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ABSTRACT

Modelling Monospony on the Labor Market with Separable Matching Models*

We model monopsony on the labor market using a separable matching model *a la* Choo and Siow (2006). We propose a simple method that estimates 1) the multidimensional determinants of productivity and non-wage preferences separately and 2) the variance of unobserved heterogeneity on both sides of the market. Simulations show the effectiveness of the method. An application to Portuguese data reveals that the variance of unobserved heterogeneity is one order of magnitude larger for workers than for firms and represents about 29% of the variance in nonwage preferences of workers, while observed characteristics of workers and firms explain 71%.

JEL Classification: C25, C78, J28, J31, J32

Keywords: monopsony, amenities, matching models, unobserved heterogeneity, Pseudo-Poisson Maximum Likelihood

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1 Introduction

An empirical regularity of the labor market is that workers only respond to a limited extent to changes in their wage (see Sokolova and Sorensen (2021) for a review). This fact suggests that labor supply is inelastic, allowing firms to extract a markup on wages. The elasticity of supply is also observed to vary depending on workers' demographics and other characteristics (see Mastrogiacomo et al. (2017) for instance) and hence may contribute to wage inequality between workers. To understand wage inequality, one must, therefore, depart from the competitive model of labor markets in which supply is perfectly elastic, and workers are paid their marginal product and instead view the labor market as monopsonistic, as first described in Manning (2003), and more recently in Card (2022).

The theory of compensating wage differentials (Rosen (1986)) provides a constructive avenue to capture these monopsonistic features. It introduces friction in the model through non-wage preferences of workers towards firms, effectively creating an inelastic labor supply while allowing for a rich setting of worker-firm interactions. Employed workers do not only enjoy their wages but also a range of job amenities that may be related to the job difficulty or risk, the firm's location, firm or occupational prestige, or interaction with colleagues. Intuitively, workers are willing to accept lower wages in exchange for a job that provides them with more amenities but get compensated with higher wages for jobs that offer disamenities. How workers perceive amenities and how much weight they attach to them might also depend on their age, education, or gender. Despite their attractiveness in theory, amenities have proven difficult to capture empirically. Lavetti (2023) discusses the challenges faced by researchers when estimating amenities from wage regressions. These challenges arise because it is difficult to disentangle amenities from productivity and other frictions in labor markets using wage differentials only. As argued in Dupuy and Galichon (2022), separable matching models in the tradition of Choo and Siow (2006) are able to tackle this concern by separately identifying worker-firm productivity from amenities, using wages and matching (who works with whom) data. Dupuy and Galichon (2022) indeed show the separate identification of productivity and amenities using the complete information given by matching and wage observations and propose a maximum likelihood estimation method to recover them. The appeal of their approach rests in their solid micro-foundations and their ability to model productivity and amenities as multidimensional.

One hurdle remains, however: because the matching and wages in the data are not determined solely by workers' and firms' observable characteristics, the model requires calibrating how much additional randomness to add to fit the data. This randomness, or unobserved heterogeneity, represents the frictions in matching that are not captured by observables. In matched employer-employee datasets, these observable characteristics usually include gender, education, experience on the workers' side, and workforce demographics on the firms' side. For the approach to be of practical use, the researcher must be able to quantify how much the observed data explain wage dispersion. It is therefore crucial to be able to determine how much wage dispersion is explained by observable characteristics, and how much is driven by unobserved heterogeneity. This is the main contribution of this paper. We use the same model as Dupuy and Galichon (2022) and extend Galichon and Salanié (2024)'s approach to propose a simple estimation method that separately identifies productivity and amenities from observable characteristics on the one hand and scales the relative importance of unobserved drivers of employment and wage on the other.

Galichon and Salanié (2024), (GS24 from now on) have shown that when the match surplus is linear-in-parameters, and the unobserved heterogeneity follows a Gumbel type I distribution, a Pseudo-Poisson Maximum Likelihood method with high-dimensional fixed effects can be used for estimation. Intuitively, the reduced form of the model[1] presents a system of equations expressing the logarithm of masses of matched agents and masses of unmatched agents on both sides of the market (that are endogenous variables) as linear functions of fixed effects in the types of agents (that are exogenous variables). This constitutes a major advantage as most software used by empirical economists includes commands to perform this type of estimation, i.e.

[1]We herewith take the perspective of the econometrician. For the econometrician, the endogenous objects are the masses of matched and unmatched agents by types and the exogenous objects are the variables defining the types of agents. The reduced form of the model represents the endogenous objects as a function of the exogenous objects.

for instance *ppmlhdfe* in Stata or *glmfit* in Matlab.

Yet, this approach requires setting the two scaling parameters of the Gumbel distributions to unity and only estimating the total match surplus, which is the sum of the value of productivity and amenities.[2] In this paper, we build on the PPML approach proposed in GS24 and show how and when one can estimate the two scaling parameters along with the amenities and productivity separately. The key idea is to use the structural form of the model instead of the reduced form used in GS24. The structural form makes use of the equilibrium relations that exist between the masses of unmatched agents and the fixed effects of the model to substitute away the fixed effects and express the (log) masses of matches in terms of the (log) masses of unmatched agents and preferences. PPML can then be applied to these structural equations to recover the relative scaling parameters together with the surplus parameters, as in GS24's leading example.[3] However, if one also has access to data on utility transfers (as wages in a labor market application, for instance), then the two scaling parameters can actually be recovered along with the amenities and productivity parameters separately in a second step by performing a simple (constrained) OLS regression of (log) wages on the (log) masses of unmatched agents. Finally, we also show that one can still recover the two scaling parameters in a situation where transfers are observed, but unmatched agents are only observed on one side of the market, i.e. labor market data where only unmatched workers (resp. jobs) are available. In this case, one can use a mixture of the reduced form and structural form, substituting away only the fixed effects on the side whose unmatched agents are observed.[4]

Next, we run four types of simulations to assess the performance of our method. In every simulation, we benchmark it against the method that fixes the scaling parameters to unity. We simulate matching and wage data based on fixed surplus parameters and

[2]The scaling parameters can also be set to different values, but this requires rescaling the data correspondingly. Note also that previous applications of these matching models to the labor market have performed grid search for the two scaling factors, i.e., Dupuy and Galichon (2022), Dupuy et al. (2021), Dupuy et al. (2020). Chiappori et al. (2017) have used multimarket data to estimate scaling factors varying by types of agents.

[3]The sum of the scaling parameters, which indicates how much heterogeneity needs to be added to the model to rationalize the matching data, is not identified. The match surplus parameters must still be interpreted with care.

[4]Note that if unmatched agents are not observed but the matching and wages are, the relative scaling parameters can still be recovered using the structural form of the matching equation. See Appendix A.2.

scaling factors and then apply our method to recover both. The first three simulations cover each of the three data availability cases described in the previous paragraph and vary the true values of the scaling factors. Our method is able to recover unbiased estimates of both the surplus parameters and scaling factors. In the fourth simulation, we vary the number of types on each side of the market and find that our method is robust to the number of types.

We showcase our method on Portuguese matched employer-employee data, which covers the universe of privately employed workers in 2017. We estimate the parameters governing production and amenities, as well as scaling factors, based on workers' potential experience, gender, and years of education, as well as the firm's size and workforce gender composition. The relationship between productivity and worker characteristics is the same as in the literature that uses only wage regressions. The way these characteristics drive amenities is a more novel finding: job amenities are also bell-shaped in potential experience and are lower for women and educated workers. The women's lower job amenities are largely compensated if they work in a firm whose workforce is more than 50% female but worsen in a small firm (less than ten employees). Educated workers tend to prefer small and mostly female firms but are more productive in the former and less productive in the latter. The scaling factor on workers' random shocks is one order of magnitude larger for workers than for firms, indicating unobservables play a more prevalent role in matching on the workers' than on the firms' side. The estimation also allows for a comparison of the variance of amenities measured through observable characteristics (explained amenities) from the variance of the random Gumbel shock (unexplained amenities). The former is about 5 times larger than the latter, which indicates observed characteristics capture most of the variance. Finally, our model allows us to measure the contribution of explained amenities and unobserved heterogeneity to wage dispersion by running counterfactuals. We alternately set amenities to zero and equalize the firms' and workers' scaling factors. We find that the absence of amenities increases wage dispersion while lowering the worker's scaling factor and increasing the firm's decreases wage dispersion.

This paper's contribution is to model monopsony on labor markets using a matching model that measures both explained and unexplained workers' non-wage preferences and their resprective contribution to wage inequality. Our paper is not the first to study monopsony on the labor market. Most previous work takes a reduced-form approach such as Azar et al. (2022) who show the inelasticity of job applications to wages (see Lavetti (2023) for a complete review), or set up supply and demand equations without micro-foundations. E.g. Fanfani (2022) introduces employer's taste-based discrimination in a monopsonistic model of the labor market to explain the gender pay gap. To our knowledge, few papers micro-found models of monopsony on the labor market. Among them, Card et al. (2018) build a stylized theoretical model that generates plausible wage dispersion, and Lamadon et al. (2022) feed two-way fixed effects estimates of worker and firm heterogeneity (Abowd et al. (1999)) to a two-sided matching model to quantify the rents earned by workers and firms on the US labor market. In Lamadon et al. (2022) a worker's type can be thought of as a single index comprising the effects of observable (age, gender, education, etc.) and unobservable (typically ability and other traits) characteristics. The advantage of this single index is that it allows for correlation between observable and unobservable characteristics. However, this single index restricts the impact of observable variables on productivity and preferences. In particular, it does not allow two different match characteristics to impact productivity and amenity in opposite directions. We view our approach as complementary to theirs: our model assumes independence between the observable and unobservable agents' characteristics, but is able to uncover richer structures of interactions between the workers' and firms' observable characteristics. Our paper is also related to Lise and Postel-Vinay (2020) who advocate for multidimensional sorting in a search model. Their search model is justified by the occurrence of productivity mismatch, which we account for through a different route, namely by allowing workers to care about job amenities. Lastly, our approach allows for a simple, off-the-shelf estimation method combining a GLM regression and a constrained OLS regression, that can easily be used by applied researchers.

Section 2 presents the model, Section 3 and 4 discusses the identification and estimation strategies, Section 5 presents the simulations, Section 6 showcases the empirical application using Portuguese matched employer-employee data, and Section 7 concludes.

2 The Model

Consider a labor market in which workers are grouped into discrete types $x \in \mathcal{X}$ and jobs are grouped into discrete types $y \in \mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{X}$ are the workers' and job's types respective state space. There is a large number of both workers and jobs. There is a mass n_x of workers of type x and a mass m_y of jobs of type y. Denote $\#\mathcal{X}$ and $\#\mathcal{Y}$ the (discrete and finite) number of worker and firm types. Types are multidimensional and stem from workers and jobs observable characteristics. Let μ_{xy} for all $(x, y) \in \mathcal{X} \times \mathcal{Y}$ denotes the mass of matches with a worker of type x and a job of type y, μ_{x0} for all $x \in \mathcal{X}$ denote the mass of unmatched workers of type x and μ_{0y} for all $y \in \mathcal{Y}$ denote the mass of vacancies of type y. Define $\mu = (\mu_{xy}, \mu_{x0}, \mu_{0y})_{x\in\mathcal{X},y\in\mathcal{Y}}$ the vector containing all these masses. By construction, there cannot be more matched agents of each type than available on the market, hence μ must satisfy the margin equations:

$$\begin{aligned} \sum_{y\in\mathcal{Y}} \mu_{xy} + \mu_{x0} = n_x \quad \forall x \in \mathcal{X} \\ \sum_{x\in\mathcal{X}} \mu_{xy} + \mu_{0y} = m_y \quad \forall y \in \mathcal{Y} \end{aligned} \tag{1}$$

The utility of a worker of type x to work in a job of type y is composed of two additive terms: a systematic part $A_{xy} + w_{xy}$ where A_{xy} is the pre-transfer utility and w_{xy} is the transfer (wage) received by the worker, and an idiosyncratic part ε_y drawn from a Gumbel type I distribution with scaling parameter σ_W. Likewise the utility of an employer with type y job when matched with a worker of type x is composed of two additive terms: a systematic utility $\Gamma_{xy} - w_{xy}$ where Γ_{xy} is the pre-transfer utility and w_{xy} the transfer made to the worker by the employer, and an idiosyncratic part η_x drawn from a Gumbel type I distribution with scaling parameters σ_F. Define $w = (w_{xy})_{x\in\mathcal{X},y\in\mathcal{Y}}$ to be the vector of all transfers. Workers who remain unemployed receive zero systematic utility and jobs that remain vacant have no profit.

It is assumed that workers and employers are price takers so that they take equilibrium transfers w as given and they are utility maximizers. Hence they solve respectively

$$\max_y \left(A_{xy} + w_{xy} + \varepsilon_y, \varepsilon_0\right),$$
$$\max_x \left(\Gamma_{xy} - w_{xy} + \eta_x, \eta_0\right)$$

where ε_0 and η_0 are drawn from the same distribution as ε_y and η_x and represent the utility of remaining unmatched.

By the Williams-Daly-Zachary theorem, these two problems yield the following solutions (see Dupuy and Galichon (2022) for more details)

$$\mu_{xy} = \exp\left(\frac{A_{xy} + w_{xy} - a_x}{\sigma_W}\right), \mu_{x0} = \exp\left(-\frac{a_x}{\sigma_W}\right), \tag{2}$$
$$\mu_{xy} = \exp\left(\frac{\Gamma_{xy} - w_{xy} - b_y}{\sigma_F}\right), \mu_{0y} = \exp\left(-\frac{b_y}{\sigma_F}\right), \tag{3}$$

where

$$a_x = \sigma_W \log\left(1 + \sum_y \exp\left(\frac{A_{xy} + w_{xy}}{\sigma_W}\right)\right) - \sigma_W \log n_x$$

and,

$$b_y = \sigma_F \log\left(1 + \sum_x \exp\left(\frac{\Gamma_{xy} - w_{xy}}{\sigma_F}\right)\right) - \sigma_F \log m_y.$$

Letting $\sigma = \sigma_W + \sigma_F$, the equilibrium matching $(\mu_{xy})_{x,y}$ is then recovered as

$$\mu_{xy} = \exp\left(\frac{\Phi_{xy} - a_x - b_y}{\sigma}\right) \tag{4}$$
$$\mu_{x0} = \exp\left(-\frac{a_x}{\sigma_W}\right) \tag{5}$$
$$\mu_{0y} = \exp\left(-\frac{b_y}{\sigma_F}\right) \tag{6}$$

where $\Phi_{xy} = A_{xy} + \Gamma_{xy}$ is the joint matching surplus, a and b satisfy the margin equations (1).

We call this system the reduced form of the model in the sense that it represents the relation between the endogenous variable μ in terms of the exogenous surplus and dummy variables, or fixed effects, for each type of agents.

Note however, that one can use equations (5) and (6) to substitute away a_x and b_y in equation (4) to obtain

$$\mu_{xy} = \exp\left(\frac{\Phi_{xy}}{\sigma} + \frac{\sigma_W}{\sigma}\left(\log\mu_{x0} - \log\mu_{0y}\right) + \log\mu_{0y}\right). \tag{7}$$

We call this equation the structural equation as it relates the endogenous objects $(\mu_{xy})_{x\in\mathcal{X},y\in\mathcal{Y}}$ to other endogenous objects $(\mu_{x0})_{x\in\mathcal{X}}$ and $(\mu_{0y})_{y\in\mathcal{Y}}$. The great benefit of representing equilibrium matching this way is to make the term $\frac{\sigma_W}{\sigma}$ appear as a proportional factor relating $\log\mu_{xy}$ and $\log\mu_{x0} - \log\mu_{0y}$, a feature that can be used for estimating the scaling parameters.

As a by product, using the equations (2-3), one can also recover the equilibrium transfers $(w_{xy})_{x,y}$ as

$$w_{xy} = \frac{\sigma_F}{\sigma}\left(a_x - A_{xy}\right) + \frac{\sigma_W}{\sigma}\left(\Gamma_{xy} - b_y\right). \tag{8}$$

This again is the reduced form representation for the equilibrium transfers and by the same procedure one can derive the structural form to obtain

$$w_{xy} = \frac{\sigma_W}{\sigma}\Phi_{xy} - A_{xy} - \frac{\sigma_W\sigma_F}{\sigma}\left(\log\mu_{x0} - \log\mu_{0y}\right). \tag{9}$$

Finally, to derive an intuitive interpretation of the scaling factors, let us return our attention to the firm's problem. Note that as σ_F increases, ceteris paribus, the difference in utility between any two types of workers x and x' becomes increasingly dependent on the difference in idiosyncratic shocks $\eta_x - \eta_{x'}$. As a consequence, firms' decisions increasingly relfect these idiosyncratic shocks, at the expense of the systematic part of the utility $\Gamma_{xy} - w_{xy}$. It follows that equilibrium matching (sorting) and wages increasingly reflect workers' systematic preferences A_{xy} at the expense of firms' systematic preferences Γ_{xy}. Of course, the reverse is true when σ_W increases, ceteris paribus. In this case, the equilibrium matching (sorting) and wages reflect increasingly firms' systematic preferences at the expense of workers' systematic preferences. This leads us to the conclusion that the relative size of the scaling factor σ_F indicates the sensitivity of equilibrium sorting and transfers to workers' systematic preferences A_{xy}.

3 Identification

Suppose one has access to data on observed matches and unmatched agents $\hat{\mu} = (\hat{\mu}_{xy}, \hat{\mu}_{x0}, \hat{\mu}_{0y})_{x\in\mathcal{X},y\in\mathcal{Y}}$ as well as on transfers $\hat{w} = (\hat{w}_{xy})_{x,y}$. In this section, we show how observed matching and wage allow for non-parametric estimation of worker amenities $A = (A_{xy})_{x\in\mathcal{X},y\in\mathcal{Y}}$ and firm productivity $\Gamma = (\Gamma_{xy})_{x\in\mathcal{X},y\in\mathcal{Y}}$. We denote the data (sample) by $(\hat{\mu}, \hat{w})$ whereas we denote (μ, w) the population assuming that the latter are generated by the model outlined above so that equations (7-9) hold.

We start by rearranging the two structural equilibrium equations (7-9). Taking the log on both sides of (7) and solving for $\frac{\Phi_{xy}}{\sigma}$ yields

$$\frac{\Phi_{xy}}{\sigma} = \log \mu_{xy} - \frac{\sigma_W}{\sigma}\left(\log \mu_{x0} - \log \mu_{0y}\right) - \log \mu_{0y}$$

which can then be plugged into (9) to substitute for $\frac{\Phi_{xy}}{\sigma}$, and after rearranging, yields

$$A_{xy} = \sigma_W \left(\log \mu_{xy} - \log \mu_{x0}\right) - w_{xy}.$$

As a result, the equilibrium equations of the model can be rearranged to express, for each pair (x, y), the firm's and the worker's preferences, i.e. Γ_{xy} and A_{xy} respectively, as functions of data and scaling parameters only, i.e.[5]

$$\begin{aligned} \Gamma_{xy} &= \sigma_F \left(\log \mu_{xy} - \log \mu_{0y}\right) + w_{xy}, \\ A_{xy} &= \sigma_W \left(\log \mu_{xy} - \log \mu_{x0}\right) - w_{xy}. \end{aligned}$$

[5] Note that adding and substracting the two equations yield

$$\Gamma_{xy} + A_{xy} = \sigma_2 \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y}\right) + \sigma_1 \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right)$$

and

$$\Gamma_{xy} - A_{xy} = \sigma_2 \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y}\right) - \sigma_1 \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right) + 2w_{xy}$$

Using our sample of data, one then obtains the following (nonparametric) estimates

$$\hat{\Gamma}_{xy} = \sigma_F \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y}\right) + \hat{w}_{xy},$$
$$\hat{A}_{xy} = \sigma_W \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right) - \hat{w}_{xy}.$$

These expressions clearly show that, for any pair (σ_W, σ_F) and for any couple type (x, y), there exist two numbers $\left(\hat{A}_{xy}, \hat{\Gamma}_{xy}\right)$ that "rationalize" the observed data $(\hat{\mu}, \hat{w})$. This implies that given the values of (σ_W, σ_F), preferences are non-parametrically identified or stated otherwise, (A_{xy}, Γ_{xy}) are point-identified by data $(\hat{\mu}, \hat{w})$ once (σ_W, σ_F) are known.

Suppose the true parameters are (σ_W, σ_F) but one wrongly assumes their values to be $(\breve{\sigma}_W, \breve{\sigma}_F)$. Under the assumption of a large sample, this wrong selection of scaling parameters leads to biased estimates of A_{xy} and Γ_{xy}, i.e. denoted $\breve{A}_{xy}$ and $\breve{\Gamma}_{xy}$ respectively, with the following expression[6]

$$bias^{\Gamma}_{xy} := \breve{\Gamma}_{xy} - \hat{\Gamma}_{xy} = \left(\breve{\sigma}_F - \sigma_F\right)\left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y}\right),$$
$$bias^{A}_{xy} := \breve{A}_{xy} - \hat{A}_{xy} = \left(\breve{\sigma}_W - \sigma_W\right)\left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right).$$

It follows that if $\breve{\sigma}_F < \sigma_F$ then $bias^{\Gamma}_{xy} = \breve{\Gamma}_{xy} - \hat{\Gamma}_{xy} > 0$ if $\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y} < 0$. A similar argument holds for the other side of the market.

4 Estimation

Non-parametric identification of amenities and productivity A and Γ rests on the knowledge of the correct (σ_W, σ_F). However, these are not know *a priori*, and we show how they can be recovered in a parametric estimation of the model. We follow GS24 and parametrize preferences as linear-in-parameters using K basis functions denoted

[6] See Appendix A.1 for more details.

$\phi^k = \left(\phi^k_{xy}\right)_{x\in\mathcal{X},y\in\mathcal{Y}}$ as follows

$$\begin{aligned}\Phi_{xy} &= \sum_k \varphi^k \phi^k_{xy},\\ A_{xy} &= \sum_k \alpha^k \phi^k_{xy},\\ \Gamma_{xy} &= \sum_k \gamma^k \phi^k_{xy},\end{aligned}$$

where by definition $\varphi^k = \alpha^k + \gamma^k$.

Two important points are noteworthy. First, each basis function ϕ^k is set to zero for unmatched agent by assumption: $\phi^k_{x0} = 0$ and $\phi^k_{0y} = 0$ for all x, y, k. Second, these basis functions have a flexible shape, in particular, they can be constant across types x or types y, i.e. be such that $\phi^k_{xy} = \phi^k_{xy\prime}$ for all $y \neq y\prime$ or such that $\phi^k_{xy} = \phi^k_{x\prime y}$ for all $x \neq x\prime$. In this case, the basis function captures the 'main effect' of a type x or y, that does not interact with the other side.[7] We differentiate main effects from fixed effects, which refer to a and b in equation 4-6. This implies that the first point is without loss of generality since systematic reservation utility for unmatched agents have been swiped out of equations (5-6) to enter as main effects into equation (4).

4.1 Data on matches and unmatched agents

We begin our analysis assuming one has access to data on matches and unmatched agents: $(\hat{\mu}_{xy}, \hat{\mu}_{x0}, \hat{\mu}_{0y})_{x,y}$. This is common on marriage markets, where couples and singles are observed, but not transfers within couples. In this setting, GS24 proposed using the system of reduced form equations (4-6), together with the assumption $\sigma_W = \sigma_F = 1$, to estimate the parameters $\left\{\left(\varphi^k\right)_k, (a_x)_x, (b_y)_y\right\}$ by Pseudo-Poisson maximum likelihood with high-dimensional fixed-effects. In essence, this is a Poisson regression of $(\mu_{xy}, \mu_{x0}, \mu_{0y})_{x,y}$ on $\left\{\left(\phi^k_{xy}\right)_{k,x,y}, (-1_x)_x, (-1_y)_y\right\}$ where 1_x is a dummy variable for type x agents and 1_y a dummy variable for type y agents. Note however that, before the estimation, one needs to rescale the observations corresponding to matches (the (x, y) rows as opposed to the $(x, 0)$ and $(0, y)$ rows) by a factor $\sigma = 2$ as required by the reduced form equation (4).

[7] These main effects are only identified when one has access to data on unmatched agents.

In contrast, we note that with the same data but using the structural equation (7) instead, one can estimate the parameters $\left\{\left(\tilde{\varphi}^k = \frac{\varphi^k}{\sigma}\right)_k, \tilde{\sigma}_W = \frac{\sigma_W}{\sigma}\right\}$ using a simple PPML. This corresponds to a Poisson regression of $(\mu_{xy})_{x,y}$ on $\left\{\left(\phi_{xy}^k\right)_{k,x,y}, (\log\mu_{x0} - \log\mu_{0y})_{x,y}, (\log\mu_{0y})_y\right\}$ where $(\log\mu_{0y})_y$ is an offset variable whose value only vary with type y agents. This method has three advantages: i) it does not require to rescale the data as the structural equation (7) applies to matches only, ii) there is no need for high dimensional fixed-effects and most importantly iii) one can estimate the relative scaling parameter $\tilde{\sigma}_W$. The parameter $\tilde{\sigma}_W$ only identifies the share of heterogeneity stemming from workers and not the absolute value of the scaling parameter σ_W.[8]

4.2 With data on transfers

Suppose that one also has access to data on transfers $(\hat{w}_{xy})_{x,y}$, in addition to the data on matching and unmatched $\hat{\mu}$. One could first perform the PPML estimation presented above to obtain estimates for $\left\{\left(\tilde{\varphi}^k = \frac{\varphi^k}{\sigma}\right)_k, \tilde{\sigma}_W = \frac{\sigma_W}{\sigma}\right\}$. It is then straightforward to compute an estimate for the variable $\sum_k \tilde{\varphi}^k \phi_{xy}^k$ and use it to substitute for $\frac{\Phi_{xy}}{\sigma}$ in the structural equation for transfers (9). This equation now reads as

$$w_{xy} = \sigma_W \sum_k \tilde{\varphi}^k \phi_{xy}^k - \sum_k \alpha^k \phi_{xy}^k, -\sigma_F \tilde{\sigma}_W (\log\mu_{x0} - \log\mu_{0y}). \tag{10}$$

It follows that the parameters $\left\{\sigma_W, \left(\alpha^k\right)_k, \sigma_F\right\}$ can be estimated using a constrained-OLS regression of $(w_{xy})_{x,y}$ on $\left\{\left(\sum_k \tilde{\varphi}^k \phi_{xy}^k\right)_{x,y}, \left(-\phi_{xy}^k\right)_{k,x,y}, -\tilde{\sigma}_W (\log\mu_{x0} - \log\mu_{0y})_{x,y}\right\}$ with the linear constraint $\sigma_F = \frac{1-\tilde{\sigma}_W}{\tilde{\sigma}_W}\sigma_W$ to satisfy that $\sigma = \sigma_W + \sigma_F$ and $\tilde{\sigma}_W = \frac{\sigma_W}{\sigma}$. Following this two-step procedure (PPML+constrained OLS), one recovers parameters $\left\{\left(\gamma^k\right)_k, \left(\alpha^k\right)_k, \sigma_W, \sigma_F\right\}$ where $\left(\gamma^k\right)_k = \left((\sigma_W + \sigma_F)\tilde{\varphi}^k\right)_k - \left(\alpha^k\right)_k$.

[8] As a side note, if one is willing to assume that $\sigma = 2$, as in GS24, then one recovers $\left\{\left(\varphi^k\right)_k, \sigma_W, \sigma_F\right\}$ by simply computing $\left\{2\left(\tilde{\varphi}^k\right)_k, 2\tilde{\sigma}_W, 2(1-\tilde{\sigma}_W)\right\}$.

4.3 With data on wages, matches and unmatched agents only on one side

Consider the case where one has access to data on transfer, but the matching data only contain unmatched agents on one side of the market. This is a common occurrence in labor, where the researcher often has access to the counts of unemployed workers (the unmatched workers), bur not to job vacancies (the unmatched jobs). Assume without loss of generality the researcher has access to unmatched workers only, that is they observe $(\hat{\mu}_{xy}, \hat{\mu}_{x0})_{x,y}$. One can then use a mixture of reduced and structural equations to recover parameters $\left\{(\gamma^k)_k, (\alpha^k)_k, \sigma_W, \sigma_F\right\}$. To proceed, start from the system of reduced form equations (4-6) and equation (8) and substitute away for the fixed-effects associated with the side of the market for which masses of unmatched agents are observed, i.e. replace a_x by its expression in terms of μ_{x0}, to obtain

$$\mu_{xy} = \exp\left(\sum_k \frac{\varphi^k}{\sigma}\phi^k_{xy} + \frac{\sigma_W}{\sigma}\log\mu_{x0} - \frac{b_y}{\sigma}\right), \tag{11}$$

$$\mu_{0y} = \exp\left(-\frac{b_y}{\sigma_F}\right), \tag{12}$$

$$w_{xy} = \sigma_W \sum_k \frac{\varphi^k}{\sigma}\phi^k_{xy} - \sum_k \alpha^k \phi^k_{xy} - \frac{\sigma_W\sigma_F}{\sigma}\log\mu_{x0} - \sigma_W\frac{b_y}{\sigma}. \tag{13}$$

Since $(\mu_{0y})_y$ are not observed we cannot use equation (12) in the estimation. However, equation (11) is a semi-structural equation that can be estimated using PPML with high dimensional fixed-effects. One can estimate the parameters $\left\{\left(\tilde{\varphi}^k = \frac{\varphi^k}{\sigma}\right)_k, \tilde{\sigma}_W = \frac{\sigma_W}{\sigma}, -\tilde{b}_y = -\frac{b_y}{\sigma}\right\}$ via a Poisson regression of $(\mu_{xy})_{x,y}$ on $\left\{\left(\phi^k_{xy}\right)_{k,x,y}, (\log\mu_{x0})_x, (1_y)_y\right\}$.

The second step consists of a constrained OLS regression of transfers as previously. First note that equation (13) rewrites in terms of the parameters estimated via PPML as

$$w_{xy} = \frac{\sigma_W}{\sigma}\Phi_{xy} - A_{xy} - \frac{\sigma_W\sigma_F}{\sigma}\left(\log\mu_{x0} - \log\mu_{0y}\right). \tag{14}$$

$$w_{xy} = \sigma_W\left(\sum_k \tilde{\varphi}^k\phi^k_{xy} - \tilde{b}_y\right) - \sum_k \tilde{\alpha}^k\phi^k_{xy} - \sigma_F\tilde{\sigma}_W\log\mu_{x0}.$$

As previously, it follows that the parameters $\left\{\sigma_W, (\alpha^k)_k, \sigma_F\right\}$ can be estimated using a constrained OLS regression of $(w_{xy})_{x,y}$ but this time on $\left\{\left(\sum_k \tilde{\varphi}^k \phi^k_{xy} - \tilde{b}_y\right)_{x,y}, \left(-\phi^k_{xy}\right)_{k,x,y}, -\tilde{\sigma}_W \left(\log \mu_{x0}\right)_x\right\}$ with again the linear constraint $\sigma_F = \frac{1-\tilde{\sigma}_W}{\tilde{\sigma}_W}\sigma_W$ to satisfy that $\sigma = \sigma_W + \sigma_F$ and $\tilde{\sigma}_W = \frac{\sigma_W}{\sigma}$. The two-step procedure (PPML+constrained OLS), allows one to recover parameters $\left\{\left(\gamma^k\right)_k, \left(\alpha^k\right)_k, \sigma_W, \sigma_F\right\}$ where $\left(\gamma^k\right)_k = \left((\sigma_W + \sigma_F) \times \tilde{\varphi}^k\right)_k - \left(\alpha^k\right)_k$.

The model can also be estimated with data on wages and matched agents only as we detail in Appendix A.2.

5 Simulations

In this section, we illustrate our method with simulations, and benchmark it against the Poisson regression strategy where σ_W and σ_F are fixed, developed in Galichon and Salanié (2024). We examine each of the three cases exposed in the previous section: first when the researcher observes a matching distribution and the unmatched, but not transfers (as is common on marriage markets), second when they have access to data on the matching distribution, the masses of unmatched agents, and the transfers (as on labor market where employed, unemployed, vacancies and wages are observed), and finally, when matching and wage distribution are observed, but the unmatched are only observed on one side (if vacancies are missing for instance). In each of these three cases, we provide consistent estimates for (σ_W, σ_F) and surplus parameters $(\varphi^k)_k$. We also exemplify the discussion the bias induced by fixing σ_W and σ_F to the wrong values, from Section 3, by estimating $(\alpha^k)_k$ and $(\gamma^k)_k$. Finally, we show the robustness of our method to varying the number of types on both sides of the market.

To simulate the data, we solve for equilibrium matching, unmatched and wages given number of types $\#X$ and $\#Y$, total masses of agents $n = (n_x)_{x\in\mathcal{X}}$, $m = (m_y)_{y\in\mathcal{Y}}$, amenities A and productivities Γ. We solve for five different models, that each differ by the value of $(\sigma_W, \sigma_F) \in \{(1., 1.), (1.7, 0.3), (2.0, 2.0), (2.0, 3.0), (0.5, 0.2)\}$. In each of these we simulate the data $(\hat{\mu}, \hat{w})$ by drawing 5,000,000 observations from the equilibrium distributions (μ, w).

In all simulations in this section, A, Γ and Φ are parametrized with two basis functions

ϕ^1, and ϕ^2, where

$$\phi_{xy}^k = |v_x^k - v_y^k|, \text{ for } k = 1, 2$$

And v_x^k, v_y^k are randomly generated numbers by type x and y. The surplus functions parameters are set to

$$(\alpha_1, \alpha_2) = (.776, .923) \text{ and } (\gamma_1, \gamma_2) = (.660, .686)$$

Therefore $\varphi^1 = \alpha^1 + \gamma^1 = 1.437$ and $\varphi^2 = \alpha^2 + \gamma^2 = 1.611$.
In the first four simulations, we set $\#X = \#Y = 9$ and $n = (3, 9, 8, 1, 3, 1, 8, 3, 3,$ $m = (9, 10, 7, 9, 5, 10, 3, 7, 2)$. The fifth simulation varies the number of types $\#X$ and $\#Y$ while keeping (σ_W, σ_F) constant to 1.

Table 1 shows estimation results assuming that matches and unmatched agents are observed but not transfers. As a result, only the shares $\left(\frac{\sigma_W}{\sigma}, \frac{\sigma_F}{\sigma}\right)$, and $\left(\frac{\varphi^k}{\sigma}\right)_k$ can be identified. Table 1 therefore presents the estimated ratios $\frac{\hat{\sigma}_W}{\hat{\sigma}_F}$ and $\frac{\hat{\varphi}_1}{\hat{\varphi}_2}$. The ratios $\frac{\hat{\sigma}_W}{\hat{\sigma}_F}$ are not reported in the third column, since the benchmarking method assumes $\sigma_W = \sigma_F = 1$. Compared to fixing $\sigma_W = \sigma_F = 1$, our method is better at estimating both $\frac{\sigma_W}{\sigma_F}$ and $\frac{\varphi_1}{\varphi_2}$. This is particularly evident in the second, fouth and fifth rows, where $\sigma_W \neq \sigma_F$ and we manage to retrieve unbiased estimates of $\frac{\varphi_1}{\varphi_2}$ while fixing the scaling factors leads to biased estimates. When $\sigma_W = \sigma_F$, both method have similar performance. Our method can also retrieve the unbiased ratios $\frac{\sigma_W}{\sigma}$, $\frac{\sigma_F}{\sigma}$ and $\frac{\varphi_k}{\sigma}$, which may be of interest to the analyst.

$[\sigma_W, \sigma_F]$	$\frac{\sigma_W}{\sigma_F}$	unit scaling parameters $\frac{\hat{\sigma}_W}{\hat{\sigma}_F}$	$\frac{\hat{\varphi}_1}{\hat{\varphi}_2}$	scaling parameters estimation $\frac{\hat{\sigma}_W}{\hat{\sigma}_F}$	$\frac{\hat{\varphi}_1}{\hat{\varphi}_2}$
$[1.0, 1.0]$	1.0	–	0.895	1.002	0.883
			(0.006)	(0.01)	(0.012)
$[1.7, 0.3]$	5.667	–	1.274	5.701	0.91
			(0.017)	(0.046)	(0.012)
$[2.0, 2.0]$	1.0	–	0.883	0.993	0.895
			(0.01)	(0.009)	(0.022)
$[2.0, 3.0]$	0.667	–	0.829	0.683	0.929
			(0.009)	(0.01)	(0.029)
$[0.5, 0.2]$	2.5	–	0.937	2.501	0.895
			(0.002)	(0.035)	(0.008)

Notes: Authors' own simulations. Matching is simulated with 9 types on each side of the market, $n_x = (3, 9, 8, 1, 3, 1, 8, 3, 3)$, $m_y = (9, 10, 7, 9, 5, 10, 3, 7, 2)$, $(\varphi_1, \varphi_2) = (1.437, 1.611)$, $\frac{\varphi_1}{\varphi_2} = .892$ and $N = 5,000,000$. Bootstrap standard errors are in brackets below the estimates.

Table 1: Varying $[\sigma_W, \sigma_F]$ - only matching and unmatched on both sides are observed

Table 2 presents the comparison in the second case, where the matches, unmatched and transfers are observed for the same five markets that only differ by the value of (σ_W, σ_F). The point estimates $(\hat{\sigma}_W, \hat{\sigma}_F)$ are not reported in the second column, since the benchmark assumes it to be $(1, 1)$. With our method, $(\hat{\sigma}_W, \hat{\sigma}_F)$ are precisely estimated. Comparing the second and fourth columns, that report estimates for the surplus parameters for both methods, we see that the assumption on (σ_W, σ_F) made by the benchmark has a substantial impact: as soon as $(\sigma_W, \sigma_F) \neq (1, 1)$, the estimates of φ are biased, and land far from their true value. Our method does not suffer from this pitfall: $\hat{\varphi}$ is close to the true φ in all five cases, and the confidence intervals always include the true value.

	unit scaling parameters		scaling parameters estimation	
$[\sigma_W, \sigma_F]$	$[\hat{\sigma}_W, \hat{\sigma}_F]$	$[\hat{\varphi}_1, \hat{\varphi}_2]$	$[\hat{\sigma}_W, \hat{\sigma}_F]$	$[\hat{\varphi}_1, \hat{\varphi}_2]$
$[1.0, 1.0]$	$-$	$[1.444, 1.612]$	$[1.0, 1.0]$	$[1.429, 1.62]$
		$[(0.005), (0.007)]$	$[(0.006), (0.006)]$	$[(0.016), (0.024)]$
$[1.7, 0.3]$	$-$	$[0.869, 0.682]$	$[1.704, 0.3]$	$[1.452, 1.6]$
		$[(0.005), (0.008)]$	$[(0.008), (0.008)]$	$[(0.012), (0.016)]$
$[2.0, 2.0]$	$-$	$[0.717, 0.811]$	$[1.991, 1.999]$	$[1.428, 1.588]$
		$[(0.005), (0.006)]$	$[(0.01), (0.01)]$	$[(0.02), (0.034)]$
$[2.0, 3.0]$	$-$	$[0.718, 0.866]$	$[2.031, 2.998]$	$[1.523, 1.669]$
		$[(0.005), (0.008)]$	$[(0.017), (0.017)]$	$[(0.046), (0.049)]$
$[0.5, 0.2]$	$-$	$[4.052, 4.323]$	$[0.502, 0.2]$	$[1.445, 1.612]$
		$[(0.006), (0.009)]$	$[(0.004), (0.004)]$	$[(0.013), (0.015)]$

Notes: Authors' own simulations. Matching is simulated with 9 types on each side of the market, $n_x = (3, 9, 8, 1, 3, 1, 8, 3, 3)$, $m_y = (9, 10, 7, 9, 5, 10, 3, 7, 2)$, $(\varphi_1, \varphi_2) = (1.437, 1.611)$ and $N = 5,000,000$. Bootstrap standard errors are in brackets below the estimates.

Table 2: Varying $[\sigma_W, \sigma_F]$ - matching, unmatched on both sides and transfers are observed

Table 3 presents the estimation results when unmatched are unobserved on one side of the market. Overall, the fixed σ_W, σ_F method suffers from the same flaws as in Table 2, and our method yields an unbiased estimation of both (σ_W, σ_F) and (φ_1, φ_2). The standard errors are larger than in Table 2 however, indicating the estimation is less precise. This is due to the larger reliance on a single side of the matching market in the absence of unmatched on the other side.

	unit scaling parameters		scaling parameters estimation	
$[\sigma_W, \sigma_F]$	$[\hat{\sigma}_W, \hat{\sigma}_F]$	$[\hat{\varphi}_1, \hat{\varphi}_2]$	$[\hat{\sigma}_W, \hat{\sigma}_F]$	$[\hat{\varphi}_1, \hat{\varphi}_2]$
$[1.0, 1.0]$	–	$[1.445, 1.616]$	$[1.0, 0.996]$	$[1.422, 1.616]$
		$[(0.006), (0.008)]$	$[(0.007), (0.007)]$	$[(0.016), (0.025)]$
$[1.7, 0.3]$	–	$[1.41, 1.652]$	$[1.704, 0.302]$	$[1.441, 1.583]$
		$[(0.006), (0.01)]$	$[(0.009), (0.009)]$	$[(0.017), (0.019)]$
$[2.0, 2.0]$	–	$[0.718, 0.814]$	$[1.991, 2.008]$	$[1.45, 1.616]$
		$[(0.006), (0.008)]$	$[(0.011), (0.011)]$	$[(0.023), (0.043)]$
$[2.0, 3.0]$	–	$[0.578, 0.644]$	$[2.031, 2.959]$	$[1.481, 1.584]$
		$[(0.005), (0.008)]$	$[(0.017), (0.017)]$	$[(0.033), (0.061)]$
$[0.5, 0.2]$	–	$[4.086, 4.598]$	$[0.502, 0.201]$	$[1.454, 1.622]$
		$[(0.007), (0.011)]$	$[(0.004), (0.004)]$	$[(0.014), (0.017)]$

Notes: Authors' own simulations. Matching is simulated with 9 types on each side of the market, $n_x = (3, 9, 8, 1, 3, 1, 8, 3, 3)$, $m_y = (9, 10, 7, 9, 5, 10, 3, 7, 2)$, $(\varphi_1, \varphi_2) = (1.437, 1.611)$ and $N = 5,000,000$. Bootstrap standard errors are in brackets below the estimates.

Table 3: Varying $[\sigma_W, \sigma_F]$ - only matching, transfers and unmatched on one side are observed

Next, Table 4 displays the estimation of $(\alpha^k)_k$ and $(\gamma^k)_k$, using both the fixed σ_W, σ_F method and the technique exposed in this paper. Both use the equation for transfers (10) to recover the split in surplus parameters and estimate it with OLS. Using transfers to estimate the parameters that accrue to the worker's versus the firm's surplus allows the researcher to distinguish between perceived amenities and productivity. Table 4 also exemplifies the discussion from Section 3 on the bias introduced by the fixed σ_W, σ_F method. In the first row, the true value of σ_W and σ_F is 1, and the true values for $(\alpha^k)_k$ and $(\gamma^k)_k$ are within both methods' confidence intervals. When both σs are different from 1, as is the case in the second to fourth row, the results from the unit scaling parameters method are far from the true values. How far they are depends both on the difference between the true σs and 1, and the ratio of matched to unmatched, as shown in equations (19). The last two rows fix σ_F to 1, while σ_W is either below or above 1. The result is a large bias on the $(\alpha^k)_k$, and a more limited one in the $(\gamma^k)_k$.

	unit scaling parameters		scaling parameters estimation	
$[\sigma_W, \sigma_F]$	$[\hat{\alpha}_1, \hat{\alpha}_2]$	$[\hat{\gamma}_1, \hat{\gamma}_2]$	$[\hat{\alpha}_1, \hat{\alpha}_2]$	$[\hat{\gamma}_1, \hat{\gamma}_2]$
$[1.0, 1.0]$	$[0.777, 0.938]$	$[0.661, 0.68]$	$[0.8, 0.95]$	$[0.66, 0.686]$
	$[(0.003), (0.006)]$	$[(0.004), (0.006)]$	$[(0.017), (0.022)]$	$[(0.003), (0.004)]$
$[1.7, 0.3]$	$[-0.015, -0.105]$	$[0.879, 0.792]$	$[0.78, 0.943]$	$[0.657, 0.685]$
	$[(0.003), (0.006)]$	$[(0.004), (0.006)]$	$[(0.012), (0.013)]$	$[(0.002), (0.006)]$
$[2.0, 2.0]$	$[0.415, 0.523]$	$[0.305, 0.279]$	$[0.721, 0.894]$	$[0.669, 0.687]$
	$[(0.004), (0.006)]$	$[(0.004), (0.005)]$	$[(0.022), (0.034)]$	$[(0.004), (0.007)]$
$[2.0, 3.0]$	$[0.563, 0.711]$	$[0.155, 0.146]$	$[0.736, 0.948]$	$[0.664, 0.682]$
	$[(0.004), (0.006)]$	$[(0.004), (0.006)]$	$[(0.042), (0.056)]$	$[(0.007), (0.011)]$
$[0.5, 0.2]$	$[1.773, 1.935]$	$[2.276, 2.396]$	$[0.771, 0.946]$	$[0.662, 0.686]$
	$[(0.005), (0.006)]$	$[(0.005), (0.006)]$	$[(0.012), (0.014)]$	$[(0.002), (0.002)]$
$[0.5, 1.0]$	$[1.31, 1.549]$	$[0.721, 0.782]$	$[0.701, 0.843]$	$[0.664, 0.694]$
	$[(0.004), (0.006)]$	$[(0.005), (0.006)]$	$[(0.062), (0.069)]$	$[(0.005), (0.005)]$
$[1.7, 1.0]$	$[0.291, 0.351]$	$[0.563, 0.534]$	$[0.781, 0.923]$	$[0.663, 0.685]$
	$[(0.004), (0.006)]$	$[(0.004), (0.005)]$	$[(0.018), (0.022)]$	$[(0.003), (0.005)]$

Notes: Authors' own simulations. Matching is simulated with 9 types on each side of the market, $n_x = (3, 9, 8, 1, 3, 1, 8, 3, 3)$, $m_y = (9, 10, 7, 9, 5, 10, 3, 7, 2)$, $(\alpha_1, \alpha_2) = (.776, .923)$, $(\gamma_1, \gamma_2) = (.660, .686)$ and $N = 5,000,000$. Bootstrap standard errors are in brackets below the estimates.

Table 4: Estimating α and γ - matching, transfers and unmatched are observed

Finally, Table 5 presents estimation results in simulations where (σ_W, σ_F) is fixed, but the number of types on each side of the market varies. The increase in the number of types on both sides of the market has two opposing effects on the size of standard errors. First, a the number of types grows, the number of observations in the Poisson regression increases, which improves precision. But then the sample variability within each $x \times y$ cell also increases which worsens precision.

	unit scaling parameters		scaling parameters estimation	
$[\#\mathcal{X}, \#\mathcal{Y}]$	$[\hat{\sigma}_1, \hat{\sigma}_2]$	$[\hat{\varphi}_1, \hat{\varphi}_2]$	$[\hat{\sigma}_1, \hat{\sigma}_2]$	$[\hat{\varphi}_1, \hat{\varphi}_2]$
$[9, 9]$	$-$	$[1.444, 1.612]$	$[1.0, 1.0]$	$[1.429, 1.62]$
		$[(0.005), (0.007)]$	$[(0.006), (0.006)]$	$[(0.016), (0.024)]$
$[3, 3]$	$-$	$[1.439, 1.601]$	$[1.0, 1.008]$	$[1.441, 1.612]$
		$[(0.004), (0.006)]$	$[(0.004), (0.004)]$	$[(0.008), (0.006)]$
$[27, 27]$	$-$	$[1.445, 1.609]$	$[1.003, 1.006]$	$[1.444, 1.61]$
		$[(0.005), (0.003)]$	$[(0.004), (0.004)]$	$[(0.012), (0.01)]$
$[3, 27]$	$-$	$[1.446, 1.616]$	$[0.986, 0.999]$	$[1.483, 1.686]$
		$[(0.012), (0.018)]$	$[(0.019), (0.019)]$	$[(0.109), (0.103)]$

Notes: Authors' own simulations. Matching is simulated with $(\sigma_1, \sigma_2) = (1., 1.)$, $(\varphi_1, \varphi_2) = (1.437, 1.611)$ and $N = 5,000,000$. Bootstrap standard errors are in brackets below the estimates.

Table 5: Varying $[\#\mathcal{X}, \#\mathcal{Y}]$ - matching, transfers and unmatched on both sides are observed

6 Empirical Application

Models of matching *a la* Choo and Siow (2006) have recently been used to answer questions on wage inequality and sorting in labor markets (Dupuy et al. (2021), Corblet (2021), Dupuy and Galichon (2022)). However, the relevance of the analysis rests on how large the scaling factors are: if they are too large, it means matching is mostly random, and the conclusions from estimation are not relevant for public policy. Besides, the variance of unobserved shocks may very well be different between workers and firms, which makes our method of particular interest. To illustrate our method, we provide in this section a proof-of-concept application by using Portuguese matched employer-employee data, *Quadros de Pessoal*, in 2017. We use the information on workers and firms in the data to estimate a Mincerian-like surplus, where we are able to differentiate between amenities and productivity.

In *Quadros de Pessoal*, we have access to workers' age, gender, years of education, and the firms' size (number of employees) and gender composition (share of women employed). We also have access to unemployment by age bin, gender, and education level from publically available data. *Quadros de Pessoal* also contains gross hourly wages paid to employed workers. In order to use these wages as the transfers paid by firms to workers in our matching model, we have to account for income tax paid by employees, as well as contributions to social security paid by both employees and employers. The gross salary reported in the data is net of social security contributions, although employees must still pay income tax on it. In other words, if w is the gross salary, workers of type x and firms of type y matched with one another receive utility and profit:

$$\begin{aligned} &A_{xy} + (1-\tau_{it})w_{xy} + \sigma_W \epsilon_y \\ &\Gamma_{xy} - (1+\tau_{ss})w_{xy} + \sigma_F \eta_x \end{aligned} \tag{15}$$

where τ_{it} is the income tax rate, and τ_{ss} is the total (employer and employee) social security contribution rate. Note that the Portuguese income tax system is progressive, but the first income tax bracket is so wide that almost all employees fall within it; therefore we approximate income tax with a linear rate[9]. We set $\tau_{it} = 14.5\%$ (the tax

[9] See the appendix for more details

rate in the first tax bracket in 2017) and $\tau_{ss} = 34.75\%$ (the sum of employees and employers' social security contributions, respectively 11% and 23.75%). Dupuy et al. (2020) introduce linear taxation in matching markets and show that the maximization of equations (15) is equivalent to

$$\begin{aligned} &A^{\tau}_{xy} + w_{xy} + \sigma^{\tau}_{W}\epsilon_{y} \\ &\Gamma^{\tau}_{xy} - w_{xy} + \sigma^{\tau}_{F}\eta_{x} \end{aligned} \tag{16}$$

where $A^{\tau}_{xy} = \frac{A_{xy}}{1-\tau_{it}}$, $\Gamma xy^{\tau} = \frac{\Gamma_{xy}}{1+\tau_{ss}}$, $\sigma^{\tau}_{W} = \frac{\sigma_{W}}{1-\tau_{it}}$ and $\sigma^{\tau}_{F} = \frac{\sigma_{F}}{1+\tau_{ss}}$. Given that τ_{it} and τ_{ss} are known, we can parametrize the model for A^{τ}_{xy} and Γ^{τ}_{xy}, and correct the estimated parameters by multiplying them by the relevant tax rate. We parametrize worker i's and firm j's type as follows:

$$\begin{aligned} x_i &= \{\text{PotExp}_i, \text{Woman}_i, \text{EducYears}_i\} \\ y_j &= \{\text{MicroFirm}_i, \text{MostlyFemale}_i\} \end{aligned} \tag{17}$$

where PotExp is potential experience in years, Woman is a binary variable equal to 1 if the worker is female, EducYears is the number of years of schooling, MicroFirm is a binary variable equal to 1 if the firm has less than 10 employees, and MostlyFemale is equal to 1 if the firm's workforce is more than 50% female. Appendix B provides more details on the construction of these variables and their distribution.

We choose a Mincerian specification for workers' non-wage amenities and job's production, with a focus on the interaction between gender and education, and firm characteristics. The basis functions ϕ^k are

$$\begin{aligned} &\{\text{PotExp}, \text{Woman}, \text{EducYears}, \\ &\text{Woman} \times \text{MicroFirm}, \text{Woman} \times \text{MostlyFemale}, \\ &\text{EducYears} \times \text{MicroFirm}, \text{EducYears} \times \text{MostlyFemale}\} \end{aligned} \tag{18}$$

In addition to estimating heterogeneity parameters σ_W and σ_F, along with the familiar wage determinants that are education and potential experience, we are focusing on how being a woman and years of education affect returns to the firm's characteristics in non-wage amenities and perceived productivity. Indeed, in the absence of actual productivity data to fit, γ is only the productivity perceived by the worker and firm. The negative coefficient on being a woman could be an actual difference in productivity or the result of discrimination. On the other side of the market, α captures job amenities that might compensate for a low wage. There exists a broad literature on the gender pay gap (Blau and Kahn (2017)) that highlights the role of compensating differentials in wage determination. By identifying non-wage amenities, our model is able to capture those differentials.

From *Quadros de Pessoal* and data on unemployment, we observe $\hat{\mu}_{xy}$, the number of matches between workers of type x and firms of type y, and $\hat{\mu}_{x0}$ the number of unemployed workers of type x. Equation (11) suggests caution in the parametrization of surplus: main effects in job type are not identified,[10] and any main effects in worker type risk collinearity with the log of the masses of unemployed workers. These two observations constrain our parametrization (18): we are only introducing main effects in a specific dimension of worker type, such as gender. Since it is unlikely that the masses of unmatched workers are collinear to this main effect, we ensure that our parameters for surplus are identified. The user should always be cautious when using the method however, and can rely on pre-build functions for PPML in most programming languages that flag any collinearity between variables.

Table 6 presents the estimation results. Estimates read in the same unit as wages, in euros per hour. Parameters for perceived productivity, under column γ have the sign and magnitudes we would expect from a classical wage regression: productivity as a function of potential experience is bell-shaped, depends positively on education, and negatively on being a woman. The negative impact of women on γ is partially offset in firms that employ mostly women, but is strengthened in micro firms. Both micro firms and female-dominated workforces profit from an additional year of education.

Parameters for non-wage amenities significantly differ from perceived productivity. The main effects of being a woman and an additional year of education are negative. The higher their education level, the more workers enjoy working in micro and mostly female firms, although the effect is small. Women strongly prefer working in mostly

[10]See Appendix A.2 for more details.

female firms and dislike being employed in micro firms. Since non-wage amenities in the model go in the opposite direction from wage, the former interaction suggests there are indeed compensating differentials, while the latter shows they depend on the characteristic of the firm considered.

It is important to note that the results in Table 6, indicate that the interaction of education with the share of female workers and with small firms are both positive for the value of amenities, but the interaction in the value of productivity is positive for the former and negative for the latter. A similar observation holds for the interactions between gender and the two firms' characteristics. These results are in sharp contrast with the restrictions imposed by a unidimensional matching model. This point is further explained in Appendix A.3.

The scale of unobserved heterogeneity is about five times larger on the workers' side than the firms', even though there are more information included on workers than firms. Only firm size and gender composition are included in the model, and the occupational composition or industry are ignored.

The estimation above allows us to measure the relative dispersion of explained versus unexplained amenities. The variance of worker's Gumbel shocks is

$$\frac{\pi^2}{6}\sigma_W^2 = 2.66$$

which corresponds to the unexplained amenities. The variance of the explained amenities is

$$V(\alpha_{xy}) = \sum_{x\in\mathcal{X},y\in\mathcal{Y}} \left(\hat{\alpha}_{xy} - \bar{\hat{\alpha}}\right)^2 = 6.64$$

The characteristics we observe on workers in the data (education, experience, gender) therefore allow us to explain $\frac{6.64}{6.64+2.66} = 71\%$ of the non-wage preferences of workers towards jobs.

We produce a series of counterfactuals to illustrate 1) the importance of amenities in the matching of workers to jobs and 2) how unequal scaling factors affect wage inequality. To this aim we first use the estimates of the model and predict the equilib-

	α	γ
Pot. Exp.	.021	.159
	(.001)	(.001)
Pot. Exp. squared	-.002	-.000
	(.000)	(.000)
Woman	-.531	-1.898
	(.026)	(.024)
Educ. Years	-.366	.636
	(.003)	(.004)
Woman $\times$ mostly female	3.271	.296
	(.040)	(.037)
Woman $\times$ micro firm	-1.224	1.446
	(.015)	(.025)
Educ. Years $\times$ mostly female	.0196	1.571
	(.005)	(.009)
Educ. Years $\times$ micro firm	.080	-.218
	(.002)	(.002)
	σ_W	σ_F
	1.271	.233
	(.014)	(.003)

Notes: Authors' own calculation based on *Quadros de Pessoal*. Standard error computed from 100 bootstraps.

Table 6: Empirical Application - Estimates

rium matching and associated wages. This corresponds to the equilibrium as observed on the market/in the data. We then run three counterfactuals. In the first one we set amenities to 0 to eliminate their role in the equilibrium matching and wages. In the second we equalize scaling factors while keeping their sum constant (to keep total heterogeneity constant). In the third, we do both: set amenities to 0 and equalize the scaling factors, keeping their sum constant. This exercise allows us to illustrate how amenities affect expected productivity and wage inequality through their impact on equilibrium matching and wages.

	Parameter choices			Outcomes	
	σ_W	σ_F	α	$\mathbb{E}_\mu[\gamma]$	V(w)
Model prediction	1.271	.233	Estimated	8.00	5.21
Counterfactual 1	1.271	.233	Set to 0	9.35	6.32
Counterfactual 2	.837	.837	Estimated	7.45	3.50
Counterfactual 3	.837	.837	Set to 0	9.26	6.02

Notes: Authors' own calculation based on *Quadros de Pessoal*.
$\mathbb{E}_\mu[\gamma] = \sum_{x,y} \mu_{xy}\hat{\gamma}_{xy}$ where μ is the predicted or counterfactual matching distribution, and $V(w)$ is computed from equation (8) using estimates for A, Γ and (σ_W, σ_F).

Table 7: Empirical Application - Counterfactuals

Comparing counterfactual 1 with the model prediction allows us to quantify the importance of non-wage amenities for productivity and wage inequality. Theory predicts that in the absence of amenities, the equilibrium matching is optimal, i.e., maximizes expected productivity. This is observed in Table 7, where average productivity per match goes from 8.00 in the model prediction to 9.35 in the counterfactual 1, where amenities are absent. It represents an increase of 17% in average productivity. Stated otherwise, the presence of amenities affects the sorting of workers to jobs, generating mismatches that together account for a 17% drop in average productivity.

Comparing counterfactual 2 with the model prediction allows us to quantify the importance of accounting for different scaling factors between workers and firms. Equalizing the scaling factors on both sides of the market dramatically decreases the variance of wages from 5.21 to 3.50, equivalent to a decrease of 33%.

These counterfactuals illustrate the role of amenities, scaling factors, and the interaction between the two: the presence of amenities mitigates productivity maximization by creating mismatches while scaling factors have a significant impact on wage inequality. This impact is highly dependent on how workers value jobs: in the absence of amenities, equalizing the scaling factors in counterfactual 3 would have slightly increased the variance of wages (from 6.32 in counterfactual 1 to 6.02 in counterfactual 3) instead of decreasing it as in the model prediction. This shows a strong interaction between amenities and scaling factors in the determination of wages.

7 Conclusion

This paper adds to the estimation techniques of matching models *a la* Choo and Siow (2006). We show how the Pseudo-Poisson Maximum Likelihood estimation developed in Galichon and Salanié (2024) can be adapted to estimate the scaling parameters of the Extreme Value Type I unobserved heterogeneity. By writing a separable matching model in its structural form, we are able to express the masses of matches in terms of the masses of unmatched agents, which allows us to recover both the scaling and the preference parameters with PPML up to a normalizing constant. Suppose transfers are also observed in the data, such as a labor market application. In that case, one can pin down the magnitude of the normalizing constant and hence recover the absolute values of the scaling parameters, as well as the preference parameters for both sides of the market.

We show the satisfactory performance of our method with a series of simulations. These simulations reveal, among other things, that wrongly assuming equal scaling factors on both sides of the market can lead to severe biases in the estimation of preferences. We then provide a proof of concept through an empirical application to the Portuguese labor market. The empirical application illustrates the importance of our methodology as we find a much larger scaling factor for workers than for firms, meaning that applying PPML while assuming equal scaling parameters on both sides would have led to biased estimates of the preference parameters. Our method is especially relevant to the study of labor markets, where monopsony is driven by workers' unequal non-wage preferences towards firms. Being able to quantify the importance of unobserved heterogeneity versus observed characteristics in employer-employee matching and the setting of wages is key to measuring how much they drive compensating wage differentials.

The most important avenue we see for future research is to link separable matching models *a la* Choo and Siow (2006) to search models and establish the extent to which unobserved heterogeneity in the former captures frictions in the latter. Doing so would bring together the two strands of literature on labor market monopsony, compensating wage differentials, and search frictions, as described in Card (2022).

A Model Appendix

A.1 Identification

Suppose the true scaling parameters are (σ_W, σ_F) but one wrongly assumes their values to be $(\breve{\sigma}_W, \breve{\sigma}_F)$.

The nonparametric estimates of A_{xy} and Γ_{xy} with this wrong set of scaling parameters are then

$$\begin{aligned} \breve{\Gamma}_{xy} &= \breve{\sigma}_F \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y}\right) + \hat{w}_{xy}, \\ \breve{A}_{xy} &= \breve{\sigma}_W \left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right) - \hat{w}_{xy}. \end{aligned}$$

This wrong selection of scaling parameters leads to biased estimates of A_{xy} and Γ_{xy} with the following expression

$$\begin{aligned} bias^{\Gamma}_{xy} &: = \breve{\Gamma}_{xy} - \hat{\Gamma}_{xy} = \left(\breve{\sigma}_F - \sigma_F\right)\left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y}\right), \\ bias^{A}_{xy} &: = \breve{A}_{xy} - \hat{A}_{xy} = \left(\breve{\sigma}_W - \sigma_W\right)\left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right), \end{aligned}$$

since given our large sample assumption, the sample biases

$$\sigma_F \left(\log \hat{\mu}_{xy} - \log \mu_{xy} - \left(\log \hat{\mu}_{0y} - \log \mu_{0y}\right)\right) + \hat{w}_{xy} - w_{xy}$$

and

$$\sigma_W \left(\log \hat{\mu}_{xy} - \log \mu_{xy} - (\log \hat{\mu}_{x0} - \log \mu_{x0})\right) - (\hat{w}_{xy} - w_{xy})$$

vanish.

Subtracting the two biases to figure out which is more important obtains

$$\begin{aligned} b^{\Gamma}_{xy} - b^{A}_{xy} &= (\check{\sigma}_F - \sigma_F)\left(2\log \hat{\mu}_{xy} - \log \hat{\mu}_{0y} - \log \hat{\mu}_{x0}\right) \\ &\quad - (\check{\sigma} - \sigma)\left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right). \end{aligned}$$

Note that when the true sigmas are $(\sigma_W, \sigma_F) = (1, 1)$ then

$$\begin{aligned} \Phi_{xy} &= \Gamma_{xy} + A_{xy} \\ &= 2\log \mu_{xy} - \log \mu_{0y} - \log \mu_{x0}. \end{aligned}$$

So in this specific situation one has

$$b^{\Gamma}_{xy} - b^{A}_{xy} = (\check{\sigma}_F - 1)\,\Phi_{xy} - (\check{\sigma} - 2)\left(\log \hat{\mu}_{xy} - \log \hat{\mu}_{x0}\right).$$

A.2 Estimation with data on matches and wages but without unmatched agents

If one has no data on unmatched agents, the parametrized equation (4)

$$\mu_{xy} = \exp\left(\frac{\sum_k \varphi^k \phi^k_{xy} - a_x - b_y}{\sigma}\right)$$

can be estimated via PPML with high dimensional fixed effects. Note however that in this case main effects (any basis function ϕ^k that is constant for all x or for all y) are not identified. To see why, let two basis functions $\phi^{k'}$ and $\phi^{k''}$ be such that $\phi^{k'}_{xy} = \phi^{k'}_{xy\prime}$ for all $y \neq y\prime$ and $\phi^{k''}_{xy} = \phi^{k''}_{x\prime y}$ for all $x \neq x\prime$. Then the previous equation is strictly equivalent to

$$\mu_{xy} = \exp\left(\frac{\sum_{k \neq k', k''} \varphi^k \phi^k_{xy} - a'_x - b'_y}{\sigma}\right)$$

where $a'_x = a_x - \varphi^{k'} \phi^{k'}_{xy}$ and $b'_y = b_y - \varphi^{k''} \phi^{k''}_{x\prime y}$.
In otherwords, main effects of the types of agents are 'absorbed' by the fixed effects and hence cannot be identified. Yet, one can estimate parameters $\left\{ \left(\tilde{\varphi}^k = \frac{\varphi^k}{\sigma} \right)_k, \left(-\tilde{a}_x = -\frac{a_x}{\sigma} \right)_x, \left(-\tilde{b}_y = -\frac{b_y}{\sigma} \right)_y \right\}$ by Pseudo-Poisson maximum likelihood with high-dimensional fixed-effects through a Poisson regression of $(\mu_{xy})_{x,y}$ on $\left\{ \left(\phi^k_{xy}\right)_{k,x,y}, (1_x)_x, (1_y)_y \right\}$.
Note that the reduced form transfer equation can be rewritten as

$$w_{xy} = \sigma_W \left(\sum_k \tilde{\varphi}^k \phi^k_{xy} - \tilde{b}_y \right) - \sum_k \alpha^k \phi^k_{xy} + \sigma_F \tilde{a}_x. \tag{19}$$

Hence, the parameters $\left\{ \sigma_W, \left(\alpha^k\right)_k, \sigma_F \right\}$ can be estimated using an unconstrained OLS regression of $(w_{xy})_{x,y}$ on $\left\{ \left(\sum_k \tilde{\varphi}^k \phi^k_{xy} - \tilde{b}_y \right)_{x,y}, \left(-\phi^k_{xy}\right)_{k,x,y}, \tilde{a}_x \right\}$.
The two-step procedure (PPML+unconstrained OLS), allows one to recover parameters $\left\{ \left(\gamma^k\right)_k, \left(\alpha^k\right)_k, \sigma_W, \sigma_F \right\}$ where $\left(\gamma^k\right)_k = \left((\sigma_W + \sigma_F) \times \tilde{\varphi}^k \right)_k - \left(\alpha^k\right)_k$.

A.3 Uni versus Multidimensional Model

In the matching model of Lamadon et al. (2022), a worker's type x is unidimensional and defined in such a way that it comprises the impact of both observed (by the analyst) and unobserved characteristics, where the former may be correlated with the latter. Hence, if $x_1, ..., x_n$ are observed and $x_{n+1}, ..., x_m$ are unobserved, a worker's type is given by $x = h(x_1, ..., x_m)$. Firms' types are defined in a similar fashion and let $y = g(y_1, ..., y_m)$ denote the type of a firm. The value of amenities of job y by a worker of type x is then given as say $\alpha(x, y)$ and the value of productivity for that type of worker in that type of job is $\gamma(x, y)$. For the sake of simplicity let us assume that $\alpha(.,.)$, $\gamma(.,.)$, $h()$ and $g()$, are continuous and twice differentiable.
It follows that the cross-partial derivative of amenities and productivity with respect to x_i and y_j read as

$$\begin{aligned} \frac{\partial^2 \alpha}{\partial x_i \partial y_j} &= \frac{\partial h}{\partial x_i} \frac{\partial g}{\partial y_j} \frac{\partial^2 \alpha}{\partial x \partial y}, \\ \frac{\partial^2 \gamma}{\partial x_i \partial y_j} &= \frac{\partial h}{\partial x_i} \frac{\partial g}{\partial y_j} \frac{\partial^2 \gamma}{\partial x \partial y}. \end{aligned}$$

Degree	Nb. years of schooling
No schooling	0
Ensino basico 1	4
Ensino basico 2	6
Ensino basico 3	9
Ensino secundario	12
Ensino secundario+	13
Bachalerato	15
Licenciatura	16
Mestrado	18
Doutoramento	21

Notes: Authors' own matching.

Table 8: Education levels

This is restricive in important ways and in particular note that one then has

$$\frac{\frac{\partial^2 \alpha}{\partial x_i \partial y_j}}{\frac{\partial^2 \gamma}{\partial x_i \partial y_j}} = \frac{\frac{\partial^2 \alpha}{\partial x \partial y}}{\frac{\partial^2 \gamma}{\partial x \partial y}} \forall i, j. \tag{20}$$

Interpreting loosely the cross-partial derivative above as the interaction between a characteristic of the worker and a characteristic of the firm, it follows that the interaction effect between x_i and y_j in amenities relative to the interaction effect between x_i and y_j in productivity must be the same for all x_i and y_j.

B Data Appendix

We use the Portuguese matched employer-employee dataset *Quadros de Pessoal* for 2017. It covers the universe of privately employed workers in Portugal. Employees's occupation, age, education level and gender are provided. Employees are matched with a firm ID, which allows the researcher to compute firm level information. Gross monthly wages (including baseline wage, extra-time pay and bonuses) and hours worked per month, are also available.

Education level is provided as the maximum degree attained by the worker. There are ten different degree levels, reported in Table 8 with their corresponding years of schooling.

	Mean	Median	p25	p75
Years of schooling	10.4	9.0	9.0	12.0
Potential experience	23.0	23.0	14.0	32.0
Female	.460			
White-collar	.144			
Hourly wage (euros)	6.38	4.65	3.83	6.83

Notes: Authors' own matching.

Table 9: Education levels

To compute the share of white-collar workers employed in each firm, we use the worker's occupation at the 1-digit level. We classify *quadros superiores* and *quadros médios* (roughly translated as executives and managers) as white-collar, and the other occupations as non white-collar.

Finally, potential experience is computed by taking the difference of age and the number of years of schooling plus six. Hourly wage is computed by dividing monthly wage by the number of hours worked in a month.

The final sample covers workers between 16 and 68, who work at least one day a week. It contains 2,365,203 employees and 242,401 employers Wage outliers, below the $.1$ percentile and above the $.9$ percentile, are excluded. Table 9 describes the distribution of the variables of interest in the sample. More than half of the sample has completed middle school (ensino basico 3) but not high school (ensino secundario). The distribution of potential experience is balanced between young and senior workers, 46% of the workers are woemn and 14.4% are employed in a white-collar occupation. Hourly wage is low compared to Portugal's EU neighbors, but the distribution is in line with the minimum wage in 2017: 649.83 euros per month for a full time job, roughly $649.83/(4.35 \times 40) = 3.74$ euros per hour.

The income tax rate $\tau_{it} = 14.5\%$ used in the empirical application corresponds to the first income bracket of the Portuguese income tax. It goes from no income to 7,091 euros in monthly income. As already suggested by Table 9, the majority of workers fall under this threshold: out of the 2,365,203 workers in the sample, only 4,678 have a monthly wage above 7,091 euros.

Unemployment data is obtained through the Portuguese statistic institute's website of available public databases.[11] The share of unemployed workers as a percentage of the active population is available at the gender, education and age bin level. Both the education level and age are less disaggregated than in *Quadros de Pessoal*. The distinction between the levels of higher education does not exist in the unemployment data, and the shares are computed by age bins (15-24 years old, 24- 34 years old, etc.). The merging of the two datasets, *quadros de pessoal* and the unemployment share, is therefore done at this more aggregated level. Unemployment shares are then used to compute the number of unemployed individuals, given the number of employees by years of schooling, age and gender.

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