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# ABSTRACT

## A Multisector Perspective on Wage Stagnation*

Low-skill workers are concentrated in sectors experiencing fast productivity growth, yet their real wages have stagnated and lagged behind aggregate productivity. We provide evidence demonstrating the importance of a multisector perspective. Central to our mechanism is the decline in the relative price of the low-skill intensive sector driven by its faster productivity growth. This dampens wage gains for low-skill workers by lowering the price of their output relative to their consumption basket, which is further reinforced by shifting them into the sector where less weight is placed on their labor. We calibrate the two-sector model to the 1980–2010 U.S. economy and find this mechanism to be quantitatively important. Our counterfactual analysis reveals that low-skill real wage growth would have nearly doubled if the observed aggregate productivity growth had been evenly distributed across sectors.

**JEL Classification:** E24, J23, J31

**Keywords:** low-skill wage stagnation, wage-productivity divergence, multisector model, relative prices, uneven productivity growth

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# 1 Introduction

Low-skill workers have experienced very little wage growth, despite working mostly in sectors with fast productivity growth. In the U.S., the real wage of non-college workers increased by about 20% between 1980–2010, which is less than half the increase in aggregate labor productivity.[1] The low-skill wage "stagnation" persists even after controlling for age, race, gender, education, and occupation, indicating it is not due to compositional changes in low-skill employment.[2] Hours worked by these workers represent two-thirds of overall hours worked, so their wage stagnation explains why the average wage is lagging behind aggregate labor productivity, despite the real wage of college graduates growing faster than aggregate labor productivity. Taken together, these observations reject the view that a rising tide lifts all boats; apparently, many boats are left behind.

Our objective is to understand why the growth of the low-skill real wage has been so low and lagging behind aggregate labor productivity. We offer a novel multisector perspective, where the key mechanism is the falling relative price driven by faster productivity growth in sectors that use low-skill workers more intensively. This mechanism dampens the positive effect of productivity on the low-skill real wage, which is the average *value* of the marginal product of low-skill workers, through two channels. First, the increase in the physical marginal output caused by faster productivity growth is *valued* at a lower price relative to their consumption basket. Second, when outputs are complements across sectors, this leads to a reallocation of low-skill workers to the high-skill intensive sector where they have a lower weight in the production function.

[1]The precise increase in the aggregate non-college real wage range from 15% to 25%, depending on the choice of price deflators, composition adjustment, the inclusion of non-wage compensation and self-employment, and whether it is only for the nonfarm business sectors. Regardless of these choices, the finding that the non-college real wage has had little growth and lags behind the aggregate labor productivity growth is robust.

[2]As documented in Acemoglu and Autor (2011), low-skill wage stagnation coexists with occupational polarization (low-wage occupations have faster wage growth than middle-wage occupations). The low-skill wage stagnation pertains to a group of workers with given education qualifications, whereas polarization is defined over given occupational groups irrespective of who is employed there. Sevinc (2019) documents the role of skill heterogeneity within an occupation in understanding these two patterns.

We provide two sets of motivating evidence from the U.S. to support this multisector mechanism. First, using a simple accounting equation for the aggregate low-skill real wage, we demonstrate that the growth in the low-skill wage would have been more than double if there were no changes in relative prices and no reallocation of low-skill hours across sectors. This simple exercise highlights the crucial role that changing relative prices play, even in the absence of labor reallocation. The dampening effect of the relative price is further reinforced by the changing hours shares. The data imply that sectors with declining relative prices are also experiencing declining shares of low-skill hours. Consistent with Buera et al. (2022), we find that these sectors are the low-skill intensive sectors.

Second, to understand the divergence between the low-skill real wage and aggregate labor productivity, we turn to an accounting identity that the total value-added of the economy equals the sum of total factor payments. This identity reveals three driving forces for the divergence: the increasing skill premium, declining labor income share, and rising relative cost of living, measured by the ratio of the consumption deflator to the output deflator. The latter two forces, which together account for 30% to 50% of the divergence, necessitate the presence of capital. In its absence, both the labor income share and the relative price of consumption would equal to one.

To quantify the proposed multisector mechanism in understanding the stagnation low-skill wages and their divergence from aggregate labor productivity, we calibrate a two-sector model to match key features of the U.S. labor market from 1980 to 2010. Production in both sectors uses low-skill, high-skill labor, and capital. The low-skill sector uses low-skill labor more intensively and has faster productivity growth. As in Buera et al. (2022), we show that the faster productivity growth in the low-skill sector leads to an increase in the skill premium, which contributes to the divergence. In addition, due to the presence of capital in our model, we find that the mutlisector mechanism also contributes to the divergence by increasing the relative cost of living. Besides our mechanism through uneven productivity growth, the calibration also allows for four other forces that are shown to be important for understanding the skill premium and the labor share. They are the falling relative price of capital in the presence of capital-skill complementarity (Krusell et al., 2000), the falling production weights of low-skill labor (Goldin and Katz, 2009), and the skill-biased demand and supply shifts (Katz and Murphy, 1992).

The uneven productivity growth, which is calibrated to match the observed changes in relative prices, is quantitatively important for both the divergence and low-skill wage stagnation. This can be demonstrated by considering what would happen to the low-skill wage if the same level of aggregate productivity growth were instead driven by a balanced increase in sectoral productivity. The result of this counterfactual analysis is that the increase in the low-skill wage would have been almost double, and the resulting divergence would have been nearly halved. This highlights that the source of the aggregate productivity growth is crucial for understanding low-skill wage stagnation.

The declining production weights of low-skill labor also play an important role, as they are a key factor driving the decrease in the labor share and the increase in the skill premium. Their contribution to the low-skill wage stagnation relies on lowering the marginal product of low-skill labor in both sectors, which fails to account for the observed differential trends. These differential trends are a result of changing relative prices when the growth of nominal low-skill wages is similar across sectors. Both the decline in the relative price of capital and the skill-biased demand shifts that increase the production weight of high-skill labor are quantitatively important for the rise in the skill premium but not for low-skill wage stagnation. These quantitative exercises demonstrate that factors contributing to the increase in the skill premium do not necessarily contribute to low-skill wage stagnation.

Our paper can be viewed as providing a framework to assess the quantitative significance of various forces underlying key aspects of labor market inequalities and their roles for understanding the low-skill wage stagnation. Since the seminal work of Katz and Murphy (1992), an extensive literature has emerged studying the effects of skill-biased demand and supply shifts on the skill premium, with a particular focus on skill-biased technical change (see Goldin and Katz, 2009, for a review). The skill-biased technical change that simply improves the relative productivity of high-skill workers, however, does not necessarily contribute to low-skill wage stagnation (Johnson, 1997; Acemoglu and Autor, 2011). This limitation has partly contributed to a growing literature on automation and declining labor shares (see Zeira, 1998; Karabarbounis and Neiman, 2014; Acemoglu and Restrepo, 2018; Martinez, 2019; Caselli and Manning, 2019; Hémous and Olsen, 2022; Moll et al., 2022; Hubmer, 2023, among others).[3] Other potential explanations include de-unionization and the decline in the minimum wage (Lee, 1999; Dustmann et al., 2009), increasing monopsony power (Manning, 2003), rising imports (Autor et al., 2013), and the decline in the urban premium for non-college workers (Autor, 2019).[4] Many of these forces can be understood within the conceptual framework of one-sector models. Our contribution to this literature is to emphasize the importance of sector-specific technological changes.

In exploring the role of uneven productivity growth on the labor market by skill groups, Buera et al. (2022) is the closest work to ours in terms of explaining the rise in the skill premium and the expansion of the high-skill intensive sector. The main contributions of our paper, relative to theirs, are to demonstrate the effects of uneven productivity growth on low-skill wage growth and to elucidate the roles of changing relative prices and sectoral reallocation of labor in driving these outcomes. In addition, capital, absent in their model, plays two crucial roles in our analysis. First, it is essential for studying the decoupling of wages and aggregate productivity through its effect on the labor share and the relative price of consumption. Second, it provides an additional mechanism for the rise in the skill premium through capital-skill complementarity and declining relative price

[3]This is accompanied by a parallel growing empirical literature on the effect of automation on employment, wages and labor income shares (see Autor and Salomons, 2018; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020; Chen et al., 2021; Kapetaniou and Pissarides, 2020, among others)

[4]To the extent that most of the expansion in high-skill services occurs in urban areas, our mechanism is consistent with the finding of Autor (2019) on the decline of the urban premium for non-college workers due to region-specific occupational changes.

Table 1: *Percentage change in the aggregate low-skill wage*

| | | Fixed at the 1980 level | | |
|---|---|---|---|---|
| | Data | Hours shares | Relative prices | Hours shares and relative prices |
| 1980–2010 | 20% | 18% | 29 % | 48% |

Note: The table reports percentage change in the aggregate low-skill wage in the data and three counterfactual scenarios (see equation 1), where hours shares, relative prices and both are fixed at the 1980 level. Real wage is equal to nominal wage deflated by PCE price index. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Appendix A1 for the construction of variables and sectors. Source: CPS, WORLD KLEMS and authors' calculations.

of capital, as in Krusell et al. (2000).

Section 2 presents motivating facts on understanding low-skill wage stagnation and the importance of a multisector perspective. It uses a simple two-sector model to illustrate the basic mechanism. Section 3 presents the full model, and Section 4 calibrates the model to quantify the multisector mechanism.

# 2 Motivation

## 2.1 Low-skill real wage in a multisector economy

The importance of a multisector perspective can be illustrated in a simple exercise by expressing the aggregate low-skill real wage as a weighted sum of the sectoral wages:

$$\frac{w_l}{P_C} = \sum_j \frac{w_{lj}}{P_C}\frac{L_j}{L} = \sum_j \frac{w_{lj}}{p_j}\left(\frac{p_j}{P_C}\right)\frac{L_j}{L}; \tag{1}$$

where $w_l$ is the aggregate low-skill nominal wage, $P_C$ is the aggregate consumption price index, $w_{lj}$ and $p_j$ are the low-skill nominal wage and value-added price, and $L_j/L$ is the share of low-skill workers in sector $j$.

The key aspects of a multisector perspective are that sectoral relative prices and hours shares are changing over time. To see the importance of these changes, Table 1 compares the observed aggregate low-skill real wage with three different counterfactual scenarios where either hours shares ($L_j/L$) or relative prices ($p_j/P_C$) or both are kept constant at the 1980 level. To highlight their impacts on the growth of the low-skill real wage.

The low-skill real wage in the data grew by 20% during the 30-year period. If hours shares are fixed but relative prices vary, the low-skill real wage would have grown by 18 percent, very similar to the data. This suggests that low-skill wage stagnation would have happened even in the absence of labor reallocation.

The picture is very different if relative prices are fixed but hours shares vary. In this case, the low-skill real wage would have grown by 29%. A simple interpretation based on equation (1) is that sectors with faster growing $w_{lj}/p_j$ are experiencing falling relative prices in the data, thus the low-skill real wage would have increased by more if we remove these changes in relative prices. Intuitively, since the ratio $w_{lj}/p_j$ serves as a proxy for the marginal product of low-skill labor ($MPL_{lj}$), this finding suggests that in sectors where low-skill workers experienced bigger increases in productivity, the faster productivity growth did not fully materialize as growth in their real wages because of the falling relative prices.

It follows from equation (1) that labor reallocation can further dampen the growth of low-skill real wage if hours shares also decrease in sectors with faster productivity growth. This is confirmed by the table, which shows that the low-skill real wage would have increased by 48% if both relative prices and hours shares are fixed. In other words, if the economy behaves like a one-sector economy with no change in relative prices or labor reallocation, the growth in the low-skill real wage would have been more than double.

## 2.2 The basic mechanism

The observations from Table 1 highlight the importance of falling relative prices for sectors with faster productivity growth in accounting for the low-skill real wage stagnation. However, this is merely a mechanical exercise, as relative prices, hours shares, and the marginal product of low-skill labor are all equilibrium outcomes. The objective here is to elucidate the core mechanisms of our multisector framework that can explain these observations.

The basic idea of how this multisector mechanism operates can be shown in a simple model with two sectors and two types of households. There is a measure $H$ of high-skill households and a measure $L$ of low-skill households. Household $i = l, h$ derives utility from consuming high-skill and low-skill goods:

$$U_i = \ln c_i; \quad c_i = \left[\psi c_{il}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\psi)\, c_{ih}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad i = h, l. \tag{2}$$

where $\varepsilon < 1$ so that low-skill and high-skill goods are complements. The budget constraint is:

$$p_h c_{ih} + p_l c_{il} = w_i, \quad i = h, l. \tag{3}$$

where $w_i$ is the wage of household $i$. The optimal relative consumption is derived from equating the marginal rate of substitution to the relative prices, which can be aggregated to derive the relative aggregate consumption (see Appendix A2.1):

$$\frac{C_h}{C_l} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \quad C_j = Lc_{lj} + Hc_{hj}, \quad j = h, l. \tag{4}$$

**The role of changing relative prices:** To illustrate the importance of changing relative prices, consider the simplest form of production function where sector $l$ only uses low-skill labor and sector $h$ only uses high-skill labor. This implies there is no sectoral reallocation, allowing us to focus solely on the role of changing relative prices. The equilibrium outputs are $Y_l = A_l L$ and $Y_h = A_h H$ and wages are $w_l = p_l A_l$ and $w_h = p_h A_h$. Let $P_C = (\psi^{\epsilon} p_l^{1-\varepsilon} + (1-\psi)^{\varepsilon} p_h^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}$ be the aggregate consumption price index, the low-skill real wage is derived as:

$$\frac{w_l}{P_C} = A_l \frac{p_l}{P_C}; \quad \frac{p_l}{P_C} = \left(\psi^{\varepsilon} + (1-\psi)^{\varepsilon}\left(\frac{p_l}{p_h}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}, \tag{5}$$

where the relative price is derived from substituting the goods market clearing condition $C_j = Y_j$ into (4):

$$\frac{p_l}{p_h} = \frac{\psi}{1-\psi}\left(\frac{A_h H}{A_l L}\right)^{1/\varepsilon}. \tag{6}$$

The equilibrium conditions (5) and (6) illustrate the basic mechanism of how changing relative prices can contribute to the low-skill real wage stagnation even in the absence of labor reallocation across sectors. As shown in (5), an increase in the productivity of the low-skill sector $A_l$ raises the marginal product of the low-skill labor (also equal to $A_l$), which has a positive effect on the real wage. However, the real wage of low-skill labor is in terms of the unit of aggregate consumption goods, which depends on the relative price of the low-skill sector. This is the first key difference from a one-sector model where the low-skill real wage is simply equal to their marginal product of labor.

As shown in (6), the relative price of the low-skill sector depends negatively on its relative productivity, as long as the two goods are not perfect substitutes. If productivity growth is the same across sectors, there will be no change in the relative price. In this case, the increase in the low-skill real wage will be the same as the increase in the marginal product of low-skill labor, which will be the case in a one-sector model. However, if productivity growth is faster in the low-skill sector ($A_l/A_h$ increases), the decline in the relative price of the low-skill sector dampens the positive effect of productivity. In other words, although low-skill workers are producing more output, the increase in their physical productivity is offset by the decrease in the price of the goods they produce relative to their consumption basket.[5]

On the other hand, increasing productivity in the high-skill sector can boost the low-skill real wage by increasing the relative price of the low-skill goods. Therefore, an important message of the multisector perspective is that the source of the aggregate productivity growth is important for understanding low-skill wage stagnation. Another key insight from this simple setup is that the dampening effect of declining relative prices on the low-skill real wage operates independently of whether the two goods are complements ($\varepsilon < 1$) or substitutes ($\varepsilon > 1$). As explained below, this condition is crucial for the effects of uneven productivity

[5] In other words, specializing in sectors with faster productivity growth works against low-skill workers, as the output they produce is getting cheaper over time. This has a similar flavor, but the mechanism is different, to the early trade literature on immiserizing growth, where faster productivity growth results in a country being worse off because of deteriorating terms of trade (Bhagwati, 1958).

growth on labor reallocation and the skill premium.

**The role of labor reallocation:** The discussion of Table 1 shows that sectoral reallocation can reinforce the dampening effects of changing relative prices on the low-skill real wage. This can be shown by allowing the sectoral production function to use both high-skill and low-skill labor:

$$Y_j = A_j \left[ \xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{7}$$

where parameter $\xi_l > \xi_h$ indicates that the low-skill sector uses the low-skill labor more intensively.

As is well-known in the structural transformation literature (Baumol, 1967; Ngai and Pissarides, 2007), if the same type of labor is used in both sectors, labor reallocates from the sector with faster productivity growth to the sector with lower productivity growth when outputs are complements ($\varepsilon < 1$). When there are two types of labor, as in the production function (7), the relative wage of the type of labor that is used more intensively in the expanding sector will increase (Ngai and Petrongolo, 2017; Buera et al., 2022). More explicitly, the sectoral reallocation generates an endogenous female-biased shift in Ngai and Petrongolo (2017) to lower the gender wage gap, and an endogenous skill-biased shift in Buera et al. (2022) to increase the skill premium. We show here that this mechanism can also contribute to lowering the growth of the low-skill real wage by shifting low-skill workers into high-skill intensive sector where less weight is placed on their labor.

Using data from 15 advanced economies, Buera et al. (2022) document the pattern of *skill-biased structural change* where the high-skill intensive sectors experience rising relative prices, and growing shares in total labor compensation and value added. Using U.S. state-level data for 11 sectors, we confirm their finding that the growth of sectoral price is skill-biased and further document that the growth of sectoral low-skill hours shares is also skill-biased. This is shown using the following simple regression:

$$g_{njt} = \theta s_{nj} + \gamma_n + \gamma_t + \epsilon_{njt}, \tag{8}$$

Table 2: *Sectoral growth and skill intensity*

| | Share of Low-skill Hours | | Sectoral Price | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Skill Intensity | | | | |
| Hours | 2.24 | | 4.98 | |
| | (0.39) | | (0.44) | |
| Compensation | | 1.47 | | 3.32 |
| | | (0.31) | | (0.29) |

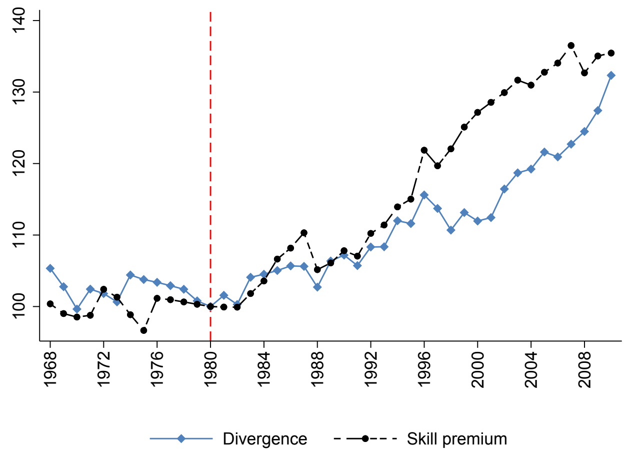
Note: The table reports the coefficients of the skill intensity variables estimated from equation (8) using state-level data for 11 one-digit sectors. The dependent variable is the annualized growth rate of the sectoral low-skill hours share in columns (1)-(2) and the sectoral value-added price in columns(3)-(4) in each decade from 1980 to 2010 by state. Skill intensity in hours is calculated as the sample mean of sectoral hours of high-skill divided by total hours in the sector. Skill intensity in labor compensation is calculated as the sample mean of sectoral compensation of high-skill divided by total compensation in the sector. Low-skill is defined as education less than a university degree. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Appendix A1 for the construction of variables. All specifications include state and decade fixed effects. The number of observations is 1683. Robust standard errors are in parentheses. All reported coefficients are significant at the 1 percent level.

where $g_{njt}$ is the growth rate of low-skill hours shares and sectoral prices for sector $j$ in state $n$ and decade $t$; $s_{nj}$ is the long-run skill-intensity of sector $j$ in state $n$, $\gamma_n$ and $\gamma_t$ are state and decade fixed effects that control for state-specific and decade-specific elements affecting the economy-wide growth rates, and $\tilde{\epsilon}_{njt}$ is the disturbance term. The slope term $\theta$ indicates the strength of conditional correlation between the growth rates and skill intensity.

Table 2 reports the estimated $\theta$ from equation (8), where the left-hand side growth variables are regressed on two alternative skill intensity measures based on hours and labor compensation. It shows that the growth in both the share of low-skill hours and value-added prices are positively correlated with skill intensity. In other words, low-skill workers are reallocating into sectors with higher skill intensities and rising relative prices. This confirms the finding in Table 1 that changes in hours shares reinforce the changes in relative prices, explaining the gap between the red-diamond line and the black-triangle line.

The basic multisector mechanism can be summarized using equation (1): sectors with faster productivity growth ($w_{lj}/p_j$) are associated with falling relative prices ($p_j/P_C$) and falling shares of low-skill hours ($L_j/L$). These falling relative

Figure 1: *The divergence, the skill premium and the low-skill real wage*



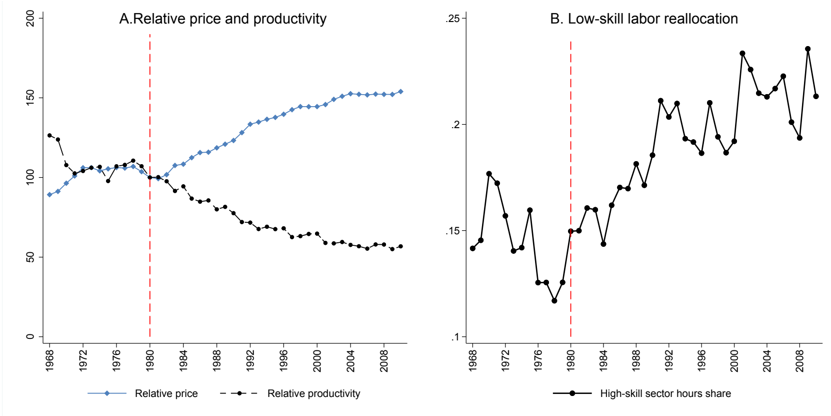

Note: Divergence is the ratio of aggregate labor productivity relative to the low-skill real wage. Skill premium is the ratio of the high-skill wage relative to the low-skill wage. Low-skill is defined as education less than a university degree. Composition-adjusted wages control for age, sex, race and education within the high-skill and the low-skill. See Data Appendix A1 for the construction of variables. Source: WORLD KLEMS and CPS.

prices and low-skill hour shares can offset the direct gain from faster productivity growth, contributing to lowering the growth of the aggregate low-skill real wage.

## 2.3 Low-skill wage and productivity divergence

To present the final set of motivating evidence, we turn to the divergence between the low-skill real wage and aggregate labor productivity using insights from the basic mechanism. To construct a consistent measure, we compute the aggregate wages by merging the WORLD KLEMS data on total compensation and hours with the distribution of demographic subgroups in the CPS. The labor compensation variable of KLEMS includes both wage and non-wage components as well as reflecting the compensation of the self-employed, and the hours variable is adjusted for the self-employed. Thus, KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. The increase in the aggregate low-skill real wage from 1980–2010 is slightly higher at 26% compared to the 20% shown in Table 1. Given that the distribution of demographic sub-

Figure 2: *Relative prices, relative productivity and labor reallocation*




Note: Panel A shows the value-added price and real labor productivity of the high-skill sector relative to the low-skill sector, normalized to 100 in 1980. Panel B shows the share of low-skill hours in the high-skill sector. See Appendix A1 for the construction of variables and sectors. Source: WORLD KLEMS and CPS.

groups is taken from the CPS, the implied relative wage is the same as the CPS.[6] The implied skill premium and the ratio of aggregate productivity relative to the low-skill real wage are shown in Figure 1. Interestingly, the low-skill real wage was growing at about the same rate as the high-skill real wage and the aggregate labor productivity prior to 1980 before it started to lag behind both series.

We aggregate sectors into two sectors according to their level of skill intensities to examine the multisector mechanism in understanding Figure 1.[7] Figure 2 shows that the multisector mechanism is consistent with the timing reported in Figure 1. Specifically, Figure 2A shows that the rise in the relative productivity of the low-skill sector started mainly after 1980 and this is mirrored by the fall in the relative price of the low-skill sector. Figure 2B shows that the reallocation of low-skill workers into the high-skill sector also started after 1980.

Finally, the divergence can be decomposed into three factors using an accounting relationship between the low-skill wage and the aggregate labor productivity. Starting with the definition of the labor income share $\beta y = w$, where $\beta$ is the

[6] As before, wages are composition adjusted for age, sex, race and education within the high-skill and the low-skill labor. See Appendix A1 for details.

[7] As explained in Appendix A1, the high-skill intensive sector includes: finance, insurance, government, health and education services, and the low-skill intensive sector includes the remaining industries.

aggregate labor income share, $y$ is the nominal aggregate labor productivity and $w$ is the average nominal wage. Let $P_Y$ be the aggregate output price index and $P_C$ be the consumption price index, we can express:

$$\underbrace{\frac{y/P_Y}{w_l/P_C}}_{\textit{Divergence}} = \underbrace{\left(\frac{P_C}{P_Y}\right)}_{\textit{Living Cost}} \underbrace{\left(\frac{1}{\beta}\right)}_{\textit{Labor Share}} \underbrace{\left(\frac{w}{w_l}\right)}_{\textit{Wage Inq}} \tag{9}$$

The divergence in the low-skill real wage and productivity is attributable to three factors: (1) a rise in the relative cost of living, (2) a decline in labor share, and (3) a rise in wage inequality, measured by the ratio of the average wage relative to the low-skill wage. The relative contributions of these three factors depends on the choice of the consumption price index. If we use PCE as a measure of $P_C$, then the contributions of the three factors are 10%, 20% and 70%. If we use CPI instead, then the contributions are 30%, 20% and 50%.[8] The main takeaway is that all three factors are quantitatively important. The presence of capital is essential for the first two factors to be present. Without capital, both the relative price of consumption and the labor income share are equal to one.

# 3 The Full Model

This section presents the full model to quantify the role of the multisector mechanism. Building on the basic two-sector model introduced in Section 2.2, we now incorporate capital as an additional factor of production. The household side is the same as before so the relative aggregate consumption is given in (4). The presence of capital changes the production function (7) and the market clearing conditions. Following Krusell et al. (2000), the production function allows for

[8] The role of different price deflators, the declining labor income share, and difference between mean and median wages have been empirically documented as sources of the decoupling between average wage and productivity (e.g., Lawrence and Slaughter, 1993; Bivens and Mishel, 2015). Here and in the next section, we deflate output by the value-added deflator and wages by alternative consumer price deflators (See Stansbury and Summers, 2019; Greenspon et al., 2021, for a similar empirical approach).

capital-skill complementarity:

$$
\begin{aligned}
Y_j &= A_j F_j \left(G_j \left(H_j, K_j\right), L_j\right) && (10) \\
F_j \left(G_j \left(H_j, K_j\right), L_j\right) &= \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + \left(1-\xi_j\right) \left[G_j \left(H_j, K_j\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} && (11) \\
G_j \left(H_j, K_j\right) &= \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + \left(1-\kappa_j\right) H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} && (12)
\end{aligned}
$$

where $H_j$ and $L_j$ are the high-skill labor and the low-skill labor used in sector $j$. The parameter $\kappa_j$ measures the importance of capital within the capital-skill composite. The elasticity of substitution across high-skill labor and capital $\rho < 1$ captures the capital-skill complementarity.

The output of the low-skill sector can be converted into $1/\phi$ unit of capital, where $\phi$ is interpreted as the price of capital relative to the low-skill intensive goods.[9] The objective of the quantitative exercise is to compare the labor market changes from 1980 to 2010 instead of studying the time path. To keep the framework simple, we assume full depreciation of capital. The market clearing conditions for goods, capital, and labor are:

$$Y_l = C_l + \phi K, \quad Y_h = C_h. \tag{13}$$

$$K = K_h + K_l. \tag{14}$$

$$H_h + H_l = H; \quad L_h + L_l = L. \tag{15}$$

## 3.1 Firm's optimization

The optimal decision of the representative firm implies that the marginal rate of technical substitution across any two inputs is equal to the ratio of their relative

[9] This two-sector model can be mapped into a three-sector model where the low-skill intensive sector is an aggregation of a consumption goods sector and a capital goods sector under the assumption that they have identical production functions except the sector-specific TFP index. In this environment, the relative price of capital $\phi$ is equal to the inverse of their relative TFPs, so a fall in $\phi$ is interpreted as an investment-specific technical change (Greenwood et al., 1997).

prices. This implies the ratio of the high-skill labor and capital satisfies:

$$\frac{H_j}{K_j} = (\chi\delta_j)^{-\rho}; \qquad \delta_j \equiv \frac{\kappa_j}{1-\kappa_j}, \quad \chi \equiv \frac{w_h}{q_k}, \quad j = h, l, \tag{16}$$

where $w_h$ is the high-skill wage and $q_k$ is the rental price of capital. Define $\tilde{I}_j$ as the ratio of the high-skill labor income relative to the sum of high-skill labor and capital income:

$$\tilde{I}_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j} = \frac{1}{1+\chi^{\rho-1}{\delta_j}^{\rho}}, \quad j = h, l, \tag{17}$$

where the last equality follows from the condition (16). Using the optimal condition across the high-skill and the low-skill labor, Appendix A2.2 shows that the relative skill-intensity is:

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta} (1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}} \tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}; \quad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}, \quad q \equiv \frac{w_h}{w_l}, \quad j = h, l, \tag{18}$$

where $q$ denotes the skill premium. Define $J_j$ as the low-skill income share and $I_j$ as the high-skill income share in sector $j = h, l$:

$$J_j \equiv \frac{w_l L_j}{q_k K_j + w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j (1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1}, \tag{19}$$

$$I_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j + w_l L_j} = (1-J_j)\,\tilde{I}_j. \tag{20}$$

Using (19) and (20), Appendix A2.2 derives the sectoral labor income share as:

$$\beta_j = I_j + J_j = J_j\left(q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j (1-\kappa_j)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right); \quad j = h, l. \tag{21}$$

## 3.2 Equilibrium prices and allocation

The equilibrium low-skill wage is equal to the value of the marginal product of low-skill labor $MPL_{lj}$ in sector $j$, which is derived in Appendix A2.2 as:

$$w_l = p_j MPL_{lj}; \qquad MPL_{lj} \equiv \frac{\partial Y_j}{\partial L_j} = A_j \left(J_j \xi_j^{-\eta}\right)^{\frac{1}{1-\eta}}, \tag{22}$$

and the real wage equation (5) is generalized as:

$$\frac{w_l}{P_c} = A_j \left(J_j \xi_j^{-\eta}\right)^{\frac{1}{1-\eta}} \frac{p_j}{P_C}. \tag{23}$$

The relative price is then derived from the free mobility of labor:

$$\frac{p_h}{p_l} = \left(\frac{A_l}{A_h}\right)\left(\frac{\xi_l}{\xi_h}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_h}{J_l}\right)^{\frac{1}{\eta-1}}, \tag{24}$$

which shows that faster productivity growth in the low-skill sector implies a falling relative price of the low-skill sector. This generates the negative relationship between relative price and relative productivity documented in Figure 2.

The equilibrium conditions derived above are functions of the relative factor prices $(\chi, q)$, where $q$ is derived as a function of $\chi$ in Appendix A2.3:

$$q = \chi \left[\left(\frac{\phi}{A_l}\right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(\chi^{1-\rho} + \delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}}. \tag{25}$$

Finally, Appendix A2.3 shows that the equilibrium of the model can be summarized by solving for $\chi$ and the share of low-skill labor in the high-skill sector $(l_h \equiv L_h/L)$ using two conditions:

$$l_h = S\left(\chi; \frac{H}{L}, \frac{\phi}{A_l}\right) \equiv \frac{\frac{H}{L} q^{\eta} \sigma_l^{-\eta}\left(1-\kappa_l\right)^{\frac{\rho(\eta-1)}{1-\rho}} \tilde{I}_l^{\frac{\eta-\rho}{\rho-1}} - 1}{\left(\frac{\sigma_h}{\sigma_l}\right)^{\eta}\left(\frac{1-\kappa_l}{1-\kappa_h}\right)^{\frac{\rho(\eta-1)}{1-\rho}}\left(\frac{\tilde{I}_l}{\tilde{I}_h}\right)^{\frac{\eta-\rho}{\rho-1}} - 1}. \tag{26}$$

$$l_h = D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right) \equiv \left[1 + \frac{J_l}{J_h}\left(\frac{1}{x\beta_l} + \frac{1-\beta_h}{\beta_l}\right)\right]^{-1}, \tag{27}$$

where the relative consumption expenditure share is derived from (4) and (24):

$$x \equiv \frac{p_h C_h}{p_l C_l} = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{J_h}{J_l}\left(\frac{\xi_l}{\xi_h}\right)^{\eta}\right)^{\frac{1-\varepsilon}{\eta-1}}; \quad \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}, \tag{28}$$

and the consumption expenditure shares are $x_l = 1/(1+x), x_h = x/(1+x)$. In a nutshell, the condition $S\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$ is derived using the labor market clearing conditions and the firm's optimization, and the condition $D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$ is derived using the goods market clearing conditions and the household's optimization. These two conditions together solve for $(\chi, l_h)$ and the skill premium $q$ is obtained from (25). Given $q$ and $\chi$, the low-skill wage is derived from (22) and the income shares are derived from (17), (19), and (20). Appendix A2.3 derives the value-added shares as:

$$v_h \equiv \frac{p_j Y_j}{\sum_j p_j Y_j} = \left[1 + \left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1}, \quad v_l = 1 - v_h, \tag{29}$$

which then deliver the aggregate labor income share as:

$$\beta = \beta_l v_l + \beta_h v_h. \tag{30}$$

## 3.3 Divergence

The accounting identity (9) shows that the divergence of the low-skill real wage from aggregate labor productivity is due to rising relative cost of living, falling labor income shares and rising wage inequality. Using the equilibrium conditions derived above, we now explain how the model can generate these three factors through faster productivity growth in the low-skill sector.

A faster productivity growth in the low-skill sector decreases the relative price of the low-skill goods (24) and increases the relative consumption share (28) given consumption complementarity ($\epsilon < 1$). This implies a reallocation of labor towards the high-skill sector (27), which acts as an endogenous skill-biased shift leading to a higher skill premium $q$ as in Buera et al. (2022).

The relative cost of living is measured by the price of aggregate consumption relative to the price of aggregate output, $P_C/P_Y$. These two price indexes can be obtained by the Tornqvist method using the consumption expenditure shares $x_j$ as weights for $P_C$ and the value-added shares $v_j$ as weights for $P_Y$. Given the consumption share of the high-skill sector exceeds its value-added share, the faster productivity growth in the low-skill sector implies a rise in the relative cost of living $P_C/P_Y$.[10]

The effect on the aggregate labor income share $\beta$ in (30) is ambiguous for two reasons. First, it predicts a rise in the skill premium which has two opposing effects on the sectoral labor income share $\beta_j$ derived in (21). More explicitly, it reduces the low-skill income share in (19) and increases the high-skill income share in (20) in both sectors. Second, there is an increase in the value-added share of the high-skill sector ($v_h$ in 29), which can lower the aggregate labor income share if $\beta_h < \beta_l$, and vice versa.

### 3.4 Low-skill wage and skill premium

The skill premium measures the high-skill wage relative to the low-skill wage. A rise in the skill premium does not necessarily imply a slower growth in the low-skill wage. In a similar vein, factors that imply a rise in the skill premium do not always imply a slower growth in the low-skill wage. Using the optimal capital-skill ratio in (16), the production function (10) can be expressed as a function of the high-skill and low-skill labor:

$$Y_j = \tilde{A}_j \left[ (1-\lambda_j) H_j^{\frac{\eta-1}{\eta}} + \lambda_j L_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{31}$$

$$\tilde{A}_j \equiv A_j \left( \xi_j + (1-\xi_j) \left( \frac{1-\kappa_j}{\tilde{I}_j} \right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)} \right)^{\frac{\eta}{\eta-1}}; \lambda_j \equiv \frac{\xi_j}{\xi_j + (1-\xi_j) \left( \frac{1-\kappa_j}{\tilde{I}_j} \right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}, \tag{32}$$

which takes a similar form as the aggregate production function used in the literature (see Katz and Murphy, 1992; Heathcote et al., 2010), where a decrease in $\lambda$ of the aggregate production function represents an aggregate skill-biased shift. Our model provides two *endogenous* sources for this aggregate skill-biased shift.

First, as in Buera et al. (2022), the predicted shift towards the high-skill sector

[10]The assumption that capital is only produced by the low-skill sector helps to simplify the model but what is necessary for the consumption share of the high-skill sector to be larger than its value-added share is that the low-skill sector contributes more to the production of capital. This is supported by findings of McGrattan (2020) which confirm that our high-skill intensive sectors provide a negligible portion of tangible and intangible capital to other industries.

implies a decrease in the aggregate $\lambda$ when $\lambda_h < \lambda_l$. This between-sector skill-biased shift is shown to be an important source for the increase in the aggregate skill intensity for understanding the rise in the skill premium. Second, as in Krusell et al. (2000), falling relative price of capital implies an increase in $\tilde{I}_j$ due to capital-skill complementarity. This implies a decrease in $\lambda_j$ acting as a within-sector skill-biased shift in both sectors.

Both shifts imply a rise in the skill premium but they have different effects on the low-skill wage. The between-sector shift induces a shift from the low-skill sector with high $\lambda_l$ to the high-skill sector with low $\lambda_h$, so it reduces the aggregate $\lambda$ contributing to a slower growth in the low-skill wage. The within-sector shift, through rising $\tilde{I}_j$, reduces $\lambda_j$ in both sectors but this effect is offset by the implied rise in the effective productivity $\tilde{A}_j$ due to the capital-skill complementarity (i.e. $\rho < 1$, see 32). Thus the falling relative price of capital contributes to a rise in the skill premium but not necessarily to the low-skill wage stagnation.

There are other sources of within-sector skill-biased shifts that can lower $\lambda_j$ through falling production weights $\kappa_j$ and $\xi_j$. A fall in $\kappa_j$ can reflect a skill-biased organizational change that increases the importance of human capital (Caroli and Van Reenen, 2001).[11] Similar to the role of the falling relative price of capital, a fall in $\kappa_j$ reduces $\lambda_j$ but also implies a rise in the effective productivity $\tilde{A}_j$, resulting in an ambiguous effect on the low-skill wage. A fall in $\xi_j$, however, implies a fall in both $\lambda_j$ and $\tilde{A}_j$ when high-skill and low-skill labor are good substitutes ($\eta > 1$). Thus, it can contribute to both a rise in the skill premium and the low-skill wage stagnation. The decline in the production weights for low-skill workers can be due to the displacement effect from automation in the task-based model of Acemoglu and Restrepo (2018) and the outsourcing of tasks performed by low-skilled workers in Grossman and Rossi-Hansberg (2008).

[11] In general, it contributes to the skill-enhancing changes in the standard canonical skill-biased technical change model (Katz and Murphy, 1992) without capital.

### 3.5 Demand shift towards high-skill intensive goods

In addition to uneven productivity growth, a demand shift towards high-skill intensive goods can also act as a source for the between-sector skill-biased shift. This demand shift can be induced by rising income if high-skill intensive goods have a higher income elasticity. As shown by Comin et al. (2021), a fall in the preference parameter $\psi$ in the homothetic CES utility function (2) can capture this income effect in a more general non-homothetic CES utility function.[12] Thus, by examining the effect of a fall in $\psi$, we can learn about the effect of a demand shift towards the high-skill sector on the low-skill wage.

Using (28), a fall in $\psi$ implies an increase in $\hat{A}_{lh}$ and a rise in relative expenditure; thus, it has a similar effect on the skill premium as the increase in the relative productivity $A_l/A_h$. However, it does not have a direct effect on the relative prices of the high-skill intensive sector as shown in equation (24), nor the low-skill real wage in (23).[13] Its contribution to the divergence is through the increase in the skill premium, which is similar to the effect of a skill-biased shift through $\xi_j$. Thus, we let the calibration of $\xi_j$ pick up its role as a skill-biased demand shift.

# 4 Quantitative Results

The model is calibrated to match the key features of the U.S. labor market from 1980 to 2010. To evaluate the quantitative role of uneven productivity growth, the baseline also includes changes in the relative price of capital, the production weights of the low-skill labor and the high-skill labor, and the relative supply of the high-skill labor. The productivity parameters are calibrated to match the rise in the relative price of the high-skill intensive sector and the aggregate labor productivity growth. The production weights are set to match the sectoral income shares while the relative supply of high-skill labor is set to match the aggregate

[12]This can be seen explicitly from comparing the relative expenditure derived in (4) with the relative expenditure derived from a non-homothetic CES utility function in Comin et al. (2021).

[13]It has an equilibrium effect on the relative price through the rise in $q$ by changing $J_h/J_l$ in (24), but the effect is small as it depends on the difference between the parameters $\xi_h$ and $\xi_l$ as shown in (19).

Table 3: *Data targets*

| | Level | | | | | | | Growth (% p.a.) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $J$ | $J_h$ | $J_l$ | $I$ | $I_h$ | $I_l$ | $q$ | $y/P_Y$ | $\frac{p_h}{p_l}$ | $\phi$ |
| 1980 | 0.41 | 0.23 | 0.46 | 0.17 | 0.33 | 0.12 | 1.44 | - | - | - |
| 2008 | 0.28 | 0.21 | 0.31 | 0.28 | 0.44 | 0.21 | 1.94 | 1.7 | 1.4 | -0.5 |

Note: $J$'s are the low-skill income share, $I$'s are the high-skill income share, $q$ is the skill premium. High-skill are those with college or a higher degree. $y/P_Y$ is the aggregate real labor productivity, $p_h/p_l$ is the price of high-skill sector relative to the low-skill sector and $\phi$ is the price of capital relative to the low-skill sector.

income of the high-skill labor relative to the low-skill labor. The predictions in the baseline are driven by changes in five sets of parameters: $\hat{A}_{lh}$ in equation (28), the relative price of capital $\phi$, the production weights $\{\xi_l, \xi_h, \kappa_l, \kappa_h\}$ in (10), and the relative supply of the high-skill labor $H/L$.[14]

## 4.1 Data targets

The construction of data targets reported in Table 3 is described in Appendix A1. Data from the five-year average 1978–1982 is used for the year 1980 and 2006–2010 for the year 2008. During this period, the high-skill income share ($I_j$) increases while the low-skill income share ($J_j$) decreases in both sectors. The total labor income share ($\beta_j = I_j + J_j$) falls in the low-skill sector but rises in the high-skill sector, and the aggregate labor income share ($I + J$) falls.

The annual growth rate of the aggregate real labor productivity is 1.7% and the relative price of the high-skill sector is 1.4%. Using the ratio of $P_K/P_Y$ from the BEA and the ratio $P_Y/p_l$ from the KLEMS, the price of capital relative to the low-skill sector ($\phi$) declines at 0.5% per year.[15]

[14]Given the definition of $\hat{A}_{lh}$ in equation (A2), we do not need to separate the preference parameter $\psi$ from $A_l/A_h$ to solve for the model.

[15]It is worth noting that the growth of $P_Y$ in KLEMS is 2.94%, which is almost identical to that of BEA at 2.86%.

## 4.2 Calibration

The elasticity of substitution across high-skill and low-skill labor, $\eta = 1.4$, is taken from Katz and Murphy (1992). The elasticity of substitution across capital and high-skill labor, $\rho = 0.67$, is taken from Krusell et al. (2000). There is no direct estimate of the elasticity of substitution across high-skill and low-skill goods, $\varepsilon$. The literature on structural transformation finds that the elasticity of substitution across agriculture, manufacturing, and services is close to zero (Herrendorf et al., 2013). Given that we re-group these three sectors into two sectors, this likely implies a higher degree of substitution. Ngai and Pissarides (2008) report a range of estimates for the price elasticity of services from -0.3 to 0, which is informative but not an exact estimate for $-\varepsilon$, the price elasticity of the high-skill sector in our model. Based on these estimates, we use $\varepsilon = 0.2$ as our baseline value for the elasticity of substitution across the two sectors.[16]

The relative supply of high-skill labor $(H/L)$ is obtained from the data on the skill premium and income shares $(q_t, I_t, J_t)$.[17] Appendix A2.4 reports the calibration procedure for the remaining parameters. The calibration strategy is as follows: the production weights $(\xi_j, \kappa_j)$ are set to match sectoral income shares in the data for any given value of $\phi/A_l$. To simplify the explanation, denote 1980 as period 0 and 2008 as period $T$. We show that $\phi_0/A_{l0}$ can be normalized to 1 and obtain all production weights in period 0. Using these parameters, condition (26) implies a value of $l_{h0}$, and condition (27) implies a value of $\hat{A}_{lh0}$ given $q_0$. For a given level of $A_{lT}/A_{l0}$, data on the fall in $\phi_t$ implies a value for $\phi_T/A_{lT}$, which pins down all production weights in period $T$. We then set the change in $A_{lhT}/A_{lh0}$ to match the increase in the relative price of the high-skill sector. Finally, $A_{lT}/A_{l0}$ is adjusted to match the change in aggregate labor productivity deflated by the price of the low-skill sector.

Table 4 reports the calibrated parameters. The implied annual growth of $\phi$,

[16] The quantitative results are not sensitive to small changes in the values of elasticity parameters $(\varepsilon, \eta, \rho)$.

[17] The $H_j$ and $L_j$ are not the raw market hours by the high-skill and low-skill workers in the data. The composition-adjusted high-skill hours $H_j$ in sector $j$ are computed as the high-skill income in sector $j$ divided by the composition-adjusted high-skill wage; similarly for $L_j$.

$A_{lh}$, $A_l$, $H/L$, and production weights $(\kappa_j, \xi_j)$ are reported in Panel B of Table 4.[18] Matching the rise in the relative price of the high-skill sector implies faster productivity growth in the low-skill sector.[19] Matching the relative aggregate income shares of the high-skill and low-skill labor implies a rise in the relative supply of high-skill labor. Matching the sectoral income shares, on the other hand, requires changes in the production weights reflecting other sources of skill-biased shifts. The growth in the relative productivity $A_l/A_h$ is governed by the rise in the relative price of the high-skill sector.[20] Using WORLD KLEMS, we find that the relative price of the high-skill sector grew by 49% during 1980–2008.[21] This, together with the observed growth in aggregate productivity, determines the growth in the sectoral productivity parameters $(A_l, A_h)$. It is reassuring to report that the baseline calibration implies labor productivity growth of 2.2% for the low-skill sector and -0.2% for the high-skill sector, closely matching the 2.3% and 0.1% observed in the data.

## 4.3 Results on sectoral shares and skill premium

As reported in row 2 of Table 5, the baseline does a good job in matching the rise in the skill premium, the pattern of sectoral reallocation and the changes in labor share in each sector. The remaining rows of Table examine each of the five forces that drive these changes by shutting them down one at a time: the uneven sectoral productivity growth (higher $A_l/A_h$) in row 3, the falling relative price of capital $(\phi)$ in row 4, the falling production weights of low-skill labor $(\xi_l, \xi_h)$ in row 5, the rising production weights of high-skill labor within the capital-skill composite

[18]The implied negative growth in $\kappa_j$ does not necessarily indicate a decrease in the usage of capital. It only implies a fall in the input weight of capital in the capital-skill composite.

[19]The calibration implies that $A_h$ is falling, which can be understood using the findings of Aum et al. (2018) and Bárány and Siegel (2021). The former paper finds negative productivity growth for the high-skill occupations (Professional and Management), while the latter finds negative growth for abstract occupations. Their findings could be the source for the falling $A_h$ given these occupations are concentrated in the high-skill intensive sector.

[20]If we were to halve the increase in the relative price of the high-skill sector, the uneven productivity growth across sectors would remain quantitatively important, albeit to a smaller extent. This result is available upon request.

[21]Buera et al. (2022) also use WORLD KLEMS to report an increase of 46% in the relative price of the high-skill sector during 1977-2005 in the U.S.

Table 4: *Calibrated parameters*

| A. Parameters from the literature | | | | |
|---|---|---|---|---|
| Parameters | Values | | | Source |
| $\varepsilon$ | 0.2 | | | Benchmark value, see main text |
| $\rho$ | 0.67 | | | Krusell et al. (2000) |
| $\eta$ | 1.4 | | | Katz and Murphy (1992) |
| **B. Calibrated parameters** | | | | |
| Parameters | 1980 | 2008 | Growth (% p.a.) | Target |
| $\phi$ | | | -0.50 | Price of capital relative to the low-skill goods |
| $A_l$ | | | 1.09 | Aggregate real labor productivity |
| $A_{lh}$ | | | 1.82 | Relative price of the high-skill sector |
| $\xi_l$ | 0.33 | 0.25 | -0.93 | Sectoral income share. See Appendix A2.4 |
| $\xi_h$ | 0.20 | 0.19 | -0.13 | Sectoral income share. See Appendix A2.4 |
| $\kappa_l$ | 0.74 | 0.69 | -0.21 | Sectoral income share. See Appendix A2.4 |
| $\kappa_h$ | 0.41 | 0.33 | -0.79 | Sectoral income share. See Appendix A2.4 |
| $H/L$ | 0.29 | 0.50 | 1.92 | Relative aggregate labor income shares $I_t/J_t$ |

$(\kappa_l, \kappa_h)$ in row 6, and the increase in the relative supply of high-skill labor (higher $H/L$) in row 7. It is important to note that in order to match the increase in aggregate productivity at 60%, the growth in $A_l$ has to be adjusted in each of row 3 to 7. More specifically, for row 3, fixing the relative productivity $A_l/A_h$ at the 1980 level requires the same productivity growth in both sectors. This implies a large growth in $A_h$ if we keep the growth in $A_l$ as in the baseline, which would imply a larger increase in aggregate productivity (85% instead of 60%). Thus, we lower the growth in $A_l$ so that the implied change in aggregate productivity growth is the same as in the baseline at 60%.

The results confirm the intuition that uneven productivity growth (row 3) is crucial for sectoral reallocation. In a world with balanced productivity growth, there would be no reallocation of low-skill labor and the value-added shares of the high-skill sector would have fallen. While the fall in production weights of low-skill labor (row 5) is essential for the decrease in the labor share in the low-skill sector, the rise in the production weights of high-skill labor (row 6) is important for the increase in the labor share in the high-skill sector. Not surprisingly, the increase in the relative supply of high-skill labor contributes to lowering the skill premium.

Consistent with the previous literature, all mechanisms are important for the rise in the skill premium: uneven productivity growth (Buera et al., 2022), a

Table 5: *Sectoral shares and the skill premium*

| | | Sectoral reallocation | | | Sectoral labor share | | Skill premium |
|---|---|---|---|---|---|---|---|
| | | $l_h$ | $h_h$ | $v_h$ | $\beta_l$ | $\beta_h$ | $q$ |
| | Data 1980 | 0.14 | 0.46 | 0.24 | 0.59 | 0.56 | 1.44 |
| (1) | Data 2008 | 0.21 | 0.46 | 0.29 | 0.53 | 0.65 | 1.94 |
| (2) | Model 2008 | 0.20 | 0.45 | 0.28 | 0.53 | 0.65 | 1.92 |
| | *Counterfactual (fixing each parameter to its 1980 value)* | | | | | | |
| (3) | $A_l/A_h$ | 0.15 | 0.36 | 0.21 | 0.52 | 0.64 | 1.79 |
| (4) | $\phi$ | 0.19 | 0.44 | 0.27 | 0.52 | 0.62 | 1.71 |
| (5) | $\xi_l, \xi_h$ | 0.18 | 0.52 | 0.31 | 0.59 | 0.64 | 1.51 |
| (6) | $\kappa_l, \kappa_h$ | 0.19 | 0.42 | 0.26 | 0.49 | 0.59 | 1.68 |
| (7) | $H/L$ | 0.24 | 0.48 | 0.31 | 0.56 | 0.68 | 3.19 |

Note: Row 3 is the relative productivity. Row 4 is the relative price of capital. Row 5 are the weights of the low-skill labor in (11). Row 6 are the weights of capital in (12). Row 7 is the relative supply of the high-skill labor. The productivity growth of the low-skill sector is adjusted in row 3 to row 7 to match the 60% increase in the aggregate productivity.

falling relative price of capital(Krusell et al., 2000), the falling production weights of low-skill workers (Goldin and Katz, 2009), and increasing production weights of high-skill labor (Katz and Murphy, 1992). However, as discussed in Section 3.4, these mechanisms can have different implications on wage-productivity divergence and the growth of low-skill wages.

## 4.4 Results on divergence

Table 6 presents the results on the wage-productivity divergence and the three contributing factors shown in (9): wage inequality, aggregate labor share, and relative cost of living. Since the KLEMS data does not contain information on consumption, we take $P_C/P_Y$ as the ratio of PCE and GDP implicit deflators from the BEA. This implies that $P_C/P_Y$ increased by 2.8%. If we were to use the CPI, the increase in $P_C/P_Y$ would be 11.5%. This alternative value would imply a larger divergence and slower real wage growth in the data but does not affect other rows. Due to the concern that CPI tends to bias the increase in the cost of living (Boskin et al., 1998), we use the $P_C/P_Y$ implied by the PCE deflator as the main data moment for comparison but keep those implied by the CPI in brackets.

Row 1 of Table 6 reports an empirical decomposition for the accounting identity in equation (9). During this 30-year period, the negative forces imposed by the rising relative cost of living, growing wage inequality, and falling aggregate labor income share largely offset the impact of rising productivity on low-skill real wages. The rise in the relative cost of living contributes 10% (=2.8/27) of the divergence, the increase in wage inequality contributes 70% (=19/27), and the fall in the aggregate labor income share accounts for the remaining 20%. If the CPI is used, the contribution of the relative cost of living increases to 30% while the contribution of the rise in wage inequality reduces to 50%.

The baseline (row 2) can account for all the rise in wage inequality and the fall in the aggregate labor share, given it matches the skill premium, sectoral shares, and sectoral labor shares in Table 5. It over-predicts (under-predicts) the relative cost of living, thus slightly over-predicts (under-predicts) the divergence, if PCE (CPI) is used as the consumption deflator.

Row 3 and row 5 demonstrate that the faster productivity growth of the low-skill sector and the falling production weights of low-skill labor (especially the fall in the low-skill sector) are the two most important factors for the divergence. In their absence, the predicted divergence would be reduced to almost half and a third of the baseline, respectively. However, the two mechanisms work through different channels. While both contribute to predicting higher wage inequality, uneven productivity is important for the rise in the relative cost of living, whereas the fall in low-skill production weights is important for the fall in the aggregate labor share. The result that sectoral reallocation induced by uneven productivity does not contribute to the fall in the aggregate labor share is consistent with the finding that the fall in the aggregate labor share in the U.S. is primarily a within-industry phenomenon (Karabarbounis and Neiman, 2014; Elsby et al., 2013; Hubmer, 2023).

The increase in the relative supply of high-skill labor in row 7 plays an important role in wage inequality. In its absence, the increase in wage inequality would

Table 6: *Divergence: low-skill real wage and aggregate real labor productivity (percentage change, 1980–2008)*

| | | | Factors for Divergence | | |
|---|---|---|---|---|---|
| | | Divergence | Wage inequality | Labor share | Living cost |
| (1) | Data | 27 (38) | 19 | -3.4 | 2.8 (12) |
| (2) | Model | 34 | 19 | -3.7 | 8.2 |
| | *Counterfactual (fixing each parameter to its 1980 value)* | | | | |
| (3) | $A_l/A_h$ | 19 | 15 | -5.9 | -2.7 |
| (4) | $\phi$ | 29 | 12 | -6.0 | 7.8 |
| (5) | $\xi_l, \xi_h$ | 10 | 6.2 | 4.9 | 8.7 |
| (6) | $\kappa_l, \kappa_h$ | 37 | 12 | -10 | 9.5 |
| (7) | $H/L$ | 47 | 36 | 2.4 | 11 |

Note: Divergence is measured as the percentage change in the ratio of real labor productivity divided by the low-skill real wage. The three factors for the divergence are shown in (9). For the data row, the real wage is calculated using PCE as $P_C$ and the number in bracket uses CPI. Living cost is $P_C/P_Y$. Row 3 is the relative productivity. Row 4 is the relative price of capital. Row 5 are the weights of the low-skill labor in (11). Row 6 are the weights of capital in (12). Row 7 is the relative supply of the high-skill labor. The productivity growth of the low-skill sector is adjusted in row 3 to row 7 to match the 60% increase in the aggregate productivity.

have been doubled, but the labor share would have increased.[22] The latter offsets some of the rise in the divergence implied by higher wage inequality. The falling relative price of capital in row 2 also contributes to the divergence by predicting a rise in wage inequality. Finally, the increasing weight of high-skill labor through falling $\kappa$ (row 6) has an insignificant impact on the divergence. In its absence, wage inequality would have increased by less, while the labor share would have fallen by more, generating two opposing effects on the divergence.

## 4.5 Results on the low-skill wage stagnation

While Table 6 shows that all parameters (except $\kappa_j$) are important for the divergence, Table 7 reveals that only two factors are responsible for low-skill wage stagnation: the faster productivity growth of the low-skill sector (row 3) and the falling production weights of low-skill labor (row 5). In the absence of these two

[22]Its impact on the labor share is due to capital-skill complementarity, where a higher relative supply of high-skill labor increases the capital income share.

Table 7: *Low-skill real wage (percentage change, 1980–2008)*

| | | Low-skill real wage | $MPL_l$ | | Rel. price |
|---|---|---|---|---|---|
| | | $w_l/P_C$ | $w_l/p_l$ | $w_l/p_h$ | $p_h/p_l$ |
| (1) | Data | 26 (16) | 44 | -3.4 | 49 |
| (2) | Model | 20 | 44 | -3.4 | matched |
| | *Counterfactual (fixing each parameter to its 1980 value* | | | | |
| **(3)** | $\boldsymbol{A_l/A_h}$ | **35** | **27** | **48** | **-15** |
| (4) | $\phi$ | 24 | 47 | 1.4 | 45 |
| **(5)** | $\boldsymbol{\xi_l, \xi_h}$ | **45** | **79** | **15** | **56** |
| (6) | $\kappa_l, \kappa_h$ | 17 | 43 | -7.0 | 54 |
| (7) | $H/L$ | 8.7 | 40 | -18 | 70 |

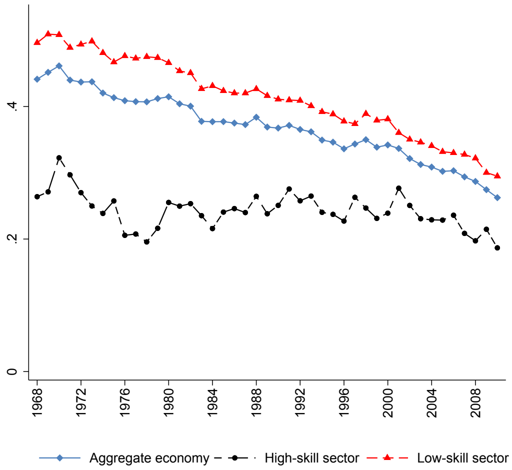
Note: For the data row, the low-skill real wage is calculated using PCE as $P_C$ and the number in bracket is when CPI is used as $P_C$. $MPL_l$ is the marginal product of low-skill labor. Row 3 is the relative productivity. Row 4 is the relative price of capital. Row 5 are the weights of the low-skill labor in (11). Row 6 are the weights of capital in (12). Row 7 is the relative supply of the high-skill labor. The productivity growth of the low-skill sector is adjusted in row 3 to row 7 to match the 60% increase in the aggregate productivity.

factors, the percentage increase in the low-skill real wage would have been more than double.

The key difference between row 3 and row 5 is their different implications for the marginal product of low-skill labor, $MPL_{lj} = w_{lj}/p_j$. In the data, $MPL_l$ rose by 44% in the low-skill sector but fell in the high-skill sector (due to the rise in the relative price of the high-skill sector). Uneven productivity growth is the main mechanism to deliver this result. In its absence, $MPL_l$ would have increased more in the high-skill sector. Another difference between the two mechanisms is the predicted timing when the low-skill wage lags behind the high-skill wage and aggregate productivity (see Figure 1). As discussed in Section 2.3, the uneven productivity growth mechanism is consistent with the beginning of the rise in the skill premium and the divergence starting in 1980. On the other hand, the production weights of low-skill labor $(\xi_l, \xi_h)$ are determined by the low-skill income shares $(J_l, J_h)$, which have been falling throughout 1968–2010 (see Figure 3).

Finally, the rise in the relative supply of high-skill labor (row 7) increases the growth of the low-skill real wage by increasing the $MPL_l$ in both sectors. In its

Figure 3: *Trends in low-skill labor income share*




Note: Panel A (B) shows the share of low-skill (high-skill) labor income in the value-added of aggregate economy, the high-skill sector, and the low-skill sector. See Appendix A1. Source: WORLD KLEMS and CPS.

absence, the growth in the low-skill real wage would have been halved. Consistent with the two opposing effects discussed in Section 3.4, the falling relative price of capital and the growth in the production weight of high-skill labor have not had a significant impact on the low-skill real wage, despite their important roles in predicting the rise in the skill premium. These quantitative exercises demonstrate that factors important for the rise in the skill premium do not necessarily contribute to low-skill wage stagnation.

## 4.6 Sources of aggregate productivity growth

As discussed in the motivation Section 2.2, one important message of the multi-sector perspective is that the source of aggregate productivity growth is crucial for understanding low-skill wage stagnation. This can be seen by comparing row 3 with row 2 in Tables 6 and 7. In the baseline, aggregate productivity is driven purely by productivity growth in the low-skill sector. The lack of productivity growth in the high-skill sector was an outcome of matching the rise in the relative price of the high-skill sector. This, as explained in Section 2.2, plays an important role in explaining low-skill stagnation. The counterfactual exercise in row 3 assumes the increase in aggregate productivity is due to *balanced* productivity growth in both sectors. This removes the rise in the relative price of the high-skill sector, predicting a much lower divergence in Table 6 because it predicts much higher growth in the low-skill real wage in Table 7. This suggests that the future of the low-skill wage relies on improving the productivity growth of the high-skill sector.

# 5 Conclusion

Despite predominantly working in sectors with fast productivity growth, low-skill workers experience slow real wage growth that lags behind the aggregate labor productivity. We argue that this phenomenon is attributable to the declining relative price of low-skill sectors, driven by their faster productivity growth.

A key insight from our multisector perspective is the importance of the source of aggregate labor productivity growth. When it originates in low-skill intensive sectors, it contributes to low-skill real wage stagnation and its divergence from aggregate labor productivity. Conversely, when it stems from high-skill intensive sectors or results from a balanced increase across both sectors, it can simultaneously boost the growth of low-skill real wages and aggregate labor productivity. In light of recent developments in artificial intelligence, which are expected to enhance productivity growth in high-skill intensive services, our view is that such development can accelerate low-skill real wage growth by decelerating the increase in the relative price of high-skill services.

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# Appendix

# A1 Data Appendix

## A1.1 Industry data and mapping

**National data:** The March 2017 Release of the WORLD KLEMS database reports industry value-added, price indexes, labor compensation, and capital compensation. The data are reported using the North American Industry Classification System (NAICS), which is the standard used by Federal statistical agencies in classifying business establishments in the U.S.

To classify sectors into the high-skill intensive sector and the low-skill intensive sector, we use the April 2013 Release of the WORLD KLEMS, which provides a labor input file that allows the computation of the low-skill and high-skill workers' shares in labor compensation and value-added. High-skill is defined as education greater than or equal to a college degree. Table A1 reports the long-run (1980–2010) average of the share of high-skill labor in the total value-added and total labor income for 15 one-digit industries. A sector is included in the high-skill intensive sector if the long-run high-skill labor income share out of the total labor income and the total value-added are above the total economy average. The high-skill intensive sector includes finance, insurance, government, health, and education services (codes J, L, M, N), and the remaining industries are grouped into the low-skill intensive sector. Due to the low number of observations in CPS we regroup the industries into 11 sectors. We merge agriculture (AtB) with mining (C) and other services (O) with private households (P), and public administration (L) with education (M), and health and social work (N) to ensure consistency in industry definitions. Our mapping across KLEMS 2013, KLEMS 2017, and CPS industries is provided in Table A2.

Using this classification we map the 65 NAICS industries of the KLEMS 2017

Table A1: *High-skill income shares by industry, 1980–2010 average*

| Industry | Code | High-skill share in Value-added | High-skill share in Labor income |
|---|---|---|---|
| Agriculture, Hunting, Forestry and Fishing | AtB | 10 | 19 |
| Mining and Quarrying | C | 11 | 32 |
| Total Manufacturing | D | 20 | 31 |
| Electricity, Gas and Water Supply | E | 9 | 30 |
| Construction | F | 14 | 16 |
| Wholesale and Retail Trade | G | 22 | 30 |
| Hotels and Restaurants | H | 14 | 18 |
| Transport and Storage and Communication | I | 16 | 25 |
| **Financial Intermediation** | **J** | **33** | **55** |
| Real Estate, Renting and Business Activity | K | 21 | 55 |
| **Public Admin** | **L** | **29** | **40** |
| **Education** | **M** | **58** | **77** |
| **Health and Social Work** | **N** | **39** | **49** |
| Other Community, Social and Personal Services | O | 23 | 31 |
| Private Households with Employed Persons | P | 16 | 16 |
| All Industries | TOT | 25 | 40 |

Note: The table reports the share of high-skill workers in total value-added and total labor income by industry. High-skill is defined as education greater than or equal to college degree. Labor income reflects total labor costs which includes compensation of employees, compensation of self-employed, and taxes on labor. Source: April 2013 Release of the WORLD KLEMS for the U.S.

Release and the three-digit *ind1990* codes of the CPS into the two broad sectors for our quantitative analysis. Sectoral value-added prices are calculated as Tornqvist indexes. For the ratio of aggregate consumption price deflator and output price deflator, we use the BEA's implicit price deflators of GDP and Personal Consumption Expenditures, respectively. The price of capital is calculated as the investment in total fixed assets divided by the chain-type quantity index for investment in total fixed assets (Tables 1.5 and 1.6 of the BEA's Fixed Assets Accounts).

**State-level data:** In Table 2, We use GDP by state from the BEA's Regional Economic Accounts for value-added sector prices at the state level. The BEA reports nominal and real GDP (chained at constant dollars) by industry for 51 states by SIC between 1963-1997, and by NAICS between 1997-2010. To calculate sectoral prices, we first aggregate the industry data into 11 sectors according to

Table A2. Next, using the common year of observation 1997, we carry forward the SIC-based series by the growth rates of the NAICS-based series. Finally, we calculate sectoral price indexes as the ratio of nominal to real GDP. Our bridging strategy produces national sectoral growth rates similar to those reported in the KLEMS data. In particular, the correlation coefficients between the long-run US-level sectoral growth rates from both sources are 0.97, 0.91, and 0.90 for nominal value-added, real value-added, and prices, respectively. In order to have sufficient number of observations at the state-level, we use Census and ACS data for sectoral hours and compensation.

## A1.2 Wages, efficiency hours, and productivity

For Table 1, we use March Current Population Survey Annual Social and Economic Supplement (ASEC) data from 1978 to 2012. Our sample includes wage and salary workers with a job aged 16-64, who are not students, retired, or in the military. Hourly wage is calculated as annual wage income divided by annual hours worked, where the latter is the product of weeks worked in the year preceding the survey and hours worked in the week prior to the survey. Top-coded components of annual wage income are multiplied by 1.5. Workers with weekly wages below $67 in 1982 dollars are dropped.

The composition-adjusted mean wages of low-skill workers are computed as follows. Within each sector, we calculate mean wages weighted by survey weights for each of 216 subgroups composed of two sexes, white and non-white categories, three education categories (high school dropout, high school graduate, some college), six age categories (16-24, 25-29, 30-39, 40-49, 50-59, 60-64 years), and three occupation categories (high-wage occupations including professionals, managers, technicians, and finance jobs, middle-wage occupations including clerical, sales, production, craft, and repair jobs, operators, fabricators, and laborers, and low-wage occupations including service jobs). The long-run hours shares of each subgroup are used as weights to calculate the low-skill wage at the industry level. Cells containing missing wages are imputed for each year of the dataset using a regression of the log of hourly wages on industry dummies and dummies including the full set of interactions of subgroups. We assign predictions from this regression to the missing wage observations while keeping the observed wages. The growth rate of sector wages with and without imputation are very close.

For Sections 2.3 and 4, the aggregate wage has to be consistent with the measure of aggregate productivity, so we use the aggregate labor compensation and aggregate hours from KLEMS. More specifically, to compute the composition-adjusted wage for the average high-skill worker and the average low-skill worker, we merge KLEMS 2013 data on total labor compensation and hours with the distribution of demographic subgroups in the CPS. We form 120 subgroups based on two sexes, two races, five education levels, and six age categories. Low-skill includes high school dropout, high school graduate, and some college; high-skill includes college graduates and post-college degree categories. Compensation for each subgroup is calculated as compensation share (from CPS) times total compensation (from KLEMS). The hours worked of each subgroup is calculated in a similar way. The wage for each subgroup is then calculated as total compensation divided by total hours. The aggregate low-skill and high-skill wages are calculated as the average of the relevant subgroups using their long-run (1980–2010) hours shares as weights. It is important to note that the labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours in KLEMS are adjusted for the self-employed. Thus, KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. This procedure is equivalent to rescaling the CPS total hours and total wage income to sum up to KLEMS total.

Efficiency hours, corresponding to $(H, L)$ in the model, are computed as the labor compensation divided by the composition-adjusted wage for high-skill workers and low-skill workers, respectively. Total efficiency hours are the sum of low- and high-skill efficiency hours. We calculate real labor productivity as total value-added divided by total efficiency hours and deflate with the output price index.

Table A2: *Industry mapping*

| NACE (KLEMS 2013) | NAICS (KLEMS 2017) | IND1990 (CPS) |
|---|---|---|
| AtB & C | Farms, Forestry, Fishing, and Related Activities, Oil and Gas Extraction, Mining, Except Oil and Gas, Support Activities for Mining | Agriculture, Forestry, and Fisheries, Mining |
| D | Wood Products, Nonmetallic Mineral Products, Primary Metals, Fabricated Metal Products, Machinery, Computer and Electronic Products, Electrical Equipment, Appliances, and Components, Motor Vehicles, Bodies and Trailers, and Parts, Other Transportation Equipment, Furniture and Related Products, Miscellaneous Manufacturing, Food and Beverage and Tobacco Products, Textile Mills and Textile Product Mills, Apparel and Leather and Allied Products, Paper Products, Printing and Related Support Activities, Petroleum and Coal Products, Chemical Products, Plastics and Rubber Products | Manufacturing |
| E | Utilities | Utilities |
| F | Construction | Construction |
| G | Wholesale Trade, Retail Trade | Wholesale Trade, Retail Trade |
| H | Accommodation, Food Services and Drinking Places | Hotels and Lodging Places, Eating and Drinking Places |
| I | Air Transportation, Rail Transportation, Water Transportation, Truck Transportation, Transit and Ground Passenger Transportation, Pipeline Transportation, Other Transportation and Support Activities, Warehousing and Storage, Publishing Industries, Except Internet (Includes Software), Motion Picture and Sound Recording Industries, Broadcasting and Telecommunications, Data Processing, Internet Publishing, and Other Information Services | Transportation, Communications |
| J | Federal Reserve Banks, Credit Intermediation, and Related Activities, Securities, Commodity Contracts, and Investments, Insurance Carriers and Related Activities, Funds, Trusts, and Other Financial Vehicles | Finance, Insurance |
| K | Real Estate, Rental and Leasing Services and Lessors of Intangible Assets, Legal Services, Computer Systems Design and Related Services, Miscellaneous Professional, Scientific, and Technical Services, Management of Companies and Enterprises, Administrative and Support Services, Waste Management and Remediation Services | Real Estate, Business Services, Professional Services* |
| L & M & N | Educational Services, Ambulatory Health Care Services, Hospitals and Nursing and Residential Care Facilities, Social Assistance, Federal General Government, Federal Government Enterprises, State and Local General Government, State and Local Government Enterprises | Public Administration, Education*, Health and Social Services* |
| O & P | Performing Arts, Spectator Sports, Museums, and Related Activities, Amusements, Gambling, and Recreation Industries, Other Services, Except Government | Sanitary and Personal Services, Private Households, Entertainment and Recreation Services, Museums, Art Galleries, and Zoos, Labor Unions, Religious Organizations, Membership Organizations, n.e.c. |

Note: The table shows the mapping of KLEMS 2013 industries to KLEMS 2017 and CPS industries. The description of KLEMS 2013 industries is provided in Table A1. Industries marked with * do not have separate sections in CPS industry classification. They are constructed as follows. Professional Services: Engineering, architectural, and surveying services, Accounting, auditing, and bookkeeping services, Research, development, and testing services, Management and public relations services, Miscellaneous professional and related services, Legal services. Education: Elementary and secondary schools, Colleges and universities, Vocational schools, Educational services, n.e.c. Health and Social Services: Offices and clinics of physicians, Offices and clinics of dentists, Offices and clinics of chiropractors, Offices and clinics of optometrists, Offices and clinics of health practitioners, n.e.c., Hospitals, Nursing and personal care facilities, Health services, n.e.c., Job training and vocational rehabilitation services, Child day care services, Family child care homes, Residential care facilities without nursing, Social services, n.e.c.

# A2 Model Appendix

## A2.1 Household optimization

Equating the marginal rate of substitution to the relative price:

$$\frac{c_{ih}}{c_{il}} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \tag{A1}$$

thus the relative consumption share is given by

$$x \equiv \frac{p_h c_{ih}}{p_l c_{il}} = \left(\frac{p_h}{p_l}\right)^{1-\varepsilon}\left(\frac{1-\psi}{\psi}\right)^{\varepsilon}. \tag{A2}$$

Using the budget constraint to derive individual's demand:

$$p_l c_{il} = x_l w_i; \quad p_h c_{ih} = x_h w_i; \quad x_l \equiv \frac{1}{1+x}, x_h \equiv \frac{x}{1+x}, \tag{A3}$$

Aggregating across households to obtain (4).

## A2.2 Equilibrium prices

Equating the marginal rate of technical substitution to the relative wage:

$$q = \sigma_j (1-\kappa_j)\left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}}\left(\frac{G_j(H_j, K_j)}{H_j}\right)^{\frac{\eta-\rho}{\rho\eta}}; \qquad \sigma_j \equiv \frac{1-\xi_j}{\xi_j} \tag{A4}$$

where, using equation (16):

$$\frac{G_j(H_j, K_j)}{H_j} = \left[\kappa_j\left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j)\right]^{\frac{\rho}{\rho-1}} = (1-\kappa_j)^{\frac{\rho}{\rho-1}}\left(\delta_j^{\rho}\chi^{\rho-1}+1\right)^{\frac{\rho}{\rho-1}} = \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}}. \tag{A5}$$

Substituting into (A4) to obtain (18). Given $I_j = (1-J_j)\,\tilde{I}_j$, using (17) and (19),

$$I_j = \frac{\tilde{I}_j}{1 + q^{\eta-1}\sigma_l^{-\eta}\left[\tilde{I}_j(1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{\rho-1}}} \tag{A6}$$

Using (17) and (19), $\beta_j$ in (21) is obtained from $\beta_j = I_j + J_j = (1-J_j)\,\tilde{I}_j + J_j$.

**Equilibrium low-skill wage $w_l$ :** Using the production function:

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j}=\left(\left(1-\xi_j\right)\left(\frac{G_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+\xi_j\right)^{\frac{\eta}{\eta-1}}=\xi_j^{\frac{\eta}{\eta-1}}\left(\sigma_j\left(\frac{G_j}{H_j}\right)^{\frac{\eta-1}{\eta}}\left(\frac{H_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+1\right)^{\frac{\eta}{\eta-1}}.$$

Substituting (A5) and (18) to obtain:

$$\frac{F_j}{L_j}=\xi_j^{\frac{\eta}{\eta-1}}\left(\sigma_j\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\left(q^{-\eta}\sigma_j^{\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}\right)^{\frac{\eta-1}{\eta}}+1\right)^{\frac{\eta}{\eta-1}}=\left(\frac{\xi_j}{J_j}\right)^{\frac{1}{\eta-1}}, \tag{A7}$$

The low-skill real wage (22) is obtained from knowing $\partial F_j/\partial L_j = A_j\xi_j(F_j/L_j)^{1/\eta}$.

## A2.3 Sectoral allocation

Using the definition $\chi = w_h/q_k, q = w_h/w_l$, and $\phi = q_k/p_l$, equation (22) can be rewritten as

$$\chi=\frac{qA_l}{\phi}\left(J_l\xi_l^{-\eta}\right)^{\frac{1}{1-\eta}}. \tag{A8}$$

Using (19) to derive:

$$\begin{aligned}\chi=q\xi_l^{\frac{\eta}{\eta-1}}\frac{A_l}{\phi}\left[1+q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}} &= \xi_l^{\frac{\eta}{\eta-1}}\frac{A_l}{\phi}\left[q^{\eta-1}+\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}}\\ \Longrightarrow q^{\eta-1}+\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}} &= \left(\frac{\phi\chi}{A_l}\right)^{\eta-1}\xi_l^{\frac{\eta}{1-\eta}}\end{aligned}$$

Using the expression for $\tilde{I}_l$ in (17) to obtain (25).

**Deriving equation for $S\left(\chi;\frac{H}{L},\frac{\phi}{A_l}\right)$:** The labor market clearing condition for the high-skill and the low-skill labor together imply:

$$\frac{H_l}{L_l}\left(L-L_h\right)+\frac{H_h}{L_h}L_h=H,$$

thus the share of low-skill labor in the high-skill sector is:

$$l_h\equiv\frac{L_h}{L}=\frac{H/L-H_l/L_l}{H_h/L_h-H_l/L_l}, \tag{A9}$$

simplify and use (18) to obtain the first equilibrium condition (26).

**Deriving equation for** $D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$ **:** The goods market clearing conditions and the relative demand imply:

$$x = \frac{p_h C_h}{p_l C_l} = \frac{P_h Y_h}{P_l \left(Y_l - \phi K\right)} \Longrightarrow \frac{p_h Y_h}{p_l Y_l} = x\left(1 - \frac{\phi K}{Y_l}\right), \tag{A10}$$

where, using relative price (24), $x$ is derived as

$$x = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{\xi_h^{-\eta} J_h}{\xi_l^{-\eta} J_l}\right)^{\frac{1-\varepsilon}{\eta-1}}; \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and using the capital market clearing condition, $K$ is derived as:

$$K = K_h + K_l = \frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)$$

so the relative demand equation (A10) can be written as

$$\frac{p_h Y_h}{x p_l Y_l} = 1 - \frac{\phi}{Y_l}\left(\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right),$$

given $\phi \equiv q_k/p_l$, rewrite it in terms of the low-skill income share $J_j$ :

$$\frac{J_l}{x J_h}\left(\frac{L_h}{L_l}\right) = 1 - \frac{q_k J_l}{q_l L_l}\left(\frac{K_h}{L_h} L_h + \frac{K_l}{L_l}\left(L - L_h\right)\right) = 1 - \frac{J_l}{L_l}\left(\frac{1-\beta_h}{J_h} L_h + \frac{1-\beta_l}{J_l}\left(L - L_h\right)\right),$$

where the equality follows from the definition of $\beta_j$. Finally (27) is derived from:

$$\frac{J_l}{x J_h}\left(\frac{l_h}{1-l_h}\right) = 1 - \frac{J_l}{1-l_h}\left[\frac{1-\beta_h}{J_h} l_h + \frac{1-\beta_l}{J_l}\left(1 - l_h\right)\right].$$

**Value-added shares :** The value-added share of the high-skill sector is:

$$v_h = \left[1 + \frac{p_l Y_l}{p_h Y_h}\right]^{-1} = \left[1 + \frac{p_l A_l F_l / L_l}{p_h F_h / L_h} \frac{L_l}{L_h}\right]^{-1}$$

Using relative prices (24) and (A7), (29) is obtained from:

$$v_h = \left[1 + \left(\frac{1-\lambda_h}{1-\lambda_l}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_l}{J_h}\right)^{\frac{1}{\eta-1}} \left(\frac{1-\lambda_l}{J_l}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h}{1-\lambda_h}\right)^{\frac{\eta}{\eta-1}} \left(\frac{L_l}{L_h}\right)\right]^{-1}.$$

### A2.4 Calibration

This section explains how the weight of each input is calibrated to match the sectoral income share for period 0 and period T.

**Normalization of $\phi/\mathbf{A}_1$ :** The initial $\frac{\phi}{A_l}$ can be normalized to 1. Note that

$$\tilde{I}_j = \left[1 + \frac{K_j}{\chi H_j}\right]^{-1} \Longrightarrow \frac{K_j}{\chi H_j} = \frac{1-\tilde{I}_j}{\tilde{I}_j},$$

which is independent of $\phi/A_l$. Also using the definition of $J$:

$$J_j^{-1} = \left[1 + \frac{K_j}{\chi H_j}\right] q \frac{H_j}{L_j} + 1$$

so $\frac{H_j}{L_j}$ is independent of $\phi/A_l$ as well. It follows from (A9) that $l_h$ is independent of $\phi/A_l$. Given $H_j/L_j$ and $K_j/H_j$ are independent of $\phi/A_1$, so the allocation of all inputs is independent of $\phi/A_1$. This shows that we can normalize $\phi/A_{l0} = 1$ as it does not affect input allocation across sectors. The value of $\phi_T/A_{lT}$ is then determined by the growth in the relative price of capital $\phi_T/\phi_0$ and the growth in the productivity of the low-skill sector $A_{lT}/A_{l0}$.

**Calibration of $\kappa_1, \xi_1$ :** Given $\phi/A_l$, equation (A8) expresses $\chi$ as a function of $\xi_l$ given data on $q$ and $J_l$ :. Substitute this into $\tilde{I}_l$ in (17) to solve for $\delta_l$ explicitly:

$$\delta_l = \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right)^{\frac{1}{\rho}},$$

which implies a value of $\kappa_l = \frac{\delta_l}{1+\delta_l}$ for any given level of $\xi_l$. Thus, the income share (19) provides an implicit function to solve for $\xi_l$ given data on $\left(\tilde{I}_l, J_l\right)$:

$$J_l = \left[ 1 + q^{1-\eta} \sigma_l^{\eta} \left[ \tilde{I}_l \left(1 - \kappa_l\right)^{-\rho} \right]^{\frac{\eta-1}{1-\rho}} \right]^{-1},$$

This procedure pins down $\chi, \xi_l$ and $\kappa_l$. More explicitly:

$$\begin{aligned} (1-\kappa_l)^{-1} = 1 + \delta_l &= 1 + \left[ \frac{1-\tilde{I}_l}{\tilde{I}_l} \chi^{1-\rho} \right]^{\frac{1}{\rho}} = 1 + \left[ \frac{1-\tilde{I}_l}{\tilde{I}_l} \left( \frac{q\phi}{A_l} J_l^{\frac{1}{1-\eta}} \xi_l^{\frac{\eta}{1-\eta}} \right)^{1-\rho} \right]^{\frac{1}{\rho}} \\ \Longrightarrow \sigma_l^{\eta} \left[ (1-\kappa_l)^{-1} \right]^{\frac{\rho(\eta-1)}{1-\rho}} &= \sigma_l^{\eta} \left[ 1 + \left( \frac{1-\tilde{I}_l}{\tilde{I}_l} \right)^{\frac{1}{\rho}} \left( q A_k J_l^{\frac{1}{1-\eta}} \right)^{\frac{1-\rho}{\rho}} \xi_l^{\frac{\eta(1-\rho)}{(\eta-1)\rho}} \right]^{\frac{\rho(\eta-1)}{1-\rho}} \end{aligned}$$

The implicit function is

$$f(\xi_l) = \left[ 1 + q^{1-\eta} \left[ \left( \frac{1-\xi_l}{\xi_l} \right)^{\frac{\eta(1-\rho)}{\rho(\eta-1)}} + \left( \frac{1-\tilde{I}_l}{\tilde{I}_l} \right)^{\frac{1}{\rho}} \left( \frac{q\phi}{A_l} J_l^{\frac{1}{1-\eta}} \right)^{\frac{1-\rho}{\rho}} (1-\xi_l)^{\frac{\eta(1-\rho)}{(\eta-1)\rho}} \right]^{\frac{\rho(\eta-1)}{1-\rho}} \right]^{-1} - J_l,$$

where

$$f'(\xi_l) > 0, \lim_{\xi_l \to 1} f(\xi_l) = 1 - J_l > 0, \lim_{\xi_l \to 0} f(\xi_l) = -J_l < 0.$$

Thus, there is a unique solution for $\xi_l \in (0,1)$.

**Calibration of $\kappa_h, \xi_h$ :** Using income share $\tilde{I}_h$ in (17):

$$\delta_h = \left[ \frac{1-\tilde{I}_h}{\tilde{I}_h} \chi^{1-\rho} \right]^{\frac{1}{\rho}} \Longrightarrow \kappa_h = \frac{\delta_h}{1+\delta_h}.$$

Given $\tilde{I}_h$ and $\chi$, $\kappa_h$ is obtained. Using $J_h$ in (19):

$$\sigma_h = \left[ \frac{1-J_h}{J_h} q^{\eta-1} \left[ \tilde{I}_h \left(1-\kappa_h\right)^{-\rho} \right]^{\frac{1-\eta}{1-\rho}} \right]^{\frac{1}{\eta}},$$

given $\kappa_h, \tilde{I}_h, J_h$ and $q$, $\xi_h$ is obtained.