I Z A Institute
of Labor Economics

Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 16870

Slouching Towards Decentralization. An Equilibrium Approach for Collective Bargaining.

Gabriele Cardullo

MARCH 2024

I Z A Institute of Labor Economics

Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 16870

Slouching Towards Decentralization. An Equilibrium Approach for Collective Bargaining.

Gabriele Cardullo
University of Genova and IZA

MARCH 2024

ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

Slouching Towards Decentralization. An Equilibrium Approach for Collective Bargaining.

Despite the strong push from European and national institutions for a more decentralized wage bargaining structure, in some countries company or establishment-level negotiations struggle to take place. This paper offers an interpretation for that based on workers' optimal choices in an strategic framework. We construct an equilibrium matching model that explains under which conditions it is best for workers to negotiate their entire wage at sectoral level (one-tier bargaining) or to let a fraction of their salary to be negotiated at company level (two-tier bargaining). Workers' strategies do not simply depend on their own characteristics or on those of their firm, but also on the decisions of all the other employees of the sector. Three alternative Nash equilibria may occur: one-tier bargaining for all workers; two-tier bargaining for all workers; two-tier bargaining for the most productive workers and one-tier bargaining for the others. The prevalence of a specific equilibrium over others hinges on some critical factors, notably the elasticity of the matching function and the properties of the productivity distribution.

JEL Classification: J50, J52, J31, J64

Keywords: collective bargaining, wage decentralization, bargaining structure, unions

Corresponding author:
Gabriele Cardullo
Department of Economics and Business
University of Genova
Via Vivaldi 5
16126 Genova
Italy

E-mail: cardullo@economia.unige.it

1 Introduction

Over the past four decades a recurring theme in discussions about industrial relations in Europe has been the necessity of greater decentralization in the bargaining process (Marginson (2015), Müller et al. (2019), Leonardi and Pedersini (2020a), and Tros (2023a)). The underlying main motivation is to enhance labour market flexibility by making wages more responsive to local and firm specific conditions. This explains why the push has been stronger after the sovereign debt crisis in 2009-2010 and in countries mostly located in Southern Europe, with a long tradition of multi-employer collective agreements[1]. That has translated into a wide array of reforms aimed at increasing the importance of company level negotiations[2].

However, despite this robust commitment of public institutions and (in some cases) the willingness demonstrated by social partners, data collected at administrative level show the spread of company-level bargaining appears mostly inadequate (Carrieri et al. (2020)) and quite heterogeneous across countries and sectors. Consider the most important Southern European countries. In Italy (see CNEL (2022), pp. 179 - 183), in 2022 only 20 % of firms were engaged in some forms of decentralized negotiation[3]. The same unsatisfying dynamics can be observed in Spain, where less than one million employees are covered by a company level agreements[4], and in Portugal[5]. In France, on the contrary, company level agreements have been on the rise for the last ten years, with a cumulative increase of about 90% (see DGT (2023), p. 216). Overall, in all these countries sectoral bargaining remains the predominant level of negotiation[6].

[1]See for instance the report of the European Commission (EC (2012)) and the country-specific recommendations of the European Council in 2015 (Council recommendation, 14 July 2015; 2015/C 272/16).

[2]The recent books of Leonardi and Pedersini (2020b) and Tros (2023b) provide a detailed account of the legislation adopted by national governments to foster decentralized bargaining in Europe in the last 15 years. For France, see for instance the so-called Macron ordinances of 2017. In Italy, it is worthwhile to mention the Law No. 148/2011 significantly entitled "Support for proximity collective bargaining" and some elements of the so-called Jobs Act adopted in 2015. In Spain reforms and counter-reforms have followed one another, from the Law 7/2011 to the Royal Decree-Law 32/2021, which restored the prevalence of the sectoral agreements over the company ones on wage related issues.

[3]For INAPP (2023, pp. 30-31), in the period 2016 - 2021 the percentage of employees that received performance related premia negotiated at local level was less than 9 %.

[4]This number has not increased over time (Muñoz Ruiz et al. (2023) and CCOO (2022)).

[5]Administrative data show that from 2011 to 2022 the percentage of employees covered by acordos de empresas (AE) remained roughly constant, between 5% and 7% (DGERT (2022)).

[6]According to the OECD/AIAS/ICTWSS (2023) database that collects information on institutional

The aim of this paper is to offer an explanation for that. The common answer advanced for such discrepancies lies on the different institutional and regulatory settings. In some of these countries the state intervention on these issues is more relevant than in others[7]. Some works also underscore the general lack of trust between sectoral and company-level players (see Hijzen et al. (2019) for the case of Portugal). For other studies the reason lies on some structural features of these economies, such as firm size. Italy and Spain are both characterized by a large share of small firms where unions or works councils may not be present or employers may not have the necessary skills to deal with complex negotiations, so that wage agreements are more difficult (CNEL (2022), Damiani and Ricci (2014), and Muñoz Ruiz et al. (2023)). If this is certainly true, one also needs to remark that the percentage of micro and small firms out of total businesses in France is not so different from the Italian and Spanish case[8].

While not disqualifying the logic of these arguments, this work takes a different stand. Instead of focusing on institutional and cultural reasons or on firm characteristics, I assume that the specific bargaining structure results from workers' optimizing behaviour. I model this choice in a framework that aims to mimic the specific elements of labour relations in France and Southern Europe countries[9]. There, sectoral bargaining may coexist with more decentralized types of negotiation (the so-called two-tier wage systems, see Cardullo et al. (2020) and Devicienti et al. (2017)). Employers and workers have the opportunity to negotiate at company or enterprise level on some specific (in some cases relevant) aspects of the labour contract. Moreover, the degree of flexibility between these two levels is still conditioned by the so-called favourability principle. According to this rule, the agreements reached at decentralized level cannot be less favourable for the employees

characteristics of trade unions and wage setting mechanisms, in all these countries negotiation predominantly takes place at the sector or industry level. See also Eurofound (2023).

[7]In France, the law has prescribed company-level bargaining since early 1980s (see among the others Rehfeldt and Vincent (2020) and Erhel (2021)).

[8]From Eurostat in 2019 the percentages of micro (0-9 persons employed) and small firms (10-49 persons employed) out of total were respectively the following: 94.7 and 4.7 in Spain, 94.9 and 4.5 in Italy, 95.5 and 3.8 in France.

[9]Adopting a new taxonomy that accounts for the varieties of industrial and labour relations in Europe, Garnero (2021) inserts France, Italy, Portugal, Slovenia, Spain, and Switzerland in the same group, denominated RCW: rather centralized and weakly coordinated.

than those negotiated at sectoral level[10].

So, in the present paper workers choose either to bargain over wages entirely at sectoral level (one-tier bargaining) or to negotiate a residual fraction of their pay at firm level (two-tier bargaining), knowing that the favourability principle holds. At centralized level, unions and the employers' confederations set a compensation based on the average productivity of the sector and take into account the negative effect of wage hikes on employment. Conversely, at firm level, employers and workers do not consider the impact of their decisions on aggregate unemployment, and the fraction of the total pay over which they negotiate just depends on the specific productivity of that job (we can consider it as a residual performance related pay).

In such a setting, I show that sectoral unions and the employers' confederation agree on a given labour share (i.e. the part of total output allocated to wages) and react to the agreements made at decentralized level by adjusting the fraction of the salary negotiated at sectoral level. This means that if, for instance, performance related pays negotiated at firm level are deemed too high, they set a lower pay at sectoral level.

Knowing that, each single employee must decide whether it is profitable to bargain at firm level or not, given the actions taken by all the other employed workers. Indeed, any agent's payoff depends not just on her specific productivity but also on the number of workers opting for a decentralized negotiation and their average productivity. The larger the number of highly productive workers choosing a two-tier bargaining scheme, the lower must be the equilibrium level of the pay decided by unions and the employers' confederation at sectoral level in order to meet their agreed labour share target.

I find that, depending on the values of some crucial parameters, three alternative Nash equilibria of the game exist: 1) one-tier bargaining for all workers (that implies an identical wage in the sector); 2) two-tier bargaining for all; 3) one-tier bargaining for the

[10] Despite many attempts (even at legislative level) to circumvent such a principle, recent research (see for instance Garnero (2021)) has highlighted how in Southern European countries exceptions to it are still quite limited. The favourability rule may be no longer a compulsory norm, but sectoral agreements remain a reference point for company bargaining and derogations are unusual (see Kahmann and Vincent (2022) for France, Armaroli and Tomassetti (2022)) for Italy and Muñoz Ruiz et al. (2023) for Spain). For Boeri (2014) and Boeri (2015) two-tier systems in which the favourability principle holds produce inferior results in terms of employment and wage dispersion.

less productive workers and two-tier bargaining for the more productive ones.

The first equilibrium is more likely to emerge if the job filling rate elasticity is small, implying that a given percentage increase in job vacancies leads to a limited change in hires, and hence little effect on the unemployment rate. In labour markets of this kind sectoral unions may demand and obtain higher wages because their negative effect on vacancy posting does not raise unemployment that much. In turn, knowing that the fraction of pay decided at sectoral level can be generous, the dominant strategy for all workers is not to engage in a bargaining process at company level. Some empirical findings on labour market elasticities seem to support this result, as European Union countries in which sectoral bargaining remains predominant also present a lower matching elasticity compared to the average (Arpaia et al. (2014)).

On the other hand, a two-tier equilibrium may occur in sectors where the mass of the productivity distribution is concentrated on the left (i.e. a right-skewed distribution) and the gap between jobs with the lowest productivity and the average is small. In these sectors, workers in the most productive jobs want to negotiate at decentralized level, so that a fraction of their salary is in line with their higher than average performance. At the same time, the rest of workers employed in the sector do not see a great difference between being paid according to their (low) productivity or the average one. This makes two-tier bargaining relatively more convenient even for them. So this second result creates a connection between the productivity distribution and wage inequality, via workers' optimal level of bargaining structure.

Finally, an equilibrium in which less productive workers opt for one-tier bargaining and the more productive ones choose a two-tier setting is also possible. This may occur in sectors where the productivity distribution is not as positively skewed as in the two-tier equilibrium for all scenario and the average is significantly larger than its lowest possible values. In this case, workers employed in less productive jobs have no interest in negotiating at decentralized level. A one-tier setting would ensure their entire wage being dependent on the average productivity of the sector.

Studying why wage decentralization may be present in some sectors/labour markets and not in others is important, because it is well known that collective bargaining affects productivity (Braun (2011) and Garnero et al. (2020)), employment, and wage inequality (Dahl et al. (2013)). This does not mean the literature has reached an unanimous conclusion on which system delivers superior results. If some works initially asserted that fully centralized (i.e. national) and decentralized systems achieve better outcomes than intermediate sectoral ones (the so-called hump-shaped hypothesis of Calmfors and Driffil (1988)), the most recent research points out that just looking at the degree of centralization is not sufficient to evaluate the performance of collective bargaining settings (see Aidt and Tzannatos (2008), Braakmann and Brandl (2016), Bechter et al. (2012),Traxler and Brandl (2012), Garnero (2021)). According to this literature, at least two other elements must be taken into account: wage coordination across sectors and companies in response to changing macroeconomic conditions and flexibility across levels of bargaining (that is, in most cases, the extent to which company agreements may change what decided at sectoral/national level). On the other hand, another vein of research stresses the importance of decentralization as a tool to champion the option voice of workers, thereby promoting dialogue between employees and employers (see the discussion in OECD (2018)).

The purpose of this paper is not to enter into this debate by indicating the optimal level of wage negotiation. Rather the opposite. Since the three equilibrium outcomes presented above alternatively arise for different values of some structural parameters of the model, comparing one with another would have little sense. Conversely, I do perform some normative analysis by comparing each equilibrium with the other non equilibrium outcomes of the game and get that each equilibrium is Pareto optimal. In a way, the results of this paper seem to suggest that a top down imposition of a specific wage system (or, borrowing a blunter expression from Marginson (2015), a frontal assault on multi-employer bargaining arrangements), as occurred in most Southern European countries in the last fifteen years, risks having little effect if the structural conditions of the sector and the labour market drive workers towards another type of bargaining process.

It must be also added that European institutions have recently taken a more nuanced approach on this issue. While in the past the main focus was on achieving greater decentralization, the EU Directive 2022/2041 on adequate minimum wages affirms that "...sectoral and cross-industry level collective bargaining came under pressure in some Member States in the aftermath of the 2008 financial crisis. However, sectoral and cross-industry level collective bargaining is an essential factor for achieving adequate minimum wage protection and therefore needs to be promoted and strengthened".

The paper is organized as follows. Section presents the basic framework. Section 3 the different wage determination settings. Section 4 illustrates the equilibrium results, while section 6 the welfare results. Finally, section 7 concludes.

2 The Model

2.1 Production and Matching Technology

Consider a continuous-time model with a continuum of infinitely-lived and risk-neutral workers who have perfect foresight and a common discount rate r. The economy is composed by one sector and a firm is composed of a single (filled or vacant) job. When a worker-firm match is formed, a certain amount of output is produced. I assume that there are two possible values for output: $y_a \cdot (1+\ell)$ and $y_b \cdot (1+\ell)$. The term $1+\ell$ denotes the amount of hours worked by each individual, whereas y_i, with $i \in \{a, b\}$ denotes the hourly productivity. I assume that $y_a > y_b$. As in Pissarides (2000, chapter 6), firms and workers realize the specific value of their match only when they make contact. Before the meeting takes place, they just know that there is a probability ϕ that a type a match with high productivity y_a can be formed.

Labour force is normalized to 1. There are frictions in the labour markets. The matching function gives the measure of matches for a certain value of unemployment u and vacancies v: $m = m(v, u)$. Function $m(.,.)$ has constant returns to scale and it is increasing and concave in each argument. As usual in the search and matching literature (see Petrongolo and Pissarides (2001)), I consider a Cobb-Douglas technology: $m = v^{1-\eta} u^{\eta}$, with $0 < \eta < 1$. Labour market tightness is defined as $\theta \equiv v/u$. A vacancy is filled according to a Poisson process with rate $q(\theta) \equiv m/v = \theta^{-\eta}$, $q'(\theta) < 0$. A job-seeker gets employed at a rate $f(\theta) \equiv m/u = \theta q(\theta) = \theta^{1-\eta}$, increasing in θ. Notice that parameter $1-\eta = \frac{dm}{dv} \cdot \frac{v}{m}$ is the elasticity of the matching function with respect to vacancies. It tells us the percentage increase in hires for a given percentage increase in job vacancies. At a certain exogenous rate δ, a filled job breaks down and the worker becomes unemployed.

In steady-state, the amount of new jobs created must be equal to the number of jobs destroyed: $e \cdot \delta = u \cdot f(\theta)$, with e denoting the level of employment. Knowing that $1 = e + u$, the steady state level of employment is equal to:

$$e = \frac{f(\theta)}{f(\theta) + \delta}. \tag{1}$$

2.2 Free-entry condition and unions' preferences

The expected discounted value of a filled job verifies the following Bellman equation:

$$r\Pi_i^E = (1+\ell)y_i - w_i + \delta\left[\Pi^V - \Pi_i^E\right] \tag{2}$$

for $i \in \{a, b\}$. Firms' expected revenues are equal to the amount of output produced net of the wage w_i, whose precise formulation will be explained in the next section. At a rate δ, the firm-worker pair splits apart and employers get a capital loss equal to the difference between the value of a filled job and the expected value of a job vacancy, Π^V. The value of a vacancy Π reads as

$$r\Pi^V = -h + q(\theta)\left[\phi\,\Pi_a^E + (1-\phi)\,\Pi_b^E - \Pi^V\right] \tag{3}$$

for $i \in \{a, b\}$. with the term h denoting the flow vacancy costs.

There is free-entry of vacancies. Firms enter the labour market as long as expected profits are nonnegative. This implies $\Pi^V = 0$. So, rearranging eqs. (2) and (3) yields:

$$\frac{(1+\ell)\,\bar{y} - \bar{w}}{r + \delta} = \frac{h}{q(\theta)}. \tag{4}$$

where $\bar{y} \equiv \phi y_a + (1-\phi) y_b$ is the average level of hourly productivity in the economy and $\bar{w}$ is the average wage. So, at the LHS one has the average expected discounted revenues of a filled job In equilibrium they must be equal to the expected costs of a vacancy: the flow cost of a vacancy multiplied by its expected duration, $1/q(\theta)$.

As concerns the other side of the market, the expected discounted utility in employment rJ_i^E can be written as follows:

$$rJ_i^E \;=\; w_i \,+\, \delta \left[\, J_i^U \,-\, J_i^E \,\right] \tag{5}$$

At a rate δ the match gets destroyed and the worker becomes unemployed, whose expected discounted utility reads as:

$$rJ^U \;=\; z \,+\, f(\theta) \left[\, \phi J_a^E + (1-\phi) J_b^E \,-\, J^U \,\right]. \tag{6}$$

Being unemployed is like holding an asset that pays you a dividend z, the value of home production. At a rate $\phi\, f(\theta)$ (resp. $(1-\phi)\, f(\theta)$) the worker gets employed and the match ensures a productivity y_a (resp. y_b). The term inside the square brackets is therefore the average capital gain obtained by finding a job. I assume that $z \,<\, y_b \,<\, y_a$. Market production is always greater than home production

I assume all the employed workers are unionised. Unions behave in utilitarian way, in the sense that they care about the sum of the utilities of all the employees in the sector. This means that the expected discounted utility of workers' union is equal to:

$$rU^W \;=\; e \cdot \bar{w} \tag{7}$$

Similarly, the utility of the employers' confederation U^F is just the sum of the revenues raised in the sector:

$$rU^F \;=\; e \cdot (1+\ell) \cdot \bar{y} \,-\, e \cdot \bar{w} \tag{8}$$

3 Wage determination

The basic assumption of the model is that workers can decide either to let unions to negotiate the wage entirely at sectoral level (one-tier setting) or to allow that a fraction of their earnings is bargained at firm level (two-tier setting).

More in detail, the real wage for workers employed in type i ($i \in \{a, b\}$) jobs is given by

$$w_i \equiv \nu + \mu_i \cdot \ell, \tag{9}$$

for $i \in \{a, b\}$. Firms pay an amount equal to ν for a fraction (normalized to 1) of the working hours. The term μ_i denotes the hourly remuneration employees receive for the remaining ℓ working hours.

Under the one-tier bargaining scheme, both ν and μ_i are negotiated at sectoral level. This implies that $\mu_a = \mu_b$. The wage is identical whatever the type of occupation: $w = \nu + \mu\ell$.

Conversely, under a two-tier wage setting process, ν is chosen at sectoral level, whereas the value for μ_i ($i \in \{a, b\}$) is determined at firm level. So μ_a is different from μ_b.

Under this formulation I am able to maintain one the crucial features of two-tier bargaining schemes present in Southern Europe and that is called "favorability principle". Under this framework, firm or plant-level agreements (the second tier of the negotiation) cannot envisage conditions that would make workers worse off than they are under the higher, sectoral, level of bargaining. In our model, a single firm-worker pair negotiates over the wage to be paid for the residual ℓ working hours but cannot change the fraction of the salary ν chosen at sectoral level.

Of course, the choice of the bargaining scheme hinges on the type of occupations in which workers are employed. Moreover, as we will see formally in the next sections, the value of ν determined at sectoral level for all the workers depends on how much employees are paid for the residual part of their working hours, μ_i. So the remuneration obtained by employees in type i jobs is affected by the kind of negotiation chosen by employees in type j jobs ($i, j \in \{a, b\}$). One can envisage four different scenarios:

1. one-tier bargaining for all workers;

2. two-tier bargaining for all workers;

3. two-tier bargaining for workers in type a jobs, one-tier for workers in type b.

4. one-tier bargaining for workers in type a jobs, two-tier for workers in type b.

Before studying the strategies and the equilibria of the game, I first present the wage setting process in the different scenarios.

3.1 One-tier bargaining

If the wage negotiation takes place just at the sectoral level, the optimal values for ν and μ are the result of the following Nash bargaining problem:

$$\max_{\nu,\, \mu} \left[U^W - \bar{U}^W \right]^\beta \cdot \left[U^F - \bar{U}^F \right]^{1-\beta} \tag{10}$$

Parameter β stands for the bargaining power of the workers' union. $\bar{U}^W$ and $\bar{U}^F$ are respectively the fall-back positions for workers' unions and employers' confederations. In detail, I assume that, in case of disagreement, workers become unemployed and enjoy an instantaneous utility equal to the value of home production, z, Therefore the fall-back position of workers' union reads as $r\bar{U}^W = z \cdot e$. By the same token, in case of failure in negotiation, firms do not produce and sell anything. This implies that the fall-back position for the firm's union is $r\bar{U}^F = 0$.

For a better comparison with the problem I will analyse under a two-tier scenario, I find it convenient to maximize with respect to the total wage w instead of the fraction ν. Using equations (7), (8), (14), and the expressions for $\bar{U}^W$ and $\bar{U}^F$, one gets:

$$\max_{w,\, \mu} \left[\frac{1}{r} \left(e \cdot w - r\bar{U}^W \right) \right]^\beta \cdot \left[\frac{1}{r} \left(e \cdot (1+\ell) \cdot \bar{y} - e \cdot w \right) \right]^{1-\beta}, \tag{11}$$

At the equilibrium, the negotiation always ends up in an agreement. The F.O.C.s for

w and μ are:

$$\beta \frac{e + w \frac{de}{dw}}{U^W - \bar{U}^W} + (1-\beta) \frac{\frac{de}{dw}(\bar{y} - w) - e}{U^F - \bar{U}^F} = 0 \tag{12}$$
$$\beta \frac{e \cdot \ell + w \frac{de}{d\mu}}{U^W - \bar{U}^W} + (1-\beta) \frac{\frac{de}{d\mu}(\bar{y} - w) - e \cdot \ell}{U^F - \bar{U}^F} = 0 \tag{13}$$

Notice that unions and the employers' confederation take into account the negative impact of a wage increase on employment, as higher labour costs dampen vacancy creation for the zero profit condition. Such effect is captured by the derivatives at the numerator in both equations. More in detail, I have:

$$\frac{de}{dw} = \frac{\partial e}{\partial \theta} \cdot \frac{\partial \theta}{\partial \nu} == -e \cdot \frac{\delta\,(1-\eta)\,(1+\ell)\,q(\theta)}{h\,\eta\,(r+\delta)\,(\delta + f(\theta))} \tag{14}$$

The first derivative at the RHS is computed using the steady state equality (1), whereas the second one is obtained totally differentiating the zero profit condition (4).

It is easy to see that $\frac{de}{dw} \cdot \ell = \frac{de}{d\mu}$. In turn, this implies that equations (12) and (13) are identical and the only possible solution is such that $\nu = \mu$. This is not surprising. Since ν and μ are jointly decided by the same unions, there is no reason any of the total $1 + \ell$ working hours should be paid differently. So, in the one-tier scenario, one gets $w_1 = \nu_1\,(1+\ell)$ with $i \in \{a, b\}$, in which the subscript 1 stands for the one-tier scenario.

Using equations (7), (8), (14), and the expressions for $\bar{U}^W$ and $\bar{U}^F$ yields:

$$w_1 = \nu_1\,(1+\ell) = \beta\,(1+\ell)\,\frac{\eta(\delta + f(\theta))}{\delta + \eta\, f(\theta)} \cdot \bar{y} + (1-\beta)\,z \tag{15}$$

As in standard matching models, the wage positively depends on workers' bargaining power β, on value of home production z, and on labour market tightness, θ. But while in a textbook matching framework labour market tightness affects the wage because it raises the opportunity cost of employment, in this scenario the mechanism is different. As an inspection of equation (14) makes clear, the negative impact of a wage increase on employment becomes smaller when the labour market is tighter[11]. Since unions exert less

[11] This is because of the concavity of the matching function, implying that the variation in employment

restraint in their wage demands when the labour market is tighter, workers end up with higher earnings.

3.2 Two-tier bargaining

Let focus on the two-tier scheme. First, unions representing all the firms with a filled position of type a and b and all the workers negotiate over ν. In a second stage, each firm-worker pair bargains over μ_i for $i \in \{a, b\}$.

I proceed backward and consider the negotiation at firm level. The value of μ_i is determined via Nash bargaining:

$$\mu_i \;=\; \text{argmax} \left[\, J_i^E \,-\, \bar{J}_i^E \,\right]^{\epsilon} \cdot \left[\, \Pi_i^E \,-\, \bar{\Pi}_i^E \,\right]^{1-\epsilon},$$

for $i \in \{a, b\}$. Parameter ϵ stands for the worker's exogenous bargaining power at local level and it is different from β, that captures the strength of employees' union at sectoral level. The terms $\bar{J}_i^E$ and $\bar{\Pi}_i^E$ stand for the expected utilities pay-offs in case of disagreement for workers and firms respectively. They are equal to:

$$r\bar{J}_i^E \;=\nu \,+\, \delta \left[\, J^U \,-\, \bar{J}_i^E \right], \qquad r\bar{\Pi}_i^E \;=\; y_i \,-\, \nu \,+ \delta \left[\, \Pi^V \,-\, \bar{\Pi}_i^E \,\right] \tag{16}$$

These two equations imply that, in case of disagreement in the second tier of the negotiation, workers remain employed but earn only the fraction ν of the salary decided at sectoral level, and firms produce less. Indeed, in the light of what discussed before about the "favorability principle" and the residual nature of the second level of the negotiation in Southern European countries, it does not seem plausible to imagine that a disagreement over a fraction of the total pay implies lay-offs or quits.
The F.O.C. of the above problem is:

$$\epsilon \cdot \left(\Pi_i^E \,-\, \bar{\Pi}_i^E \right) \;=\; (1 \,-\, \epsilon) \cdot \left(J_i^E \,-\, \bar{J}_i^E \right) \tag{17}$$

due to a tightness change is smaller the larger the value of θ.

for $i \in \{a, b\}$. Using eqs. (2), (5), and (16), one gets:

$$\mu_i \;=\; \epsilon\, y_i \tag{18}$$

for $i \in \{a, b\}$. The hourly wage μ_i is a share ϵ of productivity y_i. So the wage takes the following form: $w_i \;=\; \nu + \epsilon\, \ell\, y_i$ In turn, the average wage is $\bar{w} \;=\; \nu \,+\, \epsilon\, l\, \bar{y}$

At the first tier of the bargaining scheme, unions of workers and firms negotiate over ν. The Nash bargaining problem takes the following form:

$$\max_{\nu} \left[U^W \,-\, \bar{U}^W \right]^{\beta} \cdot \left[\, U^F \,-\, \bar{U}^F \,\right]^{1-\beta}$$

I can again solve the problem by maximizing with respect to the average wage $\bar{w}$:

$$\max_{\bar{w}} \left[\frac{1}{r} \left(\, e \cdot \bar{w} \,-\, r\bar{U}^W \,\right) \right]^{\beta} \cdot \left[\frac{1}{r} \left(e \cdot (1 + \ell) \cdot \bar{y} \,-\, e \cdot \bar{w} \,\right) \right]^{1-\beta} \tag{19}$$

Notice that such a problem is identical to the one presented under the one-tier setting, (11)[12]. This means that the average wage must be the same for both scenarios:

$$\bar{w} \;=\; \nu_1\, (1 + \ell) \;=\; \nu_2 \,+\, \epsilon\, \ell\, \bar{y} \tag{20}$$

The rationale for equation (20) goes as follows. Sectoral workers' unions and the employers' confederation have the same utility functions in both settings. Moreover, their objective is just to find the optimal division of total output between employers and employees, or, equivalently, the labour share $\bar{w} \cdot e$. They do not care about how the total wage bill is distributed between workers employed in type a or type b occupations. This can also be seen by isolating ν_2:

$$\nu_2 \;=\; \nu_1 \,+\, \ell\, (\nu_1 \,-\, \epsilon \bar{y}) \tag{21}$$

The difference between the hourly compensation decided at sectoral level between the

[12]Recall that under under a one-tier negotiation the wage is identical across occupations and $w_1 \cdot (1 + \ell) \;=\; \bar{w}$.

two scenarios considered so far is the last term at the RHS, that is a compensating term. Inside the parenthesis there is the difference between the pay workers would obtain for ℓ hours under a one-tier setting, ν_1, and the average one they earn in a two-tier scenario, $\epsilon\bar{y}$. Such a difference can be positive or negative, resulting respectively in a value for ν_2 greater or lower than ν_1. If $\epsilon\bar{y}$ is higher (resp. lower) than ν_1, sectoral unions and firms in the present regime would react by negotiating a lower (resp. higher) pay ν_2. In this scenario, the term ℓ coincides with the ratio between the number of hours whose remuneration is decided at local level, $\ell \cdot e$, and the remaining ones $1 \cdot e$. Of course, the higher this ratio is, the larger the compensating factor sectoral unions and the employers' confederation must agree upon to reach the labour share target $\bar{w} \cdot e$.

Using equation 20, one easily gets the wage equations under the two-tier bargaining scheme:

$$w_{2,\,a} \;=\; \nu_2 + \epsilon\,\ell\,y_a \;=\; w_1 + \epsilon\,\ell\,(1-\phi)\,(y_a - y_b) \tag{22}$$

$$w_{2,\,b} \;=\; \nu_2 + \epsilon\,\ell\,y_b \;=\; w_1 - \epsilon\,\ell\,\phi\,(y_a - y_b), \tag{23}$$

Wage differences depend on the second terms at the RHS of (22) and (23). Employees in type a jobs are paid more than workers in type b, as they are more productive: $y_a > y_b$. The second level of the negotiation creates a wedge in the workers' earnings. Indeed, I have $w_{2,a} - w_{2,b} = \epsilon\,\ell(y_a - y_b)$. Such a gap is wider the stronger is workers' bargaining power at firm level ϵ and the greater the amount of hours worked ℓ whose pay is not decided at sectoral level. The productivity differences between type a and type b jobs affect the earnings dispersion but not the average wage in the economy. Notice finally that $w_{2,a} > w_1 > w_{2,b}$. This is quite intuitive. If in this scenario more productive workers are paid more than low productive ones and, at the same time, the average wage must be equal to the w_1, then employees in type a (resp. type b) jobs will receive a higher (resp. lower) compared to the one obtained under one-tier bargaining.

3.3 One-tier in type b jobs, two-tier in type a jobs

Suppose workers in type a jobs opt for decentralization, that is they bargain over μ_a at firm level, whereas employees in less productive jobs leave that the negotiation is decided entirely at sectoral level.

The same passages exposed in the previous sections lead to $\mu_a = \epsilon\, \ell\, y_a$. Moreover, workers in type b jobs earn a salary $w_{3,\,b} = \nu_3 \cdot (1+\ell)$, in which ν_3 denotes the hourly pay decided at sectoral level when workers in more productive jobs opt for a two tier scheme and employees in less productive occupations choose a one-tier process. The average wage under the present scheme is therefore equal to $\bar{w} = \nu_3 \cdot (1 + (1-\phi)\,\ell) + \epsilon\,\phi\,\ell\, y_a$.

Again, the maximization problem at sectoral level is the same as in the previous two scenarios. So the average wage does not change:

$$\bar{w} \;=\; \nu_3 \cdot (1 + (1-\phi)\,\ell) \,+\, \epsilon\,\phi\,\ell\, y_a \;=\; w_1, \tag{24}$$

in which w_1 is the remuneration workers would obtain under a one-tier scenario, (15). In turn this implies that the fraction ν_3 negotiated at sectoral level under this scenario is equal to:

$$\nu_3 \;=\; \nu_1 \,+\, \frac{\phi\,\ell}{1+(1-\phi)\ell}\,(\,\nu_1 \,-\, \epsilon\, y_a\,) \tag{25}$$

The interpretation for this formula is the same given for (21). The hourly sectoral wage in this scenario ν_3 is the sum of two components, the hourly sectoral pay decided under scenario 1, ν_1, and a compensating factor, the second term at the RHS. Since sectoral unions and the employers' confederation agree on specific total wage bill $\bar{w} \cdot e = \nu_1(1+\ell)\cdot e$ (regardless of workers' decision to embark on firm level bargaining or not), the value for ν_3 must compensate for the difference between the pay for the residual ℓ hours obtained in a one-tier setting, ν_1, and the one employees in type a jobs get in this scenario, $\epsilon\, y_a$ (the difference inside the parenthesis at the RHS of (25)). If the former is lower (resp. greater) than the latter, then the pay decided at sectoral level in this third scenario will be lower (resp. greater) than ν_1. As in equation (21), the compensating factor also depends on the ratio between the number of working hours whose pay is decided at local level and those decided at sectoral level, that in this scenario is equal to $\phi\,\ell/\,(1+(1-\phi)\ell)$. A high value for this ratio means this adjustment effect must be stronger.

Using equation (25), one easily finds the wage equations for both types of jobs:

$$w_{3,\,a} \;=\; \nu_3 \,+\, \epsilon\,\ell\,y_a \;=\; \frac{w_1 \,+\, \epsilon\,\ell\,y_a\,(1-\phi)\,(1+\ell)}{1+(1-\phi)\ell}. \tag{26}$$

$$w_{3,\,b} \;=\; \nu_3\,(1+\ell) \;=\; \frac{1+\ell}{1+(1-\phi)\ell}\,(w_1 \,-\, \epsilon\,\ell\,y_a\,\phi) \tag{27}$$

3.4 One-tier in type a jobs, two-tier in type b jobs

The same passages described in the previous sections allow to find the wage equations under this fourth scenario. If workers in less productive jobs opt for a two-tier bargaining, they get a remuneration equal to $\mu_b \;=\; \epsilon\,\ell\,y_b$ for ℓ hours worked. Since earnings for the employees in type a jobs are entirely negotiated at sectoral level, the average wage in the economy is equal to $\bar{w} \;=\; \nu_4\cdot(1+\phi\,\ell) \,+\, \epsilon\,(1-\phi)\,\ell\,y_b$, in which ν_4 is the remuneration bargained at sectoral level.

With workers' unions and employers' confederations having the same preferences and facing the same maximization problem, the average wage in the economy must be equal to the one obtained in the previous three scenarios:

$$\bar{w} \;=\; \nu_4\cdot(1+\phi\,\ell) \,+\, \epsilon\,(1-\phi)\,\ell\,y_b \;=\; w_1, \tag{28}$$

Rearranging, one gets:

$$\nu_4 \;=\; \nu_1 \,+\, \frac{(1-\phi)\,\ell}{1+\phi\ell}\,(\nu_1 \,-\, \epsilon\,y_b)\,. \tag{29}$$

As observed for equations (21) and (25), the hourly pay negotiated at sectoral level in this scenario must ensure that the total wage bill in the economy is the same as in the previous ones. So ν_4 must be equal to ν_1 plus the difference between the pay for ℓ hours in the first and in the the current setting. This gap is weighted by the ratio between the amount of hours whose pay is negotiated at local level and the ones bargained at central level.

		Type b	
		1-tier	2-tier
Type a	1-tier	$w_1,\ w_1$	$\frac{(1+\ell)\,[w_1-\epsilon\,\ell\,y_b(1-\phi)]}{1+\phi\,\ell},\ \frac{w_1+\epsilon\,\ell\,y_b\,\phi\,(1+\ell)}{1+\phi\,\ell}$
	2-tier	$\frac{w_1+\epsilon\,\ell\,y_a\,(1-\phi)\,(1+\ell)}{1+(1-\phi)\ell},\ \frac{(1+\ell)\,[w_1-\epsilon\,\ell\,y_a\,\phi]}{1+(1-\phi)\ell}$	$w_1+\epsilon\,\ell\,(1-\phi)\,(y_a-y_b),\ w_1-\epsilon\,\ell\,\phi\,(y_a-y_b)$

Figure 1: The normal form game between workers in type a jobs and workers in type b jobs

From (29), one gets the wage equations for both types of jobs:

$$w_{4,\,a} \;=\; \nu_4\,(1+\ell) = \frac{1+\ell}{1+\phi\ell}\,(w_1 \;-\; \epsilon\,\ell\,y_b\,(1-\phi))\,. \tag{30}$$

$$w_{4,\,b} \;=\; \nu_4 \;+\; \epsilon\,\ell\,y_b \;=\; \frac{w_1 \;+\; \epsilon\,\ell\,y_b\,\phi\,(1+\ell)}{1+\phi\,\ell}. \tag{31}$$

4 Equilibrium

In Figure 1 I summarize the normal form game played by workers in type a and type b jobs. The payoffs are the earnings obtained in the four different scenarios presented in the previous section. The following Proposition presents the results.

Proposition 1

1. *If $\epsilon \;<\; \beta\,\eta\,\frac{\bar{y}}{y_a}$, the unique Nash equilibrium of the game implies all workers opt for a one-tier wage setting (scenario 1).*

2. *If $\epsilon \;>\; \frac{\beta\,(1+\ell)\,+\,(1-\beta)\,z}{y_b/\bar{y}\,+\,\ell}$, the unique Nash equilibrium of the game implies implies all workers opt for a two-tier wage setting (scenario 2).*

3. *If $\beta\,\eta\,\frac{\bar{y}}{y_a} \;<\; \epsilon \;<\; \frac{\beta\,(1+\ell)\,+\,(1-\beta)\,z}{y_b/\bar{y}\,+\,\ell}$, three possible equilibria may alternatively occur: one-tier bargaining (scenario 1), one-tier bargaining for players in type b jobs and two-tier bargaining for players in type a jobs (scenario 3), two-tier bargaining for all (scenario 2).*

The proof is presented in Appendix 1, while Figure 2 illustrates the results[13]. Here I

[13]It is easy to show that $\beta\,\eta\,\frac{\bar{y}}{y_a} \;<\; \frac{\beta\,(1+\ell)\,+\,(1-\beta)\,z}{y_b/\bar{y}\,+\,\ell}$. Dividing both sides of this inequality by β, one

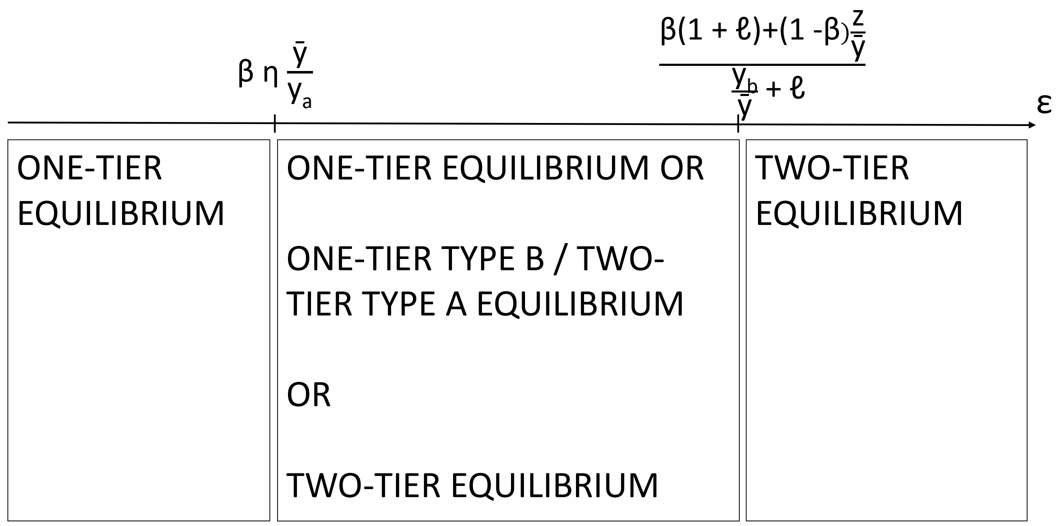

Figure 2: Proposition 1

provide the main intuition for the sufficient conditions of Proposition 1.

Not surprisingly, two relevant parameters are workers' bargaining power at central level and the one at firm level. The greater the former, the more likely the possibility that a one-tier bargaining equilibrium for all workers emerges (that is that the inequality in point 1 of Proposition 1 is respected while the one in point 2 is not). This is obvious, as workers may get a larger share of the surplus originating from the sectoral negotiation. Of course, the same logic explains why a high value for workers' bargaining power at firm level, ϵ, makes a two-tier bargaining equilibrium for all workers more likely (i.e the inequality in point 2 of Proposition 1 more likely to be fulfilled).

There are other exogenous variables that affect the probability one equilibrium arises instead of the others. The interpretation for their impact is less intuitive. I present them below.

- *If η is high, a one-tier bargaining equilibrium for all workers is more likely .*

gets at the LHS an expression that is always lower than 1 and at the RHS one that is always greater than 1, since $0 < \eta < 1$ and $y_a > \bar{y} > y_b$.

Recall that $1-\eta$ is the elasticity of the matching function with respect to vacancies. It tells us the percentage increase in hires for a given percentage increase in job vacancies. So the inequality in point 1 of Proposition 1 is more likely to be respected if the labour market has a small elasticity of the matching function with respect to vacancies (i.e. a low value for $1-\eta$, implying a high η). The intuition is the following. From the Nash bargaining problem in section 3.1 sectoral unions and the employers' confederation take into account the negative effect that wage pressures have on employment. With a low value for the matching elasticity, any given wage increase has a small impact on the unemployment rate, as the reduction in vacancy creation does not lead to a great decrease in hirings. Because of that, sectoral unions may obtain higher pays for their workers (i.e. a greater value for ν_1), making one-tier negotiation relatively more enticing (that is the inequality in point 1 of the Proposition 1 more likely to be satisfied).

There not many empirical works that compute the matching elasticity across European countries. A partial confirmation for this result may come from Arpaia et al. (2014). The find that countries in which sectoral bargaining remains predominant (France, Portugal, Spain, Slovenia) also present a lower matching elasticity compared to the average [14].

- *If $\bar{y}/y_a$ is high, a one-tier bargaining equilibrium for all workers is more likely .*

  If the share of highly productive jobs out of total is sufficiently large, the ratio between the average and the highest productivity in the sector, $\bar{y}/y_a$, is a value closer to 1 and letting all the pay dependent on $\bar{y}$ (as it is the case under sectoral negotiation) is more acceptable for employees in type a jobs. Again, the condition for a one-tier for all scenario is more likely to be fulfilled. Notice that a high value for $\bar{y}/y_a = \phi + (1-\phi)\frac{y_a}{y_b}$ is possible if the share of highly productive jobs ϕ is large, so that the mass of the distribution is mostly concentrated on the right. So,

[14] Arpaia et al. (2014) compute the job finding rate elasticity. In their (and our) specification the sum of the job finding rate and the job filling rate elasticities is equal to 1. They get therefore an average value for the job filling rate of about 0.7. The estimated value for the Spain and Slovenia is 0.6. In France it is 0.5. In Portugal it is 0.37. Results for Italy are not statistically significant.

a productivity distribution with a strongly negative skew makes workers less willing to bargain at decentralized level.

To the best of my knowledge, there are not empirical works that measure the asymmetry of the productivity distribution within sector in Europe[15]. So, unfortunately, it is not possible to get a validation (partial, at least) of this result.

- *If $y_b/\bar{y}$ is high, a two-tier bargaining equilibrium for all workers is more likely .*

  This can be easily seen by noting that the RHS of the inequality in point 2 of Proposition 1 is decreasing in $y_b/\bar{y}$. Two-tier bargaining is in principle a less attractive scenario for workers in type b jobs, because a fraction of their salary is aligned with their low productivity and not the average one, as in the one-tier wage setting. So, with a value of $y_b/\bar{y}$ close to 1, workers in type b jobs do not see a great difference between being paid according to their (low) productivity or the average one. This makes two-tier bargaining relatively more convenient for them. Notice again that the ratio $y_b/\bar{y} = \left(\phi\frac{y_a}{y_b} + 1 - \phi\right)^{-1}$ is closely related to the asymmetry of the distribution. The larger the share of low productive jobs $1 - \phi$, the higher the ratio $y_b/\bar{y}$ is, making the inequality in point 2 of Proposition 1 more likely to be respected[16]. So under a positively skewed productivity distribution a two-tier wage setting equilibrium for all workers is more likely to emerge and the wage distribution ends up being positively skewed as well, since company level negotiations make wages more in line with the specific productivity of the match. [17].

- *If ℓ is high, a two-tier bargaining equilibrium for all workers is more likely .*

  Again, I get this because the RHS of the inequality in point 2 of Proposition 1 is

[15] In their detailed analysis, Berlingieri et al. (2017) focus on productivity dispersion by industry. They compute the standard deviations and the 90-10 ratios, that are not directly connected to the variables in Proposition 1.

[16] Conversely, there is no direct connection between the variance and the prevalence of one equilibrium instead of another. In our model the variance of the productivity distribution is equal to $\phi(1-\phi)(y_a - y_b)^2$. It increases with y_a and decreases with y_b. Moreover parameter ϕ raises the variance if and only if it is lower than 0.5. So an increase in variance has an ambiguous impact on $y_b/\bar{y}$ and $\bar{y}/y_a$, the two terms that affect the sufficient conditions of Proposition 1.

[17] In our model where productivity can take only two values, it would be overreaching to delve too much into this type of analysis. Observed wage distributions are indeed positively skewed. See Mortensen (2005) and Moscarini (2005).

decreasing in ℓ. To understand why, we must focus on the best response strategy of workers in type b jobs when workers in type a ones opt for a two-tier wage setting. In other terms, one needs to compare the wage the less productive workers get in the two-tier for all scenario, $w_{2,b}$, with what they obtain in scenario 3, $w_{3,b}$, with employees in type a jobs choosing two-tier and those in type b jobs one-tier bargaining. Using equations (23) and (27), one gets that $w_{2,b} - w_{3,b}$ is increasing in ℓ: the larger the number of hours for which the pay is defined at firm level, the greater the incentive for workers in type b occupations to resort to this second tier of the negotiation. Parameter ℓ has a twofold impact on the wage equation. On the one hand, it affects the *hourly* sectoral pay, because it appears on its compensating component, the part of the wage that allows unions and the employers' confederation to reach a given labour share target, $\bar{w} \cdot e$. This component positively depends on the ratio between the number of hours whose pay is negotiated at decentralized level over those whose pay is decided at sectoral level. From equations (21) and (25) one obtains that as ℓ increases, the compensating component becomes greater in scenario 2 compared to scenario 3. This means that if ℓ is large in the two-tier bargaining for all equilibrium workers get a greater reduction in the hourly sectoral pay compared to the equilibrium in which only employees in type a jobs opt for a decentralized negotiation. This first effect makes scenario 2 less appealing. There is however a second effect, that goes in the opposite direction. In scenario 3, this compensating component applies to *all* $1 + \ell$ hours. Conversely, in the two-tier bargaining for all scenario, the reduction in the hourly sectoral pay is multiplied just by 1, as for the other ℓ hours employees in type b jobs get ϵy_b. This second effect is stronger and it explains why, in the end, the total wage $w_{2,b}$ is greater than $w_{3,b}$ as ℓ increases.

- *If $\frac{z}{\bar{y}}$ is low, a two-tier bargaining equilibrium for all workers is more likely .*

  If the ratio between the value of home production z and the average productivity $\bar{y}$ is low, workers' fall-back position at the sectoral negotiation is lower and unions obtain a lower wage a sectoral level. This makes the second tier of the negotiation relatively more attractive for all workers.

Point 3 of Proposition 1 indicates that, for a range of values of ϵ I am not able to ascertain which of three possible equilibria presented in Lemma 1 holds. So I rule out the existence of a Nash equilibrium in pure strategies for the fourth scenario, the one in which workers in high productivity jobs prefer that all their earnings are negotiated at central level, while employees in low productivity jobs opt for a decentralized negotiation. This is quite intuitive, as employees in type a occupations always have a greater incentive to engage in a negotiation at company level (and let a fraction of their pay to be aligned to their higher productivity).

Moreover, it can be shown (details area available on request) that, as ϵ gets closer to the term at the RHS of the inequality in point 2, the occurrence of a one-tier equilibrium becomes less possible. Similarly, as ϵ takes value larger but closer to $\eta\,\beta\frac{\bar{y}}{y_a}$ no two-tier for all workers equilibrium is less likely to emerge.

5 Welfare analysis

I study the welfare properties of each Nash equilibrium presented in the previous section, from the point of view of the single workers. To be clear, I do not make a comparison between the different alternative equilibria presented in Proposition 1[18]. Rather, for any game that implies one specific equilibrium, I look at the other possible payoffs to see if a Pareto improvement exists and which outcome is the first best for any type of worker.

Proposition 2 *For each of the three alternative Nash equilibria presented in Proposition* 1*, no Pareto superior outcome exists. Moreover, any equilibrium is a second best solution for workers in jobs of type* $i \in \{a, b\}$ *and a third best for the others.*

The proof is just a simple comparison of the different payoffs each worker may obtain in the game 1 and it is presented in Appendix 2.

[18]This would not be possible, as any equilibrium outcome is such for some specific values of the main parameters of the model β, ϵ, η, y_a, y_b, and ℓ. So, for instance, the hourly wage in the first scenario, ν_1, takes different values if we are in the one-tier for all, two-tier for all, or one-tier type a jobs / two-tier type a jobs equilibrium.

The intuition for the first part of the Proposition is easy to understand. Recall that sectoral players (unions and the confederation of employers) manage to reach the same average wage regardless of the choice workers make at local level. So, any possible outcome that could raise wages for workers employed in a specific type of job would inevitably imply a lower pay for the remaining ones[19]. No Pareto superior solution is available.

As concerns the fact that no worker in any equilibrium ever attain a first best, that again depends on the compensating mechanism that sectoral players set in motion. To understand this point. consider for instance the case in which a one-tier bargaining setting for all workers emerges. Recall this equilibrium occurs if sectoral unions' bargaining power β compared to bargaining power at local level ϵ or the elasticity of the matching function η are large enough, so that employees do not find convenient to negotiate at firm level. But this does not mean that the resulting equilibrium is a first best. For employees in both types of jobs, the best possible outcome would be attained if the other player would choose a two-tier wage negotiation. The hourly wage obtained at firm level would be lower than the one obtained at sectoral level. So, for the compensating mechanism explained in the previous sections, workers opting for just the sectoral negotiation would end up with a higher wage. Of course, this could never be an equilibrium outcome.

The same logic applies to the other two equilibria. The two-tier bargaining equilibrium for all workers occurs when some specific conditions of the labour market (for instance a low value for z) or of the sector (the average productivity distribution close to its lowest values) make the hourly compensation obtained if everyone choose one-tier bargaining quite low. In this case any employee in type i jobs would prefer that employees in type j (with $i, j \in \{a, b\}$) opt for a one-tier negotiation. This would spare them from the compensating mechanism that sectoral players would apply to meet the labour share target. But this cannot be an equilibrium result.

[19] In other terms, the weighted sum of the wages paid in each of the four possible outcomes in Figure1 is the same, the weight being ϕ and $1 - \phi$.

6 Concluding Remarks

I have constructed an equilibrium search framework to offer an interpretation for the limited diffusion of decentralized bargaining in Southern Europe. The crucial assumption of the model is that workers are free to choose the optimal level of wage negotiation. Unions and the employers' confederation agree on a given labour share. So, in order to meet their target they adjust the sectoral pay in response to the decisions taken by workers and firms at decentralized level. The results of this work suggest that the relatively scarce presence of company level wage agreements in Italy, Spain, and Portugal (among the others) may not just be imputed to cultural resistance from workers and employers representatives, or institutional reasons. Other factors play an important role. In labour markets where job vacancies variations have a limited impact on employment, sectoral unions may obtain higher pays and the transition towards a more decentralized bargaining structure is comparatively less convenient. Conversely, a sector where the gap between the average and the lowest levels of productivity is small may be more favourable for two-tier systems.

The present paper does not indicate which bargaining regime is the best, at least from workers' point of view. Indeed, each of the three alternative Nash equilibria presented in the model is Pareto optimal and the transition from one equilibrium to the other is possible only if some structural characteristics of the labour market and the sector change. In this sense, this work may appear as a word of caution about the effectiveness of reforms aimed at increasing decentralization. It is not sufficient to give representatives of workers more bargaining possibilities at local level (for instance in terms of topics that can be negotiated at that tier), if they do not find it convenient to use them.

With its hefty increase in company level negotiations in the last 10 years, France constitutes an exception among countries with similar collective bargaining systems. A possible explanation for that could be that the reforms in 2016/2017 involved a compulsory division of topics among different levels of negotiations. Bargaining in most workplaces is more of a mandatory requirement rather than a voluntary option (Kahmann and Vincent (2022)).

In the simple framework considered in this paper heterogeneity comes from some unexplained factor, that “transforms" firms and workers, *ex ante* identical, into high and low productive matches. One could also introduce other forms of firm and skill differentials. The impact of the productivity distribution on bargaining decentralization and, in turn, earnings inequality, could be then studied more in detail.

As discussed in the Introduction, the present model aims to convey the main characteristics of industrial and labour relations in Southern Europe, where the favourability principle is still a sort of reference point. Abandoning this principle could alter the dynamics between sectoral and company-level negotiations and change the resulting equilibria of the game. These extensions are left for future research.

References

Aidt, T. S. and Z. Tzannatos (2008). Trade unions, Collective Bargaining and Macroeconomic Performance: a Review. *Industrial Relations Journal 39*(4), 258–295.

Armaroli, I. and C. Tomassetti (2022). Decentralized bargaining in Italy: CODE-BAR-project. *available at https://aias-hsi.uva.nl/en/projects-a-z/codebar/codebar.html*.

Arpaia, A., A. Kiss, and A. Turrini (2014). Is Unemployment Structural or Cyclical? Main Features of Job Matching in the EU after the Crisis. European Economy - Economic Papers 2008 - 2015 527, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Bechter, B., B. Brandl, and G. Meardi (2012). Sectors or Countries? Typologies and Levels of Analysis in Comparative Industrial Relations. *European Journal of Industrial Relations 18*(3), 185–202.

Berlingieri, G., P. Blanchenay, and C. Criscuolo (2017). The Great Divergence(s). *OECD Science, Technology and Industry Policy Papers*, No. 39, OECD Publishing, Paris.

Boeri, T. (2014). Two-Tier Bargaining. IZA Discussion Papers 8358, Institute of Labor Economics (IZA).

Boeri, T. (2015). Perverse effects of two-tier wage bargaining structures. *IZA World of Labor*, 101.

Braakmann, N. and B. Brandl (2016). The Efficacy of Hybrid Collective Bargaining Systems: An Analysis of the Impact of Collective Bargaining on Company Performance in Europe. MPRA Paper 70025, University Library of Munich, Germany.

Braun, S. (2011). Unionisation structures, productivity and firm performance: New insights from a heterogeneous firm model. *Labour Economics 18*(1), 120–129.

Calmfors, L. and J. Driffil (1988). Bargaining Structure, Corporatism, and Macroeconomic Performance. *Economic Policy 6*, 12 – 61.

Cardullo, G., M. Conti, and G. Sulis (2020). A model of unions, two-tier bargaining and capital investment. *Labour Economics 67*(C).

Carrieri, M., M. C. Ambra, and A. Ciarini (2020). The “Resistible" Rise of Decentralised Bargaining: A Cross-Country and Inter-Sectoral Comparison. In *Multi-employer Bargaining under pressure: Decentralisation Trends in Five European countries*. Leonardi, S. and Pedersini, R. editors, European Trade Union Institute (ETUI).

CCOO (2022). Balance Negociación Colectiva. *Confederación Sindical de Comisiones Obreras, Madrid; available online at https://www.ccoo.es/*.

CNEL (2022). XXIV Rapporto del CNEL sul Mercato del lavoro e la contrattazione collettiva. *Consiglio Nazionale dell'Economia e del Lavoro, Rome; available online at https://www.cnel.it/*.

Dahl, C. M., D. le Maire, and J. R. Munch (2013). Wage dispersion and decentralization of wage bargaining. *Journal of Labor Economics 31*(3), 501–533.

Damiani, M. and A. Ricci (2014). Decentralised bargaining and performance-related pay: evidence from a panel of Italian firms. *International Journal of Manpower 35*(7), 1038–1058.

Devicienti, F., A. Manello, and D. Vannoni (2017). Technical efficiency, unions and decentralized labor contracts. *European Journal of Operational Research 260*(3), 1129–1141.

DGERT (2022). Relatório sobre regulamentação coletiva de trabalho publicada no ano de 2022 . *Direção-Geral do Emprego e das Relações de Trabalho, Lisbon; available online at https://www.dgert.gov.pt/*.

DGT (2023). La négociation collective en 2022. *Direction générale du Travail du ministère du Travail : Paris; available online at https://travail-emploi.gouv.fr/*.

EC (2012). Labour Market Developments in Europe 2012. European Commission, Publication Office of the European Union, Luxembourg, European Economy 5/12.

Erhel, C. (2021). Reforms and new challenges for work and employment in France: Social dialogue under pressure. In *The New World of Work: Challenges and opportunities for social partners and labour institutions*. Daniel Vaughan-Whitehead; Youcef Ghellab; Rafael Muñoz de Bustillo Llorente editors, Edward Edgar Publishing; International Labour Organization.

Eurofound (2023). Living and working in Europe 2022. *Publications Office of the European Union, Luxembourg.*

Garnero, A. (2021). The impact of collective bargaining on employment and wage inequality: Evidence from a new taxonomy of bargaining systems. *European Journal of Industrial Relations 27*, 185–202.

Garnero, A., F. Rycx, and I. Terraz (2020). Productivity and Wage Effects of Firm-Level Collective Agreements: Evidence from Belgian Linked Panel Data. *British Journal of Industrial Relations 58*(4), 936–972.

Hijzen, A., P. S. Martins, and J. Parlevliet (2019). Frontal assault versus incremental change: A comparison of collective bargaining in Portugal and the Netherlands. *IZA Journal of Labor Policy 9*(1), 1–26.

INAPP (2023). Lavoro, Formazione, Welfare. Un percorso di Crescita Accidentato. *Istituto Nazionale per l'analisi delle politiche pubbliche : Rome; availabel online at https://www.inapp.gov.it/pubblicazioni/rapporto*.

Kahmann, M. and C. Vincent (2022). Decentralized bargaining in France: CODE-BAR-project. *available at https://aias-hsi.uva.nl/en/projects-a-z/codebar/codebar.html*.

Leonardi, S. and R. Pedersini (2020a). Breaking through the crisis with decentralisation? Collective bargaining in the EU aft er the great recession. In *Multi-employer Bargaining under pressure: Decentralisation Trends in Five European countries*. Leonardi, S. and Pedersini, R. editors, European Trade Union Institute (ETUI).

Leonardi, S. and R. Pedersini (2020b). *Multi-employer Bargaining under pressure: Decentralisation Trends in Five European countries.* European Trade Union Institute (ETUI).

Marginson, P. (2015). Coordinated bargaining in Europe: From incremental corrosion to frontal assault? *European Journal of Industrial Relations 21*(2), 97–114.

Mortensen, D. (2005). *Wage Dispersion: Why Are Similar Workers Paid Differently?* Cambridge, Mass: MIT Press.

Moscarini, G. (2005). Job matching and the wage distribution. *Econometrica 73*(2), 481–516.

Muñoz Ruiz, A., N. R. Martín, and C. Vincent (2023). Interplay between State and Collective Bargaining , Comparing France and Spain. In *Pathways in Decentralised Collective Bargaining in Europe*. F. Tros editor, Amsterdam University Press: Cambridge, UK.

Müller, T., K. Vandaele, and J. Waddington (2019). *Collective bargaining in Europe: Towards an endgame. Volume II.* European Trade Union Institute (ETUI).

OECD (2018). The role of collective bargaining systems for good labour market performance. *OECD: Employment Outlook 2018, OECD Publishing, Paris.*

OECD/AIAS/ICTWSS (2023). Core Variables. *OECD: Paris; available online at https://www.oecd.org/employment/ictwss-database.htm.*

Petrongolo, B. and C. Pissarides (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature 39*, 716–741.

Pissarides, C. (2000). *Equilibrium Unemployment Theory.* 2nd edition, Cambridge, Mass: MIT Press.

Rehfeldt, U. and C. Vincent (2020). The decentralisation of collective bargaining in France: an escalating process. In *Multi-employer Bargaining under pressure: Decentralisation Trends in Five European countries*. Leonardi, S. and Pedersini, R. editors, European Trade Union Institute (ETUI).

Traxler, F. and B. Brandl (2012). Collective bargaining, inter-sectoral heterogeneity and competitiveness: A cross-national comparison of macroeconomic performance. *British Journal of Industrial Relations 50*(1), 73–98.

Tros, F. (2023a). Decentralisation of Collective Bargaining : Comparing Institutional Change and Company Practices in Europe. In *Pathways in Decentralised Collective Bargaining in Europe*. F. Tros editor, Amsterdam University Press: Cambridge, UK.

Tros, F. (2023b). *Pathways in Decentralised Collective Bargaining in Europe*. Amsterdam University Press: Cambridge, UK.

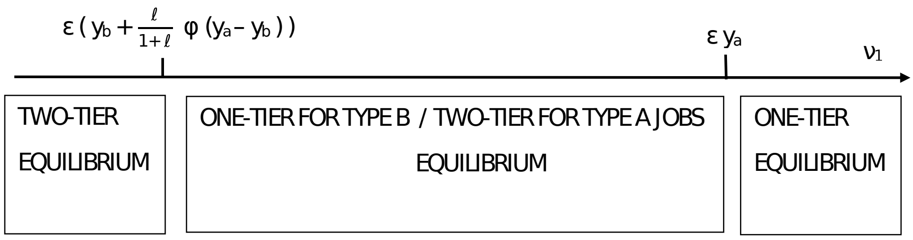

Figure 3: Lemma 1

Appendix 1: Proof of Proposition 1

I divide the proof in two steps. First, I present and prove a Lemma that presents the possible Nash equilibria when ν_1 is exogenous. Then I consider the expression for ν_1 in equation (15) to prove the results in Proposition 1.

Nash Equilibria for ν_1 given

Lemma 1

1. *If and only if $\nu_1 > \epsilon\, y_a$, the unique Nash equilibrium of the game implies all workers opt for a one-tier wage setting (scenario 1).*

2. *If and only if $\epsilon \cdot \big(y_b + \frac{\ell}{1+\ell}\,\phi(y_a - y_b)\big) < \nu_1 < \epsilon\, y_a$, the unique Nash equilibrium of the game implies workers in type a jobs opt for a two-tier mechanism whereas workers in type b jobs choose a one-tier wage setting (scenario 3).*

3. *If and only if $\nu_1 < \epsilon \cdot \big(y_b + \frac{\ell}{1+\ell}\,\phi(y_a - y_b)\big)$, the unique Nash equilibrium of the game implies all workers opt for a two-tier wage setting (scenario 2).*

Figure 3 illustrates the different outcomes of the game. To prove Lemma 1, I focus on the best response strategies of both players (workers employed in type a jobs and those employed in type b jobs) for the normal form game illustrated in Figure 1. I get the following results:

1. If workers in type b jobs choose one-tier, workers in type a jobs choose one-tier if and only if $w_1 > w_{3,a}$. Using the wage equations in (15) and (26) and doing some

algebra, I get that

$$w_1 > w_{3,a} \iff \nu_1 > \epsilon y_a \tag{32}$$

2. If workers in type b jobs choose two-tier, workers type a jobs choose one-tier if and only if $w_{4,a} > w_{2,a}$. Using the wage equations in (22) and (30) and doing some algebra, I get that:

$$w_{4,a} > w_{2,a} \iff \nu_1 - \epsilon y_a + \ell(\nu_1 - \epsilon\bar{y}) > 0 \tag{33}$$

3. If workers in type a jobs choose one-tier, workers in type b jobs choose one-tier if and only if $w_1 > w_{4,b}$. Using the wage equations in (15) and (31) and doing some algebra, I get that:

$$w_1 > w_{4,b} \iff \nu_1 > \epsilon y_b \tag{34}$$

4. If workers in type a jobs choose two-tier, workers in type b jobs choose one-tier if and only if $w_{3,b} > w_{2,b}$. Using the wage equations in (23) and (27) and doing some algebra, I get that:

$$w_{3,b} > w_{2,b} \iff \nu_1 - \epsilon y_b + \ell(\nu_1 - \epsilon\bar{y}) > 0 \tag{35}$$

Consider the inequalities at the RHS of the double arrow in (32), (33), (34), and (35). It is trivial to notice that they can be ordered as follows:

$$\nu_1 > \epsilon y_a \Rightarrow \nu_1 - \epsilon y_a + \ell(\nu_1 - \epsilon\bar{y}) > 0 \Rightarrow \nu_1 - \epsilon y_b + \ell(\nu_1 - \epsilon\bar{y}) > 0 \Rightarrow \nu_1 > \epsilon y_b$$

So, using this ordering and the necessary and sufficient conditions in (32), (33), (34), and (35), I are able to distinguish five alternative scenarios that depend on the value of ν_1:

- CASE 1: $\nu_1 > \epsilon y_a$. This inequality is the necessary and sufficient condition for a unique Nash equilibrium in which all workers choose one-tier bargaining.
- CASE 2: $\nu_1 < \epsilon y_a$ and $\nu_1 - \epsilon y_a + \ell(\nu_1 - \epsilon\bar{y}) > 0$. These two inequality imply there

exists a unique Nash equilibrium in which workers in type a jobs opt for two-tier bargaining and those in type b jobs choose one-tier bargaining.

- CASE 3: $\nu_1 - \epsilon y_a + \ell(\nu_1 - \epsilon\bar{y}) < 0$ and $\nu_1 - \epsilon y_b + \ell(\nu_1 - \epsilon\bar{y}) > 0$. These two inequalities imply there exists a unique Nash equilibrium in which workers in type a jobs opt for two-tier bargaining and those in type b jobs choose one-tier bargaining.

- CASE 4: $\nu_1 - \epsilon y_b + \ell(\nu_1 - \epsilon\bar{y}) < 0$ and $\nu_1 > \epsilon y_b$. These two inequalities imply there exists a unique Nash equilibrium in which all workers choose two-tier bargaining.

- CASE 5: $\nu_1 < \epsilon y_b$. This inequality imply there exists a unique Nash equilibrium in which all workers choose two-tier bargaining.

Putting together CASES 2 and 3, and 4 and 5 one gets:

$$\nu_1 > \epsilon y_a \iff \text{one-tier for all} \tag{36}$$

$$\nu_1 < \epsilon y_a \text{ and } \nu_1 - \epsilon y_b + \ell(\nu_1 - \epsilon\bar{y}) > 0 \iff \text{one-tier for } b \text{ and two-tier for } a \tag{37}$$

$$\nu_1 - \epsilon y_b + \ell(\nu_1 - \epsilon\bar{y}) < 0 \iff \text{two-tier for all} \tag{38}$$

Expressing the last two inequalities in terms of ν_1, I get the results in Lemma 1.

Proof of Proposition 1

Inserting the expression for ν_1 from equation (15) into the inequality (36), one gets:

$$\beta\,\eta\,\frac{\delta + f(\theta)}{\delta + \eta f(\theta)}\bar{y} + \frac{1-\beta}{1+\ell}z \;>\; \epsilon y_a \tag{39}$$

This is the necessary and sufficient condition for the existence of a one-tier for all equilibrium. It is easy to see that a sufficient condition for this inequality to be respected is

$$\beta\,\eta\,\bar{y} \;>\; \epsilon y_a \tag{40}$$

Rearranging, I get the same sufficient condition presented in point 1 of Proposition 1.

Consider the inequality (38). Again, inserting the expression for ν_1 from equation (15), I obtain:

$$\beta\,(1+\ell)\,\eta\,\frac{\delta+f(\theta)}{\delta+\eta f(\theta)}\bar{y}\;+\;(1-\beta)z\;-\;\epsilon\ell\bar{y}\;-\;\epsilon y_b\;<\;0 \tag{41}$$

Notice that a sufficient condition for this inequality to be fulfilled is:

$$\begin{aligned}
&\beta\,(1+\ell)\bar{y}\;+\;(1-\beta)z\;-\;\epsilon\ell\bar{y}\;-\;\epsilon y_b\;<\;0\;\Longleftrightarrow\\
&\epsilon\;>\;\frac{\beta(1+\ell)\bar{y}\;+\;(1-\beta)z}{\ell\bar{y}\;+\;y_b}\;\Longleftrightarrow\\
&\epsilon\;>\;\frac{\beta(1+\ell)\;+\;(1-\beta)\frac{z}{\bar{y}}}{\ell\;+\;\frac{y_b}{\bar{y}}}
\end{aligned} \tag{42}$$

This is the condition presented in point 2 of Proposition 1.

From Lemma 1 and Figure 3, we know that $\epsilon\cdot\big(y_b+\frac{\ell}{1+\ell}\,\phi(y_a-y_b)\big)\;<\;\nu_1\;<\;\epsilon\,y_a$ is the necessary and sufficient condition for a Nash equilibrium in which workers in type a jobs opt for a two-tier mechanism whereas workers in type b jobs choose a one-tier wage setting (scenario 3). Using the expression for ν_1 from equation (15), this condition can wee written as follows:

$$\begin{cases}
\beta\,\eta\,\frac{\delta+f(\theta)}{\delta+\eta f(\theta)}\bar{y}\;+\;\frac{1-\beta}{1+\ell}z\;<\;\epsilon y_a\\
\beta\,(1+\ell)\,\eta\,\frac{\delta+f(\theta)}{\delta+\eta f(\theta)}\bar{y}\;+\;(1-\beta)z\;-\;\epsilon\ell\bar{y}\;-\;\epsilon y_b\;<\;0
\end{cases} \tag{43}$$

Sufficient conditions for both inequalities in 43 to be satisfied are:

$$\begin{cases}
\beta\,\bar{y}\;+\;\frac{1-\beta}{1+\ell}z\;<\;\epsilon y_a\\
\beta\,(1+\ell)\,\eta\,\bar{y}\;+\;(1-\beta)z\;-\;\epsilon\ell\bar{y}\;-\;\epsilon y_b\;<\;0
\end{cases} \tag{44}$$

Expressing in terms of ϵ and rearranging, I have:

$$\begin{cases}
\epsilon\;>\;\beta\frac{\bar{y}}{y_a}\;+\;\frac{1-\beta}{1+\ell}\frac{z}{y_a}\\
\epsilon\;<\;\frac{\beta\eta(1+\ell)\;+\;(1-\beta)\frac{z}{\bar{y}}}{\ell\;+\;\frac{y_b}{\bar{y}}}
\end{cases} \tag{45}$$

If these two inequalities are respected, there is a unique Nash equilibrium of the game presented in Figure (1) in which workers in type a jobs opt for a two-tier mechanism whereas workers in type b jobs choose a one-tier wage setting (scenario 3). It is easy to see that the both terms at the RHS of the system (45) lie in the interval $\epsilon \in \left[\beta\eta\frac{\bar{y}}{y_a}, \frac{\beta(1+\ell)+(1-\beta)\frac{z}{\bar{y}}}{\ell+\frac{y_b}{\bar{y}}}\right]$ (see Figure (2)). So scenario 3 could be a possible equilibrium of the game. However, I am not able to ascertain whether the sum of the terms at the RHS in the first inequality in (45) is lower than the term at the RHS of the second inequality in (45). If this were the case, then the scenario 3 would be the only possible equilibrium for values of ϵ that satisfy system (45). Otherwise, I can only claim that in the interval $\epsilon \in \left[\beta\eta\frac{\bar{y}}{y_a}, \frac{\beta(1+\ell)+(1-\beta)\frac{z}{\bar{y}}}{\ell+\frac{y_b}{\bar{y}}}\right]$ one of three alternative equilibria may arise.

Appendix 2: Proof of Proposition 2

One-tier bargaining for all equilibrium

Recall from point 1 of Lemma 1 that this equilibrium occurs if and only if $\nu_1 > \epsilon y_a$. Consider first workers employed in type a jobs. Their wage in this scenario is equal to w_1. From equations (32) and (33) we know that $w_1 > w_{3,a}$ and that $w_{4,a} > w_{2,a}$.
I need to compare w_1 with $w_{4,a}$ and $w_{2,a}$. Using equations (22) and (30) and doing some algebra, it is easy to see that w_1 is always lower than $w_{2,a}$ and $w_1 > w_{4,a}$ if and only if $\nu_1 < \epsilon y_b$. This last inequality is never verified in a one-tier for all equilibrium (see point 1 of Lemma 1), so $w_1 < w_{4,a}$. Using equations (22) and (30) I also get that $w_{4,a} > w_{2,a}$ if and only if $\nu_1 > \epsilon \cdot \left(y_a - \frac{\ell}{1+\ell}(1-\phi)(y_a - y_b)\right)$. This last inequality is always verified in a one-tier for all equilibrium (see point 1 of Lemma 1), so $w_{4,a} > w_{2,a}$. So, in a one-tier bargaining equilibrium the ranking for workers employed in type a jobs is the following: $w_{4,a} > w_{2,a} > w_1 > w_{3,a}$.
Consider now workers employed in type b jobs, whose wage is equal to w_1. From equation (34) we know that in this equilibrium $w_1 > w_{4,b}$ and from equation (35) that $w_{3,b} > w_{2,b}$.
I need to compare w_1 with $w_{3,b}$ and $w_{2,b}$. Using equation (27) and doing some algebra, it is easy to see that $w_1 > w_{3,b}$ if and only if $\nu_1 < \epsilon y_a$: this last inequality is never fulfilled in a one-tier for all equilibrium (see point 1 of Lemma 1). Moreover, from equation (23) I get w_1 is always greater than $w_{2,b}$. Using equations (23) and (31) I also get that

$w_{2,b} > w_{4,b}$ if and only if $\nu_1 > \epsilon \cdot \big(y_a - \frac{\ell}{1+\ell}(1-\phi)(y_a - y_b)\big)$. This last inequality is always verified in a one-tier for all equilibrium (see point 1 of Lemma 1), so $w_{2,b} > w_{4,b}$. So, in a one-tier bargaining equilibrium the ranking for workers employed in type b jobs is the following: $w_{3,b} > w_1 > w_{2,b} > w_{4,b}$. Inspecting this ranking and the one concerning workers in type a jobs is also trivial to get that the equilibrium is Pareto optimal .

One-tier for workers in type b two-tier for workers in type a jobs equilibrium

From point 2 of Lemma 1 this equilibrium occurs if and only if $\epsilon \cdot \big(y_b + \frac{\ell}{1+\ell}\phi(y_a - y_b)\big) < \nu_1 < \epsilon\, y_a$.

Consider workers employed in type a jobs, whose wage in this equilibrium is equal to $w_{3,a}$. We know that $w_{3,a} > w_1$ (from equation (32)) and that $w_{4,a} > w_{2,a}$ (from equation (33)).

I need to compare $w_{3,a}$ with $w_{4,a}$ and $w_{2,a}$. Using equations (22) and (30) and doing some algebra, I get that $w_{3,a} > w_{2,a}$ if and only if $\epsilon \cdot \big(y_b + \frac{\ell}{1+\ell}\phi(y_a - y_b)\big) > \nu_1$: this last inequality is never verified under this equilibrium (see point 2 of Lemma 1). So $w_{3,a} < w_{2,a}$. So, the ranking for workers employed in type a jobs is $w_{4,a} > w_{2,a} > w_{3,a} > w_1$.

Consider now workers employed in type b jobs, whose wage is equal to $w_{3,b}$. From equation (34) we know that in this equilibrium $w_1 > w_{4,b}$ and from equation (35) that $w_{3,b} > w_{2,b}$. I need to compare $w_{3,b}$ with w_1 and $w_{4,b}$. Using equations (27) and (31) and doing some algebra, it is easy to see that $w_1 > w_{3,b}$ if and only if $\nu_1 < \epsilon y_a$: this last inequality is always verified under this equilibrium (see point 2 of Lemma 1). Moreover, $w_{2,b} > w_{4,b}$ if and only if $\nu_1 > \epsilon \cdot \big(y_a - \frac{\ell}{1+\ell}(1-\phi)(y_a - y_b)\big)$. This last inequality is always satisfied under such equilibrium (see point 2 of Lemma 1). So $w_{2,b} > w_{4,b}$

So, the ranking for workers employed in type b jobs is $w_1 > w_{3,b} > w_{2,b} > w_{4,b}$. Comparing the rankings for workers in type a and b jobs it is easy to see that the equilibrium is Pareto optimal .

Two-tier bargaining for all equilibrium

From point 3 of Lemma 1, we know that this equilibrium occurs if and only if $\nu_1 < \epsilon \cdot \big(y_b + \frac{\ell}{1+\ell}\phi(y_a - y_b)\big)$.

Consider workers employed in type a jobs, whose wage is equal to $w_{2,a}$. In this equilibrium we know that $w_{3,a} > w_1$ (from equation (32)) and $w_{2,a} > w_{4,a}$ (from equation (33)).

I need to compare $w_{2,a}$ with w_1 and $w_{3,a}$. Using equations (22) and (26) and doing some algebra, it is easy to see that $w_{2,a}$ is always greater than w_1 and $w_{3,a} > w_{2,a}$ if and only if $\epsilon \cdot \big(y_b + \frac{\ell}{1+\ell}\phi(y_a - y_b)\big) > \nu_1$: this last inequality is always verified under this equilibrium (see point 3 of Lemma 1). Moreover I also have that $w_1 > w_{4,a}$ if and only if $\nu_1 < \epsilon y_b$: this last inequality is always verified under this equilibrium (see point 3 of Lemma 1). So, in this equilibrium the ranking for workers employed in type a jobs is $w_{3,a} > w_{2,a} > w_1 > w_{4,a}$.

Consider now workers employed in type b jobs, whose wage is equal to $w_{2,b}$. From equation (35) we know that in this equilibrium $w_{3,b} < w_{2,b}$.

I need to compare $w_{2,b}$ with w_1 and $w_{4,b}$. Using equations (23) and (31) and doing some algebra, it is easy to see that w_1 is always greater than $w_{2,b}$. In addition, $w_{2,b} > w_{4,b}$ if and only if $\nu_1 - \epsilon y_a + \ell(\nu_1 - \epsilon\bar{y}) > 0$: this last inequality is never satisfied in this equilibrium (see point 3 of Lemma 1) .

So, under this equilibrium the ranking for workers employed in type b jobs can be either $w_1 > w_{4,b} > w_{2,b} > w_{3,b}$ or $w_{4,b} > w_1 > w_{2,b} > w_{3,b}$. Comparing the rankings for workers in type a with the two alternative rankings for workers in type b jobs it is easy to see that in neither case a Pareto superior outcome is possible.