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Intergenerational Transmission of Family Influence

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ABSTRACT

Intergenerational Transmission of Family Influence*

This paper studies intergenerational mobility—the transmission of family influence. We develop and estimate measures of lifetime resources (income and wealth) motivated by economic theory that account for generational differences in life-cycle trajectories, uncertainty, and credit constraints. These measures of lifetime resources allow us to estimate the transmission of welfare and lifetime resources at different stages of the life cycle. We compare these measures with traditional ones such as wage income and disposable income measured over narrow windows of age that are used to proxy lifetime wealth. The performance of proxy measures is poor. Parents’ expected lifetime resources are stronger predictors of many important child outcomes (including children’s own expected lifetime resources and education) than the income measures traditionally used in the literature on social mobility. Changes in patterns of educational attainment across generations explain most of the intergenerational change in life-cycle dynamics. While relative mobility is overstated by the traditional income measures, absolute upward mobility is understated. Recent generations have higher welfare and are better off compared to their parents.

JEL Classification: I24, D31, I30
Keywords: intergenerational mobility, life-cycle measures of resources, education

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1 Introduction

This paper studies the transmission of family influence across generations. We present new theory-motivated measures of lifetime resources allowing us to study the role of parents’ resources at crucial stages of children’s lives.

The conventional approach to measuring social mobility estimates intergenerational elasticities (IGEs) of income and follows the pioneering work of Becker and Tomes (1979, 1986). This approach treats childhood as a single-period stage of a three-stage overlapping generations model that is followed by adulthood (when parents invest in children) and then retirement. It ignores uncertainty and abstracts from timing considerations within the various stages of the life cycle. It attempts to compare realized lifetime resources across generations. For pragmatic reasons (such as data limitations), realized resources measured over shorter time spans are used to proxy lifetime incomes. The literature emphasizes the role of measurement errors and alignment of ages across generations.\(^1\) The literature relies on implicit assumptions of stationarity or limited forms of nonstationarity to characterize life cycles across generations.

Another approach measures the determinants of successful lives—education, health, and participation in crime—across generations.\(^2\) Recent research on human development (e.g., Cunha and Heckman, 2007; Heckman and Mosso, 2014) demonstrates the importance of critical and sensitive periods in shaping lifetime skills. In the presence of imperfect capital markets, the timing of receipt of parental income plausibly affects parental investment and the lifetime prospects of children.\(^3\) Tominey et al. (2020) show that parental income received when children are young is a better predictor of the lifetime prospects of children than parental income received at later ages. Recognizing the importance of child investment at early ages on child lifetime outcomes, it is the resources of parents at

\(^1\)The literature focuses on attenuation bias arising from measuring income over too few years and life-cycle bias if income is not measured at ages thought to approximate lifetime income flows following Mincer’s (1974) notion of the “overtaking age” (see Willis, 1986). See also Mazumder (2005) and Solon (1992) for discussion of alignment issues.

\(^2\)The first strand is connected with the second strand because parental lifetime resources help determine the resources available to invest in children. Conventional one-period-lifetime models of family influence, like Becker and Tomes (1979, 1986) and Solon (2004), provide a tight link between the two approaches.

\(^3\)See Cunha and Heckman (2007) and Caucutt and Lochner (2020).
those ages that are relevant to the transmission of family influence. In addition, when comparing lifetime resources and their valuation across generations with multiperiod life cycles, there are many possible ages of comparison. For example, García et al. (2022) show that the Perry Preschool program did not boost participant earnings at age 50, nor did it promote marriage at that age—but it promoted both measures during parental childbearing and child-rearing years with substantial resulting benefits for their children.

This paper unites and extends these two themes in the literature on social mobility by developing and estimating the intergenerational persistence in measures of lifetime resources and well-being that are most predictive of successful childhood outcomes such as education, health, and participation in crime. The present discounted value of future income (PDV) recognizes that the timing of key life events differs greatly across generations and individuals, and lifetime wealth approximates the lifetime value function and extends the PDV by taking account of both uncertainty and liquidity constraints. We introduce a crucial distinction between ex post and ex ante measures.

Lifetime income and lifetime well-being are measures of lives well lived. They are often assumed to be the objects of interest in studying social mobility. However, expected income and expected well-being at different ages are measures of resources available for consumption and child investment at those ages. They are also measures of age-specific welfare. A life well lived at age 35 may not be one well lived at 50. As individuals progress through life, anticipated income and anticipated welfare are the relevant measures of age-by-age welfare. We introduce and analyze both types of measures.

We make several contributions to the literature. First, we use PDV and lifetime wealth to analyze intergenerational dependence in lifetime resources and welfare. We compare results based on commonly used proxy measures with those based on actual lifetime resources (both ex ante and ex post).

Second, we explicitly account for agent information sets about current and future resources that govern child investment decisions and ex ante measures of lifetime resources and welfare at each age. We use estimated information sets to formally account for the evolution of uncertainty and its consequences.

Third, using the tools developed in this paper, we find that currently utilized
snapshot measures of realized incomes substantially underestimate lifetime intergenerational persistence. Exploiting rich Danish register data spanning 40 years, we document a much tighter link across generations than suggested by the measures used in the earlier literature on intergenerational mobility.

For example, the intergenerational dependence measured by log-log regressions (IGE) is 0.29 for snapshot measures of realized wage income but around 0.50 when considering expected lifetime resources and 0.35 when considering realized lifetime resources. This pattern holds for alternative measures of intergenerational income mobility such as Pearson correlations and rank–rank associations. Our results call into question common practices, based on snapshot proxy measures, used in previous studies of intergenerational income mobility. Estimates of intergenerational mobility based on incomes measured during the 30s overstate social mobility.

Contrary to the traditional estimates that find Denmark to be a highly mobile country, we find that even in a generous welfare state with substantial social insurance and redistribution through taxes and transfers, there is strong dependence in lifetime resources across generations. Relative social mobility is overstated more for children from disadvantaged families.

*Ex ante* lifetime measures of parental resources are stronger predictors of child lifetime resources and child outcomes than traditional measures of parents’ income and parents’ *ex post* lifetime resources. They better predict child cognitive skills, education, crime, and teenage pregnancy—even compared to measures of realized parental income averaged over 40 years. *Ex ante* lifetime measures better predict child outcomes because they better proxy the resources parents act on when they make investment decisions.

Moreover, expected PDV and lifetime wealth better capture intergenerational

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4As in, e.g., Landersø and Heckman (2017).
5See Aaronson and Mazumder (2008); Corak (2006); Corak and Heisz (1999); Mazumder (2005); Solon (1992) for examples of earlier studies focusing on the alignment of incomes across generations at specified ages. Black and Devereux (2011) and Jāntti and Jenkins (2015) review the literature. There are additional related studies that focus on other dimensions of intergenerational persistence, such as wealth (Charles and Hurst, 2003), consumption expenditures (Charles et al., 2014; Waldkirch et al., 2004), occupations (Bello and Morchio, 2017; Corak and Piraino, 2011), incarceration and criminal behavior (Dobbie et al., 2018; Meghir et al., 2012), health (Björkegren et al., 2019; Johnston et al., 2013), and employment and welfare dependency (Li and Goetz, 2019; Lo Bello and Morchio, 2020).
differences in income trajectories and timing of educational attainment. Differences in educational attainment mediate most of the impact of intercohort demographic changes on social mobility. In contrast, the substantial intercohort changes in educational attainment and family formation (ages of marriage, cohabitation, and birth of children) cause income measured over fixed age ranges across cohorts to be inaccurate proxies of individual expectations of lifetime resources and welfare at the ages where they are computed.

While relative mobility (measured by the intergenerational income elasticity, IGE) is overstated using traditional income measures, the opposite is true for measures of absolute upward mobility. Current generations are better off than previous generations. Once we account for redistribution through taxes and transfers and account for uncertainty and income smoothing, imperfect credit markets, reforms to financial markets, and for delayed labor market entry, the vast majority of recent cohorts are doing better than their parents. This is particularly so for children from affluent families.

The persistence of inequality across generations documented in this paper exemplifies the importance of integrating individual and family life-cycle dynamics along with accurate characterizations of policy environments in conducting studies of social mobility. Assessing intergenerational mobility through the lens used here allows for better understanding of the importance of factors such as the role of the family, changes in individual life cycles across generations, and the expectations and trajectories individuals face across their lifetimes.

This paper unfolds in the following way. Section 2 describes our data and presents descriptive statistics. Section 3 introduces our lifetime measures and discusses their identification and estimation. Section 4 documents dramatic changes in life cycles across cohorts. It also examines the best predictors of important child outcomes and demonstrates the superior predictive properties of our expected lifetime resource measures. Section 5 presents our main empirical estimates of relative intergenerational mobility. It motivates the importance of using lifetime ex ante measures over the traditional ex post measures. Section 6 presents the factors that most strongly affect estimated social mobility. Section 7 analyzes absolute mobility. Section 8 concludes. A web appendix6 presents sup-

6http://cehd.uchicago.edu//intergenerational-transmission-family-appx.
porting technical and empirical arguments.

## 2 Our Data

We use full population administrative register data from Denmark in the years 1980 through 2019. The data contain unique identifiers of individuals, which enable us to combine information on a wide range of different aspects of life across the lifetime. The data also include unique identifiers of parents and spouses, allowing us to link families throughout the entire period. In addition to information on income, assets, and liabilities of children and their parents, we also add information on completed education, household structure and demographic characteristics, 9th-grade exam scores, and crime. Appendix A provides a detailed description of all of the data sources and definitions we use.

### 2.1 Main Samples and Definitions

We primarily analyze a sample of children born in 1981 and 1982 for whom we can establish a link to parents, whose parents did not migrate, and who did not themselves migrate. We observe the birth cohorts of 1981 and 1982 from birth to age 38 and 37, respectively (in 2019). We have information on their parents in all years between 1980 and 2019. We analyze other cohorts as a complement to this paper.

For the main analyses in this paper, our specifications using log income exclude individuals with zero or negative average income for the age range over which we measure their income. We restrict the sample to native Danes. We

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7 Using the individual identifiers, we link data from registers containing educational attainment (UDDA register), income, assets, transfers, homeownership (IND register), house price (IND and EJSA registers), marital status, and fertility (BEF register) for each individual and his or her spouse and parents. We also include information on criminal convictions from the sentencing register (KRAF), and 9th-grade exam scores from the exam register (UDFK). We measure academic achievement using 9th-grade leaving exam scores in mathematics and Danish. These are national tests taken by all children in public schools. We consider both an indicator of college attainment and total years of schooling by age 35 as child educational attainment measures. We measure crime as an indicator of whether individuals have received a crime conviction by 2019, and we measure whether the child has become a parent by age 20.

8 Below, we establish the robustness of our analysis to alternative treatment of zero earnings.
also exclude individuals for whom we observe fewer than three years between ages 30 and 35 (i.e., at least three non-missing observations). We start with a sample of 105,953 native Danes who did not migrate, whose parents did not migrate, and for whom we can establish links with their parents. This reduces to 100,344 when dropping negative values and zeroes, and reduces further to 98,686 when we drop children with fewer than three observations.

Figure 1 is a schematic flowchart of the sample of cohorts used for empirical analysis throughout this paper, along with the data availability for the sample. We measure children’s income in the years 2011–2016 and 2012–2017 for the 1981 and 1982 cohorts, respectively. We measure parents’ income when they were aged 30–35 using income information between 1980 and 2000.

In Section 5.5 and Appendix C, we present estimates accounting for alternative timing of parental income, different age ranges for measuring children’s income, and alternative definitions of family units. Our principal conclusions remain unaffected. An analysis based on rank-rank measures includes zero earners and confirms our main findings.

A subset of outcome measures such as exam scores at the end of compulsory schooling are not available for the 1981 and 1982 cohorts. For our analysis of these specific outcomes, we use cohorts born between 1995 and 1997, with the sample defined in the same way as described for the 1981 and 1982 cohorts above (i.e., those native Danes for whom we can establish a link to parents, whose

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9The few observations with less than three observations when parents were aged 30–35 during this time window are dropped. The results are unaffected by this exclusion. We do not use data before age 30 because a substantial fraction of children are still in school. About 8% of the individuals born in 1981–1982 acquire additional years of schooling between ages 30 and 37. Also, earnings observations are especially noisy prior to age 30 in Scandinavian countries (see, e.g., Björklund, 1993; Landersø and Heckman, 2017).
parents did not migrate, and who did not themselves migrate). We have an initial sample of 209,603 child-parent pairs of native Danes. Our selection rules result in a final sample of 185,710 individuals.\(^\text{10}\)

### 2.2 Additional Data to Measure Lifetime Resources

We supplement our main samples with two additional data sets. First, we use information on the adult population (age 25–85) from 1980 to 2019 to construct synthetic cohort data as needed for a portion of our analysis in Section 3. For each individual in each year, we have information on total personal income, disposable income, and imputed consumption (see description below), as well as information on education, cohabitation, number of children, homeownership, and employment.

Second, we use information from the Danish Household Expenditure Survey, a diary-based survey of expenditures within the household, collected by Statistics Denmark (Browning et al., 2021; Danmarks Statistik, 1999). The survey provides detailed information on various categories of consumption expenditures for a rotating sample of individuals between 1995 and 2012. We link the survey data to the administrative register information using individual unique identifiers.\(^\text{11,12}\)

\(^{10}\)Birth cohorts from the 1980s were smaller than those from the 1990s.

\(^{11}\)We use households’ disposable income and detailed information on assets and liabilities in periods \(t\) and \(t-1\) from the register data to predict household consumption as reported in the expenditure survey (1997+). The imputation is conducted using a random forest estimator, which is a nonparametric prediction algorithm originally proposed by Ho et al. (1995). We select the number of trees using a 5-fold cross-validation approach. Among participants in the Danish Expenditure Survey, the correlation between predicted consumption and the observed consumption using a training set was 0.95. See Appendix A.3 for a full description of our imputation procedure.

\(^{12}\)While consumption can also be imputed based on information on income, assets, and liabilities across years (see, e.g., Browning and Leth-Petersen, 2003), this approach is subject to measurement error, which results in two problems in the present case: (i) approximation error leads to attenuation bias when estimating intergenerational mobility, and (ii) it inhibits a precise estimate of a Stochastic Discount Factor (SDF) for lifetime wealth (introduced in Section 3). Bruze (2018) uses an alternative strategy by imputing consumption expenditures based on asset and income flows from Danish registers, following the accounting identity imputation procedure suggested by Browning and Leth-Petersen (2003) and applying instrumental variables to correct for approximation error and Danish Expenditure Survey data to instrument the imputed consumption of parents.
3 Our Measures of Lifetime Welfare and Resources

This paper develops and estimates *ex ante* and *ex post* measures of lifetime resources (discounted income) and the utility value of lifetime income profiles (approximate value functions), as well as four conventional realized income definitions (wage income, total personal income without transfers, total personal income with public transfers, and disposable income). A discussion of the benefits of using a lifetime approach is given in Section 4. We measure individual consumption in two different ways. We also consider income and consumption-equivalized versions of these measures using the standard OECD equivalence scale to adjust for household composition (Browning et al., 2014). One novelty of this paper is that we estimate life cycle stage–specific measures of resources and welfare that reflect agent uncertainty, credit constraints, and information that govern child investment decisions. Like Tominey et al. (2020), we recognize the potential importance of resources available to parents at key developmental ages in shaping child development. Unlike them, we also estimate expected future income at or near the ages at which child investment decisions are made. Table 1 provides an overview of the measures we analyze in the paper (main text and appendix). Sections 3.1–3.3 discuss our lifetime measures in greater detail.

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13The OECD equivalence scale assigns a value of 1 to the first household member, a value of 0.5 to each additional adult, and a value of 0.3 to each child.
**Table 1: Definitions of Welfare and Income Indicators Used in this Paper**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Wage Income</td>
<td>Taxable wage earnings and fringes, labor portion of business income, non-taxable earnings, severance pay, and stock options.</td>
</tr>
<tr>
<td>(2) Income with Transfers</td>
<td>Total personal income (excluding rental value of own home). Total personal income is equal to the sum of wage income, business and self-employment income, capital income, public transfer income, property income, and other non-classifiable income that can be attributed directly to the individual person.</td>
</tr>
<tr>
<td>(3) Income without Transfers</td>
<td>Total personal income (as specified in item (2) above) minus public transfer income. The main items of public transfer income include: social assistance cash benefits, unemployment insurance benefits including leave, sickness benefits, pensions including disability pension and early retirement pay, housing allowance, and child allowance.</td>
</tr>
<tr>
<td>(4) Disposable Income</td>
<td>Total personal income including public transfers (as specified in item (2) above) and rental value of own home (for owner-occupied individuals) minus taxes, interest expenses, and child support.</td>
</tr>
<tr>
<td>(5) Family Measures (Husband and Wife or Cohabitants)</td>
<td>The sum of items (1)–(4) within households.</td>
</tr>
<tr>
<td>(6) Equivalized Family Measures</td>
<td>The family measures in item (5) adjusted by an equivalence scale for household composition.</td>
</tr>
<tr>
<td>(7) Household Consumption</td>
<td>The total household expenditures from the Danish Expenditure Survey.</td>
</tr>
<tr>
<td>(8) Survey Imputed Consumption</td>
<td>Total household expenditures, imputed from the relationship between the Danish Expenditure Survey and the Danish register.</td>
</tr>
<tr>
<td>(9) Survey Imputed Consumption with Equivalence Scale</td>
<td>The survey imputed consumption adjusted by an equivalence scale for household composition.</td>
</tr>
</tbody>
</table>
| (10) Expected Present Discounted Value | The expected present discounted value of future total income, using a deterministic discount factor ($\beta$): 
$$\text{PDV}_{i,t} = E_{i,t} \left\{ \sum_{t=1}^{T} \beta^t y_{i,t+1} | I_{i,t} \right\},$$
where $y_{i,t}$ is the total income including interest on assets, public transfers, the estimated rental value of own home for owner-occupied individuals, and unrealized capital gains from housing stock for individuals who are homeowners, minus taxes and interest expenses at age $t$. $\beta$ is a common discount factor, and $I_{i,t}$ is agent $i$’s information set. |
| (11) Realized Present Discounted Value | Same as (10) with realized lifetime measures for parents and imputed measures after mid-30s using the imputation framework of Appendix E. |
| (12) Expected Lifetime Wealth | The expected present discounted value of future total income including unrealized capital gains using a stochastic discount factor. The lifetime wealth at time $t$ for individual $i$ is 
$$\text{LW}_{i,t} = E_{i,t} \left\{ \sum_{t=1}^{T} s_{i,t+1} y_{i,t+1} | I_{i,t} \right\},$$
where $s_{i,t+1} = E_{i,t} \left\{ \frac{U'(c_{i,t+1})}{U'(c_{i,t})} | I_{i,t} \right\}$ is the stochastic discount factor, $y_{i,t}$ is the total income (see definition in item (10) above) at age $t$, $c_{i,t}$ is the survey imputed consumption at age $t$, and $I_{i,t}$ is the information set. |
| (13) Realized Lifetime Wealth | Same as (12) with realized lifetime measures for parents and imputed measures after mid-30s using the imputation framework of Appendix E. |
| (14) Equivalized Lifetime Measures | Items (10)–(13) adjusted by an equivalence scale for household composition. |
3.1 Measures of Intergenerational Mobility

This paper develops and estimates two measures of lifetime resources: the present discounted value of future income and lifetime wealth—an approximation to the lifetime value function over the relevant ages of child development. We present two versions of each: ex ante and ex post, accounting for uncertainty. The ex post measure is consistent with perfect foresight or realized lifetime incomes, which is the implicit focus in the current literature. Our ex ante measures use age-specific, anticipated resources that capture welfare as it evolves. We first introduce our measures and then discuss their identification and estimation.

For a given information set $I_{i,t}$ available to individual $i$ in period $t$, the expected PDV is

$$\text{PDV}_{i,t} = \mathbb{E}_{i,t} \left[ \frac{1}{\beta} \sum_{t=1}^{T-t} y_{i,t+t} \right] I_{i,t},$$

where $\beta$ is a fixed discount factor, and $y_{i,t}$ is income in period $t$. The expected PDV is individual $i$'s expected present value of future income flows measured at period $t$. This measure improves on the way income has been measured in the previous literature, by considering a full life-cycle perspective and by allowing for differences in age profiles across generations and individuals. For example, later generations, on average, acquire more formal education. This means that they are more likely to have lower incomes at younger ages than their parents who do not attend college, compensated by higher (and steeper) income profiles when entering the labor market after completing college (or graduate school). We show evidence on these patterns across generations in Section 4.2. While snapshots of proxies of parents’ and children’s incomes may generate a distorted picture of intergenerational persistence, the expected PDV takes into account that more highly educated individuals face steeper expected income profiles and that income profiles have changed across generations. We also report a realized lifetime version of Equation (1) which is feasible for parents but requires partial imputation for children (from age 38 onward).

With diminishing marginal utility of consumption and lifetime liquidity con-

\[\text{See Blundell et al. (2016) for a similar definition of expected PDV, which they call human capital.}\]
straints, an individual’s utility does not depend solely on expected lifetime resources but also on access to future income in the presence of imperfect credit markets. Individuals facing liquidity constraints value expected future income streams less than those who can borrow fully against their future income (see Hai and Heckman, 2017). Risk averse agents value information. The value of expected income streams—expected lifetime wealth—should account for both uncertainty and liquidity constraints.

We follow Huggett and Kaplan (2011, 2012, 2016), who present lifetime measures of an individual’s resources that incorporate uncertainty and credit constraints using stochastic discount factors and define an individual’s expected lifetime wealth at period $t$ ($LW_{i,t}$) as

$$LW_{i,t} = E_{i,t} \left[ \sum_{\tau=1}^{T-t} s_{i,t+\tau} y_{i,t+\tau} \mid I_{i,t} \right],$$

where $s_{i,t}$ (defined below) is individual $i$’s stochastic discount factor (SDF) at age $t$ when expectations are taken with respect to the information set of individual $i$ at age $t$. In principle, stochastic discount factors can be estimated nonparametrically using panel data on consumption, but in this paper, we use a parametric specification of preferences.

From the first order condition for optimal consumption, the stochastic discount factor is

$$s_{i,t+1} = E_{i,t} \left[ \frac{U(c_{i,t+1})}{U_c(c_{i,t})} \mid I_{i,t} \right],$$

where $c_{i,t}$ is individual $i$’s current consumption, $c_{i,t+1}$ is consumption at age $t + 1$, $U(c_{i,t})$ is utility at time $t$, and $U_c$ is the marginal utility of consumption. These factors account for both uncertainty and liquidity constraints, as well as the insurance value of welfare programs such as social assistance and unemployment insurance departures. Expected lifetime wealth ($LW$) is the subjective present value of lifetime income evaluated with a SDF. As a complement to this anal-

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16See Escanciano et al. (2021).

17See Appendix B.1 for a detailed derivation of the concept of lifetime measures presented here.
ysis, we also assume perfect certainty and construct a version using actual $c_{i,t}$ and $\frac{y_{i,t}}{y_{i,t}}$ to form $s_{i,t}$. This is an estimate of realized lifetime welfare, which is one way to compare lifetimes.

### 3.2 Properties of the Model, Identification, and Estimation

We briefly consider the basic structure of the model and its identification and estimation.\(^{18}\) We focus on central issues, definitions, and assumptions. Appendices B.2–5.4 provide further details.

The SDF at period $t$ for individual $i$ ($s_{i,t}$) can be decomposed into terms capturing the impacts of (i) future income uncertainty, (ii) borrowing constraints, and (iii) the market interest rate. Consider the SDF from a household’s Euler Equation:

$$E_{i,t} \left[ \beta \frac{U_c(c_{i,t+1})}{U_c(c_{i,t})} (1 + r_{i,t+1})(1 + \lambda_{i,t}) \right] = 1, \quad (3)$$

where $\lambda_{i,t}$ is the household’s Lagrange multiplier on its borrowing constraint. If $\lambda_{i,t} = 0$, the constraint is not binding. Letting $e_{i,t+1}$ be the forecast error of this equation, then

$$e_{i,t+1} = \beta \frac{U_c(c_{i,t+1})}{U_c(c_{i,t})} (1 + r_{i,t+1})(1 + \lambda_{i,t}) - 1 \implies \ln \left( \frac{\beta U_c(c_{i,t+1})}{U_c(c_{i,t})} \right) = \ln(1 + e_{i,t+1}) - \ln(1 + r_{i,t+1}) - \ln(1 + \lambda_{i,t})$$

$$= E_{i,t} [\ln(1 + e_{i,t+1})] - E_{i,t} [\ln(1 + r_{i,t+1})] - \ln(1 + \lambda_{i,t}) + \eta_{i,t+1}, \quad (4)$$

where $\eta_{i,t+1} \equiv \ln(\beta U_c(c_{i,t+1})/U_c(c_{i,t})) - E_{i,t} [\ln(\beta U_c(c_{i,t+1})/U_c(c_{i,t}))]$ arises from updated information in the next period. The first term in Equation (4) is related to precautionary savings motives and income uncertainty.\(^{19,20}\) The next two terms

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\(^{18}\)Section B.1 exposes the notion of lifetime measures in a dynastic framework.

\(^{19}\)Parker and Preston (2005) breaks consumption growth into similar components.

\(^{20}\)We can approximate the first term $E_{i,t} [\ln(1 + e_{i,t+1})] \approx -\frac{1}{2} E_{i,t} [\sigma_{e,c,i,t+1}^2] = -\frac{1}{2} \sigma_{e,c,i,t+1}^2$ following a second order Taylor expansion. $\sigma_{e,c,i,t+1}^2$ is the forecast error variance, which can be considered an expression of income uncertainty and household wealth (see Dogra and Gorbachev, 2016).
capture liquidity constraints through the expected borrowing rate and Lagrange multiplier (the former being related to a household’s idiosyncratic borrowing rate and the latter to the explicit borrowing limit).

For our main empirical analysis, we use a CRRA utility function: \( U(c_{i,t}) = \frac{c_{i,t}^{1-\rho} - 1}{1-\rho} \), where \( c_{i,t} \) denotes the adult-equivalence consumption (to adjust for family size and composition) of individual \( i \) at time \( t \). We set the risk aversion parameter at 0.67, estimated by Szpiro (1986) on Danish data using property/liability insurance.

We present further details about the estimation procedure and specification tests in Appendices B.1–B.4 and Section 5.4. Appendix B.1 presents a general statement of our model. Appendix B.2 discusses identification. Appendix B.3 discusses estimation of the SDFs. Appendix B.4 reports summary statistics of our estimates of the stochastic discount factors by education level over age, separately for different birth cohorts. Section 5.4 discusses alternative specifications of the model allowing for uncertain lifetimes with bequests following De Nardi (2004) and investigates the sensitivity of the model to alternative choices of risk aversion parameters.

### 3.3 Identifying and Estimating Information Sets

An important innovation of this paper is identification of agent information sets and their implications for social mobility. We approximate information sets \( (I_{i,t}) \) using the procedure of Cunha and Heckman (2016). For vector \( Z_{i,t} \), the key idea of the procedure is to use forecasts of future income based on \( Z \) to check if the forecast error is actually correlated with choices that depend on these forecasts. Components of income not in the information set should not predict future outcomes.

We first test whether consumption at age 30 is statistically significantly associated with the difference between realized future income at age 50 and future expected income at age 50 based on an information set estimated at age 30. If \( Z \) is defined correctly, the residualized incomes at age 50 based on characteristics \( Z \) measured at age 30 are uncorrelated with the consumption at age 30 (see \( \sigma_{\epsilon_i,t+1}^2 \) may also capture uncertainty of, e.g., health and family structure.)
for a formal presentation of the test). This is indeed what we find in Panel A of Table 2. Column (1) shows that the raw regression coefficient between consumption at age 30 and disposable income at age 50 is 0.35. When we include gender and educational attainment in the information set in column (2), the estimated coefficient drops substantially. When we further include cohabitation and homeownership status in the information set in column (3), the regression coefficient drops further. And when we use the full information set in column (4), the regression coefficient between consumption at age 30 and the residual (unexpected) income at age 50 is even lower and not statistically different from zero ($t$-statistic is equal to 0.72). Similarly, to assess whether there is any relationship between any mismeasurement of the information set and child’s outcomes, Panel B of Table 2 considers the associations between parents’ disposable income at age 50 and child outcomes. The associations are initially highly significant at first, but once we residualize parental income using the full information set we see no significant link.

Our final information set, which passes the specification tests presented in Table 2, is based on gender, education level (primary school, high school, college, and university), employment status, cohabitation, number of children, quartiles for mean income level, quartiles for mean consumption level, quartiles for mean consumption growth, quartiles for standard deviation of consumption, and homeownership status. Our nonparametric approach for forming expected values also allows for all interactions among these factors. In Section 4.2, we show how educational attainment and life-cycle profiles of family formation and income change from one generation to another. By including these characteristics in the information set, cohort effects related to changes in them are explicitly taken into account.

We interpret the results from Table 2 as evidence that our information set is correctly specified. Moreover, as parents’ residualized income based on our preferred information set is not associated with child outcomes, any minor misspecification would likely attenuate the estimated role of parents’ lifetime resources on child outcomes. Application of this approach shows that uncertainty is greater for the more recent cohorts and the more educated people. Figure 2(a) shows the standard variation of the unforecastable component of individuals’
<table>
<thead>
<tr>
<th>Panel A: Full Population</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption (Age 30)</strong></td>
<td>$\beta_{OLS}$</td>
<td>0.35</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>$T$-stat</td>
<td>(37.50)</td>
<td>(4.88)</td>
<td>(4.55)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Main Sample, Child Outcomes</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Disposable Income (Age 30)</strong></td>
<td>$\beta_{OLS}$</td>
<td>0.10</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>$T$-stat</td>
<td>(14.75)</td>
<td>(10.89)</td>
<td>(8.84)</td>
</tr>
<tr>
<td><strong>Wage Income (Age 30)</strong></td>
<td>$\beta_{OLS}$</td>
<td>0.18</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>$T$-stat</td>
<td>(31.49)</td>
<td>(19.10)</td>
<td>(13.60)</td>
</tr>
<tr>
<td><strong>College Attainment</strong></td>
<td>$\beta_{OLS}$</td>
<td>0.32</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>$T$-stat</td>
<td>(11.91)</td>
<td>(5.53)</td>
<td>(2.27)</td>
</tr>
<tr>
<td><strong>Years of Schooling</strong></td>
<td>$\beta_{OLS}$</td>
<td>2.04</td>
<td>1.23</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>$T$-stat</td>
<td>(15.28)</td>
<td>(9.02)</td>
<td>(3.60)</td>
</tr>
</tbody>
</table>

Notes: This table reports sufficiency tests using the tests described above from Cunha et al. (2005). Panel A shows the regression associations between disposable income at age 50 with own consumption at age 30 (for all individuals born in 1951), and Panel B reports regression associations between parental income at age 30 with various child outcomes (disposable income, wage income, college attainment, and years of schooling). Column (1) reports the associations using disposable income. Columns (2)–(4) report the associations using disposable income residualized with respect to different information sets ($Z_{30}^k$). $Z_{30}^1$ includes information on gender and educational attainment, $Z_{30}^2$ adds cohabitation and homeownership status to the information set, and $Z_{30}^3$ is our final information set, which includes information on gender, education level (primary school, high school, college, and university), employment status, cohabitation, number of children, quartiles for mean income level, quartiles for mean consumption level, quartiles for mean consumption growth, quartiles for standard deviation of consumption, and homeownership status. We report $t$-statistics for the null hypothesis that the OLS coefficient is zero in parenthesis.
income by age for the sample of male children and their fathers, which can be considered as a measure of uncertainty. Figure 2(b) presents the variance of unforecastable component of individuals’ income normalized by the total variance of income by age. We discuss this feature of the data below in Section 7. The finding that more educated people face greater uncertainty is consistent with research by Cunha et al. (2005) and Cunha and Heckman (2016). Uncertainty incorporates all components of labor market risk, not just unemployment.

### 3.4 Analysis under Perfect Certainty

In analyzing realized *ex post* life-cycle patterns, we replaced expected values with realized values. One interpretation of the IGE is as a measure of lifetime welfare across generations. A practical issue with this approach is that we do not have data on full life cycles for children, although we do for parents. We have to impute missing life-cycle data only for the children. Following standard imputation methods, we use synthetic cohorts adjusted for wage growth to forecast missing data for children and account for imputation-induced variability. We use multiple imputation methods (see, e.g., Little and Rubin, 2002) to impute the residuals associated with the imputed values and use the imputations so generated in our empirical analysis (see also Appendix E). We recognize the speculative element in making such imputations and address problems with doing so. At the same time, an entire literature on returns to education and on life-cycle decision-making faces this problem, although it is often less explicit about how it solves it.\(^{22}\)

\(^{21}\)To avoid any gender effects and complications arising from fertility decisions and maternity leaves, for the uncertainty analysis presented here, we focus on comparisons between fathers and their sons.

\(^{22}\)See, e.g., Mincer’s (1974) use of synthetic cohorts and his evidence in support of it for the cohorts he analyzes.
**Figure 2**: Uncertainty by Education Level

(a) Standard Deviation of Residualized Next-Period Income

(b) Variance of Residualized Next-Period Income Normalized by Total Variance of Income

Notes: The figure shows the uncertainty of next-period income over ages 25–36 for fathers and sons of the 1981–1982 cohorts, separately by education level. Figure (a) shows the standard deviation of residualized next-period income using the current approximated information set, i.e., $y_{i,j+1} - E_{i,j} \{ y_{i,j+1} \mid Z_{i,j} \}$, where $y_{i,j+1}$ denotes income of individual $i$ at $j + 1$ and $Z_{i,j}$ is the estimated information set of individual $i$ at age $j$. Figure (b) shows the corresponding variance of residualized income normalized relative to the variance of next-period income, $y_{i,j+1}$. 


4 Welfare Measures and Life Outcomes

The current literature on IGEs focuses on the intergenerational transmission of income at fixed age intervals. In this section, we argue for the importance of using income measures that explicitly take into account lifetime dynamics. First, in Section 4.1 we analyze the relationship between different measures of resources and provide evidence that snapshot measures are not accurate proxies of individuals’ lifetime incomes. In Section 4.2, we motivate the importance of lifetime measures by showing how household fertility and cohabitation decisions and educational attainment have changed significantly between the two cohorts in our study, leading to very different income profiles. Lifetime measures, rather than snapshot measures, are needed to correctly account for such effects when estimating the intergenerational mobility. In Section 4.3, we motivate the superiority of our ex ante lifetime measures of income compared to traditional realized measures of income. We also compare the performance of (lifetime) realized measures of income in predicting important child outcomes with that of our ex ante measures of lifetime income, finding ex ante measures to be more strongly correlated with life outcomes of the child. This occurs because ex ante measures based on individual information sets better estimate the resources relevant to parental-child investment decisions. This motivates our use of ex ante income measures for analyzing social mobility.23

4.1 Comparing Alternative Measures of Resources

The different life-cycle and income trajectories just documented motivate our use of lifetime measures, which consider the full life-cycle perspective and allow for those differences. To illustrate how expected PDV and lifetime wealth differ from the traditional measures of resources and their realized values, Table 3 shows correlations between the paper’s main welfare indicators, their estimated realized values, and the traditional measures of welfare.24 The two expected

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23 We discuss the importance of “outcome-based IGEs” and the timing of parental income more in a companion paper by Eshaghnia et al. (2022).

24 Tables A.1–A.3 reports summary statistics of the paper’s main welfare indicators. Also, Tables A.4–A.7 of Appendix A.4 presents the correlations between different welfare indicators for alternative samples and family units.
lifetime measures correlate most strongly with wage income and income with transfers (correlation around 0.60). In contrast, income with transfers, income without transfers, and disposable income are strongly correlated with coefficients close to 1, but their correlations with consumption, expected PDV, and lifetime wealth are only around 0.45. Realized PDV and lifetime wealth are more weakly correlated (0.3–0.4) with their expected value counterparts. Most surrogate measures of lifetime resources widely used in the literature are only weakly correlated with their decision-relevant counterparts. Consumption is only weakly correlated with expected PDV and expected wealth.
Table 3: Correlations of Income and Welfare Measures\(^1\)

<table>
<thead>
<tr>
<th></th>
<th>Wage Income</th>
<th>Income without Transfers</th>
<th>Income with Transfers</th>
<th>Disposable Income</th>
<th>Household Consumption</th>
<th>Realized Lifetime Wealth</th>
<th>Realized PDV</th>
<th>Expected Lifetime Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income without Transfers</td>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income with Transfers</td>
<td>0.50</td>
<td>0.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disposable Income</td>
<td>0.55</td>
<td>0.42</td>
<td>0.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household Consumption</td>
<td>0.45</td>
<td>0.63</td>
<td>0.61</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Realized Lifetime Wealth</td>
<td>0.39</td>
<td>0.30</td>
<td>0.30</td>
<td>0.49</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Realized PDV</td>
<td>0.37</td>
<td>0.43</td>
<td>0.42</td>
<td>0.37</td>
<td>0.37</td>
<td>0.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected Lifetime Wealth</td>
<td>0.48</td>
<td>0.51</td>
<td>0.48</td>
<td>0.36</td>
<td>0.39</td>
<td>0.35</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Expected PDV</td>
<td>0.45</td>
<td>0.45</td>
<td>0.42</td>
<td>0.35</td>
<td>0.38</td>
<td>0.30</td>
<td>0.39</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Notes: The table shows correlations between various measures used of mean values between 30 and 35 for fathers of the 1981–1982 cohorts. Traditional measures including consumption are for intervals 30–35. Household consumption expenditures are based on the procedure discussed in the text (see also row 8 of Table 1) and in Appendix A.3.\(^1\)

\(^1\)Our lifetime measures are computed for people 30–35 but include full life cycles. For realized lifetime measures, we use the data in all years between 1980 and 2019 when individuals are between 30 and 70 years old.
4.2 Nonstationarity across Cohorts

Figure 3 shows the dramatic change in educational attainment across the cohorts we analyze.\textsuperscript{25} Educational attainment for both females and males has increased substantially. The majority of parents completed at most 10 years of schooling. In contrast, for children, most females from the 1981 and 1982 cohorts have completed a college or master’s degree (15 years or higher), while most males hold either a vocational high school degree (13 years) or a college degree (15 years).

**Figure 3: Distributions of Years of Schooling for Parents and Children**

![Distributions of Years of Schooling for Parents and Children](image)

*Notes:* The figure shows distributions of years of schooling for male and female children born in 1981–1982 and their parents. The small fraction with 20 years of schooling are individuals who complete a PhD (three years on top of the 17 years it takes to complete a master’s degree).

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\textsuperscript{25}Karlson and Landersø (2021) discuss trends in the education levels and intergenerational educational mobility across cohorts. Stuhler and Nybom (2022) analyze nonstationary versions of the Becker-Tomes and Solon models and chart the interesting dynamic feedback models. They also develop some novel techniques for estimating from nonstationary economics.
Figure 4: Income and SDF across Cohorts

Disposable Income across Cohorts
(a) Non-College  (b) College

SDF across Cohorts
(c) Non-College  (d) College

Notes: Figures (a) and (b) show disposable income for the 1945, 1965, and 1981 birth cohorts respectively, by college and non-college educated individuals. Figures (c) and (d) show the SDF, $s_{i,t}$, for the 1945–46, 1965–66, and 1981–82 birth cohorts respectively, for college and non-college educated individuals. We show more levels of education in Figure B.2.

Individuals in successive generations have very different life-cycle trajectories as the timing of key life events differs substantially, which we further illustrate in Figure 5. Figures 5(a) and (b) illustrate the delay of marriage from the 1955 to the 1975 cohorts for female and males, respectively. The remainder of Figure 5 focuses on persons in our main sample and shows the distributions of age at birth of first child and age at completion of highest degree for children and their parents. For both males and females, the timing of family formation is de-
layed by 5–7 years on average. Most parents finish schooling in their late teens, while most children graduate with their final degree in their mid- to late-20s. While parents’ and children’s education and fertility behavior are associated, a simple parallel shift in timing across the two generations does not characterize cohort shifts.\textsuperscript{26} The correlation coefficients between fathers’ and sons’ ages at birth of first child and ages at completion of highest degree are 0.14 and 0.23, respectively. Similarly, the correlations for mothers and daughters are 0.25 and 0.18, respectively. Income measured over any fixed age range will inherently be an inaccurate proxy of individuals’ permanent income.

Not only has the timing of key life events changed over cohorts, but so have levels of income, uncertainty, and constraints (see Figure 4). The figure shows average disposable income (4a–b) and estimated SDFs (4c–d) by age and college education for the 1945–1946, 1965–1966, and 1980–1981 cohorts, respectively. Average disposable income has increased and income profiles have become steeper—particularly for college educated individuals. Similarly, the SDFs increase with age (as uncertainty and constraints become less pronounced, cf. Figure B.1) and across cohorts (reflecting easier access to credit, which we discuss in Section 7.1).

The economic conditions facing individuals have changed substantially over the period in question. During the late 1970s, 1980s, and early 1990s, Denmark was characterized by a high level of structural unemployment, an inflexible labor market with low productivity growth, high interest rates, and general uncertainty about the viability of the level of public expenses (Statistics Denmark, 2001). Today, virtually all these features have been reversed: Unemployment rates are low and the labor structure is more flexible following a series of labor market reforms during the 1990s and 2000s,\textsuperscript{27} credit markets have been liberalized,\textsuperscript{28} and the several welfare reforms have ensured the long-run viability of the current level of public expenditures.\textsuperscript{29}

\textsuperscript{26}As a further illustration, Figures D.1–D.4 in the Appendix show that family formation is most pronounced among the college-educated.

\textsuperscript{27}Andersen and Svarer (2007).


\textsuperscript{29}De Økonomiske Råd [The Economic Council] (2021).
Figure 5: Timing of Key Life Events across Generations

Age of First Marriage

(a) Females
(b) Males

Age of First Birth

(c) Females
(d) Males

Age of Completion of Highest Degree

(e) Females
(f) Males

Notes: Figures (a)–(b) show the distribution of the age at which individuals get married for the first time for the 1955 and 1975 birth cohorts respectively, for males and females separately. Figures (c)–(d) show the distribution of the age at which individuals have their first child for the 1981–1982 birth cohorts and their parents. Figures (e)–(f) show the distribution of the age at which individuals complete their highest degree for the 1981–1982 birth cohorts and their parents.
4.3 Parental Resources and Child Outcomes

We compare parental expected PDV and lifetime wealth, developed earlier in this paper, with their realized counterparts and traditional measures in the literature, with respect to their predictive power over important child outcomes. Figure 6 shows the correlations between different measures of parental resources, children’s exam grades, educational attainment, crime, and teen pregnancy.\textsuperscript{30}

Figures 6(a) and (b) compare the correlation between the various measures of parental resources and their children’s math and Danish test scores. The figures show a much stronger correlation for the parents’ expected lifetime measures in comparison to the traditional income measures. The correlations between test scores and expected lifetime measures at the household level range between 0.28 and 0.39. In comparison, the corresponding correlations for the traditional measures of income range between 0.12 and 0.25; about half the values estimated using expected lifetime measures. The correlation with realized lifetime measures are lower yet, ranging between 0.19 and 0.25.

Figures 6(c) and (d) show a similar pattern for the two measures of child educational attainment (whether the child has completed college and their final years of education). The correlation between education and expected lifetime measures is substantially higher than that found using traditional measures of family income. The difference in the predictive power of family resources is particularly pronounced when considering child college attainment. In contrast, the two realized lifetime measures have a weaker predictive power.

Finally, the pattern observed for the academic achievement and educational attainment measures extends to other outcomes, including risky behavior as shown by Figures 6(e) and (f). Also, the correlations between different measures of either paternal or maternal resources (instead of family resources combined) and child outcomes show a pattern similar to that presented in Figure 6. In sum, these results show that traditional measures of realized family income understate the importance of family resources in predicting a variety of dimen-

\textsuperscript{30}In light of the importance of parental investment decisions, we assess parental resources using these measures over ages 30–35, when children in our sample are, on average, 5.8 when the father is between the ages 30–35. This accords with evidence in Tominey et al. (2020) that compared to realized income in the early (ages 0–5) and late periods of childhood (ages 12–17), realized income in the middle period (ages 6–11) has relatively low productivity.
sions of child lives. The findings in this section motivate our choice of expected lifetime measures. Expected lifetime measures manifest a much tighter link between parents and children than the snapshot measures of income that are currently used in the literature on intergenerational income mobility to measure intergenerational transmission or realized life-cycle incomes.\textsuperscript{31} This is true even when we analyze the relationship between the resources of grandparents and children’s academic achievement.\textsuperscript{32}

\textsuperscript{31}Note that even though snapshot disposable income is highly correlated with realized PDV, it is not equally predictive of child outcomes.

\textsuperscript{32}Figures A.4 and A.5 of Appendix C report these results.
Figure 6: Parents’ Resources and Children’s Outcomes (Realized Measures Are Observed, No Imputation)

(a) Mathematics Problem Solving

(b) Danish Reading

(c) College Attainment

(d) Years of Education

(e) Criminal Behavior (Reversed)

(f) Having a Child by Age 20 (Reversed)

Notes: This figure shows the correlation between child outcomes and different parental resource measures. Figures (a)–(b) show 9th-grade leaving exam scores in mathematics and Danish for the 1995–1997 birth cohorts. Figures (c)–(f) show educational attainment measured at age 35 (c–d), crime (e), and an indicator of being a parent by age 20 (f) for the 1981–1982 birth cohorts.
5 Estimates of Intergenerational Mobility

Section 3 presents a catalogue of possible IGEs. This paper investigates IGEs based on measures most predictive of important child outcomes that determine successful lives. We proceed as follows. Having established in Section 4 that expected lifetime measures for parents are the strongest predictors of child outcomes, we document that IGEs estimated using our two expected lifetime measures differ from IGEs based on traditional measures. We also present rank-rank estimates and analyze nonlinear IGE estimates using the expected lifetime measures.

5.1 Intergenerational Elasticities for Expected Lifetime Measures

Figure 7 shows IGE estimates for father-child pairs and family-child pairs. The figure shows that IGE estimates based on expected lifetime measures are substantially higher than those based on standard snapshot measures of realized income measured over fixed age intervals; measures that are widely used as measures of lifetime welfare. The estimated father-son IGEs for the traditional measures range from 0.08 for disposable income to 0.23 for income excluding transfers. The estimates for the lifetime measures are substantially higher, ranging from 0.36 for lifetime wealth to 0.37 for the expected PDV. The IGE for consumption lies in the middle of the two, with a value of 0.34. We observe a similar pattern when studying family-child IGEs.

Across measures, family-based IGEs are larger than individual-based IGEs. This is intuitive, as family resources as a whole—and not just those of the father—determine how much families invest in their children, which affects the children’s outcomes later in adulthood.

\[33\text{As shown in Landersø and Heckman (2017), IGE estimates are substantially higher when excluding redistribution through taxes and transfers from measured income.}\]

\[34\text{We also observe a similar pattern when we adjust our measures of resources for family size and composition by using equivalence scales. Figure C.6 of Appendix C.2 replicates Figure 7 in the paper but, instead of individual measures, we use equivalized family measures for children and fathers. The overall pattern is similar to that in Figure 7.}\]
Figure 7: Log-Log IGE Estimates

Notes: This figure depicts IGE estimates for different measures of resources. The sample of children is restricted to the 1981–1982 cohort of native Danes. The IGE is the slope coefficient from the log-log regression of child measure on father (family) measure: \( \log(\bar{y}_c^i) = \alpha + \beta \cdot \log(\bar{y}_f^i) \), where \( \bar{y}_c^i \) denotes the average (over ages 30–35) of child measure, and \( \bar{y}_f^i \) denotes the average (over ages 30–35) of father (family) measure. Family outcomes are the sum of the mother’s and father’s outcomes.

These results show that the traditional approach, which relies on realized income measured over narrow age intervals and which is used in most of the literature to date, provides only a limited picture of the transmission of lifetime resources across generations. The traditional approach substantially overestimates intergenerational mobility and underestimates the persistence in lifetime resources across generations.

The evidence reported so far begs the question of why our estimates based on expected lifetime measures differ so markedly from other measures of income. The reason is that the lifetime measures capture a different notion than realized income at a given point in time. When we focus on expectations for the full life-cycle perspective while allowing for differences in age profiles between gener-
ations and individuals as well as uncertainty and liquidity constraints, we capture the information that agents act on when making decisions. When parents choose how much to invest in their children, they act on their (expected) yet-to-be-realized income stream. As previously shown in Section 4.3, expected PDV and lifetime wealth are significantly better predictors for children’s life courses compared to (even long-run averages of) realized values of income that average out classical measurement errors.

5.2 Intergenerational Elasticities for Realized Lifetime Measures

We next compare the intergenerational elasticities for realized lifetime measures to those for the expected lifetime measures. To this end, we use realized lifetime measures for parents and imputed measures after mid-30s for children.

Figure 8 presents IGE estimates for both ex ante and ex post measures of lifetime resources. The results suggest that IGE estimates are significantly lower for ex post lifetime measures. This might be partly due to using imputed values for the sample of children whose income measures after age 37 (for the 1982 cohorts) or 38 (for the 1981 cohort) are not observed yet. We will return to this point in Section 5.4.
Figure 8: Log-Log IGE Estimates of Realized vs. Expected Lifetime Measures

Notes: This figure depicts IGE estimates for both *ex ante* and *ex post* measures of lifetime resources. The sample of children is restricted to the 1981–1982 cohort of native Danes. The IGE is the slope coefficient from the log-log regression of child measure on father (family) measure:

\[ \log(\bar{y}_c) = \alpha + \beta \cdot \log(\bar{y}_f) \]

where \( \bar{y}_c \) denotes the average (over ages 30–35) of child measure, and \( \bar{y}_f \) denotes the average (over ages 30–35) of father (family) measure. The realized lifetime measures for children are computed using the imputed income measures of children following the imputation framework of Appendix E. Family outcomes are the sum of the mother’s and father’s outcomes.

5.3 Intergenerational Correlation and Cross-Sectional Inequality

Estimated IGEs depend on the correlation between parents’ and children’s log-income, and the ratio of the standard deviations (\( \hat{\beta} = \rho_{\text{child,father}} \frac{sd(\text{child})}{sd(\text{father})} \)). Any differences in the IGE estimates (as shown in Figure 7) must reflect differences in correlations between the children’s as well as their parents’ resources or levels and trends in cross-sectional inequality.

Table 4 decomposes the IGE estimates presented in Figure 7. While the table...
shows that child outcomes are more highly correlated with pooled family resources than with fathers’ resources alone, the table also shows that much of the difference between IGE estimates at the family-child level and those at the father-child level can be explained by differences in cross-sectional inequality.\textsuperscript{35,36} By averaging the income of the two parents, we reduce the variance of parental resources, which in turn increases the IGE. However, the table also shows that the intergenerational correlations for expected PDV and lifetime wealth are higher than for traditional measures of income. If anything, the gap between the two sets of measures is greater for the intergenerational correlation than for the IGEs. At the family level, for example, correlations are between 0.12 and 0.19 for the traditional measures and between 0.32 and 0.34 for \textit{ex ante} lifetime measures. Predictably, the measures based on realized lifetime income are substantially lower, even compared to a couple of the traditional measures.

\textsuperscript{35}Stuhler and Nybom (2022) analyze the evolution of the IGE in nonstationary environments.\textsuperscript{36} Changes in cross-sectional inequality are studied further in Table A.8 of Appendix A.4, where we present the Gini-coefficients of our main measures. Compared to traditional income measures, the Gini-coefficients are lower for consumption and our two expected lifetime measures.
Table 4: IGE Estimates (Ages 30–35 of Parents and Children)

<table>
<thead>
<tr>
<th></th>
<th>Father-Child IGE</th>
<th>Family-Child IGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{\beta} = \rho_{\text{child},\text{father}} \frac{sd(\text{child})}{sd(\text{father})} )</td>
<td>( \hat{\beta} = \rho_{\text{child},\text{family}} \frac{sd(\text{child})}{sd(\text{family})} )</td>
</tr>
<tr>
<td><strong>Traditional Measures</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage Income</td>
<td>0.125*** = 0.107 ( \text{0.030} ) ( \text{0.798} )</td>
<td>0.287*** = 0.148 ( \text{0.913} ) ( \text{0.471} )</td>
</tr>
<tr>
<td>Disposable Income</td>
<td>0.085*** = 0.078 ( \text{0.428} ) ( \text{0.402} )</td>
<td>0.239*** = 0.118 ( \text{0.434} ) ( \text{0.215} )</td>
</tr>
<tr>
<td>Income with Transfers</td>
<td>0.209*** = 0.170 ( \text{0.477} ) ( \text{0.387} )</td>
<td>0.346*** = 0.192 ( \text{0.475} ) ( \text{0.264} )</td>
</tr>
<tr>
<td>Income without Transfers</td>
<td>0.232*** = 0.162 ( \text{0.894} ) ( \text{0.623} )</td>
<td>0.405*** = 0.194 ( \text{0.879} ) ( \text{0.420} )</td>
</tr>
<tr>
<td>Household Consumption</td>
<td>0.341*** = 0.188 ( \text{0.279} ) ( \text{0.154} )</td>
<td>0.426*** = 0.210 ( \text{0.279} ) ( \text{0.135} )</td>
</tr>
<tr>
<td><strong>Lifetime Measures</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Realized Lifetime Wealth</td>
<td>0.178*** = 0.087 ( \text{0.550} ) ( \text{0.268} )</td>
<td>0.185*** = 0.087 ( \text{0.550} ) ( \text{0.255} )</td>
</tr>
<tr>
<td>Realized PDV</td>
<td>0.264*** = 0.119 ( \text{0.603} ) ( \text{0.272} )</td>
<td>0.351*** = 0.156 ( \text{0.608} ) ( \text{0.270} )</td>
</tr>
<tr>
<td>Expected Lifetime Wealth</td>
<td>0.364*** = 0.305 ( \text{0.237} ) ( \text{0.190} )</td>
<td>0.480*** = 0.323 ( \text{0.236} ) ( \text{0.155} )</td>
</tr>
<tr>
<td>Expected PDV</td>
<td>0.371*** = 0.310 ( \text{0.279} ) ( \text{0.233} )</td>
<td>0.522*** = 0.341 ( \text{0.277} ) ( \text{0.181} )</td>
</tr>
</tbody>
</table>

Notes: This table decomposes the IGE parameter into its components. The IGE is the slope coefficient from the log-log regression of the child measure on the father (family) measure: \( \log(\bar{y}_c) = \alpha + \beta \log(\bar{y}_f) \), where \( \bar{y}_c \) denotes the average (over ages 30–35) of the child measure, and \( \bar{y}_f \) denotes the average (over ages 30–35) of the father (family) measure. Family outcomes are the sum of the mother’s and father’s outcomes. The sample of children is restricted to the 1981–1982 cohort of native Danes.

\* \( p < .1 \), ** \( p < .05 \), *** \( p < .01 \).

### 5.4 Robustness of Our Findings across Ages and Cohorts

As previously noted, there is a continuum of possible IGEs depending on the age of measurement for both parents and children. Along the life cycle, individuals accumulate human capital through on-the-job-training or learning-by-doing, and usually experience an upward income trajectory through ages 40–45. This can lead to a positive relationship between the child’s age when income is measured and the IGE estimate (for a given age window for the father), and a
negative association between the father’s age and the IGE estimate (for age of child).\textsuperscript{37,38}

Motivated by this evidence, we demonstrate in Appendix C that the patterns documented in Figure 7 and Table 4 remain once we change the age of measurement. We re-estimate the IGEs for different measures of resources computed at different child and parental ages. In doing so, we have to rely on children from older birth cohorts, since we only observe income up to 2019.\textsuperscript{39} Again, we average the measures of resources over six years. We compare the IGE estimates with lifetime measures over different six-year intervals from ages 55–60 (for the 1956–1957 birth cohort) to ages 30–35 (for the 1981–1982 birth cohort). Figure C.4 reports the father-child and family-child IGE estimates across the different ages at which the parental and child incomes are measured. Across all measures, IGE estimates tend to be higher when parental resources are measured at ages 30–35 and 35–40 than when resources are measured at ages 50–55 and 55–60.\textsuperscript{40} Figure C.5 reports the father-child and family-child IGE estimates for different ages at measurement for the parents while keeping the child information fixed at 30–35. Across all measures, the IGE estimates tend to be higher when parental resources are measured when parents are younger and close to the developmentally important early ages of their children.

Crucially, however, the patterns depicted in Figure 7 remain when we change the cohorts that are used in the analysis and the age at which resources are measured. Across all ages and cohorts, the IGE estimates for the expected PDV and lifetime wealth are larger than for the traditional income measures. This emphasizes that our main findings are not driven by our choice of measuring resources at ages 30–35 or focussing on the 1981–1982 cohorts.

\textsuperscript{37}Studies that discuss a positive association between the earnings IGE estimate and the age of child at observation include, for example, Behrman and Taubman (1985); Chadwick and Solon (2002); Couch and Dunn (1997); Grawe (2006); Nilsen et al. (2008); Reville (1995); Solon (1999) Behrman and Taubman (1985), Chadwick and Solon (2002), Solon (2002), Couch and Dunn (1997), and Solon (1999).

\textsuperscript{38}The other source of bias when estimating IGE is the impact of measurement error and transitory fluctuations in measured earnings (Atkinson, 1980; Solon, 1989).

\textsuperscript{39}Also, we only observe income starting in 1980. This means that we miss the early income stream of parents of children from early cohorts. See Section 4.2 for a discussion of trends in Danish data.

\textsuperscript{40}Given the data limitations, we cannot distinguish between the age-of-measurement and cohort effects.
One possible reason why estimates based on lifetime measures differ from estimates based on traditional income measures is that long-run measures average out measurement error more than the snapshot measures. We find that the traditional income IGEs increase in value when we move from measuring individuals at a single age (for example, 30 or 35) to measuring them with the average taken over ages 30–35 (as in e.g., Mazumder, 2005; Solon, 1992), or if we move from measuring parental resources over ages 30–35 to measuring them over a 40-year average of realized values between ages 25–65 (see Appendix C). Yet, Figure 6 showed that even when we adjust the snapshot measures for measurement error by taking the average over long ranges (as we do in the realized lifetime measures), we still find that the expected PDV and lifetime wealth provide significantly better predictions of children’s outcomes.

5.5 Rank-Rank Versions

A common alternative to the IGE is rank-rank regression. The estimator avoids problems with zero earnings (see, e.g., Dahl and DeLeire, 2008). Figure 9 shows results from regressions of children’s income rank (in their cohort) on the parents’ income rank. As with the IGE, rank-rank associations are significantly higher for our two expected lifetime measures than for the traditional measures of income. Figure E.4 of Appendix E.2 presents rank-rank estimates for both ex ante and ex post measures of lifetime resources. The results show that rank-rank associations are significantly higher for ex ante lifetime measures than for their ex post counterparts.
Notes: This figure depicts rank-rank estimates for different measures of resources. The sample of children is restricted to the 1981–1982 cohort of native Danes. The rank-rank estimate is the slope coefficient from the rank-rank regression of child measure on father (family) measure: 

\[ \bar{r}_c = \alpha + \beta \cdot \bar{r}_f \]

where \( \bar{r}_c \) denotes the average percentile rank of the child for each measure averaged over ages 30 and 35, and \( \bar{r}_f \) denotes the average percentile rank of the father (family) for each measure averaged over ages 30 and 35. Family outcomes are the percentile rank of the sum of the mother’s and father’s outcomes.

5.6 Non-Linear Intergenerational Elasticities

Landersø and Heckman (2017) show that intergenerational income elasticities in Denmark, based on realized incomes, are highly nonlinear. Particularly, redistribution through taxes and transfers results in high income mobility for low-income families. This is reflected in Figure 10(a), which shows estimated non-linear IGEs using local-linear regressions for children’s and parents’ disposable income. Measured by disposable income, there is close to full mobility (locally) for children from low-income families with estimates close to zero, while there is a much greater intergenerational persistence for children from affluent families.
This pattern is, however, not found when we use consumption and the two expected lifetime measures in Figures 10(b)–(d). Intergenerational mobility in consumption is much closer to linear with local IGE estimates around 0.4 across parental consumption levels. Similar patterns are observed for the expected PDV and lifetime wealth. If anything, mobility is now lowest for children from disadvantaged backgrounds, with local IGE estimates close to 0.6, and highest for children from affluent backgrounds, with local IGEs close to 0.4. The non-linear estimates show how the traditional measures not only substantially overestimate intergenerational mobility, but do so predominantly for disadvantaged families who are likely to be most exposed to factors such as uncertainty in income profiles and liquidity constraints. The realized income IGE counterparts are smaller and show much more mobility than their ex ante counterparts. Figure E.5 of Appendix E.3 presents the results for the realized lifetime measures.
Figure 10: Local-Linear IGEs for Lifetime Measures

(a) Disposable Income

(b) Consumption

(c) Expected PDV

(d) Expected Lifetime Wealth

Notes: The figure depicts estimated nonlinear intergenerational income elasticities of disposable income, consumption, expected PDV, and lifetime wealth. The NL-IGEs are estimated using local linear regression slopes as formulated in Landersø and Heckman (2017). Dotted lines represent the 95% confidence interval from 60 bootstraps. The vertical lines indicate the 5th and 95th percentiles in the parental resource distributions.
6 Decomposing the IGE

To understand the mechanisms generating the sizeable IGEs for expected lifetime measures, we decompose the estimated IGEs into interpretable components. We explore the impact of variations in the impacts of the factors that shape individual income levels and trajectories. Particularly salient is the rise in educational attainment and the delay in the onset of family formation across cohorts. As shown in Section 4.2, life-cycle dynamics have changed dramatically across cohorts. Recent cohorts acquire more education and graduate later, they are older when they form families (marriages and stable cohabitation) and as a group, face significantly steeper income profiles when they enter the labor market associated with their rising educational level. See Appendix D for further evidence on these intercohort changes.

We conduct a simple decomposition exercise using linear approximations to explain generational income dynamics. Let $y_{i,t}^p$ and $y_{i,t}^c$ denote log income of the parents and their offspring, where $i$ indexes the specific family, and $t$ the age when income is measured. Consider the following regression specification for parents and children:

$$y_{i,t}^k = \lambda^k + (\beta^k)'X_{i,t}^k + \mu_{i,t}^k + \epsilon_{i,t}^k,$$  \hspace{1cm} (5)

where $k \in \{p, c\}$ represents the family member, $\lambda^k$ denotes an aggregate generation-specific effect, and $\beta^k$ is a vector of parameters associated with the vector of observables $X_{i,t}$. The additive shock term consists of an individual permanent component $\mu_{i,t}^k$ and serially uncorrelated shocks $\epsilon_{i,t}^k$. We focus on age 30–35 measures, as they are commonly used in the literature and we present them in this paper.

The average of log-income for ages 30 to 35 is given by:

$$\bar{y}_i^k = \lambda^k + (\beta^k)'X_i^k + \mu_i^k + \epsilon_i^k,$$

where $\bar{y}_i$ refers to the log-income averaged over ages 30–35. To assess the role of persistence due to observable characteristics, $X_{i,t}^k$ (e.g., persistence in college attainment) and persistence in permanent components $\mu_i^k$ (which include a va-
riety of unexplained factors such as the transmission of genetic potential), we decompose the intergenerational covariance of log-income into components:

$$\text{Cov}(\overline{y}_i, \overline{y}_i) = \text{Cov} \left( (\beta^c)' \overline{X}_i, \overline{y}_i \right) + \text{Cov}(\mu_i, \overline{y}_i), \quad (6)$$

where serially uncorrelated shocks are ignored to facilitate exposition.

We decompose $\overline{X}_i$ into four common elements: education (high school, college, and university dummies), experience (years of experience linear and squared), marriage and cohabitation (marriage and cohabitation dummies, and age of first marriage and cohabitation), and fertility (number of children and age at birth of first child). To avoid gender effects, we focus on comparisons between fathers and their sons. Table F.1 presents the mean differences for each listed variable for fathers and sons. Also, note that the differences in fertility is partially driven by the sample selection where we focus on fathers (who, of course, all have children) and their children who may not be a parent.

Figure 11 shows how the two expected value lifetime measures better capture important intergenerational differences in educational attainment and income trajectories. The figure presents the key results from the simple linear decomposition exercise of Equation (6), with Figure 11(a) showing the share of the covariance between fathers and their sons’ resources that can be explained by each of the child’s four observable factors as well as the unexplained component, and Figure 11(b) showing the corresponding estimates by IGE levels. There are large differences in the relative importance of the different factors for traditional and lifetime measures. Around 60% of the father-son covariance for the traditional measures of resources is unexplained. In comparison, the unexplained share is only 10%–20% for the lifetime measures. Intergenerational persistence in education and income trajectories explain the majority of the father-son covariance in both the expected PDV and lifetime wealth. Figure E.6 of Appendix E.4 presents the results for the realized lifetime measures. The figure displays a pattern similar to the expected lifetime measures in Figure 11 in terms of the role of education in explaining the intergenerational covariance.

\footnote{See Appendix F for additional details on the covariance decomposition.}
Figure 11: Decomposition of IGEs and Covariances

Notes: This figure depicts the covariance decomposition. Panel (a) plots the share of the intergenerational covariance that can be explained by each of the child’s observables $\text{Cov}((\beta_i')'X_i, \pi_{it,i})$ and the child’s unexplained component $\text{Cov}(\mu_{it,i}, \pi_{it,i})$. Panel (b) decomposes the estimated father-son IGEs into each of the components depicted in Panel (a).
7 Absolute Upward Mobility

Thus far we have discussed relative mobility. Absolute upward mobility is another important dimension of social mobility. One measure of it is the percentage of children who have better outcomes than their parents (Berman, 2018; Chetty et al., 2017; Manduca et al., 2020).

For ease of comparison and to avoid any gender effects, we focus on the comparison between fathers and their sons. For each measure of resources we estimate the percentage of male children (of the 1981—1982 birth cohort) whose resources are greater than those of their fathers where we measure resources for each individual over ages 30–35. These estimates are plotted in Figure 12 by percentile in the corresponding distribution of paternal wage income.

Focusing on the traditional income measures (e.g., income with transfers), Figure 12(a) shows a strong downward sloping relationship between absolute upward mobility and father’s income. Around 70% of those with fathers in the lowest income quintile have higher income than their fathers, while among those with fathers in the top quintile only 30% are better off than their fathers.

The message is, however, quite different if we consider expected lifetime wealth as presented in Figure 12(b). While absolute upward mobility in expected (at ages 30–35) lifetime wealth among children from low income families is similar to the level observed for income with transfers, the negative association between absolute upward mobility and father’s income is more muted for lifetime wealth. Around 50% of children with parents in the top quintile have a higher expected lifetime wealth than their parents. It should be noted, the pattern for consumption and disposable income in Figure 12(a) resembles the pattern for our lifetime measures.

The results suggest that absolute mobility is higher for the expected lifetime measures than the traditional income measures at ages 30–35. Appendix G.4 presents the results for the realized lifetime measures. Here, absolute mobility estimates are even higher. But, as noted earlier, these measures are based on imputed values for children and should be interpreted with caution.

42 Appendix G presents the results for alternative family units, i.e., where we measure both parents and children at the family level. A similar pattern emerges.
Figure 12: Absolute Mobility

Notes: The figure shows the percentage of male children (of the 1981–1982 birth cohort) whose measured resources are greater than those of their fathers by percentile in the fathers’ wage income distribution. Resources are averaged over ages 30–35 for both parents and children. Figure (a) compares the absolute mobility pattern for traditional measures. Figure (b) compares the absolute mobility pattern for expected lifetime measures (including income with transfers to ease comparison).
7.1 Roles of Disposable Income, Uncertainty, and Credit Markets in Shaping Absolute Mobility

To better understand the differences in absolute mobility across different measures of resources, we first decompose the difference between absolute mobility of income without transfers and disposable income. Disposable income (which is a component of lifetime wealth, see e.g., Table 1) includes six main components: (i) income without transfers, (ii) public transfers, (iii) rental value of the home, (iv) interest expenses, (v) paid taxes, and (vi) paid child support. For simplicity, we focus on fathers and their sons. Figure 13(a) displays absolute mobility as we include each of these components, one at at time.

Starting with a baseline measure of income without transfers we make a series of perturbations to its components starting with public transfers. We then add the rental value of the home and subtract interest expenses, taxes, and paid child support. The main drivers of absolute mobility are interest payments and to some degree tax payments (particularly for children from middle and high income families). Lower interest payment reflects a liberalization of credit markets, large interest rate decreases, and lowers asset stocks, which is a consequence of the delayed onset of graduation and family formation among younger cohorts.43 Child support and the rental value of housing play minor roles. Changes in transfers across cohorts have very small effects.

Thus, access to credit plays a central role in explaining high absolute mobility. The easing of credit markets is captured by our measure of lifetime wealth through the SDF (which is reflected by the higher SDFs for later cohorts as shown in Figure 4).44

To further clarify this point, Figure 13(b) compares absolute mobility estimates for expected lifetime wealth computed under different assumptions on the level of relative risk aversion. The figure shows that the higher the assumed

43Interest expenses include interests on bank debt, mortgages, and interest on student loans. Figure G.2 shows that, at a given age, children have lower net assets (assets minus liabilities) than their parents. Figure A.4 shows the decreasing interest rates in Denmark over time.

44This is also consistent with the findings in Section 5.6, where we show that lower-income children, who are more likely to have liquidity-constrained parents but not to be liquidity-constrained themselves, have lower IGE estimates for lifetime wealth than for PDV. Our results are also consistent with the disposable income results, where we find that a decrease in interest rates increases disposable income for middle- to high-income children.
level of risk aversion ($\rho$), the higher the estimated level of absolute mobility. This is true across family units (family and individual level, see Appendix G.3).

### 7.2 Changes in Welfare across Generations

Our integration of economic theory into the study of intergenerational mobility allows us to advance the literature and capture the change in welfare across generations. We do so by examining how the intertemporal ratio of marginal utility of consumption $\beta U_c(c_{i,t+1})/U_c(c_{i,t})$ from household Euler equations (see Equation (3)) has changed over cohorts. Figure 14 shows the cumulative distribution for parents and children measured at age 35. Consistent with Figures 4(c) and (d), Figure 14 suggests that after taking into account credit constraints and earnings uncertainty, recent cohorts discount their future incomes less heavily at their early ages (in their 30s) compared to their parents. This is reflected in their consumption profiles. The expected future consumption for individuals with binding credit constraints or significant precautionary savings (due to a great deal of income risk) is higher compared to current consumption, leading to a lower SDF for the valuation of future income. This is because future incomes are worth less for an individual if they cannot access them in advance to smooth their consumption. Consistent with this, Figure 12(b) suggests that the gains are greatest for children of educated and wealthier parents. As Hai and Heckman (2017) show, the most educated face the steepest earnings growth profiles and face intertemporal credit constraints until their early 40s.

### 7.3 Summary

Our results show the importance of considering which measures are analyzed, what they capture, and what the prevailing economic conditions are when analyzing absolute mobility patterns. Relative mobility estimates based on traditional income measures that ignore different life-cycle trajectories (among others) overstate intergenerational mobility. Absolute mobility estimates based on traditional income measures that do not take into account changes in economic environments severely underestimate the welfare gains across generations. The SDFs are statistically different across education groups for both generations.
Figure 13: (a) Components of Absolute Mobility of Disposable Income and (b) Expected Lifetime Wealth Under Alternative Levels of Risk Aversion

Notes: Figure (a) shows the percentage of male children (of the 1981–1982 birth cohort) whose disposable income is greater than that of their fathers, as we add the different components of disposable income, by percentile of the fathers’ disposable income distribution. Figure (b) shows the percentage of male children whose lifetime wealth is greater than that of their fathers using different risk aversion parameters (\( \rho \)), by percentile of fathers’ wage income distribution.
8 Conclusion

Incomes of fathers and sons measured over the same age intervals across cohorts are the most commonly utilized measure in empirical studies of intergenerational mobility. This paper shows that, for many reasons, the traditional approach gives an incomplete account of intergenerational mobility. For one thing, families, and not fathers alone, shape child’s lives, but there are many other factors at work. We present new theory-motivated measures of expected lifetime resources, allowing us to study parents’ expected lifetime resources at crucial stages of investment in the lives of children.

These measures take into account intergenerational differences in life-cycle family dynamics, earnings uncertainty, agent information sets, and borrowing constraints when children are young. Age-specific expected lifetime measures are better predictors of child human capital outcomes due to the much closer connection to the resources parents plan on in making decisions to invest in children. In this regard, our expected lifetime measures quantify long-term family
influence on children. Realized lifetime measures are considerably less correlated with decision-relevant expected lifetime measures.

Accounting for taxes and transfers, earnings uncertainty, and imperfect credit markets, we estimate significantly higher intergenerational persistence in expected lifetime measures compared to what is found using traditional snapshot measures of income claimed to approximate lifetime income and compared to realized lifetime measures. Our evidence is robust across a variety of specification checks and alternative measures of persistence.

Conventional measures of income underestimate absolute mobility compared to our expected lifetime measures. Our IGE decomposition analysis highlights the role of important intergenerational differences in educational attainment and the timing of family formation in explaining the differences between traditional and expected lifetime measures of IGEs. Changes in patterns of educational attainment are the main drivers of high IGEs. Life-cycle dynamics have changed greatly across cohorts. Recent cohorts acquire more education and, as a group, face significantly steeper income profiles in their early adulthood. Using economic theory applied to data on individual and family life-cycle dynamics gives a deeper understanding of the mechanisms shaping social mobility.

References


