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DISCUSSION PAPER SERIES

IZA DP No. 12441

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ABSTRACT

Revisiting the Hypothesis of High Discounts and High Unemployment

We revisit the hypothesis that labor market fluctuations are driven by shocks to the discount rate. Using a model in which the UE and the EU rates are endogenous, we show that an increase in the discount rate leads to a decline in both the UE and the EU rates. In the data, though, the UE and EU rates move against each other at business cycle frequency. Using a lifecycle model with human capital accumulation on the job, we show that an increase in the discount rate does indeed lead to a decline in the aggregate UE rate and to an increase in the aggregate EU rate. However, the decline in the UE rate is larger for younger workers than for older workers and the EU rate increases only for younger workers. In the data, fluctuations in the UE and EU rates at the business cycle frequency are nearly identical across age groups.

JEL Classification: E24, J63, J64

Keywords: unemployment fluctuations, discount rate, human capital, lifecycle earnings

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1 Introduction

In a standard search-theoretic model of the labor market (see, e.g., Mortensen and Pissarides 1994), agents make two decisions: firms decide whether to open a vacancy in order to hire an additional worker, and firm-worker pairs decide whether to break up their match in order to let the worker seek employment elsewhere. The firms' decision to open a vacancy trades off the upfront cost of advertisement and recruiting with the delayed benefit of employing an additional worker. The firm-worker pairs' decision to break up trades off the upfront cost of foregone wages and profits with the delayed benefit of higher wages once the worker finds better employment. Both decisions are investments, in the sense that they trade off an upfront cost with a delayed benefit.

As all investments do, the firms' decision to open a vacancy and the firm-worker pairs' decision to break up are sensitive to the rate at which agents discount future payoffs relative to current payoffs. Specifically, if the rate at which agents discount the future increases, firms will open fewer vacancies and firm-worker pairs will become more reluctant to break up. The decline in the number of open vacancies will tend to lower the tightness of the labor market and, in turn, the rate at which unemployed workers become employed (the UE rate). The tightening of the conditions under which firm-worker pairs break up will tend to lower the rate at which employed workers become unemployed (the EU rate).

We formalize the argument above and examine its implications for the hypothesis, put forward by Hall (2017), that the cyclical fluctuations in unemployment might be caused by fluctuations in the rate at which agents discount the future. We formalize the argument using a model of directed search on the job in the spirit of Menzio and Shi (2011). We use a model in which workers can search off and on the job, albeit with different intensity, because the firm-worker pairs decision to break up depends on how much faster the worker can find a better employment opportunity when he is searching off rather than on the job. We use a model in which the search process of workers is directed by the wages offered by firms because, while models of random search on the job are notoriously hard to solve in the presence of aggregate shocks, models of directed search on the job are as easy to solve as a representative agent model.

In the first part of the paper, we characterize the effect of a discount rate shock analytically. We find that an increase in the discount rate lowers the tightness of the submarket visited by unemployed workers, i.e. it lowers the ratio between the number of vacancies that offer the wage demanded by unemployed workers and the number of unemployed workers. For this reason, an increase in the discount rate lowers the UE rate. We find a necessary and sufficient condition under which an increase in the discount rate lowers the reservation quality of a firm-worker pair, i.e. it lowers the productivity level below which a firm-worker pair finds it optimal to break up. When the condition is satisfied, an increase in the discount rate lowers the EU rate. The condition under which higher discounting lowers the reservation quality is simple: the rate at which a worker can find a new match when searching off rather than on the job must exceed the rate at which the quality of a given match changes.

In the second part of the paper, we characterize the effect of a discount rate shock quantitatively. We calibrate the model to the US labor market. In particular, we ask the model to match the rate at which unemployed workers move to employment (UE rate), the rate at which employed workers move to unemployment (EU rate), and the rate at which workers move from one employer to another (EE rate). We also ask the model to match the relationship between the EU and EE rates and the length of a worker's tenure in his current job. The calibration reveals that the condition under which an increase in the discount rate lowers the reservation quality is satisfied. Using the calibrated model, we measure the effect of an increase in the discount rate from 4 to 10%. We find that the UE rate falls by 5%; the EU rate falls by 5% on impact and then keeps falling to a level that is 10% lower than before the shock; the EE rate falls by about 10%. Since the decline in the EU rate eventually dominates the decline in the UE rate, we find that the increase in the discount rate actually lowers unemployment.

These findings highlight a problem with the hypothesis of labor market fluctuations driven by discount rate shocks. In response to a discount rate shock, the UE and EU rates move in the same direction—as they are both manifestations of investment decisions—and, hence, they have offsetting effects on unemployment. Empirically, the UE and EU rates move in opposite directions at the business cycle frequency and, hence, they both contribute to unemployment fluctuations. The hypothesis of labor market fluctuations driven by aggregate productivity shocks does not suffer from this problem. Using the same model, we find that, in response to a 5% decline in aggregate productivity, the UE rate falls by 10%, the EU rate increases by 20%, and unemployment rises by 30%. Intuitively, the UE and EU rate move in opposite directions because a negative productivity shock lowers the firms' benefit from employing an additional worker and lowers the firm-worker pairs' cost of breaking up.

In the last part of the paper, we measure the effect of a discount rate shock using a version of the model in which workers accumulate human capital on the job (which is the model of Menzio, Telyukova and Visschers, 2016). The exercise is inspired by Kehoe, Midrigan and Pastorino (2017) and is motivated by two observations. If workers accumulate human capital on the job, the firm's benefit of hiring an unemployed worker becomes more backloaded. When this is the case, an increase in the discount rate will cause a larger decline in the UE rate. More importantly, if workers accumulate human capital on the job, the firm-worker pair's cost of breaking up also becomes more backloaded. And if the rate of human capital accumulation is high enough, the firm-worker pairs' decision to stay together—rather than the decision to break up—becomes an investment. When this is the case, an increase in the discount rate will raise the reservation quality and, in turn, the EU rate.

We calibrate the process of human capital accumulation on the job to match the lifecycle profile of wages. The calibration reveals that human capital grows very quickly with the first few years of experience, and then slows down sharply. Using the calibrated model, we find that, in response to an increase in the discount rate from 4 to 10%, the aggregate UE rate falls by about 20%, the aggregate EU rate increases by about 15%, and unemployment rises by almost 30%. We also find that the response of labor market outcomes varies dramatically by age. The decline in the UE rate is 4 times larger for workers aged 21-30 than for workers aged 41-50 and 51-60. The EU rate increases by 40% for workers aged 21-30, it is constant for workers aged 41-50, and it slightly declines for workers aged 51-60. The heterogeneity in the response of labor market outcomes by age is a consequence of the fact that the rate of human capital accumulation on the job is very high for young workers but negligible for older workers.

Accounting for human capital accumulation on the job appears to realign—both in terms of magnitudes and in terms of comovement patterns—the response of labor market variables to an increase in the discount rate with the behavior of the US economy in a typical recession. This is, however, only true at the aggregate level. In the data, the increase in the UE rate during a recession is nearly identical for workers in all age groups. In response to an increase in the discount rate, the UE rate increases much more for younger than for older workers. In the data, the EU rate increases in a recession for workers in all age groups and by nearly the same proportion. In response to an increase in the discount rate, the EU rate increases for younger workers but nor for older ones. In contrast, we find that the response of labor market outcomes to an aggregate productivity shock is homogeneous across age groups. The main shortcoming of the hypothesis of labor market fluctuations driven by productivity shocks is that, since 1984, the correlation between labor productivity and unemployment is effectively zero (see, e.g., Gali and van Rens, 2019).

The contribution of the paper is to use a richer search-theoretic model of the labor market to revisit the hypothesis put forward by Hall (2017) that cyclical movements in the labor market and cyclical fluctuations in stock market prices are intimately related phenomena, in the sense that movements in discount rates that rationalize the latter also rationalize the former. Hall (2017) shows that discount rate shocks can rationalize unemployment fluctuations using the basic search-theoretic model of Pissarides (1985). In Pissarides (1985), the only choice is the firms' decision of whether to open a vacancy and, hence, the only endogenous labor market outcome is the UE rate. Here, we use a model in which not only the UE rate, but also the EU and EE rates are endogenous because different firm-worker matches have different quality. The model belongs to a large class of models that have been successfully used to explain the pattern of transitions of workers between employment states as well as their wages gains (see, e.g., Burdett and Mortensen 1998, Postel-Vinay and Robin 2002). Recently, these models have been fruitfully used to study aggregate labor market fluctuations (see, e.g., Menzio and Shi 2011, Moscarini and Postel-Vinay 2013, Lise and Robin 2017). Kehoe, Midrigan and Pastorino (2017) consider a search-theoretic model with human capital accumulation and financial frictions. They show that a tightening of the financial frictions leads to an increase in the effective discount rate and, because of human capital accumulation, to large fluctuations in the UE rate and unemployment. Our model is richer because it incorporates the lifecycle of workers. Using a richer model allows us to generate additional implications of discount rate shocks. The implications that are counterfactual may be useful as a testing ground for future research on discount rate shocks (see, e.g., Borovicka and Borovickova 2018). The implications that are different from those of productivity shocks may be useful to separate different sources of labor market volatility.

2 Theory

In this section, we present a directed search model of the labor market in the spirit of Menzio and Shi (2011). In the model, the UE, EU and EE rates are endogenous. The UE and EE rates are determined by the interaction between the firms' decision on how many vacancies offering different wages to open and by the unemployed and employed workers' decision on what wages to seek. The EU rate is determined by the firm-worker pairs' decision on whether to break up their match to allow the worker to find a better match more quickly. We prove that an increase in the discount rate lowers the UE rate. We also prove that–iff the difference between the rate at which a worker can find a new match off the job rather than on the job exceeds the rate at which the quality of a given match changes–an increase in the discount rate lowers the EU rate.

2.1 Environment

The economy is populated by a measure 1 of workers. Each worker maximizes the expected present value of income, discounted at the rate r_t. When unemployed, a worker receives a flow income of b units of output, where b is a combination of unemployment benefits and leisure (converted into equivalent units of output). When employed, a worker receives a flow income of w_t units of output, where w_t is the worker's wage.

The economy is also populated by a positive measure of firms. Each firm maximizes the expected present value of profits discounted at the rate r_t. Each firm operates a constant return to scale production technology that turns the labor input of one worker into a flow of zy_t units of output, where y_t is a component of productivity that is common to all firm-worker matches and z is a component of productivity that is idiosyncratic to a particular firm-worker match. We refer to z as the *quality of the match.*

The labor market is subject to search frictions. Unemployed workers search for jobs with an intensity normalized to 1. Employed workers search for better jobs with an intensity $\chi \geq 0$. Firms search for workers by maintaining vacancies at the unit flow cost $k > 0$. The search process is directed by the terms of trade offered by firms to workers. Specifically, firms choose the lifetime utility x that is offered to a worker hired to fill a particular vacancy. Workers choose the lifetime utility x of the vacancies they seek to fill. Firms and workers take as given the vacancy-to-applicant ratio $\theta(x)$ associated with different terms of trade. We refer to $\theta(x)$ as the *tightness* of submarket x. The tightness $\theta(x)$ matters to firms and workers because it affects their meeting rates. A worker meets a vacancy in submarket x at the rate $p(\theta(x))$, where $p(\cdot)$ is a strictly increasing, strictly concave function of θ such that $p(0) = 0$. A vacancy meets a worker in submarket x at the rate $q(\theta(x))$, where $q(\theta) = p(\theta)/\theta$ and $q(\cdot)$ is a strictly decreasing function of θ.

When a worker and a vacancy meet in submarket x, the firm owning the vacancy offers the worker a contract worth x in lifetime utility. If the worker rejects the offer, the firm retains its vacancy and the worker returns to his prior employment status (either unemployment or employment at some other firm). If the worker accepts the offer, the firm and the worker start production. The firm and the worker are not immediately aware of the quality z of their match. They learn the quality of their match at the rate $\phi > 0$. The quality of the match is drawn from a twice differentiable cumulative distribution function $F(z)$ with mean $\overline{z}$ normalized to 1. The quality of the match changes at the rate $\eta \geq 0$, in which case the firm and the worker have to discover it again. The match becomes unviable for exogenous reasons at the rate $\delta \geq 0$, in which case the firm and the worker break up.

We assume that the contracts offered by firms to workers are bilaterally efficient, in the sense that they maximize the joint value of the firm-worker match. As explained in Menzio and Shi (2011), the assumption is consistent with several contractual environments. Consider two extreme cases. In the first case, a contract can specify the worker's wage, the worker's search strategy on the job (i.e. in which submarket to search) and the worker's quitting strategy (i.e. when to move into unemployment) as functions of the history of the match and the economy. In this case, the contract space is rich enough to independently control the allocative decisions of the match and the distribution of the value of the match. Clearly, the firm finds it optimal to offer a contract such that the allocative decisions maximize the joint value of the match and such that the wages provide the worker with the lifetime utility x. In the second case, a contract can specify the worker's wage as a function of the history of the match and a one-time transfer. Given the contract, the worker is then free to follow his preferred search and quitting strategy. In this case, the firm finds it optimal to offer a contract such that the worker is the residual claimant of output (and, hence, makes allocative decisions to maximize the joint value of the match) and a (negative) transfer such that the worker's lifetime utility is x.

The economy is subject to shocks to two fundamentals: the discount factor r and the aggregate component of productivity y. Specifically, at a given point in time, the fundamentals $\omega = \{r, y\}$ belong to the discrete set $\Omega = \{\omega_1, \omega_2, ...\omega_N\}$. The fundamentals change at the rate $\lambda \geq 0$ and, conditional on changing, they take on the values $\hat{\omega} \in \Omega$ with probability $\Pi(\hat{\omega}|\omega)$.

2.2 Block Recursive Equilibrium

At date t, the state of the economy is summarized by $\psi = \{\omega, u, n, G\}$, where ω is the state of the fundamentals, u is the measure of workers who are unemployed, n is the measure of workers employed in a match of unknown quality, and $G(z)$ is the measure of employed workers in a match of quality non-greater than z. Clearly, $u+n+G(\infty) = 1$. In principle, value and policy functions might depend on both the exogenous fundamentals of the economy and the endogenous distribution of workers across employment states. However, as shown in Menzio and Shi (2011), the unique equilibrium is block recursive, in the sense that the value and policy functions depend only on the state ω of the fundamentals.

Let $U(\omega)$ denote the value of unemployment to a worker. Let $V_0(\omega)$ denote the joint value to a worker and a firm of a match of unknown quality. Let $V(z, \omega)$ denote the joint value to a worker and a firm of a match of known quality z. Finally, let $\theta(x, \omega)$ denote the tightness of submarket x.

The value $U(\omega)$ of unemployment to a worker is such that

$$rU(\omega) = b + \max_x \left[p(\theta(x,\omega))\,(x - U(\omega))\right] + \lambda\mathbb{E}[U(\hat{\omega}) - U(\omega)]. \tag{2.1}$$

The flow income of an unemployed worker is b. At rate $p(\theta(x,\omega))$, the worker meets a vacancy in submarket x. In this case, the worker's continuation value is x. Since search is directed, the worker chooses x so as to maximize the product of the meeting rate, $p(\theta(x,\omega))$, and the net gain from meeting a vacancy, $x - U(\omega)$. At rate λ, the fundamentals of the economy change. In this case, the worker's continuation value is $U(\hat{\omega})$.

The value $V(z,\omega)$ to a worker and a firm of a match of quality z is such that

$$\begin{aligned} rV(z,\omega) \quad &= zy + \chi \max_x \left[p(\theta(x,\omega))\,(x - V(z,\omega))\right] + \eta\,(V_0(\omega) - V(z,\omega)) \\ &\quad +\delta\left[U(\omega) - V(z,\omega)\right] + \lambda\mathbb{E}[\max\{V(z,\hat{\omega}), U(\hat{\omega})\} - V(z,\omega)]. \end{aligned} \tag{2.2}$$

The flow income of a firm-worker match of quality z is zy. At rate $\chi p(\theta(x,\omega))$, the worker meets a vacancy in submarket x. In this case, the continuation value to the worker is x, the continuation value to the firm is 0, and their joint continuation value is x. Since search is directed and contracts are bilaterally efficient, x is chosen so as to maximize the product of the rate at which the worker meets a new vacancy, $\chi p(\theta(x,\omega))$, and the net gain to the worker and the firm from such meeting, $x - V(z,\omega)$. At rate η, the quality of the firm-worker match changes. In this case, the joint continuation value is $V_0(\omega)$. At rate δ, the firm-worker match is destroyed for exogenous reasons. In this case, the joint continuation value is $U(\omega)$. At rate λ, the fundamentals of the economy change. If the firm and the worker separate, their joint continuation value is $U(\hat{\omega})$. Otherwise, it is $V(z,\hat{\omega})$. Since contracts are bilaterally efficient, the firm and the worker separate iff $U(\hat{\omega}) > V(z,\hat{\omega})$.

The joint value $V_0(\omega)$ of a match of unknown quality is such that

$$\begin{aligned} rV_0(\omega) \quad &= y + \chi \max_x \left[p(\theta(x,\omega))\,(x - V_0(\omega))\right] + \phi\mathbb{E}\left[\max\{V(z,\omega), U(\omega)\} - V_0(\omega)\right] \\ &\quad +\delta\left[U(\omega) - V_0(\omega)\right] + \lambda\mathbb{E}[\max\{V_0(\hat{\omega}), U(\hat{\omega})\} - V_0(\omega)]. \end{aligned} \tag{2.3}$$

The flow income of a firm-worker match of unknown quality is $\bar{z}y = y$. At rate $\chi p(\theta(x,\omega))$, the worker meets a vacancy in submarket x. In this case, the joint continuation value to the worker and the firm is x. At rate ϕ, the worker and the firm learn the quality z of their match. If the firm and the worker decide to separate after learning z, their joint continuation value is $U(\omega)$. Otherwise, it is $V(z,\omega)$. Since contracts are bilaterally efficient, the firm and the worker separate iff $U(\omega) > V(z,\omega)$. The last two terms in (2.3) are analogous to the last two terms in (2.2).

The tightness $\theta(x,\omega)$ of submarket x is such that

$$k \geq q(\theta(x,\omega))\,(V_0(\omega)-x)\,, \tag{2.4}$$

and $\theta(x,\omega) \geq 0$, where the two inequalities hold with complementary slackness. The left-hand side of (2.4) is the cost to a firm from maintaining a vacancy in submarket x. The right-hand side is the benefit to a firm from maintaining a vacancy in submarket x, which is given by product between the rate at which the firm fills its vacancy, $q(\theta(x,\omega))$, and the value to the firm from filling its vacancy, $V_0(\omega)-x$. Condition (2.4) states that, if some vacancies are maintained in submarket x and, hence, $\theta(x,\omega)$ is strictly positive, then the cost of a vacancy must equal the benefit. If no vacancies are maintained in submarket x, the cost of a vacancy must exceed the benefit.

The search problems in (2.1), (2.2) and (2.3) have a common structure and can be analyzed together. Consider the search problem for a worker in an employment state with some arbitrary value v (where v is U if the worker is unemployed and V if he is employed). The search problem of the worker can be written as

$$D(v,\omega) = \max_{x,\theta} p(\theta)\,(x-v)\,, \text{ s.t. } \theta = \theta(x,\omega). \tag{2.5}$$

In words, the worker chooses the lifetime utility x offered by the vacancy and the tightness θ of the submarket in which the vacancy is located so as to maximize $p(\theta)\,(x-v)$ taking as given the constraint $\theta = \theta(x,\omega)$. For any $\theta > 0$, (2.4) implies that the value x offered by the vacancy is $V_0(\omega)-k/q(\theta)$ or, equivalently, $V_0(\omega)-k\theta/p(\theta)$. For $\theta = 0$, the worker's meeting rate is 0 and so is the value of searching, irrespective of x. Hence, (2.5) is equivalent to

$$D(v,\omega) = \max_{\theta} -k\theta + p(\theta)\,(V_0(\omega)-v)\,. \tag{2.6}$$

The formulation (2.6) makes it clear that, when search is directed, the worker internalizes both the firm's cost of maintaining vacancies and the firm's benefit from filling its vacancies.

Using (2.6), we can characterize the solution to the search problem of a worker. When unemployed, the worker searches in a submarket with tightness $\theta_u(\omega)$, where $\theta_u(\omega)$ satisfies the optimality condition

$$k \geq p'(\theta_u(\omega))\,(V_0(\omega)-U(\omega))\,, \tag{2.7}$$

and $\theta_u(\omega) \geq 0$, where the two inequalities hold with complementary slackness. Condition (2.7) is intuitive. When unemployed, the worker searches in a submarket with a tightness such that the cost to the firm from maintaining an additional vacancy is equal to the joint benefit to the worker and the firm from increasing their meeting rate.

Similarly, when employed in a match of quality z, the worker searches in a submarket with tightness $\theta_e(z,\omega)$, where $\theta_e(z,\omega)$ satisfies optimality condition

$$k \geq p'(\theta_e(z,\omega))\left(V_0(\omega) - V(z,\omega)\right), \tag{2.8}$$

and $\theta_e(z,\omega) \geq 0$, where the two inequalities hold with complementary slackness. Condition (2.8) implies that workers employed in matches of quality z such that $V(z,\omega) \geq V_0(\omega)$ search in submarkets with zero tightness, as they have nothing to gain from moving from their current match to a new one. For the same reason, workers employed in matches of unknown quality search in submarkets with zero tightness.

Lastly, let us consider the problem of a firm and a worker that discover the quality of their match and need to decide whether to break up or stay together. We define the *reservation quality* $R(\omega)$ as the quality that makes the firm-worker pair indifferent between breaking up and staying together, i.e.

$$V(R(\omega),\omega) = U(\omega). \tag{2.9}$$

As the joint value of a match $V(z,\omega)$ is strictly increasing in its quality z, it follows that the firm and the worker break up if they discover that z is smaller than the reservation quality $R(\omega)$, and they stay together if they discover that z is greater than $R(\omega)$.

It is useful to define the surplus $S(z,\omega)$ of a firm-worker match of quality z as the difference between the joint value $V(z,\omega)$ of the match to the firm and the worker and the value $U(\omega)$ of unemployment to the worker. Similarly, we define the surplus $S_0(\omega)$ of a firm-worker match of unknown quality as $V_0(\omega) - U(\omega)$. Subtracting (2.1) from (2.2) and using (2.6), we obtain the following expression for $S(z,\omega)$:

$$\begin{aligned} & (r+\delta+\lambda_\omega)S(z,\omega) \\ = \;& zy - b + \chi\left[-k\theta_e(z,\omega) + p(\theta_e(z,\omega))\left(S_0(\omega) - S(z,\omega)\right)\right] \\ - \;& \left[-k\theta_u(\omega) + p(\theta_s(\omega))S_0(\omega)\right] + \eta\left(S_0(\omega) - S(z,\omega)\right) + \lambda\mathbb{E}\left[\max\{S(z,\hat{\omega}),0\}\right]. \end{aligned} \tag{2.10}$$

Subtracting (2.1) from (2.3) and using (2.6), we obtain the following expression for $S_0(\omega)$:

$$\begin{aligned} & (r+\delta+\lambda_\omega)S_0(\omega) \\ = \;& y - b - \left[-k\theta_u(\omega) + p(\theta_u(\omega))S_0(\omega)\right] \\ + \;& \phi\mathbb{E}\left[\max\{S(z,\omega),0\} - S_0(\omega)\right] + \lambda\mathbb{E}\left[\max\{S_0(\hat{\omega}),0\}\right] \end{aligned} \tag{2.11}$$

Using (2.10) and the fact that $S(R(\omega),\omega) = 0$, we obtain a more explicit expression for the reservation quality $R(\omega)$:

$$
\begin{aligned}
yR(\omega) - b \quad &= (1-\chi)\left[-k\theta_u(\omega) + p(\theta_u(\omega))S_0(\omega)\right] \\
&\quad -\eta S_0(\omega) - \lambda \mathbb{E}[\max\{S(R(\omega),\hat{\omega}),0\}].
\end{aligned} \tag{2.12}
$$

The difference between the flow income of a match with reservation quality $R(\omega)$ and the flow income of unemployment b is equal to the sum of three terms, which reflect the difference between the option value of unemployment and the option value of employment. The first term is the difference between the option value of searching for a new match if the worker is unemployed rather than employed. As we expect $\chi < 1$, this term represents the option value of unemployment. The second and the third terms are the option values of employment in a marginal match and, hence, enter the reservation quality equation with a minus sign. The second term is the option value of a change in the quality of the match. The third term is the option value of a change in fundamentals.

As the reader can easily see, the equilibrium conditions (2.10) and (2.11) for the value functions S_0 and S and the equilibrium conditions (2.7), (2.8) and (2.12) for the policy functions θ_u, θ_e and R can be solved independently of the evolution of the distribution of workers across employment states. The evolution of the distribution of workers across employment states can then be recovered by applying the policy functions. Specifically, during any interval of time in which the fundamentals do not change, the distribution of workers across employment states evolves according to the differential equations

$$
\mathring{u} = (1-u)\delta + n\phi F(R(\omega)) - up(\theta_u(\omega)), \tag{2.13}
$$

$$
\mathring{n} = up(\theta_u(\omega)) + \int (\eta + \chi p(\theta_e(z,\omega)))\, dG(z) - n\,(\delta+\phi)\,, \tag{2.14}
$$

$$
\mathring{G}(z) = n\phi\left[F(z) - F(R(\omega))\right] - \int_{\underline{z}}^{z} (\delta + \eta + \chi p(\theta_e(\hat{z},\omega)))\, dG(\hat{z}). \tag{2.15}
$$

The above expressions are easy to understand. For instance, the change $\mathring{u}$ in the measure of unemployed workers is the flow of employed workers who become unemployed net of the flow of unemployed workers who become employed. Similarly, (2.14) and (2.15) express the change $\mathring{n}$ in the measure of workers who are employed in a match of unknown quality and the change $\mathring{G}(z)$ in the measure of workers who are employed in a match of quality non-greater than z as differences between inflows and outflows.

At a time when the state of the fundamentals changes from ω to $\hat{\omega}$, there is a discontinuity in the distribution of workers across employment states. Letting $+$ denote measures immediately after the fundamental shock, we have

$$u(+) = u + [G(R(\hat{\omega})) - G(R(\omega))], \tag{2.16}$$

$$n(+) = n, \tag{2.17}$$

$$G(z+) = G(z) - G(R(\hat{\omega})) \text{ if } z > R(\hat{\omega}), 0 \text{ else.} \tag{2.18}$$

2.3 High discounts

We now derive analytically the effect of an increase in the discount rate on the equilibrium value and policy functions and, in turn, on labor market outcomes. To this aim, we consider an economy in which fundamental shocks are unexpected and permanent (in the sense that $\lambda = 0$) and derive the effect of marginal increase in the discount rate r.

We start by examining the effect of the discount rate on the value functions. Let $\partial S_0/\partial r$ denote the derivative of the surplus of a match of unknown quality with respect to r and $\partial S(z)/\partial r$ the derivative of the surplus of a match of quality z with respect to r. The derivative $\partial S_0/\partial r$ is such that

$$(r + \delta + \phi + p(\theta_u))\frac{\partial S_0}{\partial r} = S_0 + \phi \int_R \frac{\partial S(z)}{\partial r} dF(z), \tag{2.19}$$

where the expression above is obtained by differentiating (2.11) with respect to r and by making use of the optimality condition (2.7) for θ_u. The derivative $\partial S(z)/\partial r$ is such that

$$(r + \delta + \eta + \chi p(\theta_e(z)))\frac{\partial S(z)}{\partial r} = -S(z) - (p(\theta_u) - \chi p(\theta_e(z)) - \eta)\frac{\partial S_0}{\partial r}, \tag{2.20}$$

where the expression above is obtained by differentiating (2.10) with respect to r and by making use of the optimality condition (2.8) for $\theta_e(z)$. Substituting (2.20) into (2.19), one finds $\partial S_0/\partial r < 0$. That is, an increase in the discount rate lowers the surplus of a new match between a worker and a firm. This finding is intuitive. An increase in the discount rate reduces the present value of the difference between the stream of income generated by a firm and a worker in a new match and the stream of income generated by an unemployed worker.

Next, we examine the effect of the discount rate on the policy functions. The derivative $\partial \theta_u/\partial r$ of the tightness of the submarket where unemployed workers look for vacancies is such that

$$0 = p'(\theta_u)\frac{\partial S_0}{\partial r} + p''(\theta_u)\frac{\partial \theta_u}{\partial r}. \tag{2.21}$$

Since $p'(\cdot) > 0$, $p''(\cdot) < 0$ and $\partial S_0/\partial r < 0$, (2.21) implies $\partial \theta_u/\partial r < 0$. That is, an increase in the discount rate lowers the tightness of the submarket where unemployed workers look for vacancies. This effect is easy to understand. An increase in the discount rate reduces the net value of a new match between an unemployed worker and a firm and, for this reason, firms reduce the number of vacancies targeted at unemployed workers. This is the effect of high discounts highlighted in Hall (2017). In Hall (2017), this effect is illustrated in a random search model where workers and firms bargain ex-post over the division of the surplus. The same effect is at work here, in a directed search model where firms commit ex-ante to the division of the surplus.

The derivative $\partial R/\partial r$ of the reservation quality R is such that

$$y\frac{\partial R}{\partial r} = \left[(1-\chi)p(\theta_u) - \eta\right]\frac{\partial S_0}{\partial r}, \tag{2.22}$$

where the expression above makes use of the optimality condition (2.8) for θ_u. Since $\partial S_0/\partial r$, an increase in the discount rate lowers the reservation quality if $(1-\chi)p(\theta_u)-\eta > 0$ and raises the reservation quality if $(1-\chi)p(\theta_u) - \eta < 0$. This effect is also easy to understand. The reservation quality R depends on the difference between the option value of unemployment–which is the option value of searching for a new match if the worker is unemployed rather than employed–and the option value of employment at the reservation quality–which is the option value of a change in the quality of the match. An increase in the discount rate lowers both option values. If $(1-\chi)p(\theta_u) - \eta > 0$, the decline in the option value of unemployment dominates the decline in the option value of employment at the reservation quality and, hence, R falls. If $(1-\chi)p(\theta_u) - \eta < 0$, the opposite is true. This effect is novel, as Hall (2017) only considers a model in which all matches are equally productive.

Let us look in some detail at the condition

$$(1-\chi)p(\theta_u) - \eta > 0. \tag{2.23}$$

Condition (2.23) is effectively a condition under which the decision of breaking the match is an investment, in the sense that breaking the match involves future benefits and upfront costs. The condition is certainly satisfied if workers cannot search on the job ($\chi = 0$) and the quality of a firm-worker match is permanent ($\eta = 0$). More generally, the condition is laxer when the job-finding rate $p(\theta_u)$ for unemployed workers is higher, when the relative efficiency χ of search on the job is lower, and when the rate η at which the quality of a match changes is lower. Empirically, the condition is likely to be satisfied. Indeed, the job-finding rate for unemployed worker is about 25% per month, the job-finding rate for employed workers is around 2% per month—which suggests that χ is low—and the rate at which matches break-up is very low at long tenures—which suggests that η is low.

Later on, we will make these observations precise. For now, we will proceed under the assumption that condition (2.23) holds.

Lastly, we examine the effect of an increase in the discount rate on the UE and EU rates. To this aim, suppose that, at the time of the positive shock to the discount rate, the distribution of workers across employment states is at its stationary level $\{u^*, n^*, G^*\}$. From (2.13)-(2.15), it follows that

$$u^* = \frac{h^*_{eu}}{h^*_{eu} + h^*_{ue}}, \tag{2.24}$$

$$\frac{n^*}{1-u^*} = \frac{\delta + \eta + h^*_{ee}}{\delta + \eta + \phi(1 - F(R))}, \tag{2.25}$$

$$G^*(z) = \frac{n^*\phi}{\delta + \eta}\left[F(z) - F(R)\right] - \chi \int_R^z p(\theta_e(\hat{z}))dG^*(\hat{z}), \tag{2.26}$$

where h^*_{ue}, h^*_{eu} and h^*_{ee} denote the UE, EU and EE rates at the stationary distribution and are given by

$$h^*_{ue} = p(\theta_u), \tag{2.27}$$

$$h^*_{eu} = \delta + \frac{n^*}{1-u^*}\phi F(R), \tag{2.28}$$

$$h^*_{ee} = \frac{\chi}{1-u^*}\int p(\theta_e(z))dG^*(z). \tag{2.29}$$

At the time of the positive shock to the discount factor, there is no instantaneous change in the distribution of workers across employment states. The UE and EU rates, however, do change because of the instantaneous change in the policy functions. In particular, the change in the UE and EU rates relative to h^*_{ue} and h^*_{eu} is

$$\frac{\partial h_{ue}}{\partial r} = p'(\theta_u)\frac{\partial \theta_u}{\partial r} < 0, \tag{2.30}$$

$$\frac{\partial h_{eu}}{\partial r} = \frac{n^*}{1-u^*}\phi F(R)\frac{\partial R}{\partial r} < 0. \tag{2.31}$$

At the time of the discount factor shock, both the UE and the EU rate fall. Intuitively, an increase in the discount rate lowers the tightness facing unemployed workers and leads to a decline in the UE rate. Simultaneously, an increase in the discount rate lowers the reservation quality and leads to a decline in the EU rate. In the data, however, recessions start with a sharp increase in the EU rate coupled with a decline in the UE rate (see, e.g., Figure xx in Section 4).

Over time, the distribution of workers across employment states evolves according to the laws of motion (2.13)-(2.15). Once the distribution reaches its new steady-state, the change in the UE and EU rates relative to h_{ue}^* and h_{ee}^* is

$$\frac{\partial h_{ue}^*}{\partial r} = p'(\theta_u)\frac{\partial \theta_u}{\partial r}, \tag{2.32}$$

$$\frac{\partial h_{eu}^*}{\partial r} = \frac{\phi + (\delta+\eta)(\delta+\eta+h_{ee}^*)}{(\delta+\eta+\phi(1-F(R)))^2}\phi F'(R)\frac{\partial R}{\partial r} + \frac{\phi F(R)}{\delta+\eta+\phi(1-F(R))}\frac{\partial h_{ee}^*}{\partial r} \tag{2.33}$$

The derivative in (2.32) implies that the UE rate is lower at the new than at the old steady state. The derivative in (2.33) implies that the EU rate is lower at the new than at the old steady state, as long as the EE rate declines. Overall, a positive discount rate shock cannot possibly lead to a stationary equilibrium where the UE and EE rates are lower and the EU rate is higher. In the data, though, recessions feature precisely lower UE and EE rates, and a higher EU rate (see, e.g., Menzio and Shi, 2011).

We summarize the analysis of the baseline model in the following proposition.

Proposition 1. *(High Discounts) Consider an unanticipated and permanent positive shock to the discount factor. If* $(1-\chi)p(\theta_u)-\eta > 0$*, then:*

(i) On impact, the shock lowers the UE and EU rates;

(ii) In steady-state, the shock either lowers the UE and EU rates, or it lowers the UE rate and increases the EU and EE rates.

3 Calibration

In this section, we calibrate the model to reproduce some of the central features of the US labor market, such as the average UE, EU and EE rates and the negative relationship between EU and EE rates and the tenure of a worker at a particular job. Using the calibrated model, we measure the response of the labor market to a permanent increase in the discount rate. We find that the discount rate shock leads to a small decline in both the UE and EU rates and to an even smaller and transitory increase in the unemployment rate. These findings are problematic for the hypothesis of labor market fluctuations being driven by discount rate shocks. Indeed, in the US labor market, the UE and EU rates move against each other over the business cycle and their fluctuations are much larger than what implied by reasonable discount rate shocks.

3.1 Parameters and data

The model is described by a handful of fundamentals. The search process is described by the vacancy cost k, the flow income of unemployment b, the job-finding rate function $p(\theta)$, and the relative efficiency of search on the job χ. The production process is described by the aggregate component of productivity y, which we normalize to 1, the distribution F of the idiosyncratic component of productivity, and the rate δ at which a firm-worker match is exogenously destroyed. The learning process is described by the rate ϕ at which a firm and a worker learn the quality of their match and the rate η at which the quality is reset. We assume that $p(\theta)$ is of the form θ^{α}, where $\alpha \in (0,1)$ is the elasticity of the job-finding rate to the vacancy-to-applicant ratio. We assume that $F(z)$ is a Weibull distribution with shape τ, scale σ and a location parameter chosen so that the average of z is 1. The Weibull distribution encompasses distributions with declining density, hump-shaped density and a thick right tail, and hump-shaped densities and a thick left tail.

We calibrate the parameters using moments constructed from the US Census' Survey of Income and Program Participation (SIPP) for male workers with a high school degree and no further degree. We focus on male workers to minimize the discrepancy between the model—where workers are always in the labor force—and the data—where workers transition in and out of the labor force. We focus on high-school workers because they represent the largest as well as the median education group. We refer the reader to Menzio, Telyukova and Visschers (2016) for additional details on the data.

We calibrate k so that the steady-state UE rate in the model is the same as the average UE rate in the SIPP (25% per month). We calibrate χ so that the steady-state EE rate in the model is the same as in the SIPP (1.8% per month). The EU rate in the SIPP is 0.5% per month, which implies a steady-state unemployment rate of 2.5% in the model. This is because our model abstracts from lifecycle considerations and, in particular, from the fact that young workers typically enter the labor market as unemployed. In order to reconcile our model without lifecycle and the data, we target a EU rate to 1.2%, which implies a steady-state unemployment of 5%. We use this as the target to calibrate δ. The empirical EU and EE rates are corrected for time-aggregation bias.

We calibrate the parameters ϕ, η, τ and σ to fit the empirical profile of EU and EE transitions by tenure length. Formally, we minimize the distance between the model and the data with respect to: (i) the fraction of workers with $t = 1, 2, ...60$ months of tenure in their job who, in the next month, leave for unemployment (EU rate by tenure) or for another job (EE rate by tenure); (ii) the fraction of workers who before reaching a tenure of t months in their job have left for unemployment (cumulated EU flow) or for another job (cumulated EE flow); (iii) the fraction of workers who reach a tenure of t months. Intuitively, the EU rate at short tenures is informative about $\phi F(R)$ and the rate at which the EU rate declines with tenure is informative about ϕ. The EE rate at short tenures is informative about the shape of F for z's above R and the EE rate at long tenures is informative about the rate at which the quality of a match is reset. To make the EU rate by tenure consistent with the corrected average EU rate of 1.2%, we shift up the EU tenure profile by $1.2\% - .5\% = .7\%$. The empirical EU and EE profiles are corrected for time-aggregation bias.

Lastly, we need to choose values for the parameters α and b. We set the elasticity α of the job-finding rate function with respect to the vacancy-to-applicant ratio to 1/2, which is the value typically chosen in the literature. We set the flow unemployment income b to be equal to 70% of the average productivity of labor, which has now become the standard target thanks to Hall and Milgrom (2008). Our findings are qualitatively robust to changes in the value of these two parameters.

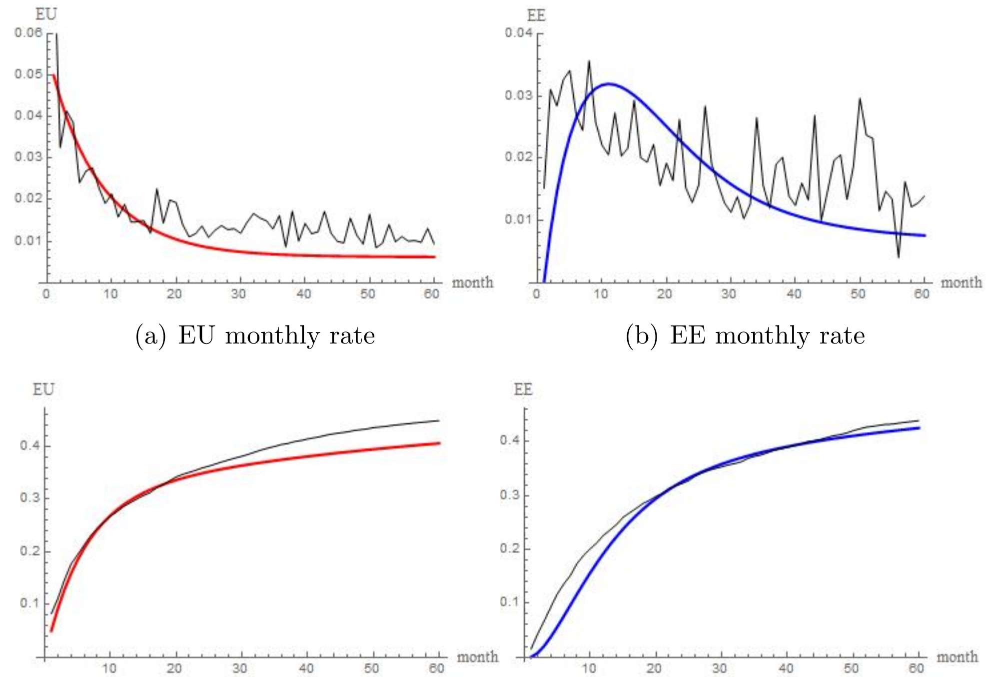
The calibrated F is a Weibull distribution with shape parameter $\tau = 1.5$ and scale parameter $\sigma = .41$. This is a distribution with a standard deviation of 25%, a skewness of 1.1, and a kurtosis of 4.3. The calibrated ϕ is 1.86, which implies that, on average, a firm and a worker learn about the quality of their match in 6 months. The calibrated η is 0.2, which implies that, on average, the quality of a match changes once every 5 years. The calibrated χ is 0.65, which means that searching on the job is 35% less effective as search off the job. The calibrated value of the other parameters are $k = .035$, $\delta = .012$, $b = .77$, $r = .04$.

Figure 1 plots the tenure profiles of the EU and EE monthly rates and the cumulated EU and EE flows by tenure. Even though the model is parsimonious, it fits quite well the tenure profiles. Intuitively, the calibrated shape and scale of the F distribution place a lot of mass to the left of the reservation quality and, thus, generate the high value of the EU rate at short tenures. The high value of ϕ generates the sharp decline of the EU rate profile (as well as the high value of the EE rate at short tenures). The shape of the F distribution generates the decline of the EE rate profile. The small value of η sustains a small but positive EE rate at long tenures.

As expected, condition (2.23) is satisfied at the calibrated parameter values. The job-finding rate for unemployed workers is $p(\theta_u) = .25 \times 12 = 3.1$ per year. The difference between the efficiency of search off and on the job is $1 - \chi = .35$. The rate at which the quality of a match is reset is $\eta = .2$ per year. Overall, $(1 - \chi)p(\theta_u) - \eta = .83 > 0$. Since condition (2.23) holds, Proposition 1 applies to the calibrated model.

3.2 Discount shock

Using the calibrated model, we compute the response of the labor market to a positive discount rate shock. We assume that the economy is at the steady state associated with



(a) EU monthly rate (b) EE monthly rate

(c) Cumulated EU flow (d) Cumulated EE flow

Notes: EU and EE tenure profiles for male workers with high school degree in SIPP (thin) and in the model (thick).

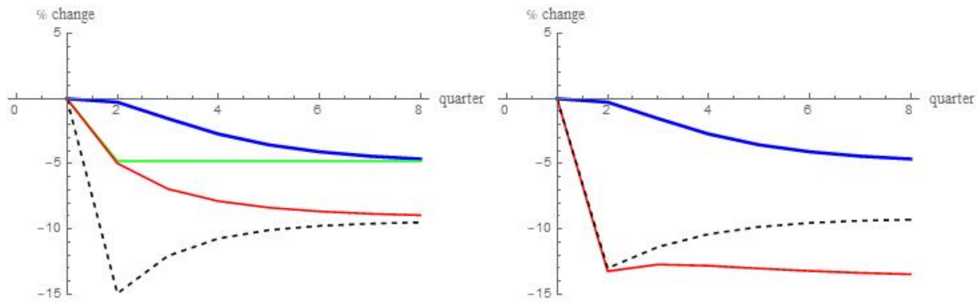
Figure 1: Transition Rates by Tenure

a discount rate r of 4% per year (the steady state at which the model is calibrated). We then hit the economy with an unanticipated and permanent increase in the discount rate from 4 to 10% per year. While the exact magnitude of the discount rate shock is somewhat arbitrary, it is similar to the change in the yearly discount rate required to rationalize the cyclical volatility of stock market prices (see Table 3 in Hall 2017).[1]

The left panel in Figure 2 shows the response of the UE, EU and EE rates to the discount rate shock. The UE rate falls by 5%. The EU rate falls by approximately 5% on impact. Over time, the EU rate keeps falling and reaches a level that is about 10% lower than before the shock. The EE rate falls by 15% on impact. Over time, the EE rate recovers and settles at a level that is about 10% lower than before the shock.

The findings in the left panel are easy to understand. The UE rate falls because, as pointed out by Hall (2017) and shown in Proposition 1, the increase in the discount rate lowers the net value of a new match between a firm and an unemployed worker and, thus,

[1] To be precise, Hall (2017) recovers from the stock market prices a vector of state-dependent marginal utilities for income which, together with the rate of transition across states, implies state-dependent discount rates. While the block recursivity of the model would have allowed to study the effect of state-dependent marginal utilities of income, we thought it would be more transparent to show the effect of a one-time unanticipated shock to the discount rate.



(a) u, UE, EU, EE rates (b) u, v and tightness

Notes: Percentage change relative to steady state for u (thick blue), UE rate (green), EU rate (red), EE rate (dashed black) v (purple), θ (dashed gray). Transition rates computed by comparing employment state at one-month intervals and then aggregated at quarterly level.

Figure 2: High Discounts

induces firms to reduce the number of vacancies targeted at unemployment workers. On impact, the EU rate falls because, as shown in Proposition 1, the increase in the discount rate lowers the option value of unemployment more than the option value of employment and, hence, induces firms and workers to lower their reservation quality. Over time, the EU rate keeps falling because—due to the decline in the EE rate—the fraction of workers employed in new matches of unknown quality keeps falling. On impact, the EE rate falls because the increase in the discount rate lowers the return from experimenting with new matches and, thus, induces firms to reduce the number of vacancies targeted at employed workers. Over time, the EE rate recovers as the composition of employed workers shifts towards matches of lower quality, which tend to have a higher EE rate.

The right panel in Figure 2 shows the response of unemployment, vacancies and aggregate market tightness. The unemployment rate is subject to two opposing forces. On the one hand, unemployment is pushed up by the decline in the UE rate. On the other hand, unemployment is pushed down by the decline in the EU rate. On impact, the two forces nearly cancel each other and unemployment barely changes. Over time, the second force becomes stronger and stronger and unemployment starts falling. Eventually, unemployment settles down to a level that is 5% *lower than before the shock*. The vacancy rate falls by about 15%. The aggregate tightness of the labor market, defined as the ratio of vacancies to unemployment, falls by 10%.

Even though the discount rate shock does not affect technology directly, it does lead to a decline in labor productivity. Intuitively, the increase in the discount rate shifts the distribution of employed workers towards matches of lower quality. First, the increase in the discount rate lowers the reservation quality and, for this reason, it lowers the rate at which workers leave low quality matches for unemployment. Second, the increase in the discount rate lowers the number of vacancies that firms target to employed workers and, for this reason, it lowers the rate at which workers leave low quality matches for other matches. Overall, labor productivity declines by .5%. Borrowing the language of Barlevy (2002), we say that an increase in the discount rate has a "sullying" effect on the economy.

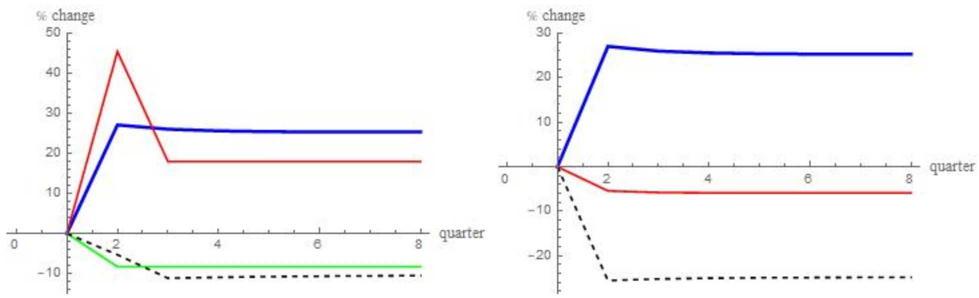
Clearly, the response of the labor market to an increase in the discount rate does not resemble the behavior of the US labor market in a typical recession. In a typical US recession, the UE rate falls by about 30%, the EU rate increases by about 20% and, as a result of both the movement in the UE and EU rates, the unemployment rate increases by around 50%. An increase in the discount rate, in contrast, leads to a decline in both the UE and the EU rate. The decline in the EU rate dominates the decline in the UE rate and the unemployment rate actually falls, albeit by only 5%.

Kehoe, Midrigan and Pastorino (2017) show that, by introducing human capital accumulation on the job, the response of the UE rate to a discount rate shock can be magnified. Intuitively, when workers accumulate human capital on the job, the payoffs to a match between a firm and an unemployed worker become more backloaded relative to the payoff to an unemployed worker and, for this reason, vacancy creation and the UE rate become more sensitive to changes in the discount rate. In Section 4, we embed human capital accumulation into our model and show that it not only amplifies the response of the UE rate to discount rate shocks, but it also switches the direction of the response of the EU rate.

3.3 Productivity shock

It is instructive to compare the responses of the labor market to an increase in the discount rate and to a negative productivity shock. We assume that the economy is at the steady state associated with an aggregate component of productivity y of 1. We then hit the economy with an unanticipated and permanent 5% decline to the aggregate component of labor productivity. While the exact magnitude of the productivity shock is arbitrary, it is approximately equal to a 2 standard deviations change.

The left panel in Figure 3 shows the response of the UE, EU and EE rates. The UE rate falls by 10%. The EU rate increases by more than 40% on impact and then falls to a level that is 20% higher than before the shock. The EE rate falls by a little more than 10%. As explained in detail in Menzio and Shi (2011), the UE rate falls because the decline in the aggregate component of productivity lowers the net value of a match between a firm and an unemployed worker and, thus, induces firms to target fewer vacancies to each unemployed worker. The response of the EU rate is also easy to understand. The decline in the aggregate component of productivity lowers the gap between the income



(a) u, UE, EU, EE rates (b) u, v and tightness

Notes: Percentage change relative to steady state for u (thick blue), UE rate (green), EU rate (red), EE rate (dashed black) v (purple), θ (dashed grey). Transition rates computed by comparing employment state at one-month intervals and then aggregated at quarterly level.

Figure 3: Low Productivity

generated by a firm-worker match and the income generated by an unemployed worker and, for this reason, the reservation quality rises. The EE rate falls because the decline in the aggregate component of productivity lowers, on average, the return to experimenting with new matches.

The right panel in Figure 3 shows the response of unemployment, vacancies and aggregate market tightness. Unemployment increases by 25%, as the result of both the decline in the UE rate and the rise in the EU rate. Vacancies fall by 5%, as the result of two opposing forces. On the one hand, firms target fewer vacancies per each unemployed worker as well as fewer vacancies per each employed worker. This tends to lower vacancies. On the other hand, the fraction of unemployed workers increases and firms target more vacancies per unemployed than per employed worker. This tends to increase vacancies. The net effect is a decline in vacancies.

We also find that labor productivity falls by 4.5%, less than the 5% shock in the aggregate component of productivity y. The decline in labor productivity is muted because, in response to the shock, firms and workers increase their reservation quality causing an increase in the average idiosyncratic component of productivity z. Borrowing the language of Mortensen and Pissarides (1994), we say that a negative shock to the aggregate component of productivity has a "cleansing" effect on the economy.

Overall, the response of the labor market to the negative productivity shock resembles qualitatively the behavior of the labor market in a typical US recession: the UE and EE rate fall and the EU rate increases, unemployment rises while vacancies fall. Moreover, as in the data, the fluctuations in labor market outcomes are much larger than the fluctuations in productivity. The response of the UE rate is 2.5 times larger than the change in productivity, the response of the EU rate is 4 times larger, the response of unemployment is 6 times larger. These findings confirm the results in Menzio and Shi (2011), even though the model considered here is richer and the calibration strategy uses more data.

The main shortcoming of the hypothesis that labor market fluctuations are driven by aggregate productivity shocks is not the lack of amplification (a point first made by Shimer 2005 and then prominently repeated by Hall 2017 and Sargent and Ljungqvist 2017), but the fact that, since 1984, the correlation between labor productivity and unemployement is effectively zero. Kaplan and Menzio (2016), Gali and van Rens (2018) and Golosov and Menzio (2018) propose alternative theories of recessions that display no correlation between labor productivity and unemployment and that, unlike the discount rate shock theory, do not generate counterfactual movements in the EU rate.

4 Human capital accumulation

In this section, we measure the effect of an increase in the discount rate using a calibrated version of the model in which workers accumulate human capital on the job. We find that a permanent increase in the discount rate leads to a large decline in the aggregate UE rate, a large increase in the aggregate EU rate, and to an even larger increase in aggregate unemployment. Thus, accounting for human capital accumulation makes the aggregate response of the labor market to the discount rate shock consistent with the aggregate behavior of the US labor market in recessions. We also find that the discount rate shock leads to fluctuations in labor market outcomes that differ across young and old workers: a decline in the UE rate for workers aged 21-31 that is four times larger than for workers aged 51-60, a large increase in the EU rate for workers aged 21-31 and a small decrease in the EU rate for workers aged 51-60. These disaggregated findings are problematic for the hypothesis that labor market fluctuations are driven by discount rate shocks since, in the data, the cyclical fluctuations of UE and EU rates are very similar across age groups.

4.1 Model and calibration

Menzio, Telyukova and Visschers (2016, henceforth MTV) contains a lifecycle version of our model in which workers accumulate human capital on the job. More specifically, workers enter the labor market when young and exit the labor market when old according to probabilistic functions $\mu(t)$ and $\nu(t)$ of the worker's age t. The labor market is organized in submarkets indexed by the value x offered by the vacancy to the worker and by the required human capital, h, and age, t, of the worker. Workers choose in which submarket to search and firms choose in which submarket to open vacancies, taking as given the equilibrium tightness function $\theta(x, h, t)$. When matched, a firm and a worker produce a flow of hzy units of output. The quality z of the match is initially unknown, it is discovered at the rate ϕ, and it is redrawn at the rate η. The wage of the worker is set as a constant fraction of the output flow, where the fraction obviously depends on the offered value x. The human capital of the worker depends on his months of work-experience e according to the function $h = g(e)$. We refer the reader to MTV for an exhaustive description of the environment, the equilibrium conditions and the welfare properties of the equilibrium.

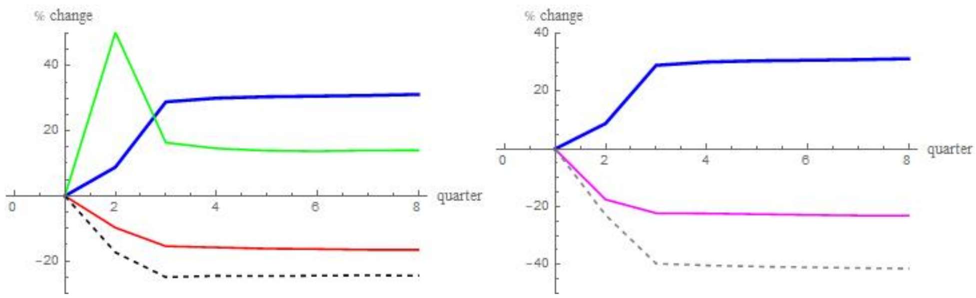
MTV calibrate the model using SIPP data on male workers with a high-school degree. As we did in Section 3, they calibrate the model to match the average UE, EU and EE rates and the cumulated EU and EE rates by workers' tenure at a given job. They calibrate the probability of entry into and exit from the labor market to match the fraction of workers of age t entering and exiting the labor market. They calibrate the human capital accumulation function, which they take to have the functional form $g(e) = (1 - \rho_1) + \rho_1(1 + e)^{\rho_2}$, to match the lifecycle profile of average wages.

Table 1 in MTV contains the calibrated value of the parameters. Here, it suffices to say that the calibrated human capital accumulation function ($\rho_1 = 4.3$ and $\rho_2 = .065$) displays steep decreasing returns to experience, in the sense that a worker's productivity nearly doubles with the first 2.5 years of experience and only increases by an additional 45% with the next 5 years of experience. Tables 2, 3 and 4 in MTV report the fit between the targeted moment and the data. Figures 12, 14 and 16 in MTV show that the calibrated model explains well the profile of the UE, EU and EE rates across different age groups. We are therefore comfortable in using their model to assess the impact of a discount rate shock for workers in different age groups.

4.2 Discount shock

Using the calibrated model, we compute the response of the labor market to a positive discount rate shock. As in Section 3, we assume that the economy is at the steady state associated with a discount rate of 4% per year. We then hit the economy with an unanticipated, permanent increase in the discount rate from 4 to 10%.

The left panel in Figure 4 shows the response of the aggregate UE, EU and EE rates to the discount rate shock. The aggregate UE rate falls by approximately 18%. The aggregate EU rate increases by almost 50% on impact and then settles down to a level that is 15% higher than before the shock. The aggregate EE rate falls by approximately 25%. The right panel in Figure 4 shows the response of aggregate unemployment, vacancies and tightness. The aggregate unemployment rate increases by almost 30% and the aggregate vacancy rate falls by about 20%.



(a) u, UE, EU, EE rates

(b) u, v and tightness

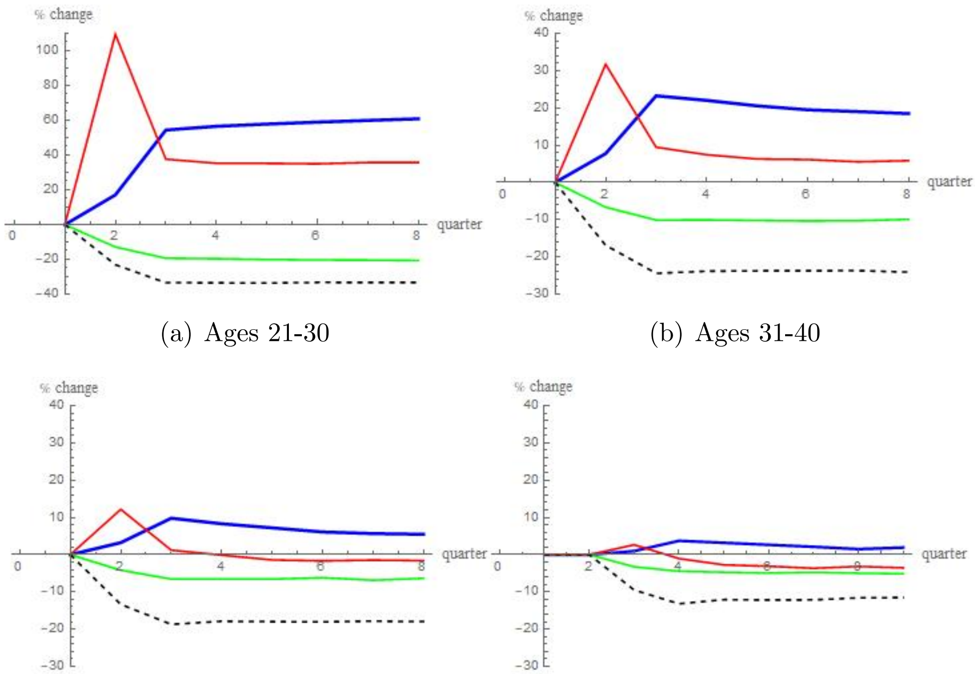
Notes: Percentage change relative to steady state for u (thick blue), UE rate (green), EU rate (red), EE rate (dashed black) v (purple), θ (dashed grey).

Figure 4: High Discounts

A comparison between Figures 2 and 4 makes it absolutely clear that the response of the labor market to a discount rate shock changes dramatically once we take into account the fact that workers accumulate human capital while employed. In response to the same discount rate shock, the aggregate UE rate declines by 18% rather than 5%. In response to the same discount rate shock, the aggregate EU rate increases by about 15% rather than falling by 10%. Overall, the unemployment rate increases by almost 30% rather than falling by 5%. Once we take into account human capital accumulation on the job, not only does the aggregate UE rate become more sensitive to discount rate shocks (a point already made by Kehoe, Midrigan and Pastorino 2017), but the aggregate EU rate responds with the opposite sign.

We now turn to examine the response to the UE, EU and EE rates at a more disaggregated level. Figure 5 reports the response of transition and unemployment for workers in the age groups 21-30, 31-40, 41-50 and 51-60. For the youngest group of workers, the UE rate falls by 20% and the EU rate nearly doubles on impact and then settles at a level that is 40% higher than before the shock. The unemployment rate for these workers rises by 60%. For workers in the age group 31-40, the UE rate falls by 10% and the EU rate increases by 30% on impact and then settles at a level that is 5% higher than before the shock. The unemployment rate for these workers increases by about 18%. For the workers in the age group 41-50, the UE rate falls by 6% and the EU rate increases by 10% on impact and then quickly falls back to about the same level as before the shock. The unemployment rate for these workers increases by about 5%. For the oldest group of workers, the UE rate falls by 5% and the EU rate *falls* by 2%. The unemployment rate for these workers increases by about 2%.

The disaggregated analysis highlights two novel implications of discount rate shocks. First, the increase in the discount rate generates much larger responses in the UE rate of



(a) Ages 21-30

(b) Ages 31-40

(c) Ages 41-50

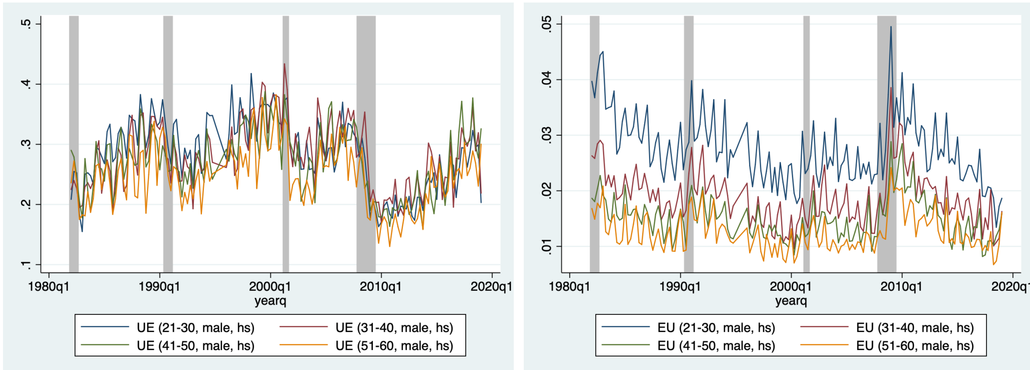
(d) Ages 51-60

Notes: Percentage change relative to steady state for u (thick blue), UE rate (green), EU rate (red), EE rate (dashed black) for different age groups.

Figure 5: High Discounts by Worker's Age

younger workers than in the UE rate of older workers. For workers aged 21-30, the UE rate falls by 20%. For workers aged 51-60, the UE rate falls by 5%. Second, the increase in the discount rate generates a positive response in the EU rate of younger workers and a negative response in the EU rate of older workers. For workers aged 21-30, the EU rate increases by 40%. For workers aged 51-60, the EU rate falls by 2%. As a result of these heterogeneous responses, the increase in the discount rate generates unemployment fluctuations that decline sharply with age.

The findings in Figures 4 and 5 are easy to understand. Consider the UE rate. The higher is the rate at which workers accumulate human capital on the job, the more back-loaded is the value of a match between a firm and an unemployed worker relative to the value of unemployment. Hence, the higher is the rate of human capital accumulation, the more sensitive is the UE rate to an increase in the discount rate. Next, consider the EU rate. When workers do not accumulate human capital on the job, the value of a marginal firm-worker match is frontloaded relative to the value of unemployment. When this is the case, breaking a marginal match is an investment and the reservation quality falls in response to an increase in the discount rate. When workers accumulate human capital on the job at a sufficiently high rate, the value of a marginal firm-worker match becomes



(a) UE monthly rate

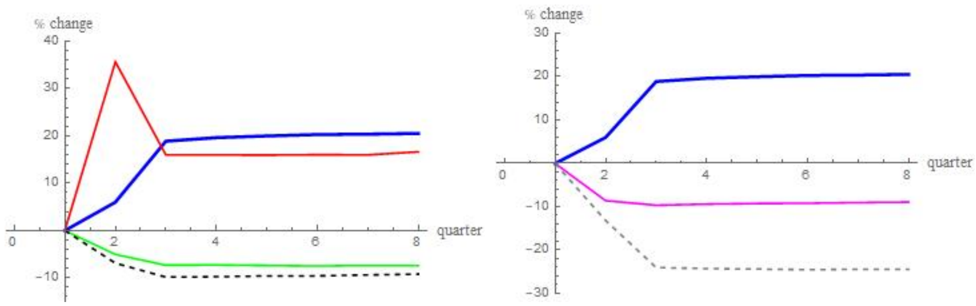
(b) EU monthly rate

Figure 6: UE and EU Rates by Worker's Age

backloaded relative to the value of unemployment. When this is the case, keeping a marginal match is an investment and the reservation quality rises in response to an increase in the discount rate.

At the aggregate level, the rate at which workers accumulate human capital is high enough for an increase in the discount rate to generate a large decline in the EU rate and a large increase in the EU rate. The rate at which workers accumulate human capital on the job, though, falls sharply over the lifecycle. For this reason, an increase in the discount rate generates a larger response in the UE rate of younger workers than in the UE rate of older workers, and a positive response in the EU rate of younger workers and a negative response in the EU rate of older workers.

The findings in Figure 4 seemingly vindicate the hypothesis of labor market fluctuations being driven by discount rate shocks. Indeed, as in a typical US recession, the response of the labor market to an increase in the discount rate features a sizeable decline in the aggregate UE rate, an equally sizeable increase in the aggregate EU rate, and an even larger increase in unemployment. The findings in Figure 5, though, highlight a new set of problems for the hypothesis. In a typical US recession, the decline in the UE rate is the same for younger and older workers (see the top panel of Figure 6). In response to an increase in the discount rate shock, the UE rate falls by 4 times more for workers aged 21-30 than for workers aged 41-60. In a typical US recession, the EU rate increases for workers of all ages, and in percentage terms by approximately the same amount. In response to an increase in the discount rate shock, the EU rate falls for workers aged 21-30 more than for workers aged 31-40, it does not change for workers aged 41-50, and it falls for workers aged 51-60. Similar counterfactual predictions of a discount rate shock could be derived by comparing the response of the UE and EU rates of worker with a high



(a) u, UE, EU, EE rates (b) u, v and tightness

Notes: Percentage change relative to steady state for u (thick blue), UE rate (green), EU rate (red), EE rate (dashed black) v (purple), θ (dashed grey).

Figure 7: Low Productivity

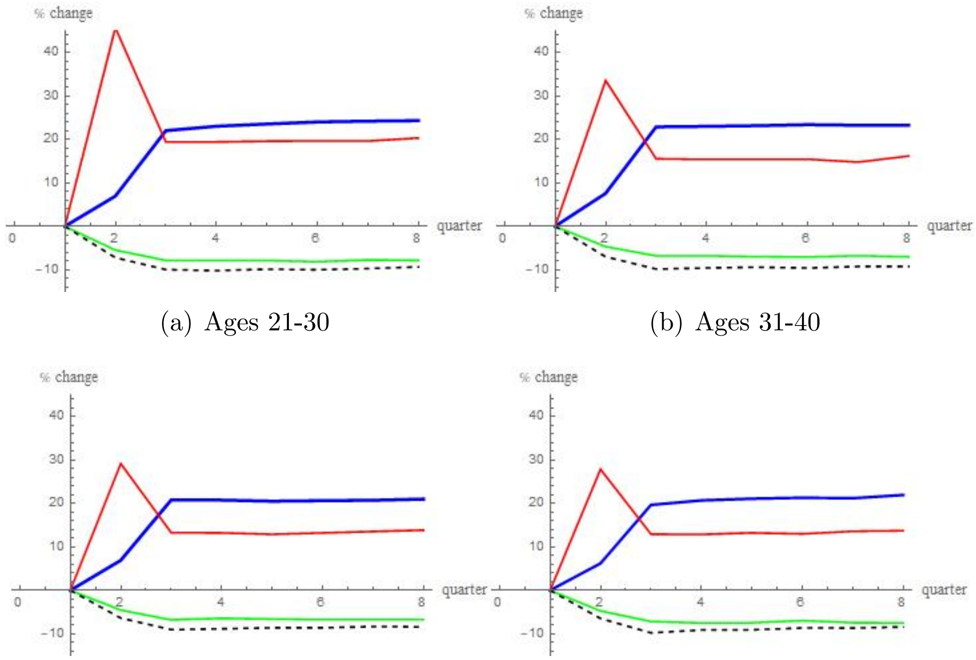
school and workers with a college degree, who have very different rates of accumulation of human capital but similar fluctuations in UE and EU rates.

4.3 Productivity shock

For the sake of comparison, we compute the response of the labor market to a negative shock to the aggregate component of productivity. As in Section 3, we assume that the economy is at the steady state associated with an aggregate component of productivity y equal to 1. We then hit the economy with an unanticipated and permanent negative 5% shock to y.

The left panel in Figure 7 shows the response of the aggregate UE, EU and EE rates to the productivity shock. The aggregate UE rate falls by about 7.5%; the aggregate EU rate increases by 35% on impact and then settles to a level that is 20% higher than before the shock; the aggregate EE rate falls by 10%. The right panel in Figure 7 shows the response of unemployment, vacancies and aggregate labor market tightness. The unemployment rate increases by about 20% and the vacancy rate falls by about 10%. Comparing Figure 7 and Figure 3, it is clear that the response to a productivity shock is very similar whether we account for human capital accumulation on the job or not.

Figure 8 shows the response of transition and unemployment rates for workers in different age groups. In contrast to a discount rate shock, the response of UE, EU, EE and unemployment rates to a productivity shock is very similar across workers in different age groups. Indeed, the decline in the UE rate is around 7.5% for workers aged 21-30, 31-40, 41-50 and 51-60. The EU rate increases for workers in all age groups, although the increase is slightly larger for younger workers than for older ones (as young workers are more likely to be in marginal matches than older ones).



(a) Ages 21-30 (b) Ages 31-40

(c) Ages 41-50 (d) Ages 51-60

Notes: Percentage change relative to steady state for u (thick blue), UE rate (green), EU rate (red), EE rate (dashed black) for different age groups.

Figure 8: Low Productivity by Worker's Age

Overall, the response of aggregate labor market outcomes to an increase in the discount rate is similar, in pattern and magnitude, to the response to a decline in the aggregate component of productivity. Both responses broadly resemble the behavior of the US labor market in a recession. However, the response to an increase in the discount rate features a great deal of heterogeneity in the response of labor market outcomes for workers of different age, while the response to a decline in the aggregate component of productivity does not. Along this disaggregated dimension, a negative shock to productivity performs better than a positive shock to the discount rate.

5 Conclusions

We revisited the hypothesis put forward by Hall (2017) that cyclical movements in the labor market and cyclical fluctuations in stock market prices are intimately related phenomena, in the sense that movements in discount rates that rationalize the latter also rationalize the former. We revisited the hypothesis using a directed search model in which the UE, EU and EE rates are all endogenous. We showed analytically that an increase in the discount rate lowers the UE rate and, under some rather natural conditions, also lowers the EU rate. We found quantitatively that, in response to an increase in the discount rate from 4 to 10%, the UE rate declines by 5%, the EU rate falls by 10%, and the unemployment rate actually falls by 5%. These implications of a discount rate shock are counterfactual. In a typical US recession, the UE rate falls but the EU rate increases.

Motivated by Kehoe, Midrigan and Pastorino (2018), we then considered a version of the model in which workers accumulate human capital on the job. We found that, in response to an increase in the discount rate from 4 to 10%, the aggregate UE rate declines by 18%, the aggregate EU rate increases by 15%, and the unemployment rate rises by almost 30%. We also showed that the response of labor market outcomes to the discount rate shock varies dramatically with the worker's age. The decline in the UE rate for workers aged 21-30 is 4 times larger than for workers aged 41-50 and 51-60. The EU rate increases by 40% for workers aged 21-30, it is constant for workers aged 41-50, and it declines for workers aged 51-60. The disaggregated implications of a discount rate shock are counterfactual. In a typical US recession, the decline in the UE rate is nearly identical across age groups, and the EU rate increases for all age groups and by a similar factor of proportionality.

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