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IZA DP No. 16873

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ISSN: 2365-9793

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ABSTRACT

An Optimal Allocation of Asylum Seekers*

We formulate a rule for allocating asylum seekers that is based on the social preferences of the native workers of the receiving countries. To derive the rule, we construct for each country a social welfare function, *SWF*, where the social welfare of a population is determined both by the population's aggregate absolute income and by the population's aggregate relative income. In a utilitarian manner, we combine the social welfare functions of the countries into a global social welfare function, *GSWF*. We look for the allocation that yields the highest value of the *GSWF*. We draw on assumptions that pertain to the manner in which the asylum seekers join the income distribution of the native workers: we consider a case in which the arrival of the asylum seekers has only a minor effect on the absolute income of the native population, and in which following their admission and integration, the asylum seekers join the income distribution of the native population "from below," namely the incomes of the asylum seekers are lower than the incomes of the low-income native workers. The arrival of asylum seekers can, however, measurably affect the relative incomes of the native population. Our rule states that the share of asylum seekers to be optimally assigned to each country depends only on the aggregate of the income excesses experienced by the native populations in the receiving countries.

JEL Classification: C54, D62, D78, E61, E65, F22, F62, F68, I31, I38, J15, J48, J68, O15

Keywords: asylum seekers, policy formation, relative deprivation, global social welfare, rule of allocation

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* We are indebted to reviewers for sound advice and thoughtful commentary.

1. Introduction

In the past decade alone, groups of countries have had to devise rules regarding how to distribute between them large numbers of asylum seekers. Obvious examples are Venezuelans in South America, and Syrians and Ukrainians in Europe. Several criteria were proposed, for example, that the numbers of asylum seekers to be admitted by EU member states should be in proportion to the size of the populations of these countries.[1]

The approach taken in this paper is that a rule of allocation of an exogenously determined number of asylum seekers to countries should be based on the social preferences of the native workers of the countries, and that the rule should aim at maximizing (a measure of) the aggregate social welfare of the native workers. Our approach draws on two main assumptions: an assumption about the manner in which the asylum seekers join the income distribution of the native workers, and an assumption as to what the social welfare function of the native workers consists of.

Our first main assumption is that the asylum seekers enter the social space of the native workers; that they join in at the lowest rung of the income hierarchy; and that the wages of the low-income native workers remain approximately constant. (The term "social space" stands for the set of individuals with whose incomes an individual compares his income or his income-based rank. Because this set constitutes a social environment, we make use of the word "social.") Reviewing, for example, European integration programs, these programs ensure that asylum seekers who might otherwise constitute an "outsider group" become an "insider group," both socially and economically.[2] That the wages of the low-income natives remain approximately constant

[1] For example, in September 2015, Jean-Claude Juncker, the then President of the European Commission, outlined a legislative proposal that would distribute 120,000 asylum seekers and migrants among the EU member states. According to a bill adopted by the European Council later that month, member states would be obliged to admit the prescribed numbers of asylum seekers and migrants, which would essentially be directly proportional to the size of their populations.

[2] In recent years, the speed and extent of the admission and integration of asylum seekers in several European countries appear not to have been left up to the asylum seekers. Policies have been put in place to ensure this. The EU leader in compelling asylum seekers to integrate is the Netherlands, which in 1998 introduced the *Newcomer Integration Act*, requiring asylum seekers and migrants to participate in language and social orientation courses or risk being fined or having their welfare benefits reduced. Recently, the Netherlands revised its *Newcomer Integration Act*, replacing it with the *Civic Integration Act 2021*, which was due to take effect in 2022. The new Act too requires asylum seekers and migrants to participate in language and social orientation courses. Consult

can be supported by empirical evidence regarding the effect of low-skill migration on the wages and / or the employment opportunities of low-skill native workers, which finds that the effect tends to be weak, neutral, or slightly positive (LaLonde and Topel, 1991; Card, 2001, 2005; Kifle, 2009; Cohen-Goldner and Paserman, 2011; Ottaviano and Peri, 2012, Manacorda et al., 2012; Foged and Peri, 2016).

Our second main assumption relates to what the social welfare of the native workers consists of and, in particular, to the type of income component of the function. In principle, we need to bear in mind that income maps into wellbeing in two distinct ways: absolute and relative. People prefer high absolute income to low absolute income, and high relative income to low relative income. The relative income of a person is a function of the difference between the incomes of the comparators of the person whose incomes are higher than his and the person's own income, and of the share of these comparators in the population. A large literature tells us that relative income matters; it affects wellbeing significantly.[3]

An approach often taken regarding the choice of a policy that affects the wellbeing of a population is to base the choice on the policy's impact on the population's aggregate absolute income, ignoring, implicitly or explicitly, the policy's impact on the population's aggregate relative income. More than 50 years ago, Amartya Sen introduced a social welfare function that accounts for both types of incomes (Sen, 1973 and 1997, Sen, 1976, and Sen, 1982). Sen defined social welfare as the product of two terms: aggregate absolute income divided by the size of the population, and 1 minus the Gini coefficient. Drawing on Sen's (1973) representation of the Gini coefficient of a population of n individuals, Sen's social welfare function can equivalently be expressed

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https://business.gov.nl/amendment/new-law-on-integration/. Austria, Denmark, France, Germany, Luxembourg, and Sweden have in place integration programs of different types, involving at least a language course, which is mandatory for all non-EU migrants. (For details and comparison of integration programs in the EU-15 countries see the summary in Hübschmann, 2015.) For example, since 2005 Germany has had a program of at least 430 hours of study consisting of a language part and a part dedicated to German history, politics, and culture. In Sweden, the government is running some 30 "fast-track" programs training asylum seekers with experience in occupations where labor is short. Both Germany and Sweden have shifted legal barriers in order to let the asylum seekers start work sooner.

[3] Parts of this literature can be found in three articles on the themes of relative deprivation, comparison groups, and social comparisons that appeared in *Kyklos*: Stark (1990), Fan and Stark (2007), and Stark and Budzinski (2021).

as the difference, divided by n, between the population's aggregate absolute income and a measure of the population's aggregate relative income. A detailed derivation of this equivalence is provided in Appendix A. "On Sen's social welfare function." As we note below, our construction of social welfare functions is guided by Sen's approach, and by our generalization of Sen's social welfare function.

In this paper, we consider a case in which a social planner needs to allocate asylum seekers to a set of countries where evidence exists that the effect of the arrival of the asylum seekers on the absolute income of the native population is small, so that as a first order approximation it can be ignored. Following their admission and integration, the asylum seekers join the income distribution of the native population "from below," namely the incomes of the asylum seekers are lower than the incomes of the low-income native workers.

At the same time, however, the arrival of asylum seekers who join the income distribution of the native population "from below" can measurably affect the relative income of the native population. In view of the preceding reasoning, we devise in this paper a rule for allocating an exogenously determined number of asylum seekers between countries when the guiding principle is to maximize the combined social welfare of the native workers of the countries that host the asylum seekers, and when the argument in the social welfare function of a country is the aggregate relative income of the country's native workers.

To derive the rule, we construct for each country a social welfare function, *SWF*, where, in the spirit of Sen's approach, the social welfare of a population is determined both by the population's aggregate absolute income and by the population's aggregate relative income. For each country, we calculate the *SWF* and then, in a utilitarian manner, we combine the social welfare functions of the countries into a global social welfare function, *GSWF*. We look for the allocation that yields the highest value of the *GSWF*.

Essentially, the approach taken in this paper is the flip side of what could be described as a common approach in which allocation is to be determined on the basis of where it brings about the biggest boost of absolute income; here we base the allocation on where it brings about the greatest lowering of a measure of relative income.

2. A rule of an efficient allocation of asylum seekers

In this section, we define the social welfare function of the native workers of each of two countries. In so doing we are inspired by Sen's idea of incorporating absolute income and relative income into a composite measure of wellbeing. The arrival of asylum seekers reduces unequivocally the relative income component of the social welfare function. Our objective is to maximize the global social welfare function (defined as the sum of the utilities of the native workers of the two countries) with respect to the distribution of the asylum seekers between the countries. We show that the optimal distribution depends on the quotient of the sums of the income excesses in the countries. Essentially, the higher the sum of the income excesses of the native workers of a country, the larger the number of asylum seekers to be assigned to a country.

Let there be a population N, and let n be the number of individuals in *N*. By $y(i)$ we denote the income of individual $i \in N$. The utility of individual $i \in N$ is defined as

$$u_N(i) \equiv (1-\alpha) y(i) - \alpha RD_N(i),$$

where $RD_N(i)$, the relative deprivation of individual *i* in population N, is defined as

$$RD_N(i) \equiv \frac{1}{n} \sum_{j \in N} \max\{y(j) - y(i), 0\},$$

the coefficient $\alpha \in (0,1)$ measures the intensity of dissatisfaction inflicted by relative deprivation, and the coefficient $(1-\alpha)$ measures the intensity of satisfaction derived from income.

In the utility of individual *i*, the weights accorded to income and to relative deprivation sum up to one. This means that, in general, individual *i* is given 100 percent of weight that he can apportion between income and relative deprivation. When relative deprivation does not matter, $\alpha = 0$, so $u_N(i) \equiv (1-\alpha) y(i) - \alpha RD_N(i)$ is reduced to $u_N(i) = y(i)$. And when, in comparison to income, relative deprivation matters greatly, say relative deprivation is twice as important as income, then $\alpha = \frac{2}{3}$ is an accurate representation of the intensity of this preference: $\alpha / (1-\alpha) = (2/3)/(1/3) = 2$.

The social welfare of a (possibly smaller than N) population of K individuals, that is, of $K \subset N$, is the sum of the utilities of the members of population K, namely

$$SWF_N(K) = \sum_{i \in K} u_N(i) = (1-\alpha)\sum_{i \in K} y(i) - \alpha \sum_{i \in K} RD_N(i), \tag{1}$$

where the utility of every member is accorded equal weight.

In a sense, the social welfare function (1) is a generalization of the social welfare function of Sen, SWF_{Sen}, which, as shown in Appendix A, can be expressed as

$$SWF_{Sen} = \frac{1}{n}\sum_{i \in N} y(i) - \frac{1}{n}\sum_{i \in N} RD_N(i). \tag{2}$$

We use "generalization" because in our setting the weight accorded to aggregate income is not necessarily the same as the weight accorded to aggregate relative deprivation; the weights sum up to 1; and the individuals for whom the social welfare function is calculated do not necessarily constitute the entire population.

Let N_A and N_B denote the populations of the native workers of countries A and B, respectively. A given population of asylum seekers, AS, is to be divided into two groups, AS_A and AS_B, to be allocated to countries A and B, respectively. We denote

$$|N_A| = n_A, |N_B| = n_B, |AS| = n_{AS}, |AS_A| = n_{ASA}, |AS_B| = n_{ASB},$$

where $|X|$ is the number of elements in the set X. Let $M_A = N_A \cup AS_A$, and let $M_B = N_B \cup AS_B$ denote the sets of native workers of country A and of country B, respectively, after the arrival in the countries of the asylum seekers. Naturally, $n_{AS} = n_{ASA} + n_{ASB}$, $|M_A| = n_A + n_{ASA}$, and $|M_B| = n_B + n_{ASB}$.

A benevolent "global" social planner seeks to maximize the utilitarian sum of the levels of social welfare of the native workers of countries A and B. We refer to this sum as the Global Social Welfare Function, *GSWF*, defined as

$$GSWF \equiv SWF_{M_A}(N_A) + SWF_{M_B}(N_B). \tag{3}$$

To streamline notation, we denote the sum of the excesses of the incomes of the native workers in country A by a, and the sum of the excesses of the incomes of the native workers in country B by b:

$$a = \sum_{i \in N_A} \sum_{j \in N_A} \max\{\, y(i) - y(j), 0\},$$

$$b = \sum_{i \in N_B} \sum_{j \in N_B} \max\{\, y(i) - y(j), 0\}\,.$$

By definition, $a \geq 0$ and $b \geq 0$. Without loss of generality, we can assume that $b > a$: the sum of the income excesses in one country, say country B, is higher than the sum of the income excesses in the other country A. Although the asylum seekers are assumed to assimilate sufficiently well enough to join the receiving countries' workforces, the income of each asylum seeker is taken to be lower than the income of any native worker. The levels of the aggregate relative deprivation in countries A and B prior to the arrival of the asylum seekers are, respectively,

$$ARD(N_A) = \sum_{i \in N_A} RD_{N_A}(i) = \frac{a}{n_A},$$

and

$$ARD(N_B) = \sum_{i \in N_B} RD_{N_B}(i) = \frac{b}{n_B}.$$

The levels of the aggregate relative deprivation of the native workers in countries A and B following the arrival of the asylum seekers are, respectively,

$$ARD(N_A, M_A) = \sum_{i \in N_A} RD_{M_A}(i) = \frac{a}{n_A + n_{ASA}},$$

and

$$ARD(N_B, M_B) = \sum_{i \in N_B} RD_{M_B}(i) = \frac{b}{n_B + n_{ASB}}.$$

Then, depending on the numbers of asylum seekers to be allocated to the two countries, (3) takes the form

$$GSWF(n_{ASA}, n_{ASB}) = (1-\alpha)\left(\sum_{i \in N_A} y(i) + \sum_{i \in N_B} y(i)\right) - \alpha\left(\frac{a}{n_A + n_{ASA}} + \frac{b}{n_B + n_{ASB}}\right)$$

or after substituting $n_{ASB} = n_{AS} - n_{ASA}$,

$$GSWF(n_{ASA}) = (1-\alpha)\left(\sum_{i \in N_A} y(i) + \sum_{i \in N_B} y(i)\right) - \alpha\left(\frac{a}{n_A + n_{ASA}} + \frac{b}{n_B + n_{AS} - n_{ASA}}\right). \tag{4}$$

Remark 1. Although n_{ASA} is not a continuous variable, because typically the numbers involved are relatively large, it is just as well to resort to a continuous representation.

In order to analyze *GSWF* as a function of n_{ASA} so as to find the optimal number of asylum seekers, $\tilde{n}_{ASA}$, to be allocated to country A, we differentiate (4) with respect to n_{ASA}

$$\frac{dGSWF}{dn_{ASA}} = \alpha\left[\frac{a}{(n_A + n_{ASA})^2} - \frac{b}{(n_B + n_{AS} - n_{ASA})^2}\right].$$

Solving $\frac{dGSWF}{dn_{ASA}} = 0$ yields

$$a(n_B + n_{AS} - n_{ASA})^2 = b(n_A + n_{ASA})^2.$$

Substituting $x = \frac{b}{a}$ we get

$$(n_B + n_{AS} - n_{ASA})^2 = x(n_A + n_{ASA})^2,$$

$$n_B + n_{AS} - n_{ASA} = \sqrt{x}(n_A + n_{ASA}),$$

and then

$$n_{ASA} = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x} + 1}.$$

Moreover,

$$\frac{d^2GSWF}{dn_{ASA}{}^2} = \alpha\left[-\frac{2a}{(n_A + n_{ASA})^3} - \frac{2b}{(n_B + n_{AS} - n_{ASA})^3}\right] < 0,$$

so that $\tilde{n}_{ASA} = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$ is the only global maximum of $GSWF$, as long as $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \in (0, n_{AS})$. The function $GSWF$ is increasing for $n_{ASA} < \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$ and decreasing for $n_{ASA} > \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$. We thus have the following claim.

Claim 1. Let $x = \frac{b}{a}$.

(i) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \le 0$, then the optimal number of asylum seekers to be allocated to country A is $\tilde{n}_{ASA} = 0$: all the asylum seekers are to be allocated to country B.

(ii) If $0 < \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} < n_{AS}$, then $GSWF$ increases with the number of asylum seekers allocated to country A as long as that number is smaller than $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$, and $GSWF$ decreases with the number of asylum seekers allocated to country A as long as that number is higher than $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$. In particular, $\tilde{n}_{ASA} = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$ is the optimal number of asylum seekers allocated to country A.

(iii) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \ge n_{AS}$, then the optimal number of asylum seekers to be allocated to country A is $\tilde{n}_{ASA} = n_{AS}$: all the asylum seekers are to be allocated to country A.

Cases (i), (ii), and (iii) cover all the relevant configurations of the parameters (a, b, n_A, n_B, n_{AS}).

Proof. The proof is in Appendix B. “Proofs.”

From Claim 1 it follows that the optimal number of asylum seekers to be allocated to country A increases when the quotient of the aggregate excess of incomes in country B and the aggregate excess of incomes in country A, namely $x = \frac{b}{a}$, decreases. When the aggregate income excesses in countries A and B are approximately the same, then more asylum seekers are to be allocated to country A than when the aggregate excess income in country B is much larger than the aggregate excess income in country A. We thus state and prove the following property.

Claim 2. The optimal number of asylum seekers to be allocated to country A, $\tilde{n}_{ASA}$, is a weakly decreasing function of *x*, the quotient of the aggregate excesses of incomes in countries B and A.

Proof. The proof is in Appendix B. "Proofs."

Remark 2. If b, the aggregate excess of incomes in country B, is held constant, then from Claim 2 it follows that the optimal number of asylum seekers to be allocated to country A is a weakly increasing function of a, the aggregate excess of incomes in country A.

3. A numerical example and a back-of-envelope real world example

3.1 A numerical example

Let there be two countries, A and B. Let $n_A = 10$ million and let $n_B = 30$ million. The sum of the income excesses in country A is $a = 40$ million units of income, and the sum of the income excesses in country B is $b = 250$ million units of income. Let $n_{AS} = 3$ million to be distributed between countries A and B. The income of each asylum seeker is lower than the income of any native worker of countries A and B. Which allocation brings *GSWF* to a maximum?

Applying Claim 1, we calculate the quotient $x = \frac{b}{a} = 6.25$, and we then calculate the number

$$\tilde{n}_{ASA} = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} = \frac{30+3-10\sqrt{6.25}}{\sqrt{6.25}+1} = \frac{8}{3.5} \approx 2.286 \text{ million.}$$

Thus, optimally (about) 2.286 million asylum seekers are to be allocated to country A, and the remainder of (about) 0.714 million asylum seekers are to be allocated to country B. In this case, the more populous country B is to receive fewer asylum seekers than the less populous country A.

To illustrate Claim 2, we consider how $\tilde{n}_{ASA}$, the optimal number of asylum seekers assigned to country A, changes when we perturb the values of a and b. That is, when $n_A = 10$ million, $n_B = 30$ million, and $n_{AS} = 3$ million, we represent the optimal number of asylum seekers assigned to country A as a function of $x = \frac{b}{a}$. For these parameter values, the function $\tilde{n}_{ASA}$, defined in Claim 2, takes the form

$$\tilde{n}_{ASA}(x) = \begin{cases} 3 \text{ million, if } x \le \left(\frac{30}{13}\right)^2, \\ \frac{33-10\sqrt{x}}{\sqrt{x}+1} \text{ million, if } \left(\frac{30}{13}\right)^2 < x < \left(\frac{33}{10}\right)^2, \\ 0, \text{ if } x \ge \left(\frac{33}{10}\right)^2. \end{cases}$$

Figure 1 presents $\tilde{n}_{ASA}$ (measured vertically in millions) as a function of *x*.

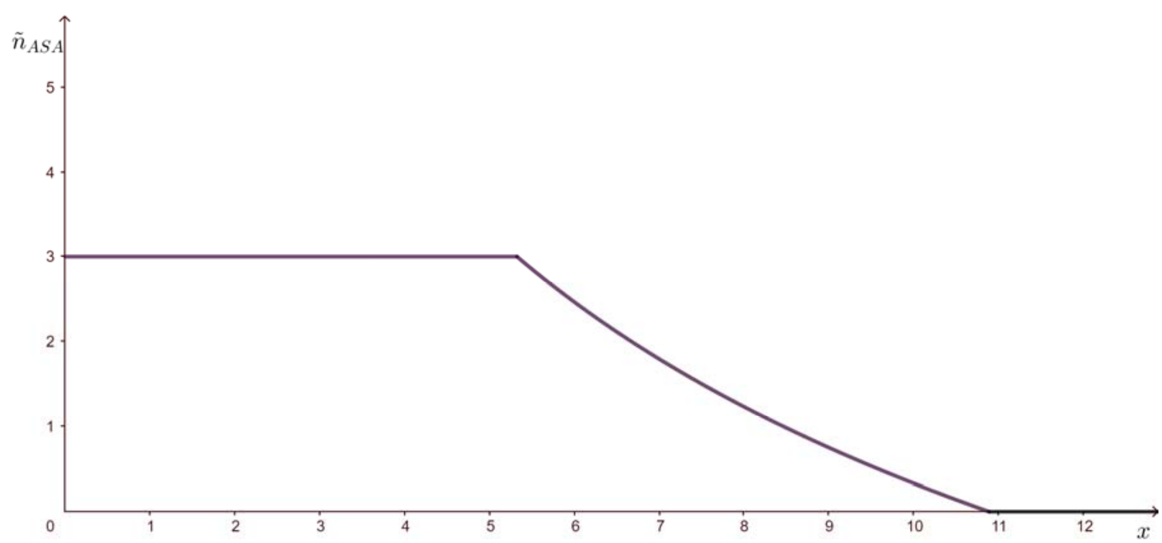

Figure 1. The optimal number of asylum seekers assigned to country A as per the numerical example.

Noticeably, $\tilde{n}_{ASA}$ is a weakly decreasing function of x, in line with the prediction of Claim 2.

3.2 A back-of-envelope real world example

Let *N* be a population consisting of *n* people such that *TI* is the sum of their incomes, hence $\frac{TI}{n}$ is the income per capita of *N*. Let *G* be the Gini coefficient of the income distribution of *N*. Because $G = \frac{ARD}{TI}$, then $ARD = G \cdot TI$. Thus, data needed to calculate $a = ARD(N_A) \cdot n_A$ and $b = ARD(N_B) \cdot n_B$ can be elicited from statistics on the Gini coefficient, *G*, and on aggregate income, *TI*.

In the wake of the Russian invasion of Ukraine, it was the case that the Czech Republic and Poland hosted between them about $n_{AS} = 2.2$ million Ukrainian asylum seekers. The number of native workers in the Czech Republic is about $n_A = 5.3$ million, and the number of native workers in Poland is about $n_B = 17$ million. The Gini coefficient of the Czech Republic is about $G_A = 0.25$, and the Gini coefficient of Poland is about $G_B = 0.26$. The GDP of the Czech Republic is close to 270 billion Euros, and the GDP of Poland is close to 660 billion Euros. We can then calculate the quotient as per Claim 1 of the aggregate excesses of incomes of Poland and of the Czech Republic. We obtain

$$x = \frac{b}{a} = \frac{ARD(N_B) \cdot n_B}{ARD(N_A) \cdot n_A} = \frac{G_B \cdot GDP_B \cdot n_B}{G_A \cdot GDP_A \cdot n_A} \approx \frac{0.26 \cdot 660 \cdot 17}{0.25 \cdot 270 \cdot 5.3} \approx 8.1543.$$

We next calculate the optimal number of asylum seekers to be allocated to the Czech Republic. From Claim 1, we obtain

$$\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x} + 1} \approx 1.0544 \text{ million.}$$

Thus, the optimal number of Ukrainian asylum seekers to be allocated to the Czech Republic should be (about) 1.0544 million, and the remainder (of about) 1.1456 million Ukrainian asylum seekers should be allocated to Poland. In reality, the Czech Republic hosted about 0.5 million Ukrainian asylum seekers, and Poland hosted about 1.7 million Ukrainian asylum seekers.

4. Discussion and conclusion

The rule proposed in this paper requires that the asylum seekers enter the social sphere of the native workers, and that in the hierarchy of earnings they do so from below. We remarked that, in that respect, the asylum seekers are not left to decide all by themselves. Fan and Stark (2007) developed a theory that can help explain why asylum seekers may want to limit their integration into the mainstream society of the country that hosts them: non-assimilation arises from a fear of enhanced relative deprivation if they reduce their distance in social space from the native workers as a reference group. The integration programs referred to in some detail in footnote 2 serve to forestall such behavior.

Remark 3. The rule developed in Section 2 of the optimal number of asylum seekers to be allocated to a country pertains to the case of two receiving countries. Application of the method of Lagrange multipliers enables us to formulate a rule that pertains to the case of more than two receiving countries. To this end, let there be $J \in \mathbb{N}$ countries, $1,2,\ldots,J$ of populations $n_1,n_2,\ldots,n_J$, respectively, and let there be aggregate income excesses of $a_1,a_2,\ldots,a_J$, respectively. Then, the optimal number of asylum seekers allocated to country $k \in \{1,2,\ldots,J\}$ is given by

$$n_{ASk} = \frac{n_{AS} + \sum_{j=1}^{J} n_j - n_k \sum_{j=1}^{J} \sqrt{\frac{a_j}{a_k}}}{\sum_{j=1}^{J} \sqrt{\frac{a_j}{a_k}}}, \tag{5}$$

as long as the values of n_{ASk} ($k \in \{1,2,\ldots,J\}$) calculated from (5) are all non-negative.

In constructing our rule, we abstracted from the consideration that the receipt and absorption of asylum seekers could entail a cost to the host country. For example, the programs described in footnote 2 entail some costs. However, because these programs expedite the entry of the asylum seekers into the host country's workforce, taxes paid on the asylum seekers' earnings could offset the program costs, and do so fairly quickly after the asylum seekers' arrival. Moreover, we can think of the cost of receiving asylum seekers as the sum of a fixed cost of setting up the administration and facilities required to process asylum seekers, and a variable cost that depends on the number of asylum seekers such that, for example, there is a given outlay (absorption package) per asylum seeker. If the fixed cost is first order and is similar across countries, while the variable cost is second order, then the cost issue will not measurably affect our rule. We might also reason that when, in the formation of the utility functions of the native workers and consequently in the formation of the social welfare function of the native workers, the weight α accorded to the relative deprivation component is relatively large, then some cost which may affect the income component of the function will not matter much.

That said, if the cost of "processing" asylum seekers is covered by a lump-sum tax, then, as long as the tax does not reduce the income of any native worker below the income of the asylum seekers, that will not change the results reported in Claims 1 and 2. That is, although the value of the global social welfare function *GSWF* will change, the optimal numbers of asylum seekers assigned to the two host countries will remain the same after enacting the lump-sum tax as it was prior to enacting the lump-sum tax. Specifically, let the cost of the receipt and absorption of an asylum seeker be λ. Then, the lump-sum tax in country A will be $\tau_A = \dfrac{\lambda n_{ASA}}{n_A}$, and the lump-sum tax in country B will be $\tau_B = \dfrac{\lambda\left(n_{AS} - n_{ASA}\right)}{n_B}$. On inserting these terms into (4) we get

$$GSWF(n_{ASA}) = (1-\alpha)\left(\sum_{i \in N_A}\left(y(i)-\tau_A\right)+\sum_{i \in N_B}\left(y(i)-\tau_B\right)\right)-\alpha\left(\frac{a}{n_A+n_{ASA}}+\frac{b}{n_B+n_{AS}-n_{ASA}}\right).$$

Following a simple transformation, this last expression becomes

$$GSWF(n_{ASA}) = (1-\alpha)\left(\sum_{i \in N_A} y(i)+\sum_{i \in N_B} y(i)\right)-(1-\alpha)\lambda n_{AS}-\alpha\left(\frac{a}{n_A+n_{ASA}}+\frac{b}{n_B+n_{AS}-n_{ASA}}\right).$$

The term $(1-\alpha)\lambda n_{AS}$ does not depend on n_{ASA}, so when calculating the derivative of $GSWF(n_{ASA})$ with respect to n_{ASA}, this term will disappear.

In Europe alone, in a post-WWII reality in which several countries needed to decide jointly how to optimally allocate asylum seekers between them, the flow of Syrians in 2015 was not the first of such crises. A similar situation arose in 1992-1995 when people were fleeing the savage conflict in Bosnia and Herzegovina. The same five European countries that were the main receivers of asylum seekers in 1992-1995 were also the lead receivers in 2015. The recent crisis calling for the allocation of Ukrainian asylum seekers across Europe (and beyond) implies that European countries could be required to play host to large numbers of asylum seekers in the future. History can repeat itself and, as in this context, it did. Viewed in this light, the rule proposed in this paper could serve as a guide in future asylum-seeking crises.

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Appendix A. On Sen's social welfare function

For ease of reference, we use in this appendix simple notations that slightly differ from the ones that we use in the main text of the paper.

In population $N = \{1, 2, \ldots, n\}$, $n \geq 2$, let $y = (y_1, ..., y_n)$ be the vector of incomes of the members of the population. Let these incomes be ordered, $0 < y_1 < y_2 < \ldots < y_n$. RD_i - by which we denote the relative deprivation of individual *i*, $i = 1, 2, ..., n-1$, whose income is y_i - is defined as

$$RD_i \equiv \frac{1}{n} \sum_{j=i+1}^{n} (y_j - y_i), \tag{6}$$

where it is understood that $RD_n \equiv 0$.

The idea here is to aggregate the income excesses (the differences between the incomes that are higher than the income of individual *i* and the income of individual *i*) and normalize this sum, that is, dividing it by the size of the population. Because the relative deprivation of an individual arises from having an income that is lower than the incomes of others (rather than from having a low absolute income), we refer to this stress as income-based relative deprivation.[4]

Observation 1. The relative deprivation index presented in (6) can be rewritten in a slightly different form. Multiplying and dividing the index by $n-i$, yields

[4] We characterize the stress that arises from having less than others as social, and we quantify this stress by (6). In taking this approach, we follow, and we are in line with, a large literature on the subject of relative deprivation and reference (comparison) groups, spanning from the pioneering 1949 two-volume study of Stouffer et al. *Studies in Social Psychology in World War II: The American Soldier*, through Akerlof (1997) and all the way to recent writings, for example, of Stark et al. (2017), and Stark (2020). These two studies include deliberations and discussions on the identity of the reference group, and they provide many references to related works. By definition and construction, the concept of relative deprivation is the dual of the concept of reference group or comparison group. Hence the term social. The work of Stouffer et al., which opened the way to research on relative deprivation and reference groups, documented the stress caused not by a low military rank and weak prospects of promotion (military police) but rather by the faster pace of promotion of others (air force). It also documented the lower dissatisfaction of Black soldiers stationed in the South who compared themselves with Black civilians in the South rather than the dissatisfaction of their counterparts stationed in the North who compared themselves with Black civilians in the North.

$$RD_i = \frac{n-i}{n}\left[\frac{1}{n-i}\sum_{k=i+1}^{n}(y_k - y_i)\right] = \frac{n-i}{n}\left(\frac{\sum_{k=i+1}^{n} y_k}{n-i} - y_i\right) = (1-\frac{i}{n})\left(\tilde{y}_i - y_i\right), \tag{7}$$

where $\tilde{y}_i \equiv \frac{1}{n-i}\sum_{k=i+1}^{n} y_k$ is the average income of the individuals whose incomes are higher than the income of individual i (these are the individuals who are positioned to the right of individual i, namely higher up, in the income distribution). In words, (7) states that the income-based relative deprivation of individual i whose income is y_i is equal to the product of two terms: the fraction of the individuals in the population of n individuals whose incomes are higher than y_i, and the mean excess income. Formula (7) reveals that even though RD_i is sensed by looking to the right of the income distribution, it is impacted by events taking place on the left of the income distribution. For example, an exit from the population of a low-income individual increases the relative deprivation of higher-income individuals (other than the richest) because the weight that the latter attach to the difference between the incomes of individuals "richer" than themselves and their own income rises. Conversely, the entry into the population of a low-income individual decreases the relative deprivation of higher-income individuals (other than the richest) because the weight that the latter attach to the difference between the incomes of individuals "richer" than themselves and their own income declines. This latter characterization features in the main text of this paper.

Observation 2. Analytical accounts of the significance of relative deprivation to the wellbeing of the native inhabitants and of the way in which the assimilation of migrants affects the wellbeing of the native inhabitants, where the effect arises from the impact of that assimilation on the relative deprivation of the native inhabitants, are provided in two papers published in this journal: "Income redistribution going awry: The reversal power of the concern for relative deprivation" by Sorger and Stark (2013), and "The impact of the assimilation of migrants on the wellbeing of native inhabitants: A theory" by Stark et al. (2015). Readers of the current paper who seek to acquaint themselves further with the topics of relative deprivation, assimilation, and social welfare could gain from studying these two papers.

We denote by *TRD* the sum (the aggregate) of the levels of RD_i in population *N*. Then,

$$TRD \equiv \frac{1}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(y_j - y_i)\,. \tag{8}$$

Following Sen (1973), the Gini coefficient of population $N=\{1,2,\dots,n\}$, $n \geq 2$, with a vector $y=(y_1,...,y_n)$ of the incomes of the members of the population, is

$$G \equiv \frac{\sum_{j=1}^{n}\sum_{i=1}^{n}\left|y_i - y_j\right|}{2n^2\overline{y}}, \tag{9}$$

where $\overline{y}=(1/n)\sum_{i=1}^{n} y_i$ is the average income of the population. In Sen's (1973, p. 8) words: "In any pair-wise comparison the man with the lower income can be thought to be suffering from some depression on finding his income to be lower. Let this depression be proportional to the difference in income. The sum total of all such depressions in all possible pair-wise comparisons takes us to the Gini coefficient." In this paper we use the terms of income-based "depression," income-based stress, and income-based relative deprivation interchangeably.

On noting that $\sum_{j=1}^{n}\sum_{i=1}^{n}\left|y_i - y_j\right| = 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(y_j - y_i)$, an equivalent representation of the Gini coefficient in (9), which disposes of the need to operate with absolute values, is

$$G = \frac{\frac{1}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(y_j - y_i)}{\sum_{i=1}^{n} y_i} = \frac{TRD}{TI}, \tag{10}$$

so the Gini coefficient in (10) is expressed as a ratio: *TRD* as defined in (8), divided by aggregate (total) income $\sum_{i=1}^{n} y_i = TI$.

Sen (1973 and 1997), Sen (1976), and Sen (1982) sought to measure social welfare by means of the function, SWF_{Sen}, formulated as $\mu(1-G)$, namely as the product of income per capita, $\mu = \frac{\sum_{i=1}^{n} y_i}{n}$, and 1 minus G, where G is as defined in (9). Income, expressed as income per capita, awards, while inequality, expressed by the Gini coefficient, penalizes. Expanding the SWF_{Sen} function while substituting from (10), we get

$$SWF_{Sen} \equiv \mu(1-G) = \frac{TI}{n}\left(1-\frac{TRD}{TI}\right) = \frac{1}{n}(TI - TRD)\,. \tag{11}$$

We see that the welfare of a population of a given size, n, is "damaged" by the population's aggregate relative deprivation. The reason why income inequality lowers welfare is not aversion to inequality per se but, rather, aversion to income-based stress; the higher the stress (the higher is TRD), the lower the welfare. The $\frac{1}{n}(TI - TRD)$ representation in (11) implies that the statistically-based social welfare function $\mu(1-G)$ is transformed into a social-psychological-based social welfare function.

We next refer briefly to the relationship between the relative-deprivation-based social welfare function that we use in our paper, and Sen's social welfare function. In a way, our social welfare function is a generalization of Sen's social welfare function or, stated the other way around, Sen's social welfare function can be perceived as a special case of the social welfare function that we define in Section 2. We recall (1), rewritten slightly to align with the notation used in this appendix:

$$SWF_N(K) = \sum_{i\in K} u_N(i) = (1-\alpha)\sum_{i\in K} y_i - \alpha \sum_{i\in K} RD_i. \tag{1'}$$

In (1'), $\alpha \in [0,1]$ is the weight accorded by individual i to relative deprivation, and $(1-\alpha) \in [0,1]$ is the weight accorded by individual i to income. We consider a special case of (1'): upon replacing K with N, and assigning equal weights to aggregate income and to aggregate relative deprivation, we obtain

$$SWF_N(N) = \frac{1}{2}\sum_{i\in N} y_i - \frac{1}{2}\sum_{i\in N} RD_i = \frac{1}{2}\left(TI - TRD\right). \tag{12}$$

Applying (11), we obtain from (12)

$$\frac{2}{n}SWF_N(N) = \frac{2}{n}\left[\frac{1}{2}\left(TI - TRD\right)\right] = \frac{1}{n}\left(TI - TRD\right) = SWF_{Sen}.$$

Therefore, when $\alpha = \frac{1}{2}$, then SWF_{Sen} is just a simple rescaling of $SWF_N(N)$ by $\frac{2}{n}$. Thus, the optimization results obtained for the social welfare function defined in (1) pertain to Sen's social welfare function SWF_{Sen} as a special case.

Appendix A. **References**

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Appendix B. Proofs

For ease of reference, we replicate the texts of Claims 1 and 2.

Claim 1. Let $x = \frac{b}{a}$.

(i) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \leq 0$, then the optimal number of asylum seekers to be allocated to country A is $\tilde{n}_{ASA} = 0$: all the asylum seekers are to be allocated to country B.

(ii) If $0 < \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} < n_{AS}$, then *GSWF* increases with the number of asylum seekers allocated to country A as long as that number is smaller than $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$, and *GSWF* decreases with the number of asylum seekers allocated to country A as long as that number is higher than $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$. In particular, $\tilde{n}_{ASA} = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$ is the optimal number of asylum seekers allocated to country A.

(iii) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \geq n_{AS}$, then the optimal number of asylum seekers to be allocated to country A is $\tilde{n}_{ASA} = n_{AS}$: all the asylum seekers are to be allocated to country A.

Cases (i), (ii), and (iii) cover all the relevant configurations of the parameters (a, b, n_A, n_B, n_{AS}).

Proof.

(i) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \leq 0$, then *GSWF* is decreasing in $[0, n_{AS}] \subset \left[\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}, +\infty\right)$. Therefore, $GSWF(n_{ASA}) < GSWF(0)$ for $n_{ASA} \in (0, n_{AS}]$ and, thus, 0 is the optimal number of asylum seekers to be allocated to country A.

(ii) If $0 < \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} < n_{AS}$, then $GSWF$ is increasing in $\left[0, \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}\right)$ and decreasing in $\left(\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}, n_{AS}\right]$. In particular, $GSWF(n_{ASA}) < GSWF\left(\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}\right)$ for $n_{ASA} \in [0, n_{AS}] \setminus \left\{\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}\right\}$. Therefore, $\tilde{n}_{ASA} = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$ is the optimal number of asylum seekers to be allocated to country A.

(iii) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \geq n_{AS}$, then, $[0, n_{AS}] \subset \left(-\infty, \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}\right]$ and $GSWF$ is increasing in $[0, n_{AS}]$. Therefore, $GSWF(n_{ASA}) < GSWF(n_{AS})$ for $n_{ASA} \in [0, n_{AS})$ and, thus, n_{AS} is the optimal number of asylum seekers to be allocated to country A. Q.E.D.

Claim 2. The optimal number of asylum seekers to be allocated to country A, $\tilde{n}_{ASA}$, is a weakly decreasing function of *x*, the quotient of the aggregate excesses of incomes in countries B and A.

Proof. From Claim 1 we know that

(i) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \leq 0$, then $\tilde{n}_{ASA}(x) = 0$. Because $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \leq 0$ for $x \geq x_1 \equiv \left(\frac{n_B + n_{AS}}{n_A}\right)^2$, it follows that $\tilde{n}_{ASA}(x) = 0$ for $x \in [x_1, +\infty)$.

(ii) If $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \geq n_{AS}$, then $\tilde{n}_{ASA}(x) = n_{AS}$. Because $\frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1} \geq n_{AS}$ for $x \leq x_0 \equiv \left(\frac{n_B}{n_A + n_{AS}}\right)^2$, it follows that $\tilde{n}_{ASA}(x) = n_{AS}$ for $x \in (-\infty, x_0]$. Moreover, $x_0 < x_1$.

(iii) If $x \in (x_0, x_1)$, then $\tilde{n}_{ASA}(x) = \frac{n_B + n_{AS} - n_A\sqrt{x}}{\sqrt{x}+1}$ and $0 < \tilde{n}_{ASA}(x) < n_{AS}$. Thus,

$$\frac{d\tilde{n}_{ASA}}{dx} = \frac{-\dfrac{n_A\left(\sqrt{x}+1\right)+n_B+n_{AS}-n_A\sqrt{x}}{2\sqrt{x}}}{\left(\sqrt{x}+1\right)^2} = -\frac{n_A+n_B+n_{AS}}{2\sqrt{x}\left(\sqrt{x}+1\right)^2} < 0$$

in (x_0, x_1). Therefore, $\tilde{n}_{ASA}$ is also decreasing in (x_0, x_1). In sum, $\tilde{n}_{ASA}$ is a weakly decreasing function of x. Q.E.D.