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DISCUSSION PAPER SERIES

IZA DP No. 11791

The Price of Capital, Factor Substitutability, and Corporate Profits

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ABSTRACT

The Price of Capital, Factor Substitutability, and Corporate Profits

The capital-to-labor ratio has steadily risen in the U.S. and elsewhere during the post-WWII period. Since the 1970s this rise has been accompanied by a rise in the level and variability of corporate profits whereas the labor share of income has declined. In this paper we ask whether these trends are related in that they can be explained by a common determinant such as the observed decline in the relative price of new capital goods, or the change in production technology towards in- creased factor substitutability. We use a dynamic stochastic equilibrium model of competitive search in the labor market augmented by a CES production function that allows firms to substitute between capital and labor at varying degrees. By assumption, firms can adjust capital more easily than labor. Profits arise from rents paid to quasi-fixed factors of production. We find that the declining relative price of capital and the increase in factor substitutability each causes the capital-to-labor ratio and the level and volatility of corporate profits to rise, but only increased factor substitutability generates the observed decrease in the labor share of income.

JEL Classification: E24, G32, J64
Keywords: factor substitutability, quasi-fixed production factor, competitive search, profits

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1 Introduction

The stock of physical capital that is used per employed worker for the production of output has steadily risen in the post-WWII period in the United States and many other industrialized countries. Since the 1970s, this positive trend has been accompanied by a steady decline in the labor share of income – a phenomenon that has received much attention recently, since it contradicts conventional wisdom regarding constant factor shares of income that was first presented in Kaldor (1961). Recent evidence further suggests that during the same period, the ratio of corporate profits to GDP has risen and become more volatile in the past two decades. Put differently, it seems that the traditionally close tie between corporate profits and labor income has disappeared.[1] Figure 1 depicts these trends for the U.S. economy during the post-WWII period.

In this paper we investigate whether these developments are possibly connected in that they can be explained by a common determinant. In particular, we ask whether and to which extent they can simultaneously be explained by the observed decline in the relative price of new capital goods that Gordon (1990) documented for the U.S., or rather by the change in the production technology that slowly, but steadily has increased the substitutability of labor by capital.[2] *A priori* either of these two fundamental changes has the potential to have contributed to the rise in the capital-to-labor share and also to the declining labor share of income. But what about their respective implication for the dynamics of firms' profits?

We address these questions in the context of a dynamic stochastic equilibrium model of competitive search in the labor market. We extend the standard model by allowing firms to use physical capital in addition to labor for producing output. By assumption capital is easier to adjust than labor. We take this view because of structural change that has transformed the U.S. economy during the period we consider towards one where services have become increasingly important for GDP production. In addition to labor, services require equipment rather than structures, and equipment is relatively easy to adjust. Moreover, the production technology allows for factor substitutability by permitting firms to employ multiple workers. Thus, we effectively abandon the Leontief

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[1] This recent phenomenon is emphasized also by the FRED blog of the Federal Reserve Bank of St. Louis on August 8, 2018. `https://fredblog.stlouisfed.org/2018/08/corporate-profits-versus-labor-income/`

[2] Gordon's analysis focuses on the change in the price of equipment rather than structures and documents that the price decline of equipment was extraordinarily strong. For the sake of our analysis we do not distinguish between the various components of physical capital, but look at total physical capital and the associated weighted average price.

production function of fixed factor proportions which is commonly used in models of labor market search. Doing so is necessary for studying the implications that varying degrees of factor substitutability – in addition to a change in the relative price of capital – have for our variables of interest.

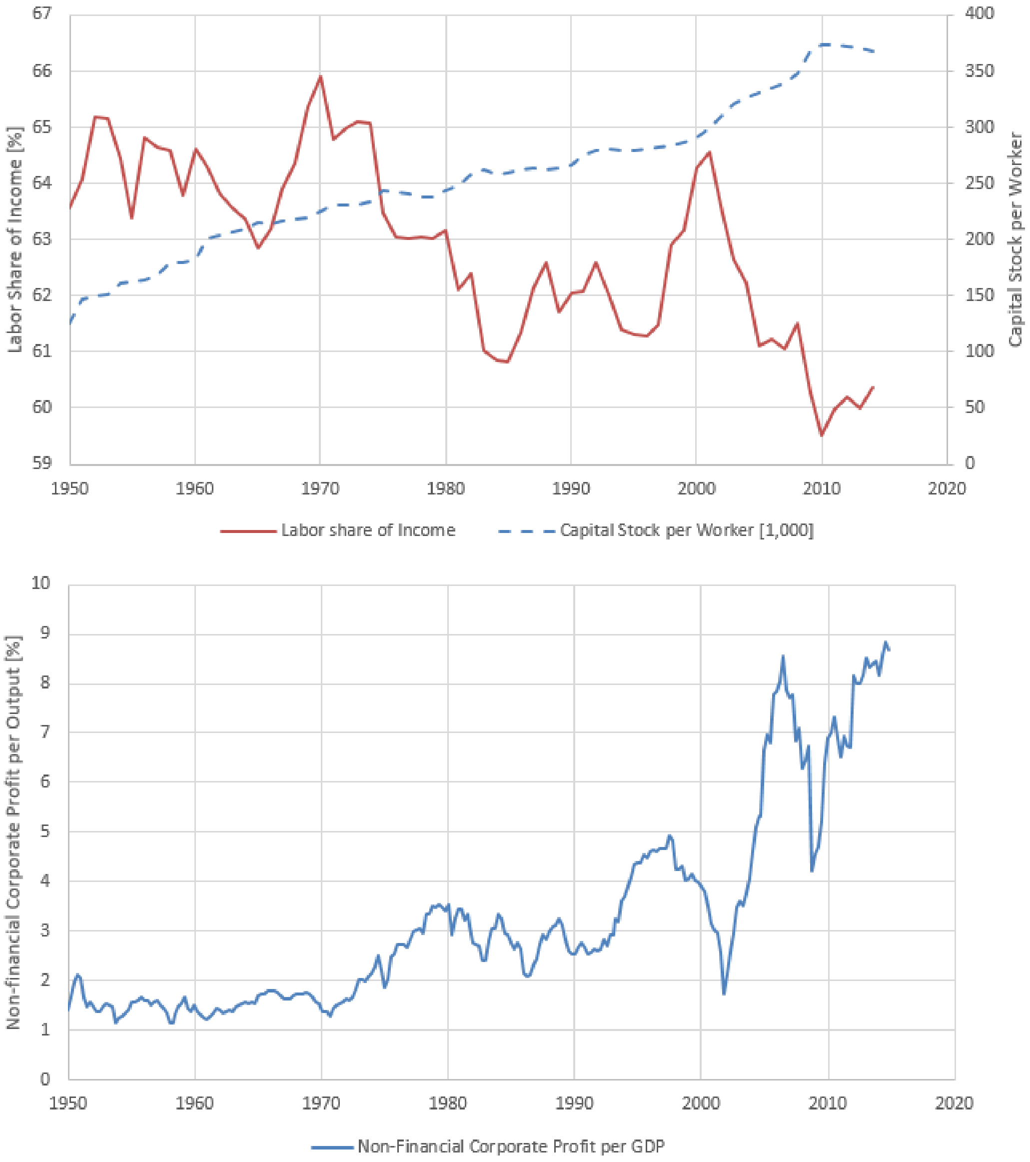
We calibrate the model to the U.S. economy in the post-WWII period, solve and simulate it. We use the model as a lab to disentangle the role that a steady decline in the relative price of new capital goods as opposed to an increase in the factor substitutability of the output production play in simultaneously explaining a declining labor share and a rise in the capital-to-labor ratio and in the level and volatility of corporate profits.

Our results show that when labor is relatively more costly to adjust than capital and the two production factors are (partial) complements, a rise in the degree of factor substitutability lets firms choose a more capital-intense input mix. The implied decrease in labor demand causes wages, employment and subsequently the labor share of income to fall. This fall in the labor share translates into rising corporate profits. When firms face shocks to total factor productivity, increased factor substitutability raises the volatility of investment and capital, but dampens that of wages and employment. In sum, corporate profits relative to output become more volatile. A decline in the relative price of capital generates identical reactions except that the labor share of income rises. Hence, our model suggests that, quantitatively speaking, the implications of a change in the production technology towards increased factor substitutability have outweighed those of a steady decline in the relative price of physical capital.

Our paper contributes to the macro literature in several respects. First, we study the declining labor share in the U.S. in conjunction with the related rise in the capital-to-labor ratio and the level and volatility of firms' profits. So, rather than looking at one trend in isolation, we study several trends that we expect to be interrelated and identify a common determinant. Second, we augment a labor market model with competitive search with a production technology that uses physical capital in addition to labor and allows for factor substitutability. Abandoning the more standard fixed-proportion input type of production function is a prerequisite for exploring the role of factor substitutability. Lastly, we can explain long-run changes in the volatility of corporate profits using changes in real economic variables only, thereby creating a bridge between a standard economic setup and finance where the dynamics of firms' profits are essential for dividends and stock price movements.

The remainder of this paper is structured as follows. In section 2 we link our work to the closely related literature. In section 3 we present our dynamic equilibrium model

Figure 1: Aggregate Trends in the U.S. Economy



Notes: Labor share relates to nonfarm business income. The capital stock is defined in millions of real U.S. Dollars (base year 2011), while employment is the total nonfarm payroll. All series were downloaded from FRED database.

of a frictional labor market. In section 4 we calibrate the model to U.S. data and perform simulation exercises to explore the implications of a change in the relative price of capital, and in the degree of factor substitutability, respectively. In section 5 we use aggregate time-series data from the U.S. on key model variables for a simple regression analysis to check for the empirical plausibility of our main arguments. Section 6 concludes.

2 Related Literature

This paper relates to several strands of the literature. First it relates to the work on factor substitutability in output production and its implications for the total economy. The distinction between short-run and long-run substitutability among input factors and the interaction with factor prices has received renewed interest in the macro literature and is discussed, e.g., in connection with increased digitization.[3] Acemoglu and Restrepo (2017) empirically study the competition between robots and workers for executing various tasks. In their environment, robots have a large negative effect on employment and wages. We do not consider tasks, but rather look at the implications of increased factor substitutability, or a declining price of capital for aggregate employment or wages in an environment where labor is subject to search frictions and, by assumption more, costly to adjust than capital. Also, wages are determined endogenously, but the price of capital is treated as a parameter. We use a CES production function and vary the parameter that reigns the degree of factor substitutability. In our environment a rise in substitutability decreases employment and wages, because firms *ceteris paribus* substitute towards the more flexible factor capital. Shim (2015) explores the implications that varying degrees of factor substitutability have for corporate profits, the associated operational risk and average stock returns of firms. His setup bridges real economic considerations and finance. Shim uses a firm valuation model that features partial capital irreversibility and external financing constraints, but treats labor as fully flexible. For the Compustat panel of U.S. firms he shows that rising factor substitutability is associated with less variable corporate profits. Shim proxies substitutability by firms' capital-labor ratios and works with a Cobb-Douglas production function that exhibits a constant unit-elasticity of substitution. Our setup nests that of Shim, but we allow for a varying degree of substitutability by altering the

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[3] A cohesive summary of this literature is beyond the scope of this paper, but readers may want to look at Brynjolfsson and Afee (2014) for a general discussion. We instead cover a selection of examples, which all closely relate to our paper.

respective parameter in a CES production function and consider a representative firm rather than a cross-section of firms. One of our main results is that in an environment where capital is easier to adjust than labor, a rise in factor substitutability increases the volatility of firms' profits.[4]

Second, it relates to the literature on labor market search when firms can hire and employ multiple workers. In order to study the relationship between factor substitutability and firm profits, we abandon the Leontief-type production commonly used in search and matching models where a firm has one job which can be filled with one worker. We use a competitive search framework and allow firms to hire multiple workers. When firms use capital in addition to labor, competitive search with wage posting does not suffer from inefficiencies arising from the hold-up problem faced by firms under bilateral wage bargaining and continues to render an efficient labor market equilibrium.[5] Hawkins (2013) is among the first to model firms that commit to a posted wage and hire multiple workers. His model has no physical capital. The same holds true for Schaal (2017) who allows for multiple workers per firm when analyzing the role of uncertainty for business cycle dynamics, and Kaas and Kircher (2015) who explore the business cycle dynamics of a model with heterogeneous firms that can employ multiple workers. Our paper differs in that it focuses on the interplay of several long-run trends, and that our model features firms that use labor and physical capital in the output production. Our setup is – to the best of our knowledge – the first to allow for physical capital in a multi-worker firm environment with competitive search in the labor market.

Lastly, our paper adds to the literature that explores alternative reasons for the decline in the labor share of income that has been observed in many OECD member countries since the mid-1970s. This observation stands in stark contrast to a supposedly constant labor share – one of the empirical facts presented in Kaldor (1961).[6] We study the declining labor share in conjunction with closely related trends, i.e. the

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[4] This finding is consistent with what Danthine and Donaldson (2002) report when treating firms' labor costs as predetermined. In that case the volatility primarily affects dividends, which are defined as sales and profits net of labor costs.

[5] Firms with multiple workers and physical capital have been studied when labor market matching is assumed to happen randomly. A recent example is Gertler et al. (2016).

[6] Blanchard (1997) was among the first to address diverging trends in unemployment and the labor share of income between some Anglo-Saxon countries including the U.S. and selected countries in continental Europe. He used a static general equilibrium model with frictional labor markets and monopolistic competition in the goods market to explore the role of supply vs. demand forces at work. He identifies alternative wage-setting mechanisms as key sources for observed cross-country differences in long-run trends. Recent contributions have examined alternative explanations, including sectoral concentration (Autor et al., 2017), automation and digitization (Arntz et al., 2016), increased markups (Loecker and Eeckhout, 2017) or international trade (Elsby et al., 2013).

rise in the capital-to-labor ratio and in the level and volatility of firms' profits and look for a common determinant. We use a setting with frictional labor markets and a production technology that incorporates factor substitutability to ask whether all trends can simultaneously be explained by a decline in the relative price of capital, or rather by a change in the production technology towards increased factor substitutability. Our work is linked to that of Karabarbounis and Neiman (2014) who find that lower prices of capital lead to a decline in the labor share. When estimating their model, they find an elasticity of substitution between capital and labor equal to 1.25. Compared to existing estimates by Chirinko (2008), or León-Ledesma et al. (2010), this value is high, but crucial for their results, as it implies that the inputs are substitutes rather than complements. We instead consider an elasticity of substitution less than one for our simulation exercises. With inputs being complements, a decrease in the price of capital leads to a rise in the labor share, whereas rising substitutability lets the share decline.

3 A Model of Competitive Search

Our model economy is populated by a unit mass of identical firms and a unit mass of identical workers. Firms post vacancies and invest in physical capital in order to maximize their profits. Due to labor market frictions, firms cannot hire workers directly, but have to post vacancies at a cost a and a corresponding wage $\tilde{w}$ that is fixed as long as the employment relationship lasts. The transition from vacancies to a filled job and from unemployed to employed depends on the number of workers applying to a vacancy and the number of vacancies posted by the firms. Firms can post vacancies in various submarkets, characterized by a wage and the ratio of jobs and jobseekers. Unemployed workers direct their search towards one of those markets, trading off the wage and the chance of getting hired. The interplay of the firms' posting behavior and the workers' application decisions generates the labor market tightness, which is defined as the ratio of vacancies to the number of applicants in a market. For ease of exposition the actual matching is governed by a standard matching function, as opposed to a specific matching algorithm.

3.1 Firms

We start the detailed description of the model at the firm as it is our core unit of analysis. There exists a unit mass of identical firms in this economy. They use capital k and labor l to produce a homogeneous output good y. The inputs are transformed into the output good according to a constant elasticity of substitution (CES) production function:

$$y(k_t, l_t, z_t) = z_t \left(\alpha k_t^{\sigma} + (1-\alpha) l_t^{\sigma}\right)^{\frac{1}{\sigma}}\ ,$$
$$\text{with } \alpha \in (0,1),\ \ \sigma \in (-\infty, 1]$$

We choose this functional form for two reasons. First, it is more general than the commonly used Cobb-Douglas function, which it nests as a special case. Second and more importantly, this functional form allows us to explicitly vary the substitutability of input factors, which enables us to address our research question. The elasticity of substitution between k and l depends on the parameter σ and is given by $\frac{1}{1-\sigma}$. As σ is a key model parameter, it is important to understand its effects on the production function. The parameter σ can vary between $-\infty$ and 1. For the limiting case of $-\infty$ the elasticity of substitution converges to zero and the production function approaches the Leontief production function with a fixed ratio of input factors. This implies that inputs are perfect complements. For $\sigma = 1$ input factors are perfect substitutes. At $\sigma = 0$ the CES nests the Cobb-Douglas case.[7] The other parameter entering the production function is α, which governs the capital intensity of production. We also include a standard Hicks-neutral TFP process z_t, which enables us to consider the variability of economic quantities.

Firms can purchase capital at a fixed price p^k per unit. Capital depreciates at rate δ every period. Because of frictional labor markets, firms can expand their labor force only by posting vacancies v_t together with a wage rate $\tilde{w}_t$ in a particular submarket, which is characterized by its respective tightness, θ_t.[8] For each vacancy posted, the firm has to pay a vacancy posting cost, a. This cost can be thought of as advertising and training newly hired employees. By assumption, a constant fraction ν of matches breaks up every period. This is the only possibility for a match to end. The firms cannot decide which workers to fire. Thus, the stock of employment l_t is a state variable for the firm in period t.

The fact that firms decide on the wage offered for a posted vacancy in every period potentially generates a distribution of wages. Since we do not focus on wage dynamics

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[7] For further discussions on the CES function and its properties see Klump et al. (2012).

[8] We choose wage-posting plus directed search – rather than random search – to avoid the holdup problem a firm would face when making investment decisions. In our competitive search setting, a higher capital stock implies higher wages and also a higher job filling rate. See Acemoglu and Shimer (1999) for more details.

*p*er se in this paper, we choose to simplify the wage setting process. New hires formed during period t become productive in period $t+1$. These new hires h_t will be paid the posted wage $\tilde{w}_t$. The wage bill that a firm has to pay in period t is given by $l_t w_t$, where w_t denotes a weighted average of the wage paid to continuing workers and new hires from the previous period. In brief, $l_{t+1}w_{t+1} = (1-\nu)l_t w_t + h_t \tilde{w}_t$. We calculate the wage bill in a recursive way, which is described in greater detail in Appendix A. We show that our recursive formulation is equivalent to keeping track of the entire history of hires and wages. Therefore, w_t is an additional state variable for the firm.

The firm discounts future profits at rate $0 < \beta < 1$. The firm's problem can be summarized as follows:

$$\max_{v_t,\theta_t,\tilde{w}_t,i_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [y(k_t, l_t, z_t) - w_t l_t - p^k i_t - a v_t]$$

subject to

$$\begin{aligned} h_t &= v_t q(\theta_t) \\ l_{t+1}w_{t+1} &= (1-\nu)l_t w_t + h_t \tilde{w}_t \\ l_{t+1} &= (1-\nu)l_t + h_t \\ k_{t+1} &= (1-\delta)k_t + i_t \\ z_{t+1} &= \rho z_t + \epsilon_t,\ \ \epsilon_t \sim \mathcal{N}(0, Var_\epsilon) \end{aligned}$$

Firms maximize the expected present discounted value of future profits. Profits consist of revenue minus wage payments, investment expenditures and hiring costs. The firm takes as given that the number of newly hired employees equals the posted vacancies multiplied by the job filling rate, the recursive formulation of the wage bill, and the laws of motion for capital, labor and exogenous total factor productivity, z_t. As we elaborate below, in equilibrium two additional constraints must be satisfied, i.e. the optimal application rule for searching workers and the requirement that the ratio of all job-vacancies to searching workers indeed equals labor market tightness in a given submarket.

3.2 Households

Workers are part of a big family, consisting of a continuum of members normalized to measure 1. Each worker can be employed or unemployed. If unemployed, she chooses to apply to a particular submarket that is characterized by vacancies and the corresponding wage-rate $\tilde{w}_t$. The worker's chances of getting matched depend on the ratio of vacancies posted to the measure of job seekers in that submarket, i.e. the labor market tightness. If employed, a worker inelastically supplies one unit of labor to the firm and receives a wage w_t in exchange. When unemployed, a worker receives the unemployment compensation b. At the end of each period the family pools all income. This implies that for each individual neither the actual labor market status, nor the individual wage rate in case of employment matter, since all equally share the family's total earnings. We effectively assume full risk-sharing. Moreover, we assume that all agents are risk-neutral and do not save. This is necessary for our recursive wage formulation to be an exact description of earnings over time.

Unemployed workers will apply for a job only if it is optimal compared to all other jobs or remaining unemployed. This implies they will select the best combination of job-finding rate and wage among all the ones offered in equilibrium. Denoting by U the value for an unemployed worker of getting a job the following condition holds:

$$U_t \leq p(\theta_t)\tilde{w}_t + (1 - p(\theta_t))b \tag{1}$$

The value U_t is the value to an unemployed individual who can apply for a job which promises the wage $\tilde{w}$ and a job-finding rate $p(\theta)$. U_t exceeds the value of the unemployment benefit b, because firms internalize this condition in their decision problem. If they were to offer just b, one firm could offer a slightly higher wage, thereby attracting all searching workers. Thus, each firm takes U_t as given, although this variable is determined endogenously in equilibrium.[9]

3.3 Matching

In each submarket, job vacancies and searching workers are randomly matched. We capture this process by a standard Cobb-Douglas matching function $m(u_t, v_t)$, which we assume to exhibit constant returns to scale:

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[9] We simplify the problem by abstracting from a continuation value for the unemployed. This makes the worker care only about current wages. However, not applying for a job will decrease the earnings by the household by an entire quarter of the annual wage bill. This loss is big, compared to the chance of a shock that would make it worthwhile for the workers to wait an entire period.

$$m(u,v) = Bv^{\gamma}u^{(1-\gamma)} \ , \ B > 0 \tag{2}$$

where $\gamma \in [0,1]$ is the elasticity of total matches with respect to vacancies, and B governs the efficiency of the matching process.

Dividing the number of matches by the measure of searching workers yields the job-finding rate $p(\theta)$, whereas dividing it by the number of vacancies delivers the job filling rate for the firm, $q(\theta)$. A firm posting vacancies v_t can expect to attract $h_t = v_t q(\theta_t)$ new workers.

3.4 Labor Market Equilibrium

Each firm enters period t with its stock of capital k_t, its workforce l_t, the average firm-level wage w_t, and the realization of the exogenous aggregate productivity process z_t. Those variables form its state vector (k_t, l_t, w_t, z_t).

When maximizing the expected present discounted value of future profits, the firm takes into account the laws of motion for each of its state variables and also the job application rule for searching workers given by equation (1). Substituting in the laws of motion for capital, employment and wages, we can summarize the firm's problem with the help of the following Lagrangian.[10]

$$\begin{aligned}\mathcal{L} = \max_{\theta_t, k_{t+1}, l_{t+1}, w_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{ & y(k_t, l_t, z_t) - w_t l_t - [k_{t+1} - (1-\delta)k_t]p^k - a\frac{l_{t+1} - (1-\nu)l_t}{q(\theta_t)}\} \\ & + \lambda_t \left[U_t - (1-p(\theta_t))b - p(\theta_t)\frac{l_{t+1}w_{t+1} - (1-\nu)l_t w_t}{l_{t+1} - (1-\nu)l_t} \right]\end{aligned}$$

The first-order-necessary conditions that need to be satisfied in equilibrium are given by

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[10] For an alternative complete formulation of the problem see Appendix B.

$$
\begin{aligned}
\frac{\partial}{\partial \theta_t} &: a(l_{t+1} - (1-\nu) l_t) \frac{q'(\theta_t)}{q(\theta_t)^2} + \lambda_t p'(\theta_t) \left[b - \frac{l_{t+1} w_{t+1} - (1-\nu) l_t w_t}{l_{t+1} - (1-\nu) l_t} \right] = 0 \\
\frac{\partial}{\partial k_{t+1}} &: p^k = \beta \left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial k_{t+1}} + p^k (1-\delta) \right] \\
\frac{\partial}{\partial l_{t+1}} &: - \frac{a}{q(\theta_t)} + \lambda_t (-p(\theta_t)) \frac{(1-\nu) l_t [w_t - w_{t+1}]}{(l_{t+1} - (1-\nu) l_t)^2} \\
&\quad + \beta \left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial l_{t+1}} - w_{t+1} + a \frac{(1-\nu)}{q(\theta_{t+1})} + \lambda_{t+1} \{ -p(\theta_{t+1}) \frac{(l_{t+2}(1-\nu)[w_{t+2} - w_{t+1}]}{(l_{t+2} - (1-\nu) l_{t+1})^2} \} \right] = 0 \\
\frac{\partial}{\partial w_{t+1}} &: \lambda_t (-p(\theta_t)) \frac{l_{t+1}}{l_{t+1} - (1-\nu) l_t} + \beta \left[-l_{t+1} + \lambda_{t+1} \{ p(\theta_{t+1}) \frac{(1-\nu) l_{t+1}}{l_{t+2} - (1-\nu) l_{t+1}} \} \right] = 0
\end{aligned}
\tag{3}
$$

As all firms are identical and so are all workers, their respective behavior can be summarized by that of a representative agent. Note that the representative firm continues to react to changes in the economy in a competitive way. Our competitive search setup in this particular environment reduces the many possible submarkets to a single market.

We close the model by enforcing that in equilibrium, the ratio of posted vacancies to the measure of unemployed workers needs to equal labor market tightness, $\frac{v}{1-l} = \theta$. Substituting v_t by $\frac{l_{t+1} - (1-\nu) l_t}{q(\theta_t)}$, and exploiting algebraic properties of our matching function, we get the following expression as additional equilibrium condition:

$$
\theta_t = \left(\frac{l_{t+1} - (1-\nu) l_t}{B(1-l_t)} \right)^{\frac{1}{\gamma}} \tag{4}
$$

In order to reach a steady state, we need a vector $(l^*, k^*, w^*, \theta^*, \lambda^*)$[11] which solves the system given by the 4 F.O.N.C.s in (3) plus equation (4). In equilibrium the value U is determined by the optimal values for wages and labor market tightness plugged into condition 1 with equality.[12] We solve the model around the deterministic steady state by second-order perturbation using Dynare.

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[11] Stars denote equilibrium values.

[12] For further discussion on the solution process of labor-search models see Rogerson et al. (2005).

4 Quantitative Analysis

4.1 Calibration

As the model cannot be solved analytically, calibration becomes an important matter. The model has a variety of parameters which need to be determined. We take certain values from the literature and perform robustness checks to ensure that these values are not driving the results. The crucial parameters are calibrated in order to match empirical targets, which are important when talking about factor substitutability and its implications for firms and workers.

We calibrate the model to quarterly data from the U.S. economy. Table 1 contains the full parametrization of the model.

Table 1: Baseline Calibration

Parameter	Interpretation	Value	Target
α	Capital intensity	0.7914	Labor share 60%
σ	Substitutability parameter	-3/2	Elasticity of substitution 0.4
p^k	Price of capital	1	Normalization
γ	Matching function elasticity	0.5	Standard
B	Matching efficiency	0.8	Unemployment rate 7%
b	Unemployment benefit	0.9	Replacement ratio 60%
a	Vacancy posting cost	4	$p(\theta) = 0.99$
β	Discount factor	0.975	Standard
δ	Depreciation rate of capital	0.026	Depreciation rate of capital
ν	Separation rate	0.075	Labor turnover

One of our central questions is what happens to firm profit, employment and investment if a firm is able to substitute more easily among capital and labor. To address this question, we vary the parameter σ, which directly relates to the elasticity of substitution between capital and labor. As a baseline value, we pick $\sigma = -\frac{3}{2}$, which corresponds to an elasticity of substitution of 0.4. This value lies at the lower end of what the literature deems plausible.[13] We will change the parameter σ to $-\frac{2}{3}$ to model increased substitutability and study its effects. We use the range provided by Chirinko (2008) as a guideline for one of the experiments we perform in the context of our model.

The parameter α which governs the efficiency of capital in the production function

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[13] For a survey of these values see Chirinko (2008). He argues that empirical estimates of the elasticity of substitution range from 0.4 to 0.6.

is central to the problem, as the technology available to the firm is key to our analysis. This parameter amounts to an additional degree of freedom in the production function, which we have to tackle in our analysis.[14] We calibrate α to ensure that the model outcomes are comparable across alternative specifications. In a standard neoclassical model with a Cobb-Douglas production function and no frictions, the parameter α corresponds to the income share of capital. We first target a labor share of income equal to 60% to inspect the key mechanism of our model. When exploring the implications of a changing price of capital, or a varying degree of factor substitutability on this share, we adjust α such that the output level remains constant across various regimes.

We normalize the price of capital, p^k, to one. This price governs the rate at which a firm can turn its output good into next period's capital. In our comparative statics exercises, we will consider what happens when we lower this price, thereby rendering investment of the firm more productive. At a price equal to one, the output good produced by the firm can simply be used as next period's capital. When lowering p^k, we implicitly make the technology via which output can be turned into capital more efficient. A falling relative price of investment goods might cause similar effects as increased factor substitutability. Whether it is cheaper to invest in capital, or whether capital can more easily be substituted for labor is hard to distinguish in reality, as both effects occur simultaneously. In our model, we can separate these two effects and study their respective effects on our variables of interest.

We set the efficiency parameter B of the matching function to target an unemployment rate of 7% and choose the unemployment benefit to match a replacement ratio equal to 0.6. The replacement ratio is defined as unemployment benefit b relative to the equilibrium wage. The vacancy posting cost a is chosen such that a worker's job-finding rate of the worker is close to 0.99, the rate implied by the monthly rate of 0.34 which Shimer (2005) reports.

The remaining parameters are taken from the literature. Many have a clear economic interpretation. Shimer (2005) shows that around 3.42 % of workers in the U.S. labor force leave their jobs each month. So we set ν equal to 0.075 for a period of three months, to also account for workers finding a job within the same quarter. The quarterly depreciation rate of 0.026 reflects the empirical equivalent. Although not explicitly targeted, our set of calibrated parameter values implies a plausible value for the cost of hiring. Blatter et al. (2012) report this value to lie between 10 to 17 weeks of wage

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[14] For a discussion of the issue of normalizing a CES production function see e.g. León-Ledesma et al. (2010).

payments. The value in our baseline-calibration is 16.7 weeks, which we calculate by dividing the expected cost to hire a worker by the yearly wage.

4.2 Results

We numerically solve the model for our benchmark calibration. Table 2 reports the corresponding results in column 1. Column 2 states the results when the parameter σ is increased from $-\frac{3}{2}$ to $-\frac{2}{3}$. This parameter change corresponds to a rise in the elasticity of substitution among input factors from 0.4 to 0.6.

Table 2: Steady State Results

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -2/3$ $p^k = 1$
k	7.0470	8.0188
l	0.9299	0.8984
w	1.5260	1.3196
θ	1.5475	0.6875
$q(\theta)$	0.6431	0.9648
$p(\theta)$	0.9952	0.6633
v	0.1085	0.0698
y	2.3651	1.9724
π	0.3290	0.3011
u	0.07	0.10
profit share	0.1391	0.1527
labor share	0.6	0.6
investment share	0.0775	0.1057
hiring cost share	0.1834	0.1416
α	0.7914	0.5298

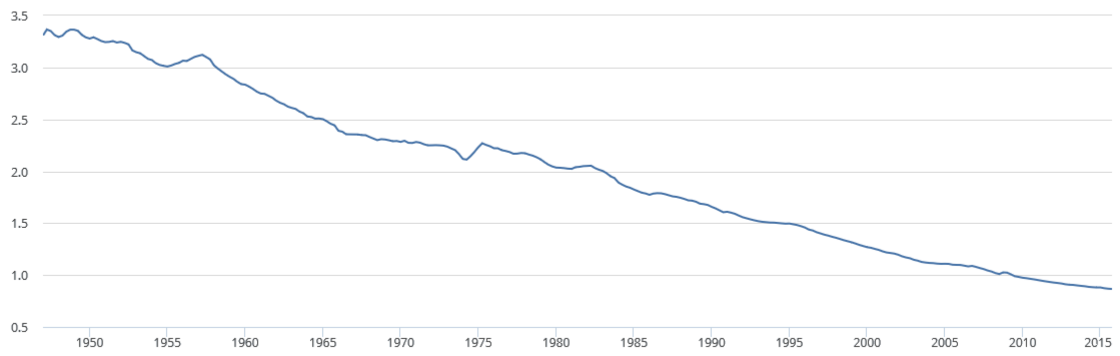
Such a rise makes production more capital-intensive while conditions for workers worsen. The job-finding rate $p(\theta)$ declines, and so do employment l and wages w. The firm spends more on investment and less on hiring, which can be seen by the decrease in the hiring cost share, which equals the costs of hiring divided by output. As the firm produces with a greater capital intensity it uses less labor and also posts fewer vacancies v. At the same time output y declines. By construction, the labor share, which is defined as the wage bill wl divided by output, remains constant, but the profit share increases. The profit share of 13% slightly exceeds what we observe in the data and increases further when factor substitutability rises.

Overall increased factor substitutability benefits firms via higher profits, while it hurts workers. They experience lower wages and a higher risk of unemployment.

4.2.1 A Lower Price of New Capital

As documented in detail by Gordon (1990) and Krusell et al. (2000), the relative price of investment goods has steadily declined for decades. Figure 2 illustrates this trend. In this section, we explore the quantitative effects of a decline in p^k for our baseline scenario ($\sigma = -\frac{3}{2}$), and also for an increased degree of factor substitutability ($\sigma = -\frac{2}{3}$). Table 3 reports the results from our numerical experiment. For both values of σ under consideration, capital and labor exhibit an elasticity of substitution less than 1 and thus are complements.

Figure 2: Relative Price of Investment Goods



Notes: Investment deflator divided by consumption deflator. The base year is 2009, seasonally adjusted. Downloaded from FRED database.

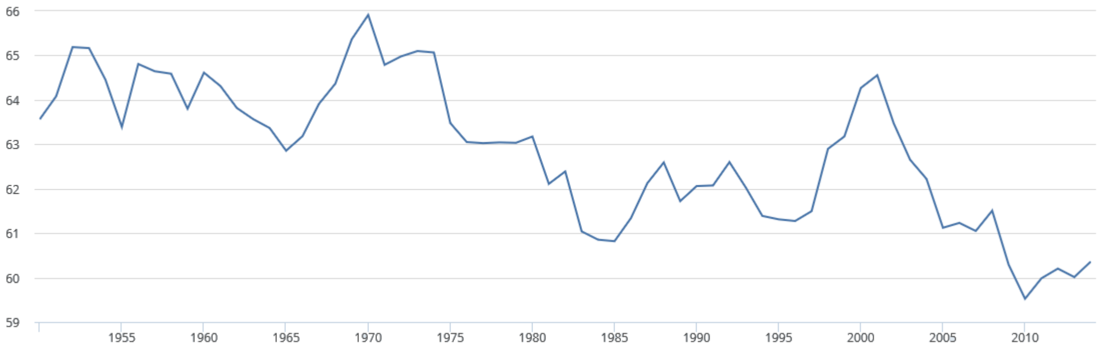
Table 3 separately reports the effects of each of these changes. Comparing the entries form the second to those from the third column, we see the implications of a decline of the price of capital, which are a lower profit share for firms and an increase of employment and wages as overall output production expands. Increased factor substitutability, on the other hand, again increases the profit share, as can be seen in the last column. These two effects push all variables in opposite directions, except for capital. In sum, when increased factor substitutability occurs together with lower prices of capital in a world of frictional labor markets, the only reliable statement we can make is that the extent of capital in use increases. However, when looking at the implied increase in profit shares, our model suggests that increased factor substitutability outweighs the cheaper price of capital.

Table 3: Steady State Results

	$\sigma = -3/2$	$\sigma = -3/2$	$\sigma = -2/3$
Variable	$p^k = 1$	$p^k = 0.7$	$p^k = 0.7$
k	7.047	9.0327	10.8953
l	0.9299	0.9415	0.9152
w	1.5260	1.6596	1.34089
θ	1.5475	2.2787	1.0228
$q(\theta)$	0.6431	0.53	0.791
$p(\theta)$	0.9952	1.2076	0.8091
v	0.1085	0.1332	0.0868
y	2.3651	2.6043	2.149
π	0.329	0.3443	0.3142
u	0.07	0.0585	0.0848
profit share	0.1391	0.1322	0.1462
labor share	0.6	0.6	0.6
investment share	0.0775	0.0631	0.0923
hiring cost share	0.1834	0.2074	0.1615
α	0.7914	0.8099	0.5351

4.2.2 Decline in the Labor Share of Income

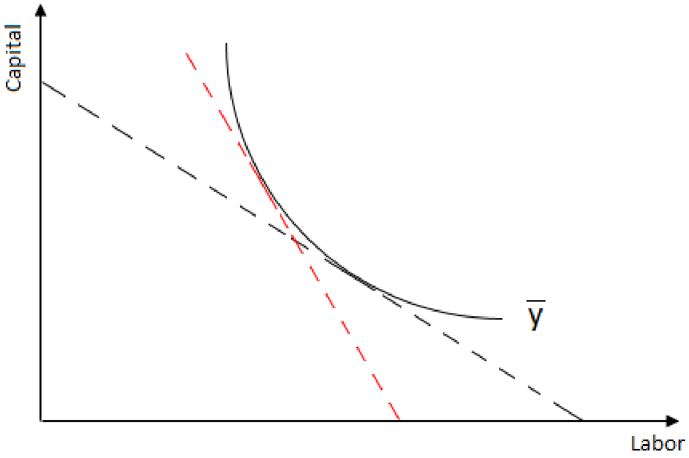
Figure 3: Labor Share in the U.S. [%]



Notes: Downloaded from the FRED database.

In all previous experiments, we recalibrated the parameter α to keep the labor share of income at 60% when we varied the degree of substitutability. This was done in accordance with the well-known empirical facts presented in Kaldor (1961). One of these facts states that the labor share is constant over long periods of time. As can be

Figure 4: A Change in the Relative Price of Capital



seen in Figure 3, the labor share has been on the decline since the 1970s.[15] Of course, a declining share of GDP accruing to labor implies that other factors benefit.

In what follows we explore how the labor share of income reacts to a decline in the price of capital, and to an increase in factor substitutability. We recalibrate α to keep steady-state output constant when varying our parameter of interest, σ. First, we consider a change in the relative price of capital, and illustrate the implications for a firm's demand for production factors in Figure 4. The slope of the straight cost lines (dashed) equals the negative ratio of input factor prices, i.e. the ratio between the wage rate w and the price of capital, p^k. A drop in the price of capital increases the steepness of the cost line which we mark in red. That is because cheaper capital increases the firm's demand for capital and also for labor. A drop in p^k let's the resulting equilibrium wage rate rise, as a higher wage is needed to attract more workers. As we keep output constant, the new equilibrium lies on the same isoquant. We observe that the point of tangency moves to the left, resulting in a higher capital-labor ratio and a more capital-intensive production.[16] This rise in the overall capital intensity in production is consistent with evidence from U.S. data.[17]

The full quantitative results of this exercise are given in Table 4. The first column of results is again the steady state obtained under our baseline calibration. We repeat

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[15] The same holds true for other OECD countries (compare Autor et al. (2017)).

[16] We refrain form illustrating the case of increased substitutability, because it would alter the shape of the production function too much, since σ and α change substantially.

[17] See Appendix D.4.

the type of numerical experiments from before holding output, y, constant, because we want to study the reaction of the labor share. A decrease in the price of capital to $p^k = 0.7$ causes the firm to use more capital and renders production more capital-intensive.[18] The price decline by 30% dominates the additional investment such that the investment share decreases. Employment and wages increase, which results in a rise of the labor share.

The last column in Table 4 shows what happens when substitutability increases. Due to frictions in the labor market, the firm decides to increasingly replace labor by capital. The decline in labor demand lets wages decrease. The labor share subsequently drops by around 4 percentage points. The drop in the job-finding rate for unemployed workers adds to the worsened situation for the factor labor.

Table 4: Steady State Results with Constant Output

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 1$
k/l	7.5782	8.7749	11.0589
w	1.5260	1.5603	1.4487
θ	1.5475	1.7220	1.1889
$q(\theta)$	0.6431	0.6096	0.7337
$p(\theta)$	0.9952	1.0498	0.8723
v	0.1085	0.1148	0.0941
y	2.3651	2.3651	2.3651
π	0.3290	0.3023	0.3898
profit share	0.1391	0.1278	0.1648
labor share	0.6	0.6157	0.564
investment share	0.0775	0.0623	0.1119
hiring cost share	0.1834	0.1942	0.1592
α	0.7914	0.7828	0.5827

We conclude that cheaper investment goods cannot be the sole source for the empirically observed decrease in the labor share in many countries, since it would imply an increase in employment and wages, and thus in the labor share. On the other hand, increased factor substitutability tends to reduce this share. Separating these two effects is important when trying to understand which aspect of the two forces under consideration leads to the observed outcomes.

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[18] As robustness checks we used other values, the results remain qualitatively similar.

Table 5: Relative Variabilities under Alternative Specifications

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 1$
y	1	1	1
k	0.4693	0.4281	0.6414
l	0.0805	0.0552	0.0708
w	0.3648	0.2257	0.2221
θ	2.2872	1.6335	1.7962
v	1.3178	0.9991	1.1157
i	5.7196	7.3915	10.8463
π	4.7561	5.9914	7.9557
profit share	4.1141	5.2581	7.6517
labor share	0.6615	0.8038	0.7875
investment share	5.6135	7.2268	10.6993
hiring cost share	0.4681	0.3874	0.4492

Notes: Ratio of coefficient of variation relative to output.

4.3 Changes in Variability

In what follows, we will investigate whether increased factor substitutability and a lower price of capital *per se* dampens or increases the variability of profits. We therefore consider a stochastic environment where the firms face shocks to total factor productivity (TFP). We assume TFP to follow an AR-(1) process with a persistence parameter of 0.9. Increments are normally distributed with mean zero and a standard deviation equal to 0.007, a standard value in the business cycle literature.

We do a second-order approximation around the deterministic steady state of our model and compute the fluctuations of the model variables. Table 5 reports the ratio of each variable's coefficient of variation, i.e. the standard deviation normalized by the mean of the variable, relative to that of output.

Again, column 2 depicts the results under our baseline calibration. While capital is more volatile than employment, the volatility of profit and the profit share are an order of magnitude larger than that of capital. Also investment and the investment share are very volatile, which is consistent with empirical evidence, as investment is the most volatile component of GDP.[19]

Columns 3 and 4 report the results when we vary the price of capital and factor substitutability, respectively. A lower relative price of new capital causes firms to

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[19] See e.g. `https://fredblog.stlouisfed.org/2015/08/gdp-components-volatility/`

maintain a more stable capital stock and employment by increasing the variability of investment. It can do so, because the price of new capital has decreased. The fluctuations in employment and wages are dampened relative to output, which lets the variability of profits increase. As the payments to the workers become more stable relative to output, the excess variability in output drives up the variability of profits. This mechanism is reminiscent of Danthine and Donaldson (2002), where wage payments are viewed as contractual obligations with the residual of the firms' earnings being paid out as dividends to the owners.

According to the results reported in the last column, increased factor substitutability causes firms to react more flexibly to stochastic fluctuations in aggregate productivity and to primarily adjust the factor which is less costly to vary. Since our model features no adjustment friction in capital, the firm reacts more strongly in capital. The volatility of investment increases in the degree of factor substitutability as does profit. At the same time, increased factor substitutability dampens fluctuations in wage and employment.

In sum, we observe that profits and investment become more volatile as the degree of factor substitutability rises and the relative price of investment goods declines.

5 Empirical Evidence

We are now in a position to subject our model to an additional plausibility check and contrast its main predictions to their real world counterparts. While our model replicates the empirically observed negative relationship between the profit share and the labor share it has difficulties explaining the behavior of investment. This is because we abstract from financing issues and corporate debt while focusing on the effects of factor substitutability on firms' profits and the labor market.

We take U.S. time-series data on key economic variables and compare their statistical moments to their counterparts generated by our model. A central equation in all of our discussion is firms' profits defined as follows:

$$\begin{aligned}\pi_t =& y_t - w_t l_t - i_t p^k - v_t a \\ \frac{\pi_t}{y_t} =& 1 - \frac{w_t l_t}{y_t} - \frac{i_t p^k}{y_t} - \frac{v_t a}{y_t}\end{aligned}$$

The second line is just a normalization by output. Once we allow for errors, ϵ_t, that we assume to be normally distributed, we can estimate the following econometric model:

$$\frac{\pi_t}{y_t} = \alpha_0 + \alpha_1 \frac{w_t l_t}{y_t} + \alpha_2 \frac{i_t p^k}{y_t} + \alpha_3 v_t a + \epsilon_t \tag{5}$$

Most of our data originate from the FRED database.[20] We take the aggregate time series of GDP, non-financial corporate profits, investment and labor share of income directly from this database.[21] Each series comes at a quarterly frequency and covers the period from the first quarter of 1947 to the last quarter of 2016. We construct investment share and profit share by dividing the respective variables by contemporaneous GDP.

For vacancies we use an updated version of the data constructed by Barnichon (2010), which we downloaded directly from the author's website.[22] The data are an index of open vacancies relative to the labor force and have been constructed from the "Help-Wanted-Index" which only relies on job openings printed in newspapers and the online Help-Wanted Index.[23]

We run OLS regressions and present the results in Table 6. The table displays the following specifications. In column (1), we estimate the regression model from equation (5). The coefficients of the labor share and the job openings each are negative. The coefficient of the investment share is significantly positive, which is not expected, given the definition of profits in our model. In the model, investment directly reduces profits. The coefficient of the investment share remains positive when we use its first lag in column (2). This is done to control for potential lags between actual investment and the implied increase in revenue.

We detect autocorrelation in the residuals using the Breusch Godfrey-Test and therefore include the first lag of the profit share in column (3). The coefficient of the investment share becomes insignificant, while the coefficients of labor share remains strongly negative, and the hiring cost share barely fails to be significant at the 5% level.[24] This is in line with the predictions of our model. Investment share and profit share are empiri-

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[20] For a detailed description see Appendix D.

[21] We take GDP instead of non-financial value added to enable comparision with our discussion on the dividend share in Appendix E, because dividends cannot be decomposed in financial and non-financial companies.

[22] `https://sites.google.com/site/regisbarnichon/data`

[23] As we do not have any data for the vacancy posting costs a, which we assume to be constant, the estimate of α_3 will actually be $\frac{\alpha_3}{a}$. However, we will also not divide vacancies by GDP, because normalizing the relatively constant index of vacancies by GDP would impose downward trends in this variable.

[24] It is, however, significantly negative if we use GDP instead of non-financial GDP.

Table 6: Regression Results, 1947Q1-2016Q4

	(1)	(2)	(3)
Labor share	-0.635***	-0.640***	-0.108***
	(0.0304)	(0.0291)	(0.0222)
Investment share	0.261***		0.0250
	(0.0341)		(0.0146)
L.Investment share		0.265***	
		(0.0335)	
Job openings	-0.249***	-0.248***	-0.0506
	(0.0745)	(0.0745)	(0.0260)
L.Profit share			0.864***
			(0.0331)
Constant	41.33***	41.55***	7.205***
	(2.171)	(2.085)	(1.475)
Observations	264	263	263
Adjusted R^2	0.859	0.863	0.980

Notes: The dependent variable is profit share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: Correlations Between Various Shares

	Model $\sigma = -3/2$				U.S. Data			
	Profit	Labor	Hiring	Investm.	Profit	Labor	Hiring	Investm.
Profit	1				1			
	(0)							
Labor	-0.1365	1			-0.2731	1		
	(0.0477)	(0)						
Hiring	-0.8016	-0.4786	1		0.4351	-0.0157	1	
	(0.0289)	(0.0055)	(0)					
Investm.	-0.912	-0.2807	0.9745	1	0.6040	-0.2725	0.7105	1
	(0.0119)	(0.0191)	(0.0044)	(0)				

Notes: The model has been simulated 100 times for the same number of periods as data points are available (264). All data are HP-filtered with a smoothing factor of 1,600.

cally highly positively correlated because of two reasons. Firstly, there is a discrepancy between the definition of profit in the model and in the data. Profits in the model represent economic profits accruing from rents, while in the data corporate profits are defined as revenues minus costs. Investment expenditures do not constitute costs in this sense, because the firm still owns the capital and only the depreciation of capital lowers profits.[25] Secondly, and perhaps more importantly, our model assumes that firms' current period's retained earnings are uset to cover investment expenditures. This stands in sharp contrast to how firms in reality pay for their investments, which might include debt or additional equity. This is in line with the arguments made by Danthine and Donaldson (2002), who use the idea that wage payments enjoy seniority over dividend and other payment, which is why the labor share and profits are negatively correlated.

Following standard practice in the business cycle literature we compare the simulation results from our model to the correlations observed in the data in Table 7. When targeting first moments, second moments are used to determine the goodness of fit of our model. Even though our model was not primarily designed to explain the business cycle, but rather to study the effects of different degrees of input substitutability on long-run trends in corporate profits and labor market variables, it performs quite well.

When we compare the correlations over the full length of our time series we get a similar picture as in the data. Investment and profit share are positively related. Our model closely matches the correlation between the labor share and the investment

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[25] We control for this by using dividends as dependent variable in Appendix E. The positive correlation remains.

share. These two variables are key elements of the firm's decision of their input mix. It also replicates a positive correlation between the hiring cost share and the investment share, although the correlation is higher than in the data. A reason for this may be lumpy investment, due to fixed costs, which are not present in the model.

Since the data and our model use different definitions for profit, the discrepancies are little surprising. In reality, firms tend to invest and hire new employees in good times when profits are high. In our model, hiring more people will decrease contemporaneous profits, while the gains only materialize in the next period. In reality firms can use debt or issue new equity to finance investments, a possibility our model does not capture.

5.1 Sub-Periods

When inspecting the time series of profit shares presented in Figure 5, different regimes stand out. Between 1947 and 1969 the share is almost flat, but rises at the beginning of the 1970s. From 2000 onwards, we see strong variability in the rate. We divide the entire period accordingly. The first period ranges from 1951 to 1970, where the start is determined by data availability and the end coincides with the end of the NBER recession in 1970. The second period lasts until the burst of the dotcom bubble in 2000, while the last sub-period ranges from 2001 to the end of 2016.

All estimation results are reported in Appendix C. Each table relates to a specific sub-period. We briefly summarize the main findings below. The coefficient associated with the labor share remains consistently negatively correlated with profit share and even increases in magnitude. This means that the tradeoff between profit share and labor share becomes stronger over time. While the investment share has a significantly negative effect on profit share in the period 1971-2000, this effect turns positive in the period 2001-2016. The variable job openings is not significant when running regressions per period. We now use fewer observations for each regression, thus standard errors tend to be bigger.

A different way to control for changes in the underlying regimes is to use dummy variables. We therefore run a regression over the entire length of the sample and control for the different regimes with time period dummies. The results are presented in Table 8. We observe that the labor share has a significant negative effect on the profit share, while the investment share is insignificant. The negative coefficient on the job openings is significant at the 10% level. The time dummies do not enter significantly, indicating that the observed relationships are stable over the entire time period.

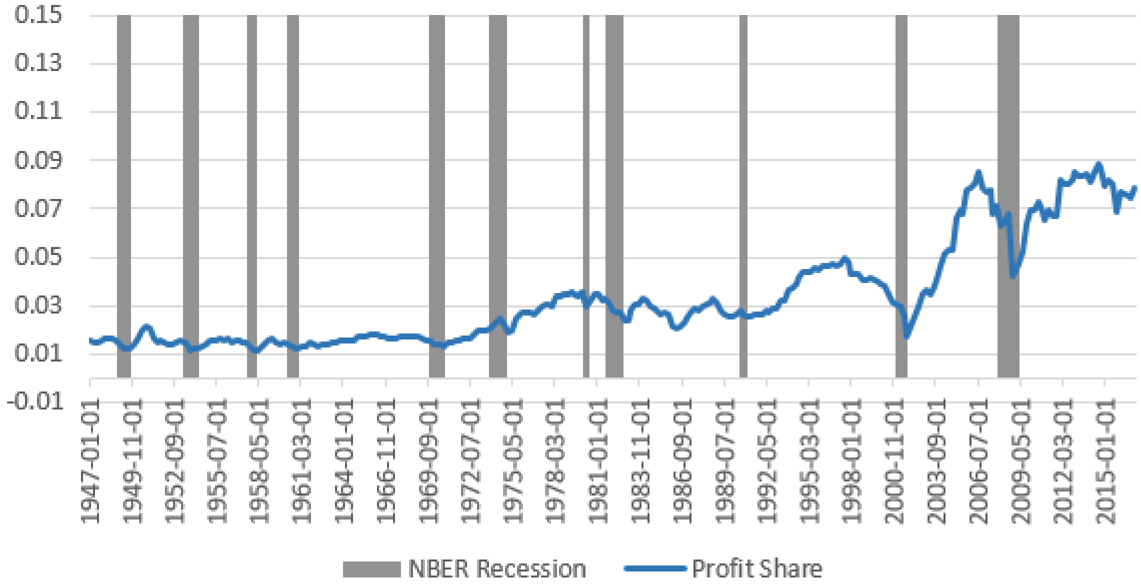
To sum up, there is a clear negative relationship between the labor share and the

Table 8: Regression over the Full Sample Period with Time Dummies

	(1)
L.Profit share	0.860***
	(0.0322)
Labor share	-0.103***
	(0.0249)
Investment share	0.0184
	(0.0179)
Job openings	-0.0388
	(0.0208)
Period1	0.0354
	(0.0492)
Period2	0.0997
	(0.130)
Constant	6.932***
	(1.624)
N	263
adj. R^2	0.980

Notes: See Table 6. We use 1951-1970 as our reference period. Period1 represents the period from 1971-2000 and Period2 stands for 2001-2016.

Figure 5: Non-Financial Profit Share in the U.S.



Notes: Corporate non-financial profits divided by non-financial GDP, seasonally adjusted. Downloaded from FRED database. The shaded areas indicate NBER recessions.

profit share. This result is robust across alternative specifications and is consistent with the results generated by our theoretical model.

5.2 Correlations

In addition to performing regression analyses, we can compare the correlations between the time series we observe in the data to their model counterparts. If we split up the time series into the three periods previously described, we get the correlation matrices observed in Table 9. We focus on the correlation between profit share and labor share. While it is strongly negative in the beginning, it grows less negative in the second period, only to become negative again from 2000 onwards.

A similar pattern can be observed for our model. When the degree of substitutability increases, the correlation between the profit share and the labor share becomes more negative. This is because a higher wage bill lowers the profit of the firm, but then the firm can more easily rely on capital in output production. However, these results should be taken with a grain of salt, because the post 2000 sample period is relatively short and includes the Great Recession.

Table 9: Empirical Correlations by Sub-Periods

	Profit	Labor	Hiring	Investment
	1951Q1 - 1970Q4 (80 obs.)			
Profit	1			
Labor	-0.6285	1		
Hiring	0.5649	-0.1566	1	
Investment	0.8161	-0.5914	0.5154	1
	1971Q1- 2000Q4 (120 obs.)			
Profit	1			
Labor	-0.2550	1		
Hiring	0.5688	0.0026	1	
Investment	0.4732	-0.198	0.8027	1
	2001Q1-2016Q4 (64 obs)			
Profit	1			
Labor	-0.3476	1		
Hiring	0.6789	0.1798	1	
Investment	0.8022	-0.1161	0.827	1

Notes: All variables except for hiring are expressed relative to output.

We also see that the correlation between the labor share and the investment share has turned less negative over time, which can be interpreted as evidence for skill-biased technological growth.[26] As firms invest more, the labor share does not decline by as much as it used to, because firms still need better qualified workers with higher wages to handle the newly installed technologies.[27] Our model replicates the positive correlation between hiring and investment. This happens because of the complementarities between capital and labor. The correlations between these two empirical series increases over time, which is consistent with what happens in our model under increased substitutability. With higher substitutability, the firm chooses a more capital-intensive input mix, thereby increasing the marginal product of an additional worker. Following positive productivity shocks, it pays to hire more workers.

6 Conclusions

We have developed a dynamic stochastic equilibrium model of a frictional labor market where firms search for suitable workers by posting vacancies and wages, and unemployed

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[26] See Krusell et al. (2000).

[27] As different skill levels are beyond the scope of this paper, we will refrain from exploring these results in greater detail.

workers search for jobs. Firms use capital and labor for producing output with the help of a technology that exhibits a constant elasticity of substitution at any point in time. This elasticity can be varied over time. Firms can flexibly adjust capital, but expanding labor is subject to search frictions. We have calibrated this model to the U.S. economy and used it to disentangle the role that a steady decline in the relative price of new capital goods or a change in production technology towards increased factor substitutability play in explaining the following empirical trends: a rise in the capital-to-labor ratio and in the level and variability of firms' profit-to-output ratio as well as a decline in the labor share of income.

Our quantitative results underline the importance of studying the decline in the price of capital and increased factor substitutability separately, but in an integrated framework. While each change can help explain the observed upward trends, only the rise in factor substitutability generates the observed decline in the labor share. Hence, a possible interpretation of the empirical facts seen through the lense of our model is that the implications of increased factor substitutability have quantitatively outweighed those of a decline in the relative price of new capital goods.

Our model of firms using capital and labor for output production while operating in frictional labor markets is rich yet tractable enough to lend itself to various extensions so that it can help study closely related issues in macro/labor, or labor/finance. The implicit assumption that firms use retained earnings to pay for investment renders a counterfactual negative correlation between investment expenditures and profit shares. Therefore, a natural next step could be to allow firms to take on debt, thereby choosing their capital structure and make this choice dependent on the structure of the labor market. When combined with firm heterogeneity, this framework can be the analytical basis for studying the cross-sectional implications for the level and variability of return on equity as examined by Shim (2015).

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A Recursive Wages

In a competitive search framework where firms post wage contracts, firms can decide to offer different wages in different periods. This can be caused by shocks, which will affect the optimal wage posted by the firm and can create a wage dispersion within a firm. To avoid keeping track of the entire wage distribution, we use the following recursive formula:

$$w_{t+1}l_{t+1} = w_t l_t(1-\nu) + \tilde{w}_t h_t$$

To show that this formulation is equivalent in terms of the total wage bill to keeping track of the entire wage history of wages posted by the firm, consider a firm in period t with l_t employees at a wage rate w_t. It hires h_t new employees at a wage rate $\tilde{w}_t$, while ν of the existing workforce leave the firm. For the firm it doesn't make a difference whether it pays a new wage rate w_{t+1} to all of its employees in period $t+1$, which are made up by $l_t(1-\nu)+h_t$ or whether it pays $(1-\nu)l_t$ of its employees a wage w_t and the other h_t receive $\tilde{w}_t$. As all earnings are pooled due to the big family assumption, also the household only cares about the total wage bill. We can now simply shift back the time index by one period, and are in the same situation as before, because w_t and l_t are state variables for the firm. We thus have shown that the recursive formulation of wages allows us to calculate the posted wages in a consistent way.

B An Alternative Formulation of the Firm's Problem

This is an alternative formulation of the problem, where all the laws of motion are written as constraints. It makes for a nice distinction between the choice variables of the firm in period t, $(v_t, \theta_t, \tilde{w}_t, i_t)$, and the endogenous state variables in the next period. However, the resulting system of equations is more complicated, but eventually determines the same equilibrium.

$$
\begin{aligned}
\mathcal{L} = \max_{v_t,\theta_t,\tilde{w}_t,i_t} \mathbb{E}_t \sum_{t=0}^{\infty} & \beta^t [z_t y(k_t, l_t) - w_t l_t - i_t p^k - a v_t] \\
& + \lambda_1 \left[U_t - (1 - p(\theta_t))b - p(\theta_t)\tilde{w}_t\right] \\
& + \lambda_2 \left[l_{t+1}w_{t+1} - (1-\nu)l_t w_t - v_t q(\theta_t)\tilde{w}_t\right] \\
& + \lambda_3 \left[l_{t+1} - (1-\nu)l_t - v_t q(\theta_t)\right] \\
& + \lambda_4 \left[k_{t+1} - (1-\delta)k_t - i_t\right]
\end{aligned}
$$

Differentiating with respect to the 4 choice variables and next period's endogenous state variables leads to the following nonlinear system of equations. As we have 4 Lagrange multipliers we denote their time indices by superscripts rather than subscripts.

$$
\begin{aligned}
\frac{\partial}{\partial v_t} :& - a - \lambda_2^t q(\theta_t)\tilde{w}_t - \lambda_3^t q(\theta_t) = 0 \\
\frac{\partial}{\partial \theta_t} :& \lambda_1^t [p'(\theta_t)b - p'(\theta_t)\tilde{w}_t] - \lambda_2^t v_t q'(\theta_t)\tilde{w}_t - \lambda_3^t v_t q'(\theta_t) = 0 \\
\frac{\partial}{\partial \tilde{w}_t} :& - \lambda_1^t p(\theta_t) - \lambda_2^t v_t q(\theta_t) = 0 \\
\frac{\partial}{\partial i_t} :& - p^k - \lambda_4^t = 0 \\
\frac{\partial}{\partial w_{t+1}} :& \lambda_2^t l_{t+1} + \beta[-l_{t+1} - \lambda_2^{t+1}(1-\nu)l_{t+1}] = 0 \\
\frac{\partial}{\partial l_{t+1}} :& \lambda_2^t w_{t+1} + \lambda_3 + \beta\left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial l_{t+1}} - w_{t+1} - \lambda_2^{t+1}(1-\nu)w_{t+1} - \lambda_3^{t+1}(1-\nu)\right] = 0 \\
\frac{\partial}{\partial k_{t+1}} :& \lambda_4^t + \beta\left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial k_{t+1}} - \lambda_4^{t+1}(1-\delta)\right] = 0
\end{aligned}
$$

The equilibrium conditions are the same, although there are 4 Lagrange multipliers, where only λ_4 can be substituted. The other have co-dependencies, which is why we decided to present the other formulation in the main part of the paper.

Table 10: Regressions 1951Q1-1970Q4

	(1)	(2)	(3)
Labor share	-0.0314**	-0.0558***	-0.0308**
	(0.0108)	(0.00942)	(0.0101)
Investment share	0.122***		0.0875***
	(0.0148)		(0.0207)
Job openings	0.0304*	0.0409*	0.0151
	(0.0126)	(0.0159)	(0.0139)
L.Investment share		0.0750***	
		(0.0152)	
L.Profit share			0.247*
			(0.119)
Constant	2.265**	4.418***	2.252**
	(0.827)	(0.674)	(0.798)
N	80	79	79
Adj. R^2	0.815	0.745	0.841

Notes: The dependent variable is profit share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

C Estimation Results by Period

We now present in greater detail the empirical analysis in each sub-period, which we briefly described in the main text. Each regression table is structured in the following manner. In column (1) we estimate the regression model described in equation (5). We see - as we expect - that the coefficients of labor share and hiring cost share enter with a negative coefficient. The coefficient of investment share is significantly positive, which is surprising. This result remains when we include lagged investment in (2). We detect autocorrelation in the residuals using the Breusch Godfrey-Test and therefore include the first Lag of profitshare in (3). We see that the coefficient of investment share changes signs in the interim period, consistent with our model predictions. However, this change is reversed in the post-2000 period.

Table 11: Regressions 1971Q1-2000Q4

	(1)	(2)	(3)
Labor share	-0.394***	-0.461***	-0.0591*
	(0.0662)	(0.0604)	(0.0231)
Investment share	0.423***		-0.0390*
	(0.0376)		(0.0172)
Job openings	-0.481***	-0.465***	0.0366
	(0.0933)	(0.0906)	(0.0297)
L.Investment share		0.413***	
		(0.0360)	
L.Profit share			0.980***
			(0.0294)
constant	24.83***	29.30***	4.377**
	(4.409)	(3.940)	(1.495)
N	120	119	119
adj. R^2	0.638	0.639	0.970

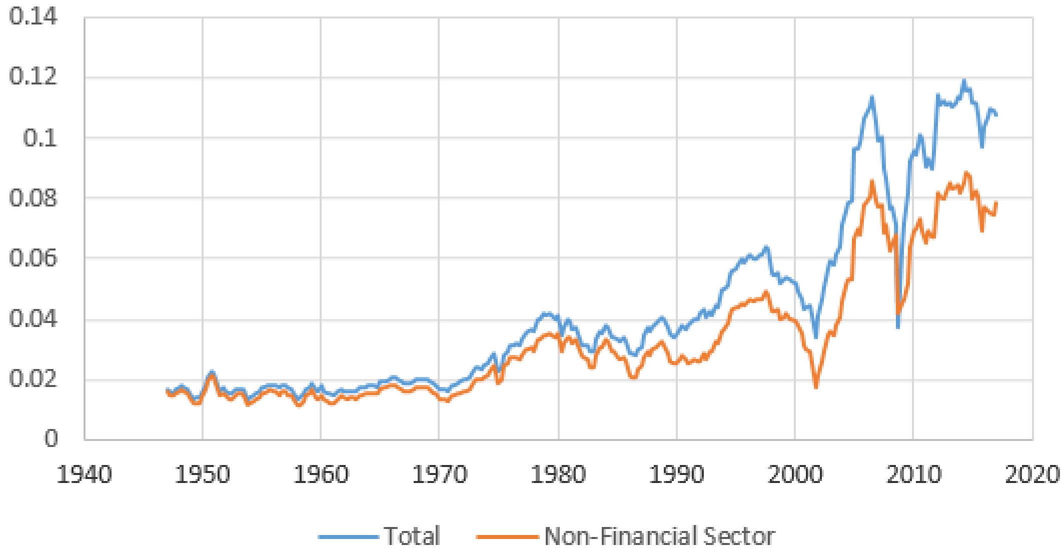
Notes: See Table 10.

Table 12: Regressions 2001Q1-2016Q4

	(1)	(2)	(3)
Labor share	-1.184***	-1.263***	-0.540***
	(0.0682)	(0.0744)	(0.149)
Investment share	0.469***		0.260*
	(0.113)		(0.116)
Job openings	0.710**	0.568	-0.0425
	(0.253)	(0.298)	(0.209)
L.Investment share		0.514***	
		(0.135)	
L.Profit share			0.596***
			(0.0955)
constant	71.62***	76.01***	32.43***
	(3.599)	(3.719)	(8.300)
N	64	63	63
adj. R^2	0.852	0.841	0.926

Notes: See Table 10.

Figure 6: Profit Shares in the U.S.



Notes: Seasonally adjusted. Profits were normalized by GDP. Downloaded from FRED database.

D Data Appendix

The data we use are of quarterly frequency. They relate to the United States and cover the period from 1951Q1 to 2016Q4. All data were downloaded from the FRED database unless noted otherwise.[28]

D.1 Output and Profit

We use the seasonally adjusted data on GDP downloaded from the FRED database. For profit, we take non-financial corporate profit, which is seasonally adjusted. We exclude the financial sector because we analyze a real model and therefore have no role for a financial sector. However, we also perform the empirical analysis using the entire corporate profit time series. The results are virtually unchanged. To illustrate this, we plot the resulting profit shares in Figure 6.

The two series track each other quite closely, but start to diverge around 1971. At this time the difference increases, meaning that the financial sector has become relatively more profitable. An interesting observation is the last quarter of 2008. In this quarter, the financial sector in total is making negative profits. Thus, the total

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[28] FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series`

profit in the U.S. is below the non-financial profit.

D.2 Investment, Price of Capital and Labor Share

As investment we use *Gross Private Domestic Investment*, which is seasonally adjusted and chained in 2009.

The price of capital which is depicted in Figure 2 is calculated by dividing the investment deflator by the consumption deflator. This is precisely the definition of the price of capital in our model and the rate at which output goods can be transformed into capital.

The labor share of income is constructed by normalizing the index of the non-financial corporate sector to its 2009 value of 60%.

D.3 Job Vacancies

For this time series we rely on the work by Barnichon (2010), who carefully combines the traditional Help-Wanted-Index taken from the print version of newspapers with the Job Openings and Labor Turnover Survey (JOLTS), which is available from 2000 onwards. The author publishes updates on his website.[29] The data are available at a monthly frequency from 1951 to 2016. We time-aggregate them to a quarterly frequency using the mean. In this way, we obtain a time series which is consistent for a long time horizon.

D.4 Capital and Labor

When comparing capital intensities, we are restricted to using yearly data due to the availability of data on the U.S. capital stock.

We use data on the capital stock at constant national prices. For employment we use two distinctive variables. One is the hours worked by full-time and part-time employees, and the other one is the employees who are on a non-farm payroll.

When calculating the capital-to-labor ratio, i.e. the capital intensity of production, we get two different series because we use different denominators. However, both series are increasing in the period from 1950 to 2014, as can be seen in Figure 7. It depicts the ratios of capital to the number of workers, and the one to total hours worked, respectively. Both ratios are steadily increasing during the period of observation.

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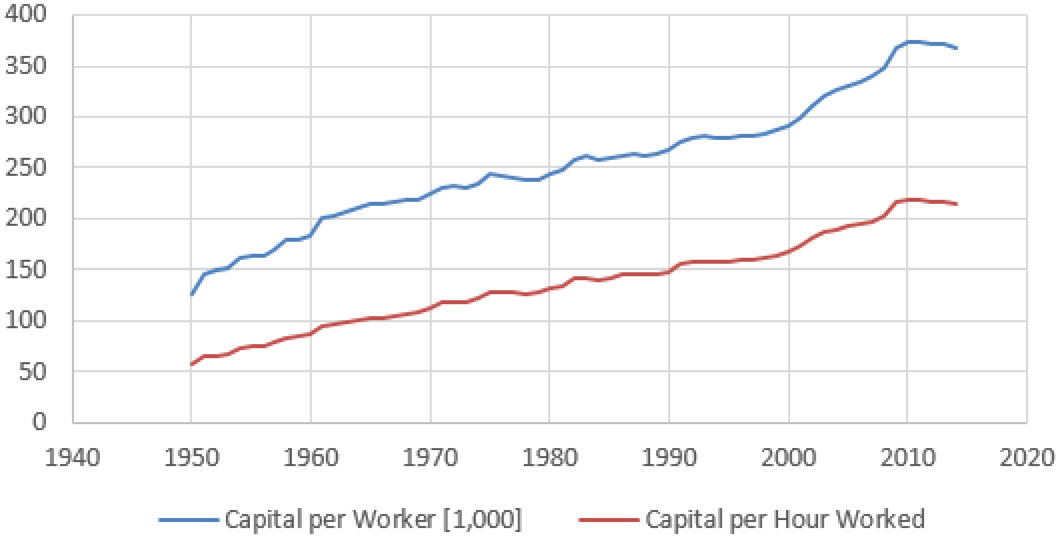
[29] `https://sites.google.com/site/regisbarnichon/data`

Figure 7: Capital Intensities in the U.S.



Notes: Capital at constant national prices in 2011 US-Dollars. Workers include those from the non-farm sectors. Hours worked are by full-time and part-time employees.

E Using Dividends and Corporate Profits

One important distinction between our model-based definition of profits and the corporate profits we observe in the data is the treatment of investment. Investment reduces profits in our model, but does not affect corporate profits which are defined according to legal accounting standards. We try to tackle this issue in two ways. First, we perform the regression analysis using dividend share rather than profit share as dependent variable. Second, we define as corporate profit in our theoretical model the sum of profit and investment and compute its correlation with the other variables.

E.1 Regressions on Dividend Share

We construct the dividend share by using FRED data on dividends and divide it by GDP. We then run regressions for the full sample and for each sub-period, corresponding to the regressions in the text.

The main changes in the full sample regressions, presented in Table 13 are not in the scaling of the coefficients. Now the number of job openings also enters with a significant negative sign, which arguably points to the fact that new hires are financed by current revenues, thus reducing profits.

Table 13: Dividend Shares

	(1)	(2)	(3)
Labor share	-0.00404***	-0.00403***	-0.000253**
	(0.000277)	(0.000247)	(0.0000821)
Investment share	0.00353***		0.000329*
	(0.000222)		(0.000136)
Job openings	-0.00566***	-0.00574***	-0.000447**
	(0.000545)	(0.000517)	(0.000156)
L.Investment share		0.00370***	
		(0.000206)	
L.Dividend Share			0.935***
			(0.0255)
Constant	0.247***	0.244***	0.0144*
	(0.0194)	(0.0171)	(0.00592)
Observations	264	263	263
Adjusted R^2	0.861	0.880	0.987

Notes: The dependent variable is dividend share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: Regressions on Dividend Share by Sub-Period

	(1)	(2)	(3)
Labor share	0.00000207	-0.0000610	-0.00106***
	(0.0000128)	(0.0000419)	(0.000297)
Investment share	0.0000152	-0.000143**	0.000553
	(0.0000256)	(0.0000459)	(0.000523)
Job openings	-0.0000160	0.000233***	0.00163
	(0.0000242)	(0.0000601)	(0.00196)
L.Dividend Share	0.980***	1.027***	0.770***
	(0.0288)	(0.00626)	(0.0949)
Constant	-0.000167	0.00507	0.0624**
	(0.00104)	(0.00304)	(0.0190)
Period	1951Q1-1970Q4	1971Q1-2000Q4	2001Q1-2016Q4
Observations	79	119	63
Adjusted R^2	0.966	0.999	0.873

Notes: See Table 13

E.1.1 Splitting up the Periods

This exercise corresponds to the one presented in Appendix C, where we divide our sample into three sub-periods, with dividend share as dependent variable. We will only report the results for the model including one lag in the dividend share, due to autocorrelation in the other variants of the regression model.

For the period 1951Q1-1970Q4, we see that the only significant variable is lagged dividend share, which suggests that dividends in that time were not very volatile and are best explained by an AR-(1) process. In the intermediate period, the coefficient on labor share is not significant, but investment enters with a negative coefficient. Although this is in line with the predictions of our model, this result disappears again in the period following 2000, when the coefficient on labor share turns significantly negative. Overall, no clear picture emerges when looking at dividends as proxy for economic rents, as dictated by our model. Our data span a long time period, and it is likely that changes in the governance of dividends have appeared over time.

Table 15: Correlations

	Profit	Labor	Hiring	Investment	Corp. Profit
Profit	1				
Labor	-0.1288	1			
Hiring	-0.8052	-0.4793	1		
Investment	-0.9137	-0.2834	0.9748	1	
Corp. Profit	0.3914	-0.9623	0.2239	0.0146	1

Notes: Approximated correlation of the model, including corporate profits.
All data are HP-filtered, with a smoothing factor of 1600.

E.2 Correlations of Corporate Profits

A different way to bridge the differences in the definition of profit between our model and the data is to define a variable *Corp. Profit*, which is revenue minus wage payments and hiring costs, and calculate its share. We present the obtained correlations in Table 15. The strong negative correlation between investment share and profit share resulting from our model makes this newly constructed share virtually uncorrelated with investment. Qualitatively, correlations now are the same as what we report in Table 7 for the U.S. economy, since all signs match their empirical counterparts. Quantitatively, there are still discrepancies, due to our model abstracting from the financing decisions of firms and other real world phenomena.