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## DISCUSSION PAPER SERIES

IZA DP No. 17061

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JUNE 2024

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## ABSTRACT

## Work Schedules*

In a new model of work schedules, employers choose the number of working hours and either dictate the exact hours to be worked or delegate that decision to workers via flextime. Workers' preferences over schedules influence their productivities. An inverted-Ushaped hours-output profile arises; flextime policies shift its peak to the right. Long hours are found to go hand-in-hand with flextime, and the employer finds flextime less appealing when wages exogenously increase. Analysis of a worker-employer matched panel of British workplaces surveyed in 2004 and 2011 reveals that flextime and other flexible work practices mitigate the productivity-eroding consequences of long hours.

JEL Classification:
J20, J23, J24, J32, M52, M50, M59
Keywords:
work hours, labor productivity, human resources management practices, flextime, work-life flexibility, workplace flexibility, work schedules, scheduling, working from home, flexible work practices, diminishing returns

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## 1. Introduction

Work schedules - both their determination and their productivity consequences - are a neglected research domain in labor economics. The bulk of scholarly attention - covered in the vast literatures on labor supply and labor demand - has focused on how many hours are worked rather than on which hours are worked. The present study offers an examination of work schedules through theoretical and empirical lenses.

I begin the inquiry with the basic observation that the longer one works, the more one can produce. While that premise is uncontroversial, it has limits that are of considerable consequence for productivity in the workplace and for social welfare. The issue is that not all hours in a worker's schedule are created equal. Hours worked at the end of a long workday, or week, are likely to be less productive than those worked earlier. That "law of diminishing marginal returns" states that the marginal product of an input to production (i.e., the additional output that is generated when a bit more of that input is used in production) shrinks when that input quantity is sufficiently high, as long as other inputs remain constant. In extreme cases, such returns may be negative as well as diminishing. A negative marginal product means that an additional work hour actually reduces total output. ${ }^{1}$

The potential mechanisms for diminishing (or negative) returns to working hours are intuitive and obvious to anyone who has worked a very long shift or workweek. Exhaustion sets in beyond a certain point, and one's physical and cognitive skills diminish, which in turn leads to a slower pace of work, to mistakes, and in extreme cases to accidents and health problems that can result in absenteeism and further productivity losses. ${ }^{2}$ Moreover, in a team setting such

[^1]negative productivity consequences of long hours can have spillover effects that hurt the productivity of other workers, including those who may not be overworked. ${ }^{3}$

These observations concern diminishing or negative returns near the end of a batch of working hours. Also relevant for productivity is which particular batch of hours is worked. Consider two alternative ten-hour shifts: an early one that begins at 7am, and a late one that begins at 2 pm . An "early bird" would prefer the early shift, whereas a "night owl" would prefer the late shift. Even if assigned to their preferred shifts where they can hit the ground running, both of these workers might well experience diminishing returns to productivity as exhaustion sets in towards the end of their shifts. If they are assigned to the shifts they dislike, however, productivity may be depressed throughout the entire shift, and diminishing returns might set in even earlier. For example, the night owl who is compelled to clock in at 7am might perform sluggishly right out of the starting gate, and the tiredness may have a compounding effect as the shift wears on.

The example highlights an employer's interests in assigning work schedules optimally. Assigning workers to the hours in which they are most rested, alert, happy, and willing to work should boost productivity and stave off the point of exhaustion and diminishing returns. Identifying those hours is tricky, however, because they are often part of the worker's private information. To make matters worse, they change over time, even day to day for the same worker. ${ }^{4}$ One solution is for the employer to delegate to the worker the authority to choose the work schedule, thereby harnessing the worker's private information to enhance both parties' welfare. That is indeed the best approach when both parties' incentives are fully aligned, but when there is misalignment then delegation can exacerbate other problems. For example, under delegation, an early bird would choose to work the early shift even if most of the firm's customers are night owls. Such considerations induce employer preferences over work hours that may differ from workers' preferences. That misalignment creates a tradeoff.

In this study I present a new theoretical framework for work schedules that captures the preceding ideas and provides an interpretation for diminishing returns and how they might be mitigated by flextime policies. The framework offers insights for interpreting empirical hours-

[^2]output profiles. For example, it explains why long hours and flextime policies tend to go hand-in-hand. The framework can be readily extended to analyze important phenomena beyond those addressed here (e.g., overtime regulations, hours caps and floors, and compensatory time off). Evidence from the most recent waves of the British Workplace Employment Relations Study is presented, suggesting a non-monotonic hours-output profile in which output increases with work hours and then decreases when hours surpass an inflection point. ${ }^{5}$ Moreover, the peak of this inverted- $U$-shaped relationship is shifted to the right by flextime policies. Long working hours and flextime are found to go hand-in-hand, as the theoretical framework predicts.
Complementary results are found for alternative flexible work practices such as working from home during normal business hours (a timely result in light of the COVID-19 pandemic) and worker-initiated shift changes, a restrictive variant of flextime.

In the theoretical model, the employer chooses the number of work hours and also the scheduling policy (i.e., either dictating which specific hours must be worked or delegating that decision to the worker via flextime). Under flextime, the worker then selects hours to maximize utility, taking the number of hours as given. Although this sequential structure resembles a standard principal-agent framework, the model has some unique features. There is no incentive pay. Rather, the wage per hour is fixed externally to the model in advance. Thus, once the employer has set the number of hours, the daily or weekly wage bill is also fixed. Rather than influencing worker behavior via choosing a (continuous) slope of an incentive contract as in the standard principal-agent model, the employer does it via choosing the (discrete) scheduling policy. The employer jointly chooses the number of hours and the scheduling policy to maximize the total amount of output the worker produces in a day or week.

The model has an interior solution because fatigue can cause total output to fall with hours and because of an assumed negative relationship between the worker's disutility of working during a particular hour of the day and a worker's productivity during that hour. The key assumption is that the worker's privately observed preferences over work schedules appear as production function parameters, such that workers are more productive when they like their schedules. The model's central tradeoff derives from a misalignment in employer and worker incentives that arises from their divergent preferences over work schedules. Other things equal, the employer prefers that work schedules coincide with the hours in which stochastic

[^3]product demand is expected to be highest, as observed by both parties. In contrast, the worker's preferences over alternative schedules are private information and independent of product demand. Delegation enables the worker to choose a work schedule that might overlap minimally with the peak hours of high demand. On the other hand, delegation has the advantage of empowering the worker to leverage their private information in a way that benefits both parties, because the worker's preferred hours are those in which they are most productive (independent of product demand). In particular, exhaustion accrues more slowly, and the point of diminishing returns is delayed, when workers are happy with their schedules.

The model delivers an inverted- $U$-shaped hours-output profile. A flextime policy pushes the peak of that profile to the right, so that long hours go hand-in-hand with flextime. Employers choose higher hours when they choose flextime for two reasons. First, workers do not tire as quickly with respect to an additional hour because they are happier with their schedules. Second, given that under flextime the inframarginal hours are generally not the high-product-demand ones the employer prefers, the expected desirability (to the employer) of an additional hour is greater under flextime than under employer-determined scheduling.

The model yields further insights that have policy implications. For example, exogenous increases in the fixed wage, such as those arising from collective bargaining agreements or changes in minimum wage legislation, make the employer less likely to choose a flextime policy. Standard labor-demand theory predicts that hours drop in response to an increase in the minimum wage, but that theory focuses on the number of hours and not on the means by which those hours are scheduled (specifically which hours in the workday are worked). The model reveals that a higher minimum wage makes employer-determined scheduling relatively more attractive, and because flextime and long hours go hand-in-hand, a switch from flextime to employer-determined scheduling implies a reduction in hours beyond that which would occur via standard labor-demand theory if the scheduling policy were to remain unchanged.

## 2. Background and Related Literature

This study's theoretical framework is designed to simultaneously address two questions: How many hours should an employee work? Given that number of hours, exactly which hours should the employee work? The first question, i.e., the determination of working hours, has been addressed in a vast literature in labor economics that has evolved over more than a half century. Pencavel (2016b) recounts the intellectual history of that literature, which dates back to Lewis
(1957), and calls attention to the central identification problem that is endemic to it. The question is whether estimated relationships between working hours and wages should be interpreted as labor supply functions (i.e., hours are chosen by workers) or as labor demand functions (i.e., hours are chosen by employers).

That identification problem was recognized in the early development of the literature (e.g., Feldstein 1968, Rosen 1969, Abbott and Ashenfelter 1976) but was then forgotten during more than 40 years of subsequent research that implicitly resolved the identification question in favor of worker-chosen hours. ${ }^{6}$ Pencavel (2016b) calls for greater recognition of a role for the influence of employers' preferences on working hours. ${ }^{7}$ This study responds to that call given that employers choose working hours in the theoretical framework. An important point, however, is that employers in the model make that choice in consideration of workers' preferences over work schedules, i.e., employers assign hours in anticipation of the implications of workers’ behavioral responses for subsequent labor productivity. This feature of the present work contrasts with the standard approach in the labor demand literature, in which employers choose working hours without regard to workers' scheduling preferences.

In contrast to the first question that opened this section, the second has been largely neglected in the literature. To my knowledge, this study is the first to provide a model of work schedules in which employers set the number of working hours and either assign the specific hours to be worked or delegate that decision to the workers via a flextime policy. ${ }^{8}$ Although

[^4]${ }^{7}$ Arguments that some workers' preferences over hours are constrained by employers, so that workers' actual hours diverge from their preferred hours at their observed wage rates, can be found in many studies, including Blundell and Walker (1982), Moffitt (1982), Altonji and Paxson (1988), Card (1990), Kahn and Lang (1991), Dickens and Lundberg (1993), Rebitzer and Taylor (1995), and Bell (1998). See Stewart and Swaffield (1997) and Golden and Gebreselassie (2007) for evidence that many workers report that their desired hours at their observed hourly wage rate diverge from those they are actually working. See Golden (2015) and the references therein for survey evidence on workers' preferences for fewer hours than they currently work. That study also provides a theoretical framework in which actual weekly hours are determined by the interaction of three forces (worker preferences, employer preferences, and the institutional environment in which workers and employers operate). DeVaro (2022) provides evidence suggesting that observed hours reflect a blend of employer and worker choices.
${ }^{8}$ This is not, however, the first attempt to address flexible work schedules in a theoretical model. Altman and Golden (2007) develop a theoretical model to "account for the persistent disequilibrium in the market for flex-time given, the evidence that many of the workers who want flexible scheduling cannot get it." The authors advance a behavioral x-efficiency explanation of "persistent excess demand" for flextime. Their analysis differs significantly from mine in a number of respects, and the reader is referred to their study for further details as well as for illuminating descriptive information concerning trends in the use of flexible work practices. Hamermesh (1999) presents a theoretical framework in which work schedules arise as a consequence of a matching process between heterogeneous workers and employers, in a compensating differentials equilibrium where night shifts are
work scheduling is a neglected research topic in economics, there is a literature on the productivity effects of different work schedules in industrial engineering and occupational health and safety journals. For points of entry into these literatures see the following examples and the references cited therein: Hanna et al. (2008), Arriagada and Alarcón (2014), McConkey and Koromyslova (2018), and Cheng, Huang, and Hutomo (2018).

Evidence of the damaging effect of long working hours on productivity is provided in Pencavel (2015), using a sample of British munition workers assembling artillery shells during the First World War. A nonlinear, "inverted- $U$-shaped" hours-output relationship is found in which long hours reduced productivity. ${ }^{9}$ The study concludes with a call to re-examine its principal finding using "contemporary data" on a broader sample. ${ }^{10}$ The present study builds on Pencavel (2015) in three ways. First, it presents a theoretical model of work schedules that predicts both an inverted- $U$-shaped hours-output profile and that flextime practices shift its peak to the right, so that long hours go hand-in-hand with flexible work practices. Second, it responds to Pencavel's call to re-examine the hours-output relationship using contemporary data spanning a wide variety of occupations and industries operating under different conditions, confirming the evidence of an inverted $U$. Third, it provides empirical evidence that flextime policies mitigate the deleterious effect of long working hours on workplace labor productivity, with corroborating results from two alternative measures of flexible work practices.

Ibanez and Toffel (2020) find evidence suggesting diminished performance near the end of work shifts (i.e., fewer violations cited by food inspectors when inspections would risk prolonging their standard workday). Brachet, David, and Drechsler (2012) find evidence that the

[^5]performance of paramedics responding to emergency medical incidents in Mississippi deteriorated towards the end of long shifts, a result that the authors attribute to fatigue. Analyzing daily data on agents in a call center in the Netherlands, Collewet and Sauermann (2017) find that the average handling time for a call (which is a measure of productivity) increased with the number of working hours, a result that the authors attribute to fatigue. Examining the effect of a workweek reduction policy on the labor productivity of manufacturing plants in Korea, Park and Park (2019) find that the policy reduced per-worker hours but increased per-worker output. Danziger, Levav, and Avnaim-Pesso (2011) find that judges delivered a higher favorable percentage in parole hearings after a meal break, a result they attribute to mental depletion (exhaustion) and replenishment.

Baucells and Zhao (2019) propose a continuous-time, fatigue disutility model that captures the notion that fatigue accumulates with effort, decays with rest, and increases marginal disutility. Their model highlights the beneficial role of breaks in reducing fatigue. That study focuses on the efficient time profile of effort, whereas the present study focuses on the determination of hours. Moreover, it does not address the tension between worker and employer preferences that is central to the determination of work schedules in the present study.

The present model's assumption that workers' preferences over hours may diverge from their employers' preferences appears in Deardorff and Stafford (1976), where there are two factors of production, the owners of which have different preferences concerning when in the workday those factors should be employed. That technology is such that production requires both factors to be simultaneously present during the workday, which in turn generates employers' preferences over hours. That model, which features no uncertainty, does not consider the employer's choice between dictating the specific hours to be worked and delegating that decision to the worker under uncertainty about the worker's preferences over hours. It focuses on a set of compensating differentials that compensate the factors of production for the unpleasantness of being simultaneously present, as the employer desires. Rather than relying on compensating differentials to resolve the misalignment in preferences between the employer and the worker, the present model treats the wage as predetermined ${ }^{11}$ and allows the employer to balance the advantages of efficient assignment (of workers to hours) against the advantages of staving off workers' fatigue and exhaustion by delegating the scheduling of hours to the worker.

[^6]Labanca and Pozzoli (2021) recognize a tradeoff firms face in that if they coordinate working hours across employees, there is a benefit to firm productivity, but imposing common schedules across heterogeneous workers requires that a compensating differential be paid to compensate workers for distasteful schedules. Using Danish, linked, employer-employee data, they document evidence of within-firm positive correlations among wages, productivity, and coordination of working hours across employees (as measured by the dispersion of hours).

As a job amenity that workers value, flextime is properly understood as a non-pecuniary component of compensation. Thus, the present study contributes to the empirical literature on fringe benefits, much of which attempts to identify a tradeoff between wages and benefits as predicted by the theory of compensating differentials, an endeavor which has proven to be challenging due to daunting data demands. ${ }^{12}$ The present study addresses compensating differentials in an extension. Its main focus, however, is on how flextime relates to long working hours and to workplace productivity, and on exploring the nature of the productivity enhancements that accompany flextime and how employers balance those against the costs that arise when workers (whose preferences over work schedules are generally misaligned with those of their employers) can choose their own schedules.

Foreshadowing the present study, Eriksson and Kristensen (2014) write of the employer's desire, "to select benefits that provide incentives for workers to perform better, that is, productivity-enhancing benefits, such as pension systems that encourage human capital investments or working-time flexibility arrangements that lower employees' costs of effort. Since benefits are costly, the employer needs to weigh the productivity gain against the increase in costs." In the present study, an increase in [expected] costs (from delegating the scheduling decision to workers via a flextime policy) arises because employers risk the possibility that workers, when empowered to set their own work schedules, might choose working hours that overlap only minimally or not at all with the high-expected-productivity "peak hours" that are of greatest value to the employer.

The particular segment of the benefits literature to which this study relates concerns socalled "family-friendly" policies or work practices, which have received increasing attention

[^7]both in the academic literature and in the popular press. ${ }^{13}$ These are flexible work practices that include flextime, working from home during normal business hours, parental leaves, job sharing, and childcare provision. Shepard et al. (1996) find that flexible work hours contributed to around a 10 percent productivity improvement in the pharmaceutical industry in U.S. manufacturing. Similarly, Bloom et al. (2015) find in an experiment involving call center workers at a Chinese travel agency that working from home led to a 13 percent performance increase. Such results are consistent with, though not implied by, the theoretical framework in the present study, which allows either flextime or employer-determined scheduling to be more productive and profitable depending on the model's parameters. ${ }^{14}$ Overall the empirical evidence is mixed concerning the productivity effects of flexible work practices (Heywood et al., 2007).

## 3. Theoretical Framework: A Model of Work Schedules

Let a unit of time be partitioned into $T$ segments of equal duration, where $T$ is a finite positive integer that is divisible by four and large enough to render the problem interesting. For concreteness, suppose that the unit of time represents a potential workday and that the segments represent hours. Consider an employment relationship between an employer and a worker. Before the workday starts, the employer offers the worker a contract involving $N$ required work hours in exchange for compensation of $w \times N$. Let $N$ be a non-negative integer that does not exceed $T$, and let $w>0$ be a predetermined hourly wage that is sufficiently high that the worker always accepts the contract. Premium pay for overtime is disallowed for simplicity but could be

[^8]incorporated into this framework. ${ }^{15}$ There is no incentive pay. ${ }^{16}$ Alternative contract forms are discussed in section 3.3.

In addition to wages and hours, the contract specifies how the work schedule is determined. If the schedule is employer-determined then the contract states which $N$ of the $T$ hours must be worked. Alternatively, if the employer offers a flextime contract then the worker chooses which $N$ hours to work. Let $\mathcal{F}$ be a binary indicator equaling 1 if flextime is used and 0 if employer-determined scheduling is used. The work schedule, $s$, is a $T$-vector, the $t^{\text {th }}$ element of which is 1 if the worker works in hour $t$ and 0 if the worker does not. Thus, $\boldsymbol{s} \boldsymbol{\prime} \boldsymbol{s}=N$. Let $h_{t}$ denote the $t^{\text {th }}$ element of $s$, and let $H_{t}$ be the number of hours that the worker has worked prior to hour $t$, i.e., $H_{t}=\sum_{\tau=1}^{t-1} h_{\tau} \cdot{ }^{17}$ Let $\boldsymbol{S}_{N}$ denote the set of all work schedules requiring exactly $N$ working hours. Therefore, the cardinality of $\boldsymbol{S}_{N}$ is $\binom{T}{N}$.

The assumption of a parametric wage, $w$, that is common to both scheduling regimes is for convenience and permits a focus on the implications of work schedules for the revenue side of the profit function. I relax it in section 3.2.2 by allowing for endogenous, regime-specific wages. In some production settings, the manager in charge may have more control over daily

[^9]
#### Abstract

${ }^{16}$ As in the literature on promotion and wage dynamics (e.g., Waldman 1984, Gibbons and Waldman 1999, 2006, Zábojník and Bernhardt 2001, and DeVaro and Waldman 2012), I abstract from incentive pay, both for simplicity and "to ease the comparison of the model with the empirical evidence" (Gibbons and Waldman 1999, p. 1328, and Gibbons and Waldman 2006, p. 67). The model should be seen as applying to settings where incentive pay is unimportant, which covers most but not all cases. For example, while telemarketers might appear to be a good illustration of the model, they would not be if their pay were commission based, in which case they would have an incentive to work during the hours that the employer prefers. In the model's favor, the incidence of incentive pay in the workplace is lower than would be suggested by the theoretical literature in economics, which since the 1970s has lavished attention on incentive contracts. This relative scarcity of performance pay contracts in the workplace is perhaps unsurprising, given the well-known downsides associated with such contracts that render them challenging to implement in practice. As DeVaro and Heywood (2017) document in the same sample analyzed here, only about $18 \%$ of establishments use pay contracts in which workers are paid on the basis of their individual performance. Even within that $18 \%$ the incidence of such contracts is not always high. If just one worker in the establishment is paid in this manner, the employer may well report that such pay is used within the establishment.


17 "Hour $t$ " refers to the $t^{\text {th }}$ hour of the $T$-hour potential workday, not the $t^{\text {th }}$ work hour of the $N$-hour actual workday. Although hour 1 is the first hour of the $T$-hour potential workday, it will be shown that the first (and only) work hour that would be assigned under employer-determined scheduling and $N=1$ is hour $0.25 T+1$.
work schedules than over pay, which changes only infrequently over time, perhaps once per year. This is especially so at the workplace level, as in the present empirical analysis, given that wages are often set at a company's headquarters and are subject to limited influence by plant managers. In such cases, the manager treats $w$ as predetermined. A parametric wage may also be appropriate for institutional reasons such as minimum wages or union contracts. If the marketclearing wages for both scheduling regimes lie below the minimum wage, then the minimum wage binds and a common parametric wage emerges. The assumption permits a comparative statics analysis on exogenous changes in the common wage, as discussed in section 3.2.1, where an increase in $w$ is found to increase the relative profitability of employer-determined scheduling. ${ }^{18}$

Subscripts " $e$ " and " $f$ " denote "employer-determined" and "flextime". Let $N_{e}$ and $N_{f}$ denote the optimal number of hours the employer would assign under either regime. Let $\boldsymbol{s}_{e}^{N}$ denote the optimal work schedule the employer would choose conditional on assigning $N$ work hours under employer-determined scheduling, so $\boldsymbol{s}_{e}^{N_{e}}$ is the optimal schedule under that regime.

The employee produces output in every hour worked. In a given hour, product demand is stochastic and can be either high or low. Neither the employer nor the worker knows in advance whether product demand in a given hour will be high or low. The worker's "unadjusted" output from working in hour $t$ is $V_{t}^{*}$, which equals $V$ if product demand is high and $v$ if it is low, where 0 $<v<V$. If a working hour is during "peak time" (i.e., the central half of the potential workday, which encompasses hours $0.25 T+1$ through $0.75 T$ ) then the probability of high product demand is $p$, and the probability of low product demand is $1-p$. If the hour is outside of peak time then the probability of high product demand is $q$, and the probability of low product demand is $1-q$, where $0<q<p<1 .{ }^{19}$ Hours of the potential workday that follow peak time are called "late", whereas those that precede peak time are called "early". Figure 1 illustrates.

[^10]Both economic agents have potentially different scheduling preferences. The worker's daily utility from schedule $\boldsymbol{s}$ is $u(\boldsymbol{s})=w N-c(\boldsymbol{s})$, where $c(\boldsymbol{s})$ is the daily disutility of working, which is defined as follows: $c(\boldsymbol{s})=\delta \lambda_{\boldsymbol{s}}\left(\boldsymbol{s}^{\prime} \boldsymbol{s}\right)^{2}$, where $\delta>0$ is known to both agents and
$\lambda_{\boldsymbol{s}} \in\left\{k \times\binom{ T}{\boldsymbol{s}^{\prime} \boldsymbol{s}}^{-1}: k=1,2, \ldots,\binom{T}{\boldsymbol{s}^{\prime} \boldsymbol{S}}\right\}$. That is, the $\lambda_{\boldsymbol{s}}$ are the $\binom{T}{N}$ evenly spaced points ranging from $\binom{T}{N}^{-1}$ to $1 .{ }^{20}$ The employer's scheduling preferences are known by the worker and are dictated by stochastic product demand. The most appealing hours for the employer to assign are those in peak time, which is when production is expected to be most profitable. Within the categories of "peak" and "late" hours, the employer is assumed to prefer earlier hours, whereas within the category of "early" hours the employer is assumed to prefer later hours. ${ }^{21}$

The hour- $t$ profit function is as follows, where $\alpha>0$ and $\beta>0$ :

$$
\Pi_{t}=\left[\frac{V_{t}^{*}}{(1+c(\boldsymbol{s}))\left(1+\alpha+\beta H_{t}\right)}-w\right] h_{t}
$$

The ratio in this expression represents hour- $t$ "adjusted" output, i.e., the unadjusted output is degraded by the denominator that captures the productivity loss arising when the worker dislikes the schedule. The assumption that $c(\boldsymbol{s})$ is a production-function parameter that reduces hourly productivity reflects the common intuition that workers are most productive when they want to work and are least productive when they do not want to work. ${ }^{22}$ The parameter $\beta$ captures

This imposition of distasteful schedules on workers requires firms that coordinate hours to pay compensating differentials.
${ }^{20}$ The model treats workers' scheduling preferences as independent of product demand. In some settings, it might be more realistic for workers to consider expected product demand - or even realized product demand - when choosing work schedules under flextime. For example, workers might prefer to avoid working the busiest hours. Such variants should not alter the model's main insights, though the relative appeal of flextime from the employer's standpoint will be diminished.
${ }^{21}$ These simplifying assumptions are without loss of generality and give rise to the continuous (employerdetermined) work schedules that are most common in practice. An alternative way to generate continuous work schedules is to assume a fixed cost of stopping and restarting work. Other assumptions that are made for expositional convenience and that are unimportant for the analysis are that $T$ is even and divisible by four, that the number of peak hours is $0.5 T$, and that the peak hours are all consecutive and located in the center of the potential workday.

[^11]exhaustion, which multiplies the cumulative daily hours worked so far. That term is augmented by the worker's daily disutility of working, so that if the worker is happy with their work schedule they have a higher tolerance for long hours and are less susceptible to exhaustion. The parameter $\alpha$ captures the idea that an undesirable work schedule can damage productivity through channels other than exhaustion, even in the first work hour of the day when there is no exhaustion yet. ${ }^{23}$ The hour-t "adjusted" output is also revenue given that the per-unit price of output is normalized at 1 .

The model's results are driven by a misalignment in preferences between the employer and worker concerning which hours to work. The main results require no private information on the worker's part and hold even if the employer fully observes the worker's preferred schedule. In a full-information benchmark version of the model the employer would observe the worker's preferences, indulging them only if the advantages of doing so (in terms of staving off exhaustion and making the worker more productive) outweighed the disadvantages of assigning the worker to less productive hours. In practice, however, asymmetric information concerning workers' scheduling preferences will typically be an important aspect of the employment relationship, particularly given how frequently and suddenly those preferences can change over time. The most complete (and current) information about the worker's preferences will typically be the worker's private information. Such information asymmetry is assumed henceforth.

Specifically, the employer faces two potential sources of uncertainty in the profit function. The first only arises under employer-determined scheduling, in which case the employer does not observe the parameter $c(\boldsymbol{s})$ in the production function. Specifically, the employer knows that the worker's daily disutility from schedule $\boldsymbol{s}$ is $c(\boldsymbol{s})$ but does not know $\boldsymbol{\lambda}_{\boldsymbol{s}}$. The employer knows that $\lambda_{s}$ is one of the evenly spaced values ranging from $\binom{T}{N}^{-1}$ to 1 but does not know which value corresponds to which work schedule. The employer therefore assumes that all one-to-one mappings between work schedules and values of $\lambda_{s}$ are equally likely. ${ }^{24}$ This

[^12]source of uncertainty does not arise under flextime, because in that case the employer knows that the worker will choose the schedule that yields the minimum value of $\lambda_{s}$, namely $\binom{T}{N}^{-1}$.

The second source of uncertainty arises under both regimes. The employer does not observe $\alpha$ and $\beta$ and treats these parameters as random variables that follow a 9-point joint probability distribution. Specifically, $\psi_{i j} \equiv \operatorname{Prob}\left(\alpha_{i}, \beta_{j}\right)$, for $i, j \in\{L, M, H\}$, where $0<\alpha_{L}<\alpha_{M}<\alpha_{H}$, $0<\beta_{L}<\beta_{M}<\beta_{H}$, and $\sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j}=1 .{ }^{25}$ For both $\alpha$ and $\beta$, the middle value is assumed to be the true value, so $\alpha_{M} \equiv \alpha$ and $\beta_{M} \equiv \beta$. Therefore, with probability $\psi_{M M} \equiv \operatorname{Prob}\left(\alpha_{M}, \beta_{M}\right)$ the employer correctly observes $\alpha$ and $\beta$, and with probability $1-\psi_{M M}$ the employer either overestimates or underestimates at least one of $\alpha$ and $\beta$. Without loss of generality, let $\alpha_{L} \equiv\left(1-\xi_{\alpha}\right) \alpha_{M}, \alpha_{H} \equiv\left(1+\xi_{\alpha}\right) \alpha_{M}, \beta_{L} \equiv\left(1-\xi_{\beta}\right) \beta_{M}$, and $\beta_{H} \equiv\left(1+\xi_{\beta}\right) \beta_{M}$, where $\xi_{\alpha} \in[0,1]$ and $\xi_{\beta} \in[0,1]$. The last two paragraphs of section 3.2. explain the (empirical) rationale for incorporating measurement error in $\alpha$ and $\beta$.

Finally, $w<\frac{q v+(1-q) V}{\left(1+\delta T^{2}\right)(1+\alpha+\beta(T-1))}$ is assumed for convenience, which ensures that hour- $t$ profit is always positive for every potential work hour.

### 3.1. Optimal scheduling policy

Next, consider the optimal scheduling policy under both regimes.

### 3.1.1. Employer-determined scheduling

Under employer-determined scheduling, the employer chooses both $N$ and $\boldsymbol{s}$ to maximize expected daily profit, under uncertainty about $\lambda_{s}, \alpha$, and $\beta$. If $N_{e} \leq 0.5 T$, the employer only assigns peak hours. If production is more profitable (e.g., if $v$ is higher), the employer prefers to assign additional hours beyond 0.5 T. These additional hours are "late" (i.e., post-peak) hours and are assigned up to $N_{e}=0.75 \mathrm{~T}$. If production is even more profitable, the employer assigns additional hours beyond 0.75 T. These additional hours are "early" (i.e., pre-peak) hours, which are the only available hours remaining.

[^13]The employer's preference for assigning all "late" hours before assigning any "early" hours derives from the term $\beta H_{t}$ in the profit function. Intuitively, the employer knows that worker exhaustion diminishes productivity throughout the workday, so ideally the worker should be as "fresh" as possible during hours of peak demand. ${ }^{26}$ Within the category of late hours, the earliest hours are assumed to be the most productive. This assumption, which is not central to the main results, gives rise to the continuous employer-determined work schedules that are commonly observed in practice. It is also equivalent to the realistic notion that there are fixed costs of stopping and restarting work. Figure 2 illustrates the optimal work schedule for all possible values of $N_{e}$.

Let $A=\{t: 0.25 T+1 \leq t \leq 0.75 T\}$, let $l_{A}(t)$ be the indicator function equaling 1 if $t \in A$ and 0 otherwise, and let $X_{t}=[p V+(1-p) v] 1_{A}(t)+[q V+(1-q) v]\left(1-1_{A}(t)\right)$. The employer determines the optimal number of work hours as follows:

$$
N_{e}=\max \left\{0, \underset{N}{\operatorname{argmax}}\left[\binom{T}{N}^{-1} \sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s} N^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}-w N\right]\right\}
$$

In the preceding expression, $h_{t}$ is the $t^{\text {th }}$ element of $\boldsymbol{s}_{e}^{N}$. Recall that $\boldsymbol{s}_{e}^{N}$ is known by both parties. Specifically, the worker knows from Figure 2 that when considering successive values of $N$ in the preceding calculation, the employer first assigns all the peak hours (ordered from earliest to latest), then all the late hours (ordered from earliest to latest) and finally all the early hours (ordered from latest to earliest).

Given $N_{e}$, expected profit is determined as follows:

$$
E\left(\Pi_{e}\right)=\binom{T}{N_{e}}^{-1} \sum_{s \in S_{N_{e}}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s} N_{e}^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}-w N_{e}
$$

In the preceding expression, $h_{t}$ is the $t^{\text {th }}$ element of $\boldsymbol{s}_{e}^{N_{e}}$.

[^14]
### 3.1.2. Flextime

Under flextime, the employer chooses $N$, and then the worker chooses $\boldsymbol{s}$ to minimize $c(\boldsymbol{s})$. Whereas under employer-determined scheduling there were only $T$ possible work schedules involving positive hours (see Figure 2), under flextime that number increases to $\sum_{k=1}^{T}\binom{T}{k}, 16$ of which are depicted in Figure 3 when $T=16$. The employer knows that under flextime the worker will choose the schedule that minimizes $c(s)$, so $\lambda_{s}=\binom{T}{N}^{-1}$ and $c(\boldsymbol{s})=\delta N^{2}\binom{T}{N}^{-1}$. The employer determines the optimal number of work hours as follows:

$$
N_{f}=\max \left\{0, \underset{N}{\operatorname{argmax}}\left[\binom{T}{N}^{-1} \sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta N^{2}\binom{T}{N}^{-1}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}-w N\right]\right\}
$$

In both the preceding expression and the forthcoming one for $E\left(\Pi_{f}\right), h_{t}$ is the $t^{\text {th }}$ element of $\boldsymbol{s}$, where $\boldsymbol{s}$ is the index in the outermost summation. ${ }^{27}$

Given $N_{f}$, expected profit is determined as follows:

$$
E\left(\Pi_{f}\right)=\binom{T}{N_{f}}^{-1} \sum_{s \in S_{N_{f}}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta N_{f}^{2}\binom{T}{N_{f}}^{-1}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}-w N_{f}
$$

The employer adopts a flextime policy if $E\left(\Pi_{f}\right)>E\left(\Pi_{e}\right)$ and otherwise adopts employerdetermined scheduling.

A key assumption of the model is that the worker's daily disutility of work appears in the production function as a parameter, implying that the worker is less productive when they prefer not to work. Under employer-determined scheduling, prior to choosing $N$ the employer observes the probability distribution of $c(\boldsymbol{s})$. In contrast, under flextime the employer observes $c(\boldsymbol{s})$ exactly. Thus, the employer influences the cost parameter that is relevant for expected profit by either assigning work schedules or delegating that responsibility to the worker. Delegation aligns the agents' interests in one sense and misaligns them in another. Delegating has the upside of

[^15]allowing the worker to harness their private information to select the most attractive value of $c(\boldsymbol{s})$ from the standpoint of both agents (independent of product demand considerations), which enhances productivity by staving off exhaustion. Delegating has the downside that the worker's preferences over schedules are independent of product demand. Whereas the worker has no particular preference for peak hours, the employer has clear preferences that are less often and less fully indulged under flextime than under employer-determined scheduling. Thus, a worker left to their own devices might well choose many nonpeak hours, perhaps including the early hours that are particularly damaging to productivity because they induce exhaustion before peak time begins. The optimal scheduling policy hinges on how the tradeoff between the two preceding considerations is resolved, given the model's parameters.

### 3.2. Analysis

Let $Q_{N}$ denote the expected daily output (i.e., revenue) corresponding to working hours
$N$. Let $b_{N}$ denote the slope of the line that connects $\left(N-1, Q_{N-I}\right)$ and $\left(N, Q_{N}\right)$, for $N=1,2, \ldots, T$.
Let $N_{l}$ denote a value of $N \in\{1,2, \ldots, T-2\}$ such that either:
(i) $Q_{N}$ is higher (lower) for $\mathscr{F}=1$ than for $\mathscr{F}=0$ when $N_{l}<N<T\left(0<N \leq N_{I}\right)$; or
(ii) $Q_{N}$ is higher (lower) for $\mathscr{F}=0$ than for $\mathscr{F}=1$ when $N_{I}<N<T\left(0<N \leq N_{I}\right)$.

Proposition 1 gives the main results (see section A1 of the appendix for the proof):

## Proposition 1:

1. $Q_{0}=0$ for $\mathscr{F}=0$ and $\mathscr{F}=1 . Q_{T}$ is the same positive number for $\mathscr{F}=0$ and $\mathscr{F}=1$. For $\mathscr{F}=0$ and $\mathscr{F}=1,\left(N, Q_{N}\right)$ lies above the line that connects $(0,0)$ and $\left(T, Q_{T}\right)$, if $0<N<T$. For $\mathscr{F}=0$ and $\mathscr{F}=$ 1 , parameterizations exist such that $b_{N}<0$ for large enough $N$. Also, $b_{N}<0$ implies $b_{N+l}<0$.
2. For $\delta$ sufficiently small (large), the graph that connects the points $\left(N, Q_{N}\right)$ for $\mathscr{F}=0$ lies everywhere above (below) that for $\mathscr{F}=1$, if $0<N<T$.
3. For some parameterizations for which $\delta$ is neither too high nor too low, $N_{l}$, exists. Whenever it exists, $Q_{N}$ is higher (lower) for $\mathscr{F}=1$ than for $\mathscr{F}=0$, for $N>N_{l}\left(N \leq N_{l}\right)$.

Proposition 1 says that although the two scheduling regimes coincide at $N=0$ and $N=T$, in general they are different. If $\delta$ is sufficiently low, more output is produced under policy $\mathscr{F}=0$ than under $\mathscr{F}=1$, for all interior values of $N$. The reverse is true if $\delta$ is sufficiently high. In either case, there are parameterizations such that for sufficiently high $N$ the $b_{N}$ becomes negative. If that happens, the subsequent values of $b_{N}$ remain negative for all higher values of $N$ up to $T$. This pattern is consistent with an inverted $-U$ shaped hours-output profile. If $\delta$ is neither too high nor
too low, the policy $\mathscr{F}=0$ dominates for low values of $N$, the policy $\mathscr{F}=1$ dominates for high values of $N$, and $N_{e}<N_{f}$. Thus, flextime goes hand-in-hand with long hours.

Numerical examples highlight Proposition 1's implications. Figure 4 displays expected daily profit as a function of assigned hours for both scheduling regimes, given the parameter values stated in the note beneath the figure. ${ }^{28}$ Expected profit peaks at $N_{e}=9$ under employerdetermined scheduling and at $N_{f}=12$ under flextime. Expected profit is slightly higher under flextime than under employer-determined scheduling, i.e., $E\left(\Pi_{e}\right) \approx 114.80$ and $E\left(\Pi_{f}\right) \approx 118.24$. Figure 5 displays hours-output profiles for both regimes, given the parameter values stated in the note beneath the figure. ${ }^{29}$ Both profiles exhibit an inverted- $U$ shape, as was found empirically in Pencavel (2015) for a sample of British munitions workers during World War I. The peak occurs further to right under the flextime regime than under employer-determined scheduling.

Figure 4 illustrates an implication of Proposition 1, i.e., long working hours go hand-inhand with flextime. The reason follows from the model's main tradeoff between two alternative strategies that the employer can use to enhance productivity: 1) guarantee the minimum value of $c(\boldsymbol{s})$ but at the expense of risking an inefficient hours assignment based on product demand, 2) guarantee an efficient hours assignment but at the expense of risking a higher value of $c(\boldsymbol{s})$. In addition to $c(\boldsymbol{s})$, the model features two types of forces that erode productivity: those relating to worker exhaustion (i.e., $\beta$ ) and those relating to other factors (i.e., $\alpha$ ), with the former being particularly salient because they intensify as the workday lengthens. Both forces amplify, via $c(\boldsymbol{s})$, when workers dislike their schedules, meaning both the number of hours worked and the particular hours worked.

Thus, the employer can mitigate the productivity erosion from $\alpha$ and $\beta$ by allowing the worker to decide how to allocate the assigned hours across the workday. The parameter $\delta$ varies the intensity of worker preferences and, therefore, the benefits to the employer of a flextime policy that indulges those preferences. When $\delta \rightarrow 0$, the benefit to the employer of choosing a flextime policy vanishes, and employer-determined scheduling is always chosen. If Figure 4 is reproduced substituting a value of $\delta$ near zero and maintaining other parameters at their values in

[^16]Figure 4's note, the graphs for the two regimes coincide at $N=0$ and $N=T$, and for all interior values employer-determined scheduling lies above flextime, with $N_{e}=11$ and $N_{f}=12$. Using that same set of parameter values, the hours-output profiles (akin to those plotted in Figure 5) coincide at $N=0$ and $N=T$, are both monotonically increasing, and for all interior values employer-determined scheduling lies above flextime.

Whereas the preference and production technology parameters $(\delta, \alpha$, and $\beta$ ) degrade the worker's unadjusted output, another relevant consideration is efficient assignment of hours (which relates to the parameters $p, q, V$, and $v$ ). Employer-determined scheduling guarantees the worker's assignment to those hours in which unadjusted output is expected to be highest. When $p$ and $q$ differ substantially (and similarly when $V$ and $v$ differ substantially) peak and nonpeak hours are very different, which increases the return to efficient assignment (and, therefore, the relative appeal of employer-determined scheduling). Given the parameter values in Figure 4, but decreasing $v$ from 40 to 30 (which increases the return to efficient hours assignment) the optimal scheduling regime switches from flextime to employer-determined scheduling, i.e., $E\left(\Pi_{e}\right) \approx$ 109.96, $E\left(\Pi_{f}\right) \approx 100.02, N_{e}=8$ and, $N_{f}=10$. On the other hand, if either $p-q$ or $V-v$ is small, there is not much distinction between peak and nonpeak hours, so the employer should prefer a flextime policy because inefficient assignment is not particularly costly. Given the parameter values just stated, i.e., maintaining $v=30$, if $q$ is increased from 0.5 to 0.8 the optimal scheduling regime switches back to flextime, with $E\left(\Pi_{e}\right) \approx 110.56, E\left(\Pi_{f}\right) \approx 118.72, N_{e}=9$, and $N_{f}=12$.

When assigned hours assume less extreme values than either 0 or $T$, the preceding effects concerning $\delta, \alpha, \beta, p, q, V$, and $v$ vary with the number of working hours. If assigned hours are low, then employer-determined scheduling offers an efficiency advantage because all assigned hours are guaranteed to be peak hours, whereas under flextime some of these hours may be nonpeak. In contrast, if hours are high, then under employer-determined scheduling the next hour is guaranteed to be a nonpeak hour, whereas under flextime it will be peak with positive probability. Moreover, the employer knows that under flextime the worker has a more desirable schedule and is, therefore, slower to incur productivity-eroding exhaustion, so it makes sense to push the worker harder and assign more hours. Thus, there are two reasons for the longer hours under flextime. One reason is that workers under flextime are slower to exhaust because of their more favorable work schedules. Another reason is that under flextime the next hour assigned will be peak with positive probability, whereas under employer-determined scheduling that
probability is zero if hours already exceed $0.5 T$. Only the first of those reasons applies when the required work schedule is short.

To summarize Proposition 1's implications, the employer can enhance productivity either by indulging the worker's preferences, thereby harnessing the worker's private information concerning productivity, or by dictating the work schedule, thereby ensuring efficient assignment of hours from the standpoint of product demand.

Two parameters remain ( $w$ and $T$ ). The next subsection discusses $w$. There is little to say about $T$, both because its role is the least interesting of all the model's parameters and because the problem quickly becomes operose when $T$ exceeds $16 .{ }^{30}$ Suffice it to say that $T$ can always be chosen to be sufficiently high so that the uninteresting case of $N_{e}=N_{f}=T$ is not empirically relevant. In fact, we are probably already there at $T=16$, given that assigned workdays of 12,13 , and 14 hours are already rather long and unusual. From an empirical standpoint, therefore, the case of "long hours" can be thought of in the context of $N<T$, so that the relative advantages of flextime over employer-determined hours apply.

### 3.2.1. Work schedules and the minimum wage

The parameter $w$ is the price to the employer of assigning an hour under either scheduling regime. Higher prices mean fewer hours scheduled under either regime. The result that flextime goes hand-in-hand with longer hours then implies that exogenous changes in the wage can induce changes in the optimal scheduling policy. For example, an increase in the U.S. federal minimum wage to $\$ 15$ per hour, which is a current policy proposal that mirrors changes that some cities and states have already enacted, could induce some employers to switch from flextime to employer-determined scheduling. The same goes for wage increases that result from collective bargaining agreements.

The intuition for the result is as follows. Suppose that the employer finds it optimal to assign long hours under flextime scheduling. When $w$ subsequently increases, hours become more expensive and the employer decides to reduce them. The relative appeal of employerdetermined scheduling then increases for potentially two reasons. First, when fewer hours are assigned there is less time for the corrosive effect of worker exhaustion to undermine

[^17]productivity throughout the workday, which means that the value of staving off exhaustion by allowing the worker to design their own schedule is diminished relative to what it was when the workday was long and exhaustion was a major factor. Second, if the reduction in hours implies a drop from greater than $0.5 T$ hours to less than that value, the employer's situation changes in the following way. Prior to the wage increase, the marginal hour assigned is certain to be nonpeak under employer-determined scheduling, whereas under flextime it could be either peak or nonpeak. After the wage increase, the marginal hour assigned is certain to be peak under employer-determined scheduling, whereas under flextime it could be either peak or nonpeak. That consideration enhances the relative appeal of employer-determined scheduling.

To elaborate, when hours are long and $N_{f}>N_{e}$, employers incur extra compensation costs in the amount of $w\left(N_{f}-N_{e}\right.$, ) by choosing $\mathcal{F}=1$. Suppose that the advantages of choosing $\mathcal{F}=1$ (on the net revenue side) are worth it, but just barely. Then if $w$ increases, the extra compensation costs required by $\mathcal{F}=1$ become prohibitively expensive, and the employer switches to $\mathcal{F}=0$. That switch is accompanied by a decrease in assigned hours, because $N_{f}>N_{e}$. That decrease is in addition to whatever drop in hours was already induced under flextime in response to the increase in $w$ (i.e., the argument from standard labor demand theory that implies a drop in $N_{f}$ in response to an increase in $w$ ). If the employer were forced to stick to $\mathcal{F}=1$, hours would drop for the reason given by standard theory, but they drop by even more because of the switch from $\mathcal{F}=$ 1 to $\mathcal{F}=0$.

Flextime is inefficient in that workers neglect product demand when choosing their hours, but the advantage is that workers tire less quickly. Thus, if hours are cheap (i.e., $w$ is low), the employer finds it desirable to push the worker harder, assigning more hours, which increases the workday's expected "coverage" of peak hours. The employer may then make less expected profit on the worker per hour (because of the inefficiency) but that is more than made up for by being able to assign longer hours. If the price of hours increases, however, that strategy becomes less appealing. An employer who is forced to pay higher compensation costs per hour might as well dictate hours to reap the efficiency advantage.

To illustrate, recall that in Figure $4, N_{e}=9, N_{f}=12, \mathrm{E}\left(\Pi_{e}\right) \approx 114.80$, and $\mathrm{E}\left(\Pi_{f}\right) \approx 118.24$, so flextime scheduling is used. If $w$ increases from 14 to 16 , these numbers change to $N_{e}=8, N_{f}$ $=10, \mathrm{E}\left(\Pi_{e}\right) \approx 98.58$, and $\mathrm{E}\left(\Pi_{f}\right) \approx 97.15$, so scheduling is employer-determined. If the employer were forced to stick with the suboptimal choice of $\mathcal{F}=1$ even after the increase in $w$, then the number of work hours would only drop from 12 to 10 , with this drop representing the usual labor
demand effect of a wage increase. Instead, the hours drop from 12 to 8 because the employer finds it optimal to switch scheduling policies in response to the exogenous wage increase.

### 3.2.2. An Extension: Endogenous, Regime-Specific Wages

The model assumes a predetermined hourly wage, $w$, that is sufficiently high that the worker always accepts the contract. Thus, there is no worker participation constraint. Imposing an undesirable work schedule hurts the employer only by making the worker less productive. The model is now generalized to incorporate regime-specific wages and an outside option for the worker that the employer must match for retention. This participation constraint is easier to meet when the worker finds the work schedule appealing. Thus, the employer is hurt by imposing an undesirable work schedule, both because the worker is less productive and because a higher hourly wage must be offered to meet the participation constraint.

Given that flextime can be understood as a nonpecuniary component of compensation that workers value, a compensating differentials equilibrium is a natural framework in which to analyze endogenous wages that differ between the two scheduling regimes. The equilibrium flextime wage is expected to be lower than the wage under employer-determined scheduling. In such an equilibrium, the main insights of the model remain. The role of the wage is to shift the relative profitability of the two regimes via a cost that is linear in hours. Given the model's parameters, in the present extension with endogenous wages that differ by scheduling regime, a compensating differential emerges with the lower (higher) wage applying to flextime (employerdetermined scheduling). The relative profitability of flextime obviously decreases when the flextime wage increases, holding the other wage constant. ${ }^{31}$ Although the logic for this compensating differential is clear, Heywood et al. (2007) document a lack of consensus in the empirical literature as to its existence. ${ }^{32}$

[^18]In the main model, the employer chooses $\left(\mathcal{F}, N_{r}\right)$ given a predetermined $w$, where $r \in\{e, f\}$. In the present extension, the employer chooses $\left(\mathcal{F}, N_{r}, w_{r}\right)$, where $w_{r}$ is a regimespecific wage offer, with $w_{r}>0$. The following discussion highlights intuition, with the technical details relegated to section A2 of the appendix. The worker has an exogenous reservation utility, $u$, where $u>0$. Although in principle the set of potential regime-specific wage offers is continuous, the only sensible wage offers form a discrete subset. Specifically, each sensible wage offer, $w_{r}(\lambda)$, corresponds to a particular value of $\lambda_{\text {. If }} \lambda_{j}$ and $\lambda_{k}$ represent adjacent values of $\lambda$, then any wage offer between $w_{r}\left(\lambda_{j}\right)$ and $w_{r}\left(\lambda_{k}\right)$ would needlessly transfer expected surplus from the employer to the worker.

The employer chooses $w_{r}$ to meet the worker's participation constraint, at least in expectation. I write "at least in expectation" because the employer only observes the worker's utility function when $\mathcal{F}=1$, in which case the employer knows that the worker will choose the schedule that yields the minimum value for $\lambda$. In that case, rendering the wage endogenous simply means substituting the participation constraint for $w_{f}$ into the expected profit function that is maximized to determine $N$. The case of $\mathcal{F}=0$ is more complicated because the employer does not observe which value of $\lambda$ applies. Therefore, the employer must consider all possible values of $\lambda$, recognizing the following familiar tradeoff. Higher wage offers (i.e., higher values of $\lambda$ ) increase the hiring probability but reduce the expected profit conditional on hiring. If a hire does not occur (because the worker's participation constraint is violated), the employer makes zero profit. ${ }^{33}$ For a particular value of $\lambda$, the hiring probability is $\frac{w_{e}(\lambda)}{w_{e}(1)}$, given that $\lambda$ follows a discrete uniform distribution on $(0,1]$.

An implication is that when hiring occurs under $\mathcal{F}=1$, all of the surplus goes to the employer in expected profit, whereas under $\mathcal{F}=0$, some of it goes to the worker in expectation because the employer "overpays" due to imperfect information. Another implication is $w_{f}<w_{e}$, i.e., given that scheduling flexibility is a nonpecuniary component of pay, the employer can depress the monetary wage under flextime, giving rise to a compensating differential.

Section 3.2.1's discussion of the interaction between minimum wages and scheduling policy requires refinement in the present context of endogenous wages. A minimum wage, $w_{\text {min }}$, shifts some surplus to the worker, so the worker gets some of it even when $\mathcal{F}=1$. There are 3 possible cases when $w_{\text {min }}$ is imposed:

[^19]Case 1: $0<w_{\text {min }}<w_{f}<w_{e}$
Case 2: $0<w_{f}<w_{\text {min }}<w_{e}$
Case 3: $0<w_{f}<w_{e}<w_{\text {min }}$
The minimum wage is irrelevant in case 1 . In case 3 , assuming compliance, $w_{f}=w_{e}=w_{\text {min }}$, so analyzing an increase in $w_{\text {min }}$ is the environment analyzed in section 3.2.1, where $w$ is common to both scheduling regimes and fixed (at effectively $w_{\text {min }}$ ). In case 2 , the compensating differential shrinks relative to its unconstrained magnitude in case 1 . This means that if $\mathcal{F}=1$ was the optimal choice before the minimum wage is introduced, the employer might find it profitable to switch to $\mathcal{F}=0$ after.

The result that a higher $w_{\text {min }}$ makes $\mathcal{F}=0$ relatively more appealing arises in the main model for reasons that are unrelated to the worker's participation constraint, which does not even appear in the main model. In the present extension with endogenous wages, however, there is an additional mechanism by which increases in $w_{\text {min }}$ make $\mathcal{F}=0$ more appealing. Under $\mathcal{F}=1$, the worker gets no surplus, and the participation constraint is met with equality. When $w_{\min }$ is introduced, however, the worker gets some surplus and there is slack in the participation constraint. This means that the employer can cut back on scheduling flexibility while still meeting (at least in expectation) the worker's participation constraint. Wage compensation is substituted for nonwage compensation, to continue (just) "meeting" the participation constraint. When wages are endogenous, the preceding mechanism operates simultaneously with the one from the main model in section 3.2.1 that is unrelated to the worker's participation constraint.

### 3.2.3. Empirical Implications

Section 5 investigates two empirical implications that emerge from the theoretical framework. ${ }^{34}$ First, under some parameterizations of the model, as the work schedule lengthens, productivity increases up to a point, before reaching a peak and subsequently declining, so that the hours-output profile has an inverted- $U$ shape as illustrated in Figure 5. Second, flextime policies mitigate the productivity-corroding effects of long working hours. In other words, if parameters are such that an inverted- $U$-shaped hours-output profile exists, flextime policies shift its peak to the right. Thus, flextime policies go hand-in-hand with longer working hours.

[^20]The model generates theoretical inverted- $U$-shaped hours-output profiles under either scheduling regime. Moreover, the peaks of the profiles in Figure 4 correspond to the optimal choices of $N$ under each regime. That raises the question of how observed deviations from the peak should be interpreted in data on hours and productivity. Why, for example, would an optimizing employer ever assign "too many" hours, i.e., operate on the downward-sloping segment of the profiles in Figure 4?

The answer lies in uncertainty about the production function, which is the rationale for incorporating uncertainty about $\alpha$ and $\beta$. In the case of flextime, such uncertainty is always present because the employer cannot observe $c(\boldsymbol{s})$. In the case of employer-determined scheduling, however, the only source of uncertainty is in $\alpha$ and $\beta$. Such uncertainty can explain why an optimizing employer might assign "too many" hours. For example, an employer who mistakenly believes that $\alpha$ and/or $\beta$ are lower than they truly are would push the worker too hard, based on an incorrect belief that the peak lies further to the right than it actually does. Thus, incorporating measurement error in the production function rationalizes observed employer behavior that would otherwise appear suboptimal, such as hours that are "too long". ${ }^{35}$

### 3.3. Alternative Contract Forms

The theoretical analysis is simplified by assumptions that restrict the contract space, but alternative contract forms merit comment. One worker, one employer, and a fixed wage are assumed, although the latter assumption is relaxed in section 3.2.2. The assumption of one worker and one employer abstracts from sorting of workers across firms and from employer contracts designed to screen workers based on their preferences over work schedules. To the extent that employers successfully screen workers, the importance of workers' assumed private information about their preferred schedules would be mitigated. One alternative contract form would require workers to propose their preferred work schedules before employers make wage

[^21]offers that workers accept or reject. If such a screening contract is wholly successful in perfectly screening workers, then flextime contracts should not emerge in equilibrium.

Screening is a relevant consideration in some production settings and could reduce the need for flextime contracts. At the same time, as will be revealed in Table 1, a third of U.K. establishments in a nationally representative sample use flextime contracts. That fact alone suggests limits to the relevance of the proposed screening contract, at least as a descriptor of employer behavior in general. Moreover, workers' preferences over schedules are not static. The hours that are most desirable and convenient for a worker in a day or week are often private information subject to frequent changes. In this respect, screening workers based on preferences over work schedules is different from screening them based on, for example, their preferences over piece-rate pay versus a straight hourly wage. Screening can be expected to be more feasible and helpful in the latter case, given that worker preferences over piece-rate pay are relatively static. In most employment relationships, pay is renegotiated only at long intervals, perhaps annually. In the proposed screening contract, employers condition their wage offers on workers’ stated preferred work schedules, but there are practical challenges associated with renegotiating the wage contract every time a worker's preferred schedule changes.

Of course, even if the model's fixed-wage assumption is maintained, considering more than one worker and one firm raises the possibility of sorting. If there is heterogeneity across employers in temporal patterns of product demand throughout the day or week, workers might sort across firms based on how their preferred work hours align with product demand. Such a setup, like the aforementioned screening contract, might call into question the optimality of flextime scheduling. Nonetheless, the same arguments just given in response to the proposed screening contract also apply here. A third of U.K. employers in a nationally representative sample use flextime contracts, despite the sorting of workers across employers based on their mutual preferences over work schedules, which must happen to some degree. Again, workers’ preferences over work schedules are non-static and can change frequently. Just as renegotiation of wage contracts on a daily basis is practically infeasible, so is worker-employer matching.

The point of this discussion is not to dismiss the potential relevance of the aforementioned alternative contracting mechanisms (particularly in certain production settings) but rather to aver that the present modeling assumptions are worthy of investigation despite restricting the space of available contracts. Neglecting sorting considerations is arguably less destructive in this study than in, for example, a study of the productivity effects of piece-rate pay
and other incentive contracts. Moreover, as discussed early in section 3, there are many production settings and situations, particularly in the short run, in which managers must take their subordinates' fixed wages as given when managing the workplace.

## 4. Data and Measures

The data are from the management and employee questionnaires of the 2004 and 2011 British Workplace Employment Relations Study (WERS), as maintained by the Department for Business, Innovation, and Skills (2013). ${ }^{36}$ An employer survey of 2295 establishments was conducted in 2004 and repeated in 2011 on a sample of 2680 employers. The 2011 sample included 989 establishments that appeared in the 2004 sample, which allows the construction of a balanced 2004-2011 panel of 1978 establishment-years. ${ }^{37}$ Sampling weights adjust for this nonrandom feature of the sampling in 2011.

In both years, in each of the sampled establishments, 5 to 25 randomly selected employees also completed surveys. ${ }^{38}$ Consistent with the theoretical model's focus on employer choices, the empirical analysis is conducted at the establishment level. The key variables from the employer survey are flextime and establishment-level labor productivity, plus establishment

[^22]characteristics. Information on work hours is measured at the individual worker level from the employee questionnaires and is aggregated to the establishment level. Missing values in certain variables, particularly labor productivity and hours worked, reduce the sample size from 1978 to 1200 establishment-years. ${ }^{39}$ The analysis uses establishment weights throughout.

## Measures

The data include a measure that closely captures the employer's choice between flextime and employer-determined scheduling, as described in the theoretical model. Survey respondents received the following instruction: "Now I'd like to ask you about different types of flexible working, leave and childcare arrangements, which some employers provide their employees to help them to balance their work and home lives." The employer was then shown a card with a number of practices related to work schedules, including the following: "Flexi time (where an employee has no set start or finish time but an agreement to work a set number of hours per week or per month)." From this information, I define the flextime measure as follows:

Flextime $=1$ if the employer reports using the preceding practice

$$
=0 \text { otherwise }
$$

For supplementary analyses, I also define two alternative measures of flexible work schedules based on practices appearing on the aforementioned show card:

Home $=1$ if the establishment allows "working at or from home in normal working hours"
$=0$ otherwise
ShiftChange $=1$ if the establishment's workers have "The ability to change set working hours (including changing shift pattern)"
$=0$ otherwise
While Flextime is the measure that matches the theoretical model most directly, the other two measures of flexible work practices are related. ShiftChange is, in fact, simply a special case of Flextime in which hours must be worked consecutively. I elaborate when presenting those results. Home might also be considered a "gateway" to flextime, ${ }^{40}$ particularly given the higher

[^23]monitoring costs that employers face when workers are offsite, though it should be noted that the survey question specifies "in normal working hours".

Information on working hours is drawn from the worker survey. Each worker was asked, "How many hours do you usually work in your job each week, including overtime or extra hours? Exclude meal breaks and time taken to travel to work." I aggregate this information from the worker survey to the establishment level by taking the fraction of the establishment's sampled workers whose usual weekly hours are at least $c$. The first empirical prediction is investigated by assessing whether a heavier right tail of the within-establishment hours distribution is associated with reduced establishment-level labor productivity, as illustrated at the right ends of either of the curves in Figure 5. The second empirical prediction is investigated by assessing whether a flextime policy mitigates the productivity-eroding effects of long working hours by shifting the peak of the hours-output profile to the right, as illustrated in the comparison of the solid and dashed graphs in Figure 5.

Managers for each establishment in the WERS were asked to compare their establishment with other workplaces in the same industry and to assess labor productivity on a five-point scale from "a lot below average" to "a lot above average." The analysis uses a binary variable, Productivity, derived from that question, which equals one for the highest-valued response (i.e., "a lot above average") and zero otherwise. A binary dependent variable permits the analysis to account for unobserved, time-invariant, establishment-level heterogeneity via establishment fixed effects in a linear probability model. ${ }^{41}$ Conditional logit models yield similar results. Table 1 displays descriptive statistics for the variables used in the empirical analysis.

The potential biases associated with the use of subjectively reported productivity measures are well known. At the same time, such measures have been found to perform well in practice. For example, Wall et al. (2004) conduct analyses in which subjective and actual company performance data are simultaneously available in the same firms, finding that the subjective and objective measures yield very similar results. The subjective measures in the WERS (and in its earlier waves) have also been successfully analyzed in a number of studies, including Machin and Stewart (1990, 1996), Pencavel (2004), DeVaro and Morita (2013), and DeVaro and Heywood (2017).

[^24]
## 5. Empirical Analysis

Two empirical questions are addressed in this section. First, is there establishment-level evidence of a non-monotonic relationship between working hours and productivity, such that output increases with hours up until an inflection point and then decreases? Second, if so, does the peak of the hours-output profile shift to the right when employers implement flextime or related policies? The reason for a rightward shift, as illustrated in Figure 5, would be that flextime mitigates and delays the onset of the exhaustion that occurs when workers are assigned to unappealing and inconvenient work schedules.

The analysis uses an indicator of establishment-level labor productivity as the dependent variable. This variable corresponds to the theoretical framework of section 3, in which the employer's chosen scheduling regime influences firm performance by directly affecting labor productivity. The theoretical framework suggests that flextime goes hand-in-hand with long working hours, and the WERS data confirm this. Consider the average (across all workers surveyed in an establishment) of the workers' usual weekly hours. As Table 2 reveals, that number is more than two hours higher, on average, between establishments that use flextime versus those that do not (i.e., 34.74 versus 32.53 ). The preceding averages are over both years. The difference is present in both years but is a bit greater in 2004 than in 2011.

Consider the following linear probability model, recalling that $c$ denotes the threshold that defines long weekly hours:

$$
\begin{equation*}
\text { Productivity }_{j t}=\alpha_{c}+\beta_{c} H_{c j t}+\lambda_{c} H_{c j t} \times \text { Flextime }_{j t}+\psi_{c} \text { Flextime }_{j t}+\boldsymbol{X}_{j \boldsymbol{t}} \boldsymbol{\delta}_{\boldsymbol{c}}+\gamma_{c} D_{t}+\eta_{c j}+\varepsilon_{c j t} \tag{1}
\end{equation*}
$$

where Productivity $_{j t}$ is a binary variable equaling one if establishment $j$ has high labor productivity in year $t$ relative to the industry average, $H_{c j t}$ is the fraction of establishment $j$ 's surveyed workers whose usual weekly hours are at least $c$ in year $t$, Flextime $_{j t}$ is a dummy equaling one if establishment $j$ uses a flextime policy in year $t$ and zero if it does not, $D_{t}$ is a dummy variable equaling one if the year is 2004 and 0 if it is $2011, \eta_{c j}$ is an establishment effect that is treated as a fixed effect in estimation, $\varepsilon_{c j t}$ is a stochastic disturbance satisfying the usual properties, and $\boldsymbol{X}_{j t}$ is a vector of control variables including the establishment's employment ${ }^{42}$

[^25]and indicators for industry, private sector, and the presence of a union. Although some of the controls might be anticipated to be time invariant (e.g., industry and union dummies), they exhibit enough temporal variation to permit the identification of their coefficients even in the presence of establishment fixed effects, albeit with modest precision. ${ }^{43}$

The regression is estimated conditional on a particular value of $c$, i.e., a particular cutoff that defines the right tail of the within-establishment hours distribution. Given that a different value of $c$ changes the values of the variable $H_{c j t}$, all of the parameters are indexed by $c$, as are the disturbance term and establishment fixed effect. The parameters of interest are $\beta_{c}$ and $\lambda_{c}$. The theoretical framework predicts that when $c$ is sufficiently large, $\beta_{c}<0$ and $\lambda_{c}>0$, so that flextime mitigates the productivity-eroding effects of long working hours by staving off exhaustion.

Table 3's Panel $A$ reports estimation results from equation (1) that impose the restriction $\lambda_{c}=\psi_{c}=0$, i.e., that exclude Flextime $_{j t}$ and $H_{c j t} \times$ Flextime $_{j t}$ from the empirical model. The first row displays estimates of $\beta_{c}$ for three values of $c$, suggesting that the slope of the hoursproductivity profile exhibits an inverted- $U$-shaped pattern, first rising (for modest levels of weekly hours) and eventually falling (as weekly hours become extremely long). The first two rows of Table 3's Panel $B$ display estimates of $\beta_{c}$ and $\lambda c$, again for three values of $c$. The result, echoing the theoretical framework, is that flextime mitigates the productivity-eroding consequences of long working hours. When $c$ is $65, \beta_{c}=-1.451, \lambda_{c}=0.924$, and both parameters are precisely estimated. The $t$-statistics for $\beta_{c}$ and $\beta_{c}+\lambda_{c}$ are 2.93 and 2.42 at $c=35,-0.13$ and -0.02 at $c=50$, and -5.14 and -1.36 at $c=65$.

To complete the picture started by the three values of $c$ considered in Table 3's Panel $B$, equation (1) is estimated 41 times, corresponding to the 41 values of $c$ ranging from 25 to 65 , inclusive. Those $c$ values appear on the horizontal axis in Figure 6, where in Panel $A$ the solid line (for employer-determined scheduling) plots the estimates of $\beta_{c}$, and the dashed line (for flextime) plots the estimates of $\beta_{c}+\lambda_{c}$. Panel $B$ plots the corresponding $t$-statistics.

The estimate of $\beta_{c}\left(\lambda_{c}\right)$ is negative (positive) for all values of $c$ above 48. Both estimates achieve statistical significance at least at the ten percent level on two-tailed tests, for all values of $c$ above 56, which is revealed to be a critical threshold. ${ }^{44}$ The $t$-statistic for $\beta_{c}$ is -1.56 at $c=56$ and -2.19 at $c=57$, whereas the corresponding $t$-statistics for $\beta_{c}+\lambda_{c}$ are -0.51 and 0.29 .

[^26]Consistent with the pattern of results from Table 3's Panel $A$, the solid graph's downward slope in Figure 6's Panel $A$ reveals that for a sufficiently high value of $c$, i.e., for a sufficiently extreme definition of the right tail of the within-establishment hours distribution, the probability of high labor productivity diminishes when a worker is shifted from below $c$ to above $c$ weekly hours. Thus, the anticipated non-monotonicity is revealed given that the graph of $\beta_{c}$ as a function of $c$ eventually decreases in $c$ beyond a certain threshold. Flextime mitigates the preceding pattern, given that the downward slope of the dashed line in Figure 6's Panel $A$ is more gradual. For more extreme definitions of a long workweek (i.e., for particularly high $c$ values) the negative productivity effect of long hours is considerably muted when a flextime policy is used.

### 5.1. Interpreting the Main Results

As it assumes one worker and one firm, the theoretical framework describes behavioral mechanisms as opposed to selection. In the empirical work, however, both behavioral and selection mechanisms may simultaneously be present. During the seven years that separate the panel's two waves, an establishment's turnover patterns might relate to the establishment's transition towards or away from flextime policies. An establishment that introduces a flextime policy during the intervening period might disproportionately attract workers who favor flextime and shed those who dislike it, changing the composition of worker types. If the composition of workers changes in a way that affects either labor productivity or workers' tolerance for working long hours, these mechanisms will be reflected in the empirical results. ${ }^{45}$

The question also arises as to how within-establishment temporal variation in Flextime should be interpreted. Given that the two survey waves are seven years apart, there is ample time for a change in management practices. The 2011 employer survey contains auxiliary information to verify the relevance of that source of variation. ${ }^{46}$ Specifically, employers are asked, "Over the past two years has management here introduced any of the changes listed on this card?" One of the possible items is, "Changes in working time arrangements". Two subsamples of the analysis sample are considered: one in which the employer lists the preceding item among its responses,

[^27]and another in which the employer does not. Within both subsamples, the following cross tabulations for Flextime are constructed:

Cross tabulations for Flextime

| Changes in working time arrangements during last 2 years ( $N=412$ ) |  |  |
| :---: | :---: | :---: |
|  | FlexTime $_{2011}=0$ | FlexTime $_{2011}=1$ |
| FlexTime $2004=0$ | 0.416 | 0.193 |
| FlexTime $2004=1$ | 0.170 | 0.221 |
| No changes in working time arrangements during last 2 years ( $N=787$ ) |  |  |
|  | FlexTime ${ }_{2011}=0$ | FlexTime ${ }_{2011}=1$ |
| FlexTime $2004=0$ | 0.549 | 0.124 |
| FlexTime $2004=1$ | 0.171 | 0.156 |

The cross tabulations reveal a good amount of temporal variation in FlexTime in both subsamples and, as anticipated, there are more switches in the smaller subsample than in the larger one. The percentage of switches is 36.4 in the top matrix and 29.5 in the lower matrix, with this difference entirely reflecting establishments adding flextime as opposed to removing it. The difference in the switch rate between the two matrices would be expected to be even larger if the survey question on which the sample is partitioned referred to more than just the last 2 years.

### 5.2. Alternative Measures of Flexible Work Schedules

Table 4 displays results akin to those in Panel $B$ of Table 3 but replacing Flextime with the two alternative measures of flexible work schedules. These are Home (Panel $A$ ) and ShiftChange (Panel B). The same three values of $c$ are considered as in Table 3.

For Home, the qualitative results for $\beta_{c}$ and $\lambda_{c}$ in Table 4's Panel $A$ match those for Flextime in Table 3's Panel $B$. At $c=35, \beta_{c}=0.564$, which is estimated with high precision, and the estimate of $\lambda_{c}$ is near zero and estimated with very low precision. Thus, shifting a worker from below 35 weekly hours to above 35 hours is associated with a higher probability of high labor productivity, but that effect is insensitive to whether the establishment allows employees to work from home during normal business hours. The parameters of interest are statistically insignificant at conventional levels when $c=50$. At $c=65, \beta_{c}=-1.471, \lambda_{c}=1.452$, and both parameters are precisely estimated. The $t$-statistics for $\beta_{c}$ and $\beta_{c}+\lambda_{c}$ are 3.43 and 2.17 at $c=35$,
-0.94 and 0.79 at $c=50$, and -5.50 and -0.08 at $c=65$. Thus, shifting a worker from below 65 weekly hours to above 65 hours is associated with a lower likelihood of high labor productivity, but that drop in the probability of high labor productivity is fully mitigated if the establishment permits employees to work from home during normal business hours.

Figure 7 plots point estimates and $t$-statistics for $\beta_{c}$ and $\beta_{c}+\lambda_{c}$ for values of $c$ from 25 to 65 for Home, as Figure 6 does for Flextime. The pattern of point estimates echoes that of Flextime, i.e., flexible work practices mitigate the productivity-eroding effects of long working hours. A critical threshold is revealed in the neighborhood of 60 weekly hours. The $t$-statistic for $\beta_{c}$ is -0.63 at $c=60$ and -5.93 at $c=61$, whereas the corresponding $t$-statistics for $\beta_{c}+\lambda_{c}$ are -0.37 and 0.60 . Thus, above 60 weekly hours there is evidence of a clear negative productivity effect of longer hours that is fully mitigated by working from home. This result complements those from the experiment on Chinese call center employees conducted by Bloom et al. (2015), which revealed a $13 \%$ increase in employee performance from working from home. ${ }^{47}$

As shown in Table 4's Panel B, for ShiftChange, both $\beta_{35}$ and $\lambda_{35}$ are precisely estimated, with $\beta_{35}=0.809$ and $\lambda_{35}=-0.531$. The result on $\lambda_{35}$, which differs qualitatively from those of Flextime and Home, says that although shifting a worker from below 35 hours per week to above 35 hours is associated with a higher probability of high labor productivity, that effect is dampened when workers have the ability to change "set" working hours, such as changing their shift pattern. Both $\beta_{50}$ and $\lambda_{50}$ are imprecisely estimated. At $c=65, \beta_{65}=-1.457$ and $\lambda_{65}=0.353$, though only the estimate of $\beta_{65}$ is statistically significant at the ten percent level. The $t$-statistics for $\beta_{c}$ and $\beta_{c}+\lambda_{c}$ are 4.32 and 1.41 at $c=35,0.23$ and -0.75 at $c=50$, and -1.74 and -2.64 at $c=65$. Figure 8 plots point estimates and $t$-statistics for $\beta_{c}$ and $\beta_{c}+\lambda_{c}$ for values of $c$ from 25 to 65 for ShiftChange, as Figure 6 does for Flextime. As was found for Flextime and Home, the ShiftChange practice mitigates the productivity-eroding effects of long hours. As was found for Home, a critical threshold is revealed in the neighborhood of 60 hours. The $t$-statistic for $\beta_{c}$ is 0.25 at $c=60$ and -2.37 at $c=61$, whereas the corresponding $t$-statistics for $\beta_{c}+\lambda_{c}$ are -1.07 and -2.68. In contrast to Home, the mitigating effect of ShiftChange is far from full.

Although the results in the right-most regions of Panels $A$ of Figures 6, 7, and 8 reveal that all three flexible work practices mitigate the productivity-eroding effect of long hours, the

[^28]mitigating role of ShiftChange is more modest than for the other two practices. Further evidence of this point can be found in the final columns of Table 4 and Table 3's Panel $B$, which reveal estimates of $\lambda_{65}$ that are positive and precisely estimated for Flextime and Home but positive and statistically insignificant at conventional levels for ShiftChange. Insights from the theoretical model offer an interpretation for the weaker mitigating effect of ShiftChange. In the context of the model, the variable ShiftChange from the data should be understood as a constrained version of a flextime policy, in which the worker is allowed to choose which $N$-hour shift to work, of $T-N+1$ possible shifts, where a "shift" is now defined as any sequence of $N$ consecutive hours. ${ }^{48}$ "Constrained" implies that much of the advantage of a flextime policy is sacrificed by a ShiftChange policy. The following example illustrates.

Consider $N=T-1$ assigned hours. Under employer-determined scheduling, the shift includes all $T$ hours of the potential workday except for hour 1, as shown in the penultimate row of Figure 2. Under a ShiftChange policy, the worker-chosen shift must be either the one just stated or the shift that includes all $T$ hours of the potential workday except for hour $T$. Whether the shift that omits hour $T$ is more or less profitable for the employer depends on parameter values. There are two competing effects. On the one hand, when the shift includes hour 1 and omits hour $T$, the worker enters the most productive (i.e., peak) part of the workday more fatigued than when the shift includes hour $T$ and omits hour 1 . That effect hurts expected profit because the term $\beta H_{t}$ in the denominator of net revenue in the profit function is larger during each peak hour than it would be under the alternative work schedule. On the other hand, if the worker chooses the shift that includes hour 1 and omits hour $T$, that particular shift must be more appealing to the worker and, therefore, associated with a lower value of $c(\boldsymbol{s})$. That effect enhances expected profit by shrinking the denominator of net revenue in the profit function.

In the preceding example, the worker gets no choice in the case of employer-determined scheduling and can choose one of two schedules under a ShiftChange policy, whereas under a flextime policy the worker enjoys a broader menu with $T$ possible schedules. It is plausible, even quite likely, that some of those $T$ schedules dominate the two schedules available under ShiftChange, from the standpoint of worker preferences and productivity. The aforementioned two schedules require $T-1$ hours of consecutive work, and the worker might well want to break

[^29]up that long haul with a rest. ${ }^{49}$ A physician working a 15 -hour shift in the emergency room might well be happier and more productive with a one-hour power nap separating the 8 -hour day shift and the 7-hour evening one. The model's insights that are captured in the preceding example can explain why the mitigating effect of ShiftChange appears weaker than that of Flextime.

Turning next to shorter workweeks, consider the results from Table 4's Panel $B$, for $c=35$. The results imply that labor productivity is increased when workers are shifted from below 35 weekly hours to above that value, but that effect is dampened by allowing workers to move their shifts. Thus, in contrast to the results for Flextime and Home, a ShiftChange policy dampens the productivity-enhancing effect of hours when weekly hours are relatively low. The model offers an interpretation for this result, and the logic is similar to the preceding. For relatively short workweeks, the ShiftChange policy carries the same negatives of the flextime policy (less expected coverage of peak hours, and pre-exhausting the worker prior to the start of peak hours) ${ }^{50}$ but with considerably smaller positives, because the requirement that shifts have consecutive hours severely limits the worker's menu of schedules.

The fact that a ShiftChange policy appears to be damaging to productivity when the workweek is short and less helpful to productivity than a flextime policy when the workweek is long paints a somewhat negative picture of the policy. In a sense, a ShiftChange policy is the "worst of both worlds", combining the worst aspects of both employer-determined scheduling and flextime. It gives workers the discretion to choose shifts that are unproductive and hurt the employer (i.e., "nonpeak" shifts), but it might not give them enough discretion to choose work schedules that are really desirable from their standpoint and that can enhance productivity by staving off exhaustion. The policy gives workers enough flexibility to damage profit but perhaps not enough flexibility to benefit significantly from a utility standpoint.

[^30]When the workweek becomes long, however, there is one advantage to employers of a ShiftChange policy relative to flextime. The fact that worker-chosen schedules imply less expected coverage of peak hours relative to employer-determined scheduling is less of a problem under a constraint requiring a shift with consecutive hours, because the worker is prevented from fragmenting a long schedule over the early and late hours while eschewing the peak hours. ${ }^{51}$ When the workweek lengthens, the expected coverage of peak hours increases under flextime, but this effect is even stronger under a ShiftChange policy because once the workweek exceeds $0.25 T$ hours the worker cannot avoid peak hours. This effect grows in importance with the length of the workweek (though vanishing at $N=T$, when all three scheduling regimes coincide), which can explain why the empirical results concerning the productivity effects of ShiftChange are positive for long workweeks while negative for short ones.

## 6. Conclusion

The contribution here is threefold. First, a flexible theoretical framework is provided for the determination of work schedules. This addresses a gap that has persisted for decades in the supply and demand literatures, which concern the number of hours worked rather than which hours are worked. The key idea underlying the model is a tradeoff that employers face between assigning workers to the hours that are expected to be most productive from the perspective of product demand, and granting workers the authority to choose their preferred work schedules, following the idea that "happy workers are productive workers". The model can generate an inverted- $U$-shaped hours-output profile. The critical inflection point, beyond which workers become exhausted (with flagging productivity) from overwork, is shifted to the right when the employer adopts flexible work practices like flextime. Among the model's other insights is the policy-relevant result that increases in the minimum wage enhance the relative appeal of employer-determined scheduling, which highlights a connection between the minimum wage and work schedules that is new to the literature.

Second, consistent with the theoretical model, evidence from the latest waves of the British Workplace Employment Relations Study suggests a non-monotonic hours-output profile,

[^31]whereby productivity increases with usual weekly work hours and then decreases when hours exceed a threshold. This result responds to the calls by Pencavel (2015) and Denison (1962) for new evidence on this subject that uses "contemporary data" on "a wide variety of occupations and industries operating under different conditions". ${ }^{52}$

Third, consistent with the theoretical model, the peak of the empirical hours-output profile is found to shift to the right when employers adopt flextime policies, as opposed to employer-determined work schedules. The result is replicated for two alternative measures of flexible work practices, namely working from home during normal business hours, and a "shift change" policy. Working from home fully mitigates the deleterious productivity effects of long working hours, a result which complements the positive productivity effects of working from home in the experiment on Chinese call center workers by Bloom et al. (2015). The result is of particular interest given the increased incidence of working-from-home in the aftermath of the COVID-19 pandemic. Consistent with intuition supplied by the theoretical model, the "shift change" policy offers the least amount of mitigation for the negative productivity effects of long hours. Shift change policies are constrained versions of flextime and suffer from some of the worst features of both flextime and employer-determined scheduling, i.e., workers are permitted potentially enough flexibility to damage profitability by choosing less profitable hours than the employer would assign, but not enough flexibility to choose the schedules that would make them happiest and most productive.

Work schedules have received little attention in the literature, despite the subject's importance for worker and firm productivity and for social welfare. More work remains. I hope that this study inspires scholars to work long hours conducting that inquiry to further advance our understanding of the nature and consequences of work schedules.

[^32]
## Appendix

Section A1 provides a proof of Proposition 1. Section A2 provides the technical details underlying the extension to endogenous, regime-specific wages in section 3.2.2.

## A1. Proof of Proposition 1

1. $Q_{0}=0$ under either scheduling regime, obviously. When $N=T$, both scheduling regimes are identical because every hour of the potential workday is worked in either case. Because each hour of work is guaranteed to produce positive output, $Q_{T}>Q_{0}$.

The remainder of point 1 is shown for the case of $\mathscr{F}=0$. The case of $\mathscr{F}=1$ is analogous.
$Q_{N}=\binom{T}{N}^{-1} \sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s} N^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}$
The equation for the line that connects $(0,0)$ and $\left(T, Q_{T}\right)$ is:
$Q_{N}($ line $)=\frac{N}{T} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta T^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}$
$Q_{N}$ is the average of $\binom{T}{N}$ terms, each of which exceeds $Q_{N}(l i n e)$. To see why, note that each of the $\binom{T}{N}$ terms differs from $Q_{N}($ line $)$ in only two respects. First, $Q_{N}($ line $)$ has $T^{2}$ in the innermost summand's denominator, whereas $Q_{N}$ has a smaller value, $\lambda_{s} N^{2}$, recalling that $N<T$. Second, $Q_{N}($ line $)$ is multiplied by $\frac{N}{T}$. Both considerations imply that each of the $\binom{T}{N}$ terms being averaged in $Q_{N}$ exceeds $Q_{N}($ line $)$. Thus, $Q_{N}>Q_{N}($ line $)$.

Figure 5 is an example that proves the penultimate sentence of point 1 . To verify the final sentence of point 1, define the inequalities (A1) and (A2), respectively, as $Q_{N}>Q_{N+1}$ and $Q_{N+1}>$ $Q_{N+2}$. These inequalities can be expressed, respectively, as follows:

$$
\begin{aligned}
&\binom{T}{N}^{-1} \sum_{s \in S_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s} N^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]} \\
&>\binom{T}{N+1}^{-1} \sum_{s \in S_{N+1}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s}(N+1)^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]} \\
&\binom{T}{N+1}^{-1} \sum_{s \in S_{N+1}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s}(N+1)^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]} \\
&>\binom{T}{N+2}^{-1} \sum_{s \in S_{N+2}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s}(N+2)^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}
\end{aligned}
$$

The result holds if (A1) $\Rightarrow$ (A2). In both (A1) and (A2), increasing $N$ by 1 , as happens when moving from an inequality's LHS to its RHS, affects the outermost summation, the number of positive terms in the innermost summation, and the innermost summand, in the following ways.

First, the outermost summation is an average over all possible work schedules; its number of terms increases in $N$ for $N<0.5 T$ and decreases in $N$ for $N \geq 0.5 T$.

Second, increasing $N$ by 1 adds one positive term to the innermost summation.
Third, increasing $N$ by 1 changes the innermost summand's denominator through 3 channels:

1) the distribution of $\lambda_{s}$ shifts either right or left;
2) the $N^{2}$ term increases;
3) $H_{t}$ increases for some terms in the innermost summation.

Channels 2) and 3) increase the innermost summand's denominator. Channel 1 increases the innermost summand's denominator if $N \geq 0.5 T$ and otherwise decreases it, because the $\lambda_{s}$ are uniformly distributed on $\left[\binom{T}{N}^{-1}, 1\right]$, meaning that the distribution of $\lambda_{s}$ shifts to the left (right) when the number of possible work schedules increases (decreases).

Case 1: $N \geq 0.5 T$. This case is easiest because the 3 channels are mutually reinforcing. In (A1), the LHS averages a larger number of terms than the RHS. Given that (A1) is assumed to hold, the terms in RHS average are smaller than those on the LHS. Each term being averaged is determined by the innermost summation, where there are competing effects. As noted, the RHS's innermost summation includes one additional positive term compared to its LHS counterpart. But a countervailing effect is that all terms in the RHS's innermost summation are smaller than their LHS counterparts, due to the 3 channels. The countervailing effect dominates given that (A1) is assumed to hold. (A2) must then also hold, by the same logic. Specifically, the effect of adding one more positive term to the innermost summation in (A2) is the same as in (A1); in either case, one positive ratio is added that has a numerator of either $p V+(1-p) v$ or $q V+(1-q) v$. But the countervailing effect from (A1) is amplified in (A2), because the quadratic term in the summand's denominator causes the summand to shrink at an increasing rate in $N$.

Case 2: $N<0.5 T-1$. The difference here compared to case 1 is that channel 1 now operates in the opposite direction to channels 2 and 3 , raising the possibility that the countervailing effect described in case 1 might not dominate. If it dominates in (A1), which it does by assumption, then it dominates even more strongly in (A2) because the strength of channel 2 relative to channel 1 is increasing in $N$. This is true because, again, the quadratic term in the summand's denominator increases in $N$ at an increasing rate, whereas the degree to which the distribution of $\lambda_{s}$ shifts to the left as $N$ increases is decreasing in $N$. To see the latter point, note that the decrease in the minimum value of $\lambda_{s}$ when $N$ increases to $N+1$ is
$\binom{T}{N}^{-1}-\binom{T}{N+1}^{-1}$, which can easily be shown to be decreasing in $N$ for $N<0.5 T-1$.
Case 3: $N=0.5 T-1$. In this case, the number of potential schedules increases when the number of work hours increases from $N$ to $N+1$ but decreases when the number of work hours increases
from $N+1$ to $N+2$. Thus, (A1) pertains to case 2 , whereas (A2) pertains to case 1 . Given that, by assumption, (A1) holds despite channel 1 operating against channels 2 and 3, (A2) holds given that channels 1,2 , and 3 are mutually reinforcing, as in case 1 .

This completes the proof of point 1 .
2. As $\delta \rightarrow 0$, the productivity advantage of $\mathscr{F}=1$ vanishes, and the condition for $Q_{N}$ (under $\mathscr{F}=$ 0 ) exceeding $Q_{N}$ (under $\mathscr{F}=1$ ) approaches the following inequality, given that $0<N<T$ :
$\sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}>\binom{T}{N}^{-1} \sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}$
Both sides of the inequality have $N$ positive terms in the innermost summation but differ in the terms that appear in that summation. Specifically, the LHS uses only the optimal $N$-hour work schedule from Figure 2, whereas the RHS averages over all $\binom{T}{N} N$-hour work schedules, all but one of which are suboptimal. For example, if $0<N \leq 0.5 T$, then $X_{t}=p V+(1-p) v$ for all positive terms in the LHS's innermost summation, whereas $X_{t}=q V+(1-q) v$ for many of the positive terms in the RHS's innermost summation. Thus, the inequality holds.

As $\delta$ increases, the advantage of $\mathscr{F}=1$ for productivity amplifies, and the condition for $Q_{N}$ (under $\mathscr{F}=0$ ) exceeding $Q_{N}$ (under $\mathscr{F}=1$ ) is as follows:

$$
\begin{aligned}
\sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} & \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s} N^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]} \\
& >\sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta N^{2}\binom{T}{N}^{-1}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}
\end{aligned}
$$

The tradeoff is visible by comparing the innermost summand's numerator to the first term in square brackets in its denominator. The inequality's LHS uses the optimal $N$-hour work schedule from Figure 2, so the numerator of its innermost summation will have relatively more " $p$ " terms (versus " $q$ " terms) than the numerator of the innermost summation on the inequality's RHS. That advantage of $\mathscr{F}=0$ is eroded by the first term in brackets in the denominator. As $\delta$ increases, the value to the employer of a small $\lambda_{s}$ amplifies. Whereas the inequality's LHS averages over all $\binom{T}{N}$ values of $\lambda_{s}$, the RHS uses the minimum $\lambda_{s}$, i.e., $\binom{T}{N}^{-1}$. If $\delta$ is sufficiently large, ceteris paribus, the inequality is reversed for all values of $N$ such that $0<N<T$.
3. Figure 5 illustrates that parameterizations exist for which $N_{I}$ exists and for which $Q_{N}$ is higher (lower) under $\mathscr{F}=1$ than under $\mathscr{F}=0$, for $N>N_{I}\left(N \leq N_{I}\right)$. Such parameterizations involve intermediate values of $\delta$, given that point 2 establishes that $N_{I}$ does not exist if $\delta$ is too high or low. It remains to show that whenever $N_{l}$ exists, $Q_{N}$ is never higher (lower) under $\mathscr{F}=0$ than under $\mathscr{F}=1$, for $N>N_{l}\left(N \leq N_{l}\right)$.

Suppose that $N_{l}$ exists and that $Q_{N}$ is lower under $\mathscr{F}=0$ than under $\mathscr{F}=1$ for $1 \leq N \leq N_{l}$. Then the following inequality holds for $1 \leq N \leq N_{l}$ :

$$
\begin{aligned}
\sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} & \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta \lambda_{s} N^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]} \\
& <\sum_{s \in \boldsymbol{S}_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \psi_{i j} \sum_{t=1}^{T} \frac{X_{t} h_{t}}{\left[1+\delta N^{2}\binom{T}{N}^{-1}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}
\end{aligned}
$$

Whether this inequality reverses or continues to hold as $N$ increases above $N_{1}$ hinges on how the numerator and denominator of the innermost summand change on both sides of the inequality. Refer to these effects as the "numerator effect" and the "denominator effect". The numerator effect is straightforward on the RHS (corresponding to $\mathscr{F}=1$ ), i.e., for all $N$, half the terms in the outermost summation have $p V+(1-p) v$ in the numerator, and half have $q V+(1-q) v$. The numerator effect on the LHS (corresponding to $\mathscr{F}=0$ ) varies with $N$. If $1 \leq N \leq 0.5 T$ then the numerator is $p V+(1-p) v$ for all terms in the sum. If $N$ exceeds $0.5 T$, however, the fraction of terms with $q V+(1-q) v$ in the numerator becomes positive and is increasing in $N$. Thus, the numerator effect works against the inequality holding. This means that the denominator effect must outweigh the numerator effect at $N=N_{l}$.

When $N$ increases from $N_{l}$ to $N_{l}+1$, the numerator effect either stays unchanged (if $1 \leq N_{l}+1$ $\leq 0.5 T$ ) or weakens (if $0.5 T<N_{I}+1 \leq T-2$ ). If it weakens, the inequality is less likely to flip. Consider the more challenging case where it stays unchanged. The denominator effect amplifies when $N$ increases from $N_{I}$ to $N_{I}+1$, both because of the quadratic in $N$ that appears in the first term in brackets and because the term $\beta_{j} H_{t}$ increases in the second term in brackets. Given that the denominator effect amplifies while the numerator effect remains unchanged (or weakens), $Q_{N_{1}+1}$ is higher under $\mathscr{F}=1$ than under $\mathscr{F}=0$. This contradicts the definition of $N_{l}$. Q.E.D.

## A2. Endogenous, regime-specific wages

This section elaborates on section 3.2 .2 of the text. When $\mathcal{F}=1$ the employer observes $c(\boldsymbol{s})$ so can make a wage bid, $w_{f}$, such that $w_{f} N_{f}-c(\boldsymbol{s})=u$. The employer makes a take-it-or-leave-it offer of $\left(\mathcal{F}=1, N_{f}, w_{f}\right)$, knowing that $c(\boldsymbol{s})=\delta\binom{T}{N_{f}}^{-1} N_{f}^{2}$, so $w_{f}=\frac{u+\delta\binom{T}{N_{f}}^{-1} N_{f}^{2}}{N_{f}}$. To find $N_{f}$, the employer chooses $N$ to maximize $E\left(\Pi_{f}\right)$, with the constraint $w_{f}=\frac{u+\delta\binom{T}{N_{f}}^{-1} N_{f}^{2}}{N_{f}}$ substituted into $E\left(\Pi_{f}\right)$. Then $N_{f}$ is substituted into the constraint to find $w_{f}$.

When $\mathcal{F}=0$, the employer only observes the distribution of $c(\boldsymbol{s})$ rather than its actual value. The employer makes a take-it-or-leave-it offer of $\left(\mathcal{F}=0, N_{e}, w_{e}\right)$. In contrast to the case of $\mathcal{F}=1$, now there is a chance that the worker chooses to "leave it", if $w_{e}$ is too low, in which case the employer makes zero profit. Offering a higher $w_{e}$ insures against that possibility but at the expense of lower profit conditional on the offer being accepted. To compute the solution:

1. Observe that the worker's participation constraint is:

$$
w_{e} \geq \frac{u+\delta \lambda N^{2}}{N}
$$

2. Write down the expression for $N_{e}$, for a given $w_{e}$, which was provided in the main model. The probability that the worker accepts the take-it-or-leave-it offer must be incorporated. Given we, the employer determines $N_{e}$ as follows: ${ }^{53}$

$$
\max \left\{0, \underset{N}{\operatorname{argmax}}\left[\frac{w_{e}}{w_{\max }}\left(\binom{T}{N}^{-1} \sum_{s \in S_{N}} \sum_{i=L}^{H} \sum_{j=L}^{H} \sum_{t=1}^{H} \frac{\psi_{i j} X_{t} h_{t}}{\left[1+\delta \lambda_{s} N^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}-w_{e} N\right)\right]\right\}
$$

where $\frac{w_{e}}{w_{\max }}=\frac{w_{e} N}{u+\delta N^{2}}$ is the probability that the worker accepts the offer, and $w_{\max }$ is the value of $w_{e}$ that solves the participation constraint at the maximum value of $\lambda$, i.e., $\lambda=1$.
3. Next, find $w_{e}$. There are only $\binom{T}{N}$ possible wage offers that make sense, corresponding to the $\binom{T}{N}$ possible values of the unknown (to the employer) parameter, $\lambda_{s}$. Solution steps: $a$. Plug the minimum value of $\lambda_{s}$, which is $\binom{T}{N}^{-1}$, into the participation constraint in step 1).
$b$. Plug the value of $w_{e}$ from $3 a$ into the $N_{e}$ expression in step 2 to get $N_{e}\left(\lambda_{s}\right)$, where the notation indicates that $N_{e}$ is conditional on a particular value of $\lambda_{s}$. Repeat steps $a$ ) and $b$ ) for all $\lambda_{s}$. $c$. Find the employer choice of $\lambda_{s}$ (which is tantamount to choosing $w_{e}$ ) that maximizes expected profit. Call that choice $\lambda_{s e}$. The argmax that defines $\lambda_{s e}$ is over the $\binom{T}{N}$ possible choices of $\lambda_{s}$, which amounts to $\binom{T}{N}$ possible choices of $w_{e}$.

$$
\underset{\lambda}{\operatorname{argmax}}\left[\frac{u+\delta \lambda N_{e}^{2}}{u+\delta N_{e}^{2}}\left(\binom{T}{N_{e}}^{-1} \sum_{k=1}^{\binom{T}{N_{e}}} \sum_{i=L}^{H} \sum_{j=L}^{H} \sum_{t=1}^{T} \frac{\psi_{i j} X_{t} h_{t}}{\left[1+\delta \lambda N_{e}^{2}\right]\left[1+\alpha_{i}+\beta_{j} H_{t}\right]}-u-\delta \lambda N_{e}^{2}\right)\right]
$$

In the preceding expression, $N_{e}$ is $N_{e}(\lambda)$, as computed in step $3 b$.
4. Plug $\lambda_{s e}$ into the constraint from step 1 to get:

$$
w_{e}=\frac{u+\delta \lambda_{s e} N^{2}}{N}
$$

5. Akin to step $3 b$, plug $w_{e}$ into the $N_{e}$ expression in step 2 , to get $N_{e}\left(\lambda_{s e}\right)$.
6. Substitute the value of $N_{e}$ found in step 5 into the $N$ in step 4 , to get:

$$
w_{e}=\frac{u+\delta \lambda_{s e} N_{e}^{2}}{N_{e}}
$$

This process is computationally intensive, but a good approximation is available at low cost. ${ }^{54}$

[^33]
## References

Abbott, M., and O. Ashenfelter. 1976. "Labour Supply, Commodity Demand, and the Allocation of Time". 43(3) Review of Economic Studies 389-411.

Altman, M., and L. Golden. 2007. "The Economics of Flexible Work Scheduling: Theoretical Advances and Contemporary Paradoxes." 17 Research in Sociology of Work 313.

Altonji, J. G., and C. H. Paxson. 1988. "Labor Supply Preferences, Hours Constraints, and Hours-Wage Trade-Offs." 6 Journal of Labor Economics 254-76.

Arriagada, R.E., and L.F. Alarcón. 2014. "Quantification of Productivity Changes Due to Work Schedule Changes in Construction Projects. A Case Study". 13(1) Journal of Construction 9-14.

Asai, Y., Kambayashi, R., and T. Kato. 2015. "Careers of Married Women and the Nature of Husbands' Work: Evidence from Japan".

Bargain, O., K. Orsini, and A. Peichl. "Comparing Labor Supply Elasticities in Europe and the United States." Journal of Human Resources, 49(3), 2014, 723-838.

Baucells, M., and L. Zhao. 2019. "It is Time to Get Some Rest". 65(4) Management Science 1717-34.

Baughman, R., DiNardi, D., and D. Holtz-Eakin. 2003. "Productivity and wage effects of 'family-friendly' fringe benefits". 24 International Journal of Manpower 247-59.

Bell, L. A. 1998. "Differences in Work Hours and Hours Preferences by Race in the U.S." 56(4) Review of Social Economy 481-500.

Bloom, N.A., Liang, J., Roberts, J., and Z.J. Ying. 2015. "Does Working from Home Work? Evidence from a Chinese Experiment." 130(1) The Quarterly Journal of Economics 165-218.

Blundell, R., and T. MaCurdy "Labor Supply: A Review of Alternative Approaches, " in Handbook of Labor Economics, Vol. 3A, edited by O. Ashenfelter and D. Card. Amsterdam, The Netherlands: North-Holland Elsevier, 1999, 1559-695.

Blundell, R., and I. Walker. 1982. "Modelling the Joint Determination of Household Labour Supplies and Commodity Demands". 92(366) The Economic Journal 351-64.

Brachet, T., David, G., and A.M. Drechsler. 2012. "The Effect of Shift Structure on Performance". 4(2) American Economic Journal: Applied Economics 219-46.

[^34]Brown, C. 1980. "Equalizing Differences in the Labor Market". 94(1) Quarterly Journal of Economics 113-34.

Card, D. 1990. "Labor Supply with a Minimum Hours Threshold". 33 Carnegie-Rochester Conference Series on Public Policy 169-92.

Chaplin, J., Mangla, J., Pardon, S., and C. Airey. 2005. The Workplace Employment Relations Survey (WERS) 2004 Technical Report (Cross-section and Panel Surveys), Prepared for Department of Trade and Industry.

Cheng, M.-Y., Huang, K.Y., and M. Hutomo. 2018. "Multiobjective Dynamic-Guiding PSO for Optimizing Work Shift Schedules". Journal of Construction Engineering and Management 144(9).

Collewet, M., and J. Sauermann. 2017. "Working Hours and Productivity". 47 Labour Economics 96-106.

Danziger, S., Levav, J., and L. Avanaim-Pesso. 2011. "Extraneous Factors in Judicial Decisions". 108(17) Proceedings of the National Academy of Sciences 6889-92.

Deardorff, A.V., and F.P. Stafford. 1976. "Compensation of Cooperating Factors". 44(4) Econometrica 671-84.

Denison, E. F. 1962. The Sources of Economic Growth in the United States and the Alternatives Before Us. Committee for Economic Development, New York.

Department for Business, Innovation and Skills, Workplace Employee Relations Survey, 2011 [computer file]. Colchester, Essex: UK Data Archive [distributor], February 2013. SN: 7226, http://dx.doi.org/10.5255/UKDA-SN-7226-1.

DeVaro, J. 2022. "Performance Pay, Working Hours, and Health-Related Absenteeism." 61(4) Industrial Relations 327-52.

DeVaro, J., and J. Heywood. 2017. "Performance-Based Pay and Work-Related Health Problems: A Longitudinal Study of Establishments." 70 Industrial and Labor Relations Review 670-703.

DeVaro, J., and N.L. Maxwell. 2014. "The Elusive Wage-Benefit Tradeoff: The Case of Employer-Provided Health Insurance". 37 International Journal of Industrial Organization 2337.

DeVaro, J., and H. Morita. 2013. "Internal Promotion and External Recruitment: A Theoretical and Empirical Analysis". 31 Journal of Labor Economics 227-69.

DeVaro, J., and M. Waldman. 2012. "The Signaling Role of Promotions: Further Theory and Empirical Evidence. 30(1) Journal of Labor Economics 91-147.

Dickens, W. T., and S. J. Lundberg. 1993. "Hours Restrictions and Labor Supply". 34(1) International Economic Review 169-92.

Duncan, G.J., and B. Holmlund. 1983. "Was Adam Smith Right After All? Another Test of the Theory of Compensating Wage Differentials". 1(4) Journal of Labor Economics 366-79.

Feldstein, M.S. 1968. "Estimating the Supply Curve of Working Hours". 20(1) Oxford Economic Papers 74-80.

Frederiksen, A., Kato, T., and N. Smith. 2018. "Working Hours and Top Management Appointments: Evidence from Linked Employer-Employee Data" IZA Discussion Paper 11675.

Gariety, B. and Shaffer, S. 2001. "Wage Differentials Associated With Flextime". 124 Monthly Labor Review 69-75.

Gibbons, R., and M. Waldman. 1999. "A Theory of Wage and Promotion Dynamics Inside Firms". 114(4) Quarterly Journal of Economics 1321-58.

Gibbons, R., and M. Waldman. 2006. "Enriching a Theory of Wage and Promotion Dynamics Inside Firms". 24(1) Journal of Labor Economics 59-107.

Gicheva, D. 2013. "Working Long Hours and Early Career Outcomes in the High-End Labor Market". 31(4) Journal of Labor Economics 785-824.

Golden, L. 2015. "FLSA Working Hours Reform: Worker Well-Being Effects in an Economic Framework". 54(4) Industrial Relations 717-49.

Golden, L., and T. Gebreselassie. 2007. "Overemployment Mismatches: The Preference for Fewer Work Hours". Monthly Labor Review, April, 18-37.

Goldin, C. 2014. "A Grand Gender Convergence: Its Last Chapter". 104(4) American Economic Review 1091-119.

Gronberg, T., and W.R. Reed. 1994. "Estimating Workers’ Marginal Willingness to Pay for Job Attributes Using Duration Data". 29(3) Journal of Human Resources 911-33.

Gruber, J. 1994. "The Incidence of Mandated Maternity Benefits". 84 American Economic Review 622-41.

Gunderson, M., and D. Hyatt. 2001. "Workplace Risks and Wages: Canadian Evidence from Alternative Models". S34 Canadian Journal of Economics 377-95.

Gunderson, M., Hyatt, D., and J. Persando. 1992. "Wage-Pension Trade-offs in Collective Agreements". 46 Industrial and Labor Relations Review 146-60.

Hamermesh, D. 1999. "The Timing of Work Over Time". 109(452) The Economic Journal 3766.

Hanna, A. S., Chang, C.-K., Sullivan, K. T., and J. A. Lackney. 2008. "Impact of Shift Work on Labor Productivity for Labor Intensive Contractor". 134(3) Journal of Construction Engineering and Management.

Heywood, J.S., Siebert, W.S., and X. Wei. 2007. "The Implicit Wage Costs of Family Friendly Work Practices". 59(2) Oxford Economic Papers 275-300.

Ibanez, M.R., and M.W. Toffel. 2020. "How Scheduling Can Bias Quality Assessment: Evidence from Food-Safety Inspections". 66(6) Management Science 2396-416.

Johnson, N.B., and K.G. Provan. 1995. "The Relationship Between Work/Family Benefits and Earnings: A Test of Competing Predictions". 24 Journal of Socio-Economics 571-84.

Kahn, S., and K. Lang. 1991. "The Effect of Hours Constraints on Labor Supply Estimates". 93 Review of Economics and Statistics 605-11.

Kato, T., Ogawa, H., and H. and Owan. 2016. "Working Hours, Promotion, and the Gender Gap in the Workplace". IZA Discussion Paper No. 10454.

Keane, M. P. 2011. "Labor Supply and Taxes: A Survey". 59(4) Journal of Economic Literature 961-1075.

Kersley, B., Alpin, C., Forth, J., Bryson, A., Bewley, H., Dix, G., and Oxenbridge, S. (2006) Inside the Workplace: Findings from the 2004 Workplace Employment Relations Survey, London: Routledge.

Kuhn, P., and F. Lozano. 2008. "The Expanding Workweek? Understanding Trends in Long Work Hours Among U.S. Men, 1979-2006". 26(2) Journal of Labor Economics 311-43.

Labanca, C., and D. Pozzoli. 2021. "Constraints on Hours Within the Firm". Journal of Labor Economics, forthcoming.

Landers, R. M., Rebitzer, J. B., and L. J. Taylor. 1996. "Rat Race Redux: Adverse Selection in the Determination of Work Hours in Law Firms". 86(3) American Economic Review 329-48.

Lewis, H. G. 1957. "Hours of Work and Hours of Leisure," Paper presented at the Proceedings of the Ninth Annual Meeting of the Industrial Relations Research Association, Madison, WI, 196-206.

Machin, Stephen J., and Mark B. Stewart. 1990. "Unions and the Financial Performance of British Private Sector Establishments". 5(4) Journal of Applied Econometrics 327-50.

Machin, Stephen J., and Mark B. Stewart. 1996. "Trade Unions and Financial Performance." 48(2) Oxford Economic Papers 213-41.

Matthews, R. C. O., Feinstein, C. H., and J. Odling-Smee. 1982. British Economic Growth 18561972. Stanford, CA: Stanford University Press.

McConkey, K., and E. Koromyslova. 2018. "A Comparative Study of Shift Work: Days, Nights, and Weekends: Which Shift Yields Higher Output and Lower Defects." Journal of Management and Strategy 9(3).

Meghir, C., and D. Phillips "Labour Supply and Taxes," in Dimensions of Tax Design: The Mirrlees Review, edited by S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Gammie, P. Johnson, G. Myles, and J. Poterba. Oxford and London: Oxford University Press, 2010, 202-74.

Moffitt, R. 1982. "The Tobit Model, Hours of Work and Institutional Constraints". 64 Review of Economics and Statistics 510-15.

Oyer, P. 2008. "Salary or Benefits?" 28 Research in Labor Economics 429-67.
Park, W., and Y. Park. 2019. "When Less Is More: The Impact of the Regulation on Standard Workweek on Labor Productivity in South Korea". 38(3) Journal of Policy Analysis and Management 681-705.

Pencavel, J. 2004. "The Surprising Retreat of Union Britain" in David Card, Richard Blundell, and Richard B. Freeman, eds., Seeking a Premier Economy: The Economic Effects of British Economic Reforms, 1980-2000 University of Chicago Press, National Bureau of Economic Research, pp. 181-232.

Pencavel, J. 2015. "The Productivity of Working Hours". 125(589) The Economic Journal 205276.

Pencavel, J. 2016a. "Recovery from Work and the Productivity of Working Hours". 83(332) Economica 545-63.

Pencavel, J. 2016b. "Whose Preferences Are Revealed in Hours of Work?" 54(1) Economic Inquiry 9-24.

Pencavel, J. 2018. Diminishing Returns at Work: The Consequences of Long Working Hours. New York, NY: Oxford University Press.

Rebitzer, J. B., and L. J. Taylor. 1995. "Do Labor Markets Provide Enough Short-Hour Jobs? An Analysis of Work Hours and Work Incentives". 33(2) Economic Inquiry 257-73.

Reiss, P., and F. Wolak. 2007. "Structural Econometric Modeling: Rationales and Examples from Industrial Organization," Chapter 64 in Handbook of Econometrics, vol. 6, Part A, pp. 4277-415.

Rosen, S. 1969. "On the Interindustry Wage and Hours Structure". 77(2) Journal of Political Economy 249-73.

Rosen, S.1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition". 82(1) Journal of Political Economy 34-55.

Rosen, S. 1986. "The Theory of Equalizing Differences". In Handbook of Labor Economics, vol.1, ed. Orley C. Ashenfelter and Richard Layard, 641-92. New York: Elsevier.

Saez, E., J. Slemrod, and S. Giertz. 2012. "The Elasticity of Taxable Income with Respect to Marginal Tax Rates". 50(1) Journal of Economic Literature 3-50.

Sato, K., Kuroda, S., and H. Owan. 2020. "Mental Health Effects of Long Work Hours, Night and Weekend Work, and Short Rest Periods". 246 Social Science \& Medicine 112774.

Shepard, E.M., Clifton, T.J., and D. Kruse. 1996. "Flexible Work Hours and Productivity: Some Evidence from the Pharmaceutical Industry". 35 Industrial Relations 123-39.

Stewart, M. B., and J. K. Swaffield. 1997. "Constraints on the Desired Hours of Work of British Men". 107(441) The Economic Journal 520-35.

Thomas, Maura. 2020. "The Downside of Flextime." Harvard Business Review, May 14, 2020.
Wall, T.D., Michie, J., Patterson, M., Wood, S.J., Sheehan, M., Clegg, C.W., and M. West. 2004. "On the Validity of Subjective Measures of Company Performance". 57(1) Personnel Psychology 95-118.

Waldman, M. 1984. "Job Assignments, Signaling and Efficiency". 15(2) RAND Journal of Economics 255-70.

Weiss, Y. 1996, "Synchronization of Work Schedules". 37(1) International Economic Review 157-79.

Woodland S., Simmonds, N., Thornby, M., Fitzgerald, R., and A. McGee. 2003. Work-Life Balance Study: Results from the Employer Survey, Employment Series No. 22, DTI, London.

Zábojník, J., and D. Bernhardt. 2001. "Corporate Tournaments, Human Capital Acquisition, and the Firm Size-Wage Relation". 68(3) Review of Economic Studies 693-716.

Table 1: Descriptive statistics on key variables

|  | Min | $25 \%$ | $50 \%$ | $75 \%$ | Max | Mean | $\sigma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Productivity | 0 | 0 | 0 | 1 | 1 | 0.394 | 0.489 |
| Flextime | 0 | 0 | 0 | 1 | 1 | 0.332 | 0.471 |
| Home | 0 | 0 | 0 | 1 | 1 | 0.304 | 0.460 |
| ShiftChange | 0 | 0 | 0 | 1 | 1 | 0.497 | 0.500 |
| year 2004 | 0 | 0 | 0.5 | 1 | 1 | 0.5 | 0.5 |
| employment | 5 | 9 | 15 | 35 | 11,566 | 49.073 | 196.645 |
| union | 0 | 0 | 0 | 1 | 1 | 0.366 | 0.482 |
| private sector | 0 | 1 | 1 | 1 | 1 | 0.779 | 0.415 |

In the above table, $x \%$ is the value of the $x^{\text {th }}$ percentile and $\sigma$ is the standard deviation.

Table 2: Average "usual weekly hours" of surveyed workers within an establishment, by year

|  | $2004-2011$ | 2004 | 2011 |
| :--- | :--- | :--- | :--- |
| Flextime $=1$ | 34.74 | 35.13 | 34.31 |
| Flextime $=0$ | 32.53 | 32.47 | 32.58 |
| Difference | 2.21 | 2.66 | 1.73 |
| $(p$-value $)$ | $(0.001)$ | $(0.026)$ | $(0.088)$ |
| \# establishments | $580($ Flextime $=1)+$ | $282($ Flextime $=1)+$ <br> $320($ Flextime $=0)=$ <br> $318($ Flextime $=0)=$ <br> $=600$ | $302($ Flextime $=1)+$ <br> 600 |

Note: All statistics incorporate sampling weights. $p$-values for the difference in means in each column are for the hypothesis test in which the null hypothesis is that there is no difference in means, and the alternative hypothesis is that hours are higher under flextime. The directional hypothesis is implied by the theoretical model.

Table 3: Linear probability models of labor productivity
Panel A: No Flextime in the model

|  | Dependent variable $=L_{j t}$ (i.e., high labor productivity) |  |  |
| :--- | :--- | :--- | :--- |
| covariate | $c=35$ hours | $c=50$ hours | $c=65$ hours |
| $H_{c}$ | $0.582(0.165)$ | $-0.057(0.258)$ | $-1.166(0.350)$ |
| Employment | $-0.039(0.111)$ | $-0.032(0.106)$ | $-0.025(0.109)$ |
| Union | $-0.222(0.114)$ | $-0.211(0.131)$ | $-0.215(0.129)$ |
| Private | $0.009(0.186)$ | $-0.036(0.186)$ | $-0.017(0.186)$ |
| $D_{t}$, i.e., Year $=2004$ | $-0.003(0.047)$ | $0.007(0.050)$ | $0.020(0.050)$ |
| Constant | $0.098(0.290)$ | $0.486(0.291)$ | $0.407(0.250)$ |

Panel B: Flextime included in the model

|  | Dependent variable $=L_{j t}$ (i.e., high labor productivity) |  |  |
| :--- | :--- | :--- | :--- |
| covariate | $c=35$ hours | $c=50$ hours | $c=65$ hours |
| $H_{c}$ | $0.550(0.188)$ | $-0.034(0.265)$ | $-1.451(0.282)$ |
| Flextime $\times H_{c}$ | $0.051(0.265)$ | $0.025(0.358)$ | $0.924(0.471)$ |
| Flextime | $0.018(0.191)$ | $0.090(0.093)$ | $0.093(0.081)$ |
| Employment | $-0.038(0.112)$ | $-0.033(0.107)$ | $-0.028(0.108)$ |
| Union | $-0.229(0.116)$ | $-0.223(0.132)$ | $-0.225(0.130)$ |
| Private | $0.001(0.184)$ | $-0.059(0.197)$ | $-0.032(0.199)$ |
| $D_{t}$, i.e., Year $=2004$ | $-0.004(0.047)$ | $0.004(0.050)$ | $0.016(0.050)$ |
| Constant | $0.124(0.301)$ | $0.494(0.294)$ | $0.390(0.247)$ |

Note: Panel $B$ reports estimates of equation (1) for 3 values of $c$. Panel $A$ does the same but with the interaction term Dropped from the right-hand side. All regressions include establishment fixed effects and also dummies for 12 Industry categories, which exhibit enough temporal variation to identify their coefficients in the presence of establishment fixed effects. Panel robust standard errors are in parentheses beside each estimate.

Table 4: Alternative Measures of Flexible Work Schedules
Panel A: Working from home during normal business hours

|  | Dependent variable $=L_{j t}$ (i.e., high labor productivity) |  |  |
| :--- | :--- | :--- | :--- |
| covariate | $c=35$ hours | $c=50$ hours | $c=65$ hours |
| $H_{c}$ | $0.564(0.164)$ | $-0.249(0.264)$ | $-1.471(0.268)$ |
| Home $\times H_{c}$ | $-0.005(0.231)$ | $0.629(0.507)$ | $1.452(0.339)$ |
| Home | $0.104(0.153)$ | $0.080(0.087)$ | $0.133(0.080)$ |
| Employment | $-0.051(0.117)$ | $-0.073(0.116)$ | $-0.050(0.112)$ |
| Union | $-0.213(0.111)$ | $-0.207(0.131)$ | $-0.206(0.128)$ |
| Private | $0.006(0.186)$ | $-0.023(0.189)$ | $-0.012(0.187)$ |
| $D_{t}$, i.e., Year $=2004$ | $0.003(0.046)$ | $0.009(0.049)$ | $0.026(0.050)$ |
| Constant | $0.096(0.283)$ | $0.472(0.281)$ | $0.364(0.235)$ |

Panel B: Ability to change set working hours (including changing shift pattern)

|  | Dependent variable $=L_{j t}$ (i.e., high labor productivity) |  |  |
| :--- | :--- | :--- | :--- |
| Covariate | $c=35$ hours | $c=50$ hours | $c=65$ hours |
| $H_{c}$ | $0.809(0.187)$ | $0.071(0.310)$ | $-1.457(0.838)$ |
| Shiftchange $\times H_{c}$ | $-0.531(0.190)$ | $-0.338(0.400)$ | $0.353(0.943)$ |
| Shiftchange | $0.326(0.127)$ | $0.022(0.081)$ | $0.017(0.080)$ |
| Employment | $-0.031(0.120)$ | $-0.030(0.110)$ | $-0.028(0.108)$ |
| Union | $-0.197(0.119)$ | $-0.204(0.130)$ | $-0.215(0.128)$ |
| Private | $-0.047(0.184)$ | $-0.043(0.187)$ | $-0.015(0.186)$ |
| $D_{t}$, i.e., Year $=2004$ | $-0.009(0.045)$ | $0.009(0.048)$ | $0.022(0.049)$ |
| Constant | $-0.068(0.294)$ | $0.486(0.296)$ | $0.376(0.242)$ |

Note: Regression estimates for equation (1), with Flextime replaced by Home in Panel $A$ and by ShiftChange in Panel $B$. All regressions include establishment fixed effects and also dummies for 12 industry categories, which exhibit enough temporal variation to identify their coefficients in the presence of establishment fixed effects. Panel robust standard errors are in parentheses beside each estimate.

Figure 1: $T$-hour Potential Workday


Figure 2: All Optimal Employer-Determined Work Schedules ( $T=16$ )
Early Hours
Peak Hours
Late Hours

| $N_{e}=0$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $=1$ |  |  |  |  | 5 |  |  |  |  |  |  |  |  |  |  |  |
| $=2$ |  |  |  |  | 5 | 6 |  |  |  |  |  |  |  |  |  |  |
| $=3$ |  |  |  |  | 5 | 6 | 7 |  |  |  |  |  |  |  |  |  |
| $=4$ |  |  |  |  | 5 | 6 | 7 | 8 |  |  |  |  |  |  |  |  |
| $=5$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 |  |  |  |  |  |  |  |
| $=6$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |  |  |  |
| $=7$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 |  |  |  |  |  |
| $=8$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |  |  |  |  |
| $=9$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |  |  |  |
| $=10$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |  |  |
| $=11$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |  |
| $=12$ |  |  |  |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $=13$ |  |  |  | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $=14$ |  |  | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $=15$ |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $=16$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |

Note: Each row depicts the optimal employer-determined work schedule given the optimal number of hours stated in the first column. $T=16$. Integers in all columns beyond the first represent the hour of the potential workday. For example, when $N_{e}=3$, hours 5,6 , and 7 of the day are worked.

Figure 3: Examples of Potential Flextime Work Schedules ( $T=16$ )

## Early Hours

Peak Hours
Late Hours

| $N_{f}=0$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $=1$ |  |  |  |  |  | 6 |  |  |  |  |  |  |  |  |  |  |
| $=2$ |  |  |  | 4 | 5 |  |  |  |  |  |  |  |  |  |  |  |
| $=3$ |  |  |  |  |  |  |  |  |  |  | 11 | 12 | 13 |  |  |  |
| $=4$ |  |  |  |  |  |  | 7 | 8 | 9 | 10 |  |  |  |  |  |  |
| $=5$ |  |  |  | 4 | 5 | 6 |  | 8 | 9 |  |  |  |  |  |  |  |
| $=6$ | 1 | 2 | 3 | 4 |  |  | 7 | 8 |  |  |  |  |  |  |  |  |
| $=7$ |  | 2 |  | 4 |  | 6 |  | 8 |  | 10 |  | 12 |  | 14 |  |  |
| $=8$ | 1 | 2 | 3 | 4 |  |  |  |  |  |  |  |  | 13 | 14 | 15 | 16 |
| $=9$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |  |  |  |  |  |  |
| $=10$ |  |  | 3 | 4 | 5 | 6 | 7 |  |  | 10 | 11 | 12 | 13 | 14 |  |  |
| $=11$ | 1 | 2 | 3 | 4 | 5 | 6 |  |  |  |  | 11 | 12 | 13 | 14 | 15 |  |
| $=12$ |  |  | 3 | 4 | 5 | 6 |  | 8 | 9 | 10 | 11 |  | 13 | 14 | 15 | 16 |
| $=13$ |  | 2 | 3 | 4 | 5 |  | 7 | 8 | 9 | 10 |  | 12 | 13 | 14 | 15 | 16 |
| $=14$ | 1 |  | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |  | 16 |
| $=15$ |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $=16$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |

Note: Each row depicts a potential flextime work schedule that the worker might choose given the optimal number of (employer-chosen) hours stated in the first column. $T=16$. Integers in all columns beyond the first represent the hour of the potential workday. For example, when $N_{f}=3$, hours 11,12 , and 13 of the day are worked. One potential schedule is depicted for each value of $N_{f}$, so only 17 of the 65,536 possible schedules are presented.

Figure 4: Optimal scheduling regime and number of hours in the workday


Note: $T=16, V=50, v=40, p=0.9, q=0.5, w=14, \alpha=0.1, \beta=0.2, \delta=0.001$, $\xi_{\alpha}=\xi_{\beta}=0.5, \psi_{L L}=\psi_{L M}=\psi_{L H}=\psi_{M L}=\psi_{M H}=\psi_{H L}=\psi_{H M}=\psi_{H H}=0, \psi_{M M}=1$.

Figure 5: Hours-output profiles by scheduling regime


Note: $T=16, V=70, v=40, p=0.9, q=0.5, w=14, \alpha=0.1, \beta=0.9, \delta=0.003$, $\xi_{\alpha}=\xi_{\beta}=0.5, \psi_{L L}=\psi_{L M}=\psi_{L H}=\psi_{M L}=\psi_{M H}=\psi_{H L}=\psi_{H M}=\psi_{H H}=0, \psi_{M M}=1$.

Figure 6: Labor Productivity and Flextime
Panel A: Point estimates


Panel $B$ : $t$-statistics


Note: Regression results for 41 estimations of equation (1), corresponding to the 41 values of $c$ (from 25 through 65) indicated on the horizontal axis. The solid line is the point estimate of $\beta_{c}$ in Panel $A$ and its $t$-statistic in Panel $B$. The dashed line is the point estimate of $\beta_{c}+\lambda_{c}$ in Panel $A$ and its $t$-statistic in Panel B. Standard errors are panel robust.


Panel B: t-statistics


Note: Regression results for 41 estimations of equation (1), corresponding to the 41 values of $c$ (from 25 through 65) indicated on the horizontal axis. The independent variable Flextime in equation (1) is replaced by Home in all 41 estimations. The solid line is the point estimate of $\beta_{c}$ in Panel $A$ and its $t$-statistic in Panel $B$. The dashed line is the point estimate of $\beta_{c}+\lambda_{c}$ in Panel $A$ and its $t$-statistic in Panel $B$. Standard errors are panel robust.

Figure 8: Labor Productivity and ShiftChange

## Panel A: Point estimates



Panel $B$ : $t$-statistics


Note: Regression results for 41 estimations of equation (1), corresponding to the 41 values of $c$ (from 25 through 65) indicated on the horizontal axis. The independent variable Flextime in equation (1) is replaced by ShiftChange in all 41 estimations. The solid line is the point estimate of $\beta_{c}$ in Panel $A$ and its $t$-statistic in Panel $B$. The dashed line is the point estimate of $\beta_{c}+\lambda_{c}$ in Panel $A$ and its $t$-statistic in Panel $B$. Standard errors are panel robust.


[^0]:    * I am indebted to John Pencavel for inspiring my interest in the subject and for extensive discussions and research collaboration that formed the foundation for some of the empirical results presented here. I acknowledge the Department of Trade and Industry, the Economic and Social Research Council, the Advisory, Conciliation and Arbitration Service and the Policy Studies Institute as the originators of the 2004 and 2011 Workplace Employee Relations Study data, and the Data Archive at the University of Essex as the distributor of the data. None of these organizations bears any responsibility for the authors' analysis and interpretations of the data. I am grateful for helpful comments from Hugh Cassidy, Lonnie Golden, Antti Kauhanen, and seminar participants at the 2024 Society of Labor Economists meeting, the LERA 72nd meeting, and Waseda University.

[^1]:    ${ }^{1}$ For some jobs, pay is convex in hours worked, as discussed in Goldin (2014) in the context of the gender wage gap. The assumption of diminishing returns to working hours does not contradict pay being convex in hours. The concept of diminishing marginal returns ignores the cost side of the profit function, but the cost side can generate pay that is convex in hours. Suppose that there are fixed costs of hiring and training workers and that there is also premium pay for overtime work. If fixed costs are sufficiently high relative to premium pay for overtime, then the employer finds it profitable to "overwork" the worker (generating pay that is convex in hours) rather than hiring more workers. That can happen even if there are diminishing returns on the revenue side of the profit function.
    ${ }^{2}$ Chapter 6 of Pencavel (2018) provides evidence on the relationship between long hours and workers' health and well-being, including cognitive function, cardiovascular disease, injuries and accidents, and household well-being. Sato et al. (2020) provide evidence that long working hours, as well as night and weekend work and insufficient rest periods, are associated with deterioration in mental health. See also DeVaro (2022) for empirical evidence on the positive relationship between long hours and health-related absenteeism.

[^2]:    ${ }^{3}$ Such negative spillovers do not even require that coworkers be physically proximate. For example, a researcher would suffer reduced productivity if their collaborator in another country were to make mistakes or be slow to respond to emails due to exhaustion from excessive working hours.
    ${ }^{4}$ An early bird who is forced to stay up late for a few nights for exogenous reasons (e.g., jetlag) might temporarily have a night owl's preferences over work schedules.

[^3]:    ${ }^{5}$ Pencavel (2015) finds such evidence in a narrow production setting from more than a century ago, i.e., women working in manufacturing plants producing artillery shells for the British military during World War I.

[^4]:    ${ }^{6}$ For surveys of this substantial literature on labor supply, see Blundell and MaCurdy (1999), Meghir and Phillips (2010), Keane (2011), Saez, Slemrod, and Giertz (2012), and Bargain, Orsini, and Peichl (2014).

[^5]:    unattractive to workers. The goal of the framework is to explain the empirical result that the prevalence of night shifts declined starting in the early 1990s. The framework disallows split shifts and, therefore, flextime. Weiss (1996) develops a model to explain why a group of interacting workers with varying degrees of aversion to work synchronize their work schedules. The model accounts for both worker and firm behavior, and work schedules are the result of an equilibration process that maximizes the welfare gains from using different combinations of workers at different times. The model generates a multiplicity of synchronized equilibria.
    ${ }^{9}$ For further discussion of the relationship between long working hours and productivity, see Denison (1962), Matthews et al. (1982), and Pencavel (2016a, 2018). Examples of research focusing on other aspects of long working hours include Landers, Rebitzer, and Taylor (1996), Kuhn and Lozano (2008), Gicheva (2013), Asai, Kambayashi, and Kato (2015), Kato, Ogawa, and Owan (2016), Frederiksen, Kato, and Smith (2018), Sato et al. (2020), and DeVaro (2022).
    ${ }^{10}$ Specifically, Pencavel (2015) concludes his study with the following quote from Denison (1962, p. 39) concerning analysis of the hours-output profile in broad samples: "Few studies offer more promise of adding to welfare and contributing to wise decisions in a matter that may greatly affect the future growth rate than a really thorough investigation of the present relationship between hours and output. Such an investigation would deal with a wide variety of occupations and industries operating under different conditions."

[^6]:    ${ }^{11}$ Endogenous wages that differ by scheduling regime are treated in an extension (section 3.2.2).

[^7]:    ${ }^{12}$ Examples, which are motivated by work on the hedonic utility model (Rosen 1974, 1986), include Brown (1980), Duncan and Holmlund (1983), Gunderson et al. (1992), Gronberg and Reed (1994), Gruber (1994), Johnson and Provan (1995), Gariety and Shaffer (2001), Gunderson and Hyatt (2001), Baughman et al. (2003), Oyer (2008), DeVaro and Maxwell (2014), and Eriksson and Kristensen (2014). Several of these empirical studies of the wagebenefits tradeoff focus specifically on flexible work practices; see section 3.2.2 for details.

[^8]:    ${ }^{13}$ See Heywood et al. (2007) for empirical evidence and a survey of the literature.
    ${ }^{14}$ In practice, there are various employer costs associated with family-friendly work practices that mitigate their productivity advantages, as noted in Heywood et al. (2007). These include forgoing the gains from complementarities when team members' schedules do not coincide (see also Labanca and Pozzoli 2021), forgoing revenue when employees' schedules do not coincide with peak hours of consumer demand (as in the present study's theoretical model), covering for absent workers, hiring additional workers, and leaving some job functions short staffed. Woodland et al. (2003, p. 246) find that 70 percent of employers who adopted family-friendly work practices experienced such problems.

[^9]:    ${ }^{15}$ Although premium pay for overtime work is a consideration in the U.S., it is not a consideration in the U.K. which is the source of my data. As discussed in section 2 of DeVaro (2022) there is no overtime law in the U.K. and no hours threshold (like 40 per week) that is relevant for institutional reasons. The model is most literally applicable to hourly-paid workers, for whom both the minimum and maximum hours, if not the exact hours, are often set by the employer in advance. In contrast, salaried workers generally have more discretion over their work schedules (both the number of hours worked and which hours are worked). Nonetheless, to the extent that salaries incorporate expectations about the number of working hours, even a salaried worker's pay is at least implicitly a function of the number of working hours. That's why investment bank analysts and associates get paid the big bucks.

[^10]:    ${ }^{18}$ That result is particularly interesting because $w$ is common to both regimes. If instead there were regime-specific wages, then an exogenous change in one while holding the other constant would obviously change the relative profitability of the two regimes.
    ${ }^{19}$ Product demand is only one possible interpretation. What matters is that the employer prefers some hours of the potential workday over others, regardless of the reason. An alternative interpretation of "peak hours" is that a second worker is employed during those hours. Complementarities in production then imply that productivity is highest when both workers on the team are simultaneously present. That interpretation evokes Deardorff and Stafford (1976), where the firm's technology requires the simultaneous presence, during the workday, of two factors of production, which could be two types of workers (perhaps skilled and unskilled, or supervisor and subordinate) or labor and capital. Relatedly, Labanca and Pozzoli (2021) argue that coordination of workers' hours within the firm enhances firm productivity but implies fixed costs and requires that heterogeneous coworkers work the same hours.

[^11]:    ${ }^{22}$ The assumption that workers are less productive when forced to work against their will is uncontroversial in most cases, but there are exceptions. For example, some workers might have a taste for packing in all of their long hours consecutively, to the point of exhaustion, just to get out of the office as early as possible. In such cases, forcing workers to take a utility-reducing lunch break might increase their hourly productivities following the break. The model of Baucells and Zhao (2019) addresses the optimal timing of effort-enhancing breaks.

[^12]:    ${ }^{23}$ For example, if the worker's favorite team is in the playoffs and the critical game is to be shown on television during scheduled work hours, the worker is distracted throughout the workday (before, during, and after the game), which could hurt productivity independently of the number of hours that have been worked so far.
    ${ }^{24}$ Thus, $E\left[\lambda_{s} \mid \mathcal{F}=0\right]=0.5\left[1+\binom{T}{N}^{-1}\right]$. The uniform assumption is for simplicity and is not pivotal. The preceding expression converges to 0.5 from above when $\binom{T}{N}$ gets large. Convergence happens extremely fast for values of $T$ and $N$ that are practically relevant, due to combinatorial explosion. For example, if $T=12$ and $N=6$,

[^13]:    there are 924 possible work schedules, and $E[\lambda(s) \mid \mathcal{F}=0] \approx 0.50054$, whereas if $T=16$ and $N=8$, there are 12,870 possible work schedules, and $E[\lambda(\boldsymbol{s}) \mid \mathcal{F}=0] \approx 0.50004$.
    ${ }^{25}$ The labels $L, M$, and $H$ stand for "low", "medium", and "high".

[^14]:    ${ }^{26}$ If $N_{e}=1$, the earliest peak hour would be assigned, namely hour $0.25 T+1$. If $N_{e}=2$, the two earliest peak hours would be assigned, and so on. If $N_{e}=0.5 T$, the assigned hours and the peak hours would coincide. If $N_{e}=0.5 T+1$, the sole nonpeak hour would be the earliest in the category of late hours, and subsequent hours would be assigned accordingly until the last hour of the potential workday. Any additional desired hours beyond $0.75 T$ would be assigned from the early (i.e., pre-peak) part of the day, starting from the latest hour in that set (i.e., hour $0.25 T$ ) and ending with the earliest (i.e., the first hour of the potential workday, namely hour 1 ).

[^15]:    ${ }^{27}$ As in the corresponding expression under employer-determined scheduling in section 3.1.1, the outermost of the four summations in the computation of expected profit under flextime involves $\binom{T}{N}$ terms. In the flextime computation, the outermost sum arises because the employer does not know which $N$-hour schedule the worker will choose, so it is necessary to sum over all $\binom{T}{N}$ possible $N$-hour schedules. In contrast, in the employer-determined scheduling computation, the outermost sum arises because the employer does not know which cost parameter, $\lambda_{s}$, the worker attaches to the assigned schedule, so it is necessary to sum over all $\binom{T}{N}$ possible values of $\lambda_{s}$.

[^16]:    ${ }^{28}$ All computations in the study assume $T=16$. Higher values of $T$ significantly increase computing time. If $T=16$, each evaluation of the flextime expected profit function involves summing more than 9.4 million terms, and that number increases to more than 3.6 billion if $T=24$.
    ${ }^{29}$ The selected parameter values differ between Figures 4 and 5. If hours-output profiles like those in Figure 5 are plotted using the parameter values from Figure 4, inverted- $U$-shaped profiles emerge for both regimes, but they are harder to visually differentiate than in the current Figure 5, and they both share the same peak (at $N=15$ ).

[^17]:    ${ }^{30}$ When the potential workday increases by a factor of 1.5 , i.e., from 16 to 24 hours, the number of possible work schedules (including the one for $N_{f}=0$ ) increases by a factor of 256 , i.e., from 65,536 to $16,777,216$, and the number of terms in the summation that defines $E\left(\Pi_{f}\right)$ increases by a factor of more than 384 , i.e., from $9,437,040$ to 3,623,878,000.

[^18]:    ${ }^{31}$ Endogenous wages have another function in a compensating differentials equilibrium, namely inducing the sorting of heterogeneous (in preferences for flextime) workers across employers who offer different scheduling practices. The present model's assumption of one worker and one firm abstracts from such sorting. Introducing heterogeneous workers and sorting across employers should preserve the model's results under an additional layer of complexity. In the present study, the preceding considerations involving workforce composition, i.e., sorting of heterogeneous workers across employers, are taken as predetermined.
    ${ }^{32}$ Baughman et al. (2003) find that the costs of flexible scheduling and child care are offset in part by employers paying lower entry-level wages. Similarly, Heywood et al. (2007) find a sizeable compensating differential associated with indicators of flexible working schedules, but surprisingly the result could not be replicated for "working at home". In contrast to the preceding two studies, Johnson and Provan (1995) find that the use of flexible work practices is associated with an increased wage. Similarly, Gariety and Shaffer (2001) find that formalized "flextime" was associated with a significantly higher wage.

[^19]:    ${ }^{33}$ In contrast, when $\mathcal{F}=1$, the hire always occurs, because the worker's participation constraint is met with equality.

[^20]:    ${ }^{34}$ Following the discussion in section 3.2.1, a third empirical prediction is that exogenous wage increases are associated with a decreased tendency for employers to use flextime policies. This prediction is not pursued in the subsequent empirical work because there is not exogenous wage variation in the data.

[^21]:    ${ }^{35}$ A similar role for measurement error appears in the literature on structural estimation of labor supply models with kinked, piecewise-linear budget constraints and convex budget sets as would arise, for example, in the case of progressive taxes. The static labor-supply model, based on the theory of consumer choice, implies that workers' choices of work hours "bunch" at the kink points of their budget constraints. In the usual data sets used to estimate such models, however, even in large samples, few if any observed hours choices are located at these kink points. Incorporating measurement error in hours of work allows the observed data to be rationalized within the theoretical model. For further discussion, see Blundell and MaCurdy (1999, p. 1633) and Reiss and Wolak (2007).

[^22]:    ${ }^{36}$ The author acknowledges the Department for Business, Innovation and Skills, the Economic and Social Research Council, the UK Commission for Employment and Skills, the Advisory, Conciliation and Arbitration Service, and the National Institute of Economic and Social Research as the originators of the 2011 Workplace Employment Relations Study data, and the UK Data Archive at the University of Essex as the distributor of the data. The National Centre for Social Research was commissioned to conduct the survey fieldwork on behalf of the sponsors. None of these organizations bears any responsibility for the author's analysis and interpretations of the data
    ${ }^{37}$ The panel is based on a 2004 stratified random sample covering British workplaces with at least 5 to 9 employees, except for local units in Northern Ireland and those in the following 2003 Standard Industrial Classification (SIC) divisions: agriculture, hunting, and forestry; fishing; mining and quarrying; private households with employed persons; and extra-territorial organizations. The sampling frame is the Inter-Departmental Business Register (IDBR) which is maintained by the Office for National Statistics (ONS). According to Chaplin et al. (2005), "The IDBR is undoubtedly the highest quality sample frame of organisations and establishments in Britain."
    ${ }^{38}$ All workers in the establishment, including supervisors, were at risk for inclusion in the worker samples. Given that the workers are randomly sampled within each establishment in each year, individual workers cannot be tracked over time. It is possible that an individual worker is sampled in both 2004 and 2011, particularly in the smaller establishments. If this were to happen, however, it would not be detectable in the data, because the worker would receive a different unique identifier in each year. The survey population for the 2011 establishment survey accounts for $35 \%$ of all establishments and $90 \%$ of all workers in Britain and includes all workplaces in Britain with 5 or more employees and that operate in Sections C-S of the Standard Industrial Classification (2007). For further details on the design of the 2011 survey, see The Workplace Employment Relations Study (WERS) 2011/12: Technical Report (Deepchand et al. 2013), or the full technical report which is available upon request from wers@bis.gsi.gov.uk. For details on the 2004 WERS, see either the technical appendix to the 2004 sourcebook (Kersley et al., 2006), or the WERS 2004 technical report (Chaplin et al., 2005).

[^23]:    ${ }^{39}$ Descriptive statistics of the key variables for the samples of 1978 and 1200 are similar.
    ${ }^{40}$ Thomas (2020) observes that, "Remote work, especially in a world affected by Covid-19, naturally leads to "flex time." Employees with small children might be getting the majority of their work done at night after the kids are in bed. Others are working early and hoping to quit early. Still others are starting late and working late."

[^24]:    ${ }^{41}$ Ordered probit and logit models are avoided because they yield inconsistent parameter estimates in the presence of establishment fixed effects.

[^25]:    ${ }^{42}$ The survey question is, "Currently how many employees do you have on the payroll at this workplace? Remember to include yourself if you are an employee at the workplace but do NOT include casual workers without a contract of employment, freelance, self-employed or agency workers."

[^26]:    ${ }^{43}$ The 7 -year span between the panel's two waves is a substantial amount of time during which changes can occur.
    ${ }^{44}$ Statements about $\lambda_{c}$ are not directly visible from Figure 6 and are intended to augment the figure's visual information about $\beta_{c}$ and $\beta_{c}+\lambda_{c}$.

[^27]:    ${ }^{45}$ Even if there were no turnover and, therefore, no compositional changes between the two survey waves, the fact that the sampled workers differ between the two waves means that sampling variability can be another source of identifying variation. Given that workers are randomly sampled, however, this should not be systematic.
    ${ }^{46}$ Measurement error must also be acknowledged as possibly contributing to the temporal variation. For example, the respondent in the employer survey may differ between the two sample years, and one (or both) of them might incorrectly report the establishment's working time arrangements.

[^28]:    ${ }^{47}$ About 9 percentage points of that $13 \%$ increase was from employees working more minutes of their assigned shifts (e.g., taking fewer breaks and sick days), and the remainder of about 4 percentage points was from higher performance per minute.

[^29]:    ${ }^{48}$ In contrast, as illustrated in Figure 3, under a flextime policy the hours in a work schedule can be nonconsecutive. When $T=16$, under flextime, there are 65,535 possible work schedules of at least one hour. Only 136 of those schedules are available to the worker under a ShiftChange policy.

[^30]:    ${ }^{49}$ See the model of Baucells and Zhao (2019) for predictions on the optimal timing of effort-rejuvenating breaks.
    ${ }^{50}$ To illustrate these negative features, first suppose that assigned hours are $0.5 T-1$. Under employer-determined scheduling, the assigned shift of $0.5 T-1$ hours would coincide with the first $0.5 T-1$ peak hours. When hours are low, as seen in Figures 4 and 5, adding an additional hour to the shift increases expected productivity, so the change in expected productivity is positive when hours increase from $0.5 T-1$ to $0.5 T$. The additional hour added to the shift is a peak hour, specifically hour $0.75 T$. Under a ShiftChange policy, however, the expected productivity increase when hours change from $0.5 T-1$ to $0.5 T$ is dampened, because the additional hour might not be a peak hour. For example, the worker might choose the (nonpeak) hour $0.25 T$. Even worse for the employer, the worker might choose to work the first 0.5 T hours of the workday, in which case only half of those hours would be peak, as opposed to all of them under employer-determined scheduling.

[^31]:    ${ }^{51}$ Suppose, for example, that $0.75 T$ hours are assigned. Under a ShiftChange policy, all peak hours are guaranteed to be covered, no matter what schedule the worker chooses, whereas under flextime it is possible that only half of the peak hours will be covered.

[^32]:    ${ }^{52}$ See footnote 10 and the surrounding text.

[^33]:    ${ }^{53}$ Recall that in the employer-determined scheduling computation, the outermost sum arises because the employer does not know which cost parameter, $\lambda_{s}$, the worker attaches to the assigned schedule, so it is necessary to sum over all $\binom{T}{N}$ possible values of $\lambda_{s}$.
    ${ }^{54}$ Given that all $\binom{T}{N}$ values of $\lambda_{s}$ are in ( 0,1$]$, they are densely packed. Moreover, $w_{e}>w_{f}$, and $w_{f}$ is easily determined. Thus, the relevant interval to search for $\lambda_{s e}$ is not $(0,1]$ but rather $\left(w_{f}, 1\right]$. Choose $z$ equally-spaced points

[^34]:    in that interval and evaluate $E\left(\Pi_{e}\right)$ at each of those points, where $z>0$ is an integer that is significantly smaller than $\binom{T}{N}$. As $z$ approaches $\binom{T}{N}$ from the left, the approximation improves and the computational burden increases. When $\lambda_{s e}$ is bounded fairly tightly, the approximation can be improved via further function evaluations between the bounds. This approach yields a good approximation. The precise answer could also be found by reverting to loop over exact (rather than approximate) values of $\lambda_{s}$ once $\lambda_{s}$ is tightly bounded.

