on with education. If $\gamma_i < \gamma^e$, the chance of obtaining a degree is so low that it cannot

[8] Derivations for marginal effects can be found in the appendix.

trade off the lower job arrival rate in education (remember $\eta_u < \lambda_u$). If $\gamma_i > \gamma^e$, the lower job arrival rate in education is set off by a sufficiently high degree achievement rate and therefore makes going to education worthwhile.

The second case is much simpler: since always $U_u < W_u$ but at the same time $W_u < E_{u,i}$ in this second case, we find $U_u < E_{u,i}$ and therefore everyone with $\gamma_i > \gamma^d$ goes to education. This is self-evident after studying the previous case: observing that $\gamma_i > \gamma^d > \gamma^e$ yields the same result.

To sum up, there are three ranges of the ability parameter and three corresponding decision rules: $\gamma_i < \gamma^e$: those with a very low ability choose to remain unemployed instead of going to education; $\gamma^e < \gamma_i < \gamma^d$: in this intermediate case, unskilled individuals choose to carry on with education but drop out of education as soon as they obtain a job offer; $\gamma_i > \gamma^d$: unskilled individuals with high ability prefer education to unemployment *and* stay in education until obtaining a degree even in the presence of job offers.

Figure 1 illustrates the possible cases. [`Figure 1 about here`]

Again, we can apply the implicit function theorem to equation (12) to see how different parameter values affect individuals at the margin of enrolling in university or remaining unemployed.[9] Holding all other parameters constant, a marginal increase in the wages of the unskilled, w_u, induces more unskilled to remain unemployed. In contrast, an increase in the wages of the skilled, w_s, or alternatively in the wage gap g, provides an incentive to more students to enroll in education. In the same way, as the job arrival rate for skilled unemployed, λ_s, goes up, more students enroll in education. An increase in the job arrival rate for the unskilled unemployed, λ_u, increases the number of people preferring to remain unemployed. An increase in b_e, i.e. a decrease in tuition fees, increases enrolment, as expected. An increase in the job arrival rate while in education, η_u, has an ambiguous effect on enrolment. The most likely case is the case of higher enrolment when η_u goes up, as one might expect. However, there are parameter constellations, in particular when w_s and λ_s are high, for which an increase in η_u has the counter-intuitive effect of decreasing enrolment. This can be explained as follows: for potential dropouts, i.e. those with $\gamma_i < \gamma^e$ the "utility" of enroling in university is

[9]The derivations are again in the appendix.

a weighted average of unskilled (outside) wage offers and skilled wage offers upon graduation. Since outside wage offers and degree arrival are 'competing 'risks', an increase in η_u makes it less likely that the degree is completed before the first job offer arrives. More weight is thus given to the unskilled wages, which makes university ceteris paribus less attractive.

2.1.3 The dropout ***rate***

In order to compute the *fraction* of dropouts, assume that γ_i is distributed over the interval $(0, \infty)$ with distribution function $F(\gamma_i)$. Then the expected fraction of dropouts is given by $\frac{F(\gamma^d)-F(\gamma^e)}{1-F(\gamma^e)}$.

Note that in a cross-section of individuals there are two margins affecting enrollment behavior: the unskilled can choose to enroll or not and the enrolled can choose to accept job offers when they arrive or to reject them. Behavior of individuals at *both* margins jointly determines total enrollment and dropout. Interestingly, comparative statics at both margins separately give an unambiguous answer on enrollment and dropout behavior. For instance, an increase in the wage gap g both increases the number of individuals who start education (*entry margin*) and increases the number of individuals who reject job offers while in education (*dropout margin*). Therefore, the unambiguous effect of an increase in skilled wages is a higher fraction of individuals in education. What is ambiguous is the implication for the dropout *rate*. To see this, consider an increase in the wage gap g: both γ^d and γ^e go down and the shift of γ^d *relative to* γ^e determines whether the dropout *rate* $\frac{F(\gamma^d)-F(\gamma^e)}{1-F(\gamma^e)}$ goes up or down.

Conditioning on the values of all other parameters (which uniquely determine the thresholds γ^e and γ^d), differences in the ability distribution $F(\gamma_i)$ will affect the *fraction* of dropouts. If the group of students holding a university-entry certificate is less able in country 1 than in country 2, then we expect more students to drop out of university. This describes the selection issue associated with university entry.

3 Conclusion and discussion of applications

We presented a job search model with two skill types, unskilled and skilled, in which the unskilled (high school graduates) can go to university, and become skilled (university graduates). The two skill levels are associated with different job market opportunities. Modeling education as a separate labor market status captures the idea of *time-to-educate* and endogenizes the (opportunity) cost of education. Depending on their expected time of completion, some individuals might drop out of education before obtaining a degree if they get a job offer. The model is able to explain transitions between education, employment, and unemployment. The *time-to-educate* model is particularly relevant in understanding job search when education/training is a separate labor market state and when obtaining a degree is time-consuming, a feature typically neglected in the job search literature.

As one striking empirical example, about 60 percent of all students in Italy drop out of university before obtaining a degree (see table 1 and Becker, 2001). In accordance with the *time-to-educate* model, entering university is the most rational thing to do when faced with the absence of job opportunities immediately after leaving high school (remember Italy's extremely high youth unemployment rate in table 1). The absence of tuition fees is also a factor in this decision.[10] For many students, however, university serves as a *parking lot*. They drop out as soon as they get the first suitable job offer but obtain a degree in case they never get a job offer throughout their studies. Obviously, some students may simply be *misguided* in going to university, i.e. are not 'college material'. As empirical evidence of the existence of the *parking lot* phenomenon, Becker (2001), using data from the 1998 survey of high school leavers (Percorsi di studio e di lavoro dei diplomati Indagine 1998) and provided by the Italian National Statistical Office (Istat), shows that the vast majority of Italian dropouts give 'accepted job offer' or 'found studies too difficult' as the main explanation for dropping out (alternative reasons being e.g. 'enlistment to compulsory military service' and 'personal motives'). Interestingly, the vast majority of those who found their studies too difficult, begin working shortly after dropping out, so a large number of them might not *only* have dropped out because the studies were too difficult

[10] Italy only very recently introduced (modest) tuition fees.

but also because their job prospects were sufficiently positive.[11] The acceptance of job offers is therefore the major motive for dropping out of university in Italy. The *time-to-educate* model rationalizes the economic mechanisms behind the *parking lot* phenomenon.

The *time-to-educate* model can also be applied to advanced (formal) training programs later in career. Workers unemployed for some exogenous reason can search for a new job or opt for a further training program to enhance their skills. When new job offers are received, a worker in a training program faces the same choice as a student in university and has to trade off the costs and benefits of accepting job offers.[12]

[11] This is a standard problem in surveys when only one answer can be given.

[12] Empirically, in Germany for instance, a considerable number of participants in active labor market training programs drop out and take up job offers (Personal communication by Marco Caliendo, DIW, Berlin, based on a yet unpublished paper, June 2005). Evaluation studies typically concentrate on employment outcomes *after* the (scheduled) end of a training program and neglect the issue of dropouts.

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Appendix

A Dropout margin

We can re-write the left-hand side of equation (10) as a function $G(x, \gamma^d)$ where γ^d is the dropout threshold and x denotes any of the parameters in the equation.

Using the implicit function theorem, we can derive

$$\gamma^{d\prime}(x) = -\frac{(\partial G/\partial x)}{(\partial G/\partial \gamma)} \tag{13}$$

Note that

$$\frac{\partial G}{\partial x} = -\frac{r}{(r+\gamma^d)^2} b_e + \frac{\gamma^d \lambda_s w_s}{(r+\gamma^d)(r+\lambda_s)}$$

is positive because $b_e \leq 0$. The denominator of (13) is thus always positive. The sign of $\gamma^{d\prime}(x)$ will be positive whenever $\partial G/\partial x$ is negative and vice versa.

- $\gamma^{d\prime}(w_u) > 0$ because $\partial G/\partial w_u = -1$.
- $\gamma^{d\prime}(w_s) < 0$ because $\partial G/\partial w_s = \frac{\gamma^d \lambda_s}{(r+\gamma^d)(r+\lambda_s)} > 0$.
- $\gamma^{d\prime}(g) < 0$ because $\partial G/\partial g = \frac{\gamma^d \lambda_s w_u}{(r+\gamma^d)(r+\lambda_s)} > 0$.
- $\gamma^{d\prime}(\lambda_s) < 0$ because $\partial G/\partial \lambda_s = \frac{\gamma^d w_s}{r+\gamma^d} \frac{r}{(r+\lambda_s)^2} > 0$.
- $\gamma^{d\prime}(r) > 0$ because $\partial G/\partial r = \frac{\gamma^d}{(r+\gamma^d)^2} b_e - \frac{\gamma^d \lambda_s w_s (2r+\lambda_s)}{[(r+\gamma^d)(r+\lambda_s)]^2} < 0$.
- $\gamma^{d\prime}(b_e) < 0$ because $\partial G/\partial b_e = \frac{r}{r+\gamma^d} > 0$.

B Entry margin

We can re-write the left-hand side of equation (12) as a function $H(x, \gamma^e)$ where γ^e is the (university) enrolment threshold and x denotes any of the parameters in the equation.

Using the implicit function theorem, we can derive

$$\gamma^{e\prime}(x) = -\frac{(\partial H/\partial x)}{(\partial H/\partial \gamma)} \tag{14}$$

Note that

$$\begin{aligned}\frac{\partial H}{\partial x} &= -\frac{b_e r}{(r+\gamma^e+\eta_u)^2} + \frac{r+\eta_u}{(r+\gamma^e+\eta_u)^2}\frac{\lambda_s}{r+\lambda_s} g w_u - \frac{\eta_u}{(r+\gamma^e+\eta_u)^2} w_u \\ &= -\frac{b_e r}{(r+\gamma^e+\eta_u)^2} + \frac{r(\lambda_s w_s - \nu_u w_u) + \eta_u \lambda_s (w_s - w_u)}{(r+\gamma^e+\eta_u)^2 (r+\lambda_s)}\end{aligned}$$

which is positive because $b_e \leq 0$ and since $\lambda_s w_s \geq \eta_u w_u > 0$. The denominator of (14) is thus always positive. The sign of $\gamma^{e\prime}(x)$ will be positive whenever $\partial H/\partial x$ is negative and vice versa.

- $\gamma^{e\prime}(w_u) > 0$ because $\partial H/\partial w_u = \frac{\eta_u}{r+\gamma_i+\eta_u} - \frac{\lambda_u}{r+\lambda_u} = \frac{(\eta_u-\lambda_u)-\lambda_u\gamma^e}{(r+\lambda_u)(r+\gamma^e+\eta_u)} < 0$.
- $\gamma^{e\prime}(w_s) < 0$ because $\partial H/\partial w_s = \frac{\gamma^e}{r+\gamma^e+\eta_u}\frac{\lambda_s}{r+\lambda_s} > 0$.
- $\gamma^{e\prime}(g) < 0$ because $\partial H/\partial g = \frac{\gamma^e}{r+\gamma^e+\eta_u}\frac{\lambda_s}{r+\lambda_s} w_u > 0$.
- $\gamma^{e\prime}(\lambda_s) < 0$ because $\partial H/\partial \lambda_s = \frac{\gamma^e w_s}{r+\gamma^e+\eta_u}\frac{r}{(r+\lambda_s)^2} > 0$.
- $\gamma^{e\prime}(\lambda_u) > 0$ because $\partial H/\partial \lambda_u = -\frac{r w_u}{r+\lambda_u^2} < 0$.
- $\gamma^{e\prime}(\eta_u) \lesseqgtr 0$ because the sign of $\partial H/\partial \eta_u = -\frac{b_e r}{(r+\gamma^e+\eta_u)^2} - \frac{\gamma^e}{(r+\gamma^e+\eta_u)^2}\frac{\lambda_s}{(r+\lambda_s)} w_s + \frac{r+\gamma^e}{(r+\gamma^e+\eta_u)^2} w_u$ is ambiguous (the first and third term are positive, the second is negative). The most likely case is the case of a positive sign of $\partial H/\partial \eta_u$, implying more enrolment when η_u goes up, as one might expect. However, there are parameter constellations, e.g. when w_s (resp. the wage gap g) and λ_s are very high, for which an increase in η_u has the counter-intuitive effect of decreasing enrolment. This is discussed in the main text.
- $\gamma^{e\prime}(b_e) < 0$ because $\partial H/\partial b_e = \frac{r}{r+\gamma^e+\eta_u} > 0$.

Figure 1: Possible cases in the model