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Clive Bell
South Asia Institute, Univerität Heidelberg

Hans Gersbach
KOF Swiss Economic Institute, ETH Zurich, CEPR and IZA

Hans Haller
Virginia Polytechnic Institute and State University

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IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Mutual Insurance in the Village and Beyond*

When formal insurance is unavailable, mutual insurance among households can serve as an alternative. This paper analyzes a game between economic agents facing uncertainty and maximizing discounted utility without enforceable contracts or access to capital markets. While autarky is always a possible outcome, under high discount factors, a mutually beneficial trigger-strategy equilibrium can be achieved. Full insurance is possible with strongly negatively correlated endowments, while partial insurance is generally feasible. The analysis highlights environments wherein varying levels of insurance can emerge, with applications to real-world institutional contexts.

JEL Classification: C72, C73, D80, G20, O11

Keywords: mutual insurance, risk sharing, group formation

Corresponding author:
Hans Gersbach
KOF Swiss Economic Institute at ETH Zurich
Leonhardstrasse 21
8092 Zurich
Switzerland
E-mail: gersbach@kof.ethz.ch

* Comments by Eric Bahel, Dongwoo Lee and Sudipta Sarangi are appreciated, with the usual absolution.

1 Introduction

One of the main purposes of financial markets is to allocate risks, usually with consequences for the intertemporal allocation of resources. It is widely recognized, however, that financial markets fail to allocate risks wholly efficiently. First, there are risks that are hard or impossible to insure against. For instance, technological progress may make human capital obsolete, leading to large losses of lifetime income, and ill health may make it impossible to pursue certain professional careers, with like losses. Second, even if extensive financial instruments are available, inadequate financial literacy may prevent people from choosing optimal portfolios from available securities. Lusardi and Mitchell (2014) survey the large literature on this subject.

Allocating risks efficiently is equally, if not more important, in less developed countries, whose financial markets are correspondingly less well-developed. This is particularly so in rural communities, wherein climactic fluctuations can be very damaging. The central question is, how do the inhabitants of such communities deal with risk? Townsend (1994), in his seminal study of three villages in India's semiarid Deccan plateau, notes that farmers do not fully exploit various diversification opportunities, such as diverse choices of soils and crops. A related question, therefore, is whether there is scope for specific policies. There are several institutions that might help in reducing risk, but these are not always successful. In India, for instance, the take-up rate of commercial index insurance against rainfall risk is extremely low. While price is important, Cole et al. (2013) find that non-price barriers, especially lack of trust, are equally so.

Formal and informal credit, including micro-credit and loans from local moneylenders, may well play an important role in smoothing consumption, but households also have other measures at their disposal. For instance, employment outside the village might compensate for idiosyncratic agricultural shortfalls, though such earnings may themselves be risky. Storage of grain from one year to the next is another possibility, albeit annual storage losses are not especially small. As for the sale of means of production, such as bullocks and land, this is a rather drastic, but not very rare, step.

The present paper analyzes the option of gifts, that is, of non-contractual giving and receiving, driven by a mutual insurance motive. The approach is based on the observation that households may insure each other by means of voluntary gifts when other forms of insurance are not available or very limited. Here, we lean on Townsend's (1994:

587) finding that most of the smoothing was effected through credit and gifts, where the terms of many informal loans might well have had a strong state-contingent component.

THE MODEL

The actors in the basic model are two economic agents, who live a countably infinite number of periods and play the strategic game described below. In this setting, the relevant trade-offs are in sharp relief. Going beyond bilateral relationships, small groups of three or more players are treated in Section 5. This extension also covers the possibility of increasing returns to the size of the group, with an associated augmentation of the group's aggregate endowment. It is applicable to formal, individual loans to a voluntary group whose members are subject to several and joint liability. The focus on small groups also reflects the fact that transactions involving outright gifts and informal credit between households are confined to relatives or good neighbors – not all fellow villagers are friends – and the transactions are often based on an ongoing relationship.

At each date, a player is exposed to uncertainty with respect to his or her future individual endowment or productivity. Players maximize expected discounted utility. Contracts between agents are not enforceable.[1] A player is not obliged to make or receive any transfers against his or her will. In this attempt to analyze the problem, we consider a model without access to exogenous capital markets. The players then have incentives to provide some mutual insurance without contractual agreement. Production is introduced indirectly in Section 5. Following some preliminaries on the key trade-off in Section 2, the game is set out in detail in Section 3.

In a "non-autarkic equilibrium", one player is willing to give in some states of nature, since it is in the other player's interest to give in return in others. Thus, the ancient principle *do ut des* is implemented. This principle is encountered in many social settings and adopted as a premise in social exchange theory; see, e.g., Roloff (1981). As an illustration, when inviting somebody to lunch or offering to do somebody a favor, we typically hope that the other person will accept and reciprocate in the future; but we can force that person neither to accept, nor to reciprocate. We are free, however, to discontinue our advances if they are rejected or not reciprocated. This compelling

[1]The players are free to conclude contracts, but a contract would only stipulate what the players would do anyhow.

resemblance between social exchange and economic exchange helps to understand the basic idea behind our formal results. Yet the analogy does not go any further. The analysis of the present paper does not explain social exchange in situations where the mutual insurance motive is missing.

THE RESULTS

The game always has an autarkic equilibrium. When endowments are perfectly negatively correlated across two players (Section 4) and discount factors are sufficiently high, full-insurance plans can be implemented as subgame perfect Nash equilibrium outcomes. Such equilibria, which both players prefer to autarky, are composed of trigger strategies, whereby the players revert to autarky forever after either has deviated from the equilibrium path.[2] Under fairly mild restrictions, full insurance is also implementable when there are three or more players, whose endowments are necessarily imperfectly correlated (see Section 5). Partial insurance plans can be implemented when the individual endowment streams are Bernoulli processes and are independent across players. When the endowment streams of two players are perfectly positively correlated, insurance may be impossible; but that is an exceptional case. In general, when there is imperfect correlation, partial insurance is feasible – and obtainable – through an implicit contract, i.e., as the outcome of a subgame perfect equilibrium. Section 6 treats a model with an arbitrary correlation structure. The two players' endowment streams are parametrized in such a way that, generically, there is a feasible insurance plan benefiting both players. In Section 7, we extend the basic model to analyze exogamy, where marriage alliances involve geographically widely separated villages, and hence go beyond smoothing mechanisms within a village.

Our findings provide a rationale why full insurance can occur in a village economy or beyond, even in the absence of capital markets. It is compatible with equilibrium play wherein a household receives a sequence of positive payments from other households, while passing through a number of periods of bad luck. In contrast to some of the literature, consumption in equilibrium tends to be uninformative about risk-bearing preferences. Households' consumption may be perfectly or strongly correlated with

[2] A similar intuition underlies a social insurance system where non-complying members are excluded forever; cf. Taub (1989). Such a scheme is actually used for voluntary mutual health insurance membership of high income individuals in Germany. Further notice that sufficiently long finite punishment phases instead of "grimm triggers" will do, at the cost of more complicated notation.

aggregate consumption, despite large differences in risk aversion. This conclusion is not restricted to the binary distributions in our leading examples. Exceptions are very impatient and risk-tolerant households, who may not participate in mutual insurance, since they do not satisfy the participation constraint.

RELATED LITERATURE

Several authors investigate risk and insurance in selected villages, just looking at income and consumption data for the households in the sample and disregarding any institutional details of borrowing, lending and insurance. Townsend (1994) assumes that individuals' preferences exhibit constant absolute risk aversion (CARA), though an earlier working paper also treats constant relative risk aversion (CRRA). Chiappori et al. (2014) use data from the Townsend Thai Monthly Survey, covering a total of 16 villages and assume CRRA. A further premise is that (a) risk-tolerant households might insure more risk-averse households against some of the aggregate risk and, as a consequence, (b) the consumption of more risk-tolerant households shows a stronger co-movement with aggregate consumption.[3] As a rule, full insurance at the village level cannot be rejected. Consumption is mainly determined by aggregate consumption and risk preferences, where there is evidence of heterogeneity in the latter.

Kimball (1988) uses a highly symmetric model to assess the plausibility of an informal system of consumption loans among farmers in medieval England that could serve as insurance against crop failure. The model of Coate and Ravallion (1993) is closest to ours. In essence, they treat the special case of *ex ante* symmetry of two agents in all respects: identical risk and time preferences, and symmetry of endowment distributions. This allows them to define the best informal insurance arrangement (best implementable contract) and the first-best solution, to compare the two, and to perform comparative statics analysis. In contrast, heterogeneity is a salient feature of our treatment. The leading case of Section 4 encompasses heterogeneous risk and time preferences, and asymmetric endowment streams. The three or more players in Section 5 exhibit like heterogeneity. Fafchamps (1992) discusses the inherent incentive and information problems of mutual insurance. He argues that because of these problems, mutual insurance may be bilateral rather than global group insurance. Friendship and kinship render some other households more trustworthy and more likely to become

[3]Despite its intuitive appeal, the assertion that (a) implies (b) need not hold; see Appendix A for details. Risk tolerance is defined to be the reciprocal value of risk aversion.

mutual insurance partners. In turn, partners can acquire relation-specific information over time and develop further mutual trust.

Among other earlier models of self-enforcing mutual insurance, Haller (1992) deals with sovereign debt and Thomas and Worrall (1988) with self-enforcing wage contracts. Udry (1994), in one of his models, considers two households – the "household" and its "partner" – who agree on oral loan contracts. The repayment obligations are not explicitly specified, but well-understood and state-contingent. There are just two periods, so that contracts cannot be self-enforcing. Rather, there exists a community or family authority entrusted with monitoring and enforcement.

To the extent that mutual insurance schemes are bilateral, networks or graphs may describe which pairs of households (or individuals) are able to engage in mutual insurance. Households form the nodes of the network. A direct link between a pair of households signifies that the pair can engage in mutual insurance. Bloch et al. (2008) start with an exogenously given network based on criteria like spatial proximity, friendship, family, social or professional contacts. While transfers between partnered households – individuals in their language – are suggested by a social norm, the insurance scheme is nonetheless self-enforcing: if a household does not make the transfers required by the social norm, some or all direct links to the household may be dropped and, consequently, the household will have fewer opportunities of mutual insurance in the future. Thus, links can be broken, but new links can never be formed. It turns out that the stable networks under this kind of dynamics tend to be either "thickly connected" or "thinly connected", while intermediate degrees of connectedness are less likely.

Gersbach and Haller (2017) evaluate, by means of examples, the risk-sharing capacity of markets versus the risk-sharing capacity of multi-member households. A household may provide insurance to some or all of its members through pooling of resources. The household may also engage in competitive exchange in a complete market setting. In the absence of aggregate risk, the possibility of various configurations is demonstrated. Insurance and risk allocation only through markets, only through households, and through both markets and households all prove to be equilibrium outcomes for specific model parameters.

2 The Basic Trade-off

Let household i's one-period income or endowment be a real-valued random variable $\widetilde{R}_i = \widetilde{R} + \widetilde{r}_i$, which depends on $\widetilde{R}$, a random variable that arises from social or aggregate risks for the community or village to which i belongs, and $\widetilde{r}_i$, a random variable that represents i's idiosyncratic component and is stochastically independent of aggregate risk.

Starting from Haller (1992), suppose players ALAN (A) and ESTHER (E) have separable inter-temporal preferences, with discount factors $\alpha \in (0,1)$ and $\varepsilon \in (0,1)$, respectively. A faces the following alternatives:

(i) Make a positive transfer to E. This sacrifice causes a current utility loss Δu, but A preserves a reputational collateral ΔU in all future periods.

(ii) Fail to make the transfer. Then A avoids the current utility loss, but loses the reputational collateral.

A would make the transfer if and only if $\Delta u \leq \sum_{t=1}^{\infty} \alpha^t \cdot \Delta U$, i.e.,

$$\Delta u \leq \frac{\alpha}{1-\alpha} \cdot \Delta U. \tag{1}$$

That is, (1) is equivalent to $\dfrac{\Delta u}{\Delta U} \leq \dfrac{\alpha}{1-\alpha}$ or $\dfrac{\Delta u/\Delta U}{1+\Delta u/\Delta U} \leq \alpha$. When $\Delta u > 0$ and $\Delta U > 0$ are given, the inequality is satisfied for α sufficiently close to 1.

Similarly, E, when trading off a potential current loss Δv caused by a transfer to A against a positive reputational collateral ΔV, would make the transfer if and only if

$$\Delta v \leq \frac{\varepsilon}{1-\varepsilon} \cdot \Delta V. \tag{2}$$

These conditions also hold when there are three or more players (see Section 5).

It is important to note that $1-\alpha$ and $1-\varepsilon$, respectively, are the players' pure impatience rates for utility, whereas the concavity of their utility functions represents both inter-temporal substitutability and preferences over lotteries. There are compelling reasons to suppose that people are impatient in this sense, but the rate itself is arguably very small when the unit period is a year, which is a natural assumption for rural communities. In the contrasting setting of the U:S., Fullerton and Rodgers (1993) calibrate a general equilibrium model wherein death is certain at the age of 80, and arrive at the rate 0.005. Villagers in poor countries face a more hazardous environment, but it seems doubtful that their impatience rates exceed 0.025. In what follows, therefore, it is assumed that the players' discount factors are indeed quite close to 1.

This fact does not, however, imply that inequalities (1) and (2) always hold. Risk-neutral players find no advantage in smoothing, but they are very likely to be somewhat impatient, so that deferring any amount of consumption in the present in exchange for the same amount in the future entails a cost. For such players, lending at any sure interest rate exceeding their impatience rates is attractive, but voluntary, reciprocal exchange is not. In the setting of Section 4, $\Delta U = 0$ if the player is risk neutral, so that (1) is then violated when the probability of a bad draw is positive.

Turning to the other aspect of players' preferences, they are assumed to be risk averse ($u'' < 0$ and $v'' < 0$). Let u and v be defined for all positive levels of consumption. A discussion of the players' coefficients of relative risk aversion, $\rho_u(c) \equiv -cu''(c)/u'(c)$ and $\rho_v(c) \equiv -cv''(c)/v'(c)$, as a measures of their respective risk aversion, is needed. Consider the normalisations $u(1) = v(1) = u'(1) = v'(1) = 1$. It is proved in Appendix B that if $\rho_u(c) \geq 1 \ \forall c \in (0, 1)$, then $u(0)$ is not defined (i.e., $u \to -\infty$ as $c \to 0$). If u (or v) is thus unbounded from below, the agent is said to exhibit 'strong' risk aversion. An ubiquitous example is $u = 1 + \ln c$. Observe, however, that introducing an additional parameter to avoid the singularity at $c = 0$ imposes a restriction on ρ_u. The form $u = k_1[\ln(c + e^{-1}) + 1]$, where k_1 satisfies $\ln(1 + e^{-1}) = (1/k_1) - 1$, yields $-u''c/u' = c/(c + e^{-1}) < 1 \, \forall c \in [0, 1]$, so that strong risk aversion is ruled out by the normalisation $u(0) = 0$.

If, instead, $1 > \rho_u(c) \geq \rho_u(1) \, \forall c \in (0, 1)$, then it is shown in Appendix B that $u(0)$ is defined if and only if cu' goes to a finite limit as $c \to 0$. These two conditions are therefore sufficient for 'weak' risk aversion, in the sense that $u(0)$ is bounded from below. The normalisation $u(0) = 0$ can now be imposed. A standard example (see Section 4.3) is $u = c^\gamma$, where $\gamma \in (0, 1)$, which satisfies $u(0) = 0$ and $u(1) = 1$, but not $u'(1) = 1$.

To sum up this preliminary, if the smallest realized endowment $b > 0$, then both strong and weak risk aversion are covered without the occurrence of a singularity. A positive value of b may be interpreted as the certainty of some minimum level of emergency support from outside, presumably in the form of government relief. If, on the contrary, $b = 0$, then the restriction on preferences expressed by ρ must be taken into account.

3 The Two-Player Game

The game is played by ALAN and ESTHER, each living infinitely many periods $t = 0, 1, 2, \ldots$. There is a single, non-storable, perfectly divisible commodity. ALAN's endowment stream is given as a sequence of independent random variables $\widetilde{A}_0$, $\widetilde{A}_1$, $\widetilde{A}_2, \ldots$ with values in $[0, 1]$, ESTHER's is a sequence of independent random variables $\widetilde{E}_0$, $\widetilde{E}_1$, $\widetilde{E}_2, \ldots$ with values in $[0, 1]$. For brevity, let $\widetilde{A}$ denote the stochastic process $(\widetilde{A}_t)$ and $\widetilde{E}$ denote the process $(\widetilde{E}_t)$. Let $(\Omega, \mathbb{F}, P)$ be the underlying probability space, i.e., $\widetilde{A}_t : \Omega \to [0, 1]$ and $\widetilde{E}_t : \Omega \to [0, 1]$ for all t. Assume all $\widetilde{A}_t$ and $\widetilde{E}_t$ to be discrete random variables. This assumption avoids measurability problems. It also permits us to narrow terminology for the purposes of this paper as follows. Given a discrete random variable $\widetilde{X} : \Omega \to [0, 1]$, call $r \in [0, 1]$ a *realization* of $\widetilde{X}$ if and only if $P(\widetilde{X} = r) > 0$. Given a finite or infinite sequence of discrete random variables, $\widetilde{X}_t : \Omega \to [0, 1]$, $t = 0, \ldots, T$ where $T \in \mathbb{N}_0$ or $T = \infty$, a *path* of the stochastic process $(\widetilde{X}_t)$ is a sequence X_t, $t = 0, \ldots, T$, in $[0, 1]$ such that X_t is a realization of $\widetilde{X}_t$ for all t.

At date t, the players realize A_t and E_t, respectively, yielding the total endowment $A_t + E_t$. ALAN consumes an amount $a_t \geq 0$ and ESTHER an amount $e_t \geq 0$ such that $a_t + e_t = A_t + E_t$ and each of a_t and e_t can take values in $[0, 2]$.

ALAN's utility derived from a consumption stream $a = (a_0, a_1, a_2, \ldots)$ is
$U(a) = \sum_t \alpha^t u(a_t)$ where $u : [0, 2] \to \mathbb{R}$ is increasing and strictly concave, $\alpha \in (0, 1)$.
ESTHER's utility derived from a consumption stream $e = (e_0, e_1, e_2, \ldots)$ is
$V(e) = \sum_t \varepsilon^t v(e_t)$ where $v : [0, 2] \to \mathbb{R}$ is increasing and strictly concave, $\varepsilon \in (0, 1)$.

At date t, when the realizations A_t and E_t are common knowledge, the players proceed like in a coordination game.[4] ALAN announces a net trade $\underline{z}_t \in [-A_t, E_t]$ and ESTHER announces a net trade $\overline{z}_t \in [-A_t, E_t]$, where all net trades are measured relative to ALAN.[5] If the announcements are incompatible, i.e., $\underline{z}_t \neq \overline{z}_t$, then $a_t = A_t, e_t = E_t$. If the announcements are compatible, i.e., $\underline{z}_t = \overline{z}_t = z_t$, then ALAN's consumption is $a_t = A_t + z_t$, ESTHER's consumption is $e_t = E_t - z_t$.

A history prior to announcements at time 0 is a pair of realizations $h_0 = (A_0, E_0)$.

[4] Alternatively, the players might proceed like in a Nash demand game. This would yield the same results while rendering description and notation a bit more complicated.

[5] A net trade $z \in \mathbb{R}$ means that the amount z of the commodity is transferred from ESTHER to ALAN so that ALAN consumes $A_t + z$ and ESTHER consumes $E_t - z$. ALAN cannot demand more than E_t from ESTHER, and she no more than A_t from ALAN, if their announcements are to be feasible.

For $t \geq 1$, a history prior to announcements at time t is a $[2(t+1)+2t]$-tuple $h_t = ((A_0, E_0), \ldots, (A_t, E_t); (\underline{b}_0, \overline{b}_0), \ldots, (\underline{b}_{t-1}, \overline{b}_{t-1}))$ where $A_0, \ldots, A_t$ is a path of the process $\widetilde{A}_0, \ldots, \widetilde{A}_t$, $E_0, \ldots, E_t$ is a path of the process $\widetilde{E}_0, \ldots, \widetilde{E}_t$, $\underline{b}_0, \ldots, \underline{b}_{t-1}$ is a history of ALAN's prior announcements and $\overline{b}_0, \ldots, \overline{b}_{t-1}$ is a history of ESTHER's prior announcements. Let H_t denote the set of histories prior to announcements at time t. Conditional on the history $h_t \in H_t$, which is common knowledge prior to moves at time t, the players make simultaneous moves at time t: ALAN chooses $\underline{b}_t \in M(h_t) = [-A_t, E_t]$, while ESTHER chooses $\overline{b}_t \in M(h_t)$. A strategy for ALAN is a sequence of mappings $S_{1t} : H_t \to [-1, 1]$ such that $S_{1t}(h_t) \in M(h_t)$ for each $t \in \mathbb{N}_0, h_t \in H_t$. A strategy for ESTHER is a sequence of mappings $S_{2t} : H_t \to [-1, 1]$ such that $S_{2t}(h_t) \in M(h_t)$ for each $t \in \mathbb{N}_0, h_t \in H_t$.

Denote $S_1 = (S_{1t})$, $S_2 = (S_{2t})$. Then for any pair of paths $A = (A_t)$ of $(\widetilde{A}_t)$ and $E = (E_t)$ of $(\widetilde{E}_t)$, the play of strategies S_1 and S_2 induces consumption streams $a = a(S_1, S_2, A, E)$ and $e = e(S_1, S_2, A, E)$, with resulting utilities $U(a)$ and $V(e)$. The players maximize their respective expected utilities:

$\bar{U}(S_l, S_2) = \mathbb{E}U(a(S_1, S_2, A, E)),$

$\bar{V}(S_1, S_2) = \mathbb{E}V(e(S_l, S_2, A, E)),$

where $\mathbb{E}$ denotes the expectation with respect to the underlying distribution of (A, E)'s. Since the $\widetilde{A}_t$'s and $\widetilde{E}_t$'s are discrete and U and V are intertemporally separable, $\bar{U}$ and $\bar{V}$ are well defined.

Let Σ_1 and Σ_2 denote the sets of ALAN's and ESTHER's strategies, respectively. Then

$$\Gamma = (\{1, 2\}, \Sigma_1, \Sigma_2, \bar{U}, \bar{V})$$

is the normal form of our strategic game of giving and receiving. A Nash equilibrium of Γ is defined in the usual way. Note that in the extensive form outlined above, each history $h_t \in H_t$ gives rise to a subgame $\Gamma(h_t)$ starting in period t. Conversely, each proper subgame is identified by its prior history h_t.[6] A subgame perfect equilibrium in the sense of Selten (1975) is defined in the usual way.

Observe that, regardless of the other player's announcement, each player can always secure the status-quo (autarky) outcome $a_t = A_t$, $e_t = E_t$ by announcing 0. This conforms to our premise that a player cannot be forced to make or to receive any transfer.

[6] Since the players make their first moves once h_0 is common knowledge, the proper subgames of Γ are the relevant ones.

Indeed, there always exists the **"autarkic" subgame perfect Nash equilibrium** given by $S_{1t} \equiv 0$, $S_{2t} \equiv 0$ for all $t \in \mathbb{N}_0$, which gives rise to autarky in each period. We are concerned above all, however, with non-autarkic, subgame-perfect Nash equilibria. We call an equilibrium (S_1, S_2) of Γ non-autarkic, if there is a path $A = (A_t)$ of the process $\widetilde{A}$ and a path $E = (E_t)$ of the process $\widetilde{E}$ with S_1, S_2, A, and E inducing consumption paths $a = (a_t) = a(S_1, S_2, A, E)$ and $e = (e_t) = e(S_1, S_2, A, E)$ such that $(a_t, e_t) \neq (A_t, E_t)$ for at least one t.

Our subsequent analysis does not rely on the theory of stochastic games, as exhibited in Friedman (1986), although the model fits into that framework. The reason is that we are interested in obtaining results not merely for the existence of an equilibrium, but rather for particular types of equilibria. We shall resort to (non-stationary) trigger strategies when investigating non-autarkic equilibria. Trigger strategy equilibria have been thoroughly studied in Friedman (1971), except for the stochastic ingredient and elaboration on mutual insurance opportunities.

4 Two Players: Perfect Negative Correlation of Endowments

The special case of perfect negative correlation between the two players' endowments at each date t is highly stylized, yet it proves to be instructive. Assume the binomial random variables $\widetilde{A}_0, \widetilde{A}_1, \widetilde{A}_2, \ldots$ to be independent and identically distributed (i.i.d.) with $P(\widetilde{A}_0 = b) = 1 - p$ and $P(\widetilde{A}_0 = 1) = p$, where $p \in (0, 1)$ and the 'bad' realization $b \in [0, 1)$. Further, let $\widetilde{E}_t(\omega) = (1 + b) - \widetilde{A}_t(\omega)$ for $\omega \in \Omega, t \in \mathbb{N}_0$. Thus the two endowments are perfectly negatively correlated, and there is only individual risk.

4.1 Mutual insurance: weak risk aversion

The players can benefit from mutual insurance. Indeed, many non-autarkic, subgame perfect equilibria are possible if the discount factors are sufficiently close to 1. Consider, in particular, the pair of certain consumption paths given by $a_t = p = \mathbb{E}\widetilde{A}_0$ and $e_t = 1 - p = \mathbb{E}\widetilde{E}_0$ for all t, where $b = 0$, with $u(0) = v(0) = 0, u(1) = v(1) = 1$. This pair has the following three properties:

1. In all periods, each player receives full and fair intra-temporal insurance, with

respective expected utility gains
$\Delta U = u(\mathbb{E}[A_t]) - \mathbb{E}[u(A_t)]) = u(p) - p,$
$\Delta V = v(\mathbb{E}[E_t]) - \mathbb{E}[u(E_t)]) = v(1-p) - (1-p).$

2. These insurance plans are feasible. When $A_t = 1$, ALAN has to transfer the amount $1-p$ to ESTHER, suffering a temporary loss $\Delta u = u(1) - u(p)$. When $A_t = 0$, ESTHER has to transfer the amount p to ALAN, suffering a temporary loss $\Delta v = v(1) - v(1-p)$.

3. The said plans $a = (p, p, p, \ldots)$ and $e = (1-p, 1-p, \ldots)$ can be implemented via a subgame perfect trigger-strategy equilibrium. To this end, define a pair of trigger strategies $S_1 = (S_{1t})$ and $S_2 = (S_{2t})$ recursively as follows:

For $h_0 \in H_0 : S_{i0}(h_0) = p - A_0$.
For $t \geq 1, h_t \in H_t$:

$$S_{it}(h_t) = \begin{cases} p - A_t & \text{if } S_1 \text{ and } S_2 \text{ were followed up to period } t-1, \\ 0 & \text{if deviation from } S_1 \text{ or } S_2 \text{ occurred in some prior period.} \end{cases}$$

Clearly, the joint strategy (S_1, S_2) yields the desired outcome. Since the autarkic equilibrium is subgame perfect, it can be used as a trigger. If the values $\Delta U, \Delta u, \Delta V, \Delta v$ satisfy (1) and (2), then standard arguments show that (S_1, S_2) is a (non-stationary) subgame perfect Nash equilibrium of Γ. The findings so far are summarized in

Proposition 1 (Full Insurance Equilibrium) *If the agents' preferences exhibit weak risk aversion and the binary variates $\widetilde{A}_t$ and $\widetilde{E}_t$ are perfectly negatively correlated, the full insurance plans $a = (p, p, \ldots)$ and $e = (1-p, 1-p, \ldots)$ can be implemented via a subgame perfect trigger-strategy equilibrium if and only if (1) and (2) hold.*

Remarks. First, consider the case where $\widetilde{A}_0, \widetilde{A}_1, \ldots,$ are arbitrary, i.i.d. discrete random variables and $\widetilde{E}_t = 1 - \widetilde{A}_t$ for all t such that 0 and 1 belong to the support of $\widetilde{A}_0$. Denote $\mu = \mathbb{E}\widetilde{A}_0$. Assume once more $u' > 0, u'' < 0, v' > 0, v'' < 0, u(0) = v(0) = 0, u(1) = v(1) = 1$. Then one obtains analogous results for the full insurance plans $a = (\mu, \mu, \ldots)$ and $e = (1-\mu, 1-\mu, \ldots)$.
Second, the full insurance plans $a = (1/2, 1/2, \ldots)$ and $e = (1/2, 1/2, \ldots)$ can be implemented when the players are identical in all respects, save that when one draws 1, the other draws $b = 0$; that is to say, the players differ only across states of nature. The latter difference, not those in their risk aversion, impatience or mean endowments, suffices as the basis for trade.

4.2 The riskiness of endowments

The riskiness of the agent's endowment has two elements: a reduction in the parameter b increases the spread and reduces the mean, whereas a reduction in p also reduces the mean, but leaves the spread unchanged. In what follows, the only assumptions are that u is strictly concave, $u(1) = 1$ and, concerning the parameter b, that ρ_u behaves in a particular way as $c \to 0$.

Let $\bar{A}_t \equiv \mathbb{E}\widetilde{A}_t = p + (1-p)b$. We have

$$\frac{\Delta u}{\Delta U} = \frac{1 - u(\bar{A}_t)}{u(\bar{A}_t) - [p + (1-p)u(b)]} = \frac{1 - u[p + (1-p)b]}{u[p + (1-p)b] - [p + (1-p)u(b)]}. \tag{3}$$

If $u(b) \to -\infty$ as $b \to 0$, then $\Delta U \to \infty$ and $\Delta u/\Delta U \to 0$, so that conditions (1) and (2) do not bind for all b sufficiently close to zero, even when the respective impatience rates are close to zero. This is the first result.

If, on the contrary, u approaches some finite lower bound as $c \to 0$, then $\Delta u/\Delta U$ is bounded away from zero, so that the said conditions may bind when the respective impatience rates are close to zero. It is seen from (3), however, that they will not bind if $|u(b)|$ is sufficiently large.

Differentiating (3) w.r.t. b, we have

$$\begin{aligned} \frac{\partial(\Delta u/\Delta U)}{\partial b} &= -\frac{1-p}{Q^2} \cdot \left(u'(\bar{A}_t)Q + [u'(\bar{A}_t) - u'(b)][1 - u(\bar{A}_t)]\right), \\ &= -\frac{1-p}{Q^2} \cdot \Big(u'(\bar{A}_t)[1 - \mathbb{E}u(\widetilde{A}_t)] - u'(b)[1 - u(\bar{A}_t)]\Big), \end{aligned} \tag{4}$$

where $Q \equiv u(\bar{A}_t) - \mathbb{E}u(\widetilde{A}_t) = u(\bar{A}_t) - [p + (1-p)u(b)]$. Hence,

$$\frac{\partial(\Delta u/\Delta U)}{\partial b} \gtrless 0 \text{ according as } \frac{u'(b)}{1 - \mathbb{E}u(\widetilde{A}_t)} \gtrless \frac{u'(\bar{A}_t)}{1 - u(\bar{A}_t)}.$$

The strict concavity of u implies $u'(b) > u'(\bar{A}_t)$; but $u(\bar{A}_t) > \mathbb{E}u(\widetilde{A}_t)$, so that the sign of the derivative is ambiguous without further assumptions. It will be positive if b is sufficiently small, $u'(b)$ is sufficiently large and p is not too close to zero. Otherwise, the said sign depends on the curvature of u over the interval $[b, 1]$.

So much for the spread parameter b. Turning to p, we can rewrite $\Delta u/\Delta U$ as

$$\frac{\Delta u}{\Delta U} = \left(\frac{(1-p)[1 - u(b)]}{1 - u(\bar{A}_t)} - 1\right)^{-1}, \tag{5}$$

which is decreasing in p if and only if $(1-p)/[1-u(\bar{A}_t)]$ is increasing in p. Differentiating w.r.t. p, we have

$$\begin{aligned} \frac{\partial\{(1-p)/[1-u(\bar{A}_t)]\}}{\partial p} &= \frac{-[1-u(\bar{A}_t)]+(1-p)\partial u(\bar{A}_t)/\partial p}{[1-u(\bar{A}_t)]^2}, \\ &= \frac{-[1-u(\bar{A}_t)]+(1-p)(1-b)u'(\bar{A}_t)}{[1-u(\bar{A}_t)]^2}. \end{aligned} \tag{6}$$

Now, $-[1-u(\bar{A}_t)]+(1-p)(1-b)u'(\bar{A}_t) \gtrless 0$ according as $(1-\bar{A}_t))u'(\bar{A}_t) \gtrless 1-u(\bar{A}_t)$, or $u'(\bar{A}_t) \gtrless [1-u(\bar{A}_t)]/(1-\bar{A}_t)$. The slope of the chord connecting the points $(\bar{A}_t, u(\bar{A}_t))$ and $(1, u(1) = 1)$ is $[1-u(\bar{A}_t)]/(1-\bar{A}_t)$, which is smaller than the slope of the tangent to u at $(\bar{A}_t, u(\bar{A}_t))$; for u is strictly concave.

4.3 Risk aversion and decisions

We examine how risk aversion affects households' mutual insurance decisions. For ease of comparison, we assume CRRA, where the value $\rho = 1$ is regarded as separating weak from strong risk aversion. For that value, $u = \ln c + 1$: u is bounded neither from below nor from above, with $u(1) = u'(1) = 1$.

Weak risk aversion. Up to a normalization, $u(c) = c^\gamma$ and $v(c) = c^\eta$ for $c \geq 0$, where $\gamma \in (0,1)$, $\eta \in (0,1)$, and $1-\gamma$ and $1-\eta$ are, respectively, ALAN's and ESTHER's constant coefficients of relative risk aversion. For the binary distributions considered above, this yields

$$\Delta U = p^\gamma - p,\ \Delta V = (1-p)^\eta - (1-p) \equiv q^\eta - q,$$

$$\Delta u = 1 - p^\gamma,\ \ \Delta v = 1-(1-p)^\eta \equiv 1 - q^\eta.$$

If the discount factors are sufficiently close to 1, so that (1) and (2) do not bind, then:

(i) The full insurance equilibrium is independent of the players' risk aversion, with an exception noted below; so that

(ii) the players consume p and $q = 1-p$, respectively, in every period; and

(iii) each player's consumption co-moves with aggregate consumption, albeit the latter is unvarying.

There is no equilibrium if either player is risk neutral; for $\Delta U = 0 \, \forall p \in (0,1)$ if $\gamma = 1$ and $\Delta V = 0 \, \forall q \in (0,1)$ if $\varepsilon = 1$. In fact, equilibrium is ruled out if either is only slightly risk averse: to be precise, if $\gamma \geq \alpha$ or $\eta \geq \varepsilon$, respectively (see Appendix C).

These findings for weak risk aversion stand in stark contrast to those of Chiappori et al. (2014), wherein, in a general equilibrium setting, less risk averse households acquire less insurance and their consumption exhibits more co-movement with aggregate supply. The same findings hold for strong risk aversion, as we now demonstrate.

Strong risk aversion. For u and v to be defined for all outcomes, let the variates $\widetilde{A}_t$ and $\widetilde{E}_t$ have the common support $[b, 1]$, $b \in (0,1)$. Then $u = 2 - 1/c^{\gamma}, v = 2 - 1/c^{\eta}$ where $\gamma > 0, \eta > 0$, and $\gamma + 1$ and $\eta + 1$ are now the respective coefficients of relative risk aversion; and $u(1) = v(1) = 1$. Hence,

$$\Delta U = \frac{1 - p(1 - b^{\gamma})}{b^{\gamma}} - \frac{1}{[p + (1-p)b]^{\gamma}}, \, \Delta V = \frac{1 - (1-p)(1 - b^{\eta})}{b^{\eta}} - \frac{1}{[(1-p) + pb]^{\eta}};$$

$$\Delta u = -1 + \frac{1}{[p + (1-p)b]^{\gamma}}, \quad \Delta v = -1 + \frac{1}{[(1-p) + pb]^{\eta}}.$$

In virtue of the strict concavity of u and v, $\Delta U > 0$ and $\Delta V > 0$. Since $b < 1$, $\Delta u > 0$ and $\Delta v > 0$. It is still the case that (1) and (2) do not bind if, *ceteris paribus*, the discount factors are sufficiently close to 1. Indeed, (1) and (2) do not bind when b is sufficiently close to 0, even when the discount factors are not close to 1, a result that follows from the fact that $\Delta U \to \infty$ and $\Delta V \to \infty$ as $b \to 0$.

4.4 Full insurance thresholds

Various aspects of risk affect decisions when (1) or (2) binds. We focus on ALAN; *mutatis mutandis*, the same findings apply to ESTHER. We call

$$\alpha^* \equiv \frac{\Delta u / \Delta U}{1 + \Delta u / \Delta U}.$$

ALAN's **full insurance threshold**, since the full-insurance equilibrium of Proposition 1 requires $\alpha^* \leq \alpha$. For any given α, ALAN is interested in playing the full-insurance equilibrium path provided α^* is small enough. Whenever α^* exceeds α, ALAN will drop out of this equilibrium and forego full insurance for good. In a sense, a player with a high α^* is less eager to obtain full and fair insurance. We now investigate how α^* responds to changes in model parameters other than α. Since α^* is an increasing function of $\Delta u / \Delta U$, it suffices to perform comparative statics on the latter.

First, there is the riskiness of endowments. It has been shown in Section 4.2 that if b is sufficiently small, $u'(b)$ is sufficiently large and p is not too close to zero, then $\Delta u/\Delta U$ is increasing in b: that is, a reduction in the spread of $\widetilde{A}_t$ then increases the threshold value α^*. Otherwise, the sign of the derivative depends on the curvature of u over the interval $[b, 1]$.

It is also shown in Section 4.2 that $\Delta u/\Delta U$ is decreasing in p, which implies $\partial\alpha^*/\partial p < 0$:

Proposition 2 *ALAN's full insurance threshold value is decreasing in p.*

It should be noted that these results concerning b and p rest on neither the assumption that ρ be constant, nor that it be less than 1.

One might expect that greater risk renders an agent more eager to be insured. That depends, however, on the specific terms of insurance. Suppose ALAN's risk increases in the sense that $1-p$ increases, and thus also reduces $\mathbb{E}\widetilde{A}_t$. Then, *ceteris paribus*, α^* increases. Hence, ALAN becomes less inclined to participate in mutual insurance on the terms specified in Proposition 1. The reason is that when p is small, full and fair insurance guarantees a low level of consumption to ALAN and a high level to ESTHER. They may end up in a different equilibrium that provides full insurance, but is unfair to ESTHER and superfair to ALAN.

Turning to the agents' preferences for risk bearing, we assume CRRA.

Weak risk aversion. As before, let $b = 0$.

Proposition 3 *If $u(c) = c^\gamma$, the full insurance threshold value is increasing in γ.*

For $\Delta u/\Delta U = (1-p^\gamma)/(p^\gamma - p)$, $p^\gamma = e^{\gamma \cdot \ln p}$, $\partial p^\gamma/\partial\gamma = \ln p \cdot p^\gamma$ yield

$$\frac{\partial(\Delta u/\Delta U)}{\partial\gamma} = \frac{1}{(p^\gamma - p)^2} \cdot [-\ln p \cdot p^\gamma \cdot (p^\gamma - p) - \ln p \cdot p^\gamma \cdot (1-p^\gamma)] = \frac{-\ln p \cdot p^\gamma \cdot (1-p)}{(p^\gamma - p)^2} > 0.$$

As ALAN becomes more risk averse, γ decreases and α^* decreases. Hence, *ceteris paribus*, a more risk-averse household is more eager to participate in mutual insurance.

With such preferences for risk bearing, a uniform change (scaling) of wealth prior to insurance has no effect on the players' eagerness to participate in the full-insurance equilibrium. To prove this claim, let $\widetilde{A}_t$ and $\widetilde{E}_t$ be replaced by $k\widetilde{A}_t$ and $k\widetilde{E}_t$, respectively, with $k > 0$. The corresponding full and fair insurance consumption paths are $a =$

$(kp, kp, \ldots)$ for ALAN and $e = (k(1-p), k(1-p), \ldots)$ for ESTHER. For ALAN,

$$\Delta U = u(kp) - pu(k) = k^\gamma p^\gamma - pk^\gamma = k^\gamma(p^\gamma - p)$$

and

$$\Delta u = u(k) - u(kp) = k^\gamma - k^\gamma p^\gamma = k^\gamma(1 - p^\gamma),$$

so that $\Delta u/\Delta U = (1-p^\gamma)(p^\gamma - p)$ is independent of k. Likewise for ESTHER.

Thus, even wealthy individuals can fully rely on informal insurance schemes when underlying inequality (arising from ALAN's $p \neq$ ESTHER's q) is constant. Equally important is that poor households can then achieve full insurance. In some communities, it has been shown that they do not purchase formal insurance (Cole et al., 2013), so that achieving full insurance through informal means is crucial.

Strong risk aversion. Let $b > 0$ and $u = 2 - 1/c^\gamma$, where the degree of risk aversion is now increasing in γ ($\rho = 1 + \gamma$). Eq. (5) specialises to

$$\frac{\Delta u}{\Delta U} = \left[(1-p)\left(\frac{(b^{-\gamma} - 1)}{-1 + \bar{A}_t{}^{-\gamma}}\right) - 1\right]^{-1} \equiv [(1-p)B(\gamma) - 1]^{-1},$$

Differentiating w.r.t. γ, we have

$$\frac{\partial B}{\partial \gamma} = -\frac{(b\bar{A}_t)^{-\gamma}}{(\bar{A}_t{}^{-\gamma} - 1)^2}\left[(1 - \bar{A}_t{}^\gamma)\ln b - (1 - b^\gamma)\ln \bar{A}_t\right],$$

so that

$$\frac{\partial B}{\partial \gamma} \gtrless 0 \text{ according as } \frac{\ln \bar{A}_t}{1 - \bar{A}_t{}^\gamma} \gtrless \frac{\ln b}{1 - b^\gamma}.$$

Now, for any given b, $\bar{A}_t$ is increasing in p, where $\bar{A}_t = b$ when $p = 0$. Hence, if $\ln \bar{A}_t/(1 - \bar{A}_t{}^\gamma)$ is increasing in $\bar{A}_t$, induced by increases in p, B will increase with γ, thus reducing the threshold value α^*. Let $x = \bar{A}_t$, where $\bar{A}_t < 1$. The derivative of $(1 - x^\gamma)^{-1}\ln x$ is

$$\frac{(1 - x^\gamma) + \gamma x^\gamma \ln x}{x(1 - x^\gamma)^2}.$$

Let $\phi = 1 - \gamma \ln x - x^{-\gamma}$. Then $\phi(1) = 0$ and $\phi' = -(\gamma/x)(1 - x^{-\gamma}) > 0\ \forall x \in (0,1)$. Hence, $-\gamma \ln x < x^{-\gamma} - 1$, $(1 - x^\gamma) + \gamma x^\gamma \ln x > 0\ \forall \gamma > 0$, and α^* is decreasing in γ, that is, increasing in the player's risk aversion, as in Proposition 3.

A definite result now holds for changes in b. We have

$$\alpha^* = \frac{(-\bar{A}^\gamma + 1)/[(\bar{A}/b)^\gamma[1 - p(1 - b^\gamma)] - 1]}{1 + (-\bar{A}^\gamma + 1)/[(\bar{A}/b)^\gamma[1 - p(1 - b^\gamma)] - 1]},$$

where $\bar{A} = p + (1-p)b = \mathbb{E}\widetilde{A}_t$. It is seen that, in the limit as $b \to 0$, $\alpha^* \to 0$: ALAN's full insurance threshold goes to zero, in keeping with intuition, since u is unbounded from below as c approaches zero.

5 Three or More Players

Let the aggregate endowment be proportional to the number of players. When there are three players, label them $i = 1, 2, 3$, where, w.l.o.g., ALAN becomes $i = 1$ and the aggregate endowment is $3/2$. In the first variation, player 1 obtains the aggregate endowment with probability p and its complement $q = 1 - p$ is allocated equally between the other two: Each obtains the aggregate endowment with probability $q_2 = q_3 = (1-p)/2$. The players' individual endowments are negatively, but now imperfectly correlated, since in each state of nature, two of them obtain zero. Routine calculations yield the following correlation coefficients: between players 1 and 2,

$$\rho_{12} \; (= \rho_{13}) = -\frac{1}{1+p}\left[\frac{2p}{1-p}\right]^{1/2},$$

and between players 2 and 3,

$$\rho_{23} = -\frac{2(1-p)}{(1+p)^2}.$$

Note that players 2 and 3 may differ in their discount factors and aversion to risk.

Proceeding as in Section 4.3 with weak risk aversion, for player 1,

$$\Delta U = (p \cdot 3/2)^\gamma - p(3/2)^\gamma, \;\; \Delta u = (3/2)^\gamma - (p \cdot 3/2)^\gamma,$$

so that the transfer for full insurance for all three players will be made if, and only if,

$$1 - p^\gamma \leq \frac{\alpha}{1-\alpha}(p^\gamma - p),$$

that is, simply condition (1) once more. For player 2, the corresponding condition is

$$1 - \left(\frac{1-p}{2}\right)^{\eta_2} \leq \frac{\varepsilon_2}{1-\varepsilon_2}\left[\left(\frac{1-p}{2}\right)^{\eta_2} - \frac{1-p}{2}\right],$$

which can be rearranged as

$$\psi(p;3) \equiv \left(\frac{1-p}{2}\right)^{\eta_2} + \varepsilon_2\left(1 - \frac{1-p}{2}\right) \geq 1. \tag{7}$$

It is proved in Appendix C that, in the game involving 1 and 2 alone, conditions (1) and (2) hold as strict inequalities $\forall \gamma \in (0, \alpha)$ and $\forall \eta_2 \in (0, \varepsilon_2)$ when p lies in some measurable interval that includes $p = 1/2$. Consider, therefore, $p = 1/2$ and $\eta_2 = 1/2$. Then $\psi(p; 3) > 1 \, \forall \varepsilon_2 > 2/3$. Likewise, for player 3: when $\eta_3 = 1/2$, $\psi(p; 3) > 1 \, \forall \varepsilon_3 > 2/3$. It follows not only that there is a full-insurance equilibrium for this constellation of parameter values, but also that there is such an equilibrium for a dense set of parameter values in the neighbourhood of $p = 1/2$, with η_3 differing somewhat from η_2 and q_2 from q_3, subject to $q_2 + q_3 = 1 - p$. When $p = 1/2$, $\rho_{12} = \rho_{13} = -2\sqrt{2}/3 \approx -0.9428$, and $\rho_{23} = -4/9$.

Generalising to n players, the aggregate endowment is $n/2$. Condition (1) continues to hold for player 1, whose average endowment is $pn/2$, and for the rest, we obtain the condition

$$\psi(p; n) \equiv \left(\frac{1-p}{n-1} \right)^{\eta_i} + \varepsilon_i \left(1 - \frac{1-p}{n-1} \right) \geq 1, \; i = 2, 3, \ldots, n.$$

This will hold, for example, for $p = 1/2$ if n is not much greater than 3, the η_i are clustered about one half and the ε_i are sufficiently close to 1.

In the second variation, player 1 obtains merely the original expected value of his endowment as additional players are introduced, the aggregate endowment being $n/2$ as in the first variation. When $n = 2$, player 1 obtains the endowment 1 with probability $p(2)$ and zero with probability $1 - p(2)$. Let $p \equiv p(2)$, so that with n players, he obtains $n/2$ with probability $2p/n$ and zero with probability $1 - 2p/n$. The spread increases with n, making him worse off and mutual insurance less attractive. With a third player, as in the first variation,

$$\rho_{12} \; (= \rho_{13}) = -\frac{6}{3 + 4p} \left[\frac{p}{3 - 2p} \right]^{1/2}$$

and

$$\rho_{23} = -\frac{1}{4} \left[\frac{3/4 - p}{(3/2 - p)(3/4 + p)^2} \right].$$

When $n = 3$, player 1 will make the required transfer if and only if

$$(3/2)^\gamma - p^\gamma \leq \frac{\alpha}{1 - \alpha} \left[p^\gamma - \frac{2p}{3} (3/2)^\gamma \right],$$

which can be rearranged as

$$\phi(p; 3) \equiv (1 - \alpha)[(1 - \alpha) + 2\alpha p/3] \leq (2p/3)^\gamma.$$

Note that $\phi(p;3) < (1-\alpha)(1-\alpha/3)\,\forall p \in (0,1)$. Since α is close to 1, this condition is easily satisfied, even when p is quite small. To illustrate with $p = 1/2$ and $\gamma = 1/2$ once more, $(1-\alpha)(1-\alpha/3) \geq (1/3)^{1/2}\ \forall \alpha > 0.79$. he endowments are also less strongly negatively correlated than in the first variation: when $p = 1/2$, $\rho_{12} = \rho_{13} = -3/5$ and $\rho_{23} = -1/25$.

For n players, the condition in question becomes

$$\phi(p;n) \equiv (1-\alpha)[(1-\alpha) + \alpha(2p/n)] \leq (2p/n)^{\gamma}.$$

As in the first variation, this condition will hold for all values of p and γ each in some substantial interval enclosing 1/2 when n is not much greater than 3.

Turning to players 2 and 3, let $q_2(3) = q_3(3) = [1 - p(3)]/2 = 1/2 - p/3$. Player 2, on realising 3/2, will make the transfer in question if and only if

$$\psi(q;3) \equiv [q_2(3)]^{\eta_2} + \varepsilon_2(1 - q_2(3)) \geq 1,$$

which has the same form as (7), with $q_2(3) = 1/2 - p/3$ replacing $1/2 - p/2$. It is seen that this condition will be satisfied if ε_2 is close to 1 and p is sufficiently close to 1/2, implying that $q_2(3)$ is close to 1/3. It follows, as before, that there is a full-insurance equilibrium when n is not much larger than 3 and p lies in some interval enclosing 1/2.

When there are just two players in the present setting, differences in the expected values of their endowments, i.e., $p \neq 1/2$, do not pose a strong barrier to trade in the form of a full insurance arrangement. Each can seek out a suitable partner, with autarky as the outside option. When there are three or more players, however, the question of whether the group is stable, or can be formed in the first place, arises in a pressing way. An examination of the two variations yields some insights.

5.1 Beyond pure insurance: group size

In the second variation, player 1 does better with just one partner than two or more. If presently a member of a group of three or more players, he might find a suitable one outside the group and so depart to form a group with her alone. Now, a key feature of both variations above is that the aggregate endowment, $n/2$, is proportional to the number of players. Yet it is quite possible that there are complementarities among players that result in increasing returns to the size of the group. If, for example, players have heterogeneous factor endowments, or engage in certain forms of collective action, then the aggregate output of the composite good available to n players can exceed $n/2$ and so create possibilities that go beyond pure insurance.

When $n = 2$, player 1 obtains an expected utility of p^γ with full insurance. Suppose the aggregate endowment with n players is $k(n)n/2$, where $k(2) = 1$, $k(n) > 1\,\forall n \geq 3$. Then player 1 obtains $p(n)[k(n)n/2]^\gamma = p(2/n)^{1-\gamma}[k(n)]^\gamma$, which exceeds p^γ if and only if $k(n) > [n/2p]^{(1-\gamma)/\gamma}$. It is clear that, *cet. par.*, this is more likely to hold if player 1 is fairly risk tolerant, i.e., γ is close to, but less than, α. Let $p = 1/3$ and $\gamma = 0.9$. Then $k(4)$ must exceed $6^{1/9} = 1.22$ if player 1 is to prefer being in a group of 4.

An interesting kind of such collective action is the voluntary formation of a small group for the purposes of obtaining formal loans, wherein its members are subject to several and joint liability for all the individual loans. Siamwalla *et al.* (1990) describe a successful programme for Thai cultivators. Under this contract, repayment in each period is enforced legally whatever be the members' individual realizations; but as the foregoing argument makes clear, the group will survive if the loans yield sufficiently large augmentations of its members' endowment streams, the alternative being autarky and hence no access to such loans and the gains they yield.

This kind of collective action is not the only means of achieving increasing returns through the formation of a group. A stream of income secured by mutual insurance also provides risk-averse members with an incentive to reallocate their productive endowments so as to obtain more output on average. Suppose that under such an arrangement, player 1's endowment in the first variation becomes the augmented variate

$$\widetilde{A}_{1t} = 1 + \kappa_1 \text{ with probability } p, \text{ and } \widetilde{A}_{1t} = 0 \text{ with probability } 1 - p,$$

where κ_1 exceeds 0 by an amount that depends on the gain ΔU. The corresponding augmentations for the other players, κ_i ($i = 2, 3, \ldots, n$), are similarly defined. Then the formation of the group yields the aggregate endowment $n/2 + \sum_{i=1}^{i=n} \kappa_i$, so that the group will be stable under conditions similar to those derived above, with the difference that the aggregate augmentation is larger when the members, as a group, are more risk-averse.

5.2 Aggregate risk

Aggregate risk has been ruled out in Section 4 and thus far: $\widetilde{R_t} = 0$ in all states of nature. Introducing a common component, let $\widetilde{R_t}$ be an i.i.d. binomial variate, taking the values 0 and $\rho > 0$, with probabilities $1-\pi$ and π, respectively. In the first variation with three players, player 1 obtains the value

$$W_1 = \{\pi[p(3/2+\rho)^\gamma + (1-p)\rho^\gamma] + (1-\pi)p(3/2)^\gamma\} /(1-\alpha)$$

under perpetual autarky. The value under full, perpetual mutual insurance is

$$S_1 = \{\pi[(p \cdot 3/2+\rho)^\gamma] + (1-\pi)(p \cdot 3/2)^\gamma\} /(1-\alpha),$$

the aggregate risk being uninsurable when storage and savings are ruled out. When $\widetilde{R_t} = \rho$, player 1 will make the required transfer if and only if

$$(3/2+\rho)^\gamma - (p \cdot 3/2+\rho)^\gamma \leq \alpha(S_1 - W_1).$$

When $\widetilde{R_t} = 0$, this condition becomes

$$(3/2)^\gamma(1-p^\gamma) \leq \alpha(S_1 - W_1),$$

which is clearly the more stringent of the two, since u is strictly concave. For it is when the common component takes its smallest value and player 1's idiosyncratic draw is favourable that his willingness to make the required transfer is most put to the test. Analogous expressions and the same argument hold for the other players. It is seen that if ρ is sufficiently small, so that autarky does not become relatively rather attractive, then all the foregoing results also hold.

6 Arbitrary Correlation of Endowments

Although full mutual insurance is not always feasible, partial insurance is mostly so. We begin the argument with a simple case. Let the random variables $\widetilde{A}_0, \widetilde{A}_1, \ldots,$ be i.i.d. with $P(\widetilde{A}_0 = 0) = 1-q$ and $P(\widetilde{A}_0 = 1) = q$. Likewise, let $\widetilde{E}_0, \widetilde{E}_1, \ldots,$ be i.i.d. with $P(\widetilde{E}_0 = 0) = 1-r$ and $P(\widetilde{E}_0 = 1) = r$, where $q, r \in (0,1)$. Assume also that the processes $\widetilde{A}$ and $\widetilde{E}$ are independent, and $u' > 0, u'' < 0, v' > 0, v'' < 0, u(0) = v(0) = 0, u(1) = v(1) = 1$.

Full insurance is excluded, for it is impossible to equalize each player's consumption across states of nature. Yet it is possible to provide partial insurance. Put

$$s = P(\widetilde{A}_0 \neq \widetilde{E}_0) = (1-q)r + q(1-r) \text{ and } p = P(\widetilde{A}_0 = 1 \mid \widetilde{A}_0 \neq \widetilde{E}_0) = q(1-r)/s,$$

and note that $p > 0$ and $s > 0$. Let the players consume $a_t = p$ and $e_t = 1-p$, whenever $\widetilde{A}_t \neq \widetilde{E}_t$. This scheme provides partial insurance, with expected utility gains $\Delta U = s \cdot [u(p) - p]$ and $\Delta V = s \cdot [v(1-p) - (1-p)]$, respectively. The (maximal) utility losses whenever a player provides insurance to the other player are $\Delta u = u(1) - u(p)$ and $\Delta v = v(1) - v(1-p)$. With these values, one obtains an analogue of Proposition 1.

When endowments across players are positively correlated, insurance opportunities are less likely to arise; indeed, they may never arise at all. Consider the sequence of i.i.d. random variables $\widetilde{A}_0, \widetilde{A}_1, \ldots$, and $\widetilde{E}_t(\omega) = f(\widetilde{A}_t(\omega))$ for $t \in \mathbb{N}_0$, $\omega \in \Omega$ where $f : [0,1] \to [0,1]$ is strictly increasing.

Example l. $P(\widetilde{A}_0 = 0) = P(\widetilde{A}_0 = 1) = 1/2, f(R) = R$ for $R \in [0,1]$, $\alpha = \varepsilon = 1/2$, $u(c) = v(c) = c^{1/2}$ for $c \in [0,2]$, a case of perfect positive correlation. Here, autarky is an optimal allocation of resources, even intertemporally. Hence, no opportunity for mutual insurance ever arises. □

To establish that even small deviations from perfect positive correlation can reopen the door to insurance, consider

Example 2. $P(\widetilde{A}_0 = 0) = 0.2, P(\widetilde{A}_0 = 0.5) = 0.4, P(\widetilde{A}_0 = 1) = 0.4, f(R) = R^2$ for $R \in [0,1], u(c) = v(c) = 5(c - c^2)/4$ for $c \in [0,2]$. Then $\widetilde{A}_t$ and $\widetilde{E}_t$ are positively correlated, with correlation coefficient 0.985. Since $v'(0.25) = 9/8, u'(0.5) = 1, u'(1) = 3/4$, there is scope for mutual insurance. Let $\delta \in (0, 0.5)$ be sufficiently small and consider the following insurance plan. When $A_t = 0.5$, ALAN transfers the amount $|z_t| = \delta$ to ESTHER. When $A_t = 1$, ALAN receives the amount $z_t = \frac{7}{5}\delta$ from ESTHER. For discount factors sufficiently close to 1, the plan can be obtained as subgame perfect Nash equilibrium outcome, again using trigger strategies. □

Finally, we allow for arbitrary correlation of endowments across players. As a rule, there are mutual insurance opportunities. It is seen from the argument in Example 1 that a necessary condition for them to arise is that the probability that one player enjoys a relatively good draw when the other does not, be positive. We consider a class of models wherein the endowment processes are parametrized as follows.

There is a finite number $\ell \geq 2$ of states $k = 1, 2, \ldots, \ell$. Let $K = \{1, 2, \ldots, \ell\}$. There is a sequence of i.i.d. random variables $\kappa_t : \Omega \to K$, $t \in \mathbb{N}_0$, with $(\Omega, \mathbb{F}, P)$ the underlying probability space. Denote $p(k) = P(\kappa_0 = k)$ and assume $p(k) > 0$ for all $k \in K$. Further let $u : [0, 2] \to \mathbb{R}$ and $v : [0, 2] \to \mathbb{R}$ satisfy $u(0) = v(0) = 0, u(1) = v(1) = 1, u' > 0, u'' < 0, v' > 0, v'' < 0$.

From now on, $K, (\Omega, \mathbb{F}, P), (\kappa_t), u$ and v are fixed. A generic element of $\mathbb{R}^\ell$ is denoted $A = (A(1), \ldots, A(\ell)), B = (B(1), \ldots, B(\ell)), Z = (Z(1), \ldots, Z(\ell))$, etc.

The **extended parameter space** is $S = [0, 1]^\ell \times [0, 1]^\ell \times (0, 1) \times (0, 1)$. Each parameter quadruplet $(A, E, \alpha, \varepsilon) \in S$, with $A = (A(1), \ldots, A(\ell)) \in [0, 1]^\ell, E = (E(1), \ldots, E(\ell)) \in [0, 1]^\ell$, $\alpha, \varepsilon \in (0, 1)$, determines a game $\Gamma = \Gamma(A, E, \alpha, \varepsilon)$. Namely,

- — u and v are the von Neumann-Morgenstern utility functions as in our general model,
- — α and ε are the discount factors,
- — the endowment processes $(\widetilde{A}_t)$ and $(\widetilde{E}_t)$ are given by $\widetilde{A}_t(\omega) = A(\kappa_t(\omega))$ and $\widetilde{E}_t(\omega) = E(\kappa_t(\omega))$ for $t \in \mathbb{N}_0$, $\omega \in \Omega$.

The **endowment parameter space** is $T = [0, 1]^\ell \times [0, 1]^\ell$, with generic elements (A, E). Haller (1992) proves

Proposition 4 *There exists an open and dense subset N^* of T such that for every $(A, E) \in N^*$, one can find sufficiently high discount factors α and ε such that the game $\Gamma(A, E, \alpha, \varepsilon)$ has a subgame perfect trigger strategy equilibrium which both players prefer to autarky.*

First, note that for $A \in [0, 1]^\ell, A(k) = 1$ may hold for some $k \in K$, but need not hold. Similarly for $E \in [0, 1]^\ell$. Second, the restriction $(A, E) \in [0, 1]^\ell \times [0, 1]^\ell$ can be relaxed. Third, the players obtain different draws in some states, which is a basis for trade.

7 Mutual Insurance through Exogamy

We apply the framework and insights to the custom of patrilocal exogamy in rural India. Households i and j reside in villages k and l, respectively. At some point in the past, one of j's daughters married one of i's sons and took up residence in i's household, as custom requires.[7] The associated material transactions, principally her dowry, are ignored for present purposes. The villages are quite some distance apart, but usually belong to the same agroclimatic region.[8]

Following the marriage, the households' endowments are given by the variates

$$\widetilde{R}_{it} = \theta_i \widetilde{R}_t + \psi_i \widetilde{G}_{kt} + \widetilde{r}_{ikt}$$

and

$$\widetilde{R}_{jt} = \theta_j \widetilde{R}_t + \psi_j \widetilde{G}_{lt} + \widetilde{r}_{jlt},$$

respectively, where θ_i and θ_j are their shares of aggregate (regional) risk $\widetilde{R}_t$ and there are village components $\widetilde{G}_{kt}$ and $\widetilde{G}_{lt}$. The common component $\widetilde{R}_t$ is i.i.d., as are the other variates, so that $\widetilde{R}_{it}$ and $\widetilde{R}_{jt}$ are positively, but imperfectly, correlated.[9] Exogamy therefore yields a particular case of the structure in the final part of Section 6. In what follows, we identify generic parameter constellations in the spirit of Proposition 4.

A marriage alliance normally involves households of roughly equal standing: let $\theta_i = \theta_j = \theta$. Suppose all variates are binomially distributed. Let $\Omega_{\theta_i R} = \{\rho^1, \rho^2\}$ and $P(\theta_i \widetilde{R}_t = \rho^2) = \pi_R$, where the superscripts $1, 2$ denote the lower and upper values.[10] Suppose, further, that the villages are similar, so that $\psi_i = \psi_j$. Then $\Omega_{\psi_i G_k} = \Omega_{\psi_j G_l} = \{g^1, g^2\}$ and $P(\psi_i \widetilde{G}_{kt} = g^2) = \pi_G$. The supports of $\widetilde{r}_{ikt}$ and $\widetilde{r}_{jlt}$ are the same, but the probabilities may differ, where $P(\widetilde{r}_{ikt} = r^2) = p$ and $P(\widetilde{r}_{jlt} = r^2) = q$. (The daughter's move changes each family's labour endowment and the members' claims on its common pot.) The assumption that the support of the distribution of an agent's endowment is

[7] See Rosenzweig and Stark (1989) for an empirical investigation of the connection between exogamy and consumption smoothing in some Indian villages.

[8] Karan and Iijima (1993) summarize the distances reported in the Indian literature, as well as describing in detail practices in the four villages they investigated. The average distance separating the two villages in the former studies was commonly about 20 km and varied somewhat by caste. In two of the latter four villages, most respondents' marriages had spans within 15 km, in the other two, 30-35 km. Distancing was notably greater among the respondents' married children.

[9] In this connection, Newbery (1989) cites two empirical estimates. The correlation of measured July rainfall at the opposite ends of ICRISAT's research station was 0.61 (Walker and Jodha, 1982). The station lies in the Deccan plateau and, at 1400 ha, would constitute a large village in its administrative block. (Those 25 villages range in size from 200 to 1700 ha [Indian Village Directory, 2022].) The main monsoon months are July and August. For the two closest rainfall stations in an area of southern India, which are 40 km apart, Seabright (1987) reports a squared correlation coefficient of 0.15 over a 30-year period.

[10] $\Omega_{\theta_i R}$ denotes the set of outcomes for i in the absence of any dealings with j. With the plethora of subscripts i, k, t, the introduction of the superscripts $1, 2$ seems a clearer alternative, no integral exponents being involved in what follows.

the positive interval $[b, 1]$ emerges as important in section 4. Hence, let $\rho^1 > 0, g^1 = r^1 = 0$ and $\rho^2 + g^2 + r^2 = 1$, reflecting the idea that some emergency relief can always be expected from outside the villages themselves, though ρ^1 may be very small. Under autarky, $\mathbb{E}A_{it} = (1-\pi_R)\rho^1+\pi_R\rho^2+\pi_G g^2+pr^2$ and $\mathbb{E}A_{jt} = (1-\pi_R)\rho^1+\pi_R\rho^2+\pi_G g^2+qr^2$ for all t.

If regional agroclimatic conditions are poor ($\theta\widetilde{R}_t = \rho^1$), realizations in which one household has an endowment of just ρ^1 and the other more arise with strictly positive probabilities. A transaction is then most attractive when i obtains $\rho^1+g^2+r^2$ and j just ρ^1, and conversely. The probability of the former outcome, conditional on $\theta\widetilde{R}_t = \rho^1$, is $\pi_G \cdot p \cdot (1 - \pi_G)(1 - q)$; in the converse, it is $\pi_G \cdot q \cdot (1 - \pi_G)(1 - p)$. Analogous expressions arise when regional conditions are good: i obtains 1 and j obtains ρ^2, and conversely, are both outcomes in which a transaction is especially attractive, though not certain to occur.

Since all realizations are assumed to be known to both parties before the decision whether to transact is made, the transaction can be made conditional on the regional draw R_t. Proposition 4 establishes that, generically, sufficiently high discount factors ensure the existence of a transaction that both households prefer to autarky. That result does not, however, delineate the generic set of parameters. The current situation can be modeled with $\ell = 32$ or fewer. The probabilities p, q and π_G are additional parameters. We shall identify parameter constellations that are conducive to beneficial mutual insurance, provided the discount factors are sufficiently large.

Consider $\theta\widetilde{R}_t = \rho^1$ and $\widetilde{G}_{kt} \neq \widetilde{G}_{lt}$. If i obtains $\rho^1 + g^2 + r^2$ and j obtains ρ^1, let i transfer $(1 - p)(g^2 + r^2)$ to j; in the converse case, let j transfer $(1 - q)(g^2 + r^2)$ to i. In all other outcomes, including $\widetilde{G}_{kt} = \widetilde{G}_{lt}$, let no transfers take place, by mutual agreement. In Appendix D, it is shown that, in each period, i will gain from these transfers relative to autarky if and only if

$$\begin{aligned} p(1-q)u[\rho^1 + p(g^2 + r^2)] + (1-p)qu[\rho^1 + (1-q)(g^2 + r^2)] \\ > p(1-q)u(\rho^1 + g^2 + r^2) + (1-p)u(\rho^1). \end{aligned} \tag{8}$$

Likewise, j will gain if and only if

$$\begin{aligned} q(1-p)v[\rho^1 + q(g^2 + r^2)] + (1-q)pv[\rho^1 + (1-p)(g^2 + r^2)] \\ > q(1-p)v(\rho^1 + g^2 + r^2) + (1-q)v(\rho^1). \end{aligned} \tag{9}$$

These results accord with intuition, the transfers taking place only when, given $\widetilde{G}_{kt} \neq \widetilde{G}_{lt}$, the individual draws match their respective village-level ones. If its realization is favourable, household i gives up $\Delta u = u(\rho^1 + g^2 + r^2) - u[\rho^1 + p(g^2 + r^2)]$ in exchange for the stream $\{\Delta U\}$. Analogous expressions hold for j when so favoured.

The arrangement is more attractive to the household with the lower probability of the good idiosyncratic draw r^2; for what is transferred to the partner is then smaller than what is received when things turn out badly. In the symmetric case $p = q$, conditions (8) and (9) specialise to

$$u[\rho^1 + p(g^2 + r^2)] + u[\rho^1 + (1-p)(g^2 + r^2)] > u(\rho^1 + g^2 + r^2) + u(\rho^1)/p,$$

and

$$v[\rho^1 + p(g^2 + r^2)] + v[\rho^1 + (1-p)(g^2 + r^2)] > v(\rho^1 + g^2 + r^2) + v(\rho^1)/p,$$

respectively. With the normalisation $u(\rho^1) = v(\rho^1) = 0$, these hold for all $p \in (0,1)$ in virtue of the strict concavity of u and v. It follows by continuity that the arrangement will be mutually advantageous for all pairs (p, q) such that p and q are sufficiently close to each other, which is likely to hold when marriage alliances always involve both caste and economic standing.

A similar argument holds when $\theta \widetilde{R}_t = \rho^2$, $\widetilde{G}_{kt} \neq \widetilde{G}_{lt}$ and the individual draws match their respective village-level ones, so that one household obtains 1 and the other ρ^2. With the said normalisation, conditions (8) and (9) become, respectively,

$$p(1-q)u[(1-p)\rho^2 + p] + (1-p)qu[(1-q) + q\rho^2] > p(1-q)u(1) + (1-p)u(\rho^2)$$

and

$$q(1-p)v[(1-q)\rho^2 + q] + (1-q)pv[(1-p) + p\rho^2] > q(1-p)v(1) + (1-q)v(\rho^2).$$

In the symmetric case, these reduce to

$$u[(1-p)\rho^2 + p] + u[(1-p) + p\rho^2] > u(1) + u(\rho^2)/p,$$

and

$$v[(1-p)\rho^2 + p] + v[(1-p) + p\rho^2] > v(1) + v(\rho^2)/p,$$

which hold for all ρ^2 sufficiently close to ρ^1.

The findings of Townsend (1994) and Chiappori et al. (2014) that, within a village, idiosyncratic risks are largely eliminated still leave exogamy with the potential advantage of mitigating those at the village level. Household i's endowment specialises, in this idealized case, to $\widetilde{R}_{it} = \theta_i \widetilde{R}_t + \psi_i \widetilde{G}_{kt} + pr^2$ and similarly for j, where their respective neighbours are likely to be in the dark about what has happened in the other village. When $\widetilde{G}_{kt} \neq \widetilde{G}_{lt}$, a transfer of $(1 - \pi_G)g^2$ from the household in the village with the draw g^2 may be attractive. When $\theta \widetilde{R}_t = \rho^1$ and $\widetilde{G}_{kt} = g^2$, we have the condition

$$u[\rho^1 + (1 - \pi_G)g^2 + pr^2] + u(\rho^1 + \pi_G g^2 + pr^2) > u(\rho^1 + g^2 + pr^2),$$

which always holds. Likewise, for j,

$$v[\rho^1 + (1 - \pi_G)g^2 + qr^2] + v(\rho^1 + \pi_G g^2 + qr^2) > v(\rho^1 + g^2 + qr^2).$$

8 Concluding Remarks

We have found that bilateral mutual insurance outcomes may not reflect differences in the partners' preferences for risk bearing: differences that arise across states of nature ex post can suffice. If their preferences do differ, then their consumption patterns can have identical co-movements with aggregate consumption, in contrast to some of the empirical findings in the literature. This can be explained by the co-existence of various ways of risk sharing. Some households may resort to mutual insurance in combination with other measures.

Under standard convexity assumptions, there can be no gains from trade if all agents have the same economic characteristics, where it must be noted that this condition is violated if, among otherwise identical agents, some agents do well and others badly in certain states of nature, and conversely in certain other states. This conclusion applies, in particular, to insurance in village economies, especially when soils, elevation and access to irrigation vary over plots, and so induce differences in the crops grown and how they are cultivated. Differences in these and other household characteristics, including those arising from differences across states, are conducive to insurance within the village. Would greater differences strengthen the case for insurance, in particular, bilateral mutual insurance? Recall that mutual insurance requires trust in, and familiarity with, partners and is, therefore, usually restricted to partners linked by spatial proximity, kinship, friendship, social or professional ties. Yet within these restrictions, a household may still have a choice of partners. Consider a rather affluent household. It can provide insurance to an extremely poor household, but would receive very little in return, so that it might prefer to partner with another affluent household, or even forego mutual insurance altogether and rely on collateralized commercial loans. Thus, while differences in household characteristics are conducive to insurance within the village, greater differences need not make the emergence of bilateral mutual insurance more likely. Marriage alliances of socially and economically similar families belonging to widely separated villages confirm the potential advantages of geographical diversification within such groups.

Furthermore, note that inter-temporal efficiency need not obtain, even if intra-temporal efficiency is achieved via mutual insurance or otherwise.

Finally, new issues arise and are addressed in Section 5 when groups of three or more households engage in mutual insurance. Since endowments within such a group are not necessarily very strongly negatively correlated, the stability of a large group becomes doubtful if the aggregate endowment of the group is proportional to group size. This problem is mitigated when the aggregate endowment increases proportionally more than group size and the group is small.

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Appendix A: Co-movement of Household and Aggregate Consumption

Suppose that there are exactly two different risk preferences with equal proportions in the population, with half of the households risk neutral and the other half risk averse. Let household i be risk-neutral, household j be risk averse, and both $\theta_i \widetilde{R}$ and $\theta_j \widetilde{R}$ be i.i.d. and uniformly distributed on $[-1, 1]$.[11] Denote $\widehat{R} = \theta_i \widetilde{R} = \theta_j \widetilde{R}$. First, consider $\widehat{C}_i = (1+\epsilon)\widehat{R}$ and $\widehat{C}_j = (1-\epsilon)\widehat{R}$ for some $\epsilon \in (0,1)$. The risk neutral household i is indifferent between $\widehat{R}$ and $\widehat{C}_i$, since both have zero mean. The risk averse household j, with von Neumann-Morgenstern utility function u satisfying $u' > 0$ and $u'' < 0$, prefers $\widehat{C}_j$ to $\widehat{R}$. This claim can be proved in two ways.

First, after suitable normalizations to ensure that u is bounded on the interval $[-1, 1]$, we can differentiate $\mathbb{E}[u(\widehat{C}_i)]$ w.r.t. ϵ:

$$\frac{d}{d\epsilon}\left[\int_{-1}^{0} u((1-\epsilon)x)dx + \int_{0}^{1} u((1-\epsilon)x)dx\right] =$$
$$-\left[\int_{-1}^{0} u'((1-\epsilon)x)xdx + \int_{0}^{1} u'((1-\epsilon)x)xdx\right] > 0,$$

for $u'((1-\epsilon)(-x)) > u'((1-\epsilon)x)\ \forall x > 0$.

Second, the assertion follows by applying Proposition 6.D.2 in Mas-Colell et al. (1995).

Now suppose that household j pays i a fixed risk premium $d > 0$ so that they end up with

$$\widetilde{C}_i = \widehat{C}_i + d = (1+\epsilon)\widehat{R} + d,\ \widetilde{C}_j = \widehat{C}_j - d = (1-\epsilon)\widehat{R} - d.$$

Then for sufficiently small d, household i prefers $\widetilde{C}_i$ to $\widehat{R}$, household j prefers $\widetilde{C}_j$ to $\widehat{R}$, and both $\widetilde{C}_i$ and $\widetilde{C}_j$ are perfectly correlated with $\widehat{R}$.

Next suppose that instead of paying a fixed risk premium, household j pays i extra compensation in good states. Let $\delta \in (0, 1-\epsilon)$ and consider $\widetilde{C}_i$ and $\widetilde{C}_j$ given by

$$\widetilde{C}_i = \begin{cases} (1+\epsilon)\widehat{R} & \text{if } \widehat{R} \leq 0, \\ (1+\epsilon+\delta)\widehat{R} & \text{if } \widehat{R} > 0; \end{cases}$$

$$\widetilde{C}_j = \begin{cases} (1-\epsilon)\widehat{R} & \text{if } \widehat{R} \leq 0, \\ (1-\epsilon-\delta)\widehat{R} & \text{if } \widehat{R} > 0. \end{cases}$$

[11]That the means are zero simplifies computations; but the conclusion would not change if we assumed a non-negative support.

Then household i prefers $\widetilde{C}_i$ to $\widehat{R}$ and for sufficiently small δ, household j prefers $\widetilde{C}_j$ to $\widehat{R}$. $\widehat{R}$ has mean $\mu = 0$ and variance $\sigma^2 = 1/3$. Hence the covariance of $\widetilde{C}_i$ and $\widehat{R}$ is

$$COV_i = \mathbb{E}[\widetilde{C}_i \widehat{R}] = \frac{1}{3}(1+\epsilon) + \frac{1}{6}\delta$$

and the covariance of $\widetilde{C}_j$ and $\widehat{R}$ is

$$COV_j = \mathbb{E}[\widetilde{C}_j \widehat{R}] = \frac{1}{3}(1-\epsilon) - \frac{1}{6}\delta.$$

$\widetilde{C}_i$ has mean $\mu_i = \dfrac{1}{4}\delta$ and variance

$$\begin{aligned}
\sigma_i^2 &= \tfrac{1}{2}\int_{-1}^{0}((1+\epsilon)x - \tfrac{1}{4}\delta)^2 dx + \tfrac{1}{2}\int_{0}^{1}((1+\epsilon+\delta)x - \tfrac{1}{4}\delta)^2 dx \\
&= \tfrac{1}{2}[\tfrac{1}{3}(1+\epsilon)^2 + \tfrac{1}{4}(1+\epsilon)\delta + \tfrac{1}{16}\delta^2 + \tfrac{1}{3}(1+\epsilon+\delta)^2 - \tfrac{1}{4}(1+\epsilon+\delta)\delta + \tfrac{1}{16}\delta^2] \\
&= \tfrac{1}{2}[\tfrac{2}{3}(1+\epsilon)^2 + \tfrac{2}{3}(1+\epsilon)\delta + \tfrac{1}{3}\delta^2 - \tfrac{1}{4}\delta^2 + \tfrac{1}{8}\delta^2] \\
&= \tfrac{1}{2}[\tfrac{2}{3}(1+\epsilon)^2 + \tfrac{2}{3}(1+\epsilon)\delta + \tfrac{5}{24}\delta^2].
\end{aligned}$$

$\widetilde{C}_j$ has mean $\mu_j = -\dfrac{1}{4}\delta$ and variance

$$\begin{aligned}
\sigma_j^2 &= \tfrac{1}{2}\int_{-1}^{0}((1-\epsilon)x + \tfrac{1}{4}\delta)^2 dx + \tfrac{1}{2}\int_{0}^{1}((1-\epsilon-\delta)x + \tfrac{1}{4}\delta)^2 dx \\
&= \tfrac{1}{2}[\tfrac{1}{3}(1-\epsilon)^2 - \tfrac{1}{4}(1-\epsilon)\delta + \tfrac{1}{16}\delta^2 + \tfrac{1}{3}(1-\epsilon-\delta)^2 + \tfrac{1}{4}(1-\epsilon-\delta)\delta + \tfrac{1}{16}\delta^2] \\
&= \tfrac{1}{2}[\tfrac{2}{3}(1-\epsilon)^2 - \tfrac{2}{3}(1-\epsilon)\delta + \tfrac{1}{3}\delta^2 - \tfrac{1}{4}\delta^2 + \tfrac{1}{8}\delta^2] \\
&= \tfrac{1}{2}[\tfrac{2}{3}(1-\epsilon)^2 - \tfrac{2}{3}(1-\epsilon)\delta + \tfrac{5}{24}\delta^2].
\end{aligned}$$

The correlation coefficients ρ_i for $\widetilde{C}_i$ and $\widehat{R}$ and ρ_j for $\widetilde{C}_j$ and $\widehat{R}$ are given by

$$\rho_i = \frac{COV_i}{\sigma_i \cdot \sigma}, \ \rho_j = \frac{COV_j}{\sigma_j \cdot \sigma}.$$

Hence $\rho_i > \rho_j$ if and only if $[COV_i]^2 \cdot \sigma_j^2 > [COV_j]^2 \cdot \sigma_i^2$ or

$$[2(1+\epsilon)+\delta]^2 \cdot \left[\frac{2}{3}(1-\epsilon)^2 - \frac{2}{3}(1-\epsilon)\delta + \frac{5}{24}\delta^2\right] > [2(1-\epsilon)-\delta]^2 \cdot \left[\frac{2}{3}(1+\epsilon)^2 + \frac{2}{3}(1+\epsilon)\delta + \frac{5}{24}\delta^2\right].$$

Disregarding higher order terms of δ caused by multiplication with δ^2 in each bracket, the left-hand side becomes

$$\begin{aligned}
&[4(1+\epsilon)^2 + 4(1+\epsilon)\delta] \cdot [\tfrac{2}{3}(1-\epsilon)^2 - \tfrac{2}{3}(1-\epsilon)\delta] \\
= \ &\tfrac{8}{3}(1-\epsilon^2)[(1-\epsilon^2) - (1+\epsilon)\delta + (1-\epsilon)\delta - \delta^2].
\end{aligned}$$

Doing the same on the right-hand side yields

$$\begin{aligned} & [4(1-\epsilon)^2 - 4(1-\epsilon)\delta] \cdot [\tfrac{2}{3}(1+\epsilon)^2 + \tfrac{2}{3}(1+\epsilon)\delta] \\ = \; & \tfrac{8}{3}(1-\epsilon^2)[(1-\epsilon^2) + (1-\epsilon)\delta - (1+\epsilon)\delta - \delta^2]. \end{aligned}$$

Hence so far, both sides are identical. Collecting the remaining terms on the left-hand side yields

$$\frac{2}{3}(1-\epsilon)^2\delta^2 - \frac{2}{3}(1-\epsilon)\delta^3 + \frac{5}{24}\delta^4 + \frac{5}{6}(1+\epsilon)^2\delta^2 + \frac{5}{6}(1+\epsilon)\delta^3.$$

Collecting the remaining terms on the right-hand side yields

$$\frac{2}{3}(1+\epsilon)^2\delta^2 + \frac{2}{3}(1+\epsilon)\delta^3 + \frac{5}{24}\delta^4 + \frac{5}{6}(1-\epsilon)^2\delta^2 - \frac{5}{6}(1-\epsilon)\delta^3.$$

The left-hand side exceeds the right-hand side if

$$\frac{1}{6}(1+\epsilon)^2\delta^2 + \frac{1}{6}(1+\epsilon)\delta^3 > \frac{1}{6}(1-\epsilon)^2\delta^2 - \frac{1}{6}(1-\epsilon)\delta^3,$$

which is the case. This shows $\rho_i > \rho_j$.

Finally suppose that instead of being overcompensated in good states, household i provides less insurance in bad states. Then similar calculations show $\rho_i > \rho_j$ again.

Appendix B: The Lower Bound of u and Relative Risk Aversion

Suppose there is a $c_1 > 0$ such that $\rho_u(c) \equiv -u''c/u' \geq 1 \, \forall c < c_1$. In the limiting case, $\rho_u = 1 \, \forall c$, which implies $u = \ln c + 1$ and $u' = 1/c$. Otherwise, and with reference to that special case, normalise u such that $u(1) = u'(1) = 1$ and suppose $\rho_u(c) > 1 \, \forall c \in [c_0, 1]$, that is, $-u''c > u'$. Integrating (the l.h.s. by parts) over the positive interval $[c_0, 1]$, we have $-u'(1) + u'(c_0)c_0 + u(1) - u(c_0) > u(1) - u(c_0)$, or $u'(c_0) > 1/c_0$, where $1/c_0$ is the slope of $\ln c + 1$ for $c = c_0$. The latter inequality holds for all $c_0 \in (0, 1)$, so that u is everywhere steeper than $\ln c + 1$ over the interval $(0, 1)$, and since $\ln c + 1$ is unbounded from below, u must be likewise. The condition $\rho_u(c) \equiv -u''c/u' \geq 1 \, \forall c \in (0, 1)$ therefore implies 'strong' risk aversion. A special case is the sub-family $u = (1 + 1/\gamma) - 1/\gamma c^\gamma$, where $\gamma > 0$ and $\rho_u = 1 + \gamma$.

Suppose, on the contrary, that there is a $c_1 > 0$ such that $1 > \rho_u(c) \geq \rho_u(c_1) \, \forall c < c_1$. Then, with $c_1 = 1$, we have $-u'(1) + c_0 u'(c_0) \geq (\rho_u(1) - 1)[u(1) - u(c_0)]$. The normalisations $u(1) = u'(1) = 1$ yield $u(c_0) \geq [1 - c_0 u'(c_0)]/[1 - \rho_u(1)]$. It follows that

$u(0)$ is bounded if and only if the $\lim_{c\to 0} cu'$ exists. The existence of the said limit and $1 > \rho_u(c) \geq \rho_u(1)\, \forall c \in (0,1)$ are therefore sufficient conditions for weak risk aversion.

Appendix C: The Transfer Decision With Two Players

Weak risk aversion. For player A, condition (1) specialises to

$$1 - p^\gamma \leq \frac{\alpha}{1-\alpha} \cdot (p^\gamma - p),$$

a rearrangement of which yields

$$\phi(p; 2) \equiv p^\gamma + \alpha(1-p) \geq 1. \tag{10}$$

Since $\alpha < 1$, this condition is violated if A is risk neutral ($\gamma = 1$) and $p < 1$. It holds, trivially, with equality $\forall \gamma \in (0,1]$ if A faces no risk ($p = 1$). In what follows, A and E are risk averse, with $\gamma < \alpha$, $\eta < \epsilon$ and $p \in (0,1)$.

The function ϕ is continuous and strictly concave $\forall p \in [0,1], \gamma \in (0,1)$; $\phi(0;2) = \alpha$, $\phi(1;2) = 1$; and $\phi'(p;2) = \gamma p^{\gamma-1} - \alpha > 0\, \forall p \in (0,1)$ if $\gamma \geq \alpha$, which implies $\phi < 1\, \forall p \in (0,1)$.

Noting that $\phi'(1;2) = \gamma - \alpha$, it is seen that the restriction $\gamma < \alpha$ implies $\phi' < 0$ for all p sufficiently close to 1. Since ϕ is strictly concave and $\phi'(p;2) \to \infty$ as $p \to 0$ and α is very close to 1, $\gamma < \alpha$ implies $\phi(p;2) > 1\, \forall p \in (p', 1)$, where p' is close to 0. The function ϕ attains a maximum for $p = (\gamma/\alpha)^{1/(1-\gamma)}$. Now, $\gamma^{1/(1-\gamma)} = \exp[(1-\gamma)^{-1} \ln \gamma]$. Since $-1 > (1-\gamma)^{-1} \ln \gamma > -1/\gamma$, $\gamma^{1/(1-\gamma)} \in (e^{-1/\gamma}, e^{-1})$. It follows from the fact that α is very close to 1 and $\phi(1;2) = 1$ that $\phi(p';2)$ exceeds 1 by quite some margin. For example, $\gamma = 1/2$ yields $(\gamma/\alpha)^{1/(1-\gamma)} = 1/4\alpha^2$. Very conservatively, put $\alpha = 0.95$. Then $p' = 0.277$ and $\phi(p';2) = 1.239$.

For player E, put $q = 1 - p$. Then condition (2) specialises to

$$1 - q^\eta \leq \frac{\varepsilon}{1-\varepsilon} \cdot (q^\eta - q),$$

a rearrangement of which yields

$$\psi(q; 2) \equiv q^\eta + \varepsilon(1-q) \geq 1. \tag{11}$$

It is seen that $\psi(q;2) > 1\, \forall q \in (q', 1)$, where $q' (\neq 1 - p')$ is close to 0, and that $\psi(q';2)$ exceeds 1 by quite some margin. It then follows there is some extensive interval $(\underline{p}, \bar{p})$

enclosing $p = 1/2$ such that conditions (10) and (11) both hold as strict inequalities for all p in that interval, thereby opening the possibility of a subgame perfect trigger-strategy equilibrium with additional players.

Strong risk aversion. For player A, condition (1) specialises to

$$-1 + \frac{1}{[p + (1-p)b]^\gamma} \leq \frac{\alpha}{1-\alpha} \cdot \left(\frac{1 - p(1-b^\gamma)}{b^\gamma} - \frac{1}{[p+(1-p)b]^\gamma} \right),$$

a rearrangement of which yields

$$[(p/b) + (1-p)]^\gamma [b^\gamma + (1-b^\gamma)\alpha(1-p)] \geq 1, \quad b \in (0,1). \tag{12}$$

This condition holds for all b sufficiently close to zero, whereby $b^\gamma + (1-b^\gamma)\alpha(1-p) \in (\alpha(1-p), 1)$.

For player E, put $q = 1 - p$. The required corresponding condition is

$$[(1-q)/b + q]^\eta [b^\eta + (1-b^\eta)\varepsilon q] \geq 1, \quad b \in (0,1), \tag{13}$$

which holds for all b sufficiently close to 0. Thus, the possibility of a subgame perfect trigger-strategy equilibrium with additional players also arises here, with b chosen such that the l.h.s. is sufficiently greater than 1.

Appendix D: Exogamy

The transfer yields i the expected utility (recall that $g^1 = r^1 = 0$)

$$\begin{aligned}\pi_G(1-\pi_G)\{pqu(\rho^1 + g^2 + r^2) + p(1-q)u[\rho^1 + p(g^2+r^2)] + (1-p)u(\rho^1 + g^2)\} \\ +(1-\pi_G)\pi_G\{pu(\rho^1 + r^2) + (1-p)qu[\rho^1 + (1-q)(g^2+r^2)]\} \\ +\pi_G^2[pu(\rho^1 + g^2 + r^2) + (1-p)u(\rho^1 + g^2)] + (1-\pi_G)^2[pu(\rho^1 + r^2) + (1-p)u(\rho^1)].\end{aligned}$$

Autarky yields

$$\pi_G pu(\rho^1 + g^2 + r^2) + \pi_G(1-p)u(\rho^1 + g^2) + (1-\pi_G)[pu(\rho^1 + r^2) + (1-p)u(\rho^1)].$$

Hence, i gains

$$\begin{aligned}\Delta U = \pi_G(1-\pi_G)\{pqu(\rho^1 + g^2 + r^2) + p(1-q)u[\rho^1 + p(g^2+r^2)] \\ +(1-p)qu[\rho^1 + (1-q)(g^2+r^2)]\} - \pi_G(1-\pi_G)[pu(\rho^1 + g^2 + r^2) + (1-p)u(\rho^1)].\end{aligned}$$

Likewise

$$\begin{aligned}\Delta V = \pi_G(1-\pi_G)\{pqv(\rho^1 + g^2 + r^2) + q(1-p)v[\rho^1 + q(g^2+r^2)] \\ +(1-q)pv[\rho^1 + (1-p)(g^2+r^2)]\} - \pi_G(1-\pi_G)[qv(\rho^1 + g^2 + r^2) + (1-q)v(\rho^1)].\end{aligned}$$