DISCUSSION PAPER SERIES

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ABSTRACT

Zero-Hours Contracts in a Frictional Labor Market*

We propose a model to evaluate the U.K.'s zero-hours contract (ZHC) – a contract that exempts employers from the requirement to provide any minimum working hours, and allows employees to decline any workload. We find quantitatively that ZHCs improve welfare by enabling firms with more volatile business conditions to create additional jobs. While weaker than job creation, substitution effects – some jobs that are otherwise viable under regular contracts are advertised as ZHCs – are sizable and likely explain negative reactions against ZHCs. Our model also assesses increased labor-force participation from ZHCs which appeal to individuals who prefer flexible work schedules.

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1 Introduction

What are the employment and welfare effects of flexible work arrangements? We answer this question using a structural model where flexible work arrangements affect the labor market through three channels. First, a job-creation effect, driven by firms endowed with more volatile business conditions which can only enter the market and/or post more vacancies by using flexible work arrangements. Second, a substitution effect, whereby some jobs which would otherwise be viable under regular employment conditions become advertised as flexible jobs. Third, a labor force participation effect, whereby workers who prefer flexible work schedules join the labor market to take advantage of the new work arrangements. We use the model to quantify the importance of these three channels in the context of the United Kingdom low-wage sector in which zero-hours contracts – contracts which exempt employers from the requirement to provide any minimum guaranteed working hours – have become a common, as well as controversial, tool to introduce more flexibility in the labor market (Wakeling [2014]; Adams et al. [2015]; Adams et al. [2018]).

The model on which we base our analysis features two-sided heterogeneity and a frictional labor market. Firms differ from each other due to, e.g., differences in fluctuating demand conditions, and flexible work offers a mean to minimize the consequences of these fluctuations on firms' profits. Firms trade off this benefit against the risk of meeting workers who dislike being employed under flexible work conditions, and would therefore reject the job offer or search on-the-job for better employment conditions and eventually leave to a competitor.[1] The relative importance of the forces that govern this trade off depend on aggregate search frictions, the vacancy posting decisions of other firms, and the distribution of workers across firms. The equilibrium exhibits partial *sorting*, in the sense that workers who strictly prefer regular employment conditions are sometimes employed at firms that only offer flexible jobs.[2] In fact, this mechanism is key to sustain an equilibrium with flexible work arrangements, as there may not be sufficiently many workers who strictly prefer flexible employment conditions to make firms with flexible jobs survive in equilibrium.

We first characterize how workers' types, which typically capture heterogeneous preferences for leisure or household production, lead to a certain ranking of employment opportunities. Likewise, we relate firms' heterogeneous types to a preferred job-posting behavior; that is to say, whether they advertise a vacant position as a regular or flexible job. We then discuss several implications of flexible work arrangements by comparing a counterfactual world where firms can only offer regular employment conditions with the benchmark equilibrium, where the two job types coexist. Some of these implications are clearly not trivial. Consider for instance the impact on the total number of workers who are employed under *regular* work conditions. First, since flexible work unlocks job creation among firms facing highly volatile business conditions, more vacancies are posted and as a result the aggregate job-finding rate goes up. Second, the vacancies posted by firms that offer regular employment conditions get partially crowded out

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[1]This trade-off is a familiar one under job posting and random search, as in Burdett and Mortensen [1998].

[2]The model also allows for sorting in the reserve direction; that is to say, workers who strictly prefer flexible work arrangements but who temporarily work at firms that only offer regular employment. However, this case turns out to be not empirically relevant in our calibration.

by those of firms with flexible work, and this lowers the number of workers who land a job at a regular firm. Third, some workers who prefer regular employment conditions now take on flexible work and carry on searching on the job. If on-the-job is not as effective as off-the-job search, this results in reduced search efficiency towards regular employment.

Ultimately, we are interested in applying our model to real-world situations. We choose to focus on the U.K.'s zero-hours contracts (ZHCs). This is an interesting case not only because the number of workers employed under ZHCs has grown during the past decade (Figure 1), but also because the main themes of the debate in the British media and political arena which has accompanied the spread of ZHCs are well captured by our model.[3]
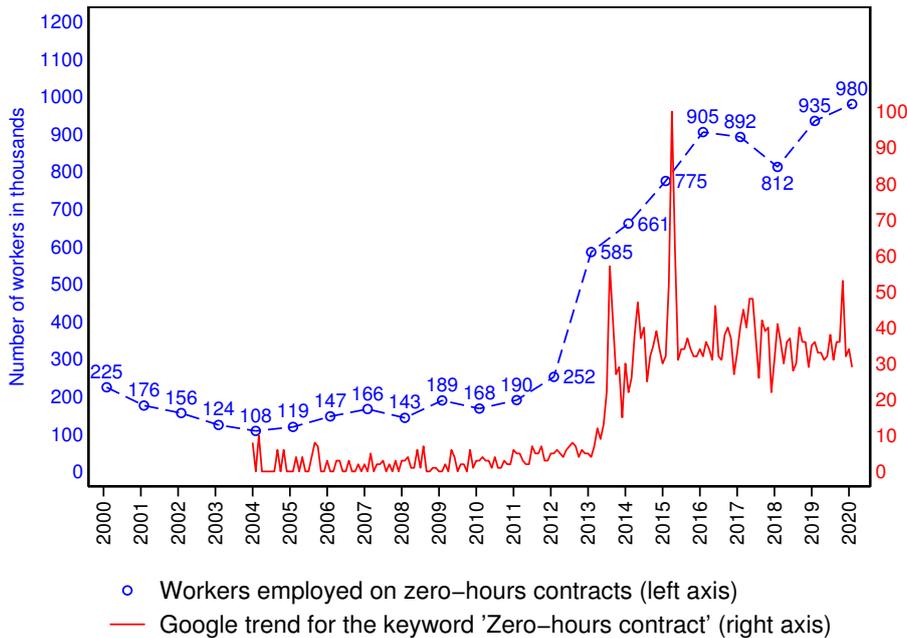

Figure 1: Workers on ZHCs and Google trend for the keyword "Zero-hours contract"

Notes: Left axis (dashed line): Office for National Statistics, calculations based on data from the Labour Force Survey weighted to official population projections. Data is annual from 2000 to 2013, semi-annual from 2014 to 2019 and quarterly in 2020. From 2014 to 2019, the number reported is the average of the two data points. For 2020, the number reported is the value for the 1st quarter. Data is not seasonally adjusted. Right axis (solid line): Google Trends, search for the keyword "Zero-hours contract" in the United Kingdom. Data is available monthly from January 2004 onwards.

We calibrate our model to match data for the U.K. low-wage sector. To uncover the preferences of workers who populate this segment of the labor market, we leverage information contained in the distribution of job tenure in regular jobs as well as in ZHCs, and of unem-

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[3]Trade unions and other commentators have repeatedly raised concerns about the potential exploitation of workers under ZHCs and spillover effects of precarious work on family life and access to credit or housing; see for instance this Press release from the TUC's General Secretary in 2017. At the same time, employers, and some employees, have pointed to the benefits of having flexible labor contracts in the face of fluctuating demand conditions. An eloquent illustration of this was the offer made in 2016 by McDonald's to 115,000 of its U.K. employees to switch to regular contracts with a minimum number of guaranteed hours every week. The offer took place amidst reports in the British press that ZHC workers were struggling to get access to loans and mortgages as a result of not having guaranteed employment each week. However, McDonald's reported that about 80% of these workers chose to remain on flexible ZHCs; see this 2017 article in The Guardian.

ployment duration. The insight is that when these distributions are better approximated by a mixture of exponential distributions rather than a single one, we identify some heterogeneity in exit rates and thus heterogeneity in worker types. To implement this insight, we adapt a statistical procedure developed by Karlis and Xekalaki [1999] to estimate the *number* of heterogeneous types which is needed to explain the data. While relatively intuitive and straightforward, this approach has not been applied before to the calibration of search models with heterogeneous workers, at least not to our knowledge. We find that our data for the low-wage sector is well accounted for by two types of workers: workers who accept flexible jobs as a stepping stone towards regular employment, and workers who only accept flexible jobs.

We then study the effects of a ban on ZHCs, which we think of as an extreme example of introducing tougher regulations on flexible work arrangements. The ban makes the unemployment rate in the low-pay labor market increase by between 2.0 and 2.7 percentage points (p.p.). The range of values reflects the underlying assumption we make about firms which post ZHC jobs. If all ZHCs are offered by firms that would otherwise find it viable to hire workers under regular employment terms, then the unemployment impact of a ZHC ban is minimized; the opposite happens when those firms cannot viably offer regular employment conditions. Meanwhile, the impact on the *employment rate* depends on the underlying heterogeneity of workers who only accept flexible work in the equilibrium with both ZHC and non-ZHC jobs. We identify a range of parameter values (i.e., disutility of work) for which these workers would change their behavior after the ban on ZHCs, namely they would accept regular jobs. When this happens, labor force participation stays put and the decrease of the employment rate mirrors the increase of the unemployment rate. On the other hand, when these workers' disutility of work is higher, they drop from the labor force in response to the ZHC ban, making the employment rate decrease by 4.8 to 5.4 p.p. Sectoral GDP falls by between 2.9 and 3.2 percent in these counterfactual experiments.

To get the full picture of the equilibrium impact of a ZHC ban, we study its effect on accession rates to regular employment. Flexible work can affect the number of those employed under regular work conditions in non-trivial ways. We find that, after the ban on ZHCs, workers spend *less* time *out* of regular employment. The broader lesson here is that jobs which can serve as a stepping stone towards regular employment, such as ZHCs, may also imply that labor market trajectories are more unstable on average. At the same time, after the ban on ZHCs, workers also face slightly longer spells of unemployment.

Given these results, the welfare consequences of a ban on ZHCs are not obvious. Focusing first on workers who strictly prefer regular employment when ZHCs are in place, we find that they suffer a welfare loss by between -0.5 and -0.6 percent of foregone consumption. This reflects the impact of longer unemployment spells, due to reduced job creation. At the same time, in the (partial equilibrium) experiment where we replace the ZHCs of these workers by regular jobs, we find welfare gains of almost +0.2 percent. This may explain the seemingly paradoxical responses to the spread of ZHCs: ZHCs have a substitution effect that is detrimental to the majority of workers in the low-wage sector, but it is counteracted by the other forces (job creation) that come into play in general equilibrium. As for workers who only accept flexible work in the equilibrium where ZHC and non-ZHC jobs coexist, their welfare decreases after the

reform by between -1.7 and -2.0 percent in consumption equivalent variation. For the subset of those workers who would remain in the labor force after the reform, the welfare loss is mitigated by the fact that they adjust their behavior and start working in regular jobs.

Related literature. Our paper contributes to a growing body of research on understanding flexible work arrangements, much of which is motivated by the advent of the online gig economy.[4] One common theme is that although alternative work arrangements have received a lot of public attention, they concern a relatively small portion of the labor force, and as a result they are difficult to study based on standard labor force survey data; see Abraham et al. [2021] and Katz and Krueger [2019b]. An alternative to make up for these data shortcomings is to use administrative data (Collins et al. [2019]; Jeon et al. [2021]), design specific questionnaire surveys (Katz and Krueger [2019a]; Boeri et al. [2020]) or conduct field experiments (Mas and Pallais [2017]; Angelici and Profeta [2020]). We undertake a different approach by combining a structural model with the empirical information provided by labor force survey data. Doing so, we direct our attention to slightly different questions than those asked in this literature. While the focus is typically on job satisfaction and valuation of flexibility on the workers' side, and productivity and cost implications of flexibility on the firms' side, we analyze the general equilibrium employment and welfare effects of flexible work arrangements.

One study closely related to ours is Scarfe [2019]. The author develops a frictional labor-market model to analyze the coexistence of "casual work" (akin to ZHCs in our analysis) and regular employment, and calibrates the model on data for Australia where casual workers account for about 10% of the labor force. Her model and ours differ along several important dimensions. Foremost, we emphasize *ex ante* heterogeneity as a key source of variation to understand workers' and firms' rankings of different labor contracts. In Scarfe [2019], agents are homogeneous *ex ante* and the choice of contracts depends much on "luck", namely the stochastic draw of match productivity at the time of meeting between firms and workers. Our assumptions are guided by our focus on linking the pros and cons of ZHCs to individuals' heterogeneous valuations of flexibility. Scarfe [2019]'s approach is rather guided by the objective of making her model tractable to theoretically analyze casual jobs. For this reason, we view the two studies as being complementary to each other.

Our paper is also related to Datta et al. [2019]'s study of the U.K. zero-hours contracts. The authors use a wealth of empirical evidence to describe the nature of ZHC work and its interaction with labor market policies: labor force survey data as in this paper, as well as an online survey data, and matched employer-employee data from an online system that records information on workers and administers payroll, training, etc. in the U.K. social care sector. Indeed, a large part of Datta et al. [2019]'s study is devoted to the use of ZHCs in the adult social care sector, and how it is impacted by the national mandated minimum wage. We likewise focus on the low-wage segment of the U.K. labor market when we bring the model to data.

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[4]However, alternatives to regular, fixed hours employment contracts are clearly not a new feature of labor markets. Other flexible work arrangements, such as those labeled "reservist", "on call", and "if and when" contracts (see Dickens [1997]) date back to the 19th century where workers hired under piece-rate contracts were not guaranteed any amount of fixed work on a daily or weekly basis, e.g. in industries involving dock labor. In this respect, the lessons of our analysis extend beyond the recent experience of the U.K. labor market.

While we do not investigate the impacts of minimum wage policies, our model's comparative statics is consistent with Datta et al. [2019] in its prediction, that an increase in the minimum wage would push firms to use ZHCs more intensively.

Last, there is a relation between our paper and the literature which studies dual labor markets, in the sense that the models therein deal with the coexistence of two types of jobs – temporary and permanent – in the context of a frictional labor market. However, this literature addresses a different set of questions, such as how this coexistence responds to changes in employment protection legislation; see, e.g., Cahuc et al. [2016, 2020], Créchet [2022], and Franceschin [2020]. We note that, besides this difference, the key features of temporary jobs analyzed in these models are much different from those of flexible jobs in our model. Temporary jobs are essentially jobs that have a fixed expiration date (see Cahuc et al. [2016]) which is in general much lower than the average duration of permanent jobs. On the other hand, flexible jobs in our model allow firms to make higher per-period profit but have a lower expected duration due to workers quitting to regular jobs.

Outline. Section 2 presents our model of the coexistence of regular and flexible work arrangements. Section 3 establishes several theoretical results by discussing the equilibrium configurations of the model. The second half of the paper specializes the analysis to the U.K. zero-hours contracts. Section 4 provides some contextual background. Section 5 proceeds with the model calibration. Section 6 analyses the policy counterfactuals. Finally, Section 7 concludes. An Online Appendix gathers proofs of the main Propositions, further details about the calibration, descriptive statistics, and robustness analyses of the results.

2 The model

This section presents our labor market model featuring regular as well as flexible work arrangements. Anticipating on Sections 4–6 in which we apply the model to zero-hours contracts, we refer to regular employment as R and to flexible work as Z. Time is discrete and runs forever.

2.1 Main assumptions

The economy is populated by a unit continuum of workers who come in different types indexed by i, and an endogenous measure of heterogeneous firms indexed by j. Workers consume and provide their labor services in exchange for wages; firms own the technology and hire workers to produce the goods. Agents discount the future at a common rate ρ.

Workers and firms come together via random search. Workers can search both off- and on-the-job, albeit with different degrees of effectiveness. The number of contacts per unit of time depends on market tightness θ, which is the ratio between the number of vacant positions and job seekers weighted by their search intensity. The contact rate for non-employed workers is $\lambda(\theta)$, where $\lambda(.)$ is an increasing, concave function of θ. For employed workers of type i, the contact rate with vacant positions is $x_{i,k}\lambda(\theta)$, where $x_{i,k} = 0$ if type-i workers do not search on-the-job and $x_{i,k} = x > 0$ if they search under employment conditions $k = R, Z$ (details

follow). The job-filling rate, i.e. the probability that a randomly chosen vacant job meets a randomly chosen job seeker, is $\lambda(\theta)/\theta$, which is a decreasing and convex function of θ. All jobs are destroyed with a per period probability δ. When this happens, the firm leaves the market. Conditional on not being hit by the δ shock, a firm's position becomes vacant if the worker quits to another firm (through on-the-job search). In this event since the job has not been destroyed, the firm remains in the market and re-advertises its now vacant position.[5]

Firms can offer either regular (R) or flexible (Z) employment conditions. The flow payoffs of workers from being employed under these conditions are denoted as ω_R^i and ω_Z^i, respectively. As the superscript i suggests, the payoffs depend on workers' types; they depend on firms' types insofar as each firm type j maps into a given offered employment condition, R or Z, in the economy's equilibrium.[6] The flow payoff from non-employment for type-i workers is denoted as ω_N^i. For type-j firms, the flow profits generated by regular and flexible employment are denoted as π_R^j and π_Z^j, respectively. It is straightforward to allow these to depend on i, the type of the worker who is employed by the firm,[7] but we choose not to so as to make the trade-offs clear. In our model's formulation, a firm cares about her worker's preference over R and Z only due to on-the-job search, as it implies that the firm may loose the worker to better employment conditions. To focus on the trade-off between choice of employment conditions and worker turnover, we additionally make one key assumption about on-the-job search: we rule out job-to-job transitions *within R and Z*. Given that we assume no within-R or within-Z dispersion of employment conditions, this essentially means that a worker quits to another firm if and only if the benefit of doing so is strictly positive – a feature that could be rationalized by introducing an epsilon cost of moving jobs.

The remaining assumptions concern firms' entry and the firms' choice of R vs. Z employment conditions. Firms enter the market until the expected value of doing so is exhausted. They pay a business creation cost K to enter the market and then discover their type. γ_j denotes the probability of drawing type j upon entry. Crucially, we assume that firms must advertise the offered conditions, R or Z, upon posting a vacancy. The flow cost of vacancy posting is homogeneous across firms and given by $\kappa > 0$. Hence, a firm's choice of R vs. Z depends chiefly on the job-filling rate as well as the distribution of workers' preferences over employment conditions and other firms' choices of employment conditions since the combination of these determine the individual firm's turnover rate and ease of hiring.

Before closing this section, we would like to point out that the flow payoffs and profits from employment, the ω_k^i's and π_k^j's, are taken as exogenous. By ruling out wage bargaining, we focus firms' attention on which type of job offer they make; and likewise, on the workers' side, we focus attention on which jobs they choose to accept. In sum, we put labor turnover at the center of the model mechanisms. Keeping the flow payoffs and profits exogenous also circumvents certain contracting issues that arise with standard Nash wage bargaining and on-the-job search

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[5]This is unlike the textbook frictional labor-market model which does not distinguish between firm-worker separations and the exit of firms from the market.

[6]In the computed equilibrium in Sections 5 and 6, we consider only two firm types j that directly map into offering either R or Z employment (with the mapping potentially depending on aggregate outcomes). Thus, a worker employed under employment condition k can back out the type j of his/her employer.

[7]A firm's asset value of employing a worker depends on the type i of the worker (see Subsection 2.3). Thus, anything that defines this asset value (profit, quit rate, etc.) could be a function of the worker's type.

(Shimer [2006]). Last, in the numerical applications, we focus on the low-pay segment of the labor market, where by definition wages are in the neighborhood of the statutory minimum wage. This certainly reduces the scope for wage bargaining between workers and firms.

2.2 Workers' asset values

We now turn to writing the asset values of workers. It is clear from what precedes that these depend on aggregate search conditions as well as the type of jobs offered by firms. We let v_k denote the measure of vacant positions advertised as $k = R, Z$, and $v = \sum_k v_k$ denote the aggregate measure of vacancies. For workers of type i, we denote as N^i the value of non-employment and W_k^i the value of being employed in $k = R, Z$. These asset values solve:

$$N^i = \omega_N^i + \frac{1}{1+\rho} \left[(1 - \lambda(\theta)) N^i + \lambda(\theta) \sum_{k'} \frac{v_{k'}}{v} \max\left\{ N^i, W_{k'}^i \right\} \right], \tag{1}$$

and

$$W_k^i = \omega_k^i + \frac{1}{1+\rho} \left[\delta N^i + (1-\delta) \left((1 - x_{i,k}\lambda(\theta)) W_k^i + x_{i,k}\lambda(\theta) \sum_{k'} \frac{v_{k'}}{v} \max\left\{ W_k^i, W_{k'}^i \right\} \right) \right], \tag{2}$$

where $k = R, Z$. The "max" operator in Equations (1) and (2) captures the decision to accept or reject an offer $k' = R, Z$ from either non-employment or employment. The variable $x_{i,k}$, which is a policy function, must be consistent with the worker's own search decisions in the sense that $x_{i,k} = x > 0$ if and only if there exists an offer k' such that $W_{k'}^i > W_k^i$. The probability that the job offer is of type $k' = R, Z$ is $v_{k'}/v$.

With these asset values at hand, we can analyze how type-i workers rank the labor market opportunities that are available to them. Proposition 1 describes useful comparative static results to this end:[8]

Proposition 1. *(Workers' ranking of labor market opportunities)*

(i) A type-i worker participates in the labor market if and only if at least one of the flow payoffs of employment is greater than the payoff of non-employment, i.e. the worker participates if and only if $\max\{\omega_R^i, \omega_Z^i\} > \omega_N^i$.

(ii) A type-i worker's most preferred employment option between regular (R) and flexible (Z) work is determined only by the difference between the flow payoffs ω_R^i and ω_Z^i, i.e. $W_R^i > W_Z^i \Leftrightarrow \omega_R^i > \omega_Z^i$.

(iii) A worker's second preferred option shifts towards non-employment if a) on-the-job is relatively ineffective ($x \to 0$), b) the labor market is tight ($\lambda(\theta) \to 1$), c) the relative availability of the worker's most preferred employment option is high ($v_k/v \to 1$).

Part (i) of Proposition 1 is a simple result that notably puts discipline on the ω_k^i's and ω_N^i. That is, if the flow payoff of non-employment is largest, then $\max\{N^i, W_{k'}^i\} = N^i$ for both $k' = R, Z$ in Equation (1) and the asset value of non-employment boils down to $N^i =$

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[8]All analytical derivations and proofs are collected in Online Appendix A.

$\omega_N^i + \frac{1}{1+\rho} N^i$. We would define this situation as 'non-participation' and consider that the worker is not searching at all, and is therefore not creating any congestion to those workers who are searching with a view to accept a job offer.

Figure 2 illustrates Parts (ii) and (iii) of Proposition 1. Part (ii), stating that the ranking between employment options boils down to comparing their flow payoffs ω_k^i, follows from the fact that the job destruction rate δ is independent of $k = R, Z$ and on-the-job search is allowed from both types of jobs. This means that, in the $(\omega_Z^i - \omega_N^i, \omega_R^i - \omega_N^i)$-plane, workers prefer R over Z in the region above the 45° line, and the reverse is true in the region below this line. Part (iii) of Proposition 1 is also intuitive. Factors which make workers better off searching for jobs from non-employment rather than from employment shift their second preferred option towards non-employment. For example in the region above the 45° line in Figure 2, workers prefer regular over flexible employment, i.e. we have $W_Z^i < W_R^i$. However, if jobs are generally hard to come by (lower θ and hence lower $\lambda(\theta)$), or if regular jobs are scarce (lower v_R/v), these workers will accept being employed in flexible work, implying that $N^i < W_Z^i$. A higher on-the-job search intensity x acts in the same direction as it allows these workers to quit flexible into regular employment at a faster rate. On the other hand, if x is low or regular jobs are easy to find, workers prefer waiting for the right job in unemployment, i.e. $W_Z^i < N^i$.
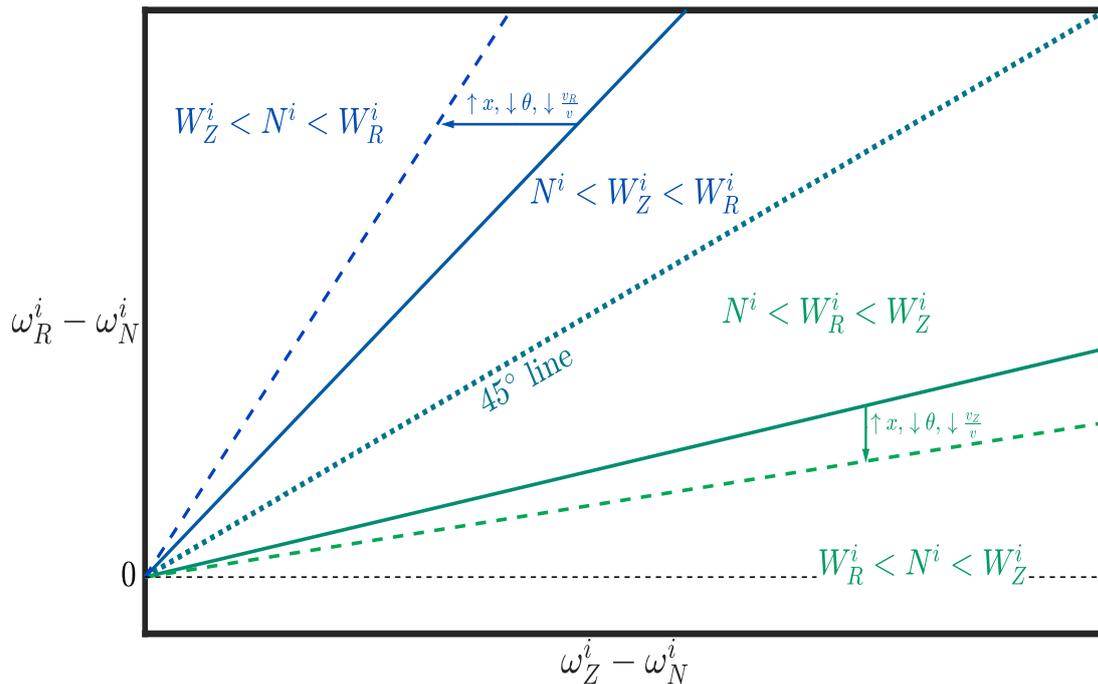
Figure 2: Workers' rankings of regular and flexible employment and non-employment

Notes: The figure shows workers' rankings of the asset values of regular (W_R^i) and flexible employment (W_Z^i) and non-employment (N^i) as a function of the flow payoffs of employment, ω_R^i and ω_Z^i, net of that of non-employment ω_N^i, and the effects of search frictions (x and θ) and job vacancies (v_k/v) on the rankings.

Taxonomy.   In sum, the above comparative statics suggest that we distinguish between four rankings (conditional on aggregate search conditions, as stated by Part (iii) of Proposition 1)

to categorize workers as follows:

$$W_Z^i < N^i < W_R^i : \quad R\text{-only workers} \qquad\qquad N^i < W_R^i < W_Z^i : \quad Z\text{-best workers}$$
$$N^i < W_Z^i < W_R^i : \quad R\text{-best workers} \qquad\qquad W_R^i < N^i < W_Z^i : \quad Z\text{-only workers}$$

(3)

R-best and Z-best workers are the ones most attached to the labor market as non-employment is their least preferred option. They search on the job while employed under respectively Z and R conditions. Notice that on-the-job search in our model has a revealed preference flavor: if $x_{i,Z} = x$, then type-i agent is a R-best worker, while if $x_{i,R} = x$, then she is a Z-best worker.

By contrast, R-only and Z-only workers are less attached in the sense that they would prefer non-employment over employment under certain working arrangements. In particular, in a world without flexible work, it could be that in equilibrium Z-only workers prefer not to participate to the labor market – this would happen if the asset values in the equilibrium without Z are such that $W_R^i < N^i$ for these workers. Thus, we can assess by how much the labor force participation rate among workers characterized by the above rankings would *change* in response to a ban on flexible work arrangements. In this sense, Z-only workers allow us to capture the so-called *participation* effect of flexible work.

2.3   Firms' asset values

To write the asset values of firms, it is useful to recall that they choose what employment conditions to offer, so $k(j) = Z, R$ is a policy function on the firms' side. Let $e_{i,j}$ denote the measure of filled jobs in firms of type j by workers of type i; u_i the measure of unemployed workers of type i; v_j the measure of vacancies of firms of type j; $e_i = \sum_j e_{i,j}$ the aggregate measure of employed workers of type i; $u = \sum_i u_i$ the aggregate measure of unemployed workers; and $v = \sum_j v_j$ the aggregate measure of vacancies.

For firms of type j, V_k^j denotes the asset value of holding a vacant position advertised as $k = Z, R$, and $J_{i,k}^j$ is the asset value of holding a job filled with a type-i worker. Firms' asset values of advertising a vacant position are given by:

$$V_k^j = -\kappa + \frac{1}{1+\rho} \left[V_k^j + \frac{\lambda(\theta)}{\theta} \sum_i \frac{u_i \mathbb{1}_{\{W_k^i > N^i\}} + \sum_{j'} x_{i,k(j')} e_{i,j'} \mathbb{1}_{\{W_k^i > W_{k(j')}^i\}}}{u + \sum_{i'} x_{i'} e_{i'}} \left(J_{i,k}^j - V_k^j \right) \right], \quad (4)$$

where $\mathbb{1}_{\{.\}}$ is the indicator function. Conditional on meeting a worker, which occurs at rate $\lambda(\theta)/\theta$ for vacant jobs, the probability that she will accept an offer k depends on her current labor market status and preferred employment contract. At the denominator, $\sum_{i'} x_{i'} e_{i'}$ is a short notation for the measure of employed workers who are searching on the job.

For filled jobs, the asset values solve:

$$J_{i,k}^j = \pi_k^j + \frac{1-\delta}{1+\rho} \left[J_k^j + x_{i,k}\lambda(\theta) \sum_{j'} \frac{v_{j'}}{v} \mathbb{1}_{\{W_{k(j')}^i > W_k^i\}} \left(V_k^j - J_{i,k}^j \right) \right]. \quad (5)$$

In this equation, the probability that the job remains filled depends on the equilibrium offers

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from other firms (through $v_{j'}/v$) and on workers' preferences over those offers. Finally, observe that in Equation (5) the continuation value is multiplied by $1 - \delta$. With probability δ the firm exits the market and its asset value becomes zero under the free entry condition (described in Subsection 2.5 below).

Solving for V_Z^j and V_R^j, we can establish results in Proposition 2 which are informative about the rankings of regular and flexible work on the firms' side:

Proposition 2. *(Firms' ranking of job posting options)*
(i) A type-j firm's preference for posting an offer $k = R, Z$ as opposed to not posting any offer increases with the flow profits π_k^j and with the share of workers who accept offers k among the pool of job seekers.
(ii) A type-j firm's preference between posting an offer R vs. Z is determined by the difference between flow profits π_R^j and π_Z^j, and by the difference between the share of job seekers who accept R vs. Z offers among the pool of job seekers.
(iii) A tighter labor market $(\lambda(\theta)/\theta \to 0)$ reduces the value of posting an offer but has an ambiguous impact on a firm's choice of R vs. Z offers. On-the job search efficiency (x) has an ambiguous impact on the value of posting any offer type.

Proposition 2 is illustrated by Figure 3 showing a firm's ranking of the asset values of posting R offers, Z offers, and not posting a vacancy, within the $(\pi_Z^j - (-\kappa), \pi_R^j - (-\kappa))$-plane. In the bottom left quadrant, the flow profits of employing a worker (net of the vacancy posting cost κ), are so low that a firm cannot profitably advertise any job offer. Above (resp. to the right-hand side of) this region, flow profits are higher, making it worth offering R (resp. Z) employment conditions. If fewer workers among job seekers accept the offer made by the firm, or if the market is tighter (meaning that the job-filling rate $\lambda(\theta)/\theta$ is lower), the threshold for posting a vacancy increases. The horizontal dashed line in Figure 3 illustrates the impact of a decrease in the share of workers in the pool of job seekers who accept R offers, denoted as $\ell_{i,R}$ (see the footnote of Figure 3). As shown by the vertical arrow, the region where firm j experiences $V_R^j > 0$ shrinks with the decrease in $\ell_{i,R}$.

In the top right quadrant of the $(\pi_Z^j - (-\kappa), \pi_R^j - (-\kappa))$-plane, a firm can profitably offer either type of employment conditions. The trade-off, then, depends on the relative profits, $\pi_R^j - \pi_Z^j$, and relative chances of meeting a job seeker who accepts the employment conditions offered by the firm, denoted as $\ell_{i,R} - \ell_{i,Z}$. For example, in the upper part of this quadrant, since the difference $\pi_R^j - \pi_Z^j$ is positive, the firm prefers to advertise a job as R rather than Z conditional on posting, as shown by the rankings $0 < V_Z^j < V_R^j$. The arrow connecting the solid to the dotted line in Figure 3 illustrates the impact of a decrease in the share of workers in the pool of job seekers who accept R offers relative to the share of workers who accept Z offers. The locus $V_Z^j = V_R^j$ rotates anticlockwise as a higher $\pi_R^j - \pi_Z^j$ is required to compensate for the decrease in $\ell_{i,R} - \ell_{i,Z}$.

Part (iii) of Proposition 2 describes the role of aggregate search conditions – tightness θ and on-the-job search efficiency x. In a tight labor market, the job-filling rate is lower, which increases the duration of a vacancy, and the job-finding rate is higher, reducing the expected duration of a filled job if the worker searches on the job. The effect on the asset values of

11
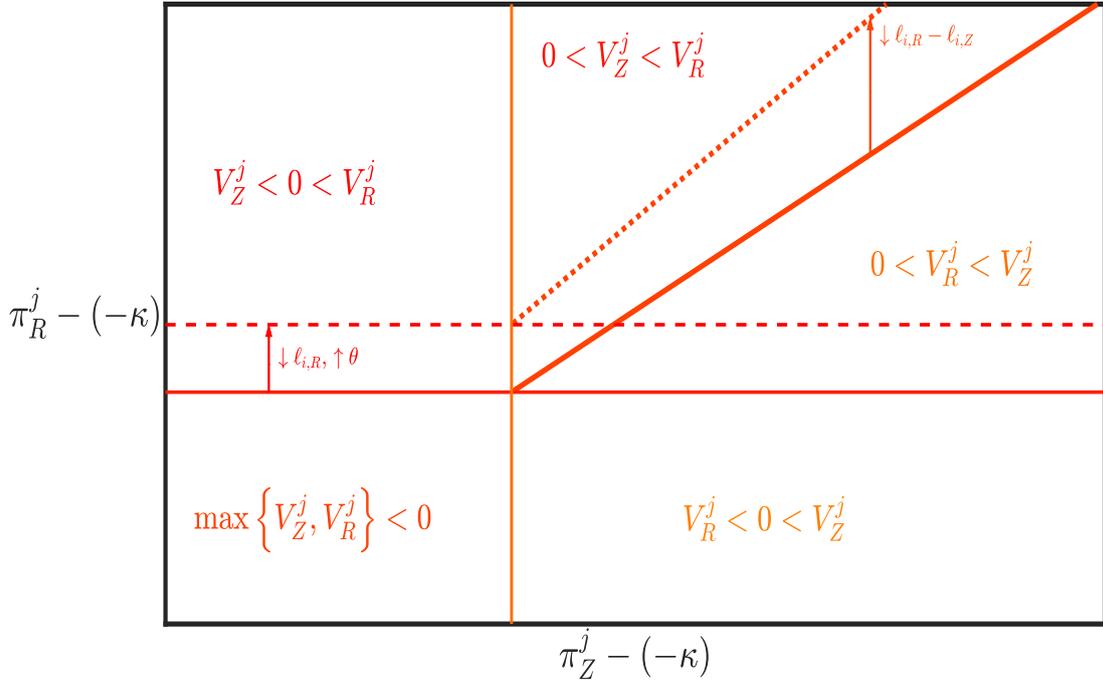
Figure 3: Firms' rankings of offering regular and flexible employment

Notes: The figure shows firms' rankings of the asset values of offering regular (V_R^j) and flexible employment conditions (V_Z^j) as a function of the flow profits of employment, π_R^j and π_Z^j, net of the vacancy posting cost κ, and the effects of market tightness (θ) and the share in the pool of job seekers of type-i workers who accept offers $k = R, Z$ (denoted as $\ell_{i,k}$) on the rankings.

posting a vacancy is negative, but given that this affects both V_R^j and V_Z^j, the effect on the choice of vacancy type is ambiguous. On the other hand, on-the-job search efficiency x has an ambiguous impact on all choices. This is because a higher on-the-job search efficiency means a larger pool of job seekers (in search units) and hence a higher chance of meeting a worker, but also lower returns to hiring a worker as it reduces the expected duration of a filled job.

Taxonomy. Similar to the taxonomy of workers, we categorize firms in the following way:

$$
\begin{array}{llll}
V_Z^j < 0 < V_R^j : & R\text{-only firms} \qquad & 0 < V_R^j < V_Z^j : & Z\text{-best firms} \\
0 < V_Z^j < V_R^j : & R\text{-best firms} & V_R^j < 0 < V_Z^j : & Z\text{-only firms}
\end{array} \tag{6}
$$

Again, the rankings are not inherent to the agents as they depend on aggregate search conditions following Part (iii) of Proposition 2. Notice that if these rankings remained the same in a world without flexible arrangements, then Z-only firms would abstain from creating any jobs in such a world. Thus, Z-only firms enable us to capture a *job creation* effect triggered by flexible work. Z-best firms, on the other hand, would profitably hire workers under R employment conditions in a world without flexible arrangements, and switch to offering Z when this option becomes available. Hence, these firms allow us to capture the so-called *substitution* effect. Finally, R-only and R-best firms would keep offering R irrespective of whether Z is included in the menu of employment conditions that firms can choose from. The expected discounted value of their profits, however, will in general depend on the presence of Z offers in the labor market. More broadly, whether flexible work is legal or not will affect equilibrium conditions, which in turn

matters for the calculations of the asset values and implies that all firm types may adjust their job creation decisions in response to changes in the legislation of flexible work.

2.4 Law of motion

In a stationary equilibrium, the distribution of job matches, non-employed workers and vacant jobs is determined by the law of motion of the economy. Using a prime ($'$) to denote the one-period ahead value of a variable, the law of motion for job matches between type-i workers and type-j firms is

$$
e'_{i,j} = \left(1 - x_{i,k(j)} \lambda(\theta) \sum_{j'} \frac{v_{j'}}{v} \mathbb{1}_{\left\{ W^i_{k(j')} > W^i_{k(j)} \right\}} \right) (1 - \delta) e_{i,j}
$$
$$
+ \frac{v_j}{v} \lambda(\theta) \left(\sum_{j'} x_{i,k(j')} \mathbb{1}_{\left\{ W^i_{k(j)} > W^i_{k(j')} \right\}} (1 - \delta) e_{i,j'} + \mathbb{1}_{\left\{ W^i_{k(j)} > N^i \right\}} u_i \right). \tag{7}
$$

The first term in Equation (7) represents workers whose jobs do not get hit by the shock δ and who do not quit to another job. Note that workers never quit to a firm that offers the same k – hence they never quit a type-j towards another type-j firm – since there is no dispersion of workers' asset values within employment conditions. The other terms are the inflows into type-j firms from on-the-job search and from non-employment.

For type-i non-employed workers, the law of motion is

$$
u'_i = \delta \sum_j e_{i,j} + \left(1 - \lambda(\theta) \sum_j \frac{v_j}{v} \mathbb{1}_{\left\{ W^i_{k(j)} > N^i \right\}} \right) u_i. \tag{8}
$$

Letting ζ_i denote the exogenous fraction of type-i workers in the population, we have $n_i + u_i + e_i = \zeta_i$ for all i, where n_i is type-i workers who do not participate in the labor market. We have: $n_i = \mathbb{1}_{\left\{ \max\{W^i_R, W^i_Z\} > N^i \right\}} \zeta_i$ (i.e., $e_i = u_i = 0$ when $n_i = \zeta_i$). $\sum_i \zeta_i = 1$ as the population size of workers is normalized to one.

Last, for vacant positions offered by type-j firms, we have

$$
v'_j = \lambda(\theta) \sum_i x_{i,k(j)} \sum_{j'} \frac{v_{j'}}{v} \mathbb{1}_{\left\{ W^i_{k(j')} > W^i_{k(j)} \right\}} (1 - \delta) e_{i,j}
$$
$$
+ \left(1 - \frac{\lambda(\theta)}{\theta} \sum_i \frac{u_i \mathbb{1}_{\left\{ W^i_{k(j)} > N^i \right\}} + \sum_{j'} x_{i,k(j')} e_{i,j'} \mathbb{1}_{\left\{ W^i_{k(j)} > W^i_{k(j')} \right\}}}{u + \sum_{i'} x_{i'} e_{i'}} \right) v_j + \delta e \gamma_j. \tag{9}
$$

The first term corresponds to the posting of replacement vacancies, when the worker quits to another job. The second term represents vacancies that fail to meet a job seeker who accepts the jobs offered by type-j firms. Last, there is a total of $\delta \times e$ firms destroyed in each period, which is replaced by an equal measure of newly-entering firms, and a fraction γ_j of these are type-j firms.

2.5 Free entry

The free entry condition closing the model reads:

$$\sum_j \gamma_j \max\left\{0, V_Z^j, V_R^j\right\} - K = 0. \tag{10}$$

The "max" operator in this equation refers to the choices of posting a vacancy and offering R vs. Z jobs employment conditions, where the latter can be expressed as: $\max\left\{V_R^j, V_Z^j\right\} = V_{k(j)}^j$. Firms enter the labor market until the above condition is satisfied. In equilibrium, this determines market tightness θ defined as the ratio between v and $u + \sum_i x_i e_i$.

3 Equilibrium configurations

In this section, we define the steady-state equilibrium of the baseline economy and that of the counterfactual world without flexible work arrangements. We then compare features of the two equilibria to highlight the role of flexible work in altering the functioning of the labor market.

3.1 Baseline equilibrium

We begin with the definition of a steady-state equilibrium of the economy with flexible work arrangements. An equilibrium is a list of asset values N^i, W_k^i, V_k^j, $J_{i,k}^j$; policy functions $x_{i,k}$ and $k(j)$; a stationary distribution of job matches $e_{i,j}$, unemployed workers u_i, inactive workers n_i, and vacancies v_j; and labor market tightness θ such that:

1. Given the policy functions $x_{i,k}$ and $k(j)$, the measures $e_{i,j}$, u_i, n_i, v_j, and market tightness θ, the asset values N^i, W_k^i, V_k^j, $J_{i,k}^j$ solve the Bellman equations (1), (2), (4), (5);

2. Given N^i and W_k^i, workers' on-the-job search decisions $x_{i,k} \in \{0, x\}$ are consistent with the rankings presented in (3);

3. Given V_k^j, firms' job offer decisions (conditional on job posting) $k(j) \in \{R, Z\}$ are consistent with the rankings presented in (6);

4. Given the policy functions $x_{i,k}$ and $k(j)$ and market tightness θ, the measures $e_{i,j}$, u_i, n_i, v_j are time-invariant with respect to the law of motion described in Equations (7)–(9);

5. Given V_k^j, the labor market tightness θ solves the free entry condition in Equation (10).

Conditions 2 and 3 of the above definition are key to understand the workings of the model. Suppose that we start off from a guess of the policy functions $x_{i,k}$ and $k(j)$. Given market tightness θ, the stationary distribution of job matches $e_{i,j}$, unemployed workers u_i, inactive workers n_i, and vacancies v_j can be found by iterating on the equilibrium stock-flow equations. Next, the policy functions allow us to simplify all the "max" operators and indicator functions in Equations (1), (2), (4), (5). For a given stationary distribution, the then simplified Bellman equations define a set of autonomous linear equations, which we can solve in one step. We then

verify that the free entry condition is met. Last, we must check whether the rankings of the asset values presented in (3) and (6) are consistent with agents' policy functions $x_{i,k}$ and $k(j)$.

3.2 Counterfactual equilibrium

To gain understanding on the impact of flexible work employment, we compare the baseline equilibrium to a labor market equilibrium where firms can only offer regular employment. Without flexible work, the equilibrium is (much) simpler because there are no incentives for on-the-job search. On workers' side, the payoffs of employment and non-employment now only affect the decision to participate in the labor market. On firms' side, without on-the-job search the composition of the pool of job seekers becomes irrelevant, and so does the composition of vacant jobs. Proposition 3 formalizes these statements:

Proposition 3. *(Equilibrium without flexible work)*

(i) In the equilibrium without flexible work, a worker's only decision is whether to participate or not. The worker does so if and only if $\omega_R^i > \omega_N^i$.

(ii) In the equilibrium without flexible work, a firm chooses to post a vacancy if and only if $\pi_R^j - (-\kappa) > \kappa/\Gamma$, where $\Gamma = \frac{\lambda(\theta)/\theta}{\rho+\delta+\lambda(\theta)/\theta}$.

Notice the relation of Proposition 3 to Figures 2 and 3. Part (i) of the Proposition analyzes a worker's decision to participate in the labor market along the vertical axis of Figure 2, where we have purposely indicated the point where $\omega_R^i - \omega_N^i > 0$. Part (ii) analyzes the decision to post a vacancy along the vertical axis of Figure 3. The latter result is formulated in a way similar to the proof of Proposition 2 (Appendix A.2), where it is established that $V_k^j = 0 \Leftrightarrow \pi_k^j + \kappa = \kappa/\tilde{\Gamma}_k$. In particular, simple comparative static analysis shows that the threshold for posting a vacancy, κ/Γ, increases with market tightness θ.

Workers' and firms' heterogeneity still matter for the equilibrium of this economy, although their role is dampened. The law of motion of job matches in this economy is

$$e'_{i,j} = (1-\delta) e_{i,j} + \lambda(\theta) \frac{v_j}{v} u_i \tag{11}$$

$$u'_i = \delta \sum_j e_{i,j} + (1-\lambda(\theta)) u_i, \tag{12}$$

together with $n_i + u_i + e_i = \zeta_i$ for all i. It is immediate to show that the *employment* rate is $e = \frac{\lambda(\theta)}{\delta+\lambda(\theta)}(1-n)$, where the labor force participation rate, $1-n$, depends on workers' heterogeneous flow payoffs of employment and non-employment through: $n = \sum_i n_i$ with $n_i = \mathbb{1}_{\{N^i \geq W_R^i\}} \zeta_i$. Meanwhile, firm heterogeneity matters for the equilibrium through the job-creation condition:

$$\sum_j \gamma_j \max\{0, V_R^j\} - K = 0. \tag{13}$$

As before, labor market tightness θ adjusts to equate expected value of vacancy posting to the business creation cost, K.

3.3 Comparative statics

Let us denote the equilibrium outcomes of the counterfactual steady-state equilibrium without flexible work with an upper tilde ($\tilde{\cdot}$). In this section, we compare them to those of the equilibrium with both flexible and regular employment. To clarify exposition, we first consider an equilibrium baseline configuration where there are no Z-best workers, i.e. there are no job-to-job transitions from R to Z. Comparing the cross-sectional distribution of the two economies, we obtain the following result:

Proposition 4a. *(Comparative statics, no Z-best workers)*
Suppose that no worker switches from regular to flexible work in the baseline equilibrium. When comparing regular employment e_R to that in the economy without flexible work, denoted by $\widetilde{e}_R$, the ratio $e_R/\widetilde{e}_R$ can be written as the product of three components:

$$\frac{e_R}{\widetilde{e}_R} = \underbrace{\frac{\lambda(\theta)}{\lambda(\widetilde{\theta})}}_{job\ creation} \times \underbrace{\frac{v_R}{v}}_{vacancy\ competition} \times \underbrace{\frac{\sum_i (1-\delta) x_{i,Z} e_{i,Z} + \mathbb{1}_{\{W_R^i > N^i\}} u_i}{\widetilde{u}}}_{search\ efficiency}. \qquad (14)$$

Equation (14) provides some relevant insights into the effects of flexible work. The first component measures the effect of job creation: flexible work unlocks job creation opportunities for firms entering the market (compare Equation (10) against (13)), which results, *ceteris paribus*, into higher labor market tightness and hence higher regular employment through a higher aggregate job-finding rate. The second component measures the effects of competition between different type of vacancies in the baseline equilibrium. In this equilibrium, R vacancies are diluted in the pool of vacancies, which makes it more difficult for firms to find workers who would accept the offered contract. Hence, vacancy competition in isolation results in lower regular employment. Lastly, if there are R-best workers who take up flexible jobs to search on the job as opposed to searching for R from unemployment, then there is reduced search efficiency towards R employment (since $x < 1$). In sum, while the job creation effect should benefit regular employment, its effects can be dampened or even overturned by vacancy competition and reduced search efficiency.

While in the quantitative application we focus on a world with no Z-best workers, it is instructive to consider the polar case where the economy is populated by such workers and has no R-best workers. Proposition 4b describes this economy:

Proposition 4b. *(Comparative statics, no R-best workers)*
Suppose that no worker switches from flexible to regular work in the baseline steady-state equilibrium. The ratio $\widetilde{e}_R/e_R$ can be written as the product of four components:

$$\frac{e_R}{\widetilde{e}_R} = \underbrace{\frac{\lambda(\theta)}{\lambda(\widetilde{\theta})}}_{job\ creation} \times \underbrace{\frac{v_R}{v}}_{vacancy\ competition} \times \underbrace{\frac{\sum_i \mathbb{1}_{\{W_R^i > N^i\}} u_i}{\widetilde{u}}}_{search\ efficiency} \times \underbrace{\frac{1}{1 + \frac{1-\delta}{\delta} \lambda(\theta) \frac{v_Z}{v} \sum_i x_{i,R} \frac{e_{i,R}}{e_R}}}_{quit\ turnover}. \qquad (15)$$

The first two components in Equation (15) are analogous to those in Equation (14). The third component, namely search efficiency, is now simpler because there are no workers searching

for R directly from Z employment, and therefore we expect this component to be closer to 1 in comparison with the economy with no Z-best workers. Meanwhile, in the economy with Z-best workers, there is a fourth term which, *ceteris paribus*, reduces regular employment: Z-best workers who are employed in R search on-the-job and eventually quit to their preferred employment option, making the "quit turnover" term in Equation (15) be lower than 1.

To summarize, in an economy with flexible work arrangements, there are several forces – vacancy competition, search efficiency, and quit turnover – that act together to reduce regular employment, potentially countervailing the effects of higher aggregate job creation. The importance of these different channels, as well as the other equilibrium and welfare consequences of flexible work arrangements, ultimately depend on specific parameter values that describe the equilibrium of an economy. We turn to the quantification exercise in the next sections.

4 Zero-hours contracts in context

Zero-hours contracts (ZHCs) in the United Kingdom provide a case in point to illustrate the mechanisms embedded in our model. We only summarize here information about ZHCs that is essential for the quantitative analysis of the next sections; we provide additional information about the regulatory framework and descriptive statistics on ZHCs in Online Appendix B.

Definition and regulatory framework of ZHCs. Zero-hours contracts are an example of flexible employment condition whereby the firm does not commit at any point to providing strictly positive hours of paid work, and the worker is not constrained to accept any hours offered by the firm. ZHCs are akin to other work arrangements labeled 'on demand', 'on call', or 'if and when' contracts (see Dickens [1997]). ZHCs are used in the U.K., predominantly in the low wage sector. Yet they are not an exclusive feature of the U.K. labor market: similar contracts can be found in Australia, Canada, Finland and Ireland, though they differ in legal status and levels of regulation.[9]

A precise description of workers' rights and entitlement to welfare is provided in the Online Appendix. In a nutshell, individuals in ZHCs are entitled to the main employment rights save for employment protection. As all adults in the U.K., they are entitled to Universal Credit, which is not conditioned on job search and tapers out as labor income grows beyond a certain threshold. This will be relevant in the quantification of our model in the next sections.

Trends and characteristics of ZHC workers. In the U.K., the number of ZHCs has surged fivefold since the beginning of the 2010s. In 2020 there were a million workers under ZHC, which represents about 3.0 percent of the U.K. labor force (versus 0.8 percent in 2012); see Figure 1. To quantify the implications of ZHCs, we use data from the quarterly U.K. Labour

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[9]Similar contracts also exist in Scandinavian countries and in Cyprus and Malta. Likewise, they are used, albeit subject to much heavier regulations, in Germany, Italy and in the Netherlands. They are either explicitly forbidden or not used in the remaining countries of the European Union. In the United States, on call working arrangements are also growing in importance. Despite the absence of federal regulation, several states operate 'show-up pay' laws, where employers are required to pay workers for a minimum number of hours, if they have been called to work, though coverage varies across these states and a number of exemptions exist.

Force Survey (LFS) from September 2018 through March 2020. We focus on the segment of the labor market where jobs pay around the minimum wage, representing about 16 percent of the U.K. total employment.[10] This leaves us with 9,342 individuals aged 16 to 69 belonging to this low-pay segment.

When we compare the cross-sectional characteristics of workers in ZHCs (denoted in short as Z) with workers in regular contracts (R)[11] and unemployed workers, we find almost no difference in terms of gender. Differences with respect to age, on the other hand, are large: Z employment is more prevalent at both ends of the working life, likely due to work while studying at one extreme, and at the other extreme the demand for flexibility among older workers to smooth out the transition to retirement. At the same time, the cross-sectional distributions of Z vs. R workers exhibit only modest differences in terms of educational attainment. Differences are much larger with regards to industries, with an over-representation of Z jobs in 'Accommodation and food services', 'Health and social work', and 'Arts, entertainment and recreation'.

Hours in ZHCs vs. regular employment. As mentioned, the flexibility afforded by ZHCs has to do with hours worked. The longitudinal dimension of the LFS allows us to compute the mean working hours for each *individual* over the 5 quarterly interviews in which they appear in the data. While individuals who are continuously employed under R jobs work on average 28.1 hours, those on Z jobs work 19.4 hours per week. The gap in hours is in large part driven by the incidence of very short hours schedules in Z employment. For instance, in the first LFS interview, 5 percent of workers in R jobs report working less than one day during the reference week vs. a staggering 17 percent of workers in Z employment.

Worker turnover in and out of ZHCs. The non-employment rate in the first interview for this low-pay segment of the U.K. labor market is 11.2 percent. 431 individuals report having a ZHC in either the first or the second interviews, amounting to 4.6 percent of our sample, about 1.5 times higher than the incidence rate of ZHCs in the overall U.K. labor market.

The distribution of job tenure within each employment type and the duration of unemployment spells yield information on the rates at which individuals exit each labor market state. As shown by Figure 4 in the next section, nearly half of Z workers report job tenures longer than 2 years, contrasting with the popular image of ZHCs as precarious jobs. Job tenures are on average shorter in Z than in regular contracts, but probably less so than expected: 9.2 percent of Z workers were recruited in the last 3 months vs. 3.4 percent of those in R employment, whereas 30.3 percent of workers under ZHCs have been with their current employer for less than a year, vs. 14.3 percent of employees in regular jobs. Last, when looking at the distribution of unemployment duration, we observe that, while over half the stock has been without job for less than 6 months, a substantial 21 percent has been unemployed for over two years.

---

[10]Low-paying occupations include 'Administrative and secretarial', 'Caring, leisure and other service', 'Sales and customer service', 'Process, plant and machine', or 'Elementary' occupations. Our sample includes individuals who either work in a low-paying occupation in at least one quarter or are 'not employed' in any of five quarters while not being recorded as 'inactive' throughout the five quarters.

[11]'Regular contracts' is somewhat of a misnomer because this category lumps together all employed workers not on ZHCs, which could include workers on part-time employment contracts, temporary jobs, etc. Thus, differences between Z and R workers might be dampened by the fact that workers employed in those 'regular' but precarious employment contracts are likely to resemble workers on ZHCs.

Finally, Table 1 in the next section displays the transition matrix between three labor-market states under consideration. Several interesting observations emerge from this matrix. First, 11 percent of exits from unemployment are to Z employment. Second, the rate of transition to unemployment is almost 50 percent larger in Z than in R employment. Third, the job-to-job transitions involving a change of contract type are predominantly from Z to R employment: 6.5 percent vs. 0.5 percent in the reverse direction. These are the key empirical findings that our model calibration tries to replicate.

5   Calibration and inference on types

Next, we turn the model presented in Sections 2 and 3 into a full quantitative tool. The calibration proceeds in two steps. In the first one, we draw information from job and worker turnover moments that are readily available from the data. We specify workers' flows and non-employment payoffs and firms' flow profits in the second step, and then calibrate the remaining parameters. Throughout the analysis, we interpret the model's period as two weeks.

5.1   Labor turnover moments

Heterogeneity in exit rates.   Workers populate different "states" of the labor market – unemployment, Z or R employment – which they leave at different rates. It is well known that when the exit rate is homogeneous, the duration of spells within that state is exponentially distributed. When workers leave at heterogeneous exit rates, the distribution of durations becomes a mixture of exponential distributions. Jewell [1982] shows that, for a given number of 'latent classes', defining the unobserved heterogeneity in exit rates, the weights and exit rates of the latent classes are all statistically identified. We can further adapt a procedure developed by Karlis and Xekalaki [1999] to estimate the number of classes that give rise to the duration distribution of unemployment, Z employment, and R employment observed in LFS data. Details of this approach are presented in Online Appendix C.1; here we provide the main intuition and results.

In terms of the worker types characterized in our theoretical model, the rates at which different workers leave each labor market state are:

	Unemployment	Z employment	R employment
R-only workers	$\lambda\left(\theta\right)\frac{v_R}{v}$	–	δ
R-best workers	$\lambda\left(\theta\right)$	$\delta+\left(1-\delta\right)x\lambda\left(\theta\right)\frac{v_R}{v}$	δ
Z-best workers	$\lambda\left(\theta\right)$	δ	$\delta+\left(1-\delta\right)x\lambda\left(\theta\right)\frac{v_Z}{v}$
Z-only workers	$\lambda\left(\theta\right)\frac{v_Z}{v}$	δ	–

By looking at the survival function of Z employment in LFS data with the method described in Online Appendix C.1, we find that it is well matched by a mixture of two exponential distributions. We thus statistically detect two distinct exit rates for Z employment. This implies that the labor market is populated by R-best workers, but as the table above shows,

this leaves open the possibility that Z-only and Z-best workers are both present as well (since they both have exit rate δ). Next, we find that a single exponential distribution approximates well the tenure duration of R employment, which means that we have only one exit rate for this state. Given that R-best workers are present, this rules out Z-best workers as they exit R employment at a different rate.[12] Last, when we look at unemployment durations, we detect again two types of workers. Since R-best and Z-only workers exit unemployment at different rates and are both present in other labor market states, this rules out R-only workers.[13] In sum, the LFS duration data for the low-wage segment of the labor market is statistically consistent with the presence of R-best and Z-only workers, but not with that of R-only or Z-best workers.

Although very useful, the above approach does not enable us to fully calibrate turnover parameters as it considers each labor market state in isolation from each other. As a result, it does not put any structure on the relations between worker types *across* states.[14] For instance, it does not require that the workers with the highest exit rate in U also have the highest exit rate in Z, or that those with the lowest exit rate in U cannot be observed in R. Developing and estimating a more general statistical model which contains these joint restrictions is beyond the scope of our calibration exercise. Instead, a simpler and transparent approach is to calibrate the turnover parameters of the above table to fit the transition matrix of worker flows across all three labor market states.

Calibration of $\lambda(\theta)$, δ, x, ζ_i's, γ_j's. In our model, the transition matrix of worker flows across unemployment and Z and R employment is implied by Equations (7) and (8), as well as Equation (9) which determines the shares of Z and R vacancies through the γ_j's. Although agent types and their rankings of different states are conceptually different objects, we temporally omit this distinction for the sake of simplicity. We assume that there are only two types of firms: those creating R vacancies, and those that create Z vacancies (either Z-only or Z-best firms, in the terminology of (6)). Given that $\gamma_R + \gamma_Z = 1$, the share of each type of firms depends on only one parameter. For workers, we have just established that there are only two types, and we have $\zeta_{R\text{-best}} + \zeta_{Z\text{-only}} = 1$, with a 100% labor force participation rate of these workers in the baseline equilibrium. This means that we have five parameters, namely: $\lambda(\theta)$, δ, x, $\zeta_{R\text{-best}}$, γ_R, to fit the transition matrix of worker flows across unemployment and Z and R employment (six data moments). However, it is also important to match the duration distribution data, given that it informs the number of worker types through the approach presented above and in Online Appendix C.1. We thus minimize the average distance between the model-implied transition matrix and duration distribution and their empirical counterparts.[15]

---

[12] Z-best workers exit R employment at a higher rate because they make job-to-job transitions to Z employment. We likewise see too few transitions from R to Z employment in Table 1.

[13] The conclusion relies on v_R/v being different from v_Z/v, which is a likely assumption.

[14] The estimation of the number of worker types also does not put constraints on the relation between the exit rates estimated separately for each labor market state. For instance, it does not require that, say, the lowest exit rate from Z matches the lowest exit rate in R.

[15] Note that the worker flows transition matrix and the distribution of unemployment duration and job tenure in Z and R employment may be not fully consistent with each other. The transition matrix is derived from measured labor market status of workers who are tracked longitudinally in the LFS; the duration distribution reflects information reported by respondents at a given point in time. They may both be affected by potential measurement errors, probably with varying degrees – measurement error may be a greater concern for the

This procedure yields: $\lambda(\theta) = 0.051$, $\delta = 0.005$, $x = 0.352$, $\zeta_{R\text{-best}} = 0.969$ and $\gamma_R = 0.950$. The calibrated on-the-job search intensity, x, is on the high end of values commonly used in the literature but not inconsistent with estimates which take account of flexible work arrangements (e.g., Lalé [2019]). The calibrated values of $\zeta_{R\text{-best}}$ and γ_R are driven by the fact that ZHCs make up for a small share of employment, even in the low-pay segment of the labor market.

Figure 4 compares the distributions of job tenure (Panels (a) and (b)) and unemployment duration (Panel (c)) in the LFS data with the model-generated ones. As can be eyeballed, the distribution of job tenure in Z employment can only be explained with heterogeneous workers within this labor market state. Some of these workers (namely R-best workers) quit at a higher rate, which accounts for the larger share of workers with a short job tenure (less than 6 months) compared with a slightly longer tenure (6 to 12 months); other workers (Z-best) remain much longer in these jobs, making up the larger shares of job tenures longer than 2 or 5 years.

Table 1 presents the results for the transition matrix. From the LFS, we have semesterly transition rates, and the model-generated data is aggregated accordingly.[16] The model over-states the transition rate from Z to R employment, which is due to a relatively high on-the-job search intensity x (Table 2), but this is needed to fit the distribution of job tenure in Z. It also overestimates the transition rate from unemployment to ZHCs. On the other hand, it matches perfectly the outflow rates of regular employment. The model also generates a non-zero semesterly transition rate from R to Z jobs. This result is a fabrication of time aggregation: there is no such transition at the model's bi-weekly frequency as the equilibrium does not feature any Z-best workers, but some R-best workers who are employed in R at some point are employed in a Z job six months later after transitioning through unemployment.

Table 1: Model fit: Transition rates between unemployment and employment states

		(a) Model				(b) Data		
	To:	**U**	**Z**	**R**	**To:**	**U**	**Z**	**R**
From:	**U**	56.0	8.8	35.2	**From:** U	62.2	4.2	33.5
	Z	4.8	83.6	11.6	Z	6.2	87.3	6.5
	R	4.4	0.3	95.3	R	4.4	0.5	95.2

Notes: The table shows the model's predicted moments in Panel (a) and their empirical counterparts based on LFS data in Panel (b). U: Unemployment. Z: Employed in a zero-hours contract. R: Employed not in a zero-hours contract. All table entries are expressed in percent.

Matching function. Next, we choose a function which maps labor market tightness θ into the contact rate $\lambda(\theta)$. We use a constant-returns Cobb-Douglas matching function:

$$\lambda(\theta) = M\theta^{\psi}, \tag{16}$$

where M denotes matching efficiency and ψ is the elasticity of the job-finding rate with respect to market tightness. We estimate ψ using U.K. vacancy data for the low-pay segment of the

duration distribution, due to recall bias and "heaping" of reported unemployment duration and job tenures.

[16]The LFS transitions are measured over a one-semester window since we only observe the response to the question: 'Do you hold a zero-hour contract?' every other quarter.
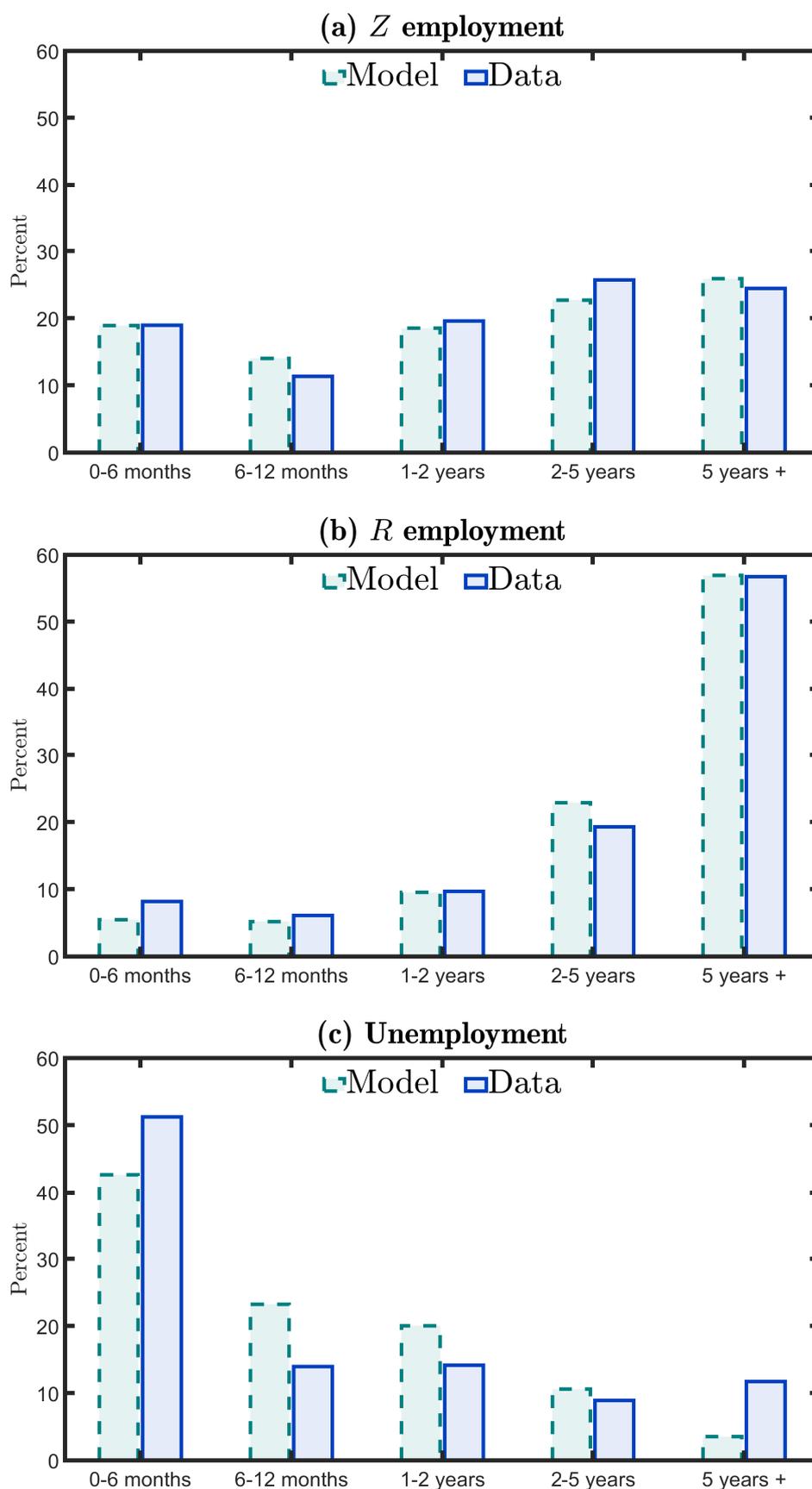
Figure 4: Model fit: Job tenure and unemployment duration

Notes: The dashed bars in this figure show the model's predicted moments and the solid bars show their empirical counterparts based on LFS data. Panel (a): distribution of job tenure in Z employment. Panel (b): distribution of job tenure in R employment. Panel (c): distribution of duration of unemployment spells.

labor market. Our procedure, presented in Online Appendix C.2, controls for occupation fixed effects as well as time variation in matching efficiency, and yields estimates of ψ between 0.60 and 0.70. These figures are higher than the value of 0.5 that is commonly used in the literature, but certainly not inconsistent with the bias-corrected estimate of ψ from Borowczyk-Martins et al. [2013]. We use the mid-point of our empirical estimates and set ψ to 0.65.

Given our estimated ψ, we need one more data moment to separately identify M and θ. For this, we use U.K. estimates of the job-filling rate (the probability of filling a vacancy) from Kuhn et al. [2021]. The authors estimate that the monthly job-filling rate for the U.K. labor market is between 0.35 to 0.38. At the bi-weekly frequency, these numbers imply that $\lambda(\theta)/\theta$ is 0.21. Given the value of $\lambda(\theta)$ that we have obtained to fit the transition matrix in Table 1, this yields $M = 0.128$ and $\theta = 0.242$, i.e. one vacancy for approximately four job seekers.

Equilibrium allocation and sorting. Table 2 reports the parameter values that come out of this first calibration step, while Table 3 presents several moments of interest describing the equilibrium allocation. We compare the first two of these moments to the ergodic distribution

Table 2: Parameter values from the calibration's first step

M	Matching function elasticity		0.128
θ	Labor market tightness		0.242
ψ	Elasticity of job-filling rate w.r.t. tightness		0.65
δ	Job destruction probability		0.005
x	On-the-job search efficiency		0.352
$\zeta_{R\text{-best}}$	Share of R-best workers		0.969
γ_R	Probability of drawing type-R upon entry		0.950

Notes: The table reports the parameter values from the first step of the calibration targeting data moments on job and worker turnover. The model period is set to be two weeks.

of the empirical transition matrix across U, Z and R shown in Table 1. The model's baseline unemployment rate is 9.2 percent, which is fairly similar to the value implied by the empirical transitions in and out of unemployment, namely 10.1 percent. The model's ZHC share of employment is lower than the value implied by the empirical transition matrix (7.2 percent), but note that the latter is high compared to that of respondents in the LFS who report being employed under a ZHC. The vacancy rate in the model is 2.8 percent, which fits well with the values typically observed in job vacancy survey data.

The other moments reported in Table 3 illustrate the sorting patterns predicted by the model, but which do not have a counterpart in our data. First, while ZHCs make up for a small share of employment (6.5 percent), they account for almost 20 percent of all vacant positions. This is due to higher worker turnover on these jobs, implying that they get re-advertised more frequently than R contracts. Notice that, in a random search environment, this implies that ZHCs exert a negative effect on R vacancies by making it more difficult for these vacancies to contact workers who prefer regular employment (congestion effect). Second, since most vacant positions are R contracts, and because of on-the-job search, there are fewer than 5 percent of employed R-best workers who hold a Z contract. Meanwhile, since R-best

Table 3: Description of baseline equilibrium

		Model	Data
$u/(u+e)$	Unemployment rate	9.2	10.1
e_Z/e	Employment share of Z jobs	6.5	7.2
$v/(v+e)$	Vacancy rate	2.8	–
v_Z/v	Share of Z vacancies	19.4	–
$e_{R\text{-best},Z}/e_{R\text{-best}}$	Share of employed R-best workers in Z jobs	4.8	–
$e_{R\text{-best},Z}/e_Z$	Share of filled Z jobs employing R-best workers	66.8	–

Notes: The table reports model-predicted moments for the baseline steady-state equilibrium, and moments computed from the LFS data based on the ergodic distribution of the transition matrix across U, Z, R (Panel (b) of Table 1). All table entries are expressed in percent.

workers make up for a very large share of the labor force, they account for two thirds of all ZHC jobs. Thus, R-best workers are key to sustain an equilibrium with ZHC jobs, as Z vacancies would not be viable without the presence of R-best workers accepting these jobs.

5.2 Flow profits and payoffs

At this point, with the limited information coming (mostly) from the LFS, we have pinned down the equilibrium allocation of the baseline economy. This means that, in terms of Figures 2 and 3, we have determined the slope of the lines separating the different regions which correspond to different rankings for workers and firms, respectively. The remaining task is essentially to choose values for firms' profit flows (the π_k^j's) and workers' flow payoffs of non-employment/employment (the ω_k^i's) consistent with the types of agents we observe in the data. We attempt to provide a microfounded rationale for the choices of these flow values, bearing in mind that there could be alternative rationales for them.

Firms' flow profits. To measure per-period profit flows, we entertain the idea that firms face shocks to their revenues which reflect fluctuations in demand and/or production shocks. Profits depend on actual hours worked, h, as well as on $\widetilde{h}$ which measures the number of working hours that would exactly meet the demand that a firm faces at a given point in time. Deviations between h and $\widetilde{h}$ are costly due to, e.g., reputation costs (if the firm is not able to produce enough to satisfy the demand of its consumers), marketing expenses (if it needs to expand marketing to sell extra units of output), etc. We capture the firm's instantaneous profit function by the simple formulation:[17]

$$\pi\left(h,\widetilde{h}\right) = (p-w)h - \frac{\phi}{2}\left(h - \widetilde{h}\right)^2. \tag{17}$$

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[17]The environment faced by firms can also be described in the following way. A firm receives orders $\widetilde{q}$ and produces output $q = ph$, where p is the hourly labor productivity. It incurs a convex reputation costs $\underline{d}\left(q - \widetilde{q}\right)^2$ when it produces less than its orders. If the firm wants to sell more than its orders, i.e. if $q > \widetilde{q}$, it needs to spend on marketing to sell the new orders, and the cost is given by $\overline{d}\left(q - \widetilde{q}\right)^2$. For simplicity, we assume that $\underline{d} = \overline{d} = d$. The instantaneous profit of firms is then $q - wh - d\left(q - \widetilde{q}\right)^2$, which boils down to $\pi(h,\widetilde{h})$ in Equation (17) after defining $\widetilde{h} = \widetilde{q}/p$ and $\phi = 2dp^2$.

$\widetilde{h}$ is stochastic and is drawn from a distribution H_j, where j denotes firm-specific types. In equilibrium, the properties of H_j – its mean μ_j and variance σ_j^2 – yield a ranking of flexible and regular contracts consistent with the taxonomy of firm types in (6).

Actual hours worked, h, depend on the type of employment contract operated by the firm. $h_Z = \widetilde{h}$ under a Z contract, so that this enables firms to avoid the quadratic losses in (17) by effectively making workers work irregular hours. Under a R contract, on the other hand, h is constant and set to an exogenous level of hours h_R. This feature of R contracts, together with the assumption of quadratic losses, implies that the firm's expected profits are negatively related to the standard deviation of $\widetilde{h}$. To put it all together in a single formula, we have: $\pi_k^j = \int \pi \left(h_k, \widetilde{h} \right) dH_j \left(\widetilde{h} \right)$.

Workers' flow payoffs.   To capture the flow payoffs from non-employment and employment, let us assume that workers derive utility from consumption and leisure. In non-employment, workers receive income b, while in employment they are paid at the national mandated minimum wage, denoted as w, and work for h hours. When earning labor income wh, workers may retain part of their non-employment income depending on the taper rate τ of welfare benefits. We further assume that earned labor income is the only source of income for workers and we rule out saving/borrowing, so that workers consume all their income within each period. As a result, consumption is:

$$c\left(h\right) = \max\left\{wh, b + \left(1-\tau\right)wh\right\}. \tag{18}$$

Hours worked generate a disutility measured by α_i, which is specific to each worker type i. Utility from consumption is derived according to a CRRA function. Putting it all together, the intra-period utility function, or flow payoff, is given by:

$$\omega^i\left(h_k\right) = \frac{c(h_k)^{1-\eta} - 1}{1 - \eta} - \alpha_i h_k, \tag{19}$$

for $k = Z, R$. Given that hours worked are constant in R employment, $\omega_R^i = \omega^i\left(h_R\right)$, while for Z employment we have $\omega_Z^i = \int \omega^i\left(\widetilde{h}\right) dH_Z\left(\widetilde{h}\right)$ where H_Z denotes the distribution of $\widetilde{h}$ for the type-j firm which posts Z jobs. Last, since $h = 0$ during non-employment, we can define ω_N^i using (18) and (19) above: $\omega_N^i = \frac{b^{1-\eta}-1}{1-\eta}$.

Parameters set externally.   We choose $\rho = 0.0015$ to yield an annual discount rate of 4 percent. The coefficient of relative risk aversion η is set to 2.0, which is a standard value in the literature. We set w to the 2017 U.K. national minimum wage for workers aged 25 and over, namely $w = £7.50$ per hour. The taper rate τ is 63 percent in line with U.K. policies in the sample period. We set $p = 8.25$, thus effectively assuming that the marginal productivity of workers in the low-pay segment is 10 percent higher than the statutory minimum wage, w. We assume that the $H_j\left(.\right)$'s are Beta distributions over the interval $[0, 50]$, where 50 refers to the upper bound on weekly hours worked. The reason we use Beta distributions is that they are flexible in terms of generating bell-shaped distributions, bi-modal distributions, etc. and the parametrization relies on the choice of the mean and variance only. We set $h_R = \mu_R$, which is a

normalization in the sense that the gap between h and $\tilde{h}$ in the profit function (17) is scaled by ϕ. From Table B2, we set $\mu_R = 28$, and choose $\mu_Z = 18$ to capture the 10-hour gap between the two contract types. Last, we set σ_R equal to 2. This is clearly an arbitrary choice but it does not affect the results to the extent that the key calibration issue is the relative dispersion of hours in Z employment, not the absolute dispersion of R hours worked. Given our choices of σ_R and functional form of $H_j(.)$'s, 90 percent of $\tilde{h}$ draws are within the interval [25, 31], centered on the mean value μ_R. The upper panel of Table 4 summarizes these parameter values.

Calibration of ϕ, κ, K, b, σ_Z. At this point, we are left with the task of choosing ϕ, κ, K, non-employment income b, and the standard deviations of Z hours, σ_Z. There is no direct empirical counterpart for ϕ, but note that any combination of values for ϕ and κ pins down the value of K through the free-entry condition (10). Thus we set up calibration targets for κ and K, and then search for the value of ϕ that allows the model to match these targets. For κ, we follow Elsby and Michaels [2013] and target an expected cost of vacancy posting (i.e. $\kappa\theta/\lambda(\theta)$) that amounts to 14 percent of average quarterly labor earnings. We find that $\kappa = £36.3$ (per week) achieves this objective.[18] As regards the startup costs of creating a business, K, estimates for the U.K. suggests that it averages around £22,500.[19] 95 percent of businesses in the U.K. have between 1 and 10 employees (House of Commons [2021]). Assuming that the average business has 5 workers, and since each firm in our model has only one job, this suggests a target for K at around £4,500. We come very close to this target by setting $\phi = 0.16$; we obtain $K = £4,376$. The calibrated ϕ implies that the cost of deviating from $\tilde{h}$ by 5 hours in a given week reduces firms' accounting profits $((p-w)h)$ by 10 percent. For non-employment income, we use a replacement ratio of 80 percent in line with 2018-2019 U.K. policies.[20] This yields $b = £148.8$ on a weekly basis.

The calibration of ϕ, κ, K, b is of course not independent of σ_Z as this will affect hours worked in Z employment and hence average earnings. We first set $\sigma_Z = 3$. Together with the gap in mean hours μ_R vs. μ_Z, this makes the coefficient of variation of $\tilde{h}$ twice higher in Z compared to R employment. When $\sigma_Z = 3$, we find that firms that post Z vacancies are Z-best firms. We further increase the value of σ_Z, and find that for $\sigma_Z = 6$ those firms become Z-only firms. In terms of Figure 3, raising the value of σ_Z pushes firms in the bottom right quadrant of the plot, as this lowers the profit flows π_R^j. We refer to the two calibrations $\sigma_Z = 3$ and $\sigma_Z = 6$ as respectively "low volatility" and "high volatility". We conduct an extensive robustness analysis of the sensitivity of our results to changes in the σ_k's and μ_k's in

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[18]Although Elsby and Michaels [2013]'s figure for vacancy posting cost is computed out of U.S. data, it seems well in line with numbers for the U.K. See, for example, https://theundercoverrecruiter.com/true-costs-hiring-uk/: they estimate that the advertising cost using social media and job sites is between £200 and £400 per new hire. Our calibration yields $\kappa\theta/\lambda(\theta)$ equal to £345.

[19]See https://www.capalona.co.uk/blog/how-much-does-it-cost-to-start-a-business/ and https://www.telegraph.co.uk/business/sme-home/start-up-costs/. We are not aware of any official statistics, but the evidence gathered from the Internet puts the value of K between £20,000 and £30,000.

[20]Non-employment income b includes the jobseeker's allowance, social assistance and housing benefits. To calculate the replacement ratio, we use the OECD's net rate calculator (https://stats.oecd.org/Index.aspx?DataSetCode=NRR), and set Family type to "Couple with 2 children - partner's earnings: Average Wage", Previous in-work earnings to "Minimum wage", and Unemployment duration to "6 months" (to match the average unemployment duration in our sample).

Table 4: Parameter values from the calibration's second step

ρ	Discount rate of 4 percent per annum		0.0015	
η	Relative risk aversion coefficient		2.0	
τ	Taper rate (U.K. policies)		0.63	
p	Productivity of hours worked		8.25	
w	Minimum hourly wage in £ (U.K. policies)		7.50	
μ_R	Mean of weekly hours for firm's offering R		28	
σ_R	St. dev. of weekly hours for firm's offering R		2	
μ_Z	Mean of weekly hours for firm's offering Z		18	
			Low	**High**
σ_Z	St. dev. of weekly hours for firm's offering R		3	6
ϕ	Marginal cost of deviating from targeted hours		0.16	0.16
κ	Flow cost of vacancy posting, in £ per week		36.3	36.3
K	Startup cost of new businesses, in £		4,376	4,376
b	Non-employment income, in £ per week		148.8	148.8

Notes: The table reports the parameter values from the second step of the calibration targeting data moments on job and worker turnover. The upper panel are parameter values based on external information. The bottom panel are parameter values that are calibrated jointly for a given choice of either lower or higher value of σ_Z. The model period is set to be two weeks.

the appendix and find that our conclusions are not sensitive to these choices. As the bottom panel of Table 4 shows, although σ_Z changes average earnings, the impact is so negligible that one cannot discern any change in the calibrated values of ϕ, κ, K, b.

6 Policy experiments

In this section, we use our structural framework to simulate the impact of a ban on ZHCs, which can be thought of as an extreme example of introducing tougher regulations on these contracts. We use the results as a basis for discussing policies related to the development, or containment, of flexible work arrangements.

6.1 Preliminaries

Some of the equilibrium effects of a ban on ZHCs depend chiefly on individuals' preferences vis-à-vis the work schedule of jobs. Given that the available data conveys only partial information about preferences, we conduct below some experiments under different assumptions regarding the disutility of work α_i that is specific to each type of workers. Figure 5 exposes the relationship between this key parameter, workers' rankings of employment options, and what their decisions would be following a ban on ZHCs. As can be seen, as the disutility of work increases, workers' preferences shift towards employment in Z jobs. Even though they prefer unemployment to R employment in an economy where the prospect of finding a ZHC exists, when these are banned, some Z-only workers would remain in the labor force. They do so if $\omega_R^i - \omega_N^i > 0$ (from Proposition 3) to take on R jobs (Part (iii) of Proposition 1). In terms of Figure 2, the bottom dashed line rotates all the way to horizontal as v_Z/v goes to zero, making Z-only

workers for whom $\omega_R^i - \omega_N^i > 0$ accept R jobs after the ban. On the other hand, Z-only workers whose disutility of work implies that $\omega_R^i - \omega_N^i < 0$ would drop from the labor force following a ban on ZHCs.
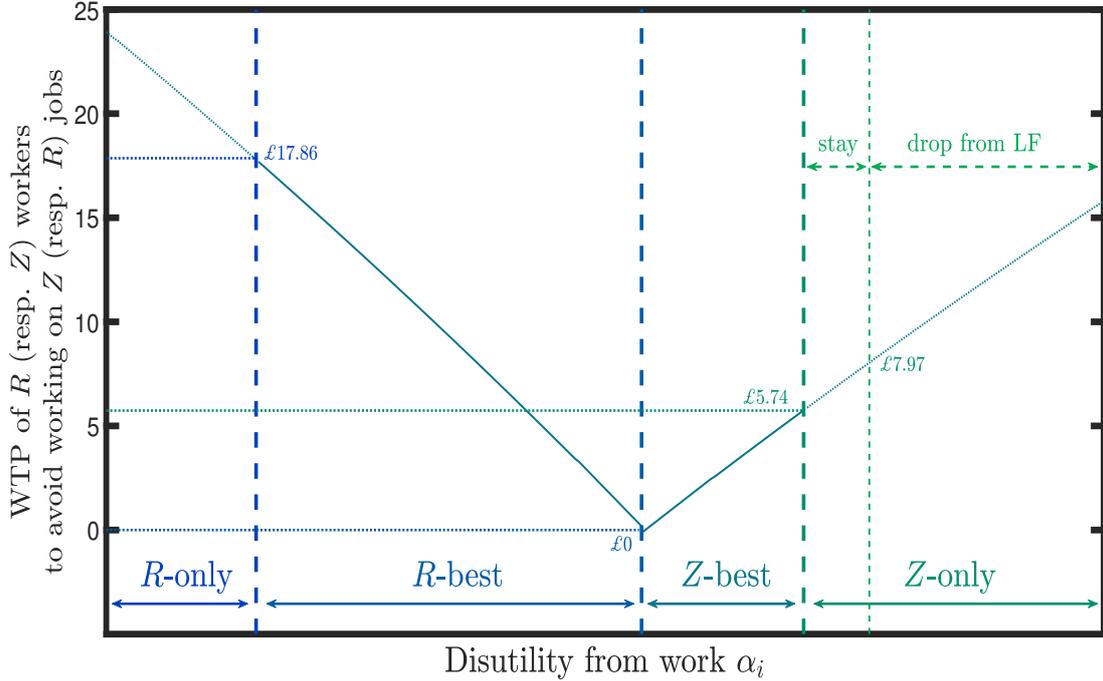

Figure 5: Disutility of work and workers' ranking of employment

Notes: The figure shows how workers' rankings of employment opportunities depend on the disutility of work, α_i, and translates this parameter into a willingness to pay (WTP) to avoid working on one's least preferred employment option. For Z-only workers, the figure indicates that there is a range of values of α_i such that these workers would stay and another range where they would drop from the labor force (LF).

In Figure 5, we translate the disutility of work α_i into agents' willingness to pay (WTP) to avoid working on their least preferred *employment* option (Z jobs for R workers, and vice versa). R-best workers would typically have a WTP between £0 (when they become indifferent to Z jobs) and £17.4. In the sequel, we select α_i in between these two values to evaluate welfare effects for these workers. Z-only workers' WTP is at least as high as £5.7. As can be seen, there is a threshold value α_i translating into a WTP of £7.9. Below this α_i, Z-workers remain in the labor force (LF) after a ban on ZHCs; when α_i yields a WTP higher than £7.9, the workers become inactive after the ban. We select α_i's symmetrically around this threshold – the implied WTPs are £6.8 and £9.1 – to illustrate these two scenarios. We call them the "low disutility" and "high disutility" economies, respectively.

6.2 Equilibrium allocations

Results of a ban on ZHCs are reported in Table 5. Consider the top left corner of the table referring to the "low volatility" calibration, namely $\sigma_Z = 3$, such that Z employers are Z-best firms, and Z-only workers have "low disutility" of work α_i. In this case, the ban on ZHCs leads to an increase of the unemployment rate by 2 percentage points (p.p.). The main driver is that Z-best firms can no longer substitute ZHCs for regular contracts, and therefore face

lower expected profits. In equilibrium, the expected benefit of creating a job is lower and there are fewer entries of firms, making the unemployment rate increase and the number of vacancies decrease. Z-only workers remain in the labor force after the ban, so that the change in employment is the mirror image of that in unemployment. Finally, GDP drops, but only marginally, because average hours worked increase and this offsets the smaller headcount in jobs. These extensive vs. intensive adjustments in total hours worked can lead to ambiguously signed effects on GDP, as shown in robustness exercices in Online Appendix D.

Next, in the top right corner of Table 5, we examine the implications of a "high volatility" economy, where Z employers are Z-only firms, and Z-only workers are of the "high disutility". This entails a larger role for the job creation effect of a ZHC ban. It causes the unemployment rate to increase by 2.7 p.p., employment to decrease by the same figure (since labor force participation stays put), and GDP to decrease by 0.14%. The reason the change in GDP is not larger is that although employment drops, fewer resources are consumed in the costly process of creating firms and posting vacancies.

The lower panel of Table 5 considers effects of the reform in the economy with "high disutility", where Z-only workers drop from the labor force after the ban on ZHCs. As can be seen in the bottom left corner corresponding with "low volatility" Z firms, while unemployment increases by 2 p.p., employment decreases by 4.8 p.p. The difference is accounted for by the labor force participation effect of ZHC. The magnitude of the effect seems plausible. At the same time, since employment is lower, fewer job matches are destroyed and then re-advertised, making the vacancy rate decrease more than in the "low disutility" economy, and fewer resources are consumed in the firm's entry process. Still, the combined effect is that GDP decreases much more than in the "low disutility" economy: it drops by 2.9 percent, due to lower labor force participation and hence lower employment rate.

The corresponding outcomes for the economy with "high volatility" Z firms and "high disutility" Z workers are reported in the bottom right corner. The employment drop is larger, at 5.4 p.p., reflecting the increase of the unemployment rate by 2.7 p.p. and decrease of the labor force participation rate by 3.1 p.p. Similar effects as before imply that the change in GDP is amplified compared with the "low disutility" economy: it drops by 2.3 percent. As for all the simulations in this section, this should be interpreted as a change in sectoral GDP since our analysis is confined to the low-pay segment of the labor market.

Accession to regular employment. It is interesting to note that, in equilibria without ZHCs shown above, even though the unemployment rate is higher, regular employment R is also higher. We use Proposition 4a to analyze the sources of this difference and gain a deeper understanding of the mechanisms affecting labor reallocation following a ban on ZHCs.

In Proposition 4a, the first component affecting regular employment is job creation: there are fewer firms entering the market after a ban on ZHCs, which reduces labor market tightness and hence regular employment through a lower aggregate job-finding rate. We find that $\lambda(\widetilde{\theta})/\lambda(\theta)$ is 71 percent, meaning that lower job creation would reduce regular employment by almost 30 percent *ceteris paribus*. The second component measures the effects of competition between different type of vacancies. According to our model, the reduction in vacancy competition in

Table 5: Equilibrium effects of a ban on Z contracts

Firms →	Low volatility		High volatility	
Workers				
↓	$e:$	-2.03 p.p.	$e:$	-2.65 p.p.
	$\frac{u}{u+e}:$	$+2.03$ p.p.	$\frac{u}{u+e}:$	$+2.65$ p.p.
Low disutility	$\frac{v}{v+e}:$	-0.92 p.p.	$\frac{v}{v+e}:$	-0.97 p.p.
	GDP :	-0.02%	GDP :	-0.14%
	$e:$	-4.78 p.p.	$e:$	-5.39 p.p.
	$\frac{u}{u+e}:$	$+2.03$ p.p.	$\frac{u}{u+e}:$	$+2.65$ p.p.
High disutility	$\frac{v}{v+e}:$	-0.97 p.p.	$\frac{v}{v+e}:$	-1.03 p.p.
	GDP :	-2.87%	GDP :	-3.24%

Notes: The table reports the equilibrium allocation effects of a ban on ZHCs. e: employment; u: unemployment; v: vacancies; GDP: gross domestic product. "Low volatility" (resp. "High volatility") refers to the calibration where the volatility of hours in Z firms is $\sigma_Z = 3$ (resp. $\sigma_Z = 6$), implying that these are Z-best (resp. Z-only) firms. "Low disutility" (resp. "High disutility") refers to the calibration where the disutility of labor α_i of Z-only workers is such that these workers prefer to stay (resp. drop from the labor force) once Z employment is banned.

isolation from the other effects would increase regular employment by 24 percent. Lastly, there is an increase in search efficiency units for R employment following a ban on ZHCs. This effect contributes an increase in R employment by 15 percent *ceteris paribus*.

A related question is: What is the overall impact of the ban on time spent *out* of regular employment? We find that for R-best workers, this duration decreases by 7 weeks when ZHCs are banned, even though the unemployment rate increases. That is, lacking a stepping stone towards regular employment, workers would face higher unemployment, but at the same time face more stable labor market trajectories on average.[21] The implications for welfare are not obvious, however. In the equilibrium with ZHCs, when R-best workers are not in regular employment, they spent only a fraction of that time in unemployment and spend the remainder in Z employment, which makes them better off compared to being unemployed. This difference must be factored in to compute the overall welfare effect. This is the issue we turn to in the next section.

6.3 Welfare effects

Table 6 provides an assessment of the welfare consequences of a ban on ZHCs.[22] It shows that a ban on ZHCs has a wholly negative impact in general equilibrium (denoted as "GE"), making R-best workers suffer a welfare loss that amounts to between -0.5 and -0.6 percent of

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[21]As pointed out in a recent paper by Güell et al. [2021], going beyond the measurement of unemployment duration is important to get a full picture of workers' trajectories in markets with atypical employment.

[22]Table 6 studies the effects in the context of the "low disutility" calibration for Z-only workers, when they remain in the labor force after the ban. The "high disutility" calibration is less interesting because it implies that the GE and PE effects are the same for Z-only workers, given that they do leave the labor market after the ban on ZHC. Quantitatively, we find that the welfare effects for the "high disutility" Z-only workers are in the same ballpark as those reported in Table 6: -2.10 percent in the "low volatility" and -1.77 percent in the "high volatility" economy.

foregone consumption, and with corresponding figures between -1.7 and -2.0 percent for Z-only workers. These are sizable welfare losses, essentially driven by the fact that workers in this segment of the labor market would face a longer expected duration of unemployment following a ban on ZHCs. For Z-only workers, the losses are larger because they also switch to a type of employment (R) which they would not even accept in the baseline equilibrium.

Table 6: Welfare effects of a ban on Z contracts

Firms →	Low volatility		High volatility	
R-best workers	GE:	-0.50%	GE:	-0.62%
	PE:	$+0.15\%$	PE:	$+0.18\%$
Z-only workers	GE:	-1.98%	GE:	-1.65%
	PE:	-3.28%	PE:	-2.94%

Notes: The table reports the welfare effects of a ban on ZHCs. GE: general equilibrium, which is the full effect of the reform; PE: partial equilibrium, which is the effect of keeping the equilibrium allocation unchanged and substitute R (resp. N) with Z employment for R-best workers (resp. Z-only workers). "Low volatility" (resp. "High volatility") refers to the calibration where the volatility of hours in Z firms is $\sigma_Z = 3$ (resp. $\sigma_Z = 6$), implying that these are Z-best (resp. Z-only) firms. All effects are computed in the context of the "Low disutility" economy, where the disutility of labor α_i of Z-only workers is such that these workers remain in the labor force once Z employment is banned. Welfare effects are in percent of consumption equivalent variation units.

In order to understand better the welfare consequences of a ban on ZHCs, we conduct a partial equilibrium experiment to isolate the role of substitution effects. We ask how R-best workers would be impacted if their ZHCs were replaced by regular contracts, holding constant the decisions of all other agents and distribution of the economy. The results of this experiment are denoted as "PE" in Table 6. R-best workers' welfare increases by about 0.2 percent in consumption equivalent variations. In equilibrium, however, the substitution effect is counteracted mostly by the job creation effect, so that (at least) in steady-state comparisons workers suffer from a ban on ZHCs.

Likewise, we can ask how Z-only workers fare if we simply replace their ZHC jobs in the baseline economy with their second most-preferred option at the time, that is to say non-employment. We find that their welfare would drop by between 3.0 and 3.3 percent, which is larger than the full general equilibrium effect. In other words, the partial equilibrium experiment overstates their losses by failing to take account of the fact that Z-only workers would prefer employment in a R job to inactivity following a ban on ZHCs.

6.4 Policy discussion

Turning back to the ongoing media and political debate about the pros and cons of ZHC, we summarize here how our approach sheds light on the arguments often put forward and propose some avenues for policy on flexible work. The debate focuses on the loss of workers' welfare due to income uncertainty and tends to assume two extreme opposite views, namely, that either (i) all those workers in Z jobs would be rehired in R jobs in the event of a ban, or (ii) all Z

jobs would be destroyed. In terms of our model taxonomy, the first assumption means that all workers in Z jobs are either R-best or "low disutility" Z-only individuals and that all firms posting Z vacancies are Z-best firms, i.e. facing "low volatility" demand conditions. The second assumption means that all firms posting Z jobs are of the Z-only type, i.e. facing "high volatility" with workers implicitly being all of the R-best type. Neither view of the debate considers general equilibrium effects or heterogeneity in workers' disutility of work.

Our counterfactual simulations show that these assumptions yield, respectively, an upper bound of the welfare gain for R-best workers in partial equilibrium and an upper bound of the welfare loss for Z-only workers. Proponents of a ban on ZHCs ignore the general equilibrium effects, whereby job creation is rendered less attractive for firms, and yield welfare losses in all cases (see Table 6).

In light of these findings, we see four policy recommendations:

(P1) ZHCs could be restricted to job matches where workers opt for Z when offered a choice of contract;

(P2) Access to ZHCs could be prioritized for workers employed in small firms rather than in large firms.

In terms of our calibration, recommendation **(P1)** would mean that the 66% of workers in Z jobs who are R-best types (see Table 3) would be in a position to choose R employment conditions provided that their firm was a "low volatility" type, or Z-best firm. Meanwhile, recommendation **(P2)** would aim to narrow down the use of Z jobs to "high volatility" firms, where the volatility of demand is proxied by an index of small size, on the grounds that small firms cannot "average away" demand shocks over a large number of jobs. Admittedly this is an imperfect proxy.

As estimated for the U.S. by Frazier [2018], there is a substantial willingness to pay for flexible working schedules in some segments of the (potential) labor force. There are also productive opportunities in sectors facing highly volatile demand which may not be viable without the ability to adjust working hours at no cost. Identifying such segments of economic activity where the existence of casual contracts conditions the viability of firms' entry and participation of workers requires the availability of richer data on firms' profitability and workers' time use and preferences. This is key to fostering gains from trade in some segments of the labor force where flexibility plays a big role both for firms and workers, without compromising the welfare of workers with low bargaining power.

Another direction for policy change would be to regulate the type of flexibility attached to ZHCs. In the above, we have assumed that the firm chooses the workload in terms of hours and the worker chooses the timing of work, and found important differences when we made different assumptions along this dimension of the model. In practice, anecdotal evidence suggests that any combination of choices of hours and timing made by the worker or the firm exist in the U.K.'s ZHCs. This brings us to the following recommendations **(P3)** and **(P4)**:

(P3) Recognize that the sharing of hours flexibility between workers and firms is often part of the incompleteness of employment contracts;

(P4) Take steps to regulate the sharing of hours flexibility between workers and firms.

Given the costs incurred on both sides when the match is producing more/fewer hours than desired, a promising avenue for welfare improvements resides in formalizing the type of flexibility offered by jobs: timing versus hours worked, offer versus obligation, firm versus worker taking the lead. Our findings point to the need to clarify the type of flexibility and rights attached to each employment relationship.

Besides, an assumption in our model is that the fixed costs of employment for the firms and worker's rights are equal across the two contract types. This is an approximation, since there are some relevant differences in workers' rights as detailed in Online Appendix B. In the media debate, ZHCs are often amalgamated with self-employment, as in the current news items on Uber drivers.[23] Despite being beyond the scope of this paper to draw general conclusions about the optimal regulation of the various forms of employment gathered under the label of 'gig economy', our analysis however sheds light on some relevant issues in this respect.

7   Conclusions

In summary, our paper presents a theoretical framework which rationalizes the coexistence of two types of working arrangements, labeled regular and flexible, on the labor market. We allow both firms and workers to have heterogeneous valuations of these working arrangements. Within a general equilibrium with random search and matching, we characterize the allocation of different worker types to different job types as well as the turnover rates between different labor market states. The counterfactual policy of interest is a potential ban of flexible work arrangements towards a market with homogeneous regular jobs.

Calibrating our model to the U.K. economy with zero-hours contracts, we find mixed welfare effects of a potential ban of ZHCs, bringing nuance to the stark media and political arguments prevailing in the current debate. Whilst partial equilibrium effects on workers who prefer regular contracts are positive, general equilibrium effects, which take into account the impact of the ban on the incentives to create jobs and on search frictions, are negative for all workers. Individuals with a large disutility of working time are particularly affected.

Our policy discussion highlights scope for welfare gains in targeting the use of such contracts onto segments on the market most likely to gain from them. At the same time, more formalization of the type of flexibility offered in these non-standard jobs and of the agent (firm or worker) with the decision power on timing and quantity of work is a likely avenue for welfare improvement in this "new" market.

Finally, it is worth noting that, with the data at hand, we are only able to quantify the impact of a ZHC ban on the population currently participating in the labor force. A relevant research question, particularly in the context of "missing labor force" in the U.K.,[24] is whether an alternative type of flexible work arrangement may be able to induce the participation of these missing workers.

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[23]See this February 2021 editorial in The Guardian.
[24]See this December 2022 editorial in The Guardian.

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Online Appendix

A   Model appendix

A.1   Proof of Proposition 1

We first prove Parts (ii) and (iii) of Proposition 1. To demonstrate Part (ii), we show that $W_R^i - W_Z^i$ and $\omega_R^i - \omega_Z^i$ are of the same sign. We rewrite workers' asset values of employment as:

$$W_R^i = \omega_R^i + \frac{1}{1+\rho}\left[W_R^i + \delta\left(N^i - W_R^i\right) + (1-\delta)\,x\lambda\left(\theta\right)\frac{v_Z}{v}\max\left\{W_Z^i - W_R^i, 0\right\}\right],$$

$$W_Z^i = \omega_Z^i + \frac{1}{1+\rho}\left[W_Z^i + \delta\left(N^i - W_Z^i\right) + (1-\delta)\,x\lambda\left(\theta\right)\frac{v_R}{v}\max\left\{W_R^i - W_Z^i, 0\right\}\right].$$

The difference between these equations gives

$$W_R^i - W_Z^i = \omega_R^i - \omega_Z^i + \frac{1}{1+\rho}\Big[(1-\delta)\left(W_R^i - W_Z^i\right)$$

$$+ (1-\delta)\,x\lambda\left(\theta\right)\left(\frac{v_Z}{v}\left(W_Z^i - W_R^i\right)\mathbb{1}_{\left\{W_Z^i > W_R^i\right\}} - \frac{v_R}{v}\left(W_R^i - W_Z^i\right)\mathbb{1}_{\left\{W_R^i > W_Z^i\right\}}\right)\Big]$$

$$= \omega_R^i - \omega_Z^i + \frac{1-\delta}{1+\rho}\left(W_R^i - W_Z^i\right)\left(1 - x\lambda\left(\theta\right)\mathbb{v}_i^W\right),$$

where

$$\mathbb{v}_i^W \equiv \frac{v_Z}{v}\mathbb{1}_{\left\{W_Z^i > W_R^i\right\}} + \frac{v_R}{v}\mathbb{1}_{\left\{W_R^i > W_Z^i\right\}} \tag{A.1}$$

measures the probability that an employed worker of type i quits upon meeting a randomly chosen vacancy. As a result, we have

$$W_R^i - W_Z^i = \xi_i\left(\omega_R^i - \omega_Z^i\right), \quad \xi_i \equiv \left[1 - \frac{1-\delta}{1+\rho}\left(1 - x\lambda\left(\theta\right)\mathbb{v}_i^W\right)\right]^{-1} \quad (> 0). \tag{A.2}$$

Since $\omega_R^i - \omega_Z^i = \left(\omega_R^i - \omega_N^i\right) - \left(\omega_Z^i - \omega_N^i\right)$, in the region above (resp. below) the 45° line in the $\left(\omega_Z^i - \omega_N^i, \omega_R^i - \omega_N^i\right)$-plane, workers' most preferred employment status is R (resp. Z).

To prove Part (iii) of Proposition 1, we first search for the locus of indifference $W_R^i = N^i$ in the region where $W_Z^i > W_R^i$. Rewrite the value of non-employment as

$$N^i = \omega_N^i + \frac{1}{1+\rho}\left[N^i + \lambda\left(\theta\right)\left(\frac{v_R}{v}\max\left\{W_R^i - N^i, 0\right\} + \frac{v_Z}{v}\max\left\{W_Z^i - N^i, 0\right\}\right)\right].$$

Consider the difference $W_R^i - N^i$. This can be written as

$$W_R^i - N^i = \omega_R^i - \omega_N^i + \frac{1}{1+\rho}\Big[(1-\delta)\left(W_R^i - N^i\right)$$

$$+ (1-\delta)\,x\lambda\left(\theta\right)\left(\frac{v_Z}{v}\left(W_Z^i - W_R^i\right)\mathbb{1}_{\left\{W_Z^i > W_R^i\right\}}\right)$$

$$- \lambda\left(\theta\right)\left(\frac{v_R}{v}\left(W_R^i - N^i\right)\mathbb{1}_{\left\{W_R^i > N^i\right\}} + \frac{v_Z}{v}\left(W_Z^i - W_R^i + W_R^i - N^i\right)\mathbb{1}_{\left\{W_Z^i > N^i\right\}}\right)\Big].$$

Re-arranging terms, we have

$$\left(W_R^i - N^i\right)\left[1 - \frac{1}{1+\rho}\left(1 - \delta - \lambda\left(\theta\right)\left(\frac{v_R}{v}\mathbb{1}_{\{W_R^i > N^i\}} + \frac{v_Z}{v}\mathbb{1}_{\{W_Z^i > N^i\}}\right)\right)\right]$$
$$= \omega_R^i - \omega_N^i + \frac{\lambda\left(\theta\right)}{1+\rho}\frac{v_Z}{v}\left[(1-\delta)\,x\mathbb{1}_{\{W_Z^i > W_R^i\}} - \mathbb{1}_{\{W_Z^i > N^i\}}\right]\left(W_Z^i - W_R^i\right). \quad \text{(A.3)}$$

If $W_Z^i > W_R^i$, then $\mathbb{1}_{\{W_Z^i > W_R^i\}} = 1$. As we search for the locus of indifference $W_R^i = N^i$, we also have $\mathbb{1}_{\{W_Z^i > N^i\}} = 1$, and the above equation yields

$$0 = \omega_R^i - \omega_N^i + \frac{\lambda\left(\theta\right)}{1+\rho}\frac{v_Z}{v}\left[(1-\delta)\,x - 1\right]\left(W_Z^i - W_R^i\right).$$

Given that $W_R^i - W_Z^i = \xi_i\left(\omega_R^i - \omega_Z^i\right)$, we have

$$0 = \omega_R^i - \omega_N^i + \frac{\lambda\left(\theta\right)}{1+\rho}\frac{v_Z}{v}\left[(1-\delta)\,x - 1\right]\xi_i\left(\omega_Z^i - \omega_N^i + \omega_N^i - \omega_R^i\right),$$

or, equivalently

$$\omega_R^i - \omega_N^i = \frac{\left[1 - (1-\delta)\,x\right]\frac{\lambda(\theta)}{1+\rho}\frac{v_Z}{v}\xi_i}{1 + \left[1 - (1-\delta)\,x\right]\frac{\lambda(\theta)}{1+\rho}\frac{v_Z}{v}\xi_i}\left(\omega_Z^i - \omega_N^i\right). \quad \text{(A.4)}$$

Since $0 < \left[1 - (1-\delta)\,x\right]\frac{\lambda(\theta)}{1+\rho}\frac{v_Z}{v}\xi_i < 1$, Equation (A.4) establishes that the locus of indifference $W_R^i = N^i$ is a straight line that lies below the 45° line in the $(\omega_Z^i - \omega_N^i, \omega_R^i - \omega_N^i)$-plane.

We can proceed in the same way to characterize the locus of indifference $W_Z^i = N^i$ in the region where $W_R^i > W_Z^i$. The difference $W_Z^i - N^i$ gives

$$\left(W_Z^i - N^i\right)\left[1 - \frac{1}{1+\rho}\left(1 - \delta - \lambda\left(\theta\right)\left(\frac{v_R}{v}\mathbb{1}_{\{W_R^i > N^i\}} + \frac{v_Z}{v}\mathbb{1}_{\{W_Z^i > N^i\}}\right)\right)\right]$$
$$= \omega_Z^i - \omega_N^i + \frac{\lambda\left(\theta\right)}{1+\rho}\frac{v_R}{v}\left[(1-\delta)\,x\mathbb{1}_{\{W_R^i > W_Z^i\}} - \mathbb{1}_{\{W_R^i > N^i\}}\right]\left(W_R^i - W_Z^i\right). \quad \text{(A.5)}$$

Using $W_R^i > W_Z^i$ and $W_Z^i = N^i$ in this equation, we get

$$0 = \omega_Z^i - \omega_N^i + \frac{\lambda\left(\theta\right)}{1+\rho}\frac{v_R}{v}\left[(1-\delta)\,x - 1\right]\left(W_R^i - W_Z^i\right).$$

Rearranging terms, we obtain

$$\omega_R^i - \omega_N^i = \frac{1 + \left[1 - (1-\delta)\,x\right]\frac{\lambda(\theta)}{1+\rho}\frac{v_R}{v}\xi_i}{\left[1 - (1-\delta)\,x\right]\frac{\lambda(\theta)}{1+\rho}\frac{v_R}{v}\xi_i}\left(\omega_Z^i - \omega_N^i\right) \quad \text{(A.6)}$$

to define the locus $W_Z^i = N^i$ when $W_R^i > W_Z^i$. This is a straight line that lies above the 45° line in the $(\omega_Z^i - \omega_N^i, \omega_R^i - \omega_N^i)$-plane.

To characterize the regions above and below, say, the locus $W_R^i = N^i$ in the region where $W_Z^i > W_R^i$, observe that at a given $\omega_Z^i - \omega_N^i$ any point North of the locus $W_R^i = N^i$ yields a higher $\omega_R^i - \omega_N^i$, which pushes up the difference $W_R^i - N^i$. Hence the region above the line $W_R^i = N^i$ in the region where $W_Z^i > W_R^i$ is such that: $W_Z^i > W_R^i > N^i$. We can proceed in a similar way to characterize the other regions of Figure 2.

To complete the proof of Part (iii) of the Proposition, let us rewrite the slope of the locus

$W_R^i = N^i$ (Equation (A.4)). In the region where $W_Z^i > W_R^i$, we have

$$\mathbb{v}_i^W = \frac{v_Z}{v} \Rightarrow \xi_i = \left[1 - \frac{1-\delta}{1+\rho} \left(1 - x\lambda\left(\theta\right) \frac{v_Z}{v} \right) \right]^{-1}$$

$$\Rightarrow \frac{\lambda\left(\theta\right)}{1+\rho} \frac{v_Z}{v} \xi_i = \left[\left(\rho+\delta\right) \left(\lambda\left(\theta\right) \frac{v_Z}{v} \right)^{-1} + \left(1-\delta\right) x \right]^{-1}$$

Substituting into (A.4), we find that the locus $W_R^i = N^i$ is defined by

$$\omega_R^i - \omega_N^i = \frac{1 - \left(1-\delta\right) x}{1 + \left(\rho+\delta\right) \left(\lambda\left(\theta\right) \frac{v_Z}{v} \right)^{-1}} \left(\omega_Z^i - \omega_N^i \right). \tag{A.7}$$

Thus, the slope decreases with x and it increases with $\lambda\left(\theta\right)$ and $\frac{v_Z}{v}$. Likewise, we can rewrite the slope of locus $W_Z^i = N^i$ in the region where $W_R^i > W_Z^i$ (Equation (A.6)). By using $\mathbb{v}_i^W = \frac{v_R}{v}$ to compute ξ_i and substitute $\frac{\lambda(\theta)}{1+\rho} \frac{v_R}{v} \xi_i$, we obtain

$$\omega_R^i - \omega_N^i = \frac{1 + \left(\rho+\delta\right) \left(\lambda\left(\theta\right) \frac{v_R}{v} \right)^{-1}}{1 - \left(1-\delta\right) x} \left(\omega_Z^i - \omega_N^i \right). \tag{A.8}$$

This shows that the locus moves anticlockwise (the slope increases) with x, and rotates towards the 45° line when $\lambda\left(\theta\right)$ or $\frac{v_R}{v}$ increase.

Part (i) of Proposition 1 follows from Equations (A.3) and (A.5) that have been derived above. Note in these two equations that

$$1 - \frac{1}{1+\rho} \left(1 - \delta - \lambda\left(\theta\right) \left(\frac{v_R}{v} \mathbb{1}_{\{W_R^i > N^i\}} + \frac{v_Z}{v} \mathbb{1}_{\{W_Z^i > N^i\}} \right) \right) > 0,$$

which implies that $W_R^i - N^i$ (resp. $W_Z^i - N^i$) has same sign as the right-hand side of (A.3) (resp. (A.5)). There are two relevant cases to analyze in Equation (A.3). If $W_Z^i > W_R^i$ and $W_Z^i > N^i$, then the right-hand side is $\omega_R^i - \omega_N^i + \frac{\lambda(\theta)}{1+\rho} \frac{v_Z}{v} \left[\left(1-\delta\right) x - 1 \right] \left(W_Z^i - W_R^i \right)$. Since $\frac{\lambda(\theta)}{1+\rho} \frac{v_Z}{v} \left[\left(1-\delta\right) x - 1 \right] \left(W_Z^i - W_R^i \right) < 0$, then $\omega_R^i - \omega_N^i < 0 \Rightarrow W_R^i - N^i < 0$. If $W_R^i > W_Z^i$ and $W_Z^i > N^i$, then we have $W_R^i > N^i$. However, using $W_R^i > W_Z^i$ and $W_R^i > N^i$ in Equation (A.5) yields $\omega_Z^i - \omega_N^i + \frac{\lambda(\theta)}{1+\rho} \frac{v_R}{v} \left[\left(1-\delta\right) x - 1 \right] \left(W_R^i - W_Z^i \right)$ on the right-hand side, which in turn implies that $W_Z^i - N^i < 0$ if $\omega_Z^i - \omega_N^i < 0$ – a contradiction of $W_Z^i > N^i$. In sum, if $\omega_R^i - \omega_N^i < 0$ and $\omega_Z^i - \omega_N^i < 0$, then $W_R^i - N^i < 0$. We can use (A.5) and proceed in the same way to show that $W_Z^i - N^i < 0$. This demonstrates that if $\omega_R^i > \omega_N^i$ or $\omega_Z^i > \omega_N^i$, then $\max\{W_R^i, W_Z^i\} > N^i$.

A.2   Proof of Proposition 2

The first step is to obtain a closed-form expression for V_k^j. We rewrite the asset values of advertising a vacant position as $k = R, Z$ in the following way:

$$V_k^j = -\kappa + \frac{1}{1+\rho} \left[V_k^j + \frac{\lambda\left(\theta\right)}{\theta} \sum_i \ell_{i,k} \left(J_{i,k}^j - V_k^j \right) \right], \tag{A.9}$$

where

$$\ell_{i,k} \equiv \frac{u_i \mathbb{1}_{\{W_k^i > N^i\}} + \sum_{j'} x_{i,k(j')} e_{i,j'} \mathbb{1}_{\{W_k^i > W_{k(j')}^i\}}}{u + \sum_{i'} x_{i'} e_{i'}}$$

39

denotes the share of type-i workers in the pool of job seekers who are willing to accept offer k. We can also simplify the asset values of employing a type-i worker by using $\mathbb{v}_i^W$ (see Equation (A.1) in Appendix A.1):

$$
\begin{aligned}
J_{i,k}^j &= \pi_k^j + \frac{1-\delta}{1+\rho}\left[J_k^j + x\lambda\left(\theta\right)\mathbb{v}_i^W\left(V_{i,k}^j - J_k^j\right)\right] \\
&= \pi_k^j + \frac{1-\delta}{1+\rho}\left[J_k^j + \left(1 - x\lambda\left(\theta\right)\mathbb{v}_i^W\right)\left(J_{i,k}^j - V_k^j\right)\right] \tag{A.10}
\end{aligned}
$$

(note that $x_{i,k}\mathbb{v}_i^W = x\mathbb{v}_i^W$). By combining (A.9) and (A.10), we obtain

$$
\begin{aligned}
J_{i,k}^j - V_k^j = \pi_k^j + \kappa + \frac{1-\delta}{1+\rho}&\left(1 - x\lambda\left(\theta\right)\mathbb{v}_i^W\right)\left(J_{i,k}^j - V_k^j\right) \\
&+ \frac{1-\delta}{1+\rho}V_k^j - \frac{1}{1+\rho}\left[V_k^j + \frac{\lambda\left(\theta\right)}{\theta}\sum_{i'}\ell_{i',k}\left(J_{i',k}^j - V_k^j\right)\right]
\end{aligned}
$$

$$
\Leftrightarrow \left(J_{i,k}^j - V_k^j\right)\left[1 - \frac{1-\delta}{1+\rho}\left(1 - x\lambda\left(\theta\right)\mathbb{v}_i^W\right)\right] = \pi_k^j + \kappa - \frac{\delta}{1+\rho}V_k^j - \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\sum_{i'}\ell_{i',k}\left(J_{i',k}^j - V_k^j\right)
$$

$$
\Leftrightarrow J_{i,k}^j - V_k^j = \xi_i \times \Gamma_k, \tag{A.11}
$$

where ξ_i has been defined previously in Equation (A.2), and

$$
\Gamma_k \equiv \pi_k^j + \kappa - \frac{\delta}{1+\rho}V_k^j - \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\sum_{i'}\ell_{i',k}\left(J_{i',k}^j - V_k^j\right). \tag{A.12}
$$

One key observation about Γ_k is that it is independent of i. Thus, we can use Equation (A.11) to solve for Γ_k. Since (A.11) gives us that

$$
\sum_i \ell_{i,k}\left(J_{i,k}^j - V_k^j\right) = \Gamma_k \times \sum_i \ell_{i,k}\xi_i, \tag{A.13}
$$

we substitute into Equation (A.12) to obtain

$$
\Gamma_k = \left(\pi_k^j + \kappa - \frac{\delta}{1+\rho}V_k^j\right) \times \left(1 + \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\sum_i \ell_{i,k}\xi_i\right)^{-1}.
$$

Plugging (A.13) into (A.9), we have

$$
V_k^j = -\kappa + \frac{1}{1+\rho}\left[V_k^j + \frac{\lambda\left(\theta\right)}{\theta}\Gamma_k\sum_i \ell_{i,k}\xi_i\right],
$$

and thus

$$
\begin{aligned}
V_k^j&\left[1 - \frac{1}{1+\rho} + \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\frac{\delta}{1+\rho}\left(1 + \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\sum_i \ell_{i,k}\xi_i\right)^{-1}\sum_i \ell_{i,k}\xi_i\right] \\
&= -\kappa + \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\left(\pi_k^j + \kappa\right)\left(1 + \frac{1}{1+\rho}\frac{\lambda\left(\theta\right)}{\theta}\sum_i \ell_{i,k}\xi_i\right)^{-1}\sum_i \ell_{i,k}\xi_i. \tag{A.14}
\end{aligned}
$$

To ease notation, let us define

$$\tilde{\Gamma}_k \equiv \frac{1}{1+\rho}\frac{\lambda(\theta)}{\theta}\left(1+\frac{1}{1+\rho}\frac{\lambda(\theta)}{\theta}\sum_i \ell_{i,k}\xi_i\right)^{-1}\sum_i \ell_{i,k}\xi_i. \tag{A.15}$$

Plugging (A.15) into (A.14), we have a closed-form solution for V_k^j:

$$V_k^j = \frac{1+\rho}{\rho+\delta\tilde{\Gamma}_k}\left[-\kappa + \left(\pi_k^j+\kappa\right)\tilde{\Gamma}_k\right]. \tag{A.16}$$

Equation (A.16) allows us to establish comparative static results to characterize different regions in the $(\pi_Z^j - (-\kappa), \pi_R^j - (-\kappa))$-plane; see Figure 3. To define the locus of indifference between not posting vs. posting an offer $k = R, Z$, we have: $V_k^j = 0 \Leftrightarrow \pi_k^j + \kappa = \frac{\kappa}{\tilde{\Gamma}_k}$. Thus, in the $(\pi_Z^j - (-\kappa), \pi_R^j - (-\kappa))$-plane, the region to the right of the vertical line $\pi_Z^j + \kappa = \frac{\kappa}{\tilde{\Gamma}_Z}$ corresponds to posting a Z offer, while the region above the horizontal line $\pi_R^j + \kappa = \frac{\kappa}{\tilde{\Gamma}_R}$ corresponds to posting a R offer. The quadrant to the left and below these regions is such that a firm would prefer not post any offer. We also have the locus of indifference between posting a Z vs. R offer, given by

$$V_R^j = V_Z^j \Leftrightarrow \pi_R^j + \kappa = \left(\pi_Z^j + \kappa\right)\frac{\left(\rho+\delta\tilde{\Gamma}_R\right)\tilde{\Gamma}_Z}{\left(\rho+\delta\tilde{\Gamma}_Z\right)\tilde{\Gamma}_R} - \kappa\delta\frac{\tilde{\Gamma}_R-\tilde{\Gamma}_Z}{\left(\rho+\delta\tilde{\Gamma}_Z\right)\tilde{\Gamma}_R}. \tag{A.17}$$

That is, the locus of indifference is an upward sloping line in the $(\pi_Z^j - (-\kappa), \pi_R^j - (-\kappa))$-plane with intercept $-\kappa\delta\frac{\tilde{\Gamma}_R-\tilde{\Gamma}_Z}{\left(\rho+\delta\tilde{\Gamma}_Z\right)\tilde{\Gamma}_R}$ and slope $\Upsilon \equiv \frac{\left(\rho+\delta\tilde{\Gamma}_R\right)\tilde{\Gamma}_Z}{\left(\rho+\delta\tilde{\Gamma}_Z\right)\tilde{\Gamma}_R}$. To characterize the direction of change of these lines, we note that

$$\tilde{\Gamma}_k = 1 - \left(1+\frac{1}{1+\rho}\frac{\lambda(\theta)}{\theta}\sum_i \ell_{i,k}\xi_i\right)^{-1}.$$

From this, it is clear that $0 \leq \tilde{\Gamma}_k < 1$ with $k = R, Z$. Also, we have:

$$\frac{\partial\tilde{\Gamma}_k}{\partial\ell_{i,k}} \geq 0 \text{ for all } i, \text{ and } \frac{\partial\tilde{\Gamma}_k}{\partial\theta} \leq 0,$$

where the latter derivative can also be expressed as a positive relation between $\tilde{\Gamma}_k$ and the job-filling rate, $\lambda(\theta)/\theta$. On the other hand, the effect of on-the-job search efficiency, x, on $\tilde{\Gamma}_k$ is ambiguous, given that $\frac{\partial\ell_{i,k}}{\partial x} \geq 0$ while $\frac{\partial\xi_i}{\partial x} \leq 0$. These results show that the locus $V_Z^j = 0$ (resp. $V_R^j = 0$) shifts to the left (resp. downwards) if $\ell_{i,Z}$ (resp. $\ell_{i,R}$) increases, or if market tightness decreases.

These results are also useful to inspect the sense of variation of the slope Υ of the locus of indifference between posting a Z vs. R job offer. Indeed, we have

$$\frac{\partial\Upsilon}{\partial\tilde{\Gamma}_Z} = \rho\frac{\rho+\delta\tilde{\Gamma}_R}{\left(\rho+\delta\tilde{\Gamma}_Z\right)^2\tilde{\Gamma}_R} > 0,$$

$$\frac{\partial\Upsilon}{\partial\tilde{\Gamma}_R} = -\rho\frac{\tilde{\Gamma}_Z}{\left(\rho+\delta\tilde{\Gamma}_Z\right)\left(\tilde{\Gamma}_R\right)^2} < 0.$$

It follows that $\frac{\partial \Upsilon}{\partial \ell_{i,Z}} > 0$ and $\frac{\partial \Upsilon}{\partial \ell_{i,R}} < 0$ for all i, i.e. the locus of indifference $V_R^j = V_Z^j$ rotates anticlockwise when the share of type-i workers in the pool of job seekers who are willing to accept offers Z increases or when that of job seekers willing to accept offers R decreases. Last, since market tightness θ shifts $\tilde{\Gamma}_Z$ and $\tilde{\Gamma}_R$ in the same direction, its impact on the slope Υ is ambiguous.

A.3 Proof of Proposition 3

First, we solve for the asset values of workers. Without flexible work, there is no on-the-job search, and as a result the asset value of employment is

$$W_R^i = \omega_R^i + \frac{1}{1+\rho} \left[W_R^i + \delta \left(N^i - W_R^i \right) \right]. \tag{A.18}$$

The asset value of non-employment is given by

$$N^i = \omega_N^i + \frac{1}{1+\rho} \left[(1 - \lambda(\theta)) N^i + \lambda(\theta) \max \left\{ N^i, W_R^i \right\} \right]. \tag{A.19}$$

Observe the difference with Equation (1): the heterogeneity of posted vacancies becomes irrelevant to the worker, but there remains the decision of whether or not to accept an offer (captured by the "max" operator). Suppose that the worker accepts job offers R. Then Equation (A.19) becomes

$$N^i = \omega_N^i + \frac{1}{1+\rho} \left[(1 - \lambda(\theta)) N^i + \lambda(\theta) W_R^i \right].$$

Together with (A.18), this defines a simple system of two linear equations, and solving for N^i we obtain:

$$N^i = \frac{1+\rho}{\rho} \left(\omega_R^i \frac{\lambda(\theta)}{\rho + \delta + \lambda(\theta)} + \omega_N^i \frac{\rho + \delta}{\rho + \delta + \lambda(\theta)} \right). \tag{A.20}$$

On the other hand, if the worker rejects the job offers R, then $\max \{N^i, W_R^i\} = N^i$ and Equation (A.19) yields

$$N^i = \frac{1+\rho}{\rho} \omega_N^i, \tag{A.21}$$

the asset value of non-participation. Since N^i in Equation (A.20) is proportional to a weighted average of ω_R^i and ω_N^i, it is greater than that in Equation (A.21) if and only if ω_R^i is greater than ω_N^i.

For Part (ii) of the Proposition, we first solve for the asset value of firms. In the equilibrium without flexible work, Equation (5) boils down to

$$J_{i,R}^j = \pi_R^j + \frac{1-\delta}{1+\rho} J_{i,R}^j \Rightarrow J_{i,R}^j = \frac{1+\rho}{\rho + \delta} \pi_R^j. \tag{A.22}$$

The asset value of posting a vacancy R is

$$V_R^j = -\kappa + \frac{1}{1+\rho} \left[V_R^j + \frac{\lambda(\theta)}{\theta} \left(J_{i,R}^j - V_R^j \right) \right]. \tag{A.23}$$

Notice that in these two equations, the subscript i plays no role as firms are indifferent to the identity of workers who match with their vacancies. Solving for V_R^j, we obtain

$$V_R^j = \frac{1+\rho}{\rho + \lambda(\theta)/\theta} \left(-\kappa + \frac{\lambda(\theta)/\theta}{\rho + \delta} \pi_R^j \right).$$

Hence $V_R^j > 0$ if and only if $-\kappa + \frac{\lambda(\theta)/\theta}{\rho+\delta}\pi_R^j > 0$, which can be expressed as $\pi_R^j - (-\kappa) > \frac{\kappa}{\Gamma}$, where $\Gamma \equiv \frac{\lambda(\theta)/\theta}{\rho+\delta+\lambda(\theta)/\theta}$.

A.4   Proof of Propositions 4a and 4b

We begin by describing the equilibrium of the economy without flexible work. The law of motion of employment is given by (11), and aggregating up across all i's and j's yields

$$\widetilde{e}_R' = (1-\delta)\,\widetilde{e}_R + \frac{\widetilde{v}_R}{\widetilde{v}}\lambda\left(\widetilde{\theta}\right)\widetilde{u} = (1-\delta)\,\widetilde{e}_R + \lambda\left(\widetilde{\theta}\right)\widetilde{u},$$

where the last equation follows from the fact that $\widetilde{v}_R = \widetilde{v}$. Thus, in the steady-state equilibrium of the economy without flexible work, we obtain

$$\widetilde{e}_R = \frac{1}{\delta}\lambda\left(\widetilde{\theta}\right)\widetilde{u}. \tag{A.24}$$

Next, we analyze the baseline equilibrium under the assumption that there are no Z-best workers. For all type-i workers, we have $x_{i,R} = 0$. Plugging this into the law of motion for $e_{i,j}$ for all j's such that $k(j) = R$, we obtain:

$$e_{i,j}' = (1-\delta)\,e_{i,j} + \frac{v_j}{v}\lambda\left(\theta\right)\left(\sum_{j'} x_{i,k(j')}\mathbb{1}_{\left\{W_R^i > W_{k(j')}^i\right\}}\left(1-\delta\right)e_{i,j'} + \mathbb{1}_{\left\{W_R^i > N^i\right\}}u_i\right)$$

$$= (1-\delta)\,e_{i,j} + \frac{v_j}{v}\lambda\left(\theta\right)\left(\sum_{j':\,k(j')=Z} x_{i,Z}\mathbb{1}_{\left\{W_R^i > W_Z^i\right\}}\left(1-\delta\right)e_{i,j'} + \mathbb{1}_{\left\{W_R^i > N^i\right\}}u_i\right)$$

$$= (1-\delta)\,e_{i,j} + \frac{v_j}{v}\lambda\left(\theta\right)\left(x_{i,Z}\left(1-\delta\right)e_{i,Z} + \mathbb{1}_{\left\{W_R^i > N^i\right\}}u_i\right).$$

Next, summing over all j's such that $k(j) = R$, we obtain

$$e_{i,R}' = (1-\delta)\,e_{i,R} + \frac{v_R}{v}\lambda\left(\theta\right)\left(x_{i,Z}\left(1-\delta\right)e_{i,Z} + \mathbb{1}_{\left\{W_R^i > N^i\right\}}u_i\right),$$

and aggregating over all i's yields

$$e_R' = (1-\delta)\,e_R + \frac{v_R}{v}\lambda\left(\theta\right)\sum_i\left(x_{i,Z}\left(1-\delta\right)e_{i,Z} + \mathbb{1}_{\left\{W_R^i > N^i\right\}}u_i\right).$$

It follows that in a steady-state equilibrium, we have

$$e_R = \frac{1}{\delta}\frac{v_R}{v}\lambda\left(\theta\right)\sum_i\left(x_{i,Z}\left(1-\delta\right)e_{i,Z} + \mathbb{1}_{\left\{W_R^i > N^i\right\}}u_i\right). \tag{A.25}$$

Divide Equation (A.25) by (A.24) to arrive at Equation (14) (Proposition 4a).

To prove Proposition 4b, consider the baseline equilibrium under the assumption that there are no R-best workers, so that $x_{i,Z} = 0$ for all type-i workers. The law of motion for $e_{i,j}$ for all

j's such that $k(j) = R$ becomes

$$e'_{i,j} = \left(1 - x_{i,R}\lambda(\theta)\sum_{j'}\frac{v_{j'}}{v}\mathbb{1}_{\left\{W^i_{k(j')}>W^i_R\right\}}\right)(1-\delta)e_{i,j} + \lambda(\theta)\frac{v_j}{v}\mathbb{1}_{\left\{W^i_R>N^i\right\}}u_i$$

$$= \left(1 - x_{i,R}\lambda(\theta)\sum_{j':k(j')=Z}\frac{v_{j'}}{v}\mathbb{1}_{\left\{W^i_Z>W^i_R\right\}}\right)(1-\delta)e_{i,j} + \lambda(\theta)\frac{v_j}{v}\mathbb{1}_{\left\{W^i_R>N^i\right\}}u_i$$

$$= \left(1 - x_{i,R}\lambda(\theta)\frac{v_Z}{v}\right)(1-\delta)e_{i,j} + \lambda(\theta)\frac{v_j}{v}\mathbb{1}_{\left\{W^i_R>N^i\right\}}u_i.$$

summing over all j's such that $k(j) = R$, we obtain

$$e'_{i,R} = \left(1 - x_{i,R}\lambda(\theta)\frac{v_Z}{v}\right)(1-\delta)e_{i,R} + \lambda(\theta)\frac{v_R}{v}\mathbb{1}_{\left\{W^i_R>N^i\right\}}u_i,$$

and aggregating over all the i's yields

$$e'_R = (1-\delta)e_R - \left(\sum_i x_{i,R}\lambda(\theta)\frac{v_Z}{v}(1-\delta)\frac{e_{i,R}}{e_R}\right)e_R + \frac{v_R}{v}\lambda(\theta)\sum_i \mathbb{1}_{\left\{W^i_R>N^i\right\}}u_i.$$

Thus, in a steady-state equilibrium, we have

$$e_R = \frac{\frac{v_R}{v}\lambda(\theta)\sum_i \mathbb{1}_{\left\{W^i_R>N^i\right\}}u_i}{\delta + (1-\delta)\lambda(\theta)\frac{v_Z}{v}\sum_i x_{i,R}\frac{e_{i,R}}{e_R}}. \tag{A.26}$$

To arrive at Equation (15) from Proposition 4b, we divide Equation (A.26) by (A.24).

B Additional information on ZHCs

B.1 Regulatory framework

To complement Section 4, this appendix reviews the legal status of individuals on ZHCs in the U.K., as well as their entitlement to welfare under these contracts.

Workers' rights. ZHCs typically give staff a "worker" employment status, which lies between the traditional categories of "employee" and "self-employed". This intermediate status confers such individuals with the following employment rights:[25]

- Right not to be discriminated against under the Equality Act 2010; right to receive pro-rata holiday pay and other working time rights (Working Time Regulations 1998); and right to receive Statutory Sick Pay (as long as they have met the Lower Earnings Limit);
- Automatic enrollment for pensions;
- Protection from unlawful deductions from wages;
- Right to receive the hourly National Minimum Wage or National Living Wage.[26]

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[25]There is, however, some controversy between trade unions and employers' associations about whether individuals on ZHCs are "workers" or "employees". While the Trade Unions Congress considers that most of them are "workers", the Chartered Institute of Personnel and Development [CPID, 2013] reports that two-thirds of employers (interviewed in a survey carried out by the institute) classify zero-hours individuals on ZHCs as "employees".

[26]In the U.K. there have been several minimum wages in place during the recent period. From April 2016 there are three minimum wage rates for young workers (those aged 16-17, 18-20, and apprentices), another

However, in contrast to employees, ZHC workers are not entitled to redundancy pay when dismissed but have more rights than the self-employed who are only entitled for protection for health and safety on a client's premises and against discrimination. It is noteworthy that exclusivity clauses in ZHCs, which stop a zero-hour worker from taking on another job, were banned in May 2015. Employers cannot enforce the clause, and since January 2016, workers have been able to claim compensation at an employment court if they are punished or dismissed for looking for work elsewhere.

Whereas in the U.K. workers under ZHCs are not obliged to provide any minimum working hours, in this respect it is noteworthy that in Ireland individuals are contractually obliged to be available for work if called by employers. By contrast, "self-employed" individuals have no employment rights besides certain discrimination rights. At the other end of the spectrum "employees" have the whole range of employment rights, such as paid maternity leave, including unfair dismissal and redundancy and family rights.

The distinction between the status of "worker" and that of "employee" has been subject to court litigation recently. A well-known case is whether companies like Uber or Deliveroo should hire under employment contracts or freelance work.[27] To the extent that some of these firms use contractors rather than employees, they do not fall into the above definition of ZHCs. The most important difference between the two categories of workers is that employers must offer "employees" work in exchange for pay, and "employees" are required to do the work, whereas "workers" can turn work down, depending on their time availability.

Moreover, whether an individual is considered to be an "employee" or a "worker" depends not just on what it is the offered contract, but also on what happens day to day. While a contract might stipulate that there is no obligation to work, if the individual is "punished" for not accepting all the offered hours offered, or consistently work a set number of hours, then a tribunal might decide that the worker is actually an "employee".

Entitlement to welfare. Since workers under ZHCs are often low-wage earners, they are entitled to means-tested benefits and tax credits. In the past, the benefits one could claim hinged on whether the individual worked more than 16 hours in a week, as in the case of the Income Support program or Jobseekers' Allowance (JSA). When working 16 hours a week or more, individuals could also claim the Working Tax Credit, Child Benefit and Housing Benefit if they needed help with the rent and had savings less than £16,000. However, the Universal Credit (UC) in 2013 replaced all of these income support schemes with a taper rate of 65 percent implemented from a typical monthly work allowance of (net of taxes) £490 for single workers. The UC taper rate was reduced to 63 percent in 2018, and a further reduction to 55 percent is expected to come into force at the end of 2021.

B.2 Descriptive statistics

Data and sample. To establish a few facts about ZHCs summarized in Section 4 and calculate data moments used in Section 5, we use data from the U.K.'s household labor force survey (LFS). This survey covers a large number of individuals, which is important for our purposes since, despite their growth, ZHCs still represent a fairly small portion of the labor market. The U.K.'s LFS has a modest longitudinal dimension as it follows individuals over five quarters, with one fifth of the sample being renewed every quarter. Among these five

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minimum wage rate for young adults (those aged 20-24), and finally the new National Living Wage (NLW) for individuals aged 25 and above. The NLW was raised in April 2019 to £8.21 an hour.

[27]The issue of whether Uber drivers are "workers" as opposed to "independent, third-party contractors" of Uber has been raised many times during the 2010s. On 19 February 2021, the U.K. Supreme Court unanimously upheld a ruling that they are workers of the company, with rights to be paid at least the national minimum wage, to holiday pay and other benefits. See this February 2021 article in The Washington Post.

interviews, respondents are asked twice (one semester apart) whether they hold a ZHC. We pool eight quarterly waves of the longitudinal LFS, up to the onset of the COVID-19 pandemic. Specifically, the survey responses that we analyze cover the period from September 2018 to March 2020. One motivation for focusing on this time period is that Figure 1 suggests that the U.K. labor market was stable during those years along the dimensions we analyze.

As our focus is on the segment of the labor market where jobs pay around the minimum wage, we restrict our data analysis to individuals who either work in a low-paying occupation in at least one quarter or are 'not employed' in any of five quarters while not being recorded as 'inactive' throughout the five quarters.[28] We define low-paying occupations as those classified as either 'Administrative and secretarial', 'Caring, leisure and other service', 'Sales and customer service', 'Process, plant and machine', or 'Elementary' occupations. We exclude all retirees and those under age 16 in the first quarterly interview. This leaves us with 9,342 individuals aged 16 to 69 belonging to the low-pay segment of the labor market.

The unemployment rate in the first interview is 11.2 percent. Every other quarter, the LFS includes a question asking respondents whether they hold a ZHC. In particular, 356 individuals, i.e. 3.8 percent of our sample, reports being employed under a ZHC in the first interview where this question is included. This represents 4.4 percent of all employees in the low-pay segment of the market. Likewise, 431 individuals, or 4.6 percent of our sample (5.2 percent of all employees), report having a ZHC in either the first or the second interviews. This is about 1.5 times the share of ZHC workers in the overall labor market. As expected, all individuals on ZHCs report being employed.

Cross-sectional characteristics. In Table B1 we start out our description of ZHCs by comparing the cross-sectional characteristics of workers in these contracts (denoted in short as Z) with workers in regular contracts (R) and unemployed workers (U). First, we note little difference in terms of gender: female employees account for 56.5 percent of ZHC employment vs. 60.4 percent of regular employment, and this difference is not statistically significant. Differences with respect to age, on the other hand, are large. The (not reported) mean age of workers under ZHCs is 40.8 vs. 46.3 years old for employees in regular contracts. In order to get a more precise view of the incidence of ZHCs over the life cycle, Panel (b) of Table B1 displays the shares of each 5-year age bands in Z and R employment and in unemployment U. This shows an increased prevalence of Z contracts at both ends of the working life. Indeed, relative to both R and U individuals, the group of Z workers displays a much lower share of workers aged 30 to 44 years old. Relative to R workers, the age distribution of workers in Z contracts is largely skewed towards younger workers; relative to U, the share of workers aged 45 and above is slightly higher. The latter fact could be related to the demand for flexibility among older workers to smooth out the transition to retirement.

Table C1: Characteristics of people by labor contracts

(a) Gender	U	Z	R
Men	44.0	43.5	39.6
Women	56.0	56.5	60.4
(b) Age	U	Z	R
16 to 19 years	21.2	16.6	3.0
20 to 24 years	9.3	11.5	4.9
25 to 29 years	5.8	7.3	6.2
30 to 34 years	6.8	6.2	7.7

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[28]This sample restriction does not exclude from our sample some long-term unemployed who are looking for a high-paying occupation job. Consequently our approach might overstate the unemployment rate of the low-pay segment of the U.K. labor market.

	U	Z	R
35 to 39 years	6.3	3.9	8.7
40 to 44 years	5.7	5.6	9.8
45 to 49 years	7.4	9.6	11.9
50 to 54 years	8.7	8.7	14.9
55 to 59 years	10.6	12.4	15.9
60 to 64 years	12.5	11.0	12.6
65 to 69 years	5.7	7.3	4.6
(c) Education	**U**	**Z**	**R**
Degree or equivalent	16.3	21.9	18.0
Higher education	6.7	8.7	8.9
GCE A level or equivalent	20.4	21.9	23.8
GCSE grades A*-C or equivalent	31.6	25.8	27.8
Other qualification	9.2	10.7	9.8
No qualification	13.9	8.2	9.2
No answer or don't know	1.8	2.8	2.7
(d) Industry		**Z**	**R**
Agriculture, forestry and fishing		1.1	0.9
Mining and quarrying		0.0	0.3
Manufacturing		3.9	7.9
Electricity, gas, AC supply		0.0	0.5
Water supply, sewerage, waste		0.0	1.0
Construction		0.8	3.7
Wholesale, retail, repair of vehicle		8.7	17.7
Transport and storage		8.7	7.9
Accommodation and food services		19.9	4.0
Information and communication		0.8	1.2
Financial and insurance activities		0.6	2.9
Real estate activities		0.3	0.7
Prof., scientific, technical activities		2.5	5.1
Admin. and support services		6.7	5.5
Public admin. and defense		2.0	7.2
Education		8.4	10.8
Health and social work		20.5	15.5
Arts, entertainment and recreation		6.7	1.9
Other service activities		3.4	3.2
Households as employers		0.8	0.3
Extraterritorial organizations		0.0	0.1
No answer		3.9	1.5

Note: Authors' calculations based on data from the Labour Force Survey. U: Unemployed, Z: Employed in a zero-hours contract, R: Employed not in a zero-hours contract. All table entries are expressed in percent.

Panel (c) of Table B1 displays the distribution of educational attainment among employed and unemployed workers. As in Datta et al. [2019], these distributions only exhibit modest differences. 21.9 percent of ZHC workers hold a degree or equivalent vs. 18 percent of employees in regular contracts, and 19 percent of ZHC workers hold no or "other" qualifications, which is the same fraction as for those in regular employment. Hence, two facts stand out from these findings: (i) a substantial fraction of employment in "low occupations" are highly qualified, corresponding perhaps to young college students and elderly college graduates who use ZHCs
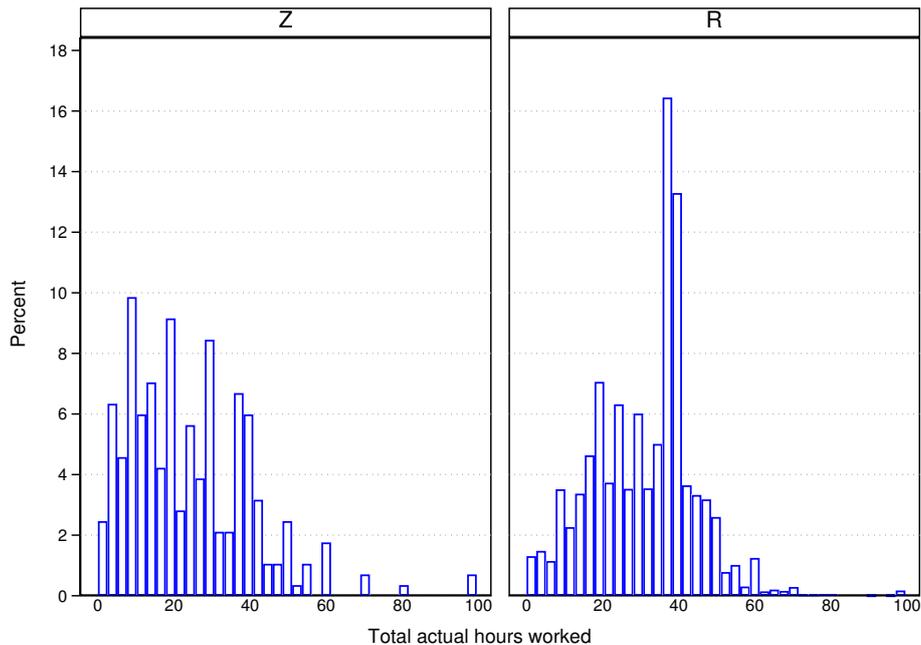
Figure B1: Distribution of actual hours worked

Notes: Authors' calculations based on data from the Labour Force Survey. Total actual hours worked exclude holidays. *Z*: employed in a zero-hour contract, *R*: employed not in a zero-hour contract.

to complement their earnings;[29] and (ii) the different contract types in this segment of the labor market are filled with very similar workers in terms of education. Next, Panel (d) of Table B1 shows the breakdown of employment in either contract type by industry. We note that industries that are over-represented in ZHC employment are 'Accommodation and food services', 'Health and social work' and 'Arts, entertainment and recreation'. Conversely, under-represented industries in ZHC employment are 'Manufacturing', 'Construction', 'Wholesale, retail, repair of vehicle' and 'Public administration and defense'. The relation between ZHCs and 'Accommodation and food services' and 'Health and social work' is typically that which has been highlighted by the COVID-19 pandemic.

Hours worked. In our dataset, the only measure of hours that does not suffer from a large fraction of missing data is "total actual hours in the main job". Since there is no information about whether the respondent is on holiday during the LFS's reference week (the time frame used to measure actual hours worked), we assume that individuals reporting hours in the lowest decile of the distribution of hours worked are on holiday. Figure B1 reports the cross-sectional distributions of hours among individuals at work with either type of contract. As can be inspected, ZHC workers spend on average fewer hours on the job, and the cross-sectional standard deviation of these hours is higher than in regular contracts.

To inspect further the source of these differences, we use the longitudinal dimension of the LFS to compute the mean and standard deviation of hours worked for each individual over the

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[29]In general, the relationship between education and the probability of being employed in a flexible job is a complex one. On the one hand, workers with more educational attainment who benefit from more bargaining power are more likely to obtain more schedule and location flexibility, and at the same time employers face a lower cost of providing flexibility in high-skilled jobs. On the other hand, less educated workers are more likely to be employed in jobs which require working overtime, having night shifts, or receiving shorter advance notice about their schedules. Mas and Pallais [2020] find a statistically significant, positive relationship between higher education and the probability of employment in some form of flexible work arrangement; see Section 1.2 of their paper and the insightful discussion therein.

five quarterly interviews in which they are included in the survey. Table B2 displays the average of the individual-level means and individual-level standard deviations obtained for those who are continuously employed under ZHCs and under regular contracts, respectively.[30] As expected, these figures suggest that ZHC jobs offer fewer hours of work and a more volatile hours schedule than regular jobs. The LFS also includes questions about an individual's willingness to work more hours. 16.6 percent of individuals in ZHCs indicate that they would like to work more hours vs. 10.1 percent in regular contracts. In a similar vein, 18.2 percent of ZHC workers report looking for another (or additional) job whereas only 5.0 percent of employees in regular jobs do so. This suggests that a higher fraction of workers in ZHCs are in underemployment, but that a majority of them are satisfied with both their job or their hours, to the extent that they are not looking to change either.

Table B2: Mean and standard deviation of actual hours worked

Continuous employment in:	Z	R
Mean	18.4	28.1
Standard deviation	7.8	7.2

Notes: Authors' calculations based on data from the Labour Force Survey. Total actual hours worked exclude holidays. Z: Employed in a zero-hours contract, R: Employed not in a zero-hours contract.

C   Estimation/Calibration appendix

C.1   Heterogeneity in exit rates

Our empirical moments include the distribution of the duration of unemployment spells (U) and job tenure in the two labor market states of employment, namely ZHCs (Z) and regular jobs (R). Our theoretical model contains up to 4 types of workers defined by their rankings of the different states. For each labor market state, we aim to retrieve the distribution of exit rates from said state, i.e. the number of types, their weights in the stock and the type-specific exit rates, from the distribution of durations/tenures. Formally, suppose that there are M types m of individuals leaving a labor market state at rate λ_m per unit of (discrete) time. The share of type m in the inflow into this state is denoted z_m. The size of the inflow is I. We can then write the distribution of durations d in this state, the total size of which is S, as:

$$S_0 = \frac{I}{S}$$

$$S_d = \frac{I}{S} \cdot \sum_{m=1}^{M} z_m \left(1 - \lambda_m\right)^d \quad \text{for } d \geq 1,$$

where S_d is the fraction of individuals with duration d in this state, $S = \sum_{d=1}^{\infty} S_d$ and $\sum_{m=1}^{M} z_m = 1$. In practice, our empirical distribution of durations has a finite number of points, with the last duration band D gathering all durations greater or equal to D, i.e. $S_D = \sum_{d=D}^{\infty} S_d$.

For a given M, we retrieve the heterogeneity distribution of exit rates from this state by

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[30]As before, we assume that the observations in the first decile of the hours distribution correspond to holidays and excluded them from these calculations.

matching these fractions to their true counterparts:

$$\left\{ \hat{z_m}, \hat{\lambda_m} \right\}_{m=1,\dots,M} = \arg \min_{\{z_m, \lambda_m\}} \sum_{d=1}^{D} \left[\frac{S_d \left(\{z_m, \lambda_m\}_{m=1,\dots,M} \right) - S_d^0}{S_d^0} \right]^2, \qquad \text{(C.1)}$$

where S_d^0 are the true moments. Jewell [1982] shows that this minimization problem has a unique global minimum, implying that $\left\{ \hat{z_m}, \hat{\lambda_m} \right\}_{m=1,\dots,M}$ are identified.

In practice, we do not observe the true S_d^0 and use their sample counterparts $\widetilde{S}_d$, the empirical fraction of individuals with duration d in this state. These frequency estimators are $\sqrt{N}$-consistent estimators of the true S_d^0.

Let us denote the best fit to the empirical moments obtained with M types as:

$$F_M = \sum_{d=1}^{D} \left[\frac{S_d \left(\{z_m, \lambda_m\}_{m=1,\dots,M} \right) - \widetilde{S}_d}{\widetilde{S}_d} \right]^2. \qquad \text{(C.2)}$$

We also need to determine the optimal number of types M to fit the data. In order to do this, we proceed by iterations: we start with a homogeneous exit rate, i.e. $M = 1$, and obtain the best fit F_1. We then obtain the best fit with $M = 2$ and test whether it is significantly better than the fit with one type. If it is, we iterate to compare the fit obtained with two and three types. The iteration process stops when the null hypothesis of M types is not rejected in favor of the alternative hypothesis of $M+1$ types. In order to compare F_{M+1} to the distribution of F_M under the null, we adapt a method developed by Karlis and Xekalaki [1999], where this distribution is simulated with 1,000 bootstrapped samples of the data. In other words, for each of these bootstrap samples, denoted s, we carry out the optimization described in (C.1) and obtain a fit for M types, denoted $\widetilde{F}_M^s$. If the fit obtained with $M + 1$ types on the original sample falls below the 5th percentile of the distribution of $\widetilde{F}_M^s$ thus obtained, we reject the null of M types in favor of the alternative of (at least) $M + 1$ types. Then we proceed with the next iteration, where the null of $M + 1$ types is tested against the alternative of $M + 2$ types. Karlis and Xekalaki [1999] show that this procedure yields the optimal number of types, M^*.[31]

Estimation results are reported in Table C1. As described in the main text, for unemployment and Z employment, we statistically detect two types, while for R employment we obtain only one type.[32] For the sake of completeness, the columns of Table C1 also report the estimated weights and exit rates for each labor market state, the $\hat{z_m}$'s and $\hat{\lambda_m}$'s. Lacking joint restrictions and relations across labor market states between these parameters, we cannot directly use them to calibrate the model, but we find it useful to compare them to the calibrated parameters. First, we find $\hat{\lambda_1} = 0.015$ in R and $\hat{\lambda_1} = 0.005$ in Z. The model counterpart is δ, whose calibrated value at the monthly frequency is $1 - (1 - 0.005)^2 = 0.010$, which falls in between the two $\hat{\lambda_1}$'s. Second, the highest estimated rate in U, namely $\hat{\lambda_2} = 0.162$ is higher that its counterpart $\lambda(\theta)$ whose monthly value is $1 - (1 - 0.051)^2 = 0.099$ in the baseline equilibrium. Third, using the estimated $\hat{\lambda_m}$'s in U and Z (the latter imply a vacancy share of R jobs of 0.94), we can back out

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[31]Karlis and Xekalaki [1999] also argue that the procedure gives the same results as a 'backward' approach which would start from a high number of type and work by iterations comparing M types with $M - 1$ types until a significant change in the criterion is found. The 'forward' procedure that they propose, i.e. incrementally increasing the number of types, has the advantage of speed.

[32]One caveat we have come across while executing the above strategy is that some results produce a worker type with an exit rate of zero (1e-16). Our understanding of the data is that all individuals leave any of these labor market states within a finite horizon of at most 40 years, so we have decided to rule out these cases by penalizing the fit $\widetilde{F}_M^s$ obtained when a type has a zero exit rate.

Table C1: Estimated number of worker types in each labor market state

Unemployment (1)	Z employment (2)	R employment (3)
p-value $(M = 1 \text{ vs. } M \geq 2) = 0.01$ p-value $(M = 2 \text{ vs. } M \geq 3) = 0.93$ $\Rightarrow M^* = 2$	p-value $(M = 1 \text{ vs. } M \geq 2) = 0.01$ p-value $(M = 2 \text{ vs. } M \geq 3) = 0.74$ $\Rightarrow M^* = 2$	p-value $(M = 1 \text{ vs. } M \geq 2) = 0.99$ $\Rightarrow M^* = 1$
$\left(\hat{z}_1, \hat{\lambda}_1\right) = (0.281, 0.010)$ $\left(\hat{z}_2, \hat{\lambda}_2\right) = (0.719, 0.162)$	$\left(\hat{z}_1, \hat{\lambda}_1\right) = (0.723, 0.015)$ $\left(\hat{z}_2, \hat{\lambda}_2\right) = (0.277, 0.081)$	$\left(\hat{z}_1, \hat{\lambda}_1\right) = (1.000, 0.005)$

Notes: Each column of the table reports, for each labor market state, the p-value of the tests to detect the optimal number of types, final outcome of the procedure, M^*, and the estimated weight(s) and (monthly) exit rate(s) of the types, $\left\{\hat{z_m}, \hat{\lambda_m}\right\}_{m=1,\ldots,M^*}$.

the implied on-the-job search efficiency as: $(0.081 - 0.015)/((1 - 0.015) \times 0.162 \times 0.94) = 0.44$, to be compared with $x = 0.35$ in our calibration. We find these differences acceptable on the grounds that the estimated $\hat{\lambda_m}$'s have fairly large standard errors and that the data used here, based on recalled job tenure and unemployment duration, likely suffers more measurement error than data on current labor market states used for the transition matrix of Table 1.

C.2 Vacancy elasticity of the matching function

We use U.K. hiring and job vacancy data from Patterson et al. [2016] to estimate the elasticity of the matching function with respect to vacancies. These data are available at the levels of 2-digit occupations, which enables us to extract information for the low-pay segment of the labor market. We use newly-formed matches ($M_{o,t}$), unemployment claims ($U_{o,t}$) and job vacancies ($V_{o,t}$) for the following occupations o: 'Administrative', 'Secretarial and related', 'Caring personal service', 'Leisure and other personal service', 'Process, plant and machine', 'Elementary trades, plant and storage related', and 'Elementary administration and service', to run the following linear regression:

$$\log\left(\frac{M_{o,t}}{U_{o,t}}\right) = \alpha_o + \varpi' g\left(t\right) + \psi \log\left(\frac{V_{o,t}}{U_{o,t}}\right) + \varepsilon_{o,t}. \tag{C.3}$$

α_o is an occupation fixed effect; $g\left(t\right)$ a polynomial that allows for a flexible time trend; $\varepsilon_{o,t}$ the regression residual; and ψ the coefficient of interest. The data is monthly, seasonally adjusted, and runs from April 2004 through June 2012. Estimation results are reported in Table C2.

Table C2: Vacancy elasticity of the matching function

	Log- job finding ($\log\left(\frac{M_{o,t}}{U_{o,t}}\right)$)			
	(1)	(2)	(3)	(4)
Log- market tightness ($\log\left(\frac{V_{o,t}}{U_{o,t}}\right)$)	0.643*** (0.027)	0.701*** (0.041)	0.586*** (0.025)	0.703*** (0.034)
R-squared	0.859	0.896	0.802	0.871
Time trend ($g\left(t\right)$)		✔		✔
Occupation fixed effect (α_o)			✔	✔

Notes: Each column of the table reports coefficients from a linear estimation of Equation (C.3) based on data from Patterson et al. [2016]. Column (1) presents the univariate regression, column (2) adds a 5-th order polynomial function $g\left(t\right)$, column (3) adds occupation fixed effects, and column (4) controls simultaneously for the time trend and occupation fixed effects. Standard errors in parentheses are clustered at the occupation level.

The coefficient on the log of market tightness (as measured by the ratio between job vacancies and unemployment claimants) is precisely estimated and statistically higher than the conventional value of 0.50. Depending on the specification used, it ranges from 0.59 to 0.70. In the model's calibration, we use the mid-point value of 0.65.

D   Additional tables and figures

Firms' distribution of $\widetilde{h}$.   Figure D1 plots the distributions $H_j(.)$'s from which each firm type draws its $\widetilde{h}$, the working hours that would meet the demand faced at a given point in time. The gap in mean hours between firms that post Z vs. R jobs is 10 hours. Firms that post Z jobs also face more volatile hours relative to R firms; at $\sigma_Z = 3$ these are Z-best firms (i.e., they advertise jobs which they can profitably filled under regular employment conditions as ZHCs), while at $\sigma_Z = 6$ they are what we call Z-only firms (i.e., they cannot profitably hire workers under regular employment conditions).
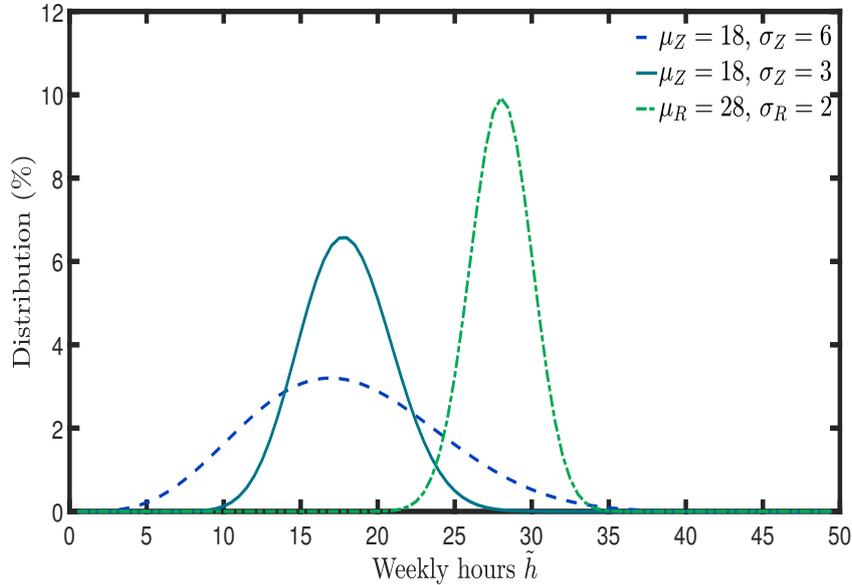

Figure D1: Firms' distribution of fluctuating weekly working hours $\tilde{h}$

Notes: The figure shows the distributions of firms' fluctuating weekly working hours used in the calibration of the baseline equilibium.

We conducted extensive robustness analyses to assess the role of μ_Z and σ_Z. We summarize the results in this section. Note that any change to μ_Z and σ_Z affects average labor earnings (through actual working hours, since $h = \widetilde{h}$ in a Z job). As a consequence, the costs parameters ϕ, κ, K, and non-employment income b, are also affected. We recalibrate them to match the targets discussed in Section 5.

The first dimension we wish to discuss is the choice of mean hours in Z employment, μ_Z. In the baseline calibration, we use the same μ_Z for Z-best and Z-only firms, assuming that the two firm types differ in terms of the standard deviation of hours, σ_Z. It is also possible to keep σ_Z the same across firm types while choosing μ_Z to make the flow profits consistent with firms' ranking of the different employment options. We set $\sigma_Z = 4.5$, which is the mid-point value of the two σ_Z's used in the baseline calibration, namely $\sigma_Z = 3$ and $\sigma_Z = 6$. We then select $\mu_Z = 20$ for Z-best firms and $\mu_Z = 16$ for Z-only firms, so the gap between hours worked at those firms vs. R firms remains 10 hours on average. Since σ_Z is the same across Z-best and Z-only types, we

I. $\mu_Z = 20$, $\sigma_Z = 4.5$ and $\mu_Z = 16$, $\sigma_Z = 4.5$		
Firms →	**High mean hours**	**Low mean hours**
Workers		
↓	$e:$ $-0.33\,$p.p.	$e:$ $-1.60\,$p.p.
	$\frac{u}{u+e}:$ $+0.33\,$p.p.	$\frac{u}{u+e}:$ $+1.60\,$p.p.
Low disutility	$\frac{v}{v+e}:$ $-0.73\,$p.p.	$\frac{v}{v+e}:$ $-0.87\,$p.p.
	GDP: $+0.65\%$	GDP: $+0.51\%$
	$e:$ $-3.14\,$p.p.	$e:$ $-4.37\,$p.p.
	$\frac{u}{u+e}:$ $+0.33\,$p.p.	$\frac{u}{u+e}:$ $+1.60\,$p.p.
High disutility	$\frac{v}{v+e}:$ $-0.79\,$p.p.	$\frac{v}{v+e}:$ $-0.93\,$p.p.
	GDP: -1.51%	GDP: -1.57%

II. $\mu_Z = 18$, $\sigma_Z = 1.0$ and $\mu_Z = 18$, $\sigma_Z = 8.0$		
Firms →	**Low volatility**	**High volatility**
Workers		
↓	$e:$ $-1.14\,$p.p.	$e:$ $-2.65\,$p.p.
	$\frac{u}{u+e}:$ $+1.14\,$p.p.	$\frac{u}{u+e}:$ $+2.65\,$p.p.
Low disutility	$\frac{v}{v+e}:$ $-0.82\,$p.p.	$\frac{v}{v+e}:$ $-0.97\,$p.p.
	GDP: $+0.35\%$	GDP: -0.10%
	$e:$ $-3.93\,$p.p.	$e:$ $-5.39\,$p.p.
	$\frac{u}{u+e}:$ $+1.14\,$p.p.	$\frac{u}{u+e}:$ $+2.65\,$p.p.
High disutility	$\frac{v}{v+e}:$ $-0.88\,$p.p.	$\frac{v}{v+e}:$ $-1.03\,$p.p.
	GDP: -1.90%	GDP: -3.24%

Notes: The table reports the equilibrium allocation effects of a ban on ZHCs under alternative assumptions about the mean and standard deviation of hours $\widetilde{h}$ in Z firms. e: employment; u: unemployment; v: vacancies; GDP: gross domestic product. In the upper panel, $\sigma_Z = 4.5$ and "High mean hours" (resp. "Low mean hours") refers to the calibration where the mean of hours in Z firms is $\mu_Z = 20$ (resp. $\sigma_Z = 16$), implying that these are Z-best (resp. Z-only) firms. In the lower panel, $\mu_Z = 18$ and "Low volatility" (resp. "High volatility") refers to the calibration where the volatility of hours in Z firms is $\sigma_Z = 1$ (resp. $\sigma_Z = 8$), implying that these are Z-best (resp. Z-only) firms. In both panels, "Low disutility" (resp. "High disutility") refers to the calibration where the disutility of labor α_i of Z-only workers is such that these workers prefer to stay (resp. drop from the labor force) once Z employment is banned.

no longer have "low volatility" and "high volatility" economies; instead, we have "high mean hours" and "low mean hours", respectively. The upper panel of Table D1 reports the results on the equilibrium allocation effects of the ZHC ban. The employment/unemployment effects turn out to be more moderate than under the baseline calibration, although the difference is not large. In these experiments, sectoral GDP increases when Z-only workers remain in the labor market, due to the increase in hours per worker and the decrease in vacancies and hence fewer resources being consumed in the costly process of job posting. The upper panel of Table D2 reports the welfare effects under this alternative calibration. As can be seen, the numbers line up closely with those obtained in the baseline calibration.

Second, we analyze the effects of changing the standard deviation of hours at Z firms, σ_Z. We set $\sigma_Z = 1$ for the "low volatility" setting, i.e. a reduction by 2 units relative to the baseline calibration. Symmetrically, we increase it by 2 units for the "high volatility"

Table D2: Robustness check: Welfare effects of a ban on Z contracts under alternative mean and standard deviation of $\widetilde{h}$ in Z firms

I. $\mu_Z = 20$, $\sigma_Z = 4.5$ and $\mu_Z = 16$, $\sigma_Z = 4.5$		
Firms $\rightarrow$	High mean hours	Low mean hours
R-best workers	GE: -0.13%	GE: -0.34%
	PE: $+0.11\%$	PE: $+0.22\%$
Z-only workers	GE: -1.69%	GE: -1.81%
	PE: -2.73%	PE: -3.39%

II. $\mu_Z = 18$, $\sigma_Z = 1.0$ and $\mu_Z = 18$, $\sigma_Z = 8.0$		
Firms $\rightarrow$	Low volatility	High volatility
R-best workers	GE: -0.30%	GE: -0.59%
	PE: $+0.13\%$	PE: $+0.19\%$
Z-only workers	GE: -2.07%	GE: -1.37%
	PE: -3.38%	PE: -2.66%

Notes: The table reports the welfare effects of a ban on ZHCs under alternative assumptions about the mean and standard deviation of hours $\widetilde{h}$ in Z firms. GE: general equilibrium, which is the full effect of the reform; PE: partial equilibrium, which is the effect of keeping the equilibrium allocation unchanged and substitute R (resp. N) with Z employment for R-best workers (resp. Z-only workers). In the upper panel, $\sigma_Z = 4.5$ and "High mean hours" (resp. "Low mean hours") refers to the calibration where the mean of hours in Z firms is $\mu_Z = 20$ (resp. $\sigma_Z = 16$), implying that these are Z-best (resp. Z-only) firms. In the lower panel, $\mu_Z = 18$ and "Low volatility" (resp. "High volatility") refers to the calibration where the volatility of hours in Z firms is $\sigma_Z = 1$ (resp. $\sigma_Z = 8$), implying that these are Z-best (resp. Z-only) firms. In both panels, all effects are computed in the context of the "Low disutility" economy, where the disutility of labor α_i of Z-only workers is such that these workers remain in the labor force once Z employment is banned. Welfare effects are in percent of consumption equivalent variation units.

economy by choosing $\sigma_Z = 8$. As in the baseline calibration, we use $\mu_Z = 18$ for Z firms. The equilibrium allocation effects, reported in the lower panel of Table D1, are consistent with those of the baseline experiment: they are dampened in the "low volatility" and amplified in the "high volatility" economy. We see a similar impact on the welfare effects reported in the lower panel of Table D2. Overall, the main results seem robust to changing μ_Z and σ_Z, partly because these changes imply recalibrating ϕ, κ, K, b, and our calibration targets for those parameters put some discipline on the responsiveness of job creation and magnitude of the welfare effects.

The job-filling rate. In the first step of the calibration, we use the mid-point value of the U.K. job-filling rates estimated by Kuhn et al. [2021] to pin down values for matching efficiency M and labor market tightness θ. We investigate the sensitivity of the results to using respectively a lower and higher target value bracketing the job-filling rate. We first recalibrate the model under the assumption that $\lambda\left(\theta\right)/\theta$ is lower by 25 percent. This yields matching efficiency M equal to 0.106 and a value of market tightness θ of 0.322 vs. $M = 0.128$ and $\theta = 0.242$ in the baseline calibration. The equilibrium and welfare effects of a ban on ZHCs under this calibration are reported in the upper panel of Table D3. The main difference with the baseline results is that job creation is less responsive to the policy reform, which in turn leads to smaller unemployment/employment effects. As a result, the welfare losses, shown the upper panel of Table D3, are smaller than under the baseline calibration. Then, we analyze

Table D3: Robustness check: Equilibrium effects of a ban on Z contracts under alternative targets for the job-filling rate $\lambda\left(\theta\right)/\theta$

I. $\lambda\left(\theta\right)/\theta = 0.16$		
Firms $\rightarrow$	**Low volatility**	**High volatility**
Workers $\downarrow$ **Low disutility**	$e:\quad -0.40\,\text{p.p.}$ $\frac{u}{u+e}:\quad +0.40\,\text{p.p.}$ $\frac{v}{v+e}:\quad -0.97\,\text{p.p.}$ $\text{GDP}:\quad +1.65\%$	$e:\quad -0.51\,\text{p.p.}$ $\frac{u}{u+e}:\quad +0.51\,\text{p.p.}$ $\frac{v}{v+e}:\quad -0.98\,\text{p.p.}$ $\text{GDP}:\quad +1.75\%$
High disutility	$e:\quad -3.20\,\text{p.p.}$ $\frac{u}{u+e}:\quad +0.40\,\text{p.p.}$ $\frac{v}{v+e}:\quad -1.05\,\text{p.p.}$ $\text{GDP}:\quad -1.09\%$	$e:\quad -3.31\,\text{p.p.}$ $\frac{u}{u+e}:\quad +0.51\,\text{p.p.}$ $\frac{v}{v+e}:\quad -1.06\,\text{p.p.}$ $\text{GDP}:\quad -1.02\%$

II. $\lambda\left(\theta\right)/\theta = 0.26$		
Firms $\rightarrow$	**Low volatility**	**High volatility**
Workers $\downarrow$ **Low disutility**	$e:\quad -4.07\,\text{p.p.}$ $\frac{u}{u+e}:\quad +4.07\,\text{p.p.}$ $\frac{v}{v+e}:\quad -0.88\,\text{p.p.}$ $\text{GDP}:\quad -2.07\%$	$e:\quad -5.51\,\text{p.p.}$ $\frac{u}{u+e}:\quad +5.51\,\text{p.p.}$ $\frac{v}{v+e}:\quad -0.96\,\text{p.p.}$ $\text{GDP}:\quad +3.37\%$
High disutility	$e:\quad -6.76\,\text{p.p.}$ $\frac{u}{u+e}:\quad +4.07\,\text{p.p.}$ $\frac{v}{v+e}:\quad -0.92\,\text{p.p.}$ $\text{GDP}:\quad -5.11\%$	$e:\quad -8.16\,\text{p.p.}$ $\frac{u}{u+e}:\quad +5.51\,\text{p.p.}$ $\frac{v}{v+e}:\quad -1.00\,\text{p.p.}$ $\text{GDP}:\quad -6.37\%$

Notes: The table reports the equilibrium allocation effects of a ban on ZHCs under alternative targets for the job-filling rate $\lambda\left(\theta\right)/\theta$, namely a 25 percent lower (resp. higher) target value in the upper (resp. lower) panel of the table. e: employment; u: unemployment; v: vacancies; GDP: gross domestic product. "Low volatility" (resp. "High volatility") refers to the calibration where the volatility of hours in Z firms is $\sigma_Z = 3$ (resp. $\sigma_Z = 6$), implying that these are Z-best (resp. Z-only) firms. "Low disutility" (resp. "High disutility") refers to the calibration where the disutility of labor α_i of Z-only workers is such that these workers prefer to stay (resp. drop from the labor force) once Z employment is banned.

the implications of a 25-percent higher calibration target for $\lambda\left(\theta\right)/\theta$, which yields recalibrated values of $M = 0.148$ and $\theta = 0.194$. This substantially amplifies the response of job creation, as shown in the lower panel of Table D3. Unemployment increases by between 4.1 and 5.5 p.p., and as a result sectoral GDP falls by large amounts even in the "low disutility" economy where Z-only workers remain in the labor force after the ZHC ban. The welfare effects are displayed in lower panel of Table D4. Due to the stronger job creation effect, the welfare losses of R-best workers become much larger, between 1.0 and 1.3 percent of foregone consumption.

We note that the baseline results seem to compare favorably to these two scenarios. On the one hand, the targeted job-filling rate of 0.16 yields expected durations of vacancies of almost one semester, which seems rather long. On the other hand, the targeted job-filling rate of 0.26 generates a reduction of sectoral output by 5-6 percent following a ban on ZHCs, which seems large given that ZHCs account for 6.5 percent of baseline employment and would be partially replaced by regular employment contracts after the policy reform.

Table D4: Robustness check: Welfare effects of a ban on Z contracts under alternative targets for the job-filling rate $\lambda\left(\theta\right)/\theta$

	I. $\lambda\left(\theta\right)/\theta = 0.16$			
Firms $\rightarrow$	**Low volatility**		**High volatility**	
R-best workers	GE:	-0.11%	GE:	-0.11%
	PE:	$+0.15\%$	PE:	$+0.18\%$
Z-only workers	GE:	-1.95%	GE:	-1.62%
	PE:	-3.27%	PE:	-2.94%

	II. $\lambda\left(\theta\right)/\theta = 0.26$			
Firms $\rightarrow$	**Low volatility**		**High volatility**	
R-best workers	GE:	-0.99%	GE:	-1.30%
	PE:	$+0.15\%$	PE:	$+0.18\%$
Z-only workers	GE:	-2.01%	GE:	-1.69%
	PE:	-3.26%	PE:	-2.94%

Notes: The table reports the welfare effects of a ban on ZHCs under alternative targets for the job-filling rate $\lambda\left(\theta\right)/\theta$, namely a 25 percent lower (resp. higher) target value in the upper (resp. lower) panel of the table. GE: general equilibrium, which is the full effect of the reform; PE: partial equilibrium, which is the effect of keeping the equilibrium allocation unchanged and substitute R (resp. N) with Z employment for R-best workers (resp. Z-only workers). "Low volatility" (resp. "High volatility") refers to the calibration where the volatility of hours in Z firms is $\sigma_Z = 3$ (resp. $\sigma_Z = 6$), implying that these are Z-best (resp. Z-only) firms. All effects are computed in the context of the "Low disutility" economy, where the disutility of labor α_i of Z-only workers is such that these workers remain in the labor force once Z employment is banned. Welfare effects are in percent of consumption equivalent variation units.