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ABSTRACT

Transmissible Diseases, Vaccination and Inequality

We build a Susceptible-Infected-Vaccinated Economic two-sector growth model to study the evolution of inequality in an economy with two groups of workers, who are differently exposed to a transmissible disease. We show that the economy can lead to various scenarios in the long run, which range from a disease-free economy to a scenario in which only the most exposed group suffers from the virus. Our numerical exercises show that the question of how a transmissible disease affects long-run inequality depends on the economic variable we use to build our inequality measure, on the infectiousness of the disease, and on whether we address individual or group measures of inequality. Under our calibration, if the share of vaccinated trespasses the 24%, then the effect of the disease on inequality in capital assets is not monotone in the exposure rate.

JEL Classification: C6, I140

Keywords: inequality, transmissible disease, vaccination

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1 Introduction

On 11 March 2020, the World Health Organization declared the Coronavirus Disease 2019 (COVID-19) to be a pandemic. The impact of the pandemic is still ongoing in 2023. As of 4 September 2023, there have been a total of 694,893,289 confirmed cases, and 6,912,384 deaths reported worldwide.[1] Governments and policymakers around the world have imposed several restrictions to manage the COVID-19 pandemic, that have temporarily succeeded in slowing it down (Chinazzi et al., 2020; Ferguson et. al., 2020; Hsiang et al., 2020), but have harmed both the society and the economy as any other epidemic (see Atkeson, 2020; Boucekkine et al., 2009; Coibion et al., 2020; Craighead et al., 2020). Vaccination emerged as the most efficient tool to fight the COVID-19 pandemic and a number of vaccines were developed in a short period to diminish the damages and save lives. We investigate how a pandemic can generate and magnify economic inequality when the economy is made of two groups of individuals who are differently exposed to a transmissible disease.

In this paper, we construct a two-sector growth model to describe an economy where one of the production sectors requires only labor and the other utilizes both physical capital and labor. The labor market will divide the population into two groups by, differentiating individuals by the sector they work in. This economy suffers from an epidemic, and the two groups differ in their exposition of the disease, which can generate different paths for the evolution of the disease and can result in magnified economic inequality among workers. The model is made of two connected blocs: the population bloc and the economic bloc. In the population bloc, the evolution of the disease is modeled as a two-group susceptible-infected-vaccinated (SIV) model, similar to the well-known Susceptible-Infected model with two novelties. First, individuals can get vaccinated and obtain protection against the disease. We solve this SIV model and use the result as an input to the economic bloc. The second novelty is that population is made of two groups, whose unique difference is their exposure to the disease. Obviously, the group that is most exposed will suffer more from the disease, and will work fewer hours as a consequence. It is assumed that the vaccination rate is identical for both groups, to avoid any unjustified judgement regarding vaccination. After obtaining the optimal equilibrium paths for the population and the economy, our analysis focuses on long-term inequality, which can be magnified by the disease.

To underline the harmful effects of a persistent pandemic, no insurance mechanism has been introduced in the model to protect the diseased. Depending on the reproduction number of the disease, we prove that the disease can affect differently both groups in the long term, generating sustained economic inequality among the two groups of workers. If the reproduction number of the disease is smaller than one, then the disease disappears in the long run. Indeed,

[1] https://www.worldometers.info/coronavirus/countries

we prove that when the disease does not spread very rapidly, then the unique steady state is the disease-free economy, and that this steady state is stable. In this case, the unique long-term difference across groups may stem from productivity differences, if any.

Then, we obtain a series of results in the long term depending on the reproduction numbers of the overall economy and that of the two groups. We find that if one of the groups is (partially) infected in the long run, so will the other group. If the overall society reproduction number is above one, then we also prove that both groups can end up in an endemic steady state and suffer from the disease forever. Worth noting, under certain conditions, the most exposed group could even face long term indeterminacy complicating the task of any policymaker aiming at controlling the epidemic.

We introduce several measures of inequality between the two groups of individuals and between any two individuals, each in one group. We focus on inequality in revenue, and then on capital assets, consumption, shares of infected labour, and overall welfare. Regarding our results on inequality, we find that if we are looking at a technologically advanced economy whose production is intense in capital, then turning towards an even more capital-intensive economy reduces inequality. Otherwise, capital intensifying the economy will always increase inequality between the two groups of individuals.

Our model does not count with a government or a policy maker aiming at maximizing the welfare of an entire economy. To explore the effects of inequality on the rate of vaccination and exposure to inequality at the steady state, we resort to numerical exercises. One could expect that the more aggressive the disease, the higher inequality. The latter affirmation might not be entirely true and as we show, it depends on how we measure inequality. Besides, the role of the vaccination rate is crucial. For instance, under our calibration, if the share of vaccinated trespasses the 24%, then the effect of the disease on inequality in capital assets is not monotone in the exposure rate. Inequality is lowest when the disease is least aggressive and highest for the second (not the first) most aggressive rate under consideration. Indeed, an extremely virulent disease will harm all individuals so strongly that it will bring (some) equality.

This article is structured as follows. Section 2 presents the most relevant literature related to epidemics, vaccination, and inequality. Then, Section 3 presents our model developing in detail both its epidemic and economic bloc, focusing on the different long-run outcomes and their implications for inequality. Section 4 illustrates numerically the case in which both groups of workers reach an endemic steady state. Finally, Section 5 presents our conclusions.

2 Literature

Our paper contributes to three different strands of the literature, namely the relationship between inequality and epidemics, vaccination and the dynamics of the COVID-19 epidemic. We present in this section the most relevant and recent works in each of these three strands.

2.1 Inequality and epidemics

The impact of pandemics on income inequality is a relatively understudied area. However, there are some notable studies that have explored this relationship, primarily focusing on historical pandemics and more recent outbreaks. Alfani (2015) and Alfani and Ammannati (2017) investigates the impact of pandemics in Italian cities during the pre-industrial era, particularly focusing on the Black Death. They find that the Black Death played a significant role in reducing income inequality in Italy during that time.

Galletta and Giommoni (2020) examine the impact of the Great Influenza on income inequality in Italian towns. Contrary to the findings of the historical studies, Galletta and Giommoni conclude that in both the short and medium term, income inequality increased in the towns most affected by the pandemic. They attributed this increase to a decline in the income share held by the poor. Moreover, Furceri et al. (2020) look at the impact of more recent epidemics that occurred between 2000 and 2020, including SARS 2003, H1N1 2009, MERS 2012, Ebola 2014, and Zika 2016. Unlike historical pandemics, these events had a relatively weak impact on the economy and society. Furceri et al. (2020) found that these recent epidemics led to a rise in income inequality in the countries they studied.

It's important to note that the impact of pandemics on income inequality can vary significantly depending on various factors, including the severity of the pandemic, government responses, and the economic and social context of the affected regions. Additionally, different methodologies and data sources can lead to varying conclusions in empirical studies.

Our paper fits in the theoretical literature related to epidemics and inequality. The closest paper to ours is Boucekkine and Laffargue (2010). Using a three-period overlapping generations model with skilled and unskilled workers, they prove that while all epidemics under consideration have significantly different demographic implications in the medium run, they all imply a worsening in the short and medium run of economic performance and income distribution.

Extending the model of Boucekkine and Laffargue (2010), Vasilakis (2012) investigates the economic and demographic impacts of the HIV-AIDS epidemic in developing countries. The author allows different effects to act either separately or together, and he investigates the marginal efficiency of health expenditures on the survival probability of individuals and demographics. He finds that first, the HIV epidemic leads adults to increase their own health expenditure and to decrease that of their children. Second, the transmissibility of the HIV virus leads parents to spend more on their children's health than on their own. The HIV epidemic reduces the productivity of young adults, and this effect dominates the other two leading adults to have fewer children. Also as a consequence, the number of unskilled workers increases and it impoverishes the economy in the short and medium run.

Sayed and Peng (2021) examines empirically the effects of pandemics on income inequality. This study includes four countries in a period of 100 years (1915-2017), and it finds a significant effect of pandemics on the reduction of income inequality. The authors discuss whether their results are in line with the pandemic of COVID-19 and they conclude that the effects of COVID-19, on inequality remain unclear so far.

Overall, while there isn't an extensive body of research on this topic, the available studies suggest that the relationship between pandemics and income inequality is complex and can yield different outcomes depending on the specific circumstances of each pandemic.

2.2 Vaccination literature

Although there is a rapid development on the economic literature regarding vaccination, the literature is still thin to date. Let us describe some of the most salient works in this domain. Bargues and Dimitrova (2021) investigate the effect of COVID-19 vaccination on psychological well-being using information from a large-scale panel survey representative of the UK population. They find that vaccination increases psychological well-being. Auld and Toxvaerd et al. (2020) support that an increase in the vaccination rate tends to be followed by a reduction in protective behaviors. Atkeson (2021) builds a model of private and public behavior to mitigate the transmission of the COVID in the United States. He finds that both private and public behaviors are key to explaining the prevalence of COVID-19 and its persistence. Absent the development of technological solutions such as vaccines or life-saving therapeutics, additional public health interventions suffer from rapidly diminishing returns in improving long-run outcomes. In contrast, Atkeson (2021) shows that the rapid implementation of non-pharmaceutical interventions, in combination with the rapid development of technological solutions, could have saved nearly 300,000 lives. Finally, there are theoretical studies like Nganmeni et al. (2022), which focuses on vaccine allocation and identify a key point: achieving inclusivity alongside popularity often requires more vaccine doses compared to merely a popular allocation. Additionally, research shows that it is feasible to design decision-making rules that ensure an inclusive and popular vaccine allocation. In this scenario, any group with veto power must represent societal diversity, promoting fairness in vaccine distribution.

2.3 COVID-19 Pandemic

Finally, let us close this review of the literature with some recent works on the COVID-19 pandemic. Toxvaerd (2020) builds an epidemiological economic model in which susceptible individuals may engage in costly social distancing in order to avoid becoming infected. He shows that social distancing stops once herd immunity sets in, and that it actually extends the duration of the epidemic. Besides, once the epidemic curve becomes flatter the disease becomes more infectious, entailing worse health consequences.

In the same vein, Alvarez et al. (2021) investigate the role of a lockdown in controlling epidemics and examines the link between the economy and an epidemic. They show that in order to control the COVID-19 fatalities while minimizing the lockdown cost, a severe lockdown should be imposed a few weeks after the outbreak, cover at least 50% of the population and last less than 4 months. In a similar epidemiological setting, Acemoglu et al. (2020) consider that only older individuals should be in a lockdown. In like manner, Bosi et al. (2021) compute the optimal lockdown policy in a dynamic general equilibrium model where households are altruistic and care about the share of infected individuals. They find that a positive lockdown is always recommended beyond a critical level of altruism. In particular, their numerical simulations show that the optimal lockdown never trespasses 60% and that eradication is not always optimal. La Torre et al. (2021) focus on the determination of the optimal intensity and duration of the social distancing policy aiming to control the spread of an infectious disease in a simple macroeconomic epidemiological model. They find that social distancing decreases the spread of the disease while preserving economic activity. They also show that the characteristics of an epidemic can determine the optimal social distance, its duration, and its economic effects. Schmitt-Grohé et al. (2020) find that the relative impact of the COVID-19 virus was considerably greater on poorer communities compared to wealthier ones during the early stages of the pandemic in the United States. This underscores how pandemics can disproportionately affect low-income populations.

Moreover Gori et al. (2021) examine how a deadly epidemic and its control measures, such as social distancing and testing-tracing-isolation (TTI), impact capital accumulation and economic development across different time frames. It combines epidemiological with economic growth models, emphasizing the trade-off between safeguarding lives and the economy. Key findings include insights into long-term interactions between infection, demographics, and capital, the limited sustainability of extended social distancing, and the value of investing in TTI to prevent future lockdowns and economic disruptions.

The theoretical work of La Torre et al. (2022) discusses the implications of geographical heterogeneities on health and macroeconomic outcomes by using an epidemiological spatial economic model. They show that the existence of cross-regional effects actually determines the long-run outcome of the economy and the disease.

Let us close this section with Davine et al. (2022), which explores whether fiscal policies, particularly public debt, can mitigate the economic and health impacts of epidemics. They use a model that considers three key factors: the vulnerability of older individuals to diseases, increased infection rates in high pollution, and the role of public health spending and debt. Their findings suggest that well-designed fiscal policies can lead to a stable, disease-free state. Even when epidemics can't be fully eliminated, public debt and income transfers can reduce infections, boost capital, and raise GDP per capita, provided pollution levels are not too high. Additionally, they propose a household subsidy policy to address income and welfare disparities between healthy and infected individuals.

3 A theoretical setup to study economic inequality

In this section, we develop a theoretical setup to study how epidemics can generate and/or magnify inequality. First, we present a two-group vaccination variant of a Susceptible-Infected-Vaccinated model. Second, we introduce a two-sector growth model where one of the production sectors requires only labor and the other, physical capital and labor. Then, we will merge both epidemics and economics and focus on the long-term of this economy using the long-run results for the population previously obtained.

3.1 Epidemics. A two group SIV model

Let us assume an economy with a constant population, divided into two groups. Groups differ in their exposition to contagion. Since population is constant, we have that $\dot{N}_1(t) + \dot{N}_2(t) = n_1(t) + n_2(t) = 0$. $n_1(t)$ and $n_2(t)$ are exogeneously given. Within each group, individuals are either infected (I), vaccinated (V) or susceptible (S). Then at every moment in time $t \geq 0$

$$N_1(t) = S_1(t) + I_1(t) + V_1(t), \tag{1}$$

$$N_2(t) = S_2(t) + I_2(t) + V_2(t). \tag{2}$$

Our economy is exposed to a Susceptible-Infected-Susceptible disease in the sense that going through the disease does not confer immunity. At every moment in time, there are $\beta_1 S_1 \frac{I_1+I_2}{N}$ and $\beta_2 S_2 \frac{I_1+I_2}{N}$ new infections in Group 1 and 2, respectively, where β_1 and β_2 measure the exposure of the groups to the disease. We can assume without loss of generality that $\beta_1 > \beta_2$. For simplicity, we assume that apart from exposure, the disease affects equally both groups. The two population groups have access to the same vaccine and have identical vaccination rates. As already mentioned, their only difference stems from their exposition to the disease, inherent to their labor occupation.

Individuals recover from the disease at a rate $c \geq 0$. Regarding the vaccine, we assume that it reduces the infection by a rate $\sigma \in [0,1]$, so that $\sigma = 1$ means that the vaccine has no effect at all, and $\sigma = 0$ that the vaccine fully protects against the disease.[2]

Susceptible individuals are vaccinated at a rate ϕ, and the vaccine wears off at a rate $\theta \geq 0$. Gathering all these elements our model writes

$$\begin{aligned}
\dot{S}_i &= n_i - \beta_i S_i \frac{I_i + I_j}{N} - \phi S_i + cI_i + \theta V_i, && (3)\\
\dot{I}_i &= \beta_i (S_i + \sigma V_i) \frac{I_i + I_j}{N} - cI_i, && (4)\\
\dot{V}_i &= \phi S_i - \sigma \beta_i V_i \frac{I_i + I_j}{N} - \theta V_i. && (5)
\end{aligned}$$

for $i, j = 1, 2,\ i \neq j$.

Assume that the size of each group reaches a constant size, that is, $n_i = 0$. Since $N_i = S_i + I_i + V_i$, we can reduce the dimension of the system above, by writing $S_i = N_i - I_i - V_i$. Then the evolution of the disease within group i is described by

$$\begin{aligned}
\dot{I}_i &= \beta_i \left[N_i - I_i - (1-\sigma) V_i\right] \frac{I_i + I_j}{N} - cI_i, && (6)\\
\dot{V}_i &= \phi \left(N_i - I_i - V_i\right) - \sigma \beta_i V_i \frac{I_i + I_j}{N} - \theta V_i, && (7)
\end{aligned}$$

where $i, j = 1, 2,\ i \neq j$.

- **Long term epidemic equilibria under constant group size**

In this section, we study the long-term behavior of the disease. In particular, we will obtain the exact description of a disease-free economy, of an economy in which one group of the population is partially infected and the other free of disease; and finally, we obtain conditions for the existence of fully endemic steady states.

At a steady state $(\bar{I}_1, \bar{I}_2, \bar{V}_1, \bar{V}_2)$, $\dot{I}_i = 0$ and $\dot{V}_i = 0$, for $i = 1, 2$. From (7), imposing $\dot{V}_i = 0$:

$$\phi \left(N_i - \bar{I}_i\right) = \left[\phi + \theta + \sigma \beta_i \frac{\bar{I}_i + \bar{I}_j}{N}\right] \bar{V}_i,$$

and

$$\bar{V}_i = \frac{\phi \left(N_i - \bar{I}_i\right)}{\phi + \theta + \sigma \beta_i \frac{\bar{I}_i + \bar{I}_j}{N}}. \tag{8}$$

Then, plugging (8) into the equation resulting from $\dot{I}_i = 0$ using (6), we obtain

$$c\bar{I}_i = \beta_i (N_i - \bar{I}_i) \frac{\sigma \phi + \theta + \sigma \beta_i \frac{\bar{I}_i + \bar{I}_j}{N}}{\phi + \theta + \sigma \beta_i \frac{\bar{I}_i + \bar{I}_j}{N}} \frac{\bar{I}_i + \bar{I}_j}{N} \tag{9}$$

[2]For simplicity reasons we will not write the time index of the variables from now onwards.

and

$$c\bar{I}_j = \beta_j(N_j - \bar{I}_j)\frac{\sigma\phi + \theta + \sigma\beta_j\frac{\bar{I}_i+\bar{I}_j}{N}}{\phi + \theta + \sigma\beta_j\frac{\bar{I}_i+\bar{I}_j}{N}}\frac{\bar{I}_i + \bar{I}_j}{N}. \tag{10}$$

Note that the zero-infection steady state always exists:

$$\bar{I}_i = \bar{I}_j = 0, \tag{11}$$

$$\bar{V}_i = \frac{\phi}{\phi + \theta}N_i, \tag{12}$$

$$\bar{V}_j = \frac{\phi}{\phi + \theta}N_j. \tag{13}$$

We prove next that the disease-free equilibrium can be asymptotically stable

Proposition 1 (Disease-free Steady State). *Let* $R_0 = \frac{\beta_i N_i + \beta_j N_j}{cN}$. *If* $R_0 < 1$, *then*

$$\limsup_{t\to\infty} [I_i(t) + I_j(t)] = 0,$$

implying that the disease free steady state $(\bar{I}_1, \bar{I}_2) = (0, 0)$ *is asymptotically stable.*

Proof. See Appendix A. □

R_0 is an economy-wide measure of the pandemic reproduction, and it divides the number of total exposed individuals over the total number of recovered. If $R_0 < 1$, then the number of exposed individuals over cured individuals is small, and the disease does not advance fast enough. In this case, the disease disappears with time. Note that when the disease is not too aggressive, the vaccination rate ϕ is not decisive, the disease disappears by itself. However, as we will see next, when the disease is more aggressive or it spreads faster, vaccination becomes a key to understanding the long term of the disease and its stability.

Let us define the group equivalent of the pandemic reproduction number R_0^i. Let R_0^i be $R_0^i = \frac{\beta_i N_i}{cN}$ for $i = 1, 2$. Note that $R_0 = R_0^1 + R_0^2$. Although Group 1 is the most exposed to the disease, its reproduction number could be lower than that of Group 2 if its population was much smaller.

In what follows we explore different situations in which $R_0 > 1$, where Proposition 1 does not hold. We prove that there exist long-term situations in which the two groups suffer differently from the disease. First, Proposition 2 shows that a steady state in which Group 2 is free of the disease while Group 1 suffers from the disease in the long run is not feasible. Then, Propositions 3 and 4 provide conditions under which the two groups suffer from the disease in the long run.

Let us begin by proving that the steady state $(\bar{I}_1, \bar{I}_2) = (\bar{x}, 0)$ cannot exist. Substituting $\bar{I}_2 = 0$ into (9) for $i = 2$, and knowing that $\bar{I}_1 \neq 0$:

$$\beta_2 \left[N_2 - (1-\sigma)V_2\right] \frac{\bar{I}_1}{N} = 0.$$

which can only be true if and only if $\sigma < 1$ and

$$V_2 = \frac{1}{1-\sigma} N_2.$$

However, this cannot be true because it would mean that $V_2 > N_2$. We conclude with the following proposition:

Proposition 2. *If one of the groups is infected in the long run, so will be the other.*

We close this section by providing conditions for the existence of an endemic steady state:

Proposition 3. *If $R_0^i < \frac{(1-\sigma)\phi}{c\sigma}$ for both $i = 1, 2$ and $R_0 > 1$, then there exists exactly one endemic steady state.*

Proof. See Appendix B. □

We can break down Proposition 3's hypothesis to better understand the society it describes. Here, we can understand $\sigma\beta_i N_i$ as the share of individuals in group i that is exposed to the disease and who are protected by the vaccine, which protects them at a rate σ. Then, if $R_0^i < \frac{(1-\sigma)\phi}{c\sigma}$, it means that for each group, the number of these protected but exposed individuals is smaller than $(1-\sigma)\phi N$, the number of individuals in the economy who are actually vaccinated but for who the vaccine did not work. Since $R_0 > 1$, the disease is virulent enough and we know that this situation arises when the share of vaccinated but unprotected individuals is sufficiently large, in particular when $(1-\sigma)\phi > \frac{c\sigma}{2}$.

Obviously, under other conditions, we could seek the existence of multiple steady states, at least for one of the groups. For example,

Proposition 4. *If $R_0^1 > \frac{2}{\sigma} R_0^2 + \frac{c\sigma+\phi+\theta+(1-\sigma)\phi}{\sigma c}$ and $R_0^2 > \frac{1}{c\sigma}\left[(1-\sigma)\phi - \frac{N_2}{N_1}(\phi+\theta)\right]$, with $R_0 > 1$, then group 1 has two endemic steady states and group 2 only 1. Hence, there is indeterminacy regarding the final state of group 1.*

Proof. See Appendix C. □

In this scenario the disease is highly infectious, $R_0 > 1$, and both groups' reproduction numbers are sufficiently large. Here, Group 1 faces indeterminacy, its long-term situation is not determined. This situation could be undesirable and difficult to tackle for a policymaker who wishes to control the disease to improve overall welfare in society.

3.2 An economic growth model

This section presents an economy made of two sectors that produce different goods using two distinct production technologies. Let us start by describing the firms, followed by the lifetime welfare problem of individuals in each group of workers. Then we present the equilibrium of the overall economy and its long run. Finally, we conclude the section with an analysis of inequality putting together the dynamics of the epidemics and of the economy.

- **Firms**

There are two economic sectors, 1 and 2, which produce two different consumption goods. Sector 1 uses a linear technology and requires only labor as input. The second sector utilizes a constant return-to-scale technology and requires both physical capital and labor as inputs. Hence, labor is split into two groups depending on the firm on which they work. According to these assumptions, production in Sectors 1 and 2, Y_1 and Y_2, are described as

$$Y_1(t) = B_1 L_1(t), \tag{14}$$

$$Y_2(t) = B_2 K(t)^{\alpha} L_2(t)^{1-\alpha}, \tag{15}$$

where K stands for physical capital, L_1 and L_2 for labor in Sectors 1 and 2, respectively. B_1 and B_2 are positive constants and stand for each of the sectors' technology. Regarding Y_2, $\alpha \in (0, 1)$, that is, production is a continuous, increasing, and concave function of both K and L_2.

Assuming that the good produced in Sector 1 is the numeraire, each firm maximizes profits at every moment in time. We denote by p the relative price of good 2 in terms of the numeraire. Then the optimal unit salary for a worker in sector 1 is $w_1(t) = B_1$ and for a worker of sector 2, $w_2(t) = B_2(1-\alpha)K(t)^{\alpha}L_2(t)^{-\alpha}$. The optimal interest rate is $R(t) = B_2\alpha K(t)^{\alpha-1}L_2(t)^{1-\alpha}$.[3]

- **Households**

Population N is divided into two groups of sizes N_1 and N_2, and this division depends on the sector they work in. For simplicity, we assume that the total population does not grow. Apart from that, all households share the same preferences over the two consumption goods and have one unit of labor that they devote entirely to labor. Households maximize overall discounted welfare over an infinite time period. Households measure their utility via the same standard utility function u, which satisfies:

Assumption 1. *The utility function* $u(c_1, c_2) : \mathbb{R}^+ \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ *is a positive, increasing and concave function of* c_1 *and* c_2*.*

[3] Again for simplicity reasons, we remove the time index in the following.

The representative household in Group i solves the following problem:

$$\max_{\{c_1^i, c_2^i\}} \int_0^\infty u(c_1^i, c_2^i) N_i e^{-\rho t} dt, \tag{16}$$

where $\rho \geq 0$ is the time discount rate (also common to all households) and c_j^i is Group i's consumption per capita of good $j = 1, 2$. Each group has an endogenous size of N_i, and depending on the evolution of the pandemic, it will have an amount $I_i(t)$ of infected individuals at time t who cannot work. Note that we are assuming that all members of Group i consume the same amounts of both consumption goods, whether they are able to work or not. Households' choices are subject to the law of accumulation of the household's assets, A_i:

$$\dot{A}_i = rA_i + w_i(N_i - I_i) - \left(c_1^i + pc_2^i\right) N_i, \tag{17}$$

where the initial value of assets, $A_i(0)$ is known.

Solving the household problem (16) subject to (17), we obtain household's i Euler equations with respect to both consumption goods:

$$\frac{-u_1''(c_1^i, c_2^i)\dot{c}_1^i}{u_1'(c_1^i, c_2^i)} = r - \rho, \tag{18}$$

$$\frac{-u_2''(c_1^i, c_2^i)\dot{c}_2^i}{u_2'(c_1^i, c_2^i)} = r - \rho. \tag{19}$$

These imply that $\frac{-u_1''(c_1^i,c_2^i)\dot{c}_1^i}{u_1'(c_1^i,c_2^i)} = \frac{-u_2''(c_1^i,c_2^i)\dot{c}_2^i}{u_2'(c_1^i,c_2^i)}$. For practical matters we will be assuming later on that the utility function is $u(c_1, c_2) = \frac{c_1^\eta c_2^{1-\eta}}{\eta(1-\eta)}$, with $\eta \in \mathbb{R}$. With this functional form for u, the first order conditions also imply that

$$c_2^i = \frac{1-\eta}{\eta}\frac{1}{p}c_1^i. \tag{20}$$

We can define the equilibrium of the economy as follows.

Definition 1. *An equilibrium is a sequence of wages and interest rates* $\{w_1(t), w_2(t), R(t)\}_{t=0}^\infty$, *a sequence of labor demand, labor supply, infected individuals, optimal choices for consumption and household assets* $\{L_i(t), N_i(t), I_i(t), c_1^i(t), c_2^i(t), A_i(t)\}_{t=0}^\infty$ *for* $i = 1, 2$ *such that at every time* $t \geq 0$

i) The epidemics evolves according to (6) and (7);

ii) Labor market clears, that is, labor demand equals labor supply for each group of individuals, that is, healthy individuals: $L_i(t) = N_i(t) - I_i(t)$, *for* $i = 1, 2$. *Total population is constant,* $N = N_1(t) + N_2(t)$.

iii) Physical capital market clears, so that physical capital demand equals the sum of all household's assets: $K(t) = A_1(t) + A_2(t)$;

iv) Goods market clear, implying that supply equals demand for each of the two goods, i.e. $c_1^1(t) + c_1^2(t) = B_1 L_1$ and $c_2^1(t) + c_2^2(t) = B_2 K(t)^\alpha L_2(t)^{1-\alpha}$.

iv) Firms behave optimally and households are paid at their marginal productivity. Accordingly, each worker of group 1 receives a unit salary $w_1(t) = B_1$, and workers of group 2, $w_2(t) = B_2(1-\alpha)K(t)^\alpha L_2(t)^{-\alpha}$. Physical capital is also paid at its marginal productivity $R(t) = B_2 \alpha K(t)^{\alpha-1} L_2(t)^{1-\alpha}$ and $r(t) = R(t) - \delta$, where $\delta \geq 0$ is physical capital's depreciation rate;

v) Households also behave optimally implying that consumption of each of the two goods follows the optimal trajectory defined in (18) and (19).

vi) Individual's capital assets evolve in time according to (17).

- **Steady State**

Denoting by $\bar{x}$ the steady state of variable x, we define the steady state of our economy as a set of equilibrium values $(\bar{N}_1, \bar{N}_2, \bar{A}_1, \bar{A}_2, \bar{c}_1^1, \bar{c}_2^1, \bar{c}_1^2, \bar{c}_2^2)$ which is solution to (17), (18), (19) once we set $\dot{A}_i = 0$, $\dot{c}_1^i = 0$ and $\dot{c}_2^i = 0$, for $i = 1, 2$.

By (18), the steady-state value of the interest rate, $\bar{R}$, satisfies that $\bar{R} = \delta + \rho$ or equivalently that $r = \rho$. Substituting $\bar{R}$ using its equilibrium value, we obtain that

$$B_2 \alpha \bar{K}^{\alpha-1} \bar{L}_2^{1-\alpha} = \delta + \rho, \tag{21}$$

with $\bar{K} = \bar{A}_1 + \bar{A}_2$, and

$$\rho \bar{A}_1 + B_1 \left[\bar{N}_1 - \bar{I}_1(\bar{N}_1)\right] = c_1^1 \frac{1}{\sigma} \bar{N}_1, \tag{22}$$

$$\rho \bar{A}_2 + (1-\alpha) B_2 \left(\frac{B_2 \alpha}{\delta + \rho}\right)^{\frac{\alpha}{1-\alpha}} \left[N - \bar{N}_1 - \bar{I}_2(\bar{N}_1)\right] = c_1^2 \frac{1}{\sigma}(N - \bar{N}_1). \tag{23}$$

where $\bar{I}_1(\bar{N}_1)$ and $\bar{I}_2(\bar{N}_1)$ are given in (9) and (10), and are functions of $\bar{N}_1$. Working with the equilibrium conditions, we find that there exists an interior steady state where the number of workers in sector 1 is implicitly given by

$$B_1 \left[\bar{N}_1 - \bar{I}_1(\bar{N}_1)\right] = \frac{\eta p}{1-\eta} B_2 \left(\frac{\alpha B_2}{\delta + \rho}\right)^{\frac{\alpha}{1-\alpha}} \left[N - \bar{N}_1 - \bar{I}_2(\bar{N}_1)\right],$$

and from (22) and (23), using (41) and the relationship between the household consumption of both goods in (20), we obtain c_1^1:

$$c_1^1 \frac{N - 2\bar{N}_1}{\eta} = B_1(N - \bar{N}_1)\left(\frac{N - \bar{N}_1}{\eta} - 1\right) - \rho \bar{K} - (1-\alpha) B_2 \bar{K}^\alpha \left[N - \bar{N}_1 - I_2(\bar{N}_1)\right]^{1-\alpha}.$$

The number of workers in sector 1 at the disease-free steady state, $\bar{N}_1^f$ is:

$$\bar{N}_1^f = \frac{1}{1 + \frac{B_1}{B_2}\frac{1-\eta}{\eta p}\left(\frac{\rho+\delta}{\alpha B_2}\right)^{\frac{\alpha}{1-\alpha}}} N.$$

Worth noting, although the long-term size of labor is obviously different in the endemic and the disease-free steady states, the relative size of the economic sectors is identical in both cases.

- **Group and Individual measures of inequality at Equilibrium**

Let us define next some simple measures of inequality, at the individual level and between groups. First, we define two functions of revenue inequality. We denote by $Q_G(t)$ revenue inequality between groups defined as the ratio between the total revenue of Group 2 and that of Group 1. Similarly, individual inequality, $Q_i(t)$, is defined as the ratio between the unit wage in Sector 2 and the unit wage in Sector 1:

$$Q_G(t) = \frac{w_2(t)N_2(t)}{w_1(t)N_1(t)}, \tag{24}$$

$$Q_i(t) = \frac{w_2(t)}{w_1(t)}. \tag{25}$$

At the steady state, the levels of inequality are given by

$$\bar{Q}_G = \frac{\bar{w}_2\bar{N}_2}{\bar{w}_1\bar{N}_1} = (1-\alpha)\frac{1-\eta}{\eta p}\frac{\alpha B_2}{\delta+\rho}, \tag{26}$$

$$\bar{Q}_i = \frac{\bar{w}_2}{\bar{w}_1} = (1-\alpha)\frac{B_2}{B_1}\left(\frac{\alpha B_2}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}. \tag{27}$$

We can perform some comparative statics exercises that reveal the role of each parameter on inequality. We summarize our results in the following lemma:

Proposition 5. *Under the model assumptions, and using the long-term definitions in (26) and (27), we can prove that*

i) If B_1 increases, then $\bar{Q}_i$ increases while $\bar{Q}_G$ remains unchanged, i.e. $\frac{\partial \bar{Q}_i}{\partial B_1} > 0$ *and* $\frac{\partial \bar{Q}_G}{\partial B_1} > 0$.

ii) If η or p increase, then $\bar{Q}_G$ decreases although $\bar{Q}_i$ does not change, that is $\frac{\partial \bar{Q}_G}{\partial \eta} < 0$ *and* $\frac{\partial \bar{Q}_i}{\partial \eta} = 0$.

iii) If α increases, then the effect on inequality depends on the base value of α itself. Indeed, $\bar{Q}_G$ increases when $\alpha < 1/2$ and decreases otherwise. The reaction of $\bar{Q}_i$ depends on the relative level of technology in Sector 2. In particular:

iii.1) If technology is relative high, i.e. $B_2 > \max\{\frac{1}{\alpha}e^{\frac{(1-\alpha)^3}{\alpha}}, (\delta+\rho)\frac{\eta}{1-\eta}\frac{p}{\alpha(1-\alpha)}\}$*, then individual inequality decrease with* α*:*

$$\frac{\partial \bar{Q}_i}{\partial \alpha} = 0.$$

iii.2) If B_2 *takes an intermediate level,* $(\delta+\rho)\frac{\eta}{1-\eta}\frac{p}{\alpha(1-\alpha)} < B_2 \leq \frac{1}{\alpha}e^{\frac{(1-\alpha)^3}{\alpha}}$*, then* $\frac{\partial \bar{Q}_i}{\partial \alpha} < 0$.

Hence, from *iii)* we can conclude that increasing the role of capital in production in Sector 2 can lead to a decrease in both measures of inequality if production is already intense in capital ($\alpha > 1/2$) and the economy is sufficiently advanced technologically speaking. Otherwise, an increase in α will always increase inequality across groups. Despite the increase across groups, it can decrease inequality among individuals if technology in Sector 2 is not sufficiently advanced.

We can also study inequality when the epidemics vanish in the long term. Here, inequality is only triggered by economic factors.

Proposition 6 (The disease-free long term economy.)**.** *If* $R_0 < 1$*, then we know from Proposition 1 that the epidemic vanishes with time. Both long-term inequalities at the group level and at the individual level coincide with the levels of the economy under an epidemic.*

Although the result in Proposition 6 may be surprising at first, it is actually not. Note that both measures of inequality depend on (relative) wages and that physical capital per capita is constant in the long-run. This constant value is identical whether there is an epidemic or not. Nevertheless, the epidemics are certainly generating inequality regarding consumption and wealth accumulation. For this reason in the numerical exercises in Section 4 we will be studying how epidemics affect inequality in capital accumulation, consumption, the share of infected, labor (i.e. shares of individuals able to work), group size, capital assets holdings, and welfare.[4]

Our model does not count with a government or a policy maker aiming at maximizing the welfare of an entire economy. However, we can explore the effects of the vaccination rate and exposure to epidemics on the steady state and on inequality. Since our results for the endemic steady state are implicit, we will illustrate in the next section these mechanisms numerically. In our exercises, we let ϕ, the vaccination rate, take values between 0 and 1 and we show the partition of labor between the two production sectors at the steady state, the values for the share of infected individuals in each group, the stock of physical capital and inequality in capital for different values of β_1 when $\beta_2 = 0.15$.

[4]See Appendix H for the definitions.

4 Numerics: Labor division and inequality in the long-run

Our numerical analysis of the endemic steady state focuses on the UK.[5] The UK is one of the European countries which has suffered the most from COVID-19, the one with the most casualties in Europe, and one of the highest death rates (2,356 deaths per 1M of the population as of 1 March 2022). Indeed, by 1 March 2022 the UK had reported a total of 18,886,701 cases and 161,361 deaths.[6] One more reason to choose the UK, as already mentioned, is that it was also the first Western country to approve both the Pfizer and the AstraZeneca vaccines, adopting an "aggressive" approach and offering the first dose of the available vaccines to as many people as possible.

Our calibration is built on data from the National Statistics Bureau of the UK and on recent research on COVID-19. First note that the majority of the population got a dose of the Astrazeneca vaccine, whose efficiency is 74% according to (Lopez Bernal et al., 2021), and as it was published in the media[7]. Accordingly, we adapt this figure to an annual basis and we assume that $\sigma = 0.74$. To bear in mind for the upcoming comparisons, the vaccination rate in the UK is around 72%, so we could have fixed ϕ to $\phi = 0.72$. We will take another approach and let vary ϕ in $(0, 1)$ and study the evolution of labor repartition and inequality levels. Given the depreciation of vaccine efficacy, it is necessary to get a booster after 3 to 6 months,[8] which leads us to consider that the vaccine depreciation rate is $\theta = 0.5$.

Based on the UK government national statistics office, we consider that the group transmission rate β_2 and the recovery rate, c, are 0.15, and 0.25 respectively. Similar values have imposed by Vasilakis (2011). Then we will choose four different values for β_1. First, a value that is 10% lower than transmission in Sector 2; second, we let transmission be equally intense in both sectors; third, β_1 will be 0.1 points higher than β_2, that is 66% higher. Finally, in the fourth case, β_1 takes the extreme value of 0.35, that is, 133% higher than β_1.

The parameters related to the economic model are taken by Barro and Sala-i-Martin (1997), Kazuo (1996), Ortigueira et al. (1997), and Bond et al. (1996). All values are gathered in Table 1.

Figure 2 shows how labour and the share of infectives evolve with ϕ for the four levels of infectiveness in Sector 2 we will be considering. Broadly speaking, we observe that the size of Group 1 and its relative labor share in the economy increase with infectiveness in the sector. Given that the economy must satisfy consumers' demand for good 1, the higher the

[5] This is also the approach of Petropoulos and Makridakis (2020) and Nikolopoulos et al. (2020).

[6] https://www.worldometers.info/coronavirus/countries.

[7] https://www.reuters.com/business/healthcare-pharmaceuticals/astrazeneca-covid-19-vaccine-shows-74-efficacy-large-us-trial-2021-09-29/

[8] https://www.england.nhs.uk/2021/12/people-40-and-over-to-get-their-lifesaving-booster-jab-three-months-on-from-second-dose/

Table 1: Parameters values

σ	Vaccine coverage	0.75
θ	Vaccine depreciation	0.5
β_2	Group 2 exposure	0.15
c	Recovery rate	0.25
p	Relative price of good 2	2
B_1	Technology in Sector 1	1
B_2	Technology in Sector 2	1
δ	Capital depreciation	0.04
ρ	Time discount	0.015
α	Production parameter	1/3
η	Utility parameter	0.3
N	Population size	1

impact of the disease in Sector 1's labor, the more workers will be allocated to the sector. By construction, the share of infected in Sector 1 increases with β_1, but the share of infected in Sector 2 increases as well. This is also natural because the more aggressive is the disease in Sector 1, the more infected will be in the economy, see equation (4), so that in particular, the more infected workers will be in Sector 2. This result is in line with Boucekkine and Laffargue (2010) and Vasilakis (2012), where the productivity of workers decreases as the transmission rate increases. As one may expect, total capital and total welfare decrease with β_1 and increase with the vaccination rate.

Figure 2 also illustrates some somewhat expected results, namely, the more infected in Sector 1, the more infected overall so that the lower labor, production, and savings, entailing a decrease in the formation of capital and in consumption, which drives welfare down. The exact opposite is observed when the vaccination rate increases. That is, total capital and welfare increase as the vaccination rate in the economy increases because it increases labor in the overall economy and in each of the sectors.

Finally, Figure 3 shows the different individual and group measures of inequality as functions of the vaccination rate, ϕ. In particular, we focus here on four measures of inequality: inequality in capital, labor income, the number of infected in each sector, and in welfare. As expected, inequality in the impact of the disease on each of the groups increases with the rate of infectiveness in Sector 1. Worth noting, that individual inequality in labor revenue remains constant under changes in ϕ and/or β_1. That is, the economy adjusts in such a manner that in the long-term equilibrium, individuals' labor revenue is always identical. At the group level, we can say that broadly speaking, inequality decreases with effectiveness, because the disease drags down labor.

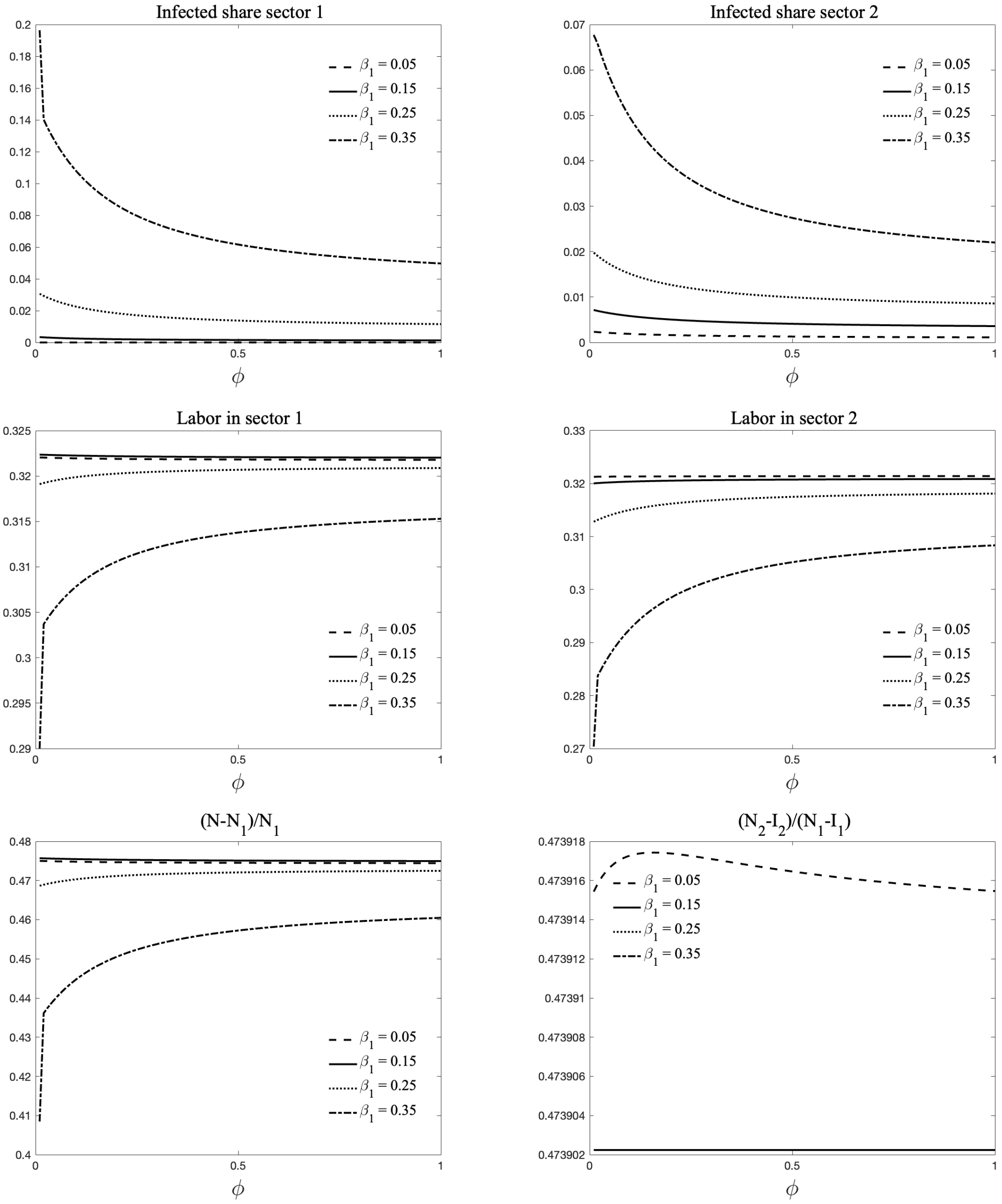

Figure 1: Shares of Infected, labor, sector size ratio and labor ratio as functions of ϕ.

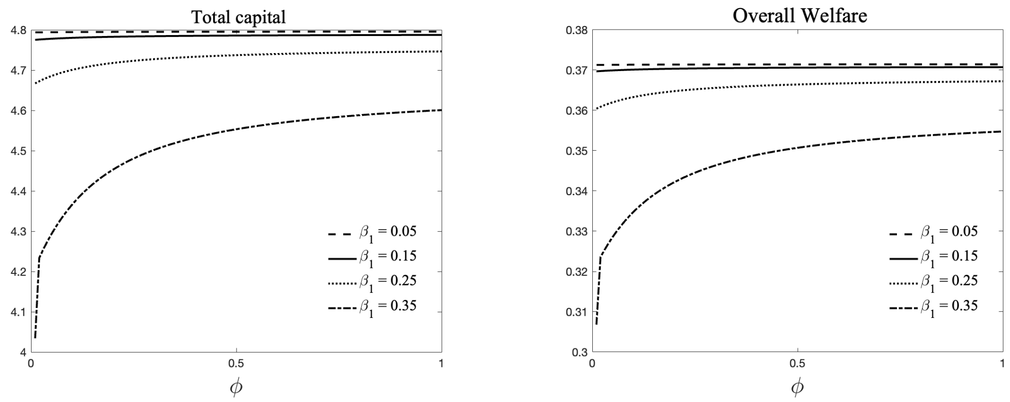

Figure 2: Capital and Overall Welfare as functions of ϕ.

Next, let us focus on inequality in capital assets. Generally speaking, we can say that inequality decreases with the vaccination rate for any level of effectiveness. However, our numerical exercises reveal some unexpected features. Indeed, while one might have expected to find that the more aggressive the disease, the higher the inequality, it is actually not true or not entirely. It is evident that both group and individual inequality in assets are at their minimum for the lowest value of β_1, that is $\beta_1 = 0.05$. Recall that at this level of infectiveness, there is almost no effect of the disease in labor. When β_1 increases, then we need to distinguish between group and individual inequality and then between high and low rates of vaccination. Let us begin with the analysis at the individual level. Figure 3 unearths the existence of a threshold value for ϕ, here $\hat{\phi} = 0.235$. Below $\hat{\phi}$, individual inequality in capital increases with the effectiveness of the vaccine, that is, the disease exacerbates inequality. However, beyond that vaccination rate, if the share of vaccinated trespasses the 24% of the total, then the effect of the disease is not monotone. Inequality still increases for moderate levels of infectiveness (from 0.15 to 0.25), but then we find out that inequality when $\beta_1 = 0.35$ is lower than when $\beta_1 = 0.25$, although still larger than for 0.15. That is when the disease becomes very aggressive and then the inequality decreases because there are more people infected, i.e. lower labor. The differences in revenue (and in investing potential) are blurred when revenue needs to be shared among relatively many and individuals become more alike. Our results quite change at the group level, i.e. when taking into account the number of people in each group. There exists a threshold value for the vaccination rate, $\tilde{\phi} = 0.11$, below which inequality in the most aggressive disease lies between the two other intermediate levels. As we saw at the individual level and simply put, more individuals become poorer when the disease is strong. At the group level, the inequality ranking is even reversed when the disease is the strongest, yielding the lowest inequality.[9]

[9]To complete the description of how infectiveness affects capital inequality, we have generated 3-dimensional

However, our exercises show that one should not rest in a unique measure of inequality. Indeed, when we look at inequality in welfare we find that inequality at the individual level increases with the disease infectiveness. At the group level, we observe that there exists a "baseline level" of inequality associated with the lowest value of β_1. When the disease gains strength from 0.05 to 0.15, inequality increases for all levels of the vaccination rate. As the disease becomes even more aggressive, group inequality starts decreasing (recall that it was increasing at the group level). Here, although each individual in Group 2 becomes more alike to each individual in Group 1, differences between the two groups increase because Group 2 becomes smaller with β_1 (see bottom-left graph in Figure 2).

Let us conclude this section regarding at the effect of a disease on the accumulation of capital in the economy and overall welfare in the long-run as functions of the vaccination rate. Figure 3 shows how the economy becomes richer and attains a higher welfare level as the vaccination rate increases, for all levels of β_1. Here, the conclusion is straightforward: the more aggressive the disease, the lower total capital and welfare.

5 Conclusions

This paper has built a Susceptible-Infected-Vaccinated Economic growth model, where population is split in two groups. Our final aim was to measure and study the evolution of inequality in an economy with two groups of workers, who are differently exposed to an epidemic. Here one could think of one of the groups made of occupations that require direct contact with consumers or patients, and the other group made of workers who can work remotely to some extent.

We develop our model in Section 3 in great detail, analyzing first the epidemic bloc and exploring various long-term outcomes. Indeed, we prove that our two-group economy can end up in different scenarios, which range from a disease-free economy to a scenario in which only the most exposed groups suffer from the virus. Then, we embed the epidemiological bloc into an economic growth model and define inequality between the groups, comparing their incomes. We complete our theoretical analysis with some numerical exercises which show that inequality increases with the share of vaccination, and it decreases with the exposure rate of the most exposed group since the economy becomes poorer. For future research, it would be interesting to develop a theoretical framework to combine the economic choices of agents and the disease dynamics to investigate inequality. After all, when there is an epidemic around, people typically modify their choices (e.g., change occupations; invest in health care or vaccination), which in turn impacts the progress of the disease. Taking into account such

graphs of inequality as a function of both ϕ and β that can be found in Appendix

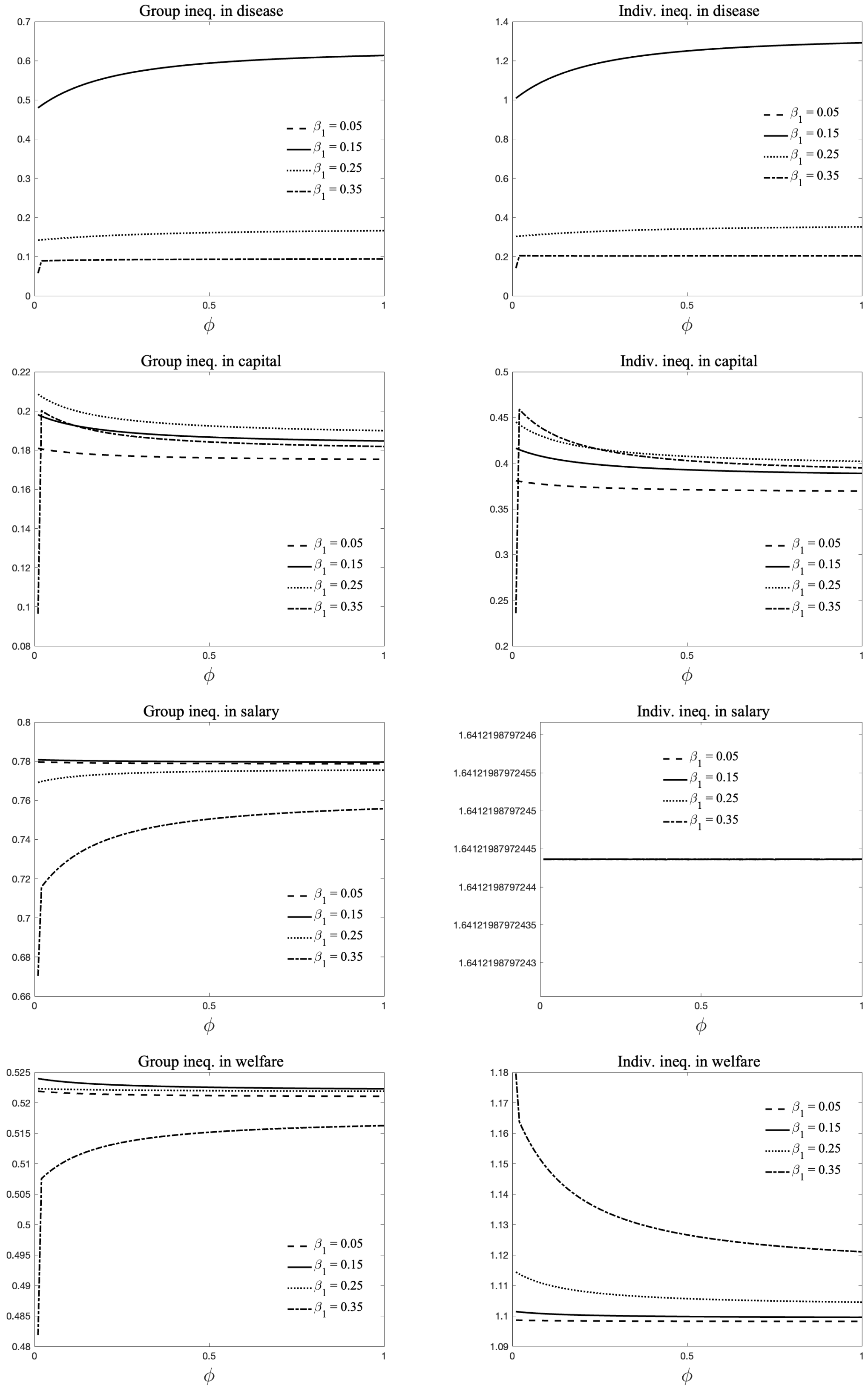

Figure 3: Individual and group measures of inequality as functions of the vaccination rate, ϕ

an interplay between the disease dynamics and endogenous economic decisions creates scope for rich and complex coevolution of the disease and economic dynamics resulting in long-run wage inequality. Finally, another extension of the current version of our model would be to model vaccination decisions and how they affect income distribution.

References

[1] Acemoglu, D., Chernozhukov, V., Werning, I., Whinston, M.D., (2020). A multi-risk SIR model with optimally targeted lockdown. In: NBER Working Paper 27102.

[2] Alfani G. (2015). Economic inequality in northwestern Italy: a long-term view (fourteenth to eighteenth centuries). J Econ Hist 75(4):1058â€"1096.

[3] Alfani G, Ammannati F. (2017) Long-term trends in economic inequality: the case of the Florentine state, c. 1300-1800. Econonomic History Review, 4,1072-1102.

[4] Alvarez, F., Argente, D., Lippi, F. (2021). A simple planning problem for COVID-19 lock-down, testing, and tracing. American Economic Review: Insights, 3(3), 367-82.

[5] Atkeson, A. (2020). What Will Be the Economic Impact of COVID-19 in the US? Rough Estimates of Disease Scenarios, National Bureau of Economic Research Working Paper Series No. 26867.

[6] Atkeson, A. (2021). A parsimonious behavioral SEIR model of the 2020 COVID epidemic in the United States and the United Kingdom (No. w28434). National Bureau of Economic Research.

[7] Auld, C., Toxvaerd, F.M.O.(2021). The Great COVID-19 Vaccine Rollout: Behavioral and Policy Responses, CWPE2136.

[8] Bargues, M. Dimitrova, V. (2021). The Psychological Gains from COVID-19 Vaccination: Who Benefits the Most? IZA DP No. 14826.

[9] Barro, R.J., Sala-i-Martin, X. (1997). Technological Diffusion, Convergence, and Growth. Journal of Economic Growth 2, 1-26.

[10] Bond,E., Wang,P., Yip,C.(1996). A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics,Journal of Economic Theory,68(1), 149-173.

[11] Bosi, S., Camacho, C., Desmarchelier, D. (2021). Optimal lockdown in altruistic economies. Journal of Mathematical Economics, 93, 102-488.

[12] Boucekkine, R., Laffargue, J.P (2010). On the distributional consequences of epidemics, 34,2, pp.231-245.

[13] Boucekkine R., Desbordes R., Latzer H.(2009). How do epidemics induce behavioral changes? Journal of Economic Growth, 14, pp. 233-264.

[14] Chinazzi, M. Davis, J. Ajelli, M. Gioannini, C. Litvinova, M. Merler, S. Piontti, A. Mu,K. Rossi, L. Sun, K. Viboud, C. Xiong, X. Yu, H. Halloran, E. Longini, I. Vespignani, A. (2020) The effect of travel restrictions on the spread of the 2019 novel coronavirus outbreak, Science, 368(6489), 395-400.

[15] Coibion, O., Gorodnichenko, Y., Weber, M. (2020). Labor markets during the COVID-19 crisis: A preliminary view (No. w27017). National Bureau of economic research.

[16] Craighead, C. Ketchen, D. Darby, J. (2020) Pandemics and Supply Chain Management Research: Toward a Theoretical Toolbox. Decision Sciences. 51, pp. 838-866.

[17] Davin, M., Fodha, M., Seegmuller, T., (2021). Environment, public debt, and epidemics. Journal of Public Economic Theory, 1â€“34. https://doi.org/10.1111/jpet.12566.

[18] Ferguson, N., Laydon, D., Nedjati-Gilani, G., Imai, N., Ainslie, K., Baguelin, M., ... Ghani, A. C. (2020). Report 9: Impact of non-pharmaceutical interventions (NPIs) to reduce COVID19 mortality and healthcare demand. Imperial College London, 10(77482), 491-497.

[19] Furceri D., Loungani P., Ostry JD., Pizzuto P. (2020) Will Covid-19 affect inequality? Evidence from past pandemics. Covid Econ, 12, 138-157.

[20] Galasso, V., Pons, V. Profeta,P. Becher, M. Brouard, S. Foucault, M. (2022). From Anti-vax Intentions to Vaccination: Panel and Experimental Evidence from Nine Countries.National Bureau of Economic Research Working Paper Series No. 29741.

[21] Galletta S., Giommoni T. (2020) the effect of the 1918 influenza pandemic on income inequality: evidence from Italy. In: CEPR discussion papers 3791, C.E.P.R. discussion papers. https://doi.org/10.2139/ ssrn.3634793.

[22] Gori, L., Manfredi, P., Marsiglio, S., Sodini, M., (2021). COVID-19 epidemic and mitigation policies: Positive and normative analyses in a neoclassical growth model. Journal of Public Economic Theory.

[23] Hsiang, S., Allen, D., Annan-Phan, S., Bell, K., Bolliger, I., Chong, T., ... Wu, T. (2020). The effect of large-scale anti-contagion policies on the COVID-19 pandemic. Nature, 584(7820), 262-267.

[24] Kazuo.K, (1996) Analysis of a Two-Sector Model of Endogenous Growth with Capital Income Taxation.â€ International Economic Review,37(1), 227-51.

[25] La Torre, D., Liuzzi, D., Marsiglio, S. (2021). "Epidemics and macroeconomic outcomes: social distancing intensity and duration", Journal of Mathematical Economics 93, 102473

[26] La Torre,D. Liuzzi, D. Marsiglio, S. (2022). Geographical heterogeneities and externalities in an epidemiological-macroeconomic framework. Journal of Public Economic Theory, Forthcoming

[27] Lopez Bernal, J., Andrews, N., Gower, C., Robertson, C., Stowe, J., Tessier, E.,Simmons, R., Cottrell, S., Roberts, R., O'Doherty, M., Brown, K., Cameron, C., Stockton, D., McMenamin, J. and Ramsay, M., (2021) Effectiveness of the Pfizer-BioNTech and Oxford-AstraZeneca vaccines on

Covid-19 related symptoms, hospital admissions, and mortality in older adults in England: test negative case-control study. BMJ Publishing Group Ltd, 373.

[28] Nganmeni, Z., Pongou, R., Tchantcho, B., Tondji, J.-B., (2022). Vaccine and inclusion. Journal of Public Economic Theory.Apr 27:10.1111/jpet.12590. doi: 10.1111/jpet.12590.

[29] Nikolopoulos, K., Punia, S. Schafers, A. Tsinopoulos, C., Vasilakis, C. (2021) Forecasting and planning during a pandemic: COVID-19 growth rates, supply chain disruptions, and governmental decisions, European Journal of Operation Research. 27, 8-36.

[30] Ortigueira, S. and Santos, M. (1997). On the Speed of Convergence in Endogenous Growth Models,The American Economic Review, 87,3, 383-399.

[31] Petropoulos, F., Makridakis, S. (2020). Forecasting the novel coronavirus COVID-19. PLOS ONE, 15(3), e0231236.

[32] Sayed, A., Peng, B.(2021). Pandemics and income inequality: a historical review. SN Bus Econ 1, 54.

[33] Toxvaerd, F. (2020), "Equilibrium Social Distancing", Cambridge-INET Working Paper Series No. 2008.

[34] Toxvaerd, F., Rowthorn R. (2020), On the Management of Population Immunity, Cambridge-INET Working Paper Series No. 2020.

[35] Vasilakis, C. (2012). The social economic impact of AIDS: Accounting for intergenerational transmission, productivity and fertility, Economic Modelling,29,2, pp. 369-381.

Appendices

A Proof of Proposition 1

From (6) and (7), taking integrals on both sides:

$$I_i = \int_{-\infty}^{t} \left(\beta_i \left[N_i - I_i - (1-\sigma)V_i \right] \frac{I_i + I_j}{N} \right) e^{-c(t-s)} ds,$$

$$V_i = \int_{-\infty}^{t} \left(\phi \left(N_i - I_i - V_i \right) - \sigma \beta_i V_i \frac{I_i + I_j}{N} \right) e^{-c(t-s)} ds.$$

We change the integration variable defining $x = t - s$:

$$I_i(t) = \int_{0}^{t} \left(\beta_i \left[N_i - I_i(t-x) - (1-\sigma)V_i(t-x) \right] \frac{I_i(t-x) + I_j(t-x)}{N} \right) e^{-cx} dx.$$

Then

$$\limsup_{t\to\infty} I_i(t) = \limsup_{t\to\infty} \int_{0}^{t} \left(\beta_i \left[N_i - I_i(t-x) - (1-\sigma)V_i(t-x) \right] \frac{I_i(t-x) + I_j(t-x)}{N} \right) e^{-cx} dx.$$

By the Fatou-Lebesgue theorem, it is true then that

$$\limsup_{t\to\infty} I_i(t) \leq \int_0^t \limsup_{t\to\infty} \left(\beta_i \left[N_i - I_i(t-x) - (1-\sigma)V_i(t-x) \right] \frac{I_i(t-x) + I_j(t-x)}{N} \right) e^{-cx} dx$$

Recall that we know that if one the two following sequences $\{s_n\}_n$ or $\{x_n\}_n$ converges, then

$$\limsup_{t\to\infty} s_n x_n = \limsup_{t\to\infty} s_n \limsup_{t\to\infty} x_n.$$

Applying the above property to our limit,

$$\int_0^t \limsup_{t\to\infty} \left[\left(\beta_i \left[N_i - I_i(t-x) - (1-\sigma)V_i(t-x) \right] \frac{I_i(t-x) + I_j(t-x)}{N} \right) e^{-cx} \right] dx$$

$$= \int_0^t \limsup_{t\to\infty} \left[N_i - I_i(t-x) - (1-\sigma)V_i(t-x) \right] \limsup_{t\to\infty} \beta_i \frac{I_i(t-x) + I_j(t-x)}{N} e^{-cx} dx$$

$$\leq N_i \int_0^t \limsup_{t\to\infty} \beta_i \frac{I_i(t-x) + I_j(t-x)}{N} e^{-cx} dx.$$

Since $\limsup_{t\to\infty}(I_i(t-x) + I_j(t-x)) = \limsup_{t\to\infty}(I_i(t) + I_j(t))$, then

$$\limsup_{t\to\infty} I_i(t) \leq \beta_i \frac{N_i}{N} \limsup_{t\to\infty} \left[I_i(t) + I_j(t) \right] \frac{1}{c} = \frac{\beta_i}{c} \frac{N_i}{N} \limsup_{t\to\infty} \left(I_i(t) + I_j(t) \right).$$

Identically,

$$\limsup_{t\to\infty} I_j(t) \leq \frac{\beta_j}{c} \frac{N_j}{N} \limsup_{t\to\infty} \left[I_i(t) + I_j(t) \right],$$

so that

$$\limsup_{t\to\infty} \left(I_i(t) + I_j(t) \right) \leq \frac{\beta_i N_i + \beta_j N_j}{cN} \limsup_{t\to\infty} \left[I_i(t) + I_j(t) \right].$$

B Proof of Proposition 3

The dynamics of the disease is described by (6) and (7). Imposing $\dot{V}_1 = \dot{V}_2 = 0$, we obtain the expressions for $\bar{V}_1$ and $\bar{V}_2$. Indeed since

$$\phi \left(N_1 - \bar{I}_1 \right) = \bar{V}_1 \left(\sigma \beta_1 \frac{\bar{I}_1 + \bar{I}_2}{N} + \theta + \phi \right),$$

or

$$\bar{V}_1 = \frac{\phi \left(N_1 - \bar{I}_1 \right)}{\theta + \phi + \sigma \beta_1 \frac{\bar{I}_1 + \bar{I}_2}{N}},$$

$$\bar{V}_2 = \frac{\phi \left(N_2 - \bar{I}_2 \right)}{\theta + \phi + \sigma \beta_2 \frac{\bar{I}_1 + \bar{I}_2}{N}}$$

Plugging them into the steady states for I_1 and I_2

$$cI_1 = \beta_1 \left(N_1 - I_1 - (1-\sigma) \frac{\phi(I_1 + I_2)}{\phi + \theta + \sigma \beta_1 \frac{I_1 + I_2}{N}} \right) \frac{I_1 + I_2}{N}, \tag{28}$$

$$cI_2 = \beta_2 \left(N_2 - I_2 - (1-\sigma) \frac{\phi(I_1 + I_2)}{\phi + \theta + \sigma \beta_2 \frac{I_1 + I_2}{N}} \right) \frac{I_1 + I_2}{N}. \tag{29}$$

Take the first one. Let us develop it

$$\frac{c}{\beta_1} I_1 \left(\phi+\theta+\sigma\beta_1 \frac{I_1+I_2}{N}\right) = \frac{I_1+I_2}{N}\left[(N_1-I_1)(\phi+\theta+\sigma\beta_1\frac{I_1+I_2}{N}) - (1-\sigma)\phi(I_1+I_2)\right]$$

Defining $x=\frac{I_1}{N}$ and $y=\frac{I_2}{N}$, we can write

$$\frac{c}{\beta_1} x\left[\phi+\theta+\sigma\beta_1(x+y)\right] = (x+y)\left[\left(\frac{N_1}{N}-x\right)\left[\phi+\theta+\sigma\beta_1(x+y)\right]-(1-\sigma)\phi(x+y)\right]$$

We can develop to obtain a third order polynomial:

$$h_1(x,y) = \sigma\beta_1 x^3 + x^2\left[c\sigma+\phi+\theta-\frac{N_1}{N}\sigma\beta_1+2\sigma\beta_1 Y+(1-\sigma)\phi\right] \tag{30}$$

$$+x\left[\sigma\beta_1 y^2 + y\left(\sigma c+\phi+\theta-2\sigma\beta_1\frac{N_1}{N}+2(1-\sigma)\phi\right)+\frac{c}{\beta_1}(\phi+\theta)-\frac{N_1}{N}(\phi+\theta)\right] \tag{31}$$

$$-\frac{N_1}{N}(\phi+\theta)y-\frac{N_1}{N}\sigma\beta_1 y^2+(1-\sigma)\phi y^2. \tag{32}$$

Similarly, developing (29) we obtain a second third order polynomial:

$$h_2(x,y) = \sigma\beta_2 y^3 + y^2\left[c\sigma+\phi+\theta-\frac{N_2}{N}\sigma\beta_1+2\sigma\beta_2 x+(1-\sigma)\phi\right] \tag{33}$$

$$+y\left[\sigma\beta_2 x^2 + x\left(\sigma c+\phi+\theta-2\sigma\beta_2\frac{N_2}{N}+2(1-\sigma)\phi\right)+\frac{c}{\beta_2}(\phi+\theta)-\frac{N_2}{N}(\phi+\theta)\right] \tag{34}$$

$$-\frac{N_2}{N}(\phi+\theta)x-\frac{N_2}{N}\sigma\beta_2 x^2+(1-\sigma)\phi x^2. \tag{35}$$

The endemic steady states are the solutions to $h_1(x,y)=0$ and $h_2(x,y)=0$ on $\left[0,\frac{N_1}{N}\right]\times\left[0,\frac{N_2}{N}\right]$.

We can start by studying h_1 (the study of h_2 is identical). We can put forward some relevant properties. Note that

i) $\lim_{x\to-\infty} h_1(x,y)=-\infty$ and $\lim_{x\to\infty} h_1(x,y)=\infty$, for every $y\in\mathbb{R}$.

ii) $h_1\left(\frac{N_1}{N},y\right)>0$, for every $y\in\mathbb{R}$. Indeed

$$h_1\left(\frac{N_1}{N},y\right)=\left(\frac{N_1}{N}\right)^2\left[c\sigma+(1-\sigma)\phi\right]+\frac{N_1}{N}y\left[c\sigma+2(1-\sigma)\phi\right]+\frac{N_1}{N}\frac{c}{\beta_1}(\phi+\theta)+(1-\sigma)\phi y^2>0.$$

iii) h_1 is a third order polynomial in x for every $y\in\mathbb{R}$. As such, it has an inflexion point, x^i. The inflexion point x^i verifies that $h_1''(x^i,y)=0$. The inflexion point of h_1 is below $\frac{N_1}{N}$. The inflexion point for h_1 is (x^i,y) for a given y, and for h_2 it is (x,y^i), for x fixed.

Imposing $h_1''(x^i,y)=0$ we obtain that

$$x^i=\frac{\frac{N_1}{N}\sigma\beta_1-2\beta_1 y-c\sigma-\phi-\theta-(1-\sigma)\phi}{3\sigma\beta_1}.$$

Indeed, if x^i was larger than $\frac{N_1}{N}$

$$\frac{\frac{N_1}{N}\sigma\beta_1-2\beta_1 y-c\sigma-\phi-\theta-(1-\sigma)\phi}{3\sigma\beta_1}>\frac{N_1}{N},$$

then it should be

$$\frac{N_1}{N}\sigma\beta_1-2\beta_1 y-c\sigma-\phi-\theta-(1-\sigma)\phi>3\sigma\beta_1\frac{N_1}{N},$$

which is not possible. Hence, $x^i<\frac{N_1}{N}$.

In particular, this result implies that there is at least one root below $\frac{N_1}{N}$ and this for every y.

iv) Next, let us prove that $h_1'(\frac{N_1}{N}, y) > 0$. If this is true, then all roots of h_1 are below $\frac{N_1}{N}$ for all y. If there are more than one endemic steady state, then we will have indeterminacy.

$$h_1'\left(\frac{N_1}{N}, y\right) = \sigma\beta_1 y^2 + y\left[2\sigma\beta_1\frac{N_1}{N} + \sigma c + \phi + \theta + 2(1-\sigma)\phi\right] + \sigma\beta_1\left(\frac{N_1}{N}\right)^2$$
$$+\frac{N_1}{N}\left[\sigma c + \phi + \theta + 2(1-\sigma)\phi\right] + c\beta_1(\phi + c),$$

which is always positive for all $y \in \left[0, \frac{N_2}{N}\right]$.

v)

$$h_1(0, y) = y\left[(1-\sigma)\phi y - R_0^1\frac{c}{\beta_1}(\phi+\theta) - R_0^1 c\sigma y\right]$$

$h_1(0, y) < 0$ if and only if

$$cR_0^1\left(\frac{\phi+\theta}{\beta_i} + \sigma y\right) > y\phi(1-\sigma),$$

or

$$cR_0^1\frac{\phi+\theta}{\beta_i} + cR_0^1\sigma y > y\phi(1-\sigma).$$

Hence, if $cR_0^1\sigma y > y\phi(1-\sigma)$, or $cR_0^1\sigma > \phi(1-\sigma)$, then $h_1(0, y) < 0$.

We can prove using an identical procedure that the conditions in Proposition 3 ensure that $h_2(x, 0) < 0$, so that there is a unique SS.

C Proof of Proposition 4

In order to have endemic steady states, we need roots on $[0, \frac{N_1}{N}]$. One way to study this is though the inflexion point. All conditions we provide are sufficient, not necessary.

With the properties of h_1 and h_2 that we have already proven, if the inflexion point for group 1 is positive, $x_1^i > 0$, then there is at least two endemic steady states for group 1. If

$$R_0^1 = \frac{N_1\beta_1}{cN} > \frac{2}{\sigma}R_0^2 + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c}, \tag{36}$$
$$R_0^2 = \frac{N_2\beta_2}{cN} > \frac{2}{\sigma}R_0^1 + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c}, \tag{37}$$

then, there are at least two endemic steady states for both groups. Note that this cannot hold. Indeed, if we use the second condition on the first:

$$R_0^1 > \frac{2}{\sigma}R_0^2 + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c} > \frac{4}{\sigma^2}R_0^1 + \frac{2}{\sigma}\frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c} + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c},$$

or

$$\left(1 - \frac{4}{\sigma^2}\right)R_0^1 > \left(1 + \frac{2}{\sigma}\right)\frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c},$$

which cannot hold since $\sigma < 1$, so that $\sigma^2 - 4 < 0$. Hence, both inflexion points cannot be larger than zero.

Let us assume then that $x_1^i > 0$ and $x_2^i < 0$. If

$$R_0^1 > \frac{2}{\sigma}R_0^2 + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c}$$

then the inflexion point for Group 1 is larger than zero and there are at least two steady states. If additionally $h_2(x, 0) < 0$, then there is only one steady state for Group 2. Note that there could also be several steady states for Group 2.

$$h_2(x, 0) = -x \left(\frac{N_2}{N}(\phi + \theta) + \frac{N_2}{N}\sigma\beta_2 x - (1-\sigma)\phi x \right).$$

Then $h_2(x, 0) < 0$ if $\left(\frac{N_2}{N}(\phi + \theta) + \frac{N_2}{N}\sigma\beta_2 x - (1-\sigma)\phi x\right) > 0$:

$$\frac{N_2}{N}(\phi + \theta) + \frac{N_2}{N}\sigma\beta_2 x - (1-\sigma)\phi x > 0,$$

$$x \left[(1-\sigma)\phi - \frac{N_2}{N}\sigma\beta_2 \right] < \frac{N_2}{N}(\phi + \theta).$$

Our condition holds if

$$\max_x x \left[(1-\sigma)\phi - \frac{N_2}{N}\sigma\beta_2 \right] < \frac{N_2}{N}(\phi + \theta),$$

that is,

$$\frac{N_1}{N} \left[(1-\sigma)\phi - \frac{N_2}{N}\sigma\beta_2 \right] < \frac{N_2}{N}(\phi + \theta),$$

which can be written as

$$(1-\sigma)\phi - c\sigma R_0^2 < \frac{N_2}{N_1}(\phi + \theta),$$

or

$$R_0^2 > \frac{1}{c\sigma} \left[(1-\sigma)\phi - \frac{N_2}{N_1}(\phi + \theta) \right], \tag{38}$$

which always holds if in particular, $\frac{N_2}{N_1} > \frac{(1-\sigma)\phi}{\phi+\theta}$. We can check whether (36) and (38) can hold at the same time:

$$R_0^1 > \frac{2}{\sigma}R_0^2 + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c} > \frac{2}{c}\left[(1-\sigma)\phi - \frac{N_2}{N_1}(\phi + \theta) \right] + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c},$$

or

$$R_0^1 > 1 + \frac{2}{c}\left[(1-\sigma)\phi - \frac{N_2}{N_1}(\phi + \theta) \right] + \frac{\phi + \theta + (1-\sigma)\phi}{\sigma c}.$$

Then, if

$$R_0^1 > 1 + \frac{2}{c}\left[(1-\sigma)\phi - \frac{N_2}{N_1}(\phi + \theta) \right] + \frac{\phi + \theta + (1-\sigma)\phi}{\sigma c}, \tag{39}$$

$$R_0^2 < \frac{2}{\sigma}R_0^1 + \frac{c\sigma + \phi + \theta + (1-\sigma)\phi}{\sigma c}, \tag{40}$$

then Group 1 has two endemic steady states, Group 2 only 1.

D Steady State

From

$$B_2\alpha\bar{K}^{\alpha-1}\bar{L}_2^{1-\alpha} = \delta + \rho$$

we can obtain that $\bar{K} = \left(\frac{B_2\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}} \bar{L}_2$.

From (9) and (10), and knowing that at the steady state equilibrium $\bar{L}_2 = N - \bar{N}_1 - \bar{I}_2(\bar{N}_1)$, we can write that at the steady state, $\bar{I}_i = \bar{I}_i(\bar{N}_1)$ for both $i = 1, 2$. Hence, we can rewrite the steady state of total physical capital as

$$\bar{K} = \left(\frac{B_2\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}} \left[N - \bar{N}_1 - \bar{I}_2(\bar{N}_1)\right]. \tag{41}$$

Let us find next the expression that will allow us to compute the steady state value for Group 1, and from there the steady state of all other variables. Using the goods market clearing conditions

$$c_1^1(t) + c_1^2(t) = B_1 L_1, \tag{42}$$

$$c_2^1(t) + c_2^2(t) = B_2 K(t)^\alpha L_2(t)^{1-\alpha}. \tag{43}$$

Using the household's first order conditions, in particular that $c_2^i = \frac{1-\eta}{\eta}\frac{1}{p}c_1^i$, we can write the second condition as

$$\frac{1-\eta}{\eta p}\left(c_1^1(t) + c_1^2(t)\right) = B_2 K(t)^\alpha L_2(t)^{1-\alpha}.$$

So that

$$\frac{1-\eta}{\eta p} B_1(\bar{N}_1 - \bar{I}_1(\bar{N}_1)) = B_2 \left(\frac{B_2\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} \left[N - \bar{N}_1 - \bar{I}_2(\bar{N}_1)\right], \tag{44}$$

which implicitly defines $\bar{N}_1$.

With this value of $\bar{N}_1$ we can recover c_1^1 starting by summing up the equilibrium conditions at the steady state related to asset accumulation. From the asset accumulation dynamic equations:

$$\rho\bar{A}_1 \quad + \quad B_1\left[\bar{N}_1 - I_1(\bar{N}_1)\right] = c_1^1 \frac{1}{\sigma}\bar{N}_1, \tag{45}$$

$$\rho\bar{A}_2 \quad + \quad (1-\alpha)B_2\left(\frac{B_2\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} \left[N - \bar{N}_1 - \bar{I}_2(\bar{N}_1)\right] = c_1^2 \frac{1}{\sigma}(N - \bar{N}_1). \tag{46}$$

Summing them up and using (41) and the relationship between the household consumption of both goods in (20), we obtain c_1^1:

$$c_1^1 \frac{N - 2\bar{N}_1}{\eta} = B_1(N - \bar{N}_1)\left(\frac{N - \bar{N}_1}{\eta} - 1\right) - \rho\bar{K} - (1-\alpha)B_2\bar{K}^\alpha\left(N - \bar{N}_1 - I_2(\bar{N}_1)\right)^{1-\alpha}.$$

E Disease free economy

Suppose that $\bar{I}_1(\bar{N}_1) = \bar{I}_2(\bar{N}_1) = 0$. Then, from (44) we have that

$$\frac{1-\eta}{\eta p} B_1 \bar{N}_1 = B_2\left(\frac{B_2\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} \left[N - \bar{N}_1\right]^{1-\alpha}.$$

Gathering the terms in $\bar{N}_1$:

$$\bar{N}_1 = \frac{1}{1 + \frac{B_1}{B_2}\frac{1-\eta}{\eta p}\left(\frac{\rho+\delta}{\alpha B_2}\right)^{\frac{\alpha}{1-\alpha}}} N.$$

F Comparative Statics

Results in Proposition 5 obtain simply taking partial derivatives of $\bar{Q}_G$ and $\bar{Q}_i$ with respect to each of the parameters.

G Inequality in Capital as a function of β_1 and ϕ.

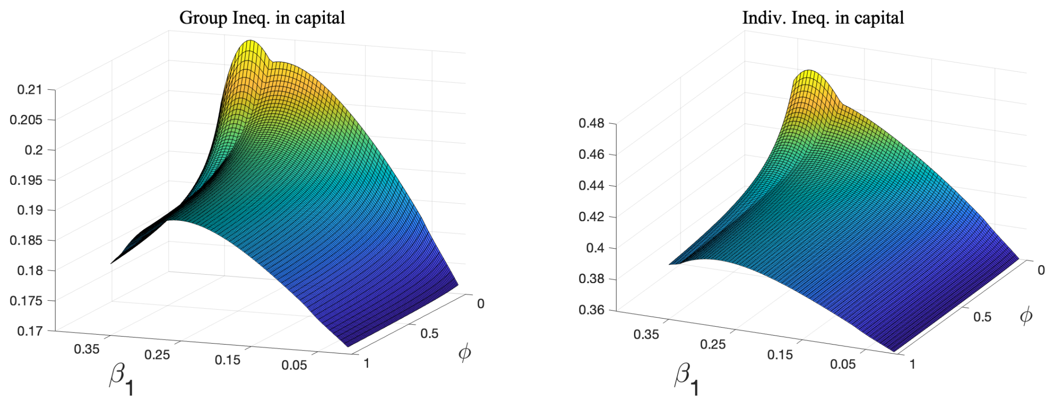

Figure 4: Group and Individual inequality in capital as functions of ϕ and β_1.

H Inequality Measures

The long-term inequality measures in capital, QK_G and QK_i, consumption, Qc_G and Qc_i, shares of infected, labor, group size, capital assets holdings and welfare are defined as

$$QK_G = \frac{A_2 N_2}{A_1 N_1} \text{ and } QK_i = \frac{A_2}{A_1}, \tag{47}$$

$$Qc_G = \frac{c_2 N_2}{c_1 N_1} \text{ and } Qc_i = \frac{c_2}{c_1}, \tag{48}$$

$$QI_G = \frac{N_2 I_2}{N_1 I_1} \text{ and } QI_i = \frac{I_2}{I_1}, \tag{49}$$

$$QL_G = \frac{N_2 - I_2}{N_1 - I_1}, \tag{50}$$

$$QS_G = \frac{N_2}{N_1}, \tag{51}$$

$$QA_G = \frac{N_2 A_2}{N_1 A_1} \text{ and } QA_i = \frac{A_2}{A_1}, \tag{52}$$

$$QW_G = \frac{N_2 W_2}{N_1 W_1} \text{ and } QA_i = \frac{W_2}{W_1}, \tag{53}$$

where W_i is welfare of an individual in Group i, $i = 1, 2$.