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ABSTRACT

Spousal Labor Supply: Decoupling Gender Norms and Earning Status*

Many household labor supply models divide couples by sex and identify separate male and female labor supply parameters. However, institutional factors in the labor market suggest that men are more likely to be primary earners in their household, meaning that intra-household gender gaps in labor supply may reflect both gender norms and earning status. I use a novel identification approach to disentangle the role of gender norms in intra-household labor supply by estimating collective labor supply models for different- and same-sex married couples. Among childless couples, I present point estimates and construct unified bounds showing that gender norms significantly increase the weight placed on women's utility by 1.1–5.1%, leading to lower labor supply. A back-of-the-envelope calculation suggests that the effect of gender norms on married, childless couples' labor supply is equivalent to a substantial widening of the gender

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1 Introduction

Many models of household labor supply explicitly or implicitly incorporate gender norms. Explicit incorporations may include secondary mover or "husband always works" assumptions and are often justified by historical labor supply trends or because it makes the empirical analysis tractable. Implicit incorporation may include set-ups in which the household is modeled as including two individuals and researchers define them by sex rather than by another characteristic. However, institutional factors in the labor market, such as the gender wage gap and traditional gender norms surrounding labor supply, suggest that husbands are more likely to be primary earners in their households, meaning that husbands' labor supply parameters, relative to their wives', may reflect both relative gender norms and earning status in the household. It is therefore unclear to what extent gender norms influence labor supply within couples separately from each spouse's earning status in the household.[1]

Disentangling the role of gender norms in household labor supply can not only illuminate the extent to which gender inequality drives intra-household inequality and economic outcomes, but can also help inform policy and expectations about labor supply elasticities. For example, if observed differences in men's and women's labor supply are entirely attributable to gender norms then social interventions aimed at adjusting gender norms may be well suited to improve welfare (e.g., as in Bursztyn, González, and Yanagizawa-Drott 2020). On the other hand, if observed labor supply differences are not affected by gender norms, then policies aimed at addressing institutional inequalities may be particularly effective (e.g., as in Blau and Kahn 2007).

Several strands of the literature have taken different approaches to identify the effect of gender norms on intra-household labor supply. Bertrand, Kamenica, and Pan (2015), Hermle, Herold, and Hildebrand (2023), and Hancock, Lafortune, and Low (2024) explore the male breadwinner norm as a particular type of gender norm in which a husband is expected to earn more than their wife. Bursztyn, Fujiwara, and Pallais (2017) and Bursztyn, González, and Yanagizawa-Drott (2020) uti-

1. Bartels and Shupe (2022) conclude that earning status in the household, rather than sex, is a more influential driver of responses to work incentives, and Baldwin, Allgrunn, and Ring (2011) suggest that the traditional male-female division in household labor supply has become less useful over time.

lize experiments to identify gender norms surrounding labor supply and their interactions with the marriage market. Cubel et al. (2016) and Flinn, Todd, and Zhang (2018) estimate the effects of personality traits on labor supply, concluding that gender differences in personality traits contribute to the gender wage gap. Finally, another approach taken by Siminski and Yetsenga (2022) and Hancock, Lafortune, and Low (2024), and one that is most closely connected to this paper, compares labor supply of different- and same-sex couples as a way of isolating and identifying the role of gender norms in intra-household labor supply.

In this paper, I estimate collective labor supply models for childless different- and same-sex married couples, accounting for non-participation and non-linear taxation, to quantify the role of gender norms in intra-household labor supply as distinct from the role of earning status within the household.[2] Rather than estimating the sharing rule, which is common when estimating collective models, I recover and estimate the relative weight on the utility functions in the household's social welfare function.[3] Identification of the effect of gender norms comes from the assumption that labor supply differences between spouses, which are driven by the relative utility weights, necessarily reflect relative gender norms and earning status between different-sex spouses but reflect only relative earning status between same-sex spouses. I present two sets of identifying assumptions that allow me to identify either point estimates or unified bounds of the effect of gender norms on labor supply of wives in different-sex couples.

I use the 2012–2019 American Community Surveys to construct a sample of childless different-sex married couples and childless, male same-sex married couples in which both spouses are between 25 and 60 years old.[4] The 2012 American Community Survey is the first of the U.S. Census Bureau surveys to explicitly identify same-sex married couples in the data, whereas prior Census Bureau surveys suffered from substantial measurement error that made it difficult to reliably iden-

2. Oreffice (2011) also estimates collective labor supply models for same-sex couples using the 2000 U.S. decennial census. However, that context pre-dates any legal access to same-sex marriage in the U.S., meaning the comparison of same-sex cohabiting partners' labor supply parameters to different-sex married spouses' parameters does not as cleanly identify the role of gender norms. To the best of my knowledge, Moreau and Donni (2002) and Bloemen (2010) are the only others to estimate a collective labor supply model allowing for both non-participation and non-linear taxation, and did so using French data from 1994 and Dutch data from 1990-2001, respectively.

3. This parameter is sometimes called the Pareto weight in other studies and can also be interpreted as wives' bargaining power over labor supply.

4. Using only male same-sex couples holds the sex of the primary earner constant across different- and same-sex couples, therefore allowing clearer identification of the effect of gender norms on relative utility weights since only the sex of the secondary earner varies. I discuss identification in more detail in Section 3.4.

tify same-sex married couples (Black et al. 2007; Gates and Steinberger 2010). I divide different-sex couples by sex, as is common in this literature, and use a machine learning LASSO approach to divide same-sex couples by predicted earning status in the household.

I estimate precise equality between the utility weights of male same-sex spouses, suggesting that earning status in the couple does not significantly affect intra-household gender gaps in labor supply. In contrast, my utility weight estimates for different-sex couples are statistically different from and larger than those for same-sex couples, suggesting that gender norms do significantly influence intra-household labor supply within different-sex couples.[5] Under the first set of identifying assumptions, I estimate that gender norms significantly increase the relative utility weight by 2.8–3.3% for working wives in different-sex couples and 19.2% for non-working wives in different-sex couples. I also show that the fact that working wives' relative utility weight is 14.7–16.9% of non-working wives' relative utility weight translates to a roughly equal 14.9–20.7% reduction in hours worked among working wives relative to working husbands, which supports the conclusion that gender norms can explain much of the intra-household gender labor supply gap through the collective model framework.[6] Under the second set of identifying assumptions, I construct unified bounds of the effect of gender norms and find that gender norms increase wives' relative utility weight by 1.1–5.1%. A back-of-the-envelope calculation suggests that the effect of gender norms on married, childless couples' labor supply is equivalent to a substantial widening of the gender wage gap by 72.0% for dual-earning couples or 111.0–219.1% for single-earner couples.

The substantial difference in relative utility weights between working and non-working wives suggests that gender norms also meaningfully interact with earning status in the couple. Indeed, other researchers have found that increases in women's relative potential wage rates lead to increased household consumption of likely female goods (Ahn and Koh 2024), decreased domestic violence against women (Aizer 2010), and fewer marriages but increased spousal quality condi-

5. Appendix A demonstrates that this result is not surprising by detailing calculations from Chiappori, Fortin, and Lacroix (2002), Moreau and Donni (2002), and Bloemen (2010) showing that the implied weight on wives' utility in these studies are also greater than one (i.e., that wives have greater weight placed on their utility relative to their husbands).

6. The traditional collective model that I use in this paper, introduced by Chiappori (1988, 1992) and expanded by Chiappori, Fortin, and Lacroix (2002) and Donni (2003), assumes a disutility from working, meaning that a larger relative utility weight for wives leads to lower labor supply. This mechanism is also consistent with Pollak (2005) who uses a Nash bargaining model instead of a collective model.

tional on marrying (Shenhav 2021), all of which are consistent with this mechanism.[7] Traditional gender norms likely discourage women from working, and my estimates imply that when these traditional norms are challenged then gender norms exert a much smaller increase in their relative utility weight. However, when these traditional norms are satisfied, so that the husband works and the wife does not, then gender norms exert a much larger positive effect on wives' relative utility weight. My results also suggest that past studies that have assumed that bargaining power is divided by sex in different-sex couples exhibit estimates of bargaining power or sharing rule parameters that are primarily driven by gender norms.

My results suggest that social interventions aimed at adjusting gender norms may be well suited to improve welfare relative to policies aimed at addressing institutional inequalities, such as the gender wage gap. This policy conclusion is consistent with Flinn, Todd, and Zhang (2018, 2024), who conclude that equalizing men's and women's personality characteristics does more to close the male-female earnings gap than equalizing market valuations of characteristics.

2 The Collective Labor Supply Model

In this paper, I estimate the collective model of labor supply presented by Donni (2003), which extends of the models from Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) to allow for non-participation and non-linear budget constraints due to taxation. The collective labor supply model is empirically useful because, by first specifying a functional form for the spouses' unrestricted labor supply functions, it is possible identify the Marshallian labor supply functions, the indirect utility functions, and the relative utility weight for each spouse's utility, as demonstrated below. In this section I provide a broad outline the collective model and reproduce the main propositions from Chiappori (1988, 1992), Chiappori, Fortin, and Lacroix (2002), and Donni (2003) where necessary.

There are two individuals in the household indexed by i ($i = 1,2$), with vectors of preference

7. Shenhav (2021) finds that increases in women's relative potential wage rate increases married and single women's hours of work, part of which may be due to changes in marital composition. I focus only on married women's labor supply in this paper and do not consider marriage or divorce decisions.

factors $\mathbf{z}$, labor supplies L^i, gross hourly wages of w_i, household non-labor income of y, and aggregate Hicksian consumption of C^i. Assume that the price of consumption is normalized to one and the total time available to each individual is normalized one, so that $1-L^i$, with $0 \leq L^i \leq 1$, denotes individual i's leisure.

Donni (2003) makes the following two assumptions:

Assumption 1. *Each household member is characterized by specific utility functions of the form $u^i(1-L^i, C^i)$. These functions are both strongly concave, infinitely differentiable, and strictly increase in all their arguments on $\mathbb{R}^3_{++}$, with $\lim_{C^i \to 0} u^i(1-L^i, C^i) = \lim_{L^i \to 1} u^i(1-L^i, C^i) = -\infty$.*

Assumption 2. *The outcome of the decision process is Pareto efficient.*

Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) also assume Pareto efficiency, and it forms the foundation of the collective approach.

Under assumptions 1 and 2, there exists a relative utility weight, μ, such that household behavior is a solution to the problem:

$$\begin{aligned} \max_{L^1, L^2, C^1, C^2} \quad & u^1(1-L^1, C^1, \mathbf{z}) + \mu u^2(1-L^2, C^2, \mathbf{z}) \\ \text{subject to} \quad & \delta : h(L^1, L^2; w_1, w_2, y) \geq C^1 + C^2 \\ & 0 \leq L^1 \leq 1, \quad 0 \leq L^2 \leq 1, \quad C^1 \geq 0, \quad C^2 \geq 0, \end{aligned} \tag{$\bar{P}$}$$

where $h(\cdot)$ is infinitely differentiable, increasing in all its arguments, and concave in L^1 and L^2.

Note that Chiappori (1988, 1992) and Chiappori, Fortin, and Lacroix (2002) provide the canonical results of the collective model under linear budget constraints and assumptions 1 and 2 when both spouses work (i.e., ignoring non-participation).[8] A key result from Chiappori (1988) is that the household problem $\bar{P}$ is equivalent to individual maximization problems in which the spouses split household non-labor income according to a sharing rule and then, conditional on the sharing rule, maximize their individual utilities subject to their relevant budget constraints. Chiappori (1992) builds upon this framework and provides testable restrictions on labor supply functions

8. Using the notation of problem $\bar{P}$, the linear budget constraint would be $h(L^1, L^2; w_1, w_2, y) = L^1 w_1 + L^2 w_2 + y$.

that allow for identification of the sharing rule. Chiappori, Fortin, and Lacroix (2002) introduce the concept of distribution factors, defined as "variables that affect the household members' bargaining position but not preferences or the joint budget set," which generate a new set of testable restrictions on labor supplies and allow for more straightforward identification of the sharing rule.

Donni (2003) extends upon the framework from Chiappori, Fortin, and Lacroix (2002) to non-participation and non-linear budget constraints. Under a non-linear budget constraint, Donni (2003) defines the shadow wages and shadow income as:

$$\omega_1(w_1, w_2, y) = \frac{\partial h(\bar{L}^1, \bar{L}^2; w_1, w_2, y)}{\partial L^1} \tag{1}$$

$$\omega_2(w_1, w_2, y) = \frac{\partial h(\bar{L}^1, \bar{L}^2; w_1, w_2, y)}{\partial L^2} \tag{2}$$

$$\eta(w_1, w_2, y) = h(\bar{L}^1, \bar{L}^2; w_1, w_2, y) - \sum_i \bar{L}^i \omega_i, \tag{3}$$

and puts forth the following lemma:

Lemma 1. *Let* $(\bar{L}^1, \bar{L}^2)$ *be a pair of labor supplies consistent with collective rationality conditionally on the budget constraint in problem* $\bar{P}$*. Then, there exist a pair of functions* $(\bar{C}^1, \bar{C}^2)$ *and a pair of functions* (ρ^1, ρ^2)*, with* $\sum_i \rho^i = \eta$*, such that* $(\bar{L}^i, \bar{C}^i)$ *is a solution to:*

$$\begin{aligned} \max_{L^i, C^i} \quad & u^i(1 - L^i, C^i, \mathbf{z}) \\ \text{subject to} \quad & \gamma : L^i \omega_i + \rho^i = C^i \\ & 0 \leq L^i \leq 1 \\ \text{for any} \quad & (w_1, w_2, y) \in \mathbb{R}^3_{++} \end{aligned} \tag{P^i}$$

At an interior solution, Donni (2003) shows that we can appeal to the Implicit Function Theorem to re-write the unrestricted labor supplies ($\bar{L}^1$ and $\bar{L}^2$) as Marshallian labor supplies that are functions of only the shadow variables (ω_1,ω_2,η):

$$\hat{L}^1(\omega_1, \omega_2, \eta) = \lambda^1(\omega_1, \varphi(\omega_1, \omega_2, \eta)) \tag{4}$$

$$\hat{L}^2(\omega_1,\omega_2,\eta) = \lambda^2(\omega_2,\eta-\varphi(\omega_1,\omega_2,\eta)), \tag{5}$$

where $\varphi(\omega_1(w_1,w_2,y),\omega_2(w_1,w_2,y),\eta(w_1,w_2,y)) = \rho(w_1,w_2,y)$.

Having defined the shadow wages, shadow income, and the unrestricted labor supplies, Marshallian labor supplies, and sharing rule (as functions of the shadow variables), the canonical results from Chiappori (1992) and Chiappori, Fortin, and Lacroix (2002) follow; namely, that the partial derivatives of the sharing rule are identifiable (with or without distribution factors) as functions of the shadow variables. In this paper, I use a distribution factor, s, for identification.[9]

Donni's (2003) extension of the collective labor supply model to non-linear budget constraints, combined with proposition 1 from Chiappori, Fortin, and Lacroix (2002) in the Appendix, constitute the theoretical results needed for identification in this paper when both household members work. In order to extend the model to non-participation, Donni (2003) first implicitly defines the reservation wage for member i as the marginal rate of substitution along the axis $L^i = 0$ (conditional on φ^i):

$$\varpi^i(\omega_1,\omega_2,\eta) = -\frac{u^i_L(1,\varphi^i(\omega_1,\omega_2,\eta)}{u^i_C(1,\varphi^i(\omega_1,\omega_2,\eta)},$$

and makes one additional contraction assumption to ensure that, for any η, there exists a single pair of wages, denoted $\hat{\omega}_1$ and $\hat{\omega}_2$, such that both household members are indifferent between working and not working, and ensure that, for each member i, there exists a function $\gamma^i(\omega_j,\eta)$ such that member i participates in the labor market if and only if $\omega_i > \gamma^i(\omega_j,\eta)$. These elements partition $\mathbb{R}^3_{++}$ into four sets: the set in which both spouses work (denoted P or the spouses' participation set), the set in which only spouse 1 works (denoted N_2 or spouse 2's non-participation set), the set in which only spouse 2 works (denoted N_1 or spouse 1's non-participation set), and the set in which neither spouse works (denoted N or the spouses' non-participation set). This allows for identification of the sharing rule up to a constant when only one spouse works.

In the next section, I assume a parametric specification for the unrestricted labor supply functions and derive the sharing rule implied by these functions.

9. Appendix B reproduces the identification proof from Chiappori, Fortin, and Lacroix (2002).

3 Estimation and Identification

For convenience, I follow Oreffice (2011) and assume the parametric specification of the unrestricted labor supplies below.[10] I first consider the situation in which both spouses work and then consider, in turn, the situations in which only one spouse works.

3.1 Parametric Specification When Both Spouses Work

$$L^1 = a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \mathbf{a}_5 \mathbf{z} \tag{6}$$

$$L^2 = b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + \mathbf{b}_5 \mathbf{z} \tag{7}$$

ω_i is individual i's net-of-tax hourly wage rate, η is the couple's virtual income, s is the age difference between the spouses (the distribution factor), and $\mathbf{z}$ includes controls for each partner's years of education, the primary earner's age, indicators for the couple's race, and year and state fixed effects.[11] The set-up of the regressions means that the constant term will effectively control for the individual's sex in different-sex couples.[12]

Let $\Delta = a_3 b_4 - a_4 b_3$. If the identification restrictions are satisfied, then the sharing rule is given by:[13]

$$\varphi = \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z}) \tag{8}$$

It is also possible to derive the Marshallian labor supplies that are consistent with the unrestricted labor supplies in equations 6 and 7. These functions should take the following form:

$$\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z} \tag{9}$$

$$\lambda^2 = \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z} \tag{10}$$

10. I have also derived and estimated the structural parameters using the parametric specification from Chiappori, Fortin, and Lacroix (2002), which adds an interaction term between spousal wage rates, but the results and conclusion are not significantly different from the main specification below. These estimates are available upon request.

11. The race groups are Black, white, Hispanic, and Asian. I control for only the primary earner's age so that the effect of the distribution factor can be identified.

12. This specification implies that the identification conditions 20a–20f in Appendix B are automatically satisfied because the derivatives are zero, but it does imply other testable restrictions. Namely, the condition that $\frac{\partial \hat{L}^1}{\partial \eta} \cdot \frac{\partial \hat{L}^2}{\partial \eta} \neq 0$ requires that $a_3 b_3 \neq 0$, and the condition that $C \neq D$ requires that $\frac{a_4}{a_3} \neq \frac{b_4}{b_3}$.

13. Appendix C presents the derivation of these partial derivatives.

Given the form of φ in equation 8, the parameters above can be recovered as: $\alpha_1 = \frac{a_1 b_4 - a_4 b_1}{b_4}$, $\alpha_2 = \frac{\Delta}{b_4}$, $\beta_1 = \frac{a_4 b_2 - a_2 b_4}{a_4}$, and $\beta_2 = -\frac{\Delta}{a_4}$.[14]

Finally, we can also recover the indirect utility functions and the relative utility weight, μ. Stern (1986) shows that the the Marshallian labor supplies in equations 9 and 10 correspond to the following indirect utility functions:

$$V^1(\omega_1, \varphi, \mathbf{z}) = \left(\frac{e^{\alpha_2 \omega_1}}{\alpha_2} \right) (\alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z}) \tag{11}$$

$$V^2(\omega_2, \eta - \varphi, \mathbf{z}) = \left(\frac{e^{\beta_2 \omega_2}}{\beta_2} \right) (\beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z}) \tag{12}$$

As noted by Chiappori (1988), and appealing to the Envelope Theorem, $\frac{\partial V^1}{\partial \varphi} = \delta$, where δ is the Legrange multiplier from the household problem $\bar{P}$. Similarly, because the utility of individual 2 is multiplied by the relative utility weight (μ), $\frac{\partial V^2}{\partial (\eta - \varphi)} = \frac{\delta}{\mu}$. It is, therefore, possible to identify the relative utility weight as:

$$\mu = \frac{\frac{\partial V^1}{\partial \varphi}}{\frac{\partial V^2}{\partial (\eta - \varphi)}} = \frac{e^{\alpha_2 \omega_1}}{e^{\beta_2 \omega_2}} \tag{13}$$

3.2 Parametric Specification When Only Spouse 1 Works

Next, consider the situation in which only spouse 1 works. Following Donni (2003), I assume that if spouse 2 does not work then spouse 1's unrestricted labor supply function switches to:

$$L^{1,s} = A_0 + A_1 \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \mathbf{A}_5 \mathbf{z}, \tag{14}$$

and the sharing rule switches to:

$$\varphi^{1,s} = K_1 \log \omega_1 + K_2 \log \omega_2 + K_3 \eta + K_4 s + \mathbf{K}(\mathbf{z}) \tag{15}$$

In order for $L^{1,s}$ and $\varphi^{1,s}$ to be continuous along spouse 2's participation frontier, it must be the

14. Appendix D presents the derivations of these parameters.

case that:

$$L^{1,s} = L^1 + g \cdot L^2 \tag{16a}$$

$$\varphi^{1,s} = \varphi + h \cdot L^2, \tag{16b}$$

where g and h are free parameters. Donni's (2003) proof of identification shows that the partial differential equation $\frac{\partial \varphi^{1,s}}{\partial \omega_2} - A\frac{\partial \varphi^{1,s}}{\partial \eta} = 0$ holds within spouse 2's non-participation set, and I show in Appendix E that this implies that $h = g\frac{b_4}{\Delta}$. Therefore, the sharing rule is identified within spouse 2's non-participation set.

The parameters of L^2 (spouse 2's unrestricted labor supply function) remain the same as in equation 7, and Appendix E shows that the parameters of $L^{1,s}$ (spouse 1's new unrestricted labor supply function) can be recovered and all follow the general form of $A_n = a_n + gb_n$ for $n=$1, 2, 3, 4, and 5.

Given this form for the A_n parameters, Appendix E shows that the sharing rule, Marshallian labor supply parameters, indirect utility functions, and the relative utility weight in this situation take the same forms as equations 8–13 with the relevant parameter definitions of A_n substituted for a_n for $n=$1, 2, 3, 4, and 5.

3.3 Parametric Specification When Only Spouse 2 Works

Finally, consider the situation in which only spouse 2 works. Following the same process as above, I assume that if spouse 1 does not work then spouse 2's unrestricted labor supply function switches to:

$$L^{2,s} = B_0 + B_1 \log \omega_1 + B_2 \log \omega_2 + B_3\eta + B_4 s + \mathbf{B}_5\mathbf{z}, \tag{17}$$

and the sharing rule switches to:

$$\varphi^{2,s} = P_1 \log \omega_1 + P_2 \log \omega_2 + P_3\eta + P_4 s + \pi(\mathbf{z}) \tag{18}$$

In order for $L^{2,s}$ and $\varphi^{2,s}$ to be continuous along spouse 1's participation frontier, it must be the case that:

$$L^{2,s} = L^2 + j \cdot L^1 \tag{19a}$$

$$\varphi^{2,s} = \varphi + k \cdot L^1, \tag{19b}$$

where j and k are free parameters. Donni's (2003) proof of identification shows that the partial differential equation $\frac{\partial \varphi^{2,s}}{\partial \omega_1} - B\frac{\partial \varphi^{2,s}}{\partial \eta} = -B$ holds within spouse 1's non-participation set, and I show in Appendix E that this implies that $k = j\frac{a_4}{\Delta}$. Therefore, the sharing rule is identified within spouse 1's non-participation set.

The parameters of L^1 (spouse 1's unrestricted labor supply function) remain the same as in equation 6, and Appendix E shows that the parameters of $L^{2,s}$ (spouse 2's new unrestricted labor supply function) can be recovered and all follow the general form of $B_n = b_n + ja_n$ for $n =$1, 2, 3, 4, and 5.

Given this form for the B_n parameters, Appendix E shows that the sharing rule, Marshallian labor supply parameters, indirect utility functions, and the relative utility weight in this situation take the same forms as equations 8–13 with the relevant parameter definitions of B_n substituted for b_n for $n =$1, 2, 3, 4, and 5.

3.4 Identification of the Effect of Gender Norms

The above sections show that the relative utility weight is identified under the situation when both spouses work (μ), when only spouse 1 works ($\mu^{1,s}$), and when only spouse 2 works ($\mu^{2,s}$). Below I outline two alternative sets of identifying assumptions that allow me to identify either point estimates or unified bounds of the effect of gender norms on the relative utility weight as distinct from the role of earning status on the relative utility weight.

Let $\tilde{\mu}$ denote structural parameters for different-sex couples and $\ddot{\mu}$ denote structural parameters for same-sex couples.

Identifying Assumption 1. *The relative utility weight within different-sex couples is affected by both earning status and gender norms, but there is no effect of gender norms on the relative utility* *weight over labor supply within same-sex couples.*

Identifying assumption 1 allows me to interpret differences in relative utility weights between higher and lower earners in same-sex couples as the result of only their different earning potentials.

Identifying Assumption 2. *The effect of earning status on μ is the same for different- and same-sex couples, but varies within each labor supply type (i.e., dual-earner, only spouse 1 works, or only spouse 2 works).*

Taken together, identifying assumptions 1 and 2 allow me to explicitly compare $\hat{\tilde{\mu}}$, $\hat{\tilde{\mu}}^{1,s}$, and $\hat{\tilde{\mu}}^{2,s}$ for different-sex couples (which are affected by earnings potential and gender norms) to $\hat{\ddot{\mu}}$, $\hat{\ddot{\mu}}^{1,s}$, and $\hat{\ddot{\mu}}^{2,s}$ for same-sex couples (which are only affected by earnings potential), respectively, conditional on labor supply. I can therefore identify and estimate point estimates of the effect of gender norms on the relative utility weight as the differences between these parameters: $\hat{\tilde{\mu}} - \hat{\ddot{\mu}}$, $\hat{\tilde{\mu}}^{1,s} - \hat{\ddot{\mu}}^{1,s}$, and $\hat{\tilde{\mu}}^{2,s} - \hat{\ddot{\mu}}^{2,s}$.

Alternatively, I can alter identifying assumption 2 to instead identify unified bounds of the effects of gender norms instead of point estimates:

Identifying Assumption 2′. *The structural parameter μ is the same within each couple type. In other words, $\tilde{\mu} = \tilde{\mu}^{1,s} = \tilde{\mu}^{2,s}$ and $\ddot{\mu} = \ddot{\mu}^{1,s} = \ddot{\mu}^{2,s}$.*

Taken together, identifying assumptions 1 and 2′ allow me to construct unified bounds on $\tilde{\mu}$ for different-sex couples (which are affected by earnings potential and gender norms) and on $\ddot{\mu}$ for same-sex couples (which are only affected by earnings potential), and use those to construct unified bounds on the effect of gender norms on bargaining.

Figure 1 presents a visual representation of the following identification argument. Define $\underline{\hat{\tilde{\mu}}}$ as the highest lower bound and $\overline{\hat{\tilde{\mu}}}$ as the lowest upper bound, respectively, of the 95% confidence intervals for $\hat{\tilde{\mu}}$, $\hat{\tilde{\mu}}^{1,s}$, and $\hat{\tilde{\mu}}^{2,s}$. Then the bounds for different-sex couples' $\tilde{\mu}$ are $[\underline{\hat{\tilde{\mu}}}, \overline{\hat{\tilde{\mu}}}]$. The bounds for same-sex couples' $\ddot{\mu}$ are similarly identified as $[\underline{\hat{\ddot{\mu}}}, \overline{\hat{\ddot{\mu}}}]$. Finally, the unified bounds of the effect of gender norms on the relative utility weight is identified as $[\underline{\hat{\tilde{\mu}}} - \overline{\hat{\ddot{\mu}}}, \overline{\hat{\tilde{\mu}}} - \underline{\hat{\ddot{\mu}}}]$.

3.5 Estimation

I estimate the parameters of the model in three steps. I first estimate the reduced form labor supply parameters among dual-earner couples for each spouse separately using OLS (equations 6 and 7). I do this separately for spouses in different-sex couples spouses in male same-sex couples, which allows these unrestricted labor supply parameters to differ. I then use these parameters to predict hours worked for spouses in single-earner couples, which produces a predicted L^1 and L^2 for each spouse. I use these predicted values to estimate the switching parameters, g and j, using OLS (equations 16a and 19a). Finally I use the estimates from the dual-earner couples and the estimates of the switching parameters to back out the reduced form labor supply parameters for single-earner couples (equations 14 and 17).

In order to quantify the role of gender norms, it is possible to compare the relative utility weight of wives in different-sex married couples to the relative utility weight of lower earners in same-sex couples. This comparison assumes that, all else equal, the only remaining unobserved influence on the relative utility weight are gender norms between different-sex spouses, which are present in the relative utility weight for wives in different-sex couples, but not present in the relative utility weight for predicted lower earners in same-sex couples.

4 Data

I use data from the 2012–2019 American Community Surveys to construct a sample of childless different-sex married couples and childless, male same-sex married couples in which both spouses are between 25 and 60 years old, so as to limit attention to labor supply of prime-age workers.[15] The utility weight is identified relative to the higher earning spouse, which is assumed to be husbands in different-sex couples, so focusing only on male same-sex couples holds constant the sex of the primary earner between different- and same-sex couples, allowing for cleaner identification

15. If a same-sex couple reports themselves to be married even though they reside in a state that does not recognize same-sex marriages, then I assume the couple married in a state that did recognize same-sex marriages. The 2013 *United States v. Windsor* Supreme Court ruling means that these marriages are recognized at the federal, including for federal tax purposes, level even if the couple does not live in a state that permitted same-sex marriages at the time.

of the effect of gender norms. My main sample includes 5,019,361 individuals across both dual- and single-earner couples: 19,788 in same-sex couples and 4,999,573 in different-sex couples. Table 1 presents demographic summary statistics for different- and same-sex couples in my sample. Different-sex couples are, on average, more likely to have spouses of the same race and are farther apart in age.

I predict earnings for each spouse in order to address several empirical issues. First, earnings are endogenous to labor supply, meaning that higher observed wage and tax rates will be correlated with higher labor supply. Second, estimating the model with labor force participation decisions requires a method of assigning wages to non-workers, which I accomplish by using predicted wage rates (Donni 2003). Third, following Pollak (2005), Aizer (2010), Shenhav (2021), and Ahn and Koh (2024), it is the potential, rather than actual, wage rate that determines the relative utility weight, particularly in settings such as marriage or labor force participation where a reservation value drives decision-making. The accuracy of these predictions is important to appropriately address endogeneity concerns and control for the influence of the non-working spouse's latent wage, which Mullainathan and Spiess (2017) note is a useful context for a machine learning LASSO approach to predict earnings since it is "effectively a prediction exercise." The LASSO is a model selection method that uses a penalized regression to select the covariates that best predict earned income using OLS (Tibshirani 2011).[16] This approach enables me to include a large number of covariates and interactions while allowing the LASSO to select the subset of variables that best fit the data. Variables that I included, but which the LASSO may have ultimately ignored, include five year age groups, four education level groups, dummies for race, sex, occupation, college major, and state, as well as pairwise interactions between all of these variables.[17]

I use the LASSO to predict earnings in levels for each spouse. I limit the prediction sample to individuals observed in 2012 with positive earnings so that predicted earnings do not reflect labor supply changes influenced by policy variation during the sample period. I use these predicted

16. The LASSO is similar to a ridge regression, but uses an L1 norm constraint rather than the L2 norm constraint of the ridge regression. Friedberg and Isaac (Forthcoming) and Isaac (Forthcoming) also use a LASSO approach in similar contexts, and more detail about the methodology can be found there.

17. Although the prediction process differs, the goal of this process is similar to that used by Delhommer and Hamermesh (2021), who predict earning potentials for same-sex spouses in order to calculate the marital surplus.

earnings for two purposes. First, I use predicted earnings to divide spouses in same-sex couples by predicted earning status, so that predicted higher earners bargain with predicted lower earners. In contrast, I divide spouses in different-sex couples by sex, as is common in this literature, so that husbands bargain with wives.

Second, I use predicted earnings to compute a predicted shadow wage rate and income for each spouse, including non-workers. I first divide predicted earnings by 2,080 (52 weeks multiplied by 40 hours per week) to obtain a predicted measure of each individual's full-time gross wage rate: $w_{it} = \frac{\text{Predicted earnings}}{2080}$.[18] To account for non-linear taxation, Donni's (2003) model requires shadow wages and shadow income, ω_1, ω_2, and η, respectively, defined in equations 1–3. I follow Moreau and Donni (2002), and define, for a household with taxable income in the k^{th} bracket, $\omega_i = w_i(1-t_k)$ and $\eta = y - T(B_k) + t_k B_k$, where t_k is the federal marginal tax rate in bracket k, B_k is the lower income limit of bracket k, and $T(B_k)$ is the federal tax revenue corresponding to B_k. I obtain the t_k, B_k, and $T(B_k)$ parameters for each tax year from the NBER TAXSIM program based on simulated married households with varying levels of earned income.[19] This process, therefore, takes into account numerous tax credits and deductions based on earned income when generating the tax brackets for each tax year, rather than using the statutory income tax brackets, which would result in much coarser measurement of the tax parameters. I also account for federal recognition of same-sex marriages when applying these definitions of ω_i and η to same-sex couples.[20]

Table 1 also displays the observed and predicted labor supply summary statistics for different- and same-sex couples in my sample. Different-sex couples tend to have lower observed log wage rates and somewhat higher virtual income. Husbands in different-sex couples tend to work more hours than predicted higher earners in same-sex couples, and wives in different-sex couples tend to work fewer hours than predicted lower earners in same-sex couples. Table 1 also makes it clear that the prediction process tends to slightly overstate wage rates for most men and understate wage rates for women, although the mean log wage rates are similar. I also obtain more compressed

18. Dividing predicted earnings by 2,080 also avoids using a noisy and endogenous measure of self-reported annual hours of work when quantifying predicted gross wage rates, although Isaac (Forthcoming) suggests that this may not be a large concern.

19. These simulated households vary only in their total earned income; I do not consider other sources of income when obtaining these tax parameters. Figures of the tax brackets generated by this process are available upon request.

20. Same-sex married couples were still required to file federal taxes as two single individuals in tax years 2011 and 2012. Same-sex married couples were required to file joint federal taxes beginning in tax year 2013 following the *United States v. Windsor* Supreme Court ruling.

variation in predicted wage rates and predicted virtual income relative to the observed values.

5 Results

I estimate collective models for different-sex and same-sex married couples following the empirical specification in equations 6 and 7 for the unrestricted labor supply equations. Section 3 outlines the derivations of the Marshallian labor supplies, the derivatives of the sharing rule, and the relative utility weight on the second household member. In what follows, I will use "husband/wife" to refer to different-sex spouses and "predicted higher/lower earner" to refer to same-sex spouses.

5.1 Unrestricted Labor Supply Parameters

Table 2 presents coefficient estimates for the unrestricted labor supply equations. Of primary importance are the coefficients on the distribution factor: the age difference between spouses. The coefficients on the distribution factor are significant and opposite-signed between spouses, which indicates that the distribution factor does differentially affect spousal labor supply in different- and same-sex couples.[21] The differential effects of the distribution factor on each spouse's labor supply provides identification of the sharing rule, as outlined in section 2.

The coefficients on the individual's predicted own net wage are all positive and mostly significant. The exception is the own wage coefficient for predicted lower earners who are the only earner in their household, which is positive but insignificant. The cross-net wage effect is mostly positive and is significant among different-sex couples. This pattern suggests that different-sex couples view their spouse's labor supply as complementary. The cross-net wage coefficients are mostly insignificant among same-sex couples, suggesting little evidence that same-sex spouses adjust their labor supply in response to their spouse's net wage.

21. Note that the coefficients on the distribution factor exhibit opposite patterns between different-sex and same-sex spouses. In different-sex spouses, the coefficient on the age difference is negative for husbands and positive for wives, whereas the coefficient is positive for predicted primary earners and negative for predicted secondary earners in same-sex couples. These patterns are consistent with Oreffice (2011), who finds that the age difference between spouses is opposite-signed for same-sex cohabiting couples relative to different-sex married couples.

5.2 Sharing Rule and Marshallian Labor Supply Parameters

Table 3 presents coefficient estimates of the sharing rule derivatives. Among different-sex couples, a \$1 increase in the husband's net wage (an increase of \$2,186–\$2,237 annually at the mean) translates into an additional \$40–238 more income to himself, but a \$1 increase in the wife's net wage (an increase of \$1,773–\$1,861 annually at the mean) translates into the transfer of \$767–3,487 to herself, which can outweigh the increase in earnings. This suggests that wives receive a larger premium (in the form a larger fraction of additional income transfered to them) when their own net wage increases relative to their husband's. In addition, across all different-sex couples, the sharing rule indicates that a \$1 increase in the couple's virtual income translates into the transfer of \$0.67–0.91 to the husband, with the remainder going to the wife. Finally, a greater age difference between an older husband and a younger wife results in a transfer to the husband. The age difference transfer is largest when he works, ranging from \$671–\$738, and drops to \$259 when he does not work.

Among same-sex couples, the derivatives of the sharing rule are not statistically significant at conventional levels. The standard errors of these coefficients are much larger relative to those from different-sex couples, and the lack of precision may be due to the demands of the theoretical model combined with smaller sample sizes of same-sex couples.

Table 4 presents coefficient estimates of the structural parameters in the Marshallian labor supply equations. The coefficient on log own net wage (α_1 and β_1 in equations 9 and 10) are positive and significant for all individuals. These coefficients imply traditional upward sloping labor supply with Marshallian elasticities of 0.03 for husbands in different-sex couples, 0.19–0.20 for wives in different-sex couples, 0.09 for predicted higher earners in male couples, and 0.04–0.05 for predicted lower earners in male couples.

The share of unearned income also only significantly affects different-sex spouses, although these coefficients are negative among same-sex couples as well. The effect of virtual income is negative, as theory predicts, indicating that a larger share of virtual income decreases hours worked resulting in negative income effects.

5.3 Relative Utility Weights and the Role of Gender Norms

Given the structural parameter estimates above, it is also possible to estimate the relative utility weight on the wife's utility ($\tilde{\mu}$) and on the predicted lower earner's utility ($\ddot{\mu}$).[22] Note, however, that spouses bargain over whether and how much to work in the context of this paper. The role of gender norms, therefore, is currently limited to this type of spousal bargaining, but future research in this area should explore interactions with child care.

Table 5 presents my relative utility weight estimates for each couple type along with the two effects of gender norms I can identify depending on the set of identifying assumptions. In all cases, I estimate relative utility weights for lower earners in same-sex couples that are precisely estimated and essentially 1, meaning that the relative utility weights are equal between lower and higher earners in same-sex couples. Because I assume that the relative utility weight within same-sex couples is only affected by earning status, a precise estimate of 1 suggests that earning status does not affect the relative utility weight within married couples. This also makes the interpretation of the relative utility weights for different-sex couples clearer: if the relative utility weight is not affected by earning status, then any remaining difference between different- and same-sex spouses must be due to gender norms.

First, identifying assumptions 1 and 2 allow me to identify point estimates of the effect of gender norms as the differences between relative utility weights conditional on labor supply. I estimate that gender norms significantly increase the relative utility weight by 2.8–3.2% for working wives in different-sex couples and 19.1% for non-working wives in different-sex couples. One interpretation of these estimates is that if wives have 19.1% larger utility weights relative to their husbands, then that is enough to reduce labor supply to zero. The effect of gender norms among working wives is $\frac{2.8}{19.1} = 14.7\%$ or $\frac{3.2}{19.1} = 16.8\%$ of that amount, which roughly corresponds to the lower observed hours worked among working wives relative to working husbands in Table 1: working husbands work 2,186–2,237 hours, whereas working wives work 1,773–1,861 hours (a 14.9–20.7% reduction). The tight connection between reductions in the relative utility weight and

22. Recall that $\mu = \frac{e^{\alpha_2 \omega_1}}{e^{\beta_2 \omega_2}}$.

hours worked supports my hypothesis that gender norms can explain lower labor supply through the collective model's bargaining framework.

The substantial difference in relative utility weights between working and non-working wives suggests that gender norms meaningfully interact with earning status in the couple. Traditional gender norms likely discourage women from working, and my estimates imply that when these traditional norms are challenged then gender norms exert a much smaller increase in their relative utility weight. However, when these traditional norms are satisfied, so that the husband works and the wife does not, then gender norms exert a much larger positive effect on wives' relative utility weight. These results suggest that working wives may sacrifice utility in the couple due to the interaction of gender norms and earning status.

Second, identifying assumptions 1 and 2′ allow me to construct unified bounds on $\tilde{\mu}$ for different-sex couples (which are affected by earnings potential and gender norms) and on $\ddot{\mu}$ for same-sex couples (which are only affected by earnings potential), and use those to construct unified bounds on the effect of gender norms on bargaining.[23] Column 4 of Table 5 presents these bounds while Figure 2 reproduces them visually.

I estimate tight unified relative utility weight bounds around 1 for same-sex couples, again suggesting that the relative utility weight is not affected by earning status and that any remaining difference between different- and same-sex spouses must be due to gender norms. I estimate unified relative utility weight bounds of $[1.014, 1.048]$ for different-sex couples. These unified bounds for different- and same-sex couples imply unified bounds of the effect of gender norms on the relative utility weight of 1.1–5.1% for wives. These bounds exclude the large the relative utility weight estimate for non-working wives because the identifying assumption is that the structural parameter μ is the same within each couple type and does not vary by labor supply.

Both sets of results suggest that gender norms increase the relative utility weight of wives in different-sex couples, leading to lower labor supply of married, childless women. The tight relative utility weight bounds around 1 for same-sex couples suggest that almost all, if not all,

23. Figure 1 presents a visual representation of the identification argument in Section 3.4.

of the remaining difference between the relative utility weights of different-sex spouses must be due to gender norms. My results also suggest that past studies that have assumed that bargaining power is divided by sex in different-sex couples exhibit estimates of bargaining power or sharing rule parameters that are primarily driven by gender norms.

6 The Wage Equivalency of Gender Norms

My results suggest that gender norms increase the relative utility weight of wives in different-sex couples, leading to lower labor supply of married, childless women, but it is possible to put the effect of gender norms into monetary terms to better contextualize the effect sizes. To do this, I perform a back-of-the-envelope calculation to back out the wage rate for the lower earner that would produce a relative utility weight of 1, all else constant, which reveals the wage effect that would be equivalent to the estimated effect of gender norms above.

Recall from Equation 13 that $\mu = \frac{e^{\alpha_2 \omega_1}}{e^{\beta_2 \omega_2}} = e^{\alpha_2 \omega_1 - \beta_2 \omega_2}$, where α_2 and β_2 are the coefficients on log(Share of virtual income) in the Marshallian labor supply functions in Table 4 and ω_1 and ω_2 are the shadow wage rates for husbands and wives, respectively, in different-sex couples. Let $\tilde{\omega}_2$ be the wife's wage rate that produces $\mu = 1$ so that $\tilde{\omega}_2 - \omega_2$ is the wage change that is equivalent to the effect of gender norms. We can back out $\tilde{\omega}_2 - \omega_2$ by setting $\mu = 1$ and taking the log of both sides of Equation 13 to obtain $\tilde{\omega}_2 - \omega_2 = \frac{\alpha_2}{\beta_2}\omega_1 - \omega_2$. A negative value of $\tilde{\omega}_2 - \omega_2$ indicates that the effect of gender norms is akin to a widening of the gender wage gap.

Intuitively, this exercise calculates the gender wage gap in married couples that could, by itself, explain the observed gender hours gap if gender norms were inconsequential. Using mean wage values, this exercise reveals that the effect of gender norms on different-sex married couples' labor supply is equivalent to a substantial widening of the gender wage gap: a \$6.02 widening (72.0%) of the gender wage gap for dual-earner couples or a \$11.53–13.35 widening (111.0–219.1%) for single-earner couples.

7 Conclusion

In this paper, I estimate collective labor supply models for different- and same-sex married couples, accounting for non-participation and non-linear taxation, to identify and estimate the effect of gender norms in intra-household labor supply as distinct from the role of earning status within the household. I recover and estimate the relative weight on the utility functions in the household's social welfare function and present two sets of identifying assumptions that allow me to identify either point estimates or unified bounds of the effect of gender norms on labor supply of wives in different-sex couples. I estimate the model using a sample of childless different-sex married couples and childless, male same-sex married couples in which both spouses are between 25 and 60 years old from the 2012–2019 American Community Surveys.

I estimate precise equality between the utility weights of male same-sex spouses, suggesting that earning status in the couple does not significantly affect intra-household gender gaps in labor supply. Under the first set of identifying assumptions, my estimates imply that gender norms significantly increase the relative utility weight by 2.8–3.3% for working wives in different-sex couples and 19.2% for non-working wives in different-sex couples. I also show that the fact that working wives' relative utility weight is 14.7–16.9% of non-working wives' relative utility weight translates to a roughly equal 14.9–20.7% reduction in hours worked among working wives relative to working husbands, which supports the conclusion that gender norms can explain lower labor supply through the collective model framework. Under the second set of identifying assumptions, I construct unified bounds of the effect of gender norms and find that gender norms increase wives' relative utility weight by 1.1–5.1%. A back-of-the-envelope calculation suggests that the effect of gender norms on married, childless couples' labor supply is equivalent to a substantial widening of the gender wage gap by 72.0% for dual-earning couples or 111.0–2.19.1% for single-earner couples.

My results suggest that social interventions aimed at adjusting gender norms may be well suited to improve welfare because intra-household gender gaps in labor supply appear to be driven by gender norms rather than earning status. This policy conclusion is consistent with Flinn, Todd, and

Zhang (2018, 2024), who conclude that equalizing men's and women's personality characteristics does more to close the male-female earnings gap than equalizing market valuations of characteristics. My estimates also suggest that past studies that have assumed that bargaining power is divided by sex in different-sex couples exhibit estimates of bargaining power or sharing rule parameters that are primarily driven by gender norms. However, it is important to note that spouses decide whether and how much to work in the context of this paper. The role of gender norms, therefore, is currently limited to this type of household decision making, but future work in this area should explore interactions with child care.

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Tables

Table 1: Summary Statistics

	Male couples				Different-sex couples			
	Both partners work				Both partners work			
	Predicted higher earners	Predicted lower earners	Only predicted higher earner works	Only predicted lower earner works	Husbands	Wives	Only husband works	Only wife works
Age	44.717	43.672	47.441	46.997	44.962	43.160	45.093	47.081
	(8.642)	(9.895)	(8.428)	(9.822)	(9.284)	(9.265)	(9.328)	(8.959)
Years of education	15.883	14.109	15.316	13.362	14.015	14.457	13.563	13.669
	(2.387)	(2.833)	(2.817)	(3.136)	(2.820)	(2.728)	(3.422)	(2.844)
Couple is Black	0.022	0.022	0.029	0.042	0.048	0.048	0.035	0.089
	(0.147)	(0.147)	(0.168)	(0.201)	(0.214)	(0.214)	(0.183)	(0.285)
Couple is white	0.065	0.065	0.097	0.090	0.083	0.083	0.163	0.085
	(0.247)	(0.247)	(0.296)	(0.286)	(0.276)	(0.276)	(0.369)	(0.279)
Couple is Hispanic	0.613	0.613	0.570	0.603	0.700	0.700	0.615	0.645
	(0.487)	(0.487)	(0.495)	(0.490)	(0.458)	(0.458)	(0.487)	(0.478)
Couple is Asian	0.025	0.025	0.041	0.037	0.052	0.052	0.074	0.053
	(0.157)	(0.157)	(0.197)	(0.188)	(0.222)	(0.222)	(0.261)	(0.224)
Partners' age difference	1.045	1.045	1.305	0.147	1.803	1.803	1.929	2.437
	(7.260)	(7.260)	(7.635)	(7.716)	(4.171)	(4.171)	(4.508)	(4.774)
Annual hours worked	2118.942	2011.820	2178.958	2117.983	2185.507	1772.530	2237.020	1861.154
	(656.353)	(697.753)	(704.407)	(727.461)	(619.064)	(700.731)	(661.444)	(693.703)
Own observed after-tax log hourly wage	3.103	2.899	3.257	3.012	2.892	2.615	3.017	2.675
	(0.803)	(0.802)	(0.891)	(0.915)	(0.740)	(0.755)	(0.805)	(0.728)
Own predicted after-tax log hourly wage	3.301	2.950	3.249	2.890	3.062	2.556	3.028	2.481
	(0.474)	(0.650)	(0.493)	(0.630)	(0.501)	(0.461)	(0.546)	(0.454)
Spouse's observed after-tax log hourly wage	2.899	3.103	.	.	2.615	2.892	.	.
	(0.802)	(0.803)	(.)	(.)	(0.755)	(0.740)	(.)	(.)
Spouse's predicted after-tax log hourly wage	2.950	3.301	2.833	3.152	2.556	3.062	2.329	2.893
	(0.650)	(0.474)	(0.637)	(0.586)	(0.461)	(0.501)	(0.574)	(0.479)
Observed virtual income ($10,000s)	1.258	1.239	1.826	2.039	1.326	1.326	1.677	2.030
	(2.934)	(2.871)	(4.252)	(4.844)	(2.210)	(2.210)	(3.237)	(3.522)
Predicted virtual income ($10,000s)	1.149	1.129	1.762	2.060	1.243	1.243	1.639	2.207
	(2.839)	(2.775)	(4.213)	(4.841)	(2.125)	(2.125)	(3.138)	(3.501)
Observations	8,873	8,873	1,307	735	2,104,716	2,104,716	642,102	148,039

Notes: The data come from the 2012–2019 American Community Surveys and include different-sex and same-sex married, childless couples in which both spouses are working and between 25-–60 years old.

Table 2: Unrestricted Labor Supply Parameters

	Male couples				Different-sex couples			
	Both partners work				Both partners work			
	Predicted higher earners	Predicted lower earners	Only predicted higher earner works	Only predicted lower earner works	Husbands	Wives	Only husband works	Only wife works
Log(predicted own net wage)	186.734***	92.770***	188.457***	37.606	62.302***	258.734***	65.632***	28.843***
	(45.864)	(26.106)	(46.192)	(62.621)	(3.387)	(5.333)	(3.499)	(3.759)
Log(predicted spouse net wage)	-1.078	13.159	11.067	92.629***	38.213***	13.717***	101.033***	268.012***
	(23.984)	(50.685)	(30.205)	(26.169)	(4.957)	(3.361)	(7.968)	(5.541)
Couple's virtual income	0.338	-1.978	0.000	-0.000	-14.688***	-14.129***	-0.002***	-0.002***
($10,000s)	(3.649)	(4.163)	(0.000)	(0.000)	(0.445)	(0.423)	(0.000)	(0.000)
Couple's age difference	3.259***	-4.752***	2.637*	-4.325***	-1.474***	3.940***	-0.517**	3.582***
	(1.207)	(1.315)	(1.530)	(1.465)	(0.180)	(0.184)	(0.207)	(0.192)
Couple is Black	-100.148*	1.968	-99.891*	-11.144	-65.149***	20.259***	-60.230***	4.441
	(59.900)	(63.668)	(60.189)	(66.795)	(4.233)	(4.114)	(4.375)	(4.510)
Couple is white	-51.719	99.128**	-38.741	92.357**	-31.470***	-40.960***	-41.415***	-48.601***
	(37.870)	(38.901)	(42.709)	(40.325)	(3.803)	(3.965)	(4.040)	(4.138)
Couple is Hispanic	2.896	60.970***	10.878	61.349***	40.515***	-6.492**	38.939***	3.345
	(18.835)	(19.768)	(22.382)	(19.834)	(2.476)	(2.551)	(2.557)	(2.790)
Couple is Asian	-189.068***	8.925	-187.900***	-15.828	-69.855***	-67.237***	-86.179***	-84.197***
	(61.703)	(69.200)	(62.090)	(78.628)	(4.298)	(4.712)	(4.723)	(5.100)
Partner 1's age	-4.028***	-5.078***	-4.693***	-5.606***	-1.889***	-5.243***	-3.162***	-5.701***
	(1.115)	(1.226)	(1.500)	(1.461)	(0.092)	(0.095)	(0.156)	(0.108)
Partner 1's years of education	-7.912	-3.591	-8.382	-4.626	8.070***	-13.326***	4.834***	-11.366***
	(7.573)	(8.404)	(7.648)	(8.564)	(0.594)	(0.598)	(0.688)	(0.644)
Partner 2's years of education	3.936	11.269**	5.411	11.784**	-5.199***	4.059***	-4.213***	2.796***
	(4.845)	(5.462)	(5.352)	(5.526)	(0.725)	(0.776)	(0.755)	(0.805)
Switching parameter			0.131	0.391			0.243***	0.092***
			(0.196)	(0.238)			(0.024)	(0.021)

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. All specifications also include year and state fixed effects. The data come from the 2012–2019 American Community Surveys.

Table 3: Sharing Rule Derivatives

	Sharing rule: $\phi = \gamma_1 \log \omega_1 + \gamma_2 \log \omega_2 + \gamma_3 \eta + \gamma_4 s + \kappa(\mathbf{z})$					
	Male couples			Different-sex couples		
	Both partners work	Only predicted higher earner works	Only predicted lower earner works	Both partners work	Only husband works	Only wife works
Derivative with respect to:						
log(partner 1 net wage)	2,994.537	2,422.932	8,558.072	113.222***	39.730**	238.081***
	(17,929.982)	(14,530.599)	(41,457.030)	(29.988)	(18.114)	(39.124)
log(partner 2 net wage)	491.825	-5,048.410	325.626	-1,318.812***	-3,486.874***	-766.658***
	(11,177.075)	(26,409.301)	(7,404.565)	(184.591)	(319.761)	(111.479)
Virtual income	-0.331	-0.077	-0.302	0.735***	0.907***	0.669***
	(4.825)	(3.926)	(4.392)	(0.027)	(0.036)	(0.033)
Age difference	-31977.437	-25873.501	-29106.072	737.807***	258.899***	670.800***
	(144682.122)	(117424.840)	(131781.686)	(68.665)	(95.911)	(55.609)

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. Section 3 details the derivations of the sharing rule derivatives above from the unrestricted labor supply equations. The derivatives with respect to ω_1 and ω_2 are calculated at the mean values of these variables.

Table 4: Structural Labor Supply Parameters

	Male couples		Different-sex couples	
Panel A: Both partners work				
	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners
Coefficient on:				
log(own net wage)	195.759***	91.198**	67.432***	360.893***
	(56.950)	(43.689)	(3.692)	(20.667)
log(Share of virtual income)	-0.000	-0.000	-0.002***	-0.005***
	(0.000)	(0.001)	(0.000)	(0.001)
Pareto weight[a]	1.000		1.033***	
	(0.002)		(0.009)	
Panel B: Only partner 1 works				
	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners
Coefficient on:				
log(own net wage)	195.759***	112.713*	67.432***	1,028.466***
	(56.950)	(64.570)	(3.692)	(347.159)
log(Share of virtual income)	-0.000	-0.000	-0.002***	-0.015***
	(0.000)	(0.001)	(0.000)	(0.006)
Pareto weight[a]	1.001		1.192*	
	(0.005)		(0.102)	
Panel C: Only partner 2 works				
	Predicted higher earners	Predicted lower earners	Predicted higher earners	Predicted lower earners
Coefficient on:				
log(own net wage)	215.071***	91.198**	74.168***	360.893***
	(70.493)	(43.689)	(4.424)	(20.667)
log(Share of virtual income)	-0.000	-0.000	-0.002***	-0.005***
	(0.001)	(0.001)	(0.000)	(0.001)
Pareto weight[a]	1.000		1.028***	
	(0.002)		(0.010)	

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. Section 3 details the derivations of the structural parameters above from the unrestricted labor supply equations. The Marshallian hours elasticity is conditional on the share of unearned income.

a: Relative utility weight test is $H_0 : \mu = 1$ so that significance stars indicate whether the relative utility weight is significantly different than 1.

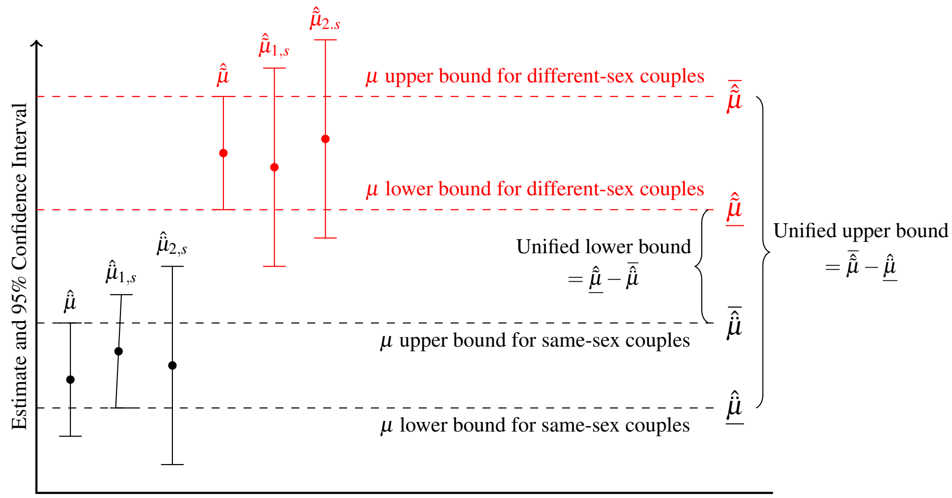
Table 5: Estimated Effect of Gender Norms on Labor Supply

	Both partners work	Only partner 1 works	Only partner 2 works	Unified bounds
Male couples	1.000	1.001	1.000	[0.997,1.003]
	(0.002)	(0.005)	(0.002)	
Different-sex couples	1.033 ***	1.192 *	1.028 ***	[1.014,1.048]
	(0.009)	(0.102)	(0.010)	
Difference	0.032 ***	0.191 *	0.028 ***	[0.011,0.051]
	(0.009)	(0.102)	(0.010)	

Notes: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are in parentheses. The significance stars in rows 1 and 2 indicate whether the estimate is significantly different than 1, whereas the significance stars in row 3 indicate whether the estimate is significantly different than 0.

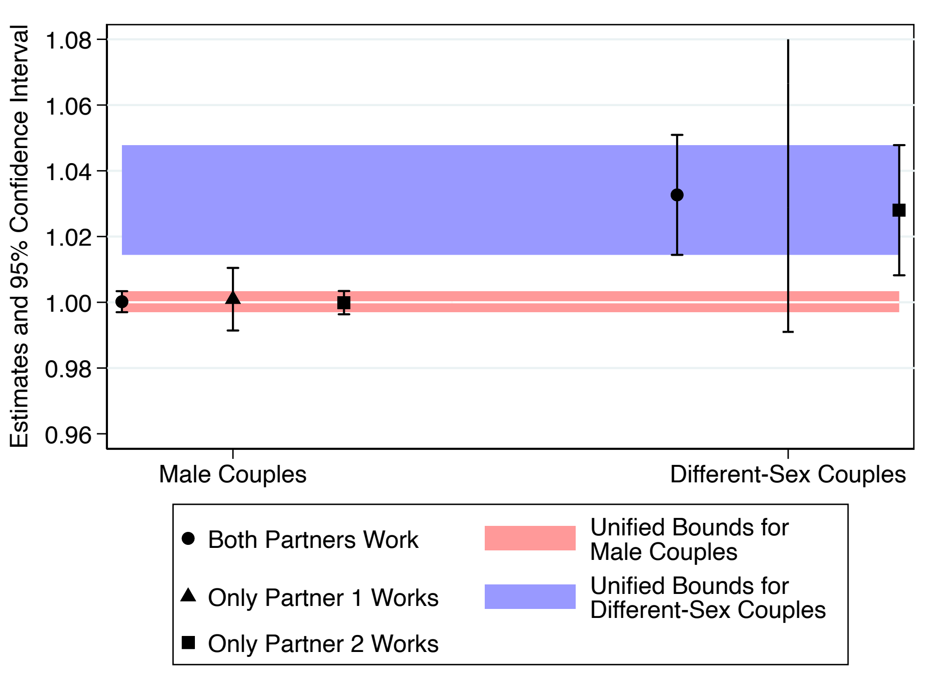
Figures

Figure 1: Illustration of Identification of Unified Bounds of the Effect of Gender Norms on Labor Supply



Notes: This figure presents an illustration of the identification argument for unified bounds of the effect of gender norms on labor supply. The black point estimates represent same-sex couples and the red estimates represent different-sex couples. $\hat{\mu}$ and $\hat{\mu}$ are the relative utility weights when both spouses work, $\hat{\mu}_{1,s}$ and $\hat{\mu}_{1,s}$ are the relative utility weights when only spouse 1 works, and $\hat{\mu}_{2,s}$ and $\hat{\mu}_{2,s}$ are the relative utility weights when only spouse 2 works. $\underline{\hat{\mu}}$ is the highest lower bound and $\overline{\hat{\mu}}$ is the lowest upper bound, respectively, of the 95% confidence intervals for $\hat{\mu}$, $\hat{\mu}^{1,s}$, and $\hat{\mu}^{2,s}$. The respective bounds for same-sex couples are defined symmetrically. The unified bounds of the effect of gender norms on the relative utility weight is identified as $[\underline{\hat{\mu}} - \overline{\hat{\mu}}, \overline{\hat{\mu}} - \underline{\hat{\mu}}]$.

Figure 2: Estimates of Unified Bounds of the Effect of Gender Norms on Labor Supply



Notes: The figure displays point estimates and 95% confidence intervals for $\hat{\mu}$, $\hat{\mu}_{1,s}$, $\hat{\mu}_{2,s}$, $\hat{\mu}$, $\hat{\mu}_{1,s}$, and $\hat{\mu}_{2,s}$ as well as the unified bounds for male and different-sex couples. These estimates are also presented in Table 5. Note that the point estimate and upper bound of $\hat{\mu}_{1,s}$ are not displayed for illustrative purposes only because they fall well outside the range of the other estimates. The point estimate for $\hat{\mu}_{1,s}$ is presented in Table 5 and is 1.192 with a 95% confidence interval of $[0.991, 1.392]$

A Relative Utility Weight Estimates in the Literature

This section shows calculations of relative utility weights implied by the models from Chiappori, Fortin, and Lacroix (2002), Moreau and Donni (2002), and Donni and Moreau (2007).

A.1 Chiappori, Fortin, and Lacroix (2002)

Chiappori, Fortin, and Lacroix (2002) use the same functional form for the reduced form labor supply functions as I use in this paper, but they add an interaction term between $\log w_1$ and $\log w_2$. However, this does not change the implied functional form for the Marshallian labor supply functions, leading to the same functional form for the indirect utility function in this paper. Therefore, the functional form for the relative utility weight in Chiappori, Fortin, and Lacroix (2002) is the same as in this paper, although the structural parameters have a slightly different construction due to the additional interaction term.

Using the reduced form labor supply parameter estimates, the derived structural parameters, and sample mean log wages of 2.47 for men and 2.04 for women from Chiappori, Fortin, and Lacroix (2002), we obtain:

$$\mu = \frac{e^{\alpha_2 w_1}}{e^{\beta_2 w_2}} = \frac{e^{\frac{\Delta}{m_4} w_1}}{e^{\frac{-\Delta}{f_4} w_2}} = \frac{e^{\frac{f_3 m_4 - f_4 m_3}{m_4} w_1}}{e^{\frac{f_4 m_3 - f_3 m_4}{f_4} w_2}} = \frac{e^{\frac{(-0.008)(0.257)-(-0.433)(-0.006)}{0.257}(e^{\ln 2.04})}}{e^{\frac{(-0.433)(-0.006)-(-0.008)(0.257)}{-0.433}(e^{\ln 2.47})}} = 0.9897$$

However, Chiappori, Fortin, and Lacroix (2002) denote the wife as individual 1, so the comparable estimate of μ should be $\mu = \frac{1}{0.9897} = 1.010$

A.2 Moreau and Donni (2002)

The functional form for the reduced form labor supply parameters in Moreau and Donni (2002) also imply the same function form for the Marshallian labor supply functions and indirect utility as used in this paper and Chiappori, Fortin, and Lacroix (2002). Using the structural parameter estimates and sample median wages of 39.36 for women and 47.56 for men, we obtain:

$$\mu = \frac{\frac{\partial V_m}{\partial(\eta-\phi)}}{\frac{\partial V_f}{\partial \phi}} = \frac{\delta}{\frac{\delta}{\mu}} \quad \Rightarrow \quad \mu = \frac{e^{B_2 w_m}}{e^{A_2 w_f}} = \frac{e^{(0.000189)(47.56)}}{e^{(-0.000477)(39.36)}} = 1.028$$

A.3 Bloemen (2010)

Bloemen (2010) uses a labor supply function that Hausman and Ruud (1984) show is derived from the utility function $u^j(h,c) = m^* \exp\left[\frac{\beta_j}{\gamma_j}(h - \delta_j - \beta_j m^*)\right]$. This, in turn, implies the indirect utility function $V_j = e^{\beta_j w_j}[y + \theta + \delta_j w_j + 0.5(\gamma_j w_j^2 + \alpha w_1 w_2)]$. Using the structural parameter estimates and sample mean wages of 23.0 for men and 18.7 for women, we obtain:

$$\mu = \frac{\frac{\partial V_m}{\partial y}}{\frac{\partial V_f}{\partial y}} = \frac{e^{\alpha_w^m w_m}}{e^{\alpha_2^f w_f}} = \frac{e^{(-0.0035)(23.0)}}{e^{(-0.016)(18.7)}} = 1.244$$

B Identification of the Sharing Rule

Below I reproduce the argument from Chiappori, Fortin, and Lacroix (2002) that proves identification of the sharing rule. Define:

$$A = \frac{\frac{\partial \hat{L}^1}{\partial \omega_2}}{\frac{\partial \hat{L}^1}{\partial \eta}}, \qquad B = \frac{\frac{\partial \hat{L}^2}{\partial \omega_1}}{\frac{\partial \hat{L}^2}{\partial \eta}}, \qquad C = \frac{\frac{\partial \hat{L}^1}{\partial s}}{\frac{\partial \hat{L}^1}{\partial \eta}}, \qquad D = \frac{\frac{\partial \hat{L}^2}{\partial s}}{\frac{\partial \hat{L}^2}{\partial \eta}},$$

Chiappori, Fortin, and Lacroix (2002) present the proposition below, which I reproduce using the shadow variable notation from above:

Proposition 1. *Take any point such that* $\frac{\partial \hat{L}^1}{\partial \eta} \cdot \frac{\partial \hat{L}^2}{\partial \eta} \neq 0$. *Then the following results hold: (i) If there exists exactly one distribution factor such that* $C \neq D$*, the following conditions are necessary for any pair* $(\hat{L}^1, \hat{L}^2)$ *to be solutions of* (P^i) *for some sharing rule* φ*:*

$$\frac{\partial}{\partial s}\left(\frac{D}{D-C}\right) = \frac{\partial}{\partial y}\left(\frac{CD}{D-C}\right) \tag{20a}$$

$$\frac{\partial}{\partial \omega_1}\left(\frac{D}{D-C}\right) = \frac{\partial}{\partial y}\left(\frac{BC}{D-C}\right) \tag{20b}$$

$$\frac{\partial}{\partial \omega_2}\left(\frac{D}{D-C}\right) = \frac{\partial}{\partial y}\left(\frac{AD}{D-C}\right) \tag{20c}$$

$$\frac{\partial}{\partial \omega_1}\left(\frac{CD}{D-C}\right) = \frac{\partial}{\partial s}\left(\frac{BC}{D-C}\right) \tag{20d}$$

$$\frac{\partial}{\partial \omega_2}\left(\frac{CD}{D-C}\right) = \frac{\partial}{\partial s}\left(\frac{AD}{D-C}\right) \tag{20e}$$

$$\frac{\partial}{\partial \omega_2}\left(\frac{BC}{D-C}\right) = \frac{\partial}{\partial \omega_1}\left(\frac{AD}{D-C}\right) \tag{20f}$$

$$\frac{\partial \hat{L}^1}{\partial \omega_1} - \frac{\partial \hat{L}^1}{\partial \eta}\left(\hat{L}^1 + \frac{BC}{D-C}\right)\left(\frac{D-C}{D}\right) \geq 0 \tag{20g}$$

$$\frac{\partial \hat{L}^2}{\partial \omega_2} - \frac{\partial \hat{L}^2}{\partial \eta}\left(\hat{L}^2 + \frac{AD}{D-C}\right)\left(-\frac{D-C}{D}\right) \geq 0 \tag{20h}$$

(ii) Under the assumption that conditions 20a–20h hold and for a given $\mathbf{z}$*, the sharing rule is defined up to an additive function* $\kappa(\mathbf{z})$ *depending only on the preference factors* $\mathbf{z}$*. The partial derivatives of the sharing rule with respect to wages, non-labor income, and the distribution factor*

are given by:

$$\begin{aligned}\frac{\partial \varphi}{\partial \eta} &= \frac{D}{D-C}\\ \frac{\partial \varphi}{\partial s} &= \frac{CD}{D-C}\\ \frac{\partial \varphi}{\partial \omega_1} &= \frac{BC}{D-C}\\ \frac{\partial \varphi}{\partial \omega_2} &= \frac{AD}{D-C}\end{aligned} \tag{21}$$

C Derivation of Sharing Rule Derivatives

The parametric specifications in equations 6 and 7 lead to the following functions for A, B, C, and D under the definitions in Appendix 2:

$$A = \frac{a_2}{a_3\omega_2}, \qquad B = \frac{b_1}{b_3\omega_1}, \qquad C = \frac{a_4}{a_3}, \qquad D = \frac{b_4}{b_3}$$

Under the assumption that conditions 20a–20h hold for a given $\mathbf{z}$, the derivatives of the sharing rule are given by:

$$\frac{\partial\varphi}{\partial\eta} = \frac{D}{D-C}$$
$$\frac{\partial\varphi}{\partial s} = \frac{CD}{D-C}$$
$$\frac{\partial\varphi}{\partial\omega_1} = \frac{BC}{D-C}$$
$$\frac{\partial\varphi}{\partial\omega_2} = \frac{AD}{D-C}$$

Recall the definitions of A, B, C, and D are:

$$A = \frac{\frac{\partial\hat{L}^1}{\partial\omega_2}}{\frac{\partial\hat{L}^1}{\partial\eta}}, \qquad B = \frac{\frac{\partial\hat{L}^2}{\partial\omega_1}}{\frac{\partial\hat{L}^2}{\partial\eta}}, \qquad C = \frac{\frac{\partial\hat{L}^1}{\partial s}}{\frac{\partial\hat{L}^1}{\partial\eta}}, D = \frac{\frac{\partial\hat{L}^2}{\partial s}}{\frac{\partial\hat{L}^2}{\partial\eta}},$$

Under the function form in Equations 6 and 7, these values are:

$$A = \frac{a_2}{a_3\omega_2}, \qquad B = \frac{b_1}{b_3\omega_1}, \qquad C = \frac{a_4}{a_3}, \qquad D = \frac{b_4}{b_3}$$

Note that the denominator of the sharing rule derivates are the same $(D-C)$, which can be written:

$$D - C = \frac{b_4}{b_3} - \frac{a_4}{a_3} = \frac{a_3b_4}{a_3b_3} - \frac{a_4b_3}{a_3b_3} = \frac{a_3b_4 - a_4b_3}{a_3b_3} = \frac{\Delta}{a_3b_3}$$

Where $\Delta \equiv a_3b_4 - a_4b_3$. Using this expression for $D-C$, the sharing rule derivatives can be written as:

$$\frac{\partial\varphi}{\partial\eta} = \frac{D}{D-C} = \frac{\frac{b_4}{b_3}}{\frac{\Delta}{a_3b_3}} = \frac{a_3b_4}{\Delta} \tag{22a}$$

$$\frac{\partial\varphi}{\partial s} = \frac{CD}{D-C} = \frac{\frac{a_4b_4}{a_3b_3}}{\frac{\Delta}{a_3b_3}} = \frac{a_4b_4}{\Delta} \tag{22b}$$

$$\frac{\partial\varphi}{\partial\omega_1} = \frac{BC}{D-C} = \frac{\frac{a_4b_1}{a_3b_3\omega_1}}{\frac{\Delta}{a_3b_3}} = \frac{a_4b_1}{\omega_1\Delta} \tag{22c}$$

$$\frac{\partial\varphi}{\partial\omega_2} = \frac{AD}{D-C} = \frac{\frac{a_2b_4}{a_3b_3\omega_2}}{\frac{\Delta}{a_3b_3}} = \frac{a_2b_4}{\omega_2\Delta} \tag{22d}$$

Using the above expressions to solve this system of differential equations leads to the sharing rule in Equation 8.

D Derivation of the Marshallian Labor Supply Parameters

Recall that the sharing rule is:

$$\varphi = \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z})$$

The Marshallian labor supplies take the following form:

$$\lambda^1 = \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z}$$
$$\lambda^2 = \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z}$$

Beginning with λ^1, let $\alpha_2 = \frac{\Delta}{b_4}$. Expanding the expression for φ, we obtain:

$$\begin{aligned}
\lambda^1 &= \alpha_1 \log \omega_1 + \alpha_2 \varphi + \alpha_3 \mathbf{z} \\
&= \alpha_1 \log \omega_1 + \frac{a_4 b_1}{b_4} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \frac{\Delta}{b_4} \kappa(\mathbf{z}) + \alpha_3 \mathbf{z} \\
&= \underbrace{\left(\alpha_1 + \frac{a_4 b_1}{b_4}\right)}_{=a_1} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \underbrace{\frac{\Delta}{b_4} \kappa(\mathbf{z}) + \alpha_3 \mathbf{z}}_{=\tilde{\alpha}_3(\mathbf{z})}
\end{aligned}$$

In order for λ^1 to be consistent with L^1, it must be the case that $\alpha_1 + \frac{a_4 b_1}{b_4} = a_1$, implying that $\alpha_1 = a_1 - \frac{a_4 b_1}{b_4} = \frac{a_1 b_4 - a_4 b_1}{b_4}$.

Similarly, moving to λ^2, let $\beta_2 = -\frac{\Delta}{a_4}$. Expanding the expression for φ, we obtain:

$$\begin{aligned}
\lambda^2 &= \beta_1 \log \omega_2 + \beta_2 (\eta - \varphi) + \beta_3 \mathbf{z} \\
&= \beta_1 \log \omega_2 - \frac{\Delta}{a_4} \eta + b_1 \log \omega_1 + \frac{a_2 b_4}{a_4} \log \omega_2 + \frac{a_3 b_4}{a_4} \eta + b_4 s + \frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3 \mathbf{z} \\
&= b_1 \log \omega_1 + \left(\beta_1 + \frac{a_2 b_4}{a_4}\right) \log \omega_2 + \left(\frac{a_3 b_4 - a_3 b_4 + a_4 b_3}{a_4}\right) \eta + b_4 s + \frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3 \mathbf{z} \\
&= b_1 \log \omega_1 + \underbrace{\left(\beta_1 + \frac{a_2 b_4}{a_4}\right)}_{=b_2} \log \omega_2 + b_3 \eta + b_4 s + \underbrace{\frac{\Delta}{a_4} \kappa(\mathbf{z}) + \beta_3 \mathbf{z}}_{=\tilde{\beta}_3(\mathbf{z})}
\end{aligned}$$

In order for λ^2 to be consistent with L^2, it must be the case that $\beta_1 + \frac{a_2 b_4}{a_4} = b_2$, implying that $\beta_1 = b_2 - \frac{a_2 b_4}{a_4} = \frac{a_4 b_2 - a_2 b_4}{a_4}$.

E Derivation of the Switching Parameters

Consider the situation in which only spouse 1 works. Following Donni (2003), I assume that if spouse 2 does not work then spouse 1's unrestricted labor supply function switches to:

$$L^{1,s} = A_0 + A_1 \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \mathbf{A}_5 \mathbf{z},$$

and the sharing rule switches to:

$$\varphi^{1,s} = K_1 \log \omega_1 + K_2 \log \omega_2 + K_3 \eta + K_4 s + \mathbf{K}(\mathbf{z})$$

In order for $L^{1,s}$ and $\varphi^{1,s}$ to be continuous along spouse 2's participation frontier, it must be the case that:

$$L^{1,s} = L^1 + g \cdot L^2$$

$$\varphi^{1,s} = \varphi + h \cdot L^2,$$

where g and h are free parameters. These conditions imply that:

$$\frac{\partial L^{1,s}}{\partial \omega_2} = \frac{\partial L^1}{\partial \omega_2} + g \frac{\partial L^2}{\partial \omega_2} \tag{23a}$$

$$\frac{\partial L^{1,s}}{\partial \eta} = \frac{\partial L^1}{\partial \eta} + g \frac{\partial L^2}{\partial \eta} \tag{23b}$$

$$\frac{\partial \varphi^{1,s}}{\partial \omega_2} = \frac{\partial \varphi}{\partial \omega_2} + h \frac{\partial L^2}{\partial \omega_2} \tag{23c}$$

$$\frac{\partial \varphi^{1,s}}{\partial \eta} = \frac{\partial \varphi}{\partial \eta} + h \frac{\partial L^2}{\partial \eta} \tag{23d}$$

Combined with the partial differential equation $\frac{\partial \varphi^{1,s}}{\partial \omega_2} - A \frac{\partial \varphi^{1,s}}{\partial \eta} = 0$, which holds within spouse 2's non-participation set, the functional forms of L^1 in equation 6 and L^2 in equation 7, and the sharing rule parameters in equations 22a–22d we can see that:

$$\frac{\partial \varphi}{\partial \omega_2} + h\frac{\partial L^2}{\partial \omega_2} = \frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\left[\frac{\partial \varphi}{\partial \eta} + h\frac{\partial L^2}{\partial \eta}\right]$$

$$\Rightarrow h = \frac{\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial \varphi}{\partial \eta} - \frac{\partial \varphi}{\partial \omega_2}}{\frac{\partial L^2}{\partial \omega_2} - \frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}$$

$$\Rightarrow h = \frac{\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{a_3 b_4}{\Delta} - \frac{a_2 b_4}{\omega_2 \Delta}}{\frac{\partial L^2}{\partial \omega_2} - \frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}$$

$$\Rightarrow h = \frac{\frac{b_4}{\Delta}\left[\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}a_3 - \frac{a_2}{\omega_2}\right]}{\frac{\partial L^2}{\partial \omega_2} - \frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}$$

$$\Rightarrow h = \frac{\frac{b_4}{\Delta}\left[\frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^1}{\partial \eta} - \frac{\partial L^1}{\partial \omega_2}\right]}{\frac{\partial L^2}{\partial \omega_2} - \frac{\frac{\partial L^{1,s}}{\partial \omega_2}}{\frac{\partial L^{1,s}}{\partial \eta}}\frac{\partial L^2}{\partial \eta}}$$

$$\Rightarrow h = \frac{b_4}{\Delta}\frac{\frac{\partial L^{1,s}}{\partial \omega_2}\frac{\partial L^1}{\partial \eta} - \frac{\partial L^1}{\partial \omega_2}\frac{\partial L^{1,s}}{\partial \eta}}{\frac{\partial L^2}{\partial \omega_2}\frac{\partial L^{1,s}}{\partial \eta} - \frac{\partial L^{1,s}}{\partial \omega_2}\frac{\partial L^2}{\partial \eta}}$$

Plugging in equations 23a and 23b, we obtain:

$$h = \frac{b_4}{\Delta}\frac{\left[\frac{\partial L^1}{\partial \omega_2} + g\frac{\partial L^2}{\partial \omega_2}\right]\frac{\partial L^1}{\partial \eta} - \frac{\partial L^1}{\partial \omega_2}\left[\frac{\partial L^1}{\partial \eta} + g\frac{\partial L^2}{\partial \eta}\right]}{\frac{\partial L^2}{\partial \omega_2}\left[\frac{\partial L^1}{\partial \eta} + g\frac{\partial L^2}{\partial \eta}\right] - \left[\frac{\partial L^1}{\partial \omega_2} + g\frac{\partial L^2}{\partial \omega_2}\right]\frac{\partial L^2}{\partial \eta}}$$

$$\Rightarrow h = \frac{b_4}{\Delta}\frac{g\frac{\partial L^2}{\partial \omega_2}\frac{\partial L^1}{\partial \eta} - g\frac{\partial L^1}{\partial \omega_2}\frac{\partial L^2}{\partial \eta}}{\frac{\partial L^2}{\partial \omega_2}\frac{\partial L^1}{\partial \eta} - \frac{\partial L^1}{\partial \omega_2}\frac{\partial L^2}{\partial \eta}}$$

$$\Rightarrow h = g\frac{b_4}{\Delta}$$

The restriction that $L^{1,s} = L^1 + g \cdot L^2$ implies that:

$$\begin{aligned}
L^{1,s} =& L^1 + gL^2 \\
=& a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \mathbf{a}_5 \mathbf{z} \\
& + gb_0 + gb_1 \log \omega_1 + gb_2 \log \omega_2 + gb_3 \eta + gb_4 s + g \mathbf{b}_5 \mathbf{z} \\
=& \underbrace{(a_0 + gb_0)}_{A_0} + \underbrace{(a_1 + gb_1)}_{A_1} \log \omega_1 + \underbrace{(a_2 + gb_2)}_{A_2} \log \omega_2 + \underbrace{(a_3 + gb_3)}_{A_3} \eta + \underbrace{(a_4 + gb_4)}_{A_4} s + \underbrace{(a_5 + gb_5)}_{A_5} \mathbf{z}
\end{aligned}$$

The restriction that $\varphi^{1,s} = \varphi + h \cdot L^2$ and using $h = g\frac{b_4}{\Delta}$ implies that:

$$\begin{aligned}
\varphi^{1,s} =& \varphi + g\frac{b_4}{\Delta} L^2 \\
=& \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z}) \\
& + g\frac{b_4}{\Delta}(b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + \mathbf{b}_5 \mathbf{z}) \\
=& \frac{1}{\Delta}[\underbrace{(a_4 b_1 + gb_1 b_4)}_{A_4 b_1} \log \omega_1 + \underbrace{(a_2 b_4 + gb_2 b_4)}_{A_2 b_4} \log \omega_2 + \underbrace{(a_3 b_4 + gb_3 b_4)}_{A_3 b_4} \eta + \underbrace{(a_4 b_4 + gb_4^2)}_{A_4 b_4} s] + \underbrace{\kappa(\mathbf{z}) + g\frac{b_4}{\Delta}\mathbf{b}_5 \mathbf{z}}_{\tilde{\kappa}^{1,s}(\mathbf{z})}
\end{aligned}$$

Recall that the Marshallian labor supplies take the following form:

$$\lambda^1 = \alpha_1^{1,s} \log \omega_1 + \alpha_2^{1,s} \varphi^{1,s} + \alpha_3^{1,s} \mathbf{z}$$

$$\lambda^2 = \beta_1^{1,s} \log \omega_2 + \beta_2^{1,s} (\eta - \varphi^{1,s}) + \beta_3^{1,s} \mathbf{z}$$

Beginning with λ^1, let $\alpha_2^{1,s} = \frac{\Delta}{b_4}$. Expanding the expression for $\varphi^{1,s}$, we obtain:

$$\begin{aligned}
\lambda^1 &= \alpha_1^{1,s} \log \omega_1 + \alpha_2^{1,s} \varphi^{1,s} + \alpha_3^{1,s} \mathbf{z} \\
&= \alpha_1^{1,s} \log \omega_1 + \frac{A_4 b_1}{b_4} \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \frac{\Delta}{b_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{1,s} \mathbf{z} \\
&= \underbrace{\left(\alpha_1^{1,s} + \frac{A_4 b_1}{b_4} \right)}_{=A_1} \log \omega_1 + A_2 \log \omega_2 + A_3 \eta + A_4 s + \underbrace{\frac{\Delta}{b_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{1,s} \mathbf{z}}_{=\tilde{\alpha}_3^{\mathbf{1,s}}(\mathbf{z})}
\end{aligned}$$

In order for λ^1 to be consistent with $L^{1,s}$, it must be the case that $\alpha_1^{1,s} + \frac{A_4 b_1}{b_4} = A_1$, implying that $\alpha_1^{1,s} = A_1 - \frac{A_4 b_1}{b_4} = \frac{A_1 b_4 - A_4 b_1}{b_4}$.

Similarly, moving to λ^2, let $\beta_2^{1,s} = -\frac{\Delta}{A_4}$. Expanding the expression for $\varphi^{1,s}$, we obtain:

$$\begin{aligned}
\lambda^2 &= \beta_1^{1,s}\log\omega_2 + \beta_2^{1,s}(\eta-\varphi^{1,s}) + \beta_3^{1,s}\mathbf{z} \\
&= \beta_1^{1,s}\log\omega_2 - \frac{\Delta}{A_4}\eta + b_1\log\omega_1 + \frac{A_2b_4}{A_4}\log\omega_2 + \frac{A_3b_4}{A_4}\eta + b_4 s + \frac{\Delta}{A_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{1,s}\mathbf{z} \\
&= b_1\log\omega_1 + \left(\beta_1^{1,s} + \frac{A_2b_4}{A_4}\right)\log\omega_2 + \left(\frac{A_3b_4 - a_3b_4 + a_4b_3}{A_4}\right)\eta + b_4 s + \frac{\Delta}{A_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{1,s}\mathbf{z} \\
&= b_1\log\omega_1 + \left(\beta_1^{1,s} + \frac{A_2b_4}{A_4}\right)\log\omega_2 + \left(\frac{A_3b_4 - A_3b_4 + A_4b_3}{A_4}\right)\eta + b_4 s + \frac{\Delta}{A_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{1,s}\mathbf{z} \\
&= b_1\log\omega_1 + \underbrace{\left(\beta_1^{1,s} + \frac{A_2b_4}{A_4}\right)}_{=b_2}\log\omega_2 + b_3\eta + b_4 s + \underbrace{\frac{\Delta}{A_4}\kappa(\mathbf{z}) + \beta_3^{1,s}\mathbf{z}}_{=\tilde{\beta}_3^{\mathbf{1,s}}(\mathbf{z})}
\end{aligned}$$

In order for λ^2 to be consistent with L^2, it must be the case that $\beta_1^{1,s} + \frac{A_2b_4}{A_4} = b_2$, implying that $\beta_1^{1,s} = b_2 - \frac{A_2b_4}{A_4} = \frac{A_4b_2 - A_2b_4}{A_4}$.

Finally, the indirect utility functions in this situation become:

$$V_{1,s}^1(\omega_1, \varphi^{1,s}, \mathbf{z}) = \left(\frac{e^{\alpha_2^{1,s}\omega_1}}{\alpha_2^{1,s}}\right)(\alpha_1^{1,s}\log\omega_1 + \alpha_2^{1,s}\varphi^{1,s} + \alpha_3^{1,s}\mathbf{z}) \tag{24}$$

$$V_{1,s}^2(\omega_2, \eta-\varphi^{1,s}, \mathbf{z}) = \left(\frac{e^{\beta_2^{1,s}\omega_2}}{\beta_2^{1,s}}\right)(\beta_1^{1,s}\log\omega_2 + \beta_2^{1,s}(\eta-\varphi^{1,s}) + \beta_3^{1,s}\mathbf{z}), \tag{25}$$

and the relative utility weight becomes:

$$\mu^{1,s} = \frac{\frac{\partial V_{1,s}^1}{\partial\varphi^{1,s}}}{\frac{\partial V_{1,s}^2}{\partial(\eta-\varphi^{1,s})}} = \frac{e^{\alpha_2^{1,s}}\omega_1}{e^{\beta_2^{1,s}}\omega_2} \tag{26}$$

Now consider the situation in which only spouse 2 works. Following Donni (2003), I assume that if spouse 1 does not work then spouse 2's unrestricted labor supply function switches to:

$$L^{2,s} = B_0 + B_1\log\omega_1 + B_2\log\omega_2 + B_3\eta + B_4 s + \mathbf{B}_5\mathbf{z},$$

and the sharing rule switches to:

$$\varphi^{2,s} = P_1 \log \omega_1 + P_2 \log \omega_2 + P_3 \eta + P_4 s + \mathbf{P}(\mathbf{z})$$

In order for $L^{2,s}$ and $\varphi^{2,s}$ to be continuous along spouse 1's participation frontier, it must be the case that:

$$L^{2,s} = L^2 + j \cdot L^1$$

$$\varphi^{2,s} = \varphi + k \cdot L^1,$$

where j and k are free parameters. These conditions imply that:

$$\frac{\partial L^{2,s}}{\partial \omega_1} = \frac{\partial L^2}{\partial \omega_1} + j \frac{\partial L^1}{\partial \omega_1} \tag{27a}$$

$$\frac{\partial L^{2,s}}{\partial \eta} = \frac{\partial L^2}{\partial \eta} + j \frac{\partial L^1}{\partial \eta} \tag{27b}$$

$$\frac{\partial \varphi^{2,s}}{\partial \omega_1} = \frac{\partial \varphi}{\partial \omega_1} + k \frac{\partial L^1}{\partial \omega_1} \tag{27c}$$

$$\frac{\partial \varphi^{2,s}}{\partial \eta} = \frac{\partial \varphi}{\partial \eta} + k \frac{\partial L^1}{\partial \eta} \tag{27d}$$

Combined with the partial differential equation $\frac{\partial \varphi^{2,s}}{\partial \omega_1} - B \frac{\partial \varphi^{2,s}}{\partial \eta} = -B$, which holds within spouse 1's non-participation set, the functional forms of L^1 in equation 6 and L^2 in equation 7, and the sharing rule parameters in equations 22a–22d we can see that:

$$\frac{\partial\varphi}{\partial\omega_1}+k\frac{\partial L^1}{\partial\omega_1}=\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\left[\frac{\partial\varphi}{\partial\eta}+k\frac{\partial L^1}{\partial\eta}\right]-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}$$

$$\Rightarrow k=\frac{\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial\varphi}{\partial\eta}-\frac{\partial\varphi}{\partial\omega_1}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}}{\frac{\partial L^1}{\partial\omega_1}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial L^1}{\partial\eta}}$$

$$\Rightarrow k=\frac{\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{a_3b_4}{\Delta}-\frac{a_4b_1}{\omega_1\Delta}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}}{\frac{\partial L^1}{\partial\omega_1}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial L^1}{\partial\eta}}=\frac{\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\left[1+\frac{a_4b_3}{\Delta}\right]-\frac{a_4b_1}{\omega_1\Delta}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}}{\frac{\partial L^1}{\partial\omega_1}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial L^1}{\partial\eta}}$$

$$\Rightarrow k=\frac{\frac{a_4}{\Delta}\left[\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}b_3-\frac{b_1}{\omega_1}\right]}{\frac{\partial L^1}{\partial\omega_1}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial L^1}{\partial\eta}}$$

$$\Rightarrow k=\frac{\frac{a_4}{\Delta}\left[\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial L^2}{\partial\eta}-\frac{\partial L^2}{\partial\omega_1}\right]}{\frac{\partial L^1}{\partial\omega_1}-\frac{\frac{\partial L^{2,s}}{\partial\omega_1}}{\frac{\partial L^{2,s}}{\partial\eta}}\frac{\partial L^1}{\partial\eta}}$$

$$\Rightarrow k=\frac{a_4}{\Delta}\frac{\frac{\partial L^{2,s}}{\partial\omega_1}\frac{\partial L^2}{\partial\eta}-\frac{\partial L^2}{\partial\omega_1}\frac{\partial L^{2,s}}{\partial\eta}}{\frac{\partial L^1}{\partial\omega_1}\frac{\partial L^{2,s}}{\partial\eta}-\frac{\partial L^{2,s}}{\partial\omega_1}\frac{\partial L^1}{\partial\eta}}$$

Plugging in equations 27a and 27b, we obtain:

$$k=\frac{a_4}{\Delta}\frac{\left[\frac{\partial L^2}{\partial\omega_1}+j\frac{\partial L^1}{\partial\omega_1}\right]\frac{\partial L^2}{\partial\eta}-\frac{\partial L^2}{\partial\omega_1}\left[\frac{\partial L^2}{\partial\eta}+j\frac{\partial L^1}{\partial\eta}\right]}{\frac{\partial L^1}{\partial\omega_1}\left[\frac{\partial L^2}{\partial\eta}+j\frac{\partial L^1}{\partial\eta}\right]-\left[\frac{\partial L^2}{\partial\omega_1}+j\frac{\partial L^1}{\partial\omega_1}\right]\frac{\partial L^1}{\partial\eta}}$$

$$\Rightarrow k=\frac{a_4}{\Delta}\frac{j\frac{\partial L^1}{\partial\omega_1}\frac{\partial L^2}{\partial\eta}-j\frac{\partial L^2}{\partial\omega_1}\frac{\partial L^1}{\partial\eta}}{\frac{\partial L^1}{\partial\omega_1}\frac{\partial L^2}{\partial\eta}-\frac{\partial L^2}{\partial\omega_1}\frac{\partial L^1}{\partial\eta}}$$

$$\Rightarrow k=j\frac{a_4}{\Delta}$$

The restriction that $L^{2,s} = L^2 + j \cdot L^1$ implies that:

$$
\begin{aligned}
L^{2,s} =& L^2 + jL^1 \\
=& b_0 + b_1 \log \omega_1 + b_2 \log \omega_2 + b_3 \eta + b_4 s + \mathbf{b}_5 \mathbf{z} \\
& + ja_0 + ja_1 \log \omega_1 + ja_2 \log \omega_2 + ja_3 \eta + ja_4 s + j\mathbf{a}_5 \mathbf{z} \\
=& \underbrace{(b_0 + ja_0)}_{B_0} + \underbrace{(b_1 + ja_1)}_{B_1} \log \omega_1 + \underbrace{(b_2 + ja_2)}_{B_2} \log \omega_2 + \underbrace{(b_3 + ja_3)}_{B_3} \eta + \underbrace{(b_4 + ja_4)}_{B_4} s + \underbrace{(b_5 + ja_5)}_{B_5} \mathbf{z}
\end{aligned}
$$

The restriction that $\varphi^{2,s} = \varphi + k \cdot L^1$ and using $k = j\frac{a_4}{\Delta}$ implies that:

$$
\begin{aligned}
\varphi^{2,s} =& \varphi + j\frac{a_4}{\Delta} L^1 \\
=& \frac{1}{\Delta}(a_4 b_1 \log \omega_1 + a_2 b_4 \log \omega_2 + a_3 b_4 \eta + a_4 b_4 s) + \kappa(\mathbf{z}) \\
& + j\frac{a_4}{\Delta}(a_0 + a_1 \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \mathbf{a}_5 \mathbf{z}) \\
=& \frac{1}{\Delta}[\underbrace{(a_4 b_1 + ja_1 a_4)}_{a_4 B_1} \log \omega_1 + \underbrace{(a_2 b_4 + ja_2 a_4)}_{a_2 B_4} \log \omega_2 + \underbrace{(a_3 b_4 + ja_3 a_4)}_{a_3 B_4} \eta + \underbrace{(a_4 b_4 + ja_4^2)}_{a_4 B_4} s] + \underbrace{\kappa(\mathbf{z}) + j\frac{a_4}{\Delta}\mathbf{a}_5 \mathbf{z}}_{\tilde{\kappa}^{2,s}(\mathbf{z})}
\end{aligned}
$$

Recall that the Marshallian labor supplies take the following form:

$$
\lambda^1 = \alpha_1^{2,s} \log \omega_1 + \alpha_2^{2,s} \varphi^{2,s} + \alpha_3^{2,s} \mathbf{z}
$$

$$
\lambda^2 = \beta_1^{2,s} \log \omega_2 + \beta_2^{2,s} (\eta - \varphi^{2,s}) + \beta_3^{2,s} \mathbf{z}
$$

Beginning with λ^1, let $\alpha_2^{2,s} = \frac{\Delta}{B_4}$. Expanding the expression for $\varphi^{2,s}$, we obtain:

$$
\begin{aligned}
\lambda^1 &= \alpha_1^{2,s} \log \omega_1 + \alpha_2^{2,s} \varphi^{2,s} + \alpha_3^{2,s} \mathbf{z} \\
&= \alpha_1^{2,s} \log \omega_1 + \frac{a_4 B_1}{B_4} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \frac{\Delta}{B_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{2,s} \mathbf{z} \\
&= \underbrace{\left(\alpha_1^{2,s} + \frac{a_4 B_1}{B_4} \right)}_{=a_1} \log \omega_1 + a_2 \log \omega_2 + a_3 \eta + a_4 s + \underbrace{\frac{\Delta}{B_4} \tilde{\kappa}(\mathbf{z}) + \alpha_3^{2,s} \mathbf{z}}_{=\tilde{\alpha}_3^{2,s}(\mathbf{z})}
\end{aligned}
$$

In order for λ^1 to be consistent with $L^{1,s}$, it must be the case that $\alpha_1^{2,s} + \frac{a_4 B_1}{B_4} = a_1$, implying that $\alpha_1^{2,s} = a_1 - \frac{a_4 B_1}{B_4} = \frac{a_1 B_4 - a_4 B_1}{B_4}$.

Similarly, moving to λ^2, let $\beta_2^{2,s} = -\frac{\Delta}{a_4}$. Expanding the expression for $\varphi^{2,s}$, we obtain:

$$\begin{aligned}
\lambda^2 &= \beta_1^{2,s}\log\omega_2 + \beta_2^{2,s}(\eta - \varphi^{2,s}) + \beta_3^{2,s}\mathbf{z} \\
&= \beta_1^{2,s}\log\omega_2 - \frac{\Delta}{a_4}\eta + B_1\log\omega_1 + \frac{a_2B_4}{a_4}\log\omega_2 + \frac{a_3B_4}{a_4}\eta + B_4 s + \frac{\Delta}{a_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{2,s}\mathbf{z} \\
&= B_1\log\omega_1 + \left(\beta_1^{2,s} + \frac{a_2B_4}{a_4}\right)\log\omega_2 + \left(\frac{a_3B_4 - a_3b_4 + a_4b_3}{a_4}\right)\eta + B_4 s + \frac{\Delta}{a_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{2,s}\mathbf{z} \\
&= B_1\log\omega_1 + \left(\beta_1^{2,s} + \frac{a_2B_4}{a_4}\right)\log\omega_2 + \left(\frac{a_3B_4 - a_3B_4 + a_4B_3}{a_4}\right)\eta + B_4 s + \frac{\Delta}{a_4}\tilde{\kappa}(\mathbf{z}) + \beta_3^{2,s}\mathbf{z} \\
&= B_1\log\omega_1 + \underbrace{\left(\beta_1^{2,s} + \frac{a_2B_4}{a_4}\right)}_{=B_2}\log\omega_2 + B_3\eta + B_4 s + \underbrace{\frac{\Delta}{a_4}\kappa(\mathbf{z}) + \beta_3^{2,s}\mathbf{z}}_{=\tilde{\beta}_3^{2,s}(\mathbf{z})}
\end{aligned}$$

In order for λ^2 to be consistent with L^2, it must be the case that $\beta_1^{2,s} + \frac{a_2B_4}{a_4} = B_2$, implying that $\beta_1^{2,s} = B_2 - \frac{a_2B_4}{a_4} = \frac{a_4B_2 - a_2B_4}{a_4}$.

Finally, the indirect utility functions in this situation become:

$$V_{2,s}^1(\omega_1, \varphi^{2,s}, \mathbf{z}) = \left(\frac{e^{\alpha_2^{2,s}\omega_1}}{\alpha_2^{2,s}}\right)(\alpha_1^{2,s}\log\omega_1 + \alpha_2^{2,s}\varphi^{2,s} + \alpha_3^{2,s}\mathbf{z}) \tag{28}$$

$$V_{2,s}^2(\omega_2, \eta - \varphi^{2,s}, \mathbf{z}) = \left(\frac{e^{\beta_2^{2,s}\omega_2}}{\beta_2^{2,s}}\right)(\beta_1^{2,s}\log\omega_2 + \beta_2^{2,s}(\eta - \varphi^{2,s}) + \beta_3^{2,s}\mathbf{z}), \tag{29}$$

and the relative utility weight becomes:

$$\mu^{2,s} = \frac{\frac{\partial V_{2,s}^1}{\partial\varphi^{2,s}}}{\frac{\partial V_{2,s}^2}{\partial(\eta - \varphi^{2,s})}} = \frac{e^{\alpha_2^{2,s}\omega_1}}{e^{\beta_2^{2,s}\omega_2}} \tag{30}$$