I Z A Institute
of Labor Economics

Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 17556

On the Relative Sequencing of Internal and External Rent-Seeking Contests

Indraneel Dasgupta
Dhritiman Gupta

DECEMBER 2024

I Z A Institute of Labor Economics

Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 17556

On the Relative Sequencing of Internal and External Rent-Seeking Contests

Indraneel Dasgupta
Indian Statistical Institute and IZA

Dhritiman Gupta
Indian Institute of Management

DECEMBER 2024

ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

On the Relative Sequencing of Internal and External Rent-Seeking Contests

We consider rent-seeking contests between and within two equal-sized groups. Each group adopts one of three sequences: first internal then external contest, first external then internal contest, and simultaneous internal and external contests. Groups cannot unilaterally postpone a contest without losing. We rank the nine possible combinations according to rent-seeking expenditure and expected utilities. Rent-seeking is maximum when both internal contests either precede, or occur simultaneously with, the external contest. These forms have identical, Pareto-dominated, welfare consequences. Among contest forms which offer both groups a positive win probability, rent-seeking is minimized if the between-group contest precedes both within-group contests; this also induces Pareto-efficiency. When the groups independently choose the contest sequence, the unique Nash equilibrium involves simultaneous occurrence of all contests. Results due to Warneryd (1998) and Amegashie (1999) fall out. When a multi-member group battles a single-member one, unity against the common enemy (an efficient sequence choice) can be sustained if the larger group can resolve its internal coordination problem. With unequal groups and symmetric contest sequencing, the one-tier contest form may be Pareto-efficient, despite generating greater rent-seeking than all symmetric two-tier forms.

JEL Classification: D70, D72, D74

Keywords: Tullock contest, internal and external rent-seeking, rent dissipation, contest design, war

Corresponding author:
Indraneel Dasgupta
Economic Research Unit
Indian Statistical Institute
203 Barrackpore Trunk Road
Kolkata 700108
West Bengal
India

E-mail: indraneel@isical.ac.in

1. Introduction

On September 18, 1931, a Japanese force blew up a stretch of the South Manchurian Railway five miles north of the town of Mukden. Within a few days, they had taken over large parts of Manchuria. In March 1932, the Japanese proclaimed 'Manchukuo' as an independent state. The next year they concluded a truce with Chinese military representatives that confirmed Japan's control over Manchuria and Inner Mongolia. The Chinese put up little resistance to the Japanese takeover of Manchuria. As soon as he heard of the Mukden bombing, the Chinese Guomingdang leader Chiang Kai-shek advised his general on the spot, Zhang Xueliang, not to respond militarily, despite the latter's troops substantially outnumbering the Japanese. He continued this policy of passivity for a further six years, in the face of repeated Japanese territorial incursions. Throughout 1931-1937, Chiang stuck to his maxim, 'First internal pacification, then external resistance'. He concentrated his rhetorical fire on the 'Red bandits' (the Communists) rather than the 'dwarf bandits' (the Japanese) and insisted that until the 'internal disease has . . . been eliminated, the external disorder cannot be cured'. Even as the Japanese tightened their grip on Manchuria, fighting raged between Nationalists and Communists, culminating in the protracted campaign to oust the Communists from their Jiangxi stronghold.[1]

When groups fight one another for some prize, and members of at least one group have conflicting interests, how does the temporal sequencing of the two contests – external and internal – matter for conflict outcomes? Can it be better for combatants to first engage in within-group conflict, to seek to eliminate rivals inside the group, before proceeding to engage another group, as Chiang Kai-shek sought to do during 1931-1937? Or is it better to defeat the other group before initiating a confrontation with one's initial allies, as Chiang Kai-shek and Mao Zedong sought to do after 1937, when they agreed on a temporary truce for the duration of a common war against the Japanese? Or is it best to engage both in-group and out-group enemies at the same time, as Tito's Communist Partisans did during WW II in Yugoslavia, when they fought both the invading German army and the Serbian Chetniks? Which sequence minimizes social losses from investment in expropriation? The purpose of this paper is to answer these questions in the context of basic models of internal versus external rent-seeking contests.

A large literature exists on two-tier, i.e., internal (or within-group) and external (or between-group), appropriative contests between two groups, featuring first internal then external, first external then internal, and simultaneously external and internal, contest sequences.[2] The three strands of this

[1] See Ferguson (2006, chap. 8).

[2] For models of first internal then external contests, see, for example, Amegashie (1999, 2000), Baik and Lee (2000) and Bhattacharya and Rampal (2024). The vast literature on first external then internal contests includes, for example, Katz and Tokatlidu (1996), Warneryd (1998), Muller and Warneryd (2001), Konrad (2004), Inderst *et al.* (2005, 2007), Lopez Cruz and Torrens (2019) and Bakshi and Dasgupta (2022). Simultaneous internal and

literature, developing the three alternative symmetric temporal sequences in which external and internal conflicts can be organized, have however proceeded independently of one another. Furthermore, asymmetric combinations, where the sequence of internal and external contests differs across groups have not been studied at all, to the best of our knowledge. Our objective is to study the consequences of different possible ways of arranging contest sequences in a comparative setting. We do this by identifying complete rankings of the nine alternative contest sequences – both symmetric and asymmetric - in terms of key outcome variables of interest such as resource investment in appropriation (or, equivalently, rent dissipation), external success probabilities and expected utility of the contestants.

In the existing literature, attempts are usually not made to theoretically justify the choice of a particular two-tier contest sequence, rather than another. Instead, the prior choice of such a contest sequence is typically motivated, when at all, by reference to empirical relevance. Thus, for example, the study of situations where groups first engage in internal contestation to choose their respective champions, with the between-group contest occurring subsequently among the group champions, is typically justified by their occurrence in athletic tournaments, faculty recruitment, R&D competitions or electoral systems with intra-party primary contests (e.g., Amegashie 1999, Bhattacharya and Rampal 2024). Studies of the opposite sequence often refer for motivation to war (e.g., Lopez Cruz and Torrens 2019), manager-shareholder conflicts in firms (e.g., Muller and Warneryd 2001), hierarchies and inter-division conflicts within organizations (e.g., Inderst *et al.* 2005, 2007), or lobbying by regional jurisdictions (e.g., Warneryd 1998) or ethnic groups (e.g., Bakshi and Dasgupta 2022) in a federal framework, with a subsequent contest among the constituent sub-groups of the winning coalition. Analyses of simultaneous internal and external conflict situations are motivated by their common occurrence in firms (e.g., Glazer 2002), civil war or ethnic conflict situations (e.g., Bakshi and Dasgupta 2020), or capital-labour conflicts with ethnic divisions among workers (e.g., Dasgupta 2009). However, the prior normative question of the grounds on which a social planner or an institution designer might prefer one two-tier contest sequence over another – whether the particular two-tier contest sequence under scrutiny is indeed optimal according to some intuitively appealing criterion compared to others – is typically left unanswered. At best, a few contributions, which we discuss below, compare an arbitrarily chosen two-tier contest form with a unified, i.e., single-tier, contest. There is also an unanswered positive counterpart to the normative question - which contest sequence is most likely to emerge endogenously in the absence of a contest designer, as the equilibrium outcome of independent choices by the antagonistic groups? This positive question is natural to ask, for example, in contexts of war. Additionally, in the absence of a contest designer, would the contest form that might be expected to arise endogenously, as an equilibrium outcome, generate socially optimal consequences, whether in

external contests are investigated by, for example, Glazer (2002), Hausken (2005), Munster (2007), Dasgupta (2009), Munster and Staal (2011, 2012), Choi *et al.* (2016), and Bakshi and Dasgupta (2020).

terms of investment in expropriation or the welfare of the participants in the contest? These are the questions, left open in the literature, that we seek to answer in this paper.

The contributions closest in spirit to our analysis are by Warneryd (1998), Inderst *et al.* (2005, 2007), and Amegashie (1999). Warneryd (1998) compares rent-seeking outcomes under a 'unitary' framework (where all jurisdictions compete against one another at the same time for control over a federal fiscal transfer), and a 'federal' or 'devolved' fiscal set-up with a symmetric external-internal contest structure (where higher-level jurisdictions first lobby against one another and lower-level constituents of the winning jurisdiction subsequently engage in mutual contestation). Inderst *et al.* (2005, 2007) carry out similar comparisons of two-tier external-internal rent-seeking with single-tier rent-seeking inside organizations. Amegashie (1999) compares two-tier symmetric internal-external rent-seeking with single-tier rent-seeking. We depart from these contributions by: (a) considering all possible two-tier contest sequences, not just the symmetric external-internal and internal-external ones, and (b) comparing them all with each other, as well as the single-tier case.

We consider a Tullock (1980) contest between two equal-sized groups with multiple members over an exogenously given rent. Group members have conflicting interests, and contest costs are linear. Alongside the between-group contest, a contest also occurs within each group for possession of the rent. We successively consider three alternative sequences in which the internal and external contests can be organized within a group. The first is internal-external, where group members first contest one another to choose a champion; subsequently, the group champion contests the other group's champion. The second sequence is external-internal, where all group members first contest the other group to determine the winning group; if successful, thereafter they contest one another to determine the individual recipient of the rent. The final sequence is simultaneously external-internal, where all agents simultaneously engage in both the between-group and the within-group contest. Apart from the three symmetric cases, where the two groups follow identical sequences, we also consider the six asymmetric cases, where the sequence adopted varies across groups. We assume that no group can unilaterally postpone external conflict without losing the prize for sure.

Comparing equilibrium outcomes among the nine cases, and using aggregate resource allocation to any type of contest as the measure of its intensity, we find that the following complete rankings obtain. Between-group conflict is maximum, and within-group conflict minimum, in the symmetric internal-external case. Thus, internal and external conflicts are affected in opposite directions by a switch in the sequence between internal and external contestation. Nonetheless, overall investment in appropriation (or rent dissipation) is highest in the symmetric internal-external and simultaneous internal-external cases; it is lowest in the symmetric external-internal case (among all the non-trivial contest forms which provide both groups a positive chance of winning in equilibrium). Thus, if the objective of a contest organizer is to minimize overall rent dissipation without eliminating one group by design, she should choose the contest form where between-group conflict precedes within-group conflict for both groups. Examining the expected utility ranking of agents, we also find this contest form to be Pareto-efficient. If, conversely, the contest-designer's objective is to maximize aggregate expenditure of contest effort, then internal contest should either precede, or be simultaneous with, external contest, within both groups. This however entails a Pareto-inferior outcome – these two symmetric contest forms (which have identical welfare and rent-dissipation consequences) are Pareto-dominated by the symmetric external-internal form. The case for fiscal federalism and devolution developed by Warneryd (1998) follows as a corollary of our normative analysis, as does a result due to Amegashie (1999).

We then turn to the question of decentralized choice of contest forms when each group can coordinate internally with regard to the choice of contest sequence. When the contest sequence is chosen independently by the groups, specifically by a group leader acting on behalf of each group, we find that simultaneous internal and external contestation by both groups constitutes the unique Nash equilibrium. Thus, the unique two-tier contest sequence that arises endogenously as the equilibrium outcome in a decentralized setting maximizes aggregate rent-seeking, and leads to a Pareto-inferior outcome.

We proceed to examine how our conclusions change if a multi-member group faces a single member group intent on attacking – a situation that is of special interest in cases of war or ethnic conflict (e.g., Dasgupta 2009, Bakshi and Dasgupta 2022). We find that, for the larger group, engaging in external conflict prior to the internal conflict maximizes aggregate ex-ante group payoff. Thus, the efficient sequence choice can be sustained as the unique Nash equilibrium when it is chosen by a group leader – i.e., when the larger group can somehow resolve its internal coordination problem. Fiscal devolution benefits the larger group and hurts the smaller one, while reducing aggregate rent-seeking.

We also compare the outcomes among all symmetric two-tier contest forms and the one-tier unified form when both groups are multi-member but unequal in size. We find that, in this case, the one-tier contest form may be Pareto-efficient, despite generating greater rent-seeking loss than all symmetric two-tier contests. Thus, no general welfare case can be made for decentralization when the groups vary in size, without taking on board between-group distributional concerns.

Our results suggest that the free-rider disadvantage of larger groups (Olson 1960) can be mitigated by preponing the internal contest within a group. It is entirely eliminated when both groups initially engage in internal contestation. It is reduced if the two contests proceed simultaneously within a group, instead of the external contest preceding the internal one.

Section 2 lays out the basic framework and notation, with two equal-sized groups, where unilateral postponement of conflict leads to certain loss of the prize. Section 3 characterizes the three symmetric cases, where the two groups follow identical contest sequences. Section 4 identifies the equilibria for the asymmetric cases, where the two groups adopt different contest sequences. Section 5 catalogues our comparative conclusions: we present complete rankings of the nine alternative contest forms with regard to internal conflict, external conflict, overall conflict, group success probabilities, and ex-ante expected utilities. We also discuss how our results yield those due to Warneryd (1998). Section 6 discusses both normative applications and positive implications of the results presented in section 5. Sections 7, 8 and 9 discuss alternative variants of the benchmark framework, with groups of unequal size. Sections 7 and 8 both consider a multi-member group facing a single-member opponent. Section 7 examines the case where unilateral postponement of conflict leads to certain loss of the prize, as in the benchmark model. Section 8 relaxes this restriction. Section 9 compares symmetric two-tier contests and a unified single-tier contest when both groups are multi-member but vary in size. Section 10 offers concluding comments. Detailed proofs of propositions are presented in an appendix.

2. Preliminaries

Consider two equal-sized groups M and N. We shall use g to denote the generic group, $g \in \{M, N\}$, while $-g$ will denote its opponent. The population size of each group is p_g; $p_M = p_N = p$, where $p \in \{2,3,4,\ldots\}$. All agents contest one another for a prize whose value is normalized to unity. The contest can be organized in three alternative ways within any given group: I, E and S. The first possibility, I, is an internal-external contest. In this case, the members of the group first contest one another to choose a group 'champion'; the group champion subsequently contests the other group for the prize. The second possibility, E, is an external-internal contest. Then all members of a group first engage in a contest against the other group to determine which group will receive the prize; subsequently, all members of the winning group contest one another for the prize. The third possibility, S, is a simultaneous internal-external contest: all group members simultaneously engage in a between-group contest and a within-group contest. The amount of resources committed to the internal, i.e., within-group, contest by the i-th member of group g will be denoted by x_{ig}; $i \in \{1,\ldots,p\}$. Total resource expenditure on internal conflict within group g is given by $x_g = \sum_{i \in \{1,2,\ldots,p\}} x_{ig}$. The amount of resources committed to the external, i.e., between-group, contest by the i-th member of group g will be denoted by y_{ig}. Total resource expenditure on external conflict by group g is given by $y_g = \sum_i y_{ig}$. Aggregate resource expenditure on between-group contest is $y = y_M + y_N$; y will also serve as our measure of the intensity or extent of external conflict.

All members of a group choose their within-group conflict allocations simultaneously. The probability that the i-th member of group g will win the within-group contest is given, in standard fashion, by the Tullock (1980) contest success function:

$$t_{ig} = \frac{x_{ig}}{x_g} \text{ if } x_g > 0;$$

$$= \frac{1}{p_g} \text{ otherwise.} \tag{1}$$

Analogously, the probability that a group g will win the between-group contest is given by:

$$s_g = \frac{y_g}{y} \text{ if } y > 0,$$

$$= \frac{1}{2} \text{ otherwise.} \tag{2}$$

All individuals live for two periods and are expected utility maximizers; there is no discounting of the future. $\pi_{1,ig}$ will denote the ex-ante individual expected utility from participating in the two-part contest. If a group g engages in the contest sequence $I,$ i.e., the internal-external sequence is given by:

$$\pi_{1,ig}{}^{I} = s_g t_{ig} - x_{ig} - t_{ig} y_g. \tag{3}$$

In the case E, i.e., the external-internal case,

$$\pi_{1,ig}{}^{E} = s_g t_{ig} - s_g x_{ig} - y_{ig}. \tag{4}$$

In the case S, i.e., the simultaneous external-internal case,

$$\pi_{1,ig}{}^{S} = s_g t_{ig} - x_{ig} - y_{ig}. \tag{5}$$

Thus, the individual expected gross benefit is simply the prize weighted by the individual probability of winning in all cases. However, the probability weighting of the costs is affected by the sequence in which the contests occur within the group.

A way of organizing the two-tier contest is an ordered pair $\langle m, n\rangle$; $\langle m, n\rangle \in A = [\{I, E, S\} \times \{I, E, S\}]$, where the first term of the ordered pair refers to the contest sequence followed by group *M* and the second term by the group *N*. The two-part contest can thus be organized in nine alternative ways. Three of these are symmetric, where both groups follow the same contest sequence: $\langle I, I\rangle$ (where each group first chooses its champion), $\langle E, E\rangle$ (where the two groups first contest one another), and $\langle S, S\rangle$ (where both groups simultaneously engage in external and internal contests). These are the cases that have been explored at length in the literature (see footnote 2). The rest are asymmetric, where the sequence of engaging in internal and external contests varies between the groups. The asymmetric possibilities are: $\langle E, S\rangle, \langle S, E\rangle, \langle I, S\rangle, \langle I, E\rangle, \langle E, I\rangle, \langle S, I\rangle$. We are not aware of any contribution except the present paper which analyses asymmetric cases.

We assume that, in case a group follows a sequence that involves initial external conflict (i.e., E or S), it receives the prize with probability 1 and at zero external conflict expenditure when its opponent postpones engagement with it (i.e., it adopts I). Thus, no group can unilaterally postpone external conflict without handing over the prize for free to its opponent. Given this assumption, the asymmetric possibilities $\langle I, S\rangle, \langle I, E\rangle, \langle E, I\rangle$ and $\langle S, I\rangle$ become indistinguishable in terms of equilibrium outcomes from a trivial contest where only one group is allowed to participate and awarded the prize, so that it engages solely in internal contest among group members over sharing of the prize. Our substantive interest with regard to contest design lies in non-trivial between-group contests, where both groups must have a positive probability of acquiring the prize in equilibrium. However, as we shall discuss below, the asymmetric possibilities where one group loses the prize with certainty are relevant for identifying the Nash equilibrium outcome when groups independently choose their sequence.

3. Symmetric sequencing

We first characterize the equilibrium outcomes for the three cases of symmetric sequencing: $\langle I, I\rangle$, $\langle E, E\rangle$ and $\langle S, S\rangle$. While the basic structures of these cases are well-known in the literature (see, for example, respectively Amegashie 1999, Katz and Tokatlidu 1996 and Munster 2007), we reproduce them here for ease of reference in later sections, where we present our substantive results.

3.1. $\langle I, I\rangle$ sequencing

First consider the case of $\langle I, I\rangle$ sequencing. In period 2, the champion from group $g \in \{M, N\}$ solves:

$$\max_{y_g} [s_g - y_g]; \tag{6}$$

subject to the contest technology given by (2). Using the superscript II to denote the equilibrium values, we then get the total external conflict investment:

$$y^{II} = \frac{1}{2}; \tag{7}$$

and the equilibrium external success probabilities:

$$s_g^{II} = \frac{1}{2}. \tag{8}$$

Period 2 payoffs to the champions are given by:

$$\pi_{2g}{}^{II} = \frac{1}{4}. \tag{9}$$

Now consider the period 1, internal, contest within g. The group members choose their internal conflict investments x_{ig} simultaneously to solve:

$$\max_{x_{ig}} \left(\frac{1}{4}\right) t_{ig} - x_{ig}; \tag{10}$$

where t_{ig} is given by (1). The first order conditions then yield the equilibrium intra-group conflict allocation:

$$x_g{}^{II} = \left(\frac{1}{4}\right)\left(\frac{p-1}{p}\right). \tag{11}$$

Aggregate resource expenditure on internal conflict is thus:

$$x^{II} = \left(\frac{1}{2}\right)\left(\frac{p-1}{p}\right). \tag{12}$$

Using (7) and (12), total resource allocation on conflict, i.e., the extent of *rent dissipation*, is:

$$D^{II} = x^{II} + y^{II} = \left(\frac{1}{2}\right)\left(\frac{p-1}{p}\right) + \frac{1}{2} = \frac{2p-1}{2p}. \tag{13}$$

Noting that

$$x_{ig}{}^{II} = \frac{x_g{}^{II}}{p}, \tag{14}$$

ex-ante expected payoff to any member of group g is:

$$\pi_{1g}{}^{II} = \left(\frac{1}{4p^2}\right). \tag{15}$$

3.2. $\langle E, E\rangle$ sequencing

First consider period 2. Consider the internal contest in the period 1 winning group, $g \in \{M, N\}$. All members of g simultaneously choose their internal conflict allocations x_{ig} to solve the problem:

$$\max_{x_{ig}} [t_{ig} - x_{ig}]; \tag{16}$$

where t_{ig} is defined by (1). Using the superscript EE to denote the equilibrium values, we then get, using the first order conditions, the internal conflict investment:

$$x_g^{EE} = \left(\frac{p-1}{p}\right), \tag{17}$$

so that the individual internal conflict effort allocations are:

$$x_{ig}^{EE} = \left(\frac{p-1}{p^2}\right). \tag{18}$$

Thus, in case the group g wins in period 1, individual period 2 pay-offs are given by:

$$\pi_{2g}^{EE} = \left(\frac{1}{p^2}\right). \tag{19}$$

In period 1, each member i of group g chooses her external conflict effort allocation y_{ig} to solve:

$$\max_{y_{ig}} [\left(\frac{1}{p^2}\right) s_g - y_{ig}], \tag{20}$$

where s_g is as defined by (2). Then, the first order conditions yield:

$$s_g^{EE} = \frac{1}{2}; \tag{21}$$

$$y^{EE} = \left(\frac{1}{2p^2}\right); \tag{22}$$

We shall focus on the within-group symmetric equilibrium for external conflict, where all members of a group make identical contributions to the external contest. In a within-group symmetric equilibrium, individual external conflict contributions are:

$$y_{ig}^{EE} = \frac{1}{4p^3}. \tag{23}$$

Expected total resource expenditure on internal conflict is:

$$X^{EE} = s_m^{EE} x_m^{EE} + s_n^{EE} x_N^{EE} = \left(\frac{p-1}{p}\right). \tag{24}$$

By (22) and (24), expected total resource expenditure on conflict, i.e. expected rent dissipation, is:

$$D^{EE} = y^{EE} + X^{EE} = \left(\frac{1}{2p^2}\right) + \left(\frac{p-1}{p}\right). \tag{25}$$

Ex ante (period 1) expected pay-offs in a within-group symmetric equilibrium are:

$$\pi_{1g}^{EE} = \left(\frac{2p-1}{4p^3}\right). \tag{26}$$

3.3. $\langle S, S \rangle$ sequencing

Now consider the case where all agents choose their internal and external contest allocations simultaneously. Thus, the representative member i of group g solves the problem:

$$\max_{x_{ig}, y_{ig}} [\, s_g\, t_{ig} - x_{ig} - y_{ig}]; \tag{27}$$

where s_g,t_{ig} are given by (2) and (1), respectively. Using the superscript SS to denote the equilibrium values, the first order conditions yield:

$$s_g^{SS} = \frac{1}{2}; \tag{28}$$

$$y^{SS} = \left(\frac{1}{2p}\right); \tag{29}$$

$$x^{SS} = \left(\frac{p-1}{p}\right). \tag{30}$$

Using (29) and (30),

$$D^{SS} = x^{SS} + y^{SS} = \left(\frac{2p-1}{2p}\right). \tag{31}$$

We focus, as before, on the within-group symmetric, i.e., equal contribution, equilibrium in the external contest. Period 1 payoffs in the within-group symmetric equilibrium are given by:

$$\pi_{1g}^{SS} = \left(\frac{1}{4p^2}\right). \tag{32}$$

4. Asymmetric sequencing

We now proceed to examine the six cases of asymmetric sequencing, where the contest sequence adopted differs between groups. To the best of our knowledge, these cases have not been discussed in the literature.

4.1. $\langle E, S\rangle$ sequencing

Suppose that M engages in external contest first, while N engages simultaneously in external and internal contests. Then, in case M wins in period 1, each member of M solves in period 2 the problem in (16). Then, using (17),

$$x_M^{ES} = \left(\frac{p-1}{p}\right).$$

Thus, in case the group g wins in period 1, individual period 2 pay-offs of its members are given by:

$$\pi_{2M}^{ES} = \left(\frac{1}{p^2}\right).$$

In period 1, each member of M chooses her external conflict effort allocation y_{ig} to solve:

$$\max_{y_{ig}} [\left(\frac{1}{p^2}\right) s_g - y_{ig}].$$

The first order condition yields:

$$\left(\frac{1}{p^2}\right)\left(\frac{y_N}{y^2}\right) = 1.$$

Now consider N. The representative member i of N solves the problem in (27):

$$\max_{x_{i-g}, y_{i-g}} [s_{-g}\, t_{i,-g} - x_{i,-g} - y_{i,-g}].$$

The first order conditions yield:

$$x_N = s_N\left(\frac{p-1}{p}\right);$$

$$\left(\frac{y_M}{y^2}\right) = p.$$

Recalling that:

$$\left(\frac{y_N}{y^2}\right) = p^2,$$

we then have:

$$s_M^{ES} = \frac{1}{p+1}. \tag{33}$$

Thus,

$$x_N^{ES} = \left(\frac{p-1}{p+1}\right),$$

$$y^{ES} = \frac{1}{p^2+p}. \tag{34}$$

$$y_M^{ES} = \left(\frac{1}{p+1}\right)\left(\frac{1}{p^2+p}\right);$$

$$y_N^{ES} = \left(\frac{p}{p+1}\right)\left(\frac{1}{p^2+p}\right).$$

Total expected expenditure on internal conflict is:

$$X^{ES} = s_M^{ES}x_M^{ES} + x_N^{ES} = \left(\frac{p-1}{p}\right). \tag{35}$$

Total expected rent-seeking expenditure is:

$$D^{ES} = X^{ES} + y^{ES} = \left(\frac{p}{p+1}\right). \tag{36}$$

Expected period 1 pay-offs under equal contributions to the external contest by group members are:

$$\pi_{1M}^{ES} = s_M^{ES}\pi_{2M}^{ES} - y_{iM}^{ES} = \left(\frac{1}{p}\right)\left(\frac{1}{p+1}\right)^2; \tag{37}$$

$$\pi_{1N}^{ES} = [\,s_N^{ES}\left(\frac{1}{p}\right) - x_{iN}^{ES} - y_{iN}^{ES}] = \left(\frac{1}{1+p}\right)^2. \tag{38}$$

4.2. $\langle S, E\rangle$ sequencing

Suppose now that *M* engages simultaneously in internal and external contests, while *N* engages first in external contestation. Since the two groups are of equal size, it is obvious that this case is the mirror image of the $\langle E, S\rangle$case. Thus:

$$s_M^{ES} = s_N^{SE}, y^{ES} = y^{SE}, X^{ES} = X^{SE}, D^{ES} = D^{SE}, \pi_{1M}^{ES} = \pi_{1N}^{SE}, \pi_{1N}^{ES} = \pi_{1M}^{SE}. \tag{39}$$

4.3. $\langle I, S\rangle, \langle I, E\rangle, \langle E, I\rangle$ and $\langle S, I\rangle$ sequencing

Recall that, when a group follows a sequence that involves initial external conflict (i.e., E or S), it receives the prize with probability 1 and at zero external conflict expenditure when its opponent postpones engagement with it (i.e., it adopts I). Hence, the asymmetric possibilities $\langle I, S\rangle, \langle I, E\rangle, \langle E, I\rangle$ and $\langle S, I\rangle$ become indistinguishable in terms of equilibrium outcomes from a contest where only one group is allowed to participate and awarded the prize, so that it engages solely in internal contest among group members over sharing of the prize. In the cases $\langle I, S\rangle$ and $\langle I, E\rangle$, then, N's problem is:

$$\max_{x_{iN}} [t_{iN} - x_{iN}];$$

so that:

$$D^{IE} = D^{IS} = D^{EI} = D^{SI} = x_N^{IE} = x_N^{IS} = x_M^{EI} = x_M^{SI} = \left(\frac{p-1}{p}\right), \tag{40}$$

$$\pi_{1N}^{IE} = \pi_{1N}^{IS} = \pi_{1M}^{EI} = \pi_{1M}^{SI} = \left(\frac{1}{p^2}\right); \tag{41}$$

$$\pi_{1M}^{IE} = \pi_{1M}^{IS} = \pi_{1N}^{EI} = \pi_{1N}^{SI} = 0. \tag{42}$$

5. Ranking contest forms

We are now ready to rank our nine alternative contest forms according to alternative equilibrium outcome measures, for the case of equal-sized groups. We first provide complete rankings according to resource allocated to between-group, within-group and overall rent-seeking.

Proposition 1. Suppose $p_N = p_M \geq 2$. Then:

(i) $y^{II} > y^{SS} > y^{ES} = y^{SE} > y^{EE} > 0$;

(ii) $x^{II} < x^{SS} = X^{ES} = X^{ES} = X^{EE} = \left(\frac{p-1}{p}\right)$;

(iii) $D^{II} = D^{SS} > D^{ES} = D^{SE} > D^{EE} > \left(\frac{p-1}{p}\right)$.

Proof. See the appendix.

Recall that $[y^{IE} = y^{IS} = y^{EI} = y^{SI} = 0]$. By Proposition 1(i), external, i.e., between-group, conflict is most intense if preceded by internal (or within-group) conflict within both groups. It is lowest in non-trivial two-tier inter-group contests if the sequence is reversed. This reflects the fact that individual incentive to contribute to between-group conflict is highest when such conflict occurs after the resolution of within-group conflict, and least if the sequence is reversed. Now recall (40). Proposition 1(ii) implies that internal conflict is least when it occurs before external conflict within both groups. Thus, a change in the sequence in which conflict occurs affects between and within-group conflict in opposite ways. What happens to aggregate conflict, then? Recalling (40), by Proposition 1(iii), overall conflict (or aggregate rent dissipation) is greatest when both internal contests occur prior to or at the same time as the external contest. For non-trivial two-group contests, it is least when external conflict occurs first. Parts (ii) and (iii) of Proposition 1 imply that the share of the between-group component in aggregate conflict is greatest when such conflict follows within-group conflict.

Our next set of results identify how external success probabilities, and first period expected utilities (which capture the ex-ante expected individual benefits from participating in the two-tier contest), are affected by the contest sequence.

Proposition 2. Suppose $p_N = p_M \geq 2$. Then:

(i) $1 > s_M^{SE} > s_M^{SS} = s_M^{II} = s_M^{EE} = \frac{1}{2} > s_M^{ES} > 0;$

(ii) $\left(\frac{1}{p^2}\right) > \pi_{1M}^{SE} > {\pi_{1M}}^{EE} > {\pi_{1M}}^{II} = \pi_{1M}^{SS} > \pi_{1M}^{ES} > 0;$

(iii) $\left(\frac{1}{p^2}\right) > \pi_{1N}^{ES} > {\pi_{1N}}^{EE} > {\pi_{1N}}^{II} = \pi_{1N}^{SS} > \pi_{1N}^{SE} > 0.$

Proof. See the appendix.

Recall that, if its opponent chooses to resolve its internal contest first, then a group can ensure the prize for itself by unilaterally choosing to engage in the external contest, whether prior to or alongside its internal contest. Proposition 2(i) states that, among all the non-trivial cases where both groups have a positive probability of winning the prize in equilibrium, a group's success probability is highest if it engages simultaneously in external and internal contests (*S*), while its opponent postpones its own internal conflict (*E*). Recalling (41) and (42), Proposition 2((ii) and (iii)) implies that this case also maximizes the symmetric ex-ante payoff to each member of the group which adopts the sequence S. Among the non-trivial cases, symmetric ex-ante payoffs are minimized within a group if that group adopts E while its opponent adopts S.

Warneryd (1998) compares the outcomes under ‘unification’, where multiple jurisdictions contest simultaneously for resource transfer from a central government, with those under ‘federalism’, where larger regions first contest one another for the rent, and subsequently constituent jurisdictions in the winning region contest one another. His federalism case corresponds to our external-internal contest – the contest form we characterize in section 3. We now show how his findings can be generalized.

In the unitary case, all jurisdictions $i \in \{1, \ldots, 2p\}$ solve:

$$\max_{d_i} \left(\frac{d_i}{\sum_{j\in\{1,\ldots,2p\}} d_j}\right) - d_i.$$

Then aggregate rent-seeking expenditure is given by:

$$D^U = \left(\frac{2p-1}{2p}\right); \tag{43}$$

while payoffs are given by:

$$\pi_U = \left(\frac{1}{4p^2}\right). \tag{44}$$

Then, using (13), (15), (43) and (44), we have the following.

Proposition 3. Suppose $p_N = p_M \geq 2$. Then:

(i) $[D^U = D^{II}]$;

(ii) $\pi_{1N}{}^{II} = \pi_U = \pi_{1M}{}^{II}$.

Together, Propositions 1-3 yield the following corollary.

Corollary 1. Suppose $p_N = p_M \geq 2$. Then:

(i) $D^U = D^{II} = D^{SS} > D^{ES} = D^{SE} > D^{EE}$.

(ii) $\pi_{1M}^{SE} > \pi_{1M}{}^{EE} > \pi_{1M}{}^{II} = \pi_{1M}^{SS} = \pi_U > \pi_{1M}^{ES}$.

(iii) $\pi_{1N}^{ES} > \pi_{1N}{}^{EE} > \pi_{1N}{}^{II} = \pi_{1N}^{SS} = \pi_U > \pi_{1N}^{SE}$.

Warneryd (1998) shows that aggregate rent-seeking is higher in the unitary case, than in his federal case of external-internal rent-seeking, i.e, $\langle E, E\rangle$. He also shows that, with equal sized groups, expected utility is higher in his federal external-internal case. Together, Proposition 1(iii) and Proposition 3(i) above yield his first result (Corollary 1(i)). Corollary 1(i) also shows that this result generalizes weakly to all possible non-trivial ways of organizing rent-seeking contests among jurisdictions in a federal set-up. Proposition 2((ii) and (iii)) and Proposition 3(ii) yield his second result (Corollary 1((ii) and (iii)) as a special case of our complete characterization. The finding in Corollary 1(i) that $D^{U} = D^{II}$ replicates a finding in Amegashie (1999).

6. Discussion

The ranking among alternative contest forms presented in section 5 above carry a number of implications both for normative contest design and positive predictions regarding endogenously generated patterns of between-group conflict.

6.1. Normative contest design

Suppose the objective of a contest designer is to maximize aggregate effort, which is productive. Proposition 1(iii) suggests that this objective is best served by choosing a contest structure where either (a) within-group contestation precedes contestation between group champions, or (b) within and between-group contests proceed simultaneously. This rationalizes the standard way of organizing athletic contests, where group champions move on to contest one another in the next stage. When primary contests yield important information for voters, this also rationalizes electoral systems where prior intra-party primary contests produce candidates for the subsequent inter-party contest. This also suggests that scientific discoveries may be best facilitated by internal competition among multiple research groups within an organization to determine which research team would be chosen as that institution's representative to participate in the competition for an external grant or patent.

Now suppose, instead, that effort spent on winning the prize is wasteful – as may be the case for lobbying among multiple jurisdictions to influence federal policy on the location of an infrastructure project. Then the objective of a social planner would be to minimize total contest effort. Proposition 1(iii) suggests that such an objective is best served by a contest structure that involves external contest preceding internal contest. Consider, for example, a political process where higher level jurisdictions, (say provinces), first lobby the federal government to determine which province would receive the project, and lower level jurisdictions (say, districts) within the winning province subsequently lobby the provincial government to determine where the project would be located. Proposition 1(iii) suggests that this process would be least wasteful, relative to other possible sequences. Analogous considerations apply to cases where different, internally fragmented or factionalized, ethnic groups compete for resource transfers from the state, or multi-unit departments within firms or other organizations compete for resources devolved from the organizational budget.

It is often argued that persistent resource conflicts between regions or ethnic groups constituting a country have a tendency to tip over into civil war and secession, especially when different ethnic groups are concentrated in different regions. When different regions or ethnic groups contest one another for federal transfers, the federal government might therefore wish to minimize inter-region, or between-group, conflict. Proposition 1(i) suggests that the federal government may achieve this objective as well (alongside minimizing aggregate lobbying expenditure) if higher-level jurisdictions first lobby the central government for resources, and constituent lower-level units of the winning jurisdiction subsequently lobby against one another for control over the devolved resources. Conversely, identitarian politicians interested in minimizing conflict within their respective identity in-groups, in order to enhance in-group cohesion and maximize deployment against out-groups, would prefer a devolution contest structure where constituent units of a group first determine which unit would receive any devolved resources, and the group winners subsequently lobby the federal government against one another (Proposition 1((i) and (ii)) for the prize.

As already discussed, Propositions 1-3 together generalize the case for fiscal federalism and devolution developed by Warneryd (1998), yielding his results as corollaries. Suppose the objective of a contest designer, who is constrained to choosing among symmetric contest structures (and thus identical ex-ante expected payoffs), is to maximize the ex-ante expected payoffs. Then, by Proposition 2((ii) and (iii)), the contest designer should implement the $\langle E, E\rangle$ sequence, which Pareto-dominates the other two-stage (or 'federal') symmetric sequences, $\langle I, I\rangle$ and $\langle S, S\rangle$ (as well as the unitary contest U). In so doing, the contest designer would also end up minimizing aggregate rent-seeking expenditure among all non-trivial alternatives (Corollary 1(i)). The same conclusion applies to the problem of organization design considered by Inderst *et al.* (2005, 2007).

6.2. Endogenous contest sequence

What happens instead if there is no contest designer, so that the adoption of a contest sequence within each group is a strategic decision by that group? What kind of sequence structure can we expect to emerge as an equilibrium outcome in that case? We now turn to this question.

Suppose that each group has a leader, who first chooses among the three alternatives I, E and S for her own group. The two leaders choose simultaneously, so that the ordered sequence $\langle m, n\rangle \in [\{I, E, S\} \times \{I, E, S\}]$ is determined as the joint outcome of their choices; m referring, as before, to the choice made by the leader of M. Once the contest sequence is determined, the two groups engage in the corresponding contests, as defined in sections 3 and 4 above. The objective of each leader is to maximize the (symmetric equilibrium) ex-ante payoff of the representative member of her group.

It is then easy to check, in light of Proposition 2((ii) and (iii)), that $\langle S, S\rangle$ constitutes the unique Nash equilibrium of this game of sequence choice. By Proposition 1(iii), the Nash equilibrium maximizes total contest effort. It does not generate a Pareto-optimal outcome: the sequence combination $\langle E, E\rangle$ Pareto-dominates $\langle S, S\rangle$, but cannot be supported as an equilibrium. This is so because, given, its opponent's choice of E, a group can increase its payoff by choosing S instead of E. Thus, in the absence of a contest designer, as for example in the case of war between internally fragmented groups, one would expect internal and external contests to proceed simultaneously within both groups when the two groups are of equal size, assuming within-group coordination with regard to sequence choice. Notice that, by Corollary 1, aggregate rent dissipation and individual pay-offs would be identical to that in the unitary case, where all agents engage in a single, unified, free for all contest among themselves, instead of a two-part contest with separate internal and external components.

7. Extension: unequal groups

We now extend our analysis to the case of unequal groups. For clarity of exposition, we confine ourselves to the case where one group is single-member (e.g., Bakshi and Dasgupta 2022). This case is of special interest in contexts of ethnic conflict, war and revolution, where a disparate coalition often shares a unified common enemy, but little else. The primary questions of interest here are: (a) whether postponing internal conflict in favour of unity against a common enemy maximizes the expected payoff to members of the fragmented group or its external success probability, and (b) whether such unity can indeed come about through decentralized decision-making within the group.

Consider a variant of our benchmark framework, where the group N is single-member, while the group M has p_M members: $p_M \in \{2,3,\dots\}$. Since N is single-member, it can only engage in external conflict. M can adopt the sequence E, S or I, as in our benchmark formulation. As before, we assume that M cannot postpone its contest with N without losing the prize with probability 1. Thus, if M adopts the sequence I, it loses the prize with probability 1, so that the payoff of its representative member is 0 (analogous to the situation in our benchmark model), while the payoff to N is 1, since, being single-member, it has no internal conflict. We thus need to consider only the two non-trivial cases, where M adopts either E or S.

First suppose M adopts E. Then, an exercise analogous to that in Section 3.2 yields the aggregate expected conflict investment, i.e. expected rent dissipation in equilibrium:

$$D^E = \left(\frac{1}{{p_M}^2+1}\right)\left(\frac{2p_M-1}{p_M}\right); \tag{45}$$

and the equilibrium period 1 expected pay-offs:

$$\pi_{1M}^{E} = \left(\frac{1}{p_M{}^2+1}\right)^2 \left(\frac{p_M{}^2+1-p_M}{p_M{}^2}\right); \tag{46}$$

$$\pi_{1N}^{E} = \left(\frac{1}{1+\frac{1}{p_M{}^2}}\right)^2. \tag{47}$$

Next, suppose M adopts S. Then, an exercise analogous to that in Section 3.3 yields the aggregate expected conflict investment, i.e. expected rent dissipation in equilibrium:

$$D^{S} = \left(\frac{1}{p_M+1}\right)\left(\frac{2p_M-1}{p_M}\right); \tag{48}$$

and the period 1 payoffs:

$$\pi_{1M}^{S} = \left(\frac{1}{p_M}\right)^2 \left(\frac{1}{p_M+1}\right)^2; \tag{49}$$

$$\pi_{1N}^{S} = \left(\frac{1}{1+\frac{1}{p_M}}\right)^2. \tag{50}$$

The relationship among alternative contest forms with regard to rent dissipation and ex-ante payoffs are summarized in Proposition 4 below.

Proposition 4. Suppose $p_N = 1, p_M > 1$. Then:

- **(i)** $D^I < D^E < D^S$.
- **(ii)** $\pi_{1M}^{E} > \pi_{1M}^{S} > \pi_{1M}^{I}$.
- **(iii)** $\pi_{1N}^{I} > \pi_{1N}^{E} > \pi_{1N}^{S}$.

Proof. See the appendix.

By Proposition 4(i), in the non-trivial case where a multi-member group does not withdraw from contesting a single-member opposing group, aggregate contest investment (or rent dissipation) is lower if the larger group engages in the external contest first (instead of simultaneously engaging in both internal and external contest). Recall that, by Proposition 1(iii), $D^{SE} > D^{EE}$ when both groups are multi-member and of equal size. Hence, the rent-dissipation ranking between these two alternative sequences remain unchanged in the two situations. Since the single member group is constrained to attacking its opponent, Proposition 4(ii) implies that choosing to engage in external conflict first is the best response of the larger group, when group responses are determined by a leader intent on maximizing the average ex ante payoff to group members. Thus, with endogenous sequence choice, $\langle E, E\rangle$ is the unique Nash equilibrium when a multi-member group faces a single-member one and can coordinate its sequence choice, unlike the case in our benchmark framework with equal-sized groups (recall the discussion in section 6). Furthermore, unlike the case in our benchmark model, the unique Nash equilibrium $\langle E, E\rangle$ is Pareto-optimal – as is evident from Proposition 4((ii) and (iii); it also implies lowest rent dissipation under non-trivial between-group conflict.

Proposition 4((ii) and (iii) thus provides an explanation as to why internally diverse political coalitions sometimes manage to keep internal tensions in check while battling a unified opponent, with internal conflict flaring up as soon as the external conflict is won – why successful revolutions often 'devour their own', or why the West and the USSR united against Germany during WW II, only to engage in the Cold War thereafter.

What about the external success probabilities? Recalling (6), (20) and (27), it can be checked that: $s_M^E = \frac{1}{p_M^2+1}$ and $s_M^S = \frac{1}{p_M+1}$; by assumption, $s_M^I = 0$. Thus, recalling $p_M \geq 2$, we have:

$$0 < s_M^E = \frac{1}{p_M^2+1} < s_M^S = \frac{1}{p_M+1} < \frac{1}{2}. \tag{51}$$

Remark 1. A large body of literature, stemming from the seminal work of Olson (1965), has investigated the conditions under which larger groups are less successful in rent-seeking, due to greater within-group free-riding. This literature has typically highlighted factors such as strictly convex contest costs and in-group consumption externalities as possibly mitigating the group size disadvantage (see, e.g., Esteban and Ray 2001 for a synthesis). Our analysis, as summarized in (51), highlights a simple alternative channel for mitigating the group size disadvantage, viz., the contest sequence. By (51), occurrence of the in-group contest at the same time as the between-group contest implies less disadvantage than the case when the in-group contest follows the between-group contest. Individual internalization of group gains is partial, and falling in group size, when individuals have to choose their external conflict investment in ignorance of the outcome of the within-group contest. Hence, external conflict allocation is inefficient both in the simultaneous internal-external and external-internal cases; the larger the group size the greater the inefficiency. Individual internalization of group gains is higher in the first case, leading to less inefficiency in group allocation to external conflict.

Remark 2. Later occurrence of the in-group contest relative to the between-group one implies lower individual internalization of group gains, and thus higher within-group free-riding with regard to conflict against the other group. Despite this, however, later relative occurrence of the in-group contest implies higher ex ante payoffs (Proposition 4(ii)). Thus, sequencing affects external success probability and group payoff in opposite directions. This happens essentially because later relative occurrence of the in-group conflict implies lower within-group conflict.

Lastly, for completeness, consider the unitary case of Warneryd (1998) discussed in section 5, where all agents contest one another at the same time. Then, aggregate rent-seeking expenditure is:

$$D^U = \left(\frac{p_M}{p_M+1}\right);$$

payoffs are given by:

$$\pi_U = \left(\frac{1}{p_M+1}\right)^2.$$

Using (46), (48) and (50), we therefore have the following from Proposition 4.

Corollary 2. Suppose $p_N = 1, p_M > 1$. Then:

(i) $D^I < D^E < D^S < D^U$.

(ii) $\pi_U > \pi_{1M}^E > \pi_{1M}^S > \pi_{1M}^I$.

(iii) $\pi_{1N}^I > \pi_{1N}^E > \pi_{1N}^S > \pi_U$.

Any two-dimensional contest, where a multi-member group engages in an internal contest in addition to an external contest against a single-member group, involves less rent-seeking than the corresponding unified contest (Corollary 2(i)). This holds regardless of whether the internal contest occurs alongside, or subsequent, to the external contest. Unification is however better for the larger group (Corollary 2(ii)) and worse for the smaller group (Corollary 2(iii)), compared to any two-dimensional contest, regardless of the sequence.

Remark 3. In light of Corollary 1, Corollary 2 implies that the case for fiscal federalism and devolution discussed in section 5 under equal-sized jurisdictions continues to hold when one jurisdiction is internally united, if the policy objective is to minimize overall rent-seeking. However, fiscal federalism does not Pareto-dominate the unitary arrangement when the jurisdictions differ in the extent of internal fragmentation – the number of constituent interest groups. Hence, a move to fiscal federalism from a unitary framework, or vice versa, may raise equity concerns when the jurisdictions differ in terms of internal fragmentation. Recall that, by Corollary 1, this concern does not arise with equally fragmented jurisdictions. Thus, Corollary 1 shows that the neglect of relative fragmentation is not normatively innocuous in the context of contest design.

8. Postponing external conflict with unequal groups

So far, we have assumed that a group cannot evade, or postpone, between-group conflict if attacked, without losing the prize with probability 1. This appears a realistic assumption in most real-life contexts.

There may however be cases, especially in the context of military conflicts, where a group may be able to postpone a battle by withdrawing. How would our conclusions change in such cases? We now turn to this question. For simplicity, we confine ourselves to the case of a multi-member group facing a single-member opponent, discussed in section 7 above. In this setting, M can now choose to engage in internal contestation first, to choose a group champion. This group champion then engages in contestation with N. The period 2 problem is then as in (6), while the initial internal contest within M is given by (10). It is easy to check that equilibrium rent dissipation and ex ante payoffs are now given by:

$$D^{I*} = \left(\frac{3p_M - 1}{4p_M}\right); \tag{52}$$

$$\pi_{1N}{}^{I*} = \frac{1}{4}; \tag{53}$$

$$\pi_{1M}{}^{I*} = \left(\frac{1}{4p_M{}^2}\right). \tag{54}$$

Proposition 5. Suppose $p_N = 1, p_M > 1$. Then:

(i) $D^E < D^S < D^{I*}$;

(ii) $\pi_{1M}^S < \pi_{1M}^E < \pi_{1M}{}^{I*}$;

(iii) $\pi_{1N}^E > \pi_{1N}^S > \pi_{1N}{}^{I*}$.

Proof. See the appendix.

By Proposition 5(i), when a multi-member group can postpone contesting a single-member opposing group without losing the prize, aggregate contest investment (or rent dissipation) is lowest if the larger group engages in the external contest first (instead of simultaneously engaging in both internal and external contest). It is highest when the larger group chooses to resolve its internal contest first. Proposition 5(ii) implies that choosing to engage in internal conflict first now becomes the collective best response of the larger group. Thus, with endogenous sequence choice and the possibility of evasion (or postponement), $\langle I, E\rangle$ is the unique Nash equilibrium when a multi-member group faces a single-member one and can cooperatively choose its contest sequence. The larger group evades the attack by the smaller group and thereby postpones between-group conflict, preferring to resolve its internal conflict first. The unique Nash equilibrium $\langle I, E\rangle$ is Pareto-optimal – as is clear from Proposition 5((ii) and (iii)).

Proposition 5((ii) explains why, when it is possible to postpone a conflict with an external enemy, members of an internally fragmented group might choose to engage in internecine warfare first.

Furthermore, as can be easily checked, $s_M^{I*} = \frac{1}{2}$. Thus, recalling (51) and Remark 1, it follows that the larger group's probability of winning the external contest is maximized if it can resolve its internal conflict before engaging in external conflict. There is complete individual internalization of the expected group gains from external conflict by the group champion. Hence, external conflict allocation by a group is efficient, and thus independent of group size, when it follows the resolution of internal conflict. These two findings provide ex post justifications, for example, for the choice of Chiang Kai-shek to concentrate on battling the Communists, even as Japanese incursions increased in China, discussed in section 1.

It however seems likely that, in practice, prolonged evasion of external conflict would lead to certain defeat for the evading group, as assumed in our benchmark model and in the extension discussed in section 7. If the internal conflict drags on without resolution, the evading group would eventually lose the prize to the external enemy, unless its members agree to postpone their internecine conflict till the elimination of the common threat (as Chiang Kai-shek and the Communists did in China after 1937, after each failed to decisively defeat the other), or at least respond to the common threat alongside battling one another (as Tito's Communist Partisans and the Serbian Chetniks did during WW II in Yugoslavia).

9. Symmetric sequencing with unequal groups

In our benchmark model, we have considered both symmetric and asymmetric sequences, while restricting ourselves to equal-sized groups. Conversely, in the variants presented in sections 7 and 8, we have considered unequal groups, but without examining the symmetric contest sequences $\langle I, I\rangle$ and $\langle S, S\rangle$ in a non-trivial manner. Since in some situations a contest designer may be confined to a choice among symmetric contest sequences, with possibly unequal groups, this case is of interest. It is also of interest because the literature has confined itself to the three symmetric contest forms, apart from the unified single-tier form. We therefore now turn to this case.

Consider then the situation where the group N has p_N members, while the group M has p_M members: $p_M, p_N \in \{2,3, \ldots\}$ and $p_M > p_N$. A contest-designer has to choose among the three symmetric two-tier contest forms and the unified single-tier form: $\langle I, I\rangle$,$\langle E, E\rangle$, $\langle S, S\rangle$ and U. Generalizing the treatment in sections 3 and 5, we then get the rankings for aggregate rent-dissipation, external success probabilities, and average ex-ante payoffs.

Proposition 6. Suppose $p_M > p_N \geq 2$. Then:

(i) $y^{II} > y^{SS} > y^{EE}$;

(ii) $D^U > D^{II} > D^{SS} > D^{EE}$;

(iii) $\frac{1}{2} = s_M^{II} > s_M^{SS} > s_M^{EE}$;

(iv) there exists $\ddot{\theta} \in (1,2)$ such that: $\pi_{1M}^{SS} < \pi_{1M}{}^{II} < \pi_{1M}^{EE}$ (resp. $\pi_{1M}{}^{II} > \pi_{1M}^{EE} > \pi_{1M}^{SS}$) if $\left(\frac{p_M}{p_N}\right) < \ddot{\theta}$ (resp. $> \ddot{\theta}$);

(v) $\pi_{1N}{}^{II} < \pi_{1N}^{SS} < \pi_{1N}^{EE}$;

(vi) $\pi_{1M}{}^{II} < \pi_U < \pi_{1N}{}^{II}$.

Proof. See the appendix.

Proposition 6(i) implies that, as in the case of equal-sized groups (Proposition 1(i)), between group conflict is highest among all symmetric two-tier contest forms if internal conflict precedes external conflict; it is minimized by the opposite sequence. By Proposition 6(ii), given unequal multi-member groups, among all symmetric two-tier contest forms, overall conflict (or aggregate rent dissipation) is uniquely greatest when internal conflict occurs first. It is least when external conflict occurs first. Rent dissipation is always higher in a unified single-tier contest, than in any symmetric two-tier contest. Thus, the finding due to Warneryd (1998) discussed earlier generalizes strongly to all possible symmetric two-tiered contests when the groups are of unequal size, and weakly otherwise (recall Corollary 1). Proposition 6(ii) implies that the multiple ties we get in the case of equal-sized groups (recall Corollary 1) all break down, yielding a strong order, when the groups are of unequal size. Analogously, by Proposition 6((iii), (iv) and (v)), a strong order among the symmetric contest forms replaces the weak order for external success probabilities and payoffs derived for equal-sized groups (recall Proposition 2). Proposition 6(iii) states that the group with the larger population is always more successful in its contest against the smaller one if its internal contest is resolved earlier. Proposition 6(iv) states that, if the relative size of the majority exceeds a threshold value greater than 1, then the period 1 expected utility of any member of that group is highest when internal conflict precedes external conflict. Otherwise, such expected utility is maximum when external conflict precedes internal conflict (analogous to the case for equal-sized groups – recall Proposition 2(ii)). This happens essentially because, when the majority is relatively similar to the minority, its gain from reduced internal conflict outweighs its loss from a reduced external success probability if the contest sequence is changed to external-internal from any of the other two alternatives. In all cases, expected utility of the members of the majority group is the least when internal and external contests occur simultaneously. By Proposition 6(v), members of the minority group are necessarily worst off if internal conflict precedes external conflict; they are best off if the sequence is reversed. For members of the smaller group, the welfare ranking of contest forms is identical to the ranking according to the smaller group's external success probabilities (Proposition 6(iii)), and is explained by the same. By Proposition 6((iv) and (v)), among the three symmetric contest forms, $\langle E,E\rangle$ leads to Pareto-efficiency, and $\langle S,S\rangle$ is always inefficient, regardless of the relative group size. This result mirrors the finding for equal-sized groups (Proposition 2(ii) and (iii)). $\langle I,I\rangle$ is Pareto efficient if the majority is sufficiently larger than the minority, but Pareto-dominated by $\langle E,E\rangle$ otherwise. When $\langle I,I\rangle$ is Pareto-efficient, *any* symmetric two-tiered contest makes the smaller group better off relative to a single-tier unified contest, but the larger group worse off (Proposition 6(vi)).

Remark 4. If the majority is sufficiently larger than the minority, the single-tier contest form is Pareto-efficient, despite generating the maximum rent-seeking loss (recall Proposition 6(ii) and Propsition 6(vi)). Thus, no general welfare case can be made for decentralization when the groups vary greatly in size, without taking on board between-group distributional concerns.

Remark 5. Consider any arbitrary pair of symmetric sequences, say, $\langle I,I\rangle$ and $\langle S,S\rangle$. By Propositions 5 and 6, the rankings between $\langle I,I\rangle$ and $\langle S,S\rangle$ that obtain with two unequal multi-member groups when $\frac{p_M}{p_N} \geq 2$ are the same as those that hold between $\langle I,E\rangle$ and $\langle S,E\rangle$ when a multi-member group, facing a single-member group intent on attacking, can unilaterally postpone external contestation. This happens because: (a) formally, Proposition 6 continues to hold if $p_N = 1$, and (b) when $p_N = 1$, and the external contest can be postponed, the equilibrium depends only on the sequence choice of the larger group. Thus, the results in Proposition 5 can also be formally derived as a corollary of a generalized version of Proposition 6, where N is permitted to be single-valued. The intuitive interpretations of the two situations are however very different, as are the motivations for studying them. We have therefore chosen to analyse them independently.

10. Concluding remarks

This paper examines how the relative sequencing of within-group and between-group rent-seeking contests affect key outcomes, such as rent dissipation, external success probability and expected utilities of contestants. We have considered the alternative sequences in which internal and external contests can be organized within a group, and their possible combinations between two groups, both symmetric and asymmetric. Our benchmark framework focuses on equal-sized multi-member groups, when no group can unilaterally postpone external conflict, without losing the prize for sure. Comparing equilibrium outcomes among the possible nine cases, we find that the following complete rankings obtain. Between-group conflict is maximum, and within-group conflict minimum, in the symmetric internal-external case. Overall investment in appropriation is highest in the symmetric internal-external and simultaneous internal-external cases; it is lowest in the symmetric external-internal case (among all the non-trivial contest forms which provide both groups a positive chance of winning in equilibrium). Examining the expected utility ranking of agents, we find the latter contest form to be also Pareto-efficient. The other two symmetric contest forms are Pareto-dominated by the symmetric external-internal form. The case for fiscal federalism and devolution developed by Warneryd (1998) falls out as a corollary of our analysis, as does a result due to Amegashie (1999). When the contest sequence is chosen by a group leader acting on behalf of each group, simultaneous internal and external contestation by both groups constitutes the unique Nash equilibrium. Thus, the unique two-tier contest sequence that arises endogenously as the equilibrium outcome in a decentralized setting maximizes aggregate rent-seeking, and is a Pareto-inferior outcome.

We also examine three variants of our benchmark framework. When a multi-member group faces a single member group intent on attacking, we find that, for the larger group, engaging in external conflict prior to the internal conflict maximizes aggregate ex-ante group payoff. Thus, the efficient sequence choice can be sustained as the unique Nash equilibrium when the larger group can somehow resolve its internal coordination problem. We also examine how our conclusions change when the larger group can unilaterally postpone the external contest. Lastly, we compare the outcomes among all symmetric two-tier contest forms and the one-tier unified form when both groups are multi-member but unequal in size. We find that, in this case, the one-tier contest form may be Pareto-efficient, despite generating greater rent-seeking loss than all symmetric two-tier contests. Thus, no general welfare case can be made for decentralization when the groups vary in size, without taking on board between-group distributional concerns. In general, our results suggest that the free-rider disadvantage of larger groups can be mitigated by preponing the internal contest within a group.

Our results carry both normative and positive implications. The rankings we derive among the alternative contest forms for different outcome indicators offer a useful normative guide to contest design. They also help rationalize different sequences of conflict between and among groups in decentralized settings, especially in contexts of ethnic division, war and revolution.

For ease of exposition, we have abstracted from possible differences in conflict efficiency between and within groups. Furthermore, we have confined ourselves to the summative conflict technology, where aggregate group conflict effort is just the sum of individual contributions. Future work relaxing these restrictions may yield useful insights.

Appendix

Proof of Proposition 1.

(i) Since $p \geq 2$, part (i) of Proposition 1 follows from (7), (22), (29), (34) and (39).

(ii) Part(ii) of Proposition 1 follows from (12), (24), (30), (35) and (39).

(iii) Parts (i) and (ii) of Proposition 1 imply: $D^{SS} > D^{ES} = D^{SE} > D^{EE}$. By (13) and (31), $D^{II} = D^{SS}$. Part (iii) of Proposition 1 follows. ■

Proof of Proposition 2.

(i) Part (i) of Proposition 2 follows from (8), (21), (28), (33) and (39).

(ii) By (37), noting that $(1 + p^2 > 2p)$ since $p \geq 2$,

$$0 < \pi_{1M}^{ES} = \frac{1}{p+2p^2+p^3} < \frac{1}{4p^2}.$$

Thus, recalling (15) and (32),

$$\pi_{1M}{}^{II} = \pi_{1M}^{SS} > \pi_{1M}^{ES} > 0. \tag{N1}$$

Now suppose $\pi_{1M}^{EE} \geq \pi_{1M}^{SE}$. Recall that, by (39), $\pi_{1N}^{ES} = \pi_{1M}^{SE}$. Then, by (26) and (38),

$$\left(\frac{2p-1}{4p^3}\right) \geq \left(\frac{1}{1+p}\right)^2;$$

or

$$2p^3 - 3p^2 + 1 \leq 0.$$

Let $A = 2p^3 - 3p^2 + 1$. Hence, $\frac{dA}{dp} = 6p(p-1) > 0$. Now, at $p = 2$, $A = 16 - 12 + 1 > 0$. Hence, for all $p \geq 2$, A>0. We thus have a contradiction, which implies:

$$\pi_{1M}^{SE} > \pi_{1M}^{EE}. \tag{N2}$$

Now suppose $\pi_{1M}^{EE} \leq \pi_{1M}{}^{II}$. Then, using (15) and (26),

$$p \leq 1,$$

a contradiction. Hence,

$$\pi_{1M}^{EE} > \pi_{1M}{}^{II}. \tag{N3}$$

Together, (N1), (N2) and (N3) yield part (ii) of Proposition 2.

(iii) Recall that, by (39), $\pi_{1M}^{ES} = \pi_{1N}^{SE}, \pi_{1N}^{ES} = \pi_{1M}^{SE}$. Then, in light of (15), (26) and (32), part (iii) of Proposition 1 follows from part (ii) of Proposition 1. ■

Proof of Proposition 4.

(i) Recalling that $D^I = 0$, and that $p_M \geq 2$, part (i) of Proposition 4 follows immediately from (45) and (48).

(ii)Suppose $\pi^E_{1M} \leq \pi^S_{1M}$. Then, using (46) and (49), we have:

$$Z = \frac{1}{p_M^2}\left(\frac{1}{p_M+1}\right)^2 - \frac{1}{p_M^2}\left(\frac{1}{p_M{}^2+1}\right)\left(1-\left(\frac{p_M}{p_M{}^2+1}\right)\right) = \frac{1}{p_M^2}\left(\frac{1}{p_M+1}\right)^2 [1-\left(1+\frac{2p_M}{p_M{}^2+1}\right)\left(1-\left(\frac{p_M}{p_M{}^2+1}\right)\right)] \geq 0.$$

Now let $\hat{Z} = [1-\left(1+\frac{2p_M}{p_M{}^2+1}\right)\left(1-\left(\frac{p_M}{p_M{}^2+1}\right)\right)]$. Then: $\hat{Z} = \left(\frac{p_M}{p_M{}^2+1}\right)\left[\left(\frac{2p_M}{p_M{}^2+1}\right)-1\right] < 0$, since $p_M{}^2 > p_M$ as $p_M \geq 2$. Hence, $Z = \frac{1}{p_M^2}\left(\frac{1}{p_M+1}\right)^2 \hat{Z} < 0$: a contradiction. Hence $\pi^E_{1M} > \pi^S_{1M}$. Recalling that $\pi^I_{1M} = 0$ by construction, and $\pi^S_{1M} > 0$ by (49), part (ii) of Proposition 4 follows.

(iii) Recalling that $\pi^I_N = 1$, part (iii) of Proposition 4 follows immediately from (47) and (50), since $p_M \geq 2$. ■

Proof of Proposition 5.

(i) Suppose $D^S \geq D^{I*}$. Then, using (48) and (52), $\left[\left(\frac{1}{p_M+1}\right)\left(\frac{2p_M-1}{p_M}\right) \geq \left(\frac{3p_M-1}{4p_M}\right)\right]$, or $[Z = 2p_M - 1 - p_M{}^2 \geq 0]$ or Z= -$(p_M-1)^2 \geq 0$. A contradiction given $p_M \geq 2$. This contradiction establishes that $D^S < D^{I*}$. By Proposition 4(i), $D^E < D^S$. Combining, we get: $D^E < D^S < D^{I*}$.

(ii) Suppose $\pi^E_{1M} \geq \pi_{1M}{}^{I*}$. Then, using (46) and (54), we have: $[4-4p_M \geq (p_M^2-1)^2]$, a contradiction, since $p_M \geq 2$. Hence, $\pi^E_{1M} < \pi_{1M}{}^{I*}$. By Proposition 4(ii), $\pi^E_{1M} > \pi^S_{1M}$. Combining, we get: $\pi^S_{1M} < \pi^E_{1M} < \pi_{1M}{}^{I*}$.

(iii) Since $p_M \geq 2$, it follows immediately from (50) and (53) that: $\pi^S_{1N} > \pi_{1N}{}^{I*}$. By Proposition 4(iii), $\pi^E_{1N} > \pi^S_{1N}$. Combining, we get: $\pi^E_{1N} > \pi^S_{1N} > \pi_{1N}{}^{I*}$. ■

Proof of Proposition 6.

Generalizing the treatment in section 3, we have:

$$D^{II} = \left(\frac{1}{4}\right)\left[\frac{p_M-1}{p_M} + \frac{p_N-1}{p_N} + 2\right]; \tag{N4}$$

$$D^{EE} = \left(\frac{1}{p_M{}^2+p_N{}^2}\right)\left[\left(\frac{p_M-1}{p_M}\right)p_N{}^2 + \left(\frac{p_N-1}{p_N}\right)p_M{}^2 + 1\right]; \tag{N5}$$

$$D^{SS} = \left(\frac{1}{p_M+p_N}\right)\left[p_N\left(\frac{p_M-1}{p_M}\right) + p_M\left(\frac{p_N-1}{p_N}\right) + 1\right]; \qquad \text{(N6)}$$

for all $g \in \{M, F\}$: $[s_g^{II} = \frac{1}{2}, s_g^{EE} = \frac{p_{-g}{}^2}{p_g{}^2+p_{-g}{}^2}$, and $s_g^{SS} = \frac{p_{-g}}{p_g+p_{-g}}]$; (N7)

for all $g \in \{M, N\}$: $\pi_{1g}{}^{II} = \left(\frac{1}{4p_g{}^2}\right)$; (N8)

for all $g \in \{M, N\}$: $\pi_{1g}^{EE} = \left(\frac{1}{p_g{}^2}\right)\left(\frac{p_{-g}{}^2}{p_g{}^2+p_{-g}{}^2}\right)\left(\frac{p_g{}^2+p_{-g}{}^2-p_g}{p_g{}^2+p_{-g}{}^2}\right)$; (N9)

for all $g \in \{M, N\}$: $\pi_{1g}^{SS} = \left(\frac{1}{p_g}\right)\left(\frac{p_{-g}}{p_g+p_{-g}}\right)\left[\left(\frac{1}{p_g}\right) - \left(\frac{1}{p_M+p_N}\right)\right]$. (N10)

Generalizing the treatment of the unified single-tier contest in section 5, we have:

$$D^U = \left(\frac{p_M+p_N-1}{p_M+p_N}\right); \qquad \text{(N11)}$$

while payoffs are given by:

$$\pi_U = \left(\frac{1}{p_M+p_N}\right)^2. \qquad \text{(N12)}$$

Let $\theta = \frac{p_M}{p_N}$. By assumption, $\theta > 1$.

(i) It can be checked that:

$$y^{II} = \frac{1}{2};\; y^{SS} = \left(\frac{1}{p_M+p_N}\right), y^{EE} = \left(\frac{1}{p_g{}^2+p_{-g}{}^2}\right).$$

Since $p_M > p_N \geq 2$, part (i) of Proposition 6 is immediate.

(ii) Let

$$Z_1 = \left(\frac{1}{p_M+p_N}\right)\left[p_N\left(\frac{p_M-1}{p_M}\right) + p_M\left(\frac{p_N-1}{p_N}\right)\right] - \left(\frac{1}{p_M{}^2+p_N{}^2}\right)\left[\left(\frac{p_M-1}{p_M}\right)p_N{}^2 + \left(\frac{p_N-1}{p_N}\right)p_M{}^2\right].$$

Then:

$$Z_1 = \left[\left(\frac{1}{1+\theta}\right) - \left(\frac{1}{1+\theta^2}\right)\right]\left(1 - \frac{1}{p_M}\right) + \left[\left(\frac{\theta}{1+\theta}\right) - \left(\frac{\theta^2}{1+\theta^2}\right)\right]\left(1 - \frac{1}{p_N}\right) = \left(\frac{(\theta-1)^2}{1+\theta}\right)\left(\frac{1}{p_N}\right).$$

Using (N5) and (N6),

$$D^{SS} - D^{EE} = Z_1 + [\left(\frac{1}{p_M+p_N}\right) - \left(\frac{1}{p_M{}^2+p_N{}^2}\right)].$$

Now $Z_1 > 0$ since $\theta > 1$; and $\left[\left(\frac{1}{p_M+p_N}\right) - \left(\frac{1}{p_M{}^2+p_N{}^2}\right)\right] > 0$ since $p_M \geq 2$. Hence:

$$D^{SS} > D^{EE}. \tag{N13}$$

Now let

$$Z_2 = \left(\frac{1}{4}\right)\left[\frac{p_M-1}{p_M} + \frac{p_N-1}{p_N}\right] - \left(\frac{1}{p_M+p_N}\right)\left[p_N\left(\frac{p_M-1}{p_M}\right) + p_M\left(\frac{p_N-1}{p_N}\right)\right].$$

Then,

$$Z_2 = \left[\left(\frac{p_M-1}{p_M}\right)\left(\frac{1}{4} - \left(\frac{1}{\theta+1}\right)\right) + \left(\frac{p_N-1}{p_N}\right)\left(\frac{1}{4} - \left(\frac{\theta}{\theta+1}\right)\right)\right] = -\left[\left(\frac{1}{2}\right) + \left(\frac{1}{p_N\theta}\right)\left(\frac{2\theta-3-3\theta^2}{4(1+\theta)}\right)\right].$$

Then, recalling (N4) and (N6),

$$D^{II} - D^{SS} = Z_2 + \left(\frac{1}{2}\right) - \left(\frac{1}{p_M+p_N}\right) = \left(\frac{1}{p_N}\right)\left(\frac{3}{1+\theta}\right)\left(\frac{(\theta-1)^2}{4\theta}\right).$$

Thus, since $\theta > 1$,

$$D^{II} > D^{SS}. \tag{N14}$$

Now, using (N4) and (N11),

$$D^{U} - D^{II} = \left(\frac{1}{4p_M} + \frac{1}{4p_N} - \frac{1}{p_M+p_N}\right) = \left(\frac{1}{p_N}\right)\left(\frac{(\theta-1)^2}{4\theta(1+\theta)}\right).$$

Then, since $\theta > 1$,

$$D^{U} > D^{II}. \tag{N15}$$

Part (ii) of Proposition 6 follows from (N13), (N14) and (N15).

(iii) By (N7), $s_g^{II} = \frac{1}{2}$, $s_M^{EE} = \frac{1}{\theta^2+1}$, and $s_N^{SS} = \frac{1}{\theta+1}$. Since $\theta > 1$, part (iii) of Proposition 6 is immediate.

(iv) First consider $[\pi_{1M}{}^{II} - \pi_{1M}^{EE}]$. By (N8) and (N9),

$$\left[\pi_{1g}{}^{II} - \pi_{1g}^{EE}\right] = \left(\frac{1}{p_g{}^2}\right)\left[\left(\frac{1}{4}\right) - \left(\frac{p_{-g}{}^2}{p_g{}^2+p_{-g}{}^2}\right)\left(\frac{p_g{}^2+p_{-g}{}^2-p_g}{p_g{}^2+p_{-g}{}^2}\right)\right]. \tag{N16}$$

Hence,

$$[\pi_{1M}{}^{II} - \pi_{1M}^{EE}] = \left(\frac{1}{p_M{}^2}\right)\left[\left(\frac{1}{4}\right) - \left(\frac{1}{\theta^2+1}\right)\left(1 - \left(\frac{1}{p_N}\right)\left(\frac{\theta}{\theta^2+1}\right)\right)\right].$$

Now, consider $Z \equiv \left(\frac{1}{\theta^2+1}\right) - \left(\frac{\theta}{p_N(1+\theta^2)^2}\right)$.

$$\frac{dZ}{d\theta} = -\left(\frac{2\theta}{(\theta^2+1)^2}\right) - \left(\frac{1}{p_N(1+\theta^2)^2}\right) + \left(\frac{4\theta^2}{p_N(1+\theta^2)^3}\right)$$

$$= -\left(\frac{1}{(\theta^2+1)^2 p_N}\right)\left[2\theta\left(\frac{p_N(1+\theta^2)-2\theta}{(1+\theta^2)}\right) + 1\right].$$

The term $[p_N(1+\theta^2)-2\theta]$ is positive at $\theta=1$ and increasing in θ for $\theta>1$. Hence, $\frac{dZ}{d\theta}<0$. At $\theta=1$, $Z=\left(\frac{1}{2}\right)-\left(\frac{1}{4p_N}\right)>\frac{1}{4}$, since $p_N>1$. Furthermore, $\lim_{\theta\to\infty} Z=0$, $\lim_{\theta\to 2} Z=\frac{1}{5}\left(1-\left(\frac{2}{5p_N}\right)\right)<\frac{1}{5}$, and $\frac{dZ}{d\theta}<0$. Hence, if $p_N\geq 2$, then there exists $\ddot{\theta}\in(1,2)$ such that:

$$Z>\frac{1}{4}\text{ (resp. }<\frac{1}{4}\text{) if }\theta<\ddot{\theta}\text{ (resp. }>\ddot{\theta}\text{).}$$

Therefore,

$$\text{there exists }\ddot{\theta}\in(1,2)\text{ such that: }{\pi_{1M}}^{II}<\pi_{1M}^{EE}\text{ (resp. }>\pi_{1M}^{EE}\text{) if }\theta<\ddot{\theta}\text{ (resp. }>\ddot{\theta}\text{).}\tag{N17}$$

Now, using (N8) and (N10),

$${\pi_{1g}}^{II}-\pi_{1g}^{SS}=\left(\frac{1}{{p_g}^2}\right)\left[\left(\frac{1}{4}\right)-\left(\frac{p_{-g}}{p_M+p_N}\right)^2\right].\tag{N18}$$

Since $p_N<p_M$,

$${\pi_{1M}}^{II}>\pi_{1M}^{SS}.\tag{N19}$$

By (N9) and (N10),

$$\pi_{1g}^{SS}-\pi_{1g}^{EE}=\left(\frac{1}{{p_g}^2}\right)\left[\left(\frac{p_{-g}}{p_g+p_{-g}}\right)^2-\left(\frac{{p_{-g}}^2}{{p_g}^2+{p_{-g}}^2}\right)\left(\frac{{p_g}^2+{p_{-g}}^2-p_g}{{p_g}^2+{p_{-g}}^2}\right)\right].\tag{N20}$$

Then,

$$\pi_{1M}^{SS}-\pi_{1M}^{EE}=\left(\frac{1}{{p_M}^2}\right)\left[\left(\frac{1}{\theta+1}\right)^2-\left(\frac{1}{\theta^2+1}\right)\left(1-\left(\frac{1}{p_N}\right)\left(\frac{\theta}{1+\theta^2}\right)\right)\right]$$

$$=\left(\frac{1}{{p_M}^2}\right)\left(\frac{1}{\theta+1}\right)^2\left[1-\left(1+\frac{2\theta}{\theta^2+1}\right)\left(1-\left(\frac{1}{p_N}\right)\left(\frac{\theta}{1+\theta^2}\right)\right)\right]$$

$$=\left(\frac{1}{{p_M}^2}\right)\left(\frac{1}{\theta+1}\right)^2\left(\frac{\theta}{1+\theta^2}\right)\left[\left(1+\frac{2\theta}{\theta^2+1}\right)\left(\frac{1}{p_N}\right)-2\right].$$

Let $\hat{Z}=\left[\left(\frac{1}{p_N}\right)\left(\frac{2\theta}{1+\theta^2}+1\right)-2\right]$. Since $p_N\geq 2, \lim_{\theta\to 1}\hat{Z}=\left(\frac{2}{p_N}\right)-2<0$. Furthermore, $\frac{d\left(\frac{2\theta}{1+\theta^2}\right)}{d\theta}=\frac{2(1-\theta^2)}{(1+\theta^2)^2}<0$ since $\theta>1$. Hence, $\hat{Z}<0$. Thus:

$$\pi_{1M}^{SS}<\pi_{1M}^{EE}.\tag{N21}$$

Part (iv) of Proposition 6 follows from (N17), (N19) and (N21).

(v) Recalling (N18), since $p_N<p_M$, we have:

$$\pi_{1N}{}^{II} < \pi_{1M}^{SS}. \tag{N22}$$

Using (N20),

$$\pi_{1N}^{SS} - \pi_{1N}^{EE} = \left(\frac{1}{p_N{}^2}\right)\left[\left(\frac{1}{1+\theta}\right)^2 - \left(\frac{1}{1+\theta^2}\right)\left(1 - \left(\frac{1}{p_N}\right)\left(\frac{1}{1+\theta^2}\right)\right)\right]$$

$$= \left(\frac{1}{p_N{}^2}\right)\left(\frac{1}{1+\theta}\right)^2\left(\frac{1}{1+\theta^2}\right)\left[\left(\frac{1}{p_N}\right) + 2\theta\left(\left(\frac{1}{1+\theta^2}\right)\left(\frac{1}{p_N}\right) - 1\right)\right].$$

Let $\hat{Z} = \left[\left(\frac{1}{p_N}\right) + 2\theta\left(\left(\frac{1}{1+\theta^2}\right)\left(\frac{1}{p_N}\right) - 1\right)\right]$. Since $p_N \geq 2$, $\lim_{\theta\to 1} \hat{Z} = \left(\frac{2}{p_N}\right) - 2 < 0$. Furthermore, since $p_N \geq 2, \theta > 1$, we have: $\frac{d\hat{Z}}{d\theta} = 2\left(\left(\frac{1}{1+\theta^2}\right)\left(\frac{1}{p_N}\right) - 1\right) - \left(\frac{1}{p_N}\right)\left(\frac{4\theta^2}{(1+\theta^2)^2}\right) < 0$. Thus, $\hat{Z} < 0$. Hence:

$$\pi_{1N}^{SS} < \pi_{1N}^{EE}. \tag{N23}$$

Part (v) of Proposition 6 follows from (N22) and (N23).

(vi) Since $[2p_M > p_M + p_N > 2p_N]$ when $p_M > p_N$, part (vi) of Proposition 6 immediately follows from (N8) and (N12). ■

Competing Interests

The authors have no competing interests to declare that are relevant to the content of this article.

References

Amegashie, J.A. (1999). The design of rent-seeking competitions: committees, preliminary and final contests. *Public Choice* 99, 63–76.

Amegashie, J.A. (2000). Some results on rent-seeking contests with shortlisting. *Public Choice* 105, 245–253.

Baik, K.H. and Lee, S. (2000). Two-stage rent-seeking contests with carryovers. *Public Choice* 103, 285-296.

Bakshi, D. and Dasgupta, I. (2020). Identity conflict with cross-border spillovers. *Defence and Peace Economics* 31, 786-809.

Bakshi, D. and Dasgupta, I. (2022). Can extremism reduce conflict? *Economics Letters* 215, art. 10482.

Bhattacharya, P. and Rampal, J. (2024). Contests within and between groups: theory and experiment. *Games and Economic Behavior* 145, 467-492.

Choi, J.P., Chowdhury, S.M., and Kim, J. (2016). Group contests with internal conflict and power asymmetry. *Scandinavian Journal of Economics* 118, 816–840.

Lopez Cruz, I. and Torrens, G. (2019) The paradox of power revisited: internal and external conflict. *Economic Theory* 68, 421-460.

Dasgupta, I. (2009). 'Living' wage, class conflict and ethnic strife. *Journal of Economic Behavior and Organization* 72, 750-765.

Esteban, J and Ray, D (2001). Collective action and the group size paradox. *American Political Science Review* 95, 663-672.

Ferguson, N. (2006). *The War of the World*. New York, Penguin.

Glazer, A. (2002). Allies as rivals: internal and external rent seeking. *Journal of Economic Behavior and Organization* 48, 155-162.

Hausken, K. (2005). Production and conflict models versus rent-seeking models. *Public Choice* 123, 59– 93.

Inderst, R., Müller, H.M. and Wärneryd, K (2007). Distributional conflict in organizations. *European Economic Review* 51, 385-402.

Inderst, R., Müller, H.M. and Wärneryd, K (2005). Influence costs and hierarchy. *Economics of Governance* 6, 177–197.

Katz, E and Tokatlidu, J (1996). Group competition for rents. *European Journal of Political Economy* 12, 599-607.

Konrad, K (2004). Bidding in hierarchies. *European Economic Review* 48, 1301-1308.

Muller, H.M. and Wärneryd, K (2001). Inside versus outside ownership: a political theory of the firm. *Rand Journal of Economics* 32, 527-541.

Münster, J (2007). Simultaneous inter- and intra-group conflicts. *Economic Theory* 32, 333–352.

Münster, J. and Staal, K. (2011). War with outsiders makes peace inside. *Conflict Management and Peace Science* 28, 91-110.

Münster, J. and Staal, K (2012). How organizational structure can reduce rent-seeking. *Public Choice* 150, 579–594.

Olson, M. (1965). *The Logic of Collective Action: Public Goods and the Theory of Groups*. Cambridge Mass., Harvard University Press.

Tullock, G. (1980). Efficient rent seeking, in J.M. Buchanan, R.D. Tollison and G. Tullock (eds.) *Toward a Theory of the Rent-seeking Society*. College Station, Texas A and M University Press: 97-112.

Wärneryd, K (1998). Distributional conflict and jurisdictional organization. *Journal of Public Economics* 69, 435-450.