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ABSTRACT

Intertemporal Labor Supply: A Household Collective Approach*

This paper proposes an extension of the collective model for labor supply developed by Chiappori, Fortin and Lacroix (2002) to an intertemporal setting. We first develop a theoretical model to analyze the intra-household distribution of wealth in a multi-period framework, with a focus on labor supply and marriage markets. The model allows us to derive a sharing rule for non-labor income under a set of testable conditions. Second, using data from the Panel Study of Income Dynamics from years 1997 to 2015, we estimate the model using a semi-log parametrization of labor supply. Our empirical results do not reject the restrictions of the model, and point to the validity of the collective framework in an intertemporal setting. We show that wages are positively related to household labor supply, although cross and lagged effects show negative correlates. Furthermore, the ability of wives to negotiate the intra-household allocation of non-labor income is mainly driven by wages, with wives behaving altruistically, and husbands egoistically. Sex ratios appear to be nonsignificant in this relationship, although counteracting effects between labor and marriage markets may influence estimates.

JEL Classification: D15, J22

Keywords: household labor supply, collective model, intra household behavior, sex ratios, panel study of income dynamics

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1. Introduction

This paper proposes an extension of the collective model for labor supply with distribution factors, of Chiappori, Fortin and Lacroix (2002), to an intertemporal setting. We test the model using data from the Panel Study of Income Dynamics (PSID) from years 1997 to 2015. We estimate the labor supply of husbands and wives of US households, using a semi-log parametrization, in terms of couple's own and cross wages, non-labor income, preference factors, and sex ratios (by age and race), as a measure of distribution factors, all of which allows us to derive from the labor supply estimates the intra-household sharing rule of non-labor income (up to an integration constant).

Traditionally, the household has been studied following the so-called "unitary" approach, that takes the family as a whole unit, i.e., defines a unique utility function for the whole household that is maximized subject to constraints, and the only interest is on the total amount of resources available for the household as a single unit. This unitary approach entails certain difficulties and limitations, which includes the lack of a robust theoretical foundation or not meeting the neoclassical microeconomic rules (e.g., individualism). In this framework, family internal behavior is considered as a black box and household formation and/or dissolution is not considered, which prevents from individual welfare considerations or the analysis of intra-household inequality. The unitary approach is characterized by the lack of convincing empirical results (e.g., the income pooling property, which says that individual non-labor income does not affect household behaviors).[1]

Becker (1973, 1974a, 1974b, 1981a, 1981b) first demonstrated the need for a new approach to the study of the family, allowing for different preferences for each household member. Several models of household behavior appeared in the literature in the 1980s, considering individual preferences, although there was no consensus on the mechanisms behind intra-household behaviors (e.g., Manser and Brown, 1980; Ashworth and Ulph, 1981; McElroy and Horney, 1981; Apps, 1981, 1982; Bourguignon, 1984; Apps and Jones, 1986; Ulph, 1988; and Woolley, 1988). Chiappori (1988, 1992), Bourguignon et al. (1993), and Browning and Chiappori (1998) then generalized the collective framework. In particular, it was in Chiappori (1992) where a collective model of household labor supply was first proposed, and Browning and Chiappori (1998) introduced the concept of "distribution

[1] There are three alternatives in the literature that aim to consider household member preferences in a unitary approach: Samuelson's welfare index (Samuelson, 1956), the Rotten Kid theorem (Becker, 1974b), and the Transferable utility assumption. However, these alternatives do not provide convincing results.

factors". Chiappori et al. (2002) formally included distribution factors in a general collective framework, and provided empirical support to the model. Since then, several studies have pointed to the validity of the collective model (e.g., Browning et al., 1994; Haddad and Hoddinott, 1994; Lundberg et al., 1997; Browning and Chiappori, 1998; Duflo, 2000; Chiappori et al., 2002), with many authors estimating the collective model (see, for example, Rapoport et al. (2011) and Lyssiotou (2017) for two recent empirical works). Two surveys of the literature on models of household behavior can be read in Donni and Chiappori (2011), and Chiappori and Mazzocco (2018).

The general collective models developed in the 1990s were mostly static, in the sense that only one period of time was considered. Thus, family members were assigned intra-family weights, and then negotiated some commitment plan to allocate family wealth and goods. Using the second fundamental welfare theorem, household members optimize their own utility function, in terms of their own preferences, subject to individual constraints, and also subject to the commitment plan negotiated within the household. However, Chiappori and Mazzocco (2018) argue that static models of household behavior have two main limitations: they cannot evaluate policies (which generally have an intertemporal aspect), and they cannot analyze the evolution of intra-household processes. Thus, a static framework appears to be not optimal in studying intertemporal behaviors, and a multi-period analysis seems more realistic. It is important to remark that the two previous limitations are not independent, since the analysis of intra-household processes (e.g., distribution of wealth, bargaining powers, or intra-family inequality) is important from the point of view of public policy, as social programs related to household welfare and wealth, or programs about gender equality, could be more efficiently implemented if intra-household processes were better understood (Mazzocco, 2007).

Despite the importance of the intertemporal approach, most of the theoretical and empirical literature on household intertemporal decisions has remained in the unitary field, considering households as whole units, with few exceptions. Among these exceptions, Aura (2005) develops a collective model with "limited intertemporal commitment". Lich-tyler (2001) examines theoretically three perspectives of multi-period household bargaining problems, and uses the PSID to find that different households use different procedures. Mazzocco (2007) proposes a test of intra-household commitment using consumption data from the US Consumer Expenditure Survey, and finds that household intertemporal behaviors are subject to the ability of household members to commit. Kalugina et al. (2009a, 2009b) study intra-household inequality using the Russia Longitudinal Monitoring Survey. Voena (2015) studies how US divorce laws affect household intertemporal choices and wellbeing, using the PSID. Blau and Goodstein (2016) study the effect of inheritances on the labor force participation of spouses in the US, using the Health and Retirement Study data. Finally, Chiappori and Mazzocco (2018) provide a survey of the literature about static and intertemporal household decisions, discuss the benefits of the collective models (against the unitary approach), and pay particular attention to the different intertemporal models of household behavior.

Within this framework, this paper adapts the collective framework for labor supply to an intertemporal setting. We follow the theoretical model of Chiappori et al. (2002), and propose a version adapted to T discrete periods of time, and a setting of limited intertemporal commitment. The interpretation of the model is mostly similar, and although it is first proposed using a general specification, we then adopt a particular semi-log parametrization for the subsequent empirical analysis. The model, both in its general version and in its concrete parametric form, yields similar results to those of Chiappori et al. (2002), given that we find a similar, but more complex, set of necessary and sufficient conditions to be fulfilled by the labor supply equations for the model to be solved, that also lead to a general sharing rule of non-labor income. Using data from the PSID of the United States for years 1997 to 2015, we then estimate the parametric form of the model.

We find that both wives' and husbands' wages are positively related to wives' labor supply, while husbands' labor supply is mainly affected by their own wage; however, cross-wages show negative correlations with both labor supplies, and non-labor income only shows a small and negative correlation with husbands' hours of work. Furthermore, wives' hours of work are negatively related to wives' lagged wage rates, while husbands' hours of work are related to both wives and husbands' lagged wages. Finally, we derive the sharing rule of non-labor income, which is determined significantly on present and lagged wages, cross-wages, and non-labor income. However, partial derivatives show that only marginal changes in present wages are followed by significant changes in the sharing rule, and that wives are, in general, altruistic in the intra-family behaviors, but husbands are egoistic. Sex ratios appear to be non-significant in this relationship, although the counteracting effects of marriage and labor markets may be influencing estimates, given that sex ratios are significant, and of opposite directions for wives and husbands, when labor supplies are estimated as conditional on the sharing rule.

The contributions of the paper are twofold. First, the paper extends the general collective model with distribution factors proposed by Chiappori et al. (2002) to a limited intertemporal commitment setting. In doing so, we make use of the development of the original model, and also use a similar semi-log parametrization. We find a similar, but more complex, set of necessary and sufficient testable conditions to be verified by the labor supply equations, and a formula to integrate the sharing rule of non-labor income in terms of wages, past-wages, non-labor income, distribution factors, and the parameters of the household labor supply. Second, we estimate the labor supply equations using data from the PSID from years 1997 to 2015. Estimates do not reject the validity of the model, and thus we can impose the set of testable conditions, and obtain an estimated equation for the sharing rule of non-labor income. We then compute the derivatives of the sharing rule, and also labor supply elasticities. Finally, we find that the estimates of the collective model are consistent with estimates of individual labor supply, conditional on the sharing rule, in line with the second fundamental welfare theorem.

The remainder of the paper is organized as follows. Section 2 shows an overview about intertemporal household models. The theoretical model is developed in Section 3. The concrete parametric form of the model to be estimated is proposed in Section 4, and the theoretical model is reformulated under that parametrization. Section 5 describes the data and empirical strategy, and Section 6 shows estimation results. Finally, Section 7 presents our main conclusions.

2. Intertemporal household decisions

Chiappori and Mazzocco (2018) argue that, in the study of the family, "the choice of a specific model of household behavior is never irrelevant, and almost never innocuous". In this sense, it is important to choose the proper model to study family behaviors, and results, conclusions and policy implications may depend on that choice. In this section, we provide a brief overview about the convenience of intertemporal models for household behaviors, and we compare the two main models that coexist in the literature: the full intertemporal commitment (FIC) model, and the limited intertemporal commitment (LIC) model.[2] A more complete and detailed survey of the literature about household decisions, including the unitary approach, non-cooperative models, household models, and the different intertemporal approaches, can be read in Chiappori and Mazzocco (2018).

[2] Intertemporal non-cooperative models of household behavior (e.g., models based on Nash equilibrium) are beyond the scope of this paper.

The static models of household behavior, including the unitary approach, the non-cooperative models, and the collective models, have two related limitations, arising from its static nature: they cannot be used to evaluate policies (which generally are associated to intertemporal effects), and they cannot explain the evolution of intra-household processes, such as household formation, divorce, income and wealth transfers, or equity. Then, in the last two decades, some models of intertemporal household behaviors emerged to fill this gap. Although in the static framework there is consensus in the validity of the collective model, against the classical unitary approach, the most common intertemporal models of household behavior are unitary models. These models have then the limitation of their static counterparts: they are not well suited to study intra-household processes, and will then lead to imprecise results.

Alternatively, during the 2000s and the 2010s, some intertemporal collective models have been developed to study the intertemporal dimension of household decisions that could not be studied from the unitary approach. For instance, there are two main (and nested) families of intertemporal household models: LIC models, and FIC models (with the FIC being a particular case of the LIC). The difference between these two models resides in the ability of household members to commit to future allocation plans. This ability may be of few relevance in a static framework, where no temporal trends are considered. However, in an intertemporal setting, to claim that household members can commit to a future and invariant allocation plan, and that this plan cannot be affected by shocks of any type (as is the case of the FIC), is a too strong hypothesis. Then, the LIC model relaxes this hypothesis, and allows the existence of an "outside option", and a renegotiation of spouses' bargaining powers to guarantee the efficiency of household decisions. [3] A more detailed description of the LIC model can be read in Section 3.

Some tests have been proposed in the literature to evaluate the different household models and, in particular, the intertemporal models. The most common test is proposed for intertemporal unitary models, and based on Euler equations: it checks whether households consume, and choose future consumption, according to Euler equations evaluated at the time the decision is taken. However, this test does not consider intra-household aspects, and then it is not suitable for intertemporal collective models. In this line, Mazzoco (2007) tests the

[3] The definition of the outside option may be crucial in the study of household decisions (Chiappori and Mazzocco, 2018). For instance, the most commonly used definition of the outside option is the value of being divorced, although other authors define it as the individual welfare in a non-cooperation marriage. In our case of study, although both definitions are compatible, we do not consider the non-cooperation option, and we will refer to the former definition.

validity of the unitary intertemporal model, against intertemporal commitment models (the FIC and the LIC), using information about present and future consumption. Results reject the validity of the unitary intertemporal approach, in favor of the LIC and FIC models. Finally, Mazzocco (2007) also tests the validity of the LIC, against the FIC, assuming that in the second case, household Euler equations in terms of future consumption can be only determined by present bargaining powers, and using data for the United States rejects the validity of the FIC, in favor of the LIC. Lise and Yamada (2014) also reject the validity of the FIC, against the LIC, using a similar test with the Japanese Panel Survey of Consumers.

Finally, in addition to the validity of models using concrete data, another important characteristic of household models is that they should let to identify and recover intra-household processes (e.g., identification and testable conditions). However, as argued by Chiappori and Mazzocco (2018), the study of the identification of LIC models is an important topic that needs further research. We contribute to this topic by proposing a LIC model of household behavior based on Chiappori et al. (2002), and obtaining a set of testable conditions, similar to the set obtained in the benchmark model, to study its identification and validity.

3. The model

We develop a theoretical framework for household behavior, with respect to consumption of private goods and hours of work and/or leisure, taking Chiappori et al. (2002) as our reference. Assume a household formed by two members, $i = 1, 2$, living for T discrete periods of time, with preference factors $\mathbf{z}$, distinct utility functions, and full information about spouses' preferences.[4] We assume that the household decision process leads to Pareto-efficient outcomes, which is given by the knowledge of spouses' preferences (Chiappori, 1992; Chiappori et al., 2002). That is to say, we assume that every decision is on the Pareto frontier.

In each period $t = 0, \ldots, T$, each household member $i = 1, 2$ consumes a Hicksian good with unitary price, c_{it}. We assume that aggregate consumption c_t is observed in each period, but not individual private consumption (Chiappori, 1992). Each member i also chooses the amount of time devoted to work activities in each period, $h_{it} \in [0, 1]$. Wage rates at t are

[4] The assumption of complete information may be strong, but it has been taken for granted in the literature about household behaviors (Chiappori and Mazzocco, 2018).

defined as w_{1t} and w_{2t}, vary across periods, and are exogenous. Household members get utility from own consumption and leisure, defined as the total time (normalized to 1) minus the time devoted to work. Preferences of household members are then egoistic (in the sense of assumption *E* of Chiappori, 1992) and the utility function forms are time-invariant, although the function arguments vary over time:

$$u_{it} = u_i(c_{it}, 1 - h_{it}), \;\; i = 1, 2, \;\; t = 0, \dots, T. \tag{1}$$

Note that this egoistic specification of preferences is just a particular type of "caring" preferences (where *i*'s utility depends on *j*'s felicity, but not directly on *j*'s preferences). Although caring preferences may be more general, egoistic preferences have been almost exclusively considered in collective models. We assume unitary discount rates. Household members are allowed to save money each period, s_t, for $t = 1, \dots, T$, with $s_0 = 0$. We assume unitary interest rates. Households are also allowed to have sources of non-labor income each period t, y_t (or, equivalently, we allow for marriage gains, net of savings). These hypotheses lead us to the budget constraint defined by Equation (2):

$$c_{1t} + c_{2t} = w_{1t}h_{1t} + w_{2t}h_{2t} + y_t + s_{t-1}. \tag{2}$$

Since households are assumed to reach Pareto-efficient outcomes, the household must solve the following program:

$$\max_{\{h_{it}, c_{it}\}_{i=1,2}^{t=0,\dots,T}} \sum_{i=1,2} \left\{ \mu_i E_0 \sum_{t=0}^{T} u_i(c_{it}, 1 - h_{it}) \right\}$$

$$\text{s.t.: } c_t + s_t = w_{1t}h_{1t} + w_{2t}h_{2t} + y_t + s_{t-1}, \qquad t = 0, \dots, T \tag{P$_1$}$$

where the household as a whole must maximize a utility function defined as the weighted sum of individual utilities, pondered by Pareto-weights, $\mu_1, \mu_2 \in [0,1]$ such that $\mu_1 + \mu_2 = 1$. That is to say, we can define a unique Pareto-weight μ such that $\mu_1 = \mu$ and $\mu_2 = 1 - \mu$. In particular, we can define these weights as an unobservable function of variables defined at the time the marriage is formed, including a vector $\mathbf{z}$ of sociodemographic characteristics, and a distribution factor *d*, defined as factors that can affect the intra-household allocation, but cannot influence individual preferences, nor the joint consumption set, and guarantees the efficiency of the problem even in an intertemporal framework (Browning and Chiappori, 1998; Chiappori et al., 2002). [5] In contrast to the non-observability of μ, distribution factors

[5] The general collective model developed in Chiappori et al. (2002) makes use of either one or several distribution factors, relying on two distribution factors, sex ratios and an index of divorce laws, in the empirical

are usually observed, which will allow for empirical tests. Thus, μ must be a function of factors known at the moment when the marriage is formed, i.e., of exogenous, time-fixed, or $t = 0$ factors:

$$\mu = \mu(w_{10}, w_{20}, y_0, \mathbf{z}, d). \tag{3}$$

It is important to note that the distribution factor d only affects the position of the solution on the Pareto-frontier, but does not affect the Pareto-frontier itself (Chiappori et al., 2002).

In an intertemporal setting, the assumption of Pareto-efficiency means that family members must commit a non-renegotiable future allocation plan where bargaining powers are equal to Pareto-weights, which are time-fixed (the FIC model). FIC models assume that household decisions are always on the *ex-ante* Pareto-frontier, i.e., in the Pareto-frontier *at the time of household formation* (Chiappori and Mazzocco, 2018). However, this perquisite hypothesis of the FIC models concerning full commitment is too strong, and we then allow for renegotiation, i.e., we relax the full-commitment hypothesis, and propose a LIC model. In doing so, we introduce an extra restriction in (P_1) representing the benefits derived from the best outside option, or what is often called a household participation restriction, as shown in (P_2):

$$\max_{\{h_{it}, c_{it}\}_{i=1,2}^{t=0,\dots,T}} \sum_{i=1,2} \left\{ \mu_i E_0 \sum_{t=0}^{T} u_i(c_{it}, 1 - h_{it}) \right\}$$

$$\text{s.t.: } c_t + s_t = w_{1t}h_{1t} + w_{2t}h_{2t} + y_t + s_{t-1}, \qquad t = 0, \dots, T \tag{P_2}$$

$$\lambda_{it}\text{: } E_\tau \sum_{t=0}^{T-\tau} u_i(c_{it}, 1 - h_{it}) \geq u_{i\tau}^*, \qquad i = 1, 2;\ \tau = 1, \dots, T$$

The following Problem (P_3) is a reformulation of Problem (P_2), but includes the participation constraint in the objective function to maximize, with its corresponding Kuhn-Tucker multiplier, λ_{it}:

$$\max_{\{h_{it}, c_{it}\}_{i=1,2}^{t=0,\dots,T}} \sum_{i=1,2} \left\{ E_0 \sum_{t=0}^{T} \left((M_{it}) u_i(c_{it}, 1 - h_{it}) - \lambda_{it} u_{it}^* \right) \right\}$$

$$\text{s.t.: } c_t + s_t = w_{1t}h_{1t} + w_{2t}h_{2t} + y_t + s_{t-1}, \qquad t = 0, \dots, T \tag{P_3}$$

analysis. Nevertheless, the use of one or more distribution factor does not qualitatively depend on the development of the model.

Where $M_{it} = \mu_i + \lambda_{it}$, for each i = 1, 2, and t = 0, …, T. The Kuhn-Tucker multiplier, λ_{it}, remains null (and then $M_{it} = \mu_i$, and the problem is identical to (P_1)) until one of the household members participation constraint binds. Thus, spouses remain (or participate) in the household, subject to the initial bargaining powers, while the participation restriction remains true for both household members. On the contrary, when at least one of the participation constraints binds (because of a large enough shock in the variables that were initially used to fix the Pareto-weights), three options emerge. First, the case where both members prefer the outside option (i.e., the constraint binds for both household members). We omit this case, which is not of interest, since, if this happened, renegotiation would be impossible. Second, the preference for the outside option for only one spouse, leading to the outside option, or the end of the marriage, which would only occur when a renegotiation plan is not possible, i.e., in the case of non-commitment. Third, the case of interest, when the participation constraint of only one spouse binds, but there exists the possibility of renegotiating the bargaining power of household members. According to this, household decisions would be in the intertemporal Pareto frontier, subject to the participation constraint, in the sense that spouses can choose to take the best outside option, or remain in the marriage by renegotiating the bargaining powers, and then to restore individual rationality.

Assume a particular case when participation constraints do not bind in the first $\tau - 1$ periods (with $\tau > 1$). Then, $M_{it} = \mu_i$ for $t = 0, \ldots, \tau - 1$ (the Kuhn-Tucker multipliers λ_{it} remains null for $t = 0, \ldots, \tau - 1$). However, at $t = \tau$, spouse 1's participation constraint binds for some reason. Then, the previous bargaining powers are no more valid for the household, since then spouse 1 would be better off with the best outside option, and household decisions would not be Pareto-efficient. It is then necessary to renegotiate the bargaining plan. In particular, spouse 1 bargaining power increases by $\lambda_{1\tau} \neq 0$, which makes him/her indifferent between the best outside option and remaining in the marriage, which is the optimal change (lower increases would make the outside option preferable, and higher increases would lead to suboptimal decisions (Ligon et al., 2002)). The intuition behind this process is clear. If spouse 2 is interested in remaining in the family (and he/she is, since his/her participation constraint does not bind), he/she allows individual 1's bargaining power to increase by $\lambda_{1\tau}$, to make him/her indifferent between divorce or remaining within the household. It is important to note that this renegotiation process may be iterative, i.e., participation constraints may bind again.

Now, since egoistic preferences are assumed and efficiency is guaranteed by Pareto-weights and the renegotiation process under the presence of limited intertemporal commitment, we can apply Proposition 1 of Chiappori (1992), and then obtain the following result:

Proposition 1: Problem (P_3), assuming that renegotiation is always possible, is equivalent to the existence of a sharing rule of non-labor income, $\varphi_t \in [0, y_t + s_{t-1}]$, such that members 1 and 2 solve Problem (P_4), formed by the following Problems (P_4^1) and (P_4^2):

$$\max_{\{h_{1t},c_{1t}\}_{t=0}^{T}} E_0 \sum_{t=0}^{T} u_1(c_{1t}, 1-h_{1t})$$

$$\text{s.t.: } c_{1t} + s_{1t} = w_{1t}h_{1t} + \varphi_t, \qquad t = 0, \dots, T \qquad (P_4^1)$$

$$\max_{\{h_{2t},c_{2t}\}_{t=0}^{T}} E_0 \sum_{t=0}^{T} u_2(c_{2t}, 1-h_{2t})$$

$$\text{s.t.: } c_{2t} + s_{2t} = w_{2t}h_{2t} + y_t + s_{t-1} - \varphi_t, \qquad t = 0, \dots, T \qquad (P_4^2)$$

Proof: see Appendix A.

This result is based on the second fundamental welfare theorem, since efficient outcomes can be decentralized, and individual utilities are egoistic, i.e., i's utility function does not depend on j's consumption, for $i \neq j$ (Chiappori et al., 2002). The intuition is as in the case of the general collective model: the problem can always be considered as a two-stage process. First, household members negotiate how non-wage household income is allocated between them. This negotiation results in the commitment of the function φ_t, and depends on wages, non-labor income, preferences, and distribution factor(s). Second, each member individually maximizes their lifetime utility as a function of individual consumption and labor supply (given the egoistic functional form of the individual utilities), subject to their corresponding individual budget constraints.

Note that φ_t represents the amount of non-labor income associated with household member 1, and then the rest of the household non-labor income is assigned to member 2. That way, we can define equivalently $\varphi_{1t} = \varphi_t$ and $\varphi_{2t} = y_t + s_{t-1} - \varphi_t$. For each t, we define the sharing rule as a function of wages, non-labor income, individual preferences, and distribution factors. Furthermore, for $t \geq 1$, we also consider the value of the sharing rule in the previous time period as an argument of the current sharing rule function, in order to control for intra-household behaviors that may be persistent/resistant to changes in wages and/or non-labor income. Thus, we define the sharing rule as follows:

$$\varphi_t = \varphi(w_{1t}, w_{2t}, y_t, w_{1t-1}, w_{2t-1}, y_{t-1}, \mathbf{z}, d). \quad (4)$$

We note that this specification of the sharing rule, oppositely to the definition of the initial Pareto-weights μ_i, does not mean that households allocate, at the beginning of the problem, a fixed rate of non-labor income for each household member. Rather, household members allocate, at the beginning of the problem, a general rule to divide non-labor income, φ, but this sharing rule is sensitive to shocks in present features (hence notated φ_t), and then it is directly allowed to vary across time periods, as happens with parameters M_{it}.

Consider a family of labor supply functions $\{h_{it}\}_{i=1,2}^{t=0,\dots,T}$ that solve (P_4) for a given sharing rule. These functions can be written in a Marshallian form:

$$h_{1t} = H^{1t}(w_{1t}, \varphi_{1t}, \mathbf{z}), \quad (5)$$

$$h_{2t} = H^{2t}(w_{2t}, \varphi_{2t}, \mathbf{z}). \quad (6)$$

This specification resembles that of Chiappori et al. (2002). In particular, at $t = \tau$, changes in spouse i wage $w_{i\tau}$ have a direct effect over the labor behavior of i, plus an indirect effect through φ_{it}. However, this change in $w_{i\tau}$ may also have an impact on household member $j \neq i$, but only indirectly, through φ_{jt}, i.e., through the distribution of the household non-labor income. In the following section, we develop a series of necessary and sufficient conditions on the partial derivatives of the sharing rule to be fulfilled for the family of (general, non-parametric) functions $\{h_{it}\}_{i=1,2}^{t=0,\dots,T}$ to be a solution of the household problem (P_4), for the previous sharing rule. Thus, given an estimation of household members' labor supply functions, whatever the parametrization of such functions happens to be, we would recover the sharing rule of non-labor income from the labor supply estimates.

3.1 Derivatives of the sharing rule, and necessary and sufficient conditions

As in the general collective model (Chiappori, 1988, 1992; Chiappori et al., 2002), we must impose certain restrictions in order to make the problem identifiable. These conditions consist of a series of necessary and sufficient conditions on the partial derivatives of the sharing rule function that must be fulfilled for the family of the labor supply functions to be solutions of the household Problem (P_4), for the given sharing rule that fulfills those restrictions. (See Proposition 3 in Chiappori et al. (2002) for the identification of the sharing rule in the case of the general collective model.)

Assume that the family of Marshallian labor supply functions $\{H^{it}\}$ is continuously differentiable, and the sharing rule φ_t is two times continuously differentiable. For each t, using the chain rule, let us consider the following derivatives of the labor supply functions:

$$\frac{\partial h_{1t}}{\partial w_{2t}} = H^{1t}_{\varphi_t}\varphi_{t,w_{2t}}, \tag{7}$$

$$\frac{\partial h_{1t}}{\partial y_t} = H^{1t}_{\varphi_t}\varphi_{t,y_t}, \tag{8}$$

$$\frac{\partial h_{1t}}{\partial d} = H^{1t}_{\varphi_t}\varphi_{t,d}, \tag{9}$$

$$\frac{\partial h_{2t}}{\partial w_{1t}} = -H^{2t}_{\varphi_t}\varphi_{t,w_{1t}}, \tag{10}$$

$$\frac{\partial h_{2t}}{\partial y_t} = H^{2t}_{\varphi_t}\left(1 - \varphi_{t,y_t}\right), \tag{11}$$

$$\frac{\partial h_{2t}}{\partial d} = -H^{2t}_{\varphi_t}\varphi_{t,d}, \tag{12}$$

$$\frac{\partial h_{1t}}{\partial w_{2t-1}} = H^{1t}_{\varphi_t}\varphi_{t,w_{2t-1}}, \tag{13}$$

$$\frac{\partial h_{2t}}{\partial w_{1t-1}} = -H^{2t}_{\varphi_t}\varphi_{t,w_{1t-1}}, \tag{14}$$

$$\frac{\partial h_{1t}}{\partial y_{t-1}} = H^{1t}_{\varphi_t}\varphi_{t,y_{t-1}}, \tag{15}$$

$$\frac{\partial h_{2t}}{\partial y_{t-1}} = -H^{2t}_{\varphi_t}\varphi_{t,y_{t-1}}. \tag{16}$$

We can define, for each t:

$$A_t = \frac{h^{1t}_{w_{2t}}}{h^{1t}_{y_t}},\ B_t = \frac{h^{2t}_{w_{1t}}}{h^{2t}_{y_t}},\ A'_t = \frac{h^{1t}_{w_{2t-1}}}{h^{1t}_{y_{t-1}}},\ B'_t = \frac{h^{2t}_{w_{1t-1}}}{h^{2t}_{y_{t-1}}},\ C_t = \frac{h^{1t}_{d}}{h^{1t}_{y_t}},\ C'_t = \frac{h^{1t}_{d}}{h^{1t}_{y_{t-1}}},\ D_t = \frac{h^{2t}_{d}}{h^{2t}_{y_t}}.$$

Assume that $C_t \neq D_t$ (otherwise, all the partial derivatives of the sharing rule would be non-finite). This assumption is equivalent to the hypothesis of Proposition 3 in Chiappori et al. (2002). Now, when we incorporate (7) to (16) in the previous parameters, we can rewrite:

$$A_t = \frac{\varphi_{t,w_{2t}}}{\varphi_{t,y_t}} \Rightarrow \varphi_{t,w_{2t}} = A_t\varphi_{t,y_t}, \tag{17}$$

$$B_t = \frac{-\varphi_{t,w_{1t}}}{1-\varphi_{t,y_t}} \Rightarrow \varphi_{t,w_{1t}} = B_t\left(\varphi_{t,y_t} - 1\right), \tag{18}$$

$$C_t = \frac{h^{1t}_{d}}{h^{1t}_{y_t}} = \frac{\varphi_{t,d}}{\varphi_{t,y_t}} \Rightarrow \varphi_{t,d} = C_t\varphi_{t,y_t}, \tag{19}$$

$$D_t = \frac{-\varphi_{t,d}}{1-\varphi_{t,y_t}} \Rightarrow \varphi_{t,d} = D_t\left(\varphi_{t,y_t} - 1\right), \tag{20}$$

$$A_t' = \frac{\varphi_{t,w_{2t-1}}}{\varphi_{t,y_{t-1}}} \Rightarrow \varphi_{t,w_{2t-1}} = A_t'\varphi_{t,y_{t-1}}, \tag{21}$$

$$B_t' = \frac{\varphi_{t,w_{1t-1}}}{\varphi_{t,y_{t-1}}} \Rightarrow \varphi_{t,w_{1t-1}} = B_t'\varphi_{t,y_{t-1}}, \tag{22}$$

$$C_t' = \frac{\varphi_{t,d}}{\varphi_{t,y_{t-1}}} \Rightarrow \varphi_{t,d} = C_t'\varphi_{t,y_{t-1}}. \tag{23}$$

From (19) and (20), we find that:

$$\varphi_{t,y_t} = \frac{D_t}{D_t - C_t}, \tag{24}$$

which is well-defined by hypothesis, since $C_t \neq D_t$. Now, using (24) in (17) to (20):

$$\varphi_{t,w_{2t}} = A_t \frac{D_t}{D_t - C_t}, \tag{25}$$

$$\varphi_{t,w_{1t}} = B_t \frac{C_t}{D_t - C_t}, \tag{26}$$

$$\varphi_{t,d} = C_t \frac{D_t}{D_t - C_t}. \tag{27}$$

On the other hand, from (19), (23), and (24) we can define:

$$\varphi_{t,y_{t-1}} = \frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}, \tag{28}$$

and incorporating (28) to (21) and (22), we obtain that:

$$\varphi_{t,w_{2t-1}} = \frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}, \tag{29}$$

$$\varphi_{t,w_{1t-1}} = \frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}, \tag{30}$$

Note that Equations (24) to (30) provide the partial derivatives of the sharing rule function, with respect to all of the arguments except for preferences **z**. Thus, given concrete parametrization and estimates of the labor supply functions, one would easily recover the corresponding parametric estimated sharing rule by integrating these equations, as shown in Section 4.

Finally, these partials are compatible if and only if they satisfy, for each *t*, the following 21 cross-derivative restrictions:

$$\frac{\partial}{\partial w_{2t}} \left(B_t \frac{C_t}{D_t - C_t} \right) = \frac{\partial}{\partial w_{1t}} \left(A_t \frac{D_t}{D_t - C_t} \right), \tag{31}$$

$$\frac{\partial}{\partial d} \left(B_t \frac{C_t}{D_t - C_t} \right) = \frac{\partial}{\partial w_{1t}} \left(C_t \frac{D_t}{D_t - C_t} \right), \tag{32}$$

$$\frac{\partial}{\partial y_t}\left(B_t \frac{C_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{1t}}\left(\frac{D_t}{D_t - C_t}\right), \tag{33}$$

$$\frac{\partial}{\partial w_{1t-1}}\left(B_t \frac{C_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{1t}}\left(\frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{34}$$

$$\frac{\partial}{\partial w_{2t-1}}\left(B_t \frac{C_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{1t}}\left(\frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{35}$$

$$\frac{\partial}{\partial y_{t-1}}\left(B_t \frac{C_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{1t}}\left(\frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{36}$$

$$\frac{\partial}{\partial d}\left(A_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{2t}}\left(C_t \frac{D_t}{D_t - C_t}\right), \tag{37}$$

$$\frac{\partial}{\partial y_t}\left(A_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{2t}}\left(\frac{D_t}{D_t - C_t}\right), \tag{38}$$

$$\frac{\partial}{\partial w_{1t-1}}\left(A_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{2t}}\left(\frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{39}$$

$$\frac{\partial}{\partial w_{2t-1}}\left(A_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{2t}}\left(\frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{40}$$

$$\frac{\partial}{\partial y_{t-1}}\left(A_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{2t}}\left(\frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{41}$$

$$\frac{\partial}{\partial y_t}\left(C_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial d}\left(\frac{D_t}{D_t - C_t}\right), \tag{42}$$

$$\frac{\partial}{\partial w_{1t-1}}\left(C_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial d}\left(\frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{43}$$

$$\frac{\partial}{\partial w_{2t-1}}\left(C_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial d}\left(\frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{44}$$

$$\frac{\partial}{\partial y_{t-1}}\left(C_t \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial d}\left(\frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{45}$$

$$\frac{\partial}{\partial w_{1t-1}}\left(\frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial y_t}\left(\frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{46}$$

$$\frac{\partial}{\partial w_{2t-1}}\left(\frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial y_t}\left(\frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{47}$$

$$\frac{\partial}{\partial y_{t-1}}\left(\frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial y_t}\left(\frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{48}$$

$$\frac{\partial}{\partial w_{2t-1}}\left(\frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{1t-1}}\left(\frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{49}$$

$$\frac{\partial}{\partial y_{t-1}}\left(\frac{B_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{1t-1}}\left(\frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{50}$$

$$\frac{\partial}{\partial y_{t-1}}\left(\frac{A_t' C_t}{C_t'} \frac{D_t}{D_t - C_t}\right) = \frac{\partial}{\partial w_{2t-1}}\left(\frac{C_t}{C_t'} \frac{D_t}{D_t - C_t}\right), \tag{51}$$

and the two non-negativity equations (see Appendix B):

$$h_{w_1}^{1t} - h_{y_t}^{1t}\left(h^{1t} + \frac{B_t C_t}{D_t - C_t}\right)\frac{D_t - C_t}{D_t} \geq 0. \tag{52}$$

$$h_{w_2}^{2t} - h_{y_t}^{2t}\left(h^{2t} - \frac{A_t D_t}{D_t - C_t}\right)\frac{C_t - D_t}{C_t} \geq 0. \tag{53}$$

These equations are analogous to Proposition 3 (part *i*) of Chiappori et al. (2002), and thus are also analogous to the Slutsky conditions, i.e., they provide necessary and sufficient conditions of the labor supply functions to be a solution to the problem, and do not depend on the functional form of either the Marshallian labor supply functions or the sharing rule function.

Note that, as argued in Chiappori (1992) and Chiappori et al. (2002), these equations are necessary and sufficient conditions, and the sharing rule is defined up to a function of the individual preferences, $k(\mathbf{z})$. We must acknowledge the use of only one distribution factor in the theoretical specification of the model, which changes the identification of the model with respect to the original generalized collective model of Chiappori (1992), and hence second-order derivatives are not required, which leads to a more robust identification than in the case of no distribution factors (as explained in Chiappori et al., 2002). Additional distribution factors would impose more equations on the set of necessary and sufficient conditions. (See Chiappori et al. (2002) for the solution in the case of more than one distribution factor, in a single-period framework.

4. Parametrization of the model

4.1 The labor supply equations

We propose the following *semi-log* parametrization of Equations (5) and (6), resembling Chiappori's (2002) empirical model in a static framework. Assume without loss of generality that marriages are formed by a "wife" and a "husband", and that i = 1 refers to wives, and i = 2 refers to husbands.[6] Thus, parameters f_j and m_j refer to wives and husbands, respectively:

$$\begin{aligned} h_{1t} = {} & f_0^t + f_1^t \log w_{1t} + f_2^t \log w_{2t} + f_3^t y_t + f_4^t \log w_{1t} \log w_{2t} + \\ & + f_5^t \log w_{1t-1} + f_6^t \log w_{2t-1} + f_7^t y_{t-1} + f_8^t \log w_{1t-1} \log w_{2t-1} + \\ & + f_9^t d + \mathbf{f}_{10}^t \mathbf{z}, \end{aligned} \tag{54}$$

[6] This identification must be understood without loss of generality, and has been chosen to be representative of ordinary marriages, and in line with most of the literature on collective models.

$$h_{2t} = m_0^t + m_1^t \log w_{1t} + m_2^t \log w_{2t} + m_3^t y_t + m_4^t \log w_{1t} \log w_{2t} +$$

$$+ m_5^t \log w_{1t-1} + m_6^t \log w_{2t-1} + m_7^t y_{t-1} + m_8^t \log w_{1t-1} \log w_{2t-1} +$$

$$+ m_9^t d + \mathbf{m}_{10}^t \mathbf{z}, \tag{55}$$

Note that $f_0{}^t$, …, $f_9{}^t$, $m_0{}^t$, …, $m_9{}^t$ are scalars, against $\mathbf{f}_{10}^t$ and $\mathbf{m}_{10}^t$, that are k-vectors of parameters, where k is the dimension of the set of preference factors $\mathbf{z}$. This semi-log parametrization is specified in other empirical work, such as Rapoport et al. (2011) and Lyssiotou (2017). The convenience of this parametrization is that it satisfies a series of desirable properties: it does not *a priori* impose the restrictions of the model, and allows them to be tested; the collective restrictions of the model do not impose unrealistic constraints under this parametric form; it is easy to recover the sharing rule; the linearity (on parameters) allows for conventional estimates (Chiappori et al., 2002); and the log form of wages is often preferred in empirical studies. Nonetheless, the semi-log parametrization also yields certain limitations, such as the limited set of restrictions that arise from the complex set of restrictions of the general (non-parametrized) specification.

Proposition 2: With the specific parametrization of labor supply given by Equations (54) and (55), if $f_9^t / f_3^t \neq m_9^t / m_3^t$ (equivalent to $C_t \neq D_t$), t = 0, …, T, the necessary and sufficient conditions provided by Equations (31) to (53) are equivalent to:

$$f_9^t / m_9^t = f_4^t / m_4^t; \quad f_8^t / m_8^t = f_7^t / m_7^t. \tag{56}$$

Proof: See Appendix C.

Thus, the previous two equations represent the necessary and sufficient conditions of the sharing rule to be satisfied for the labor supply equations, parametrized as (54) and (55), to be the solution of the collective model, i.e., they provide testable conditions for rational collectivity behavior within households. As pointed out in the general collective model, and as noted above, the reduction from 21 conditions (in the case of a general non-parametric specification) to only two testable conditions, under the proposed semi-log form, indicates that the semi-log form of labor supply is quite adequate to estimate the LIC model proposed in Section 3. The intuition behind these restrictions is as follows. First, in each period t, the ratio of wives to husbands marginal effect of the distribution factor on labor supply must be equal to the corresponding ratio of the marginal effect of non-labor income. This condition is equivalent to Condition (8) of the general collective model presented in Chiappori et al.

(2002), given that the parametric form here resembles the latter: the cross-wage term and the distribution factor enter the labor supply functions system only through the sharing rule. The second restriction is mostly analogous, given that it relates the ratio of marginal effects of past cross-wages and past non-labor incomes, which also enter as arguments of the parametrized labor supply functions through the sharing rule.

Finally, the condition $m_3^t/f_3^t \neq m_9^t/f_9^t$ of Proposition 2 is equivalent to the general hypothesis $C_t \neq D_t$, that must be fulfilled for the model to be consistent. This condition is equivalent to condition "$m_3/f_3 \neq m_5/f_5$" of Chiappori et al. (2002), and indicates that, in each time period *t*, the ratio of the income effect of husbands over wives (which is expected to be positive by definition of the model) must be different from the corresponding sex ratio, that indicates the number of males per one female in the marriage market (that must be negative, by definition of the distribution factors).

4.2 The sharing rule

From Equations (24) to (30), under the concrete form of the labor supply specified in Equations (54) and (55), we obtain the following particular expressions for the partial derivatives of the sharing rule, with respect to all of its arguments (except preference factors **z**):

$$\varphi_{t,y_t} = \frac{m_9^t/m_3^t}{m_9^t/m_3^t - f_9^t/f_3^t} = K_1^t, \tag{57}$$

$$\varphi_{t,w_{2t}} = K_1^t \frac{f_2^t + f_4^t \log w_{1t}}{w_{2t} f_3^t}, \tag{58}$$

$$\varphi_{t,w_{1t}} = \frac{f_9^t/f_3^t}{m_9^t/m_3^t - f_9^t/f_3^3} \frac{m_1^t + m_4^t \log w_{2t}}{w_{1t} m_3^t} = (K_1^t - 1) \frac{m_1^t + m_4^t \log w_{2t}}{w_{1t} m_3^t}, \tag{59}$$

$$\varphi_{t,d} = K_1^t \frac{f_9^t}{f_3^t}, \tag{60}$$

$$\varphi_{t,y_{t-1}} = K_1^t \frac{f_7^t}{f_3^t}, \tag{61}$$

$$\varphi_{t,w_{2t-1}} = K_1^t \frac{f_7^t}{f_3^t} \frac{f_6^t + f_8^t \log w_{1t-1}}{w_{2t-1} f_7^t}, \tag{62}$$

$$\varphi_{t,w_{2t-1}} = K_1^t \frac{f_7^t}{f_3^t} \frac{m_5^t + m_8^t \log w_{2t-1}}{w_{1t-1} m_7^t}, \tag{63}$$

By integrating these partial derivatives, which constitute a system of immediate differential equations, and applying $f_9^t/m_9^t = f_4^t/m_4^t$ and $f_8^t/m_8^t = f_7^t/m_7^t$ for all *t*, we find the following parametrization of the sharing rule, in terms of the coefficients of the labor supply:

$$\varphi_t = \frac{1}{m_9^t f_3^t - f_9^t m_3^t}\Big(f_9^t m_1^t \log w_{1t} + m_9^t f_2^t \log w_{2t} + m_9^t f_3^t y_t + 2m_9^t f_4^t \log w_{1t} \log w_{2t} + \frac{m_9^t f_7^t m_5^t}{m_7^t} \log w_{1t-1} + m_9^t f_6^t \log w_{2t-1} + m_9^t f_7^t y_{t-1} + 2m_9^t f_8^t \log w_{1t-1} \log w_{2t-1} + m_9^t f_9^t d\Big) + k(\mathbf{z}), \quad (64)$$

where $k(\mathbf{z})$ represents the integrating constant, in terms of preferences $\mathbf{z}$, and cannot be identified. Then, the sharing rule function can be recovered up to an integration constant, as in Chiappori et al. (2002).

4.3 Individual labor supply

Finally, by definition (see Problem (P_4)) each household member labor supply at time period *t* is defined as a function of own wages in *t*, non-labor income in *t* (after the division provided by the sharing rule), and the vector of preference factors:

$$h_{1t} = a_1^t \log w_{1t} + a_2^t \varphi_t + A^t(\mathbf{z}), \quad (65)$$

$$h_{2t} = b_1^t \log w_{2t} + b_2^t (y_t + s_{t-1} - \varphi_t) + B^t(\mathbf{z}), \quad (66)$$

Proposition 3: For each *t*, parameters of Equations (65) and (66) can be recovered from parameters of Equations (54) and (55), as follows: $a_1^t = f_1^t + \frac{m_1^t f_9^t}{m_9^t}$, $a_2^t = \frac{f_3^t}{K_1^t}$, $b_1^t = m_2^t + \frac{m_9^t f_2^t}{f_9^t}$, and $b_2^t = \frac{m_3^t}{K_1^t - 1}$.

Proof: On the one hand, by definition of (65) and (66), $h_{w_{1t}}^{1t} = \frac{a_1^t}{w_{1t}}$, $h_{\varphi_t}^{1t} = a_2^t$, $h_{w_{2t}}^{2t} = \frac{b_1^t}{w_{2t}}$, and $h_{\varphi_t}^{2t} = -b_2^t$. Then, using the partial derivatives of the labor supply equations from (54) and (55), and from (8) and (11), one gets that $h_{y_t}^{1t} = a_2^t \varphi_{t,y_t} = a_2^t K_1^t = f_3^t$, and $h_{y_t}^{2t} = b_2^t(\varphi_{t,y_t} - 1) = b_2^t (K_1^t - 1) = m_3^t$. Finally, using that $h_{w_{1t}}^{1t} = \frac{a_1^t}{w_1^t} = \frac{f_1^t}{w_{1t}} + a_2^t (K_1^t - 1) \frac{m_1^t}{w_{1t} m_3^t}$, and $h_{w_{2t}}^{2t} = \frac{b_1^t}{w_{2t}} = \frac{m_2^t}{w_{2t}} + b_2^t K_1^t \frac{f_2^t}{w_{2t} f_3^t}$, and applying the definition of K_1^t, the results follows straightforwardly. Note that parameters A^t and B^t are not identifiable, since they depend on the integrating constant of the sharing rule equation. □

5. Data

We use data from the Panel Study of Income Dynamics (PSID) from years 1997 to 2015. The PSID is "the longest running longitudinal household survey in the world" (https://psidonline.isr.umich.edu/). It is conducted every two years by the University of Michigan (since 1968), and consists of a representative sample of more than 5,000 US families. The PSID contains data on a range of factors, including employment, income, wealth, and marriage, among others, and covers information at the family level, and also at the individual level, for all individuals in each of the interviewed households. Thus, the PSID contains all the required information to empirically test the model of Sections 3 and 4.[7]

We take data from all waves of the PSID interviews from year 1997 to year 2015 (interviews refer always to information from the previous year). We restrict the sample to two-member households formed by a husband and a wife (or cohabiting unmarried partners) between 18 and 65 years old (the collective model cannot be tested for singles, for obvious reasons). Furthermore, we remove households for whom at least one member (husband or wife) reports no labor supply, or no labor-related income. We also eliminate families whose composition (in relation to husbands and/or wives only, not to children or elders) has changed in the analyzed period, i.e., we eliminate those families in which there has been a divorce, and/or a wife or husband has engaged in a new marriage or cohabitation. We retain only households for whom information is available in uninterrupted periods.[8] These restrictions leave us with a sample of 13,244 observations, corresponding to 4,078 families. Each observation corresponds to a specific household in a specific year, and each family appears in the sample, on average, 3.25 times. Specifically, 1,238 observations correspond to 1999; 1,302 to 2001; 1,418 to 2003; 1,447 to 2005; 1,503 to 2007; 1,603 to 2009; 1,614 to 2011; 1,572 to 2013; and 1,547 to 2015.

The PSID allows us to directly define the labor supply of wives and husbands as the total annual hours of (market) work (on all jobs). With regard to wages, the PSID provides information on the total annual labor income (in dollars) of individuals (on all jobs). As in Chiappori et al. (2002), we define the wage rates of wives and husbands as the rate of total labor income over total hours of work. The PSID provides the following demographics at

[7] For instance, the empirical work in Chiappori et al. (2002) makes use of the PSID from year 1988.

[8] Thus, we eliminate some potential biases arising from an unbalanced panel data sample, and aim to retain only "stable households" in terms of both family and labor supply behaviors. Nonetheless, we acknowledge the role of sample selection biases by the restriction to two-member households where both the wife and the husband report positive labor supply and income. As in Chiappori et al. (2002), we do not treat this bias.

individual level: age (measured in years); the number of completed years of education (measured in years); the education of the father (coded as follows: 1) 0-5 grades, 2) grade school, 3) some high school, 4) completed high school, 5) high school plus non-academic training, 6) some college but no degree, 7) college BA and no advanced degree, and 8) college, advanced or professional degree, some graduate work); race (we characterize the sample in two groups, whites and non-whites); and religion (we divide the sample in four groups, Catholic, Jewish, Protestant, and atheist and members of other religions).

The PSID also provides information at the family level, i.e., information that refers to households as units. For instance, we have information about the total annual income (in dollars) of every interviewed family (including taxable income, transfer income, and Social Security income of the household). We can define non-labor (annual) income as total family income, minus the sum of labor incomes of family members. We consider potential savings as part of non-labor income, given that the model considers savings as part of the non-labor income of the household that is shared between household members. Thus, negative values of non-labor income would mean indebtedness of families. The PSID also provides data on the region in which the household resides, and we define four dummies, classifying households in four regions: Northeast, North, West, and South. Furthermore, the PSID contains information on all the family members, and we consider the age of those members, and in particular the number of children in each household. Given that the age of the children may condition the behavior of mothers and fathers (Miller and Mulvey, 2000; Silver, 2000; Campaña et al., 2016), we define two variables at family level: the number of children age 6 or younger in the household, and the number of children between ages 7 and 17.

Regarding the distribution factor of the model, we define the sex ratio as the number of males for each female, by age, State of residence, and year of the survey, and separately for whites and non-whites (assuming that marriage markets are, in general, limited to own race and own regional territory), using data from the United States Current Population Survey (CPS) of the Integrated Public Use Microdata Series (IPUMS), from the corresponding years (https://www.ipums.org/).[9] The measure of the sex ratio from the CPS, at age, State and race level, is then merged with the sample from the PSID using the State of residence of the household, and the race and age of the husband. The use of sex ratios as distribution factors is

[9] We classify individuals from the CPS in 5-year age groups, from 20 to 65 for males (males aged 18 and 19 are considered in the same group than males between 20 and 24, inclusive), and from 18 to 62 for females (females aged 63, 64 and 65 are considered in the same group than females between 58 and 63). We then compare males with females with a difference of two years (given that, according to the sample, wives are on average two years younger than husbands).

taken from Chiappori et al. (2002) and Rapoport et al. (2011), since the spatial and, in this concrete case of study, temporal variation in the number of males to females can determine the bargaining power of husbands and wives within families. However, the relationship between sex ratios and labor markets is more complex, and contrary effects may emerge from different explanations. See Chiappori et al. (2002) for a complete review of the alternative explanations of sex ratios as distribution factors. Other authors have considered alternative sets of distribution factors, such as inheritance (Blau and Goodstein, 2016), and child benefits (Lyssiotou, 2017).

Table 1 shows descriptive statistics of our variables, differentiating between wives and husbands in the case of variables defined at the individual level (Panel A). These descriptives are computed over the complete sample, consisting of all observations for all the households in the sample, and pondered by specific weights provided by the PSID at the household level. Wives work, on average, 1,749 hours per year, compared to 2,218 hours for husbands. Then, males in the sample work, on average, 27% more hours than females, with this difference being statistically significant at standard levels. In terms of annual labor income, husbands report higher income than wives, $64,942 vs $39,866, on average, a difference of almost 63%. Consequently, husbands also report a higher wage rate than wives, $30.10/hour vs $22.30/hour, with this difference being significant at standard levels.

With regard to the rest of variables defined at the individual level, we find that wives are, on average, two years younger than their husbands (specifically, the average age of wives is 41.8 years, vs 43.5 years of husbands), justifying the definition of the sex ratio. In spite of the higher wage rate of husbands, wives report, on average, a slightly higher, but significant, number of complete years of education (an examination of potential explanations for gender gaps in wages in the US is beyond the scope of this paper). The average education level of the fathers of wives is also slightly, but significantly, higher than that of fathers of their husbands. In terms of race, 76.9% of the wives in the sample are whites, vs 75.8% of the husbands.

Panel B shows summary statistics of our variables defined at the household level. The average total family income of households is $116,610/year, and the average non-labor income is $12,995/year. As expected, most of the total family income comes, on average, from the labor income of husbands and wives. Each household has, on average, 0.4 children under 6 years, and 0.9 children between 7 and 17 years. Finally, there are, on average, 0.93 (0.80) males in the marriage market, for each female, for whites (non-whites), according to the US CPS.

6. Empirical results

6.1 Estimates on labor supplies

We first estimate the system of labor supply equations of wives and husbands, parametrized from the semi-log specification of Equations (54) and (55), including the following preference factors **z**: the number of children of age ≤ 6, the number of children between 7 and 17, the years of education, age, race (of the correspondent individual), and the region of residence, to control for spatial variations (taking South as the region of reference). We estimate these equations simultaneously using the full-information Generalized Method of Moments (GMM). This estimation process takes into account heteroskedasticity of any form, and is more efficient than other methods of estimation. A direct estimation of the system of Equations (54) and (55) would be biased by spurious correlations between the dependent variable and some of the regressors, given the potential endogeneity between labor supply and income. Following Chiappori et al. (2002), Rapoport et al. (2011), and Lyssiotou (2017), we instrument wages and non-labor income using a second-order polynomial in age and years of education, father's education, and religion (taking atheist as the reference group). Correlations between lagged income variables and present labor supply are not considered as endogenous, given the different time periods. We also include lagged hours of work as a GMM-style instrument. According to Hansen test p-values, the use of these instruments does not reject that the model is correctly specified in any of the estimated models (see Table 2).

Table 2 shows estimates of Equations (54) and (55). Columns (1) and (2) estimate directly Equations (54) and (55), i.e., the "unrestricted model" of Chiappori et al. (2002). This model is called unrestricted because the restrictions imposed by the theoretical model are not considered, although a post estimation χ^2 test does not reject any of the restrictions. Columns (3) and (4) show estimates of the restricted model, where the two testable restrictions on labor supply parameters provided by Proposition 2 are imposed on the model. Then, these columns are based on the two testable restrictions $f_9^t/m_9^t = f_4^t/m_4^t$ and $f_8^t/m_8^t = f_7^t/m_7^t$ (Equation (56)). Parameters do not meaningfully vary from the unrestricted to the restricted estimates. Thus, GMM estimates do not reject the validity of the collective model in an intertemporal setting.

GMM estimates on labor supply show that wives' wage rates have a positive and significant effect on their own labor supply, but a smaller and slightly significant positive effect on husbands' hours of work. On the other hand, one more dollar per hour of work of husbands is related to significant increases in the annual hours of work of both the wife and the husband. For instance, a 1% increase in the wage rate of wives (husbands) is related to an increase of wives' annual labor supply of 123 (86) hours, vs an increase of 78 (120) hours for husbands. The effect of the cross-wage (that enters in the equations only through the sharing rule) goes in the opposite direction, and its effect on labor supply is negative and significant for both wives and husbands, although stronger for wives, indicating that even when present wages have a positive relationship with hours of work, simultaneous increases in both wife's and husband's wage also have a negative impact on both household members' labor supply. The effect of husband's wages on wives' labor supply is stronger than the effect of wives' wages on husbands' labor supply. Finally, non-labor family income shows a non-significant negative relationship to wives' annual hours of work, but also a negative but significant effect on husbands' hours of work. Then, household sources of non-labor income appear to influence household labor supply, but only from the husband's side.

In terms of the lagged income variables, that enter in the labor supply equations through the sharing rule, as happened with cross wages, Table 2 shows that they follow opposite relations to the respective regressors at the present time, but these relationships are weaker, in the sense that they are significant at lower levels, and parameters are quantitatively smaller. A 1% increase in lagged wife wage rates have a negative impact on both wives' and husbands' present hours of work of around 40 annual hours. In the case of husbands' past wages, they only affect husbands' present hours of work, but not those of wives, and a 1% increase in husbands' wages is related to a decrease for husbands of 72 annual hours of work. The lagged cross wage again has the opposite effect to the lagged wages, i.e., a positive effect on present hours of work, although the relationship is significant only for husbands, and the lagged family non-labor income has a non-significant effect on household labor supply.

Finally, in terms of the remaining set of explanatory variables, GMM estimates show that sex ratios are non-statistically related to household labor supply (at standard levels). This indicates that the number of males to females does not affect the labor supply of households and, then, that household behaviors are not apparently affected by marriage markets. However, this does not mean that sex ratios do not contribute to the model, given their role in the formal development of the model, and given its role in relation to the rest of the parameters (e.g., the rational collective behavior conditions that the model satisfies, in the case of the unrestricted model, or that are imposed in the restricted model). The number of children (≤ 6 years) only reflects a significant negative correlation with wives' labor supply, consistent with the Household Responsibilities Hypothesis (i.e., females carry out most of the domestic work and childcare activities; Aguiar and Hurst, 2007; Gimenez-Nadal and Sevilla, 2012, Gimenez-Nadal and Molina, 2016); wives with a higher level of formal education work fewer hours than less well-educated wives, although the effect of education is not significant for husbands; and older individuals work fewer hours, especially among males.

Columns (5) and (6) show estimates on the sharing rule, i.e., on the amount of non-labor income (divided by 1,000) assigned to wives in the intra-family allocation process of non-labor income. In particular, Column (5) shows the coefficients associated with the sharing rule derived from the semi-log parametrization, and then provides the form of the sharing rule in Equation (64) that corresponds to the sample used in the analysis. Increases in the present and lagged wage rates of wives or husbands are associated with direct positive, plus cross-negative correlations. In terms of lagged wages, the cross-effect appears to be non-significant. Column (6) shows the partial derivatives of the sharing rule, i.e., the change of the non-labor income (divided by 1,000) assigned to females, relative to the marginal changes of the explanatory variables (derivatives are computed with respect to wages, not with respect to log-wages). An increase in the average wife wage rate is associated with a transfer of non-labor income to her husband. Oppositely, an increase in the average husband wage is associated with a smaller, but still significant, transfer of non-labor income to him. Thus, wives show an altruistic behavior with respect to husbands, against an egoistic behavior shown by husbands. That is to say, when wives earn more from labor, they are more willing to share a higher rate of non-labor income, but when husbands earn more from labor, they are willing to share a lower rate of non-labor income. The partial derivatives with respect to non-labor income and the lagged explanatory variables are non-significant, even when the parameters associated with these variables are significant and of relevant magnitude in the sharing rule equation, according to Column (5). However, derivatives with respect to lagged wages are also negative, indicating some level of persistence in the altruistic vs egoistic behavior of wives and husbands. The derivative with respect to sex ratios is found to be positive, and more males in the marriage market, relative to females, induce a transfer of non-labor income to wives. However, the parameter is also statistically non-different from zero, and then sex ratios do not significantly determine the ability of wives to negotiate. This means that, according to the data used throughout the empirical analysis, the role of sex ratios is not clear, and the effects within labor markets and marriage markets may be counteractive.

6.2 Elasticities, and estimates conditional on $\varphi_t(.)$

Table 3 shows estimates of Equations (65) and (66), conditional on $\varphi_t(.)$, where non-labor income has been defined according to the sharing rule from the restricted model of Column (5) of Table 2. We compare ordinary least square (OLS) estimates (Columns (1) and (2)) and GMM simultaneous estimates (Columns (3) and (4)). Estimates quantitatively depend on the estimation method. However, estimates on the main explanatory variables (log-wage rates and non-labor income, conditional on $\varphi_t(.)$), are in both cases statistically significant at standard levels, and do not qualitatively differ. For instance, according to GMM estimates, for wives, a 1% increase in wage rates is associated with an increase of 1.81 more hours of work per year, but for husbands, a 1% increase in wages is associated with a decrease of 0.39 annual hours of work. Non-labor income also goes in different directions for wives (positive relation) and husbands (negative relation), and then estimates reject the income pooling property. Interestingly, we find that sex ratios are positively correlated with husbands' labor supply, but not with wives', where the coefficient is negative and non-significant. This result, against estimates in Table 2, sheds light on the relationship between sex ratios, marriage markets, and labor markets. Once marriage markets have been taken into account and we estimate the individual labor supply, conditional on the sharing rule, we find that sex ratios show opposite directions for husbands and wives. In particular, one more male per each female is associated with an increase of between 102 and 148 hours of work per year for males (vs non-significant decreases of around 50 hours per year for females). Then, estimates conditional on $\varphi_t(.)$ are in line with those estimates in the general collective model of Chiappori et. al. (2002).

Finally, Table 4 shows annual hours of work elasticities, using the values of the parameter estimates, and the mean values of variables, in the cases of the unrestricted model (Columns (1) and (2)), the restricted model (Columns (3) and (4)), and the model conditional on $\varphi_t(.)$ (Columns (5) and (6)). Wage elasticities are qualitatively different in the two versions of the collective model, being considerably smaller in the restricted version than in the unrestricted. Wives' wage elasticities and lagged wage elasticities are positive and significant, but husbands' wage elasticities are very small and non-significant (Chiappori et al., 2002), while husbands' lagged wage elasticities are also small and positive, but significant. Cross-wage elasticities are non-significant for wives, but significant and positive for husbands. Moreover, non-labor income elasticities are positive and significant, but lagged non-labor income elasticity is non-significant. In the case of the labor supply elasticities conditional on $\varphi_t(.)$, wives' and husbands' wage elasticities are negative and significant, and there are no cross-terms. Further, non-labor income elasticity is positive and significant for wives, but negative for husbands.

7. Conclusions

This paper has the purpose of conceptually and empirically extending the general collective model with distribution factors proposed by Chiappori, Fortin and Lacroix (2002) to a limited intertemporal commitment (LIC) setting. We propose a collective model in a multi-period framework, with a single distribution factor, and derive a sharing rule equation in terms of household member labor supply equations, subject to a set of necessary and sufficient conditions to be fulfilled. This model is based on efficient outcomes in a partial-commitment setting where renegotiation of intra-household bargaining powers is always possible, with a focus on marriage markets, measured through sex ratios. We propose a semi-log parametrization of labor supplies, which allows us to derive testable conditions on labor supply to derive the sharing rule and guarantee the validity of the model. Finally, we use United States data from the PSID from years 1997 to 2015 to estimate the parametric form of the household labor supply equations provided by the model. We estimate the model using GMM and control for endogenous correlates using an instrumental variables approach. Model estimates do not reject the validity of the collective model in an intertemporal setting. Further, estimates show that husband and wife wages have a direct positive correlation with labor supplies, but negative cross and lagged effects emerge. In terms of the sharing rule, it is defined in terms of significant parameters associated with present and lagged wages, non-labor income, and sex ratios, although partial derivatives show that only marginal changes in wages are followed by changes in the sharing rule, and that wives behave altruistically, but husbands show an egoistic behavior.

The empirical evidence points to the validity of the collective model and, in particular, to the existence of intra-household processes, and has important consequences in terms of policy-making. Results provide a background for an understanding of intra-household inequalities, and family behaviors in relation to wealth and income, in a multi-period setting. For instance, the different behaviors found for husbands and wives in terms of the relationship between wages and the allocation of resources may be relevant to the development of social programs about household welfare and gender equality.

The paper is subject to certain limitations. First, we acknowledge the role of sample selection biases, given the restriction to only two-member households where both the husband and the wife work. Second, as in Chiappori et al. (2002), we assume that sex ratios are an exogenous measure of marriage markets, although spatial and temporal variations of sex ratios may be caused by unobserved factors, thus biasing estimates, and we are also assuming that time not spend in market work is devoted only to leisure. Third, we are only considering the case where renegotiation is possible. Finally, we must also consider the role of unobserved heterogeneity in the empirical analysis.

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Table 1. Summary Statistics

A. Individual variables	Wives		Husbands		Diff.
	Mean	S.D.	Mean	S.D.	p-value
Hours of work/1,000	1.749	0.638	2.218	0.602	(<0.001)
Labor income/1,000	39.866	35.597	64.942	87.203	(<0.001)
Wage rate	22.319	23.499	30.097	51.470	(<0.001)
Log-wage rate	9.789	0.698	10.063	0.683	(<0.001)
Age	41.804	10.102	43.531	10.255	(<0.001)
Years of education	14.120	2.202	13.835	2.348	(<0.001)
Education of the father	4.014	2.358	3.836	2.328	(<0.001)
White	0.769	0.422	0.758	0.428	(0.041)
Catholic	0.230	0.421	0.219	0.414	(0.039)
Jewish	0.020	0.141	0.028	0.164	(<0.001)
Protestant	0.603	0.489	0.510	0.499	(<0.001)
Other religion or atheist	0.146	0.353	0.242	0.429	(<0.001)

B. Family variables		
	Mean	S.D.
Total family income/1,000	116.610	111.797
Non-labor income/1,000	12.995	39.057
N. children ≤ 6 years	0.389	0.699
N. children 7-17 years	0.878	1.049
Northeast	0.201	0.401
North	0.245	0.430
South	0.332	0.471
West	0.220	0.414
Sex ratio: whites	0.930	0.137
Sex ratio: non-whites	0.798	0.232
N. Families	4,078	
Total N. Observations	13,244	

Note: Summary statistics are considered by specific weights provided by the PSID. Statistics are computed over the overall number of observations. T-type test p-values for the differences between husbands and wives in parentheses. The sample (PSID 1997-2015) is restricted to families in which both the husband and the wife report positive labor supply and positive labor income. Hours of work is measured in hours worked per year. Labor income is measured in dollars per year. Wage rate is defined as the rate of labor income per hour of work. Age and years of education are measured in years. Total family income is measured in dollars. Non-labor income is defined as total family income, minus the labor incomes of the husband and the wife. The sex ratio is defined as the number of males for each female, by age and race (e.g., whites and non-whites).

Table 2. GMM Estimates

VARIABLES	Unrestricted model[1] (1) Wives	(2) Husbands	Restricted model (3) Wives	(4) Husbands	Sharing rule (5) Parameters	(6) Derivatives
Log wage rate (w, t)	13.12***	7.081	12.27***	7.779*	346.4***	-8.764**
	(4.394)	(4.810)	(4.344)	(4.616)	(45.03)	(3.806)
Log wage rate (h, t)	9.443**	11.44**	8.644**	12.08**	333.6***	-2.477*
	(4.275)	(4.978)	(4.229)	(4.785)	(45.43)	(1.284)
Log cross wage (t)	-1.049**	-0.776	-0.961**	-0.850*	-0.102***	-
	(0.438)	(0.497)	(0.432)	(0.476)	(0.0244)	
Non-labor income/1,000 (t)	0.0030	-0.0206**	-0.0012	-0.0169*	-92.10***	0.246
	(0.0083)	(0.0105)	(0.0066)	(0.0088)	(4.792)	(0.290)
Log wage rate (w, t – 1)	-4.651*	-4.515	-4.335*	-4.797*	163.8***	-0.0043
	(2.450)	(2.882)	(2.442)	(2.783)	(3.980)	(0.0026)
Log wage rate (h, t – 1)	-3.157	-6.977**	-2.849	-7.243**	161.8***	-0.0014
	(2.398)	(2.968)	(2.389)	(2.869)	(7.717)	(0.0012)
Log cross wage (t – 1)	0.349	0.476	0.313	0.506*	0.0110	-
	(0.247)	(0.301)	(0.245)	(0.305)	(0.0306)	
Non-labor income/1,000 (t – 1)	0.0003	0.0031	0.0010	0.0017	0.000	0.137
	(0.0015)	(0.0026)	(0.0011)	(0.0016)	(0.000)	(0.229)
Sex ratio	0.0335	-0.116	-0.0295	-0.0261	-0.120	29.50
	(0.103)	(0.119)	(0.0803)	(0.0885)	(2.192)	(23.43)
N. children ≤ 6 years	-0.221***	-0.0314	-0.223***	-0.0231		
	(0.0251)	(0.0319)	(0.0246)	(0.0304)		
N. children 6-17 years	0.0231	0.0109	0.0280	0.0033		
	(0.0192)	(0.0249)	(0.0178)	(0.0230)		
Years of education	-0.131***	-0.110***	-0.129***	-0.0231		
	(0.0139)	(0.0173)	(0.0137)	(0.0304)		
Age	-0.0099**	-0.00331	-0.0084**	-0.201***		
	(0.0039)	(0.0044)	(0.0033)	(0.0579)		
White (vs non-white)	-0.0270	-0.182***	-0.0136	-0.0262		
	(0.0424)	(0.0534)	(0.0411)	(0.0449)		
Northeast	-0.188***	-0.195***	-0.184***	-0.202***		
	(0.0454)	(0.0606)	(0.0442)	(0.0540)		
North	0.0045	-0.0229	0.00747	-0.108***		
	(0.0373)	(0.0464)	(0.0368)	(0.0167)		
West	-0.221***	-0.197***	-0.212***	-0.0032		
	(0.0429)	(0.0564)	(0.0415)	(0.0040)		
Constant	-73.02***	-36.14	-68.05***	-39.91*		
	(21.24)	(22.26)	(20.79)	(21.32)		
N. Families	4,078	4,078	4,078	4,078		
Total N. Observations	13,244	13,244	13,244	13,244		
Hansen's test p-value	0.122		0.138			

Note: Robust standard errors in parentheses. The sample (PSID 1997-2015) is restricted to families in which both the husband and the wife report positive labor supply and positive labor income. The dependent variable is the hours of work of wives (Columns (1) and (3)) and husbands (Columns (2) and (4)), divided by 1,000. Index "w" refers to wives; index "h" refers to husbands; indices "t" and "t – 1" refer to present and (two-years) past, respectively. Derivatives are computed with respect to wages, not to log-wages.

Instruments: labor supply (w-h; t – 1), log wage rate (w-h; t – 1), log cross wage (t – 1), non-labor income (t – 1), second order polynomial on age and years of education (w-h), n. children ≤ 6, n. children 7-17, education of father (w-h), white (w-h), Northeast, North, West, religion (Catholic, Jewish, Protestant; w-h; reference category: other religions and atheists).

[1] χ^2 tests do not reject the two testable conditions established by Proposition 2 at standard levels, with associated p-values of 0.236 and 0.379 for each of the conditions, in the case of the unrestricted model.

*** p<0.01, ** p<0.05, * p<0.1.

Table 3. Estimates conditional on $\varphi_t(.)$

VARIABLES	OLS		GMM	
	(1) Wives	(2) Husbands	(3) Wives	(4) Husbands
Log wage rate (w)	0.234***	-	0.181***	-
	(0.0189)		(0.0176)	
Log wage rate (h)	-	-0.0464***	-	-0.0387**
		(0.0171)		(0.0159)
Non-labor income/1,000	0.0003***	-0.0001***	0.0003***	-0.0001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Sex ratio	-0.0055	0.148***	-0.0419	0.102***
	(0.0410)	(0.0431)	(0.0337)	(0.0360)
Years of education	0.0067*	0.0223***	0.0141***	0.0218***
	(0.0034)	(0.0031)	(0.0031)	-0.0028
Age	0.0006	-0.0039***	0.0014**	-0.0031***
	(0.0007)	(0.0006)	(0.0006)	(0.0006)
White (vs non-white)	-0.0459**	0.0328*	-0.0711***	0.0509***
	(0.0190)	(0.0192)	(0.0138)	(0.0148)
North-East	-0.0879***	-0.0411**	-0.0836***	-0.0503***
	(0.0184)	(0.0173)	(0.0164)	(0.0159)
North	0.0076	-0.0652***	0.0170	-0.0727***
	(0.0161)	(0.0152)	(0.0140)	(0.0138)
West	-0.111***	-0.105***	-0.100***	-0.111***
	(0.0182)	(0.0169)	(0.0161)	(0.0152)
Constant	-0.353**	2.507***	0.0546	2.431***
	(0.174)	(0.153)	(0.160)	(0.141)
N. Families	4,078	4,078	4,078	4,078
Total N. Observations	13,244	13,244	13,244	13,244

Note: Robust standard errors in parentheses. The sample (PSID 1997-2015) is restricted to families in which both the husband and the wife report positive labor supply and positive labor income. The dependent variable is hours of work of wives (Columns (1) and (3)) and husbands (Columns (2) and (4)), divided by 1,000. Index "w" refers to wives; index "h" refers to husbands.
*** p<0.01, ** p<0.05, * p<0.1.

Table 4. Elasticities

VARIABLES	Unrestricted Model		General Collective Model		Conditional on $\varphi_t(.)$	
	(1) Wives	(2) Husbands	(3) Wives	(4) Husbands	(5) Wives	(6) Husbands
Wage rate (w, t)	0.471**	0.247**	0.0234*	0.0696**	-0.408***	-
	(0.235)	(0.123)	(0.0121)	(0.0349)	(0.128)	
Wage rate (w, t)	0.0819	0.0422	0.00396	0.0113	-	-0.319**
	(0.0736)	(0.0385)	(0.0037)	(0.0108)		(0.130)
Non-labor income/1,000 (t)	0.128***	0.0671***	0.0069***	0.0190***	0.0532***	-0.102***
	(0.0456)	(0.0239)	(0.002)	(0.006)	(0.0013)	(0.0012)
Wage rate (w, t – 1)	0.0013***	0.0007***	0.0008***	0.0002***	-	-
	(0.0003)	(0.0001)	(0.0001)	(0.0000)		
Wage rate (h, t – 1)	0.0003*	0.0001*	0.0001*	0.0004*	-	-
	(0.0001)	(0.0000)	(0.0000)	(0.0002)		
Non-labor income/1,000 (t – 1)	0.0518	0.0275	0.0026	0.0080	-	-
	(0.0393)	(0.0206)	(0.0020)	(0.0058)		

Note: Robust standard errors in parentheses. The sample (PSID 1997-2015) is restricted to families in which both the husband and the wife report positive labor supply and positive labor income. Index "w" refers to wives; index "h" refers to husbands; indices "t" and "t – 1" refer to present and (two-years) past, respectively. Elasticities are computed with respect to wages, not to log-wages.
*** p<0.01, ** p<0.05, * p<0.1.

Appendix A: Proof of Proposition 1

We define the following problem, which is a reformulation and thus equivalent to Problem (P_1) (Chiappori, 1992; Chiappori et al., 2002):

$$\max_{\{h_{1t},c_{1t}\}_{t=0}^{T}} E_0 \sum_{t=0}^{T} u_1(c_{1t}, 1-h_{1t})$$

$$\text{s.t.:} \quad \mu^*: E_0 \sum_{t=0}^{T} u_2(c_{2t}, 1-h_{2t}) \geq \overline{u_2}, \qquad (P_0)$$

$$c_t + s_t = w_{1t}h_{1t} + w_{2t}h_{2t} + y_t + s_{t-1}, \qquad t = 0, \dots, T$$

The Lagrange multiplier μ^* represents the weight of member 2 utility in the household decision process, that is, his/her bargaining power, relative to a unitary weight of member 1. The utility $\overline{u_2}$ is a function of the *expected* environment $\overline{u_2} = \overline{u_2}(w_{10}, w_{20}, \dots, w_{1T}, w_{2T}, y_0, \dots, y_T)$. If we add the participation constraint of (P_2) to (P_0), it would be then equivalent to (P_2) (and (P_3)).

If $\{c_{it}, h_{it}\}_{i=1,2}^{t=0,\dots,T}$ is a family of functions solving (P_0), then $\{c_{it}, h_{it}\}_{t=0}^{T}$ solve (P_2^i), $i = 1, 2$, for the extreme case of $\varphi_t(y_t + s_{t-1}) = w_{1t}h_{1t} - c_{1t}$. If they do not, it would not be a Pareto outcome, since member 1's utility could increase without changing member 2's individual outcomes, which would result in a contradiction (Chiappori, 1992).

Conversely, if $\{c_{1t}, h_{1t}\}_{t=0}^{T}$ solve (P_2^1) and $\{c_{2t}, h_{2t}\}_{t=0}^{T}$ solve (P_2^2), for a given sharing rule φ_t, we can define the reserve utility $\overline{u_2} = \sum_{t=0}^{T} u_2(c_{2t}, 1-h_{2t})$, and then the pair $\{c_{1t}, h_{1t}\}_{t=0}^{T}$ and $\{c_{2t}, h_{2t}\}_{t=0}^{T}$ are together a solution of (P_0). Further, $w_2(1-h_{2t}) + c_{2t} = e_{2t}(u_{2t})$, the expenditure function associated with the utility $\overline{u_2}$, and then the cost of a family $\{c'_{2t}, h'_{2t}\}_{t=0}^{T}$ providing the same utility $\overline{u_2}$ must satisfy $\sum_{t=0}^{T} e_{2t} \geq \sum_{t=0}^{T} w_2 + y_t - \varphi_t$. Given a family of functions $\{c'_{1t}, h'_{1t}\}_{t=0}^{T}$ such that $\{c'_{it}, h'_{it}\}_{i=1,2}^{t=0,\dots,T}$ solves (P_1), if $\sum_{t=0}^{T} u_1(c'_{1t}, 1-h'_{1t}) > \sum_{t=0}^{T} u_1(c_{1t}, 1-h_{1t})$, $\{c'_{1t}, h'_{1t}\}_{t=0}^{T}$ does not cost more than $\{c_{1t}, h_{1t}\}_{t=0}^{T}$, which is a contradiction. See Chiappori (1992) for the original demonstration of the static problem. □

Appendix B: The non-negativity equations

The non-negativity equations (37) and (38) are derived from the RMS between work hours and consumption. In particular, for $i = 1$ (without loss of generality), it must be that $\mathrm{RMS}_t = w_{1t} > 0$, which is trivial, and $\mathrm{RMS}_{t,h_{1t}} - w_{1t}\mathrm{RMS}_{t,c_{1t}} < 0$. The RMS is defined as the projection in the first component of the inverse of a function

$$\theta_t \colon \begin{pmatrix} w_{1t} \\ w_{2t} \\ y_t \end{pmatrix} \rightarrow \begin{pmatrix} h_{1t} \\ c_{1t} \\ y_t \end{pmatrix}, \tag{C1}$$

whose Jacobian matrix,

$$\begin{pmatrix} h^{1t}_{w_{1t}} & h^{1t}_{w_{2}t} & h^{1t}_{y_t} \\ c^{1t}_{w_{1t}} & c^{1t}_{w_{2t}} & c^{1t}_{y_t} \\ 0 & 0 & 1 \end{pmatrix}, \tag{C2}$$

is assumed to be of full rank, i.e., with determinant $h^{1t}_{w_{1t}} c^{1t}_{w_{2t}} - h^{1t}_{w_{2}t} c^{1t}_{w_{1t}} \neq 0$, and thus θ_t is invertible for all *t*. The Jacobian matrix of θ_t^{-1} is given by

$$\frac{1}{h^{1t}_{w_{1t}} c^{1t}_{w_{2t}} - h^{1t}_{w_{2}t} c^{1t}_{w_{1t}}} \begin{pmatrix} c^{1t}_{w_{2t}} & -h^{1t}_{w_{2}t} & h^{1t}_{w_{2}t} c^{1t}_{y_t} - h^{1t}_{y_t} c^{1t}_{w_{2t}} \\ -c^{1t}_{w_{1t}} & h^{1t}_{w_{1t}} & h^{1t}_{w_{2}t} c^{1t}_{y_t} - h^{1t}_{y_t} c^{1t}_{w_{2t}} \\ 0 & 0 & 1 \end{pmatrix} \tag{C3}$$

Then, using that $c_{1t} = w_{1t} h_{1t} + \varphi_t$, from the constraint in (P_2^1), and considering that the amount of non-labor income available for each member increases with his/her wage, i.e., $\frac{\partial \varphi_t}{\partial w_{2t}} \leq 0$, the inequalities follow straightforwardly:

$$\frac{c^{1t}_{w_{2t}} - w_{1t} h^{1t}_{w_{2t}}}{h^{1t}_{w_{1t}} c^{1t}_{w_{2t}} - h^{1t}_{w_{2}t} c^{1t}_{w_{1t}}} < 0 \Leftrightarrow \frac{\varphi_{t,w_{2t}}}{h^{1t}_{w_{1t}} \varphi_{t,w_{2t}} - h^{1t}_{w_{2t}} (h^{1t} + \varphi_{t,w_{1t}})} = \frac{\varphi_{t,w_{2t}}}{h^{1t}_{w_1} - h^{1t}_{y_t} \left(h^{1t} + \frac{A_t^2 C_t^1}{C_t^2 - C_t^1} \right) \frac{C_t^2 - C_t^1}{C_t^2}} < 0 \Leftrightarrow h^{2t}_{w_2} - h^{2t}_{y_t} \left(h^{1t} - \frac{A_t D_t}{D_t - C_t} \right) \frac{C_t - D_t}{C_t} \geq 0. \tag{C4}$$

For $i = 2$, the development is analogous, by symmetries. Note that the intuition behind the restrictions in the RMS is clear. The more work hours, the higher wage income, and the more consumption, and thus the RMS must be positive. However, it must be of decreasing returns since, in other cases, household members would always choose to work all the available time, $h_{it} = 1$ for all *t* and $i = 1, 2$, due to the increasing returns of work hours, and the household problem would be trivial.

Appendix C: Proof of Proposition 2

Under the parametric form of labor supply equations provided by (54) and (55), the partial derivatives of interest of the labor supply functions are given by:

$$h^{1t}_{w_{2t}} = \frac{f_2^t + f_4^t \log w_{1t}}{w_{2t}}, \tag{D1}$$

$$h^{2t}_{w_{1t}} = \frac{m_1^t + m_4^t \log w_{2t}}{w_{1t}}, \tag{D2}$$

$$h^{1t}_{y_t} = f_3^t, \tag{D3}$$

$$h^{2t}_{y_t} = m_3^t, \tag{D4}$$

$$h^{1t}_{d} = f_9^t, \tag{D5}$$

$$h^{2t}_{d} = m_9^t, \tag{D6}$$

$$h^{1t}_{w_{2t-1}} = \frac{f_6^t + f_8^t \log w_{1t-1}}{w_{2t-1}}, \tag{D7}$$

$$h^{2t}_{w_{1t-1}} = \frac{m_5^t + m_8^t \log w_{2t-1}}{w_{1t-1}}, \tag{D8}$$

$$h^{1t}_{y_{t-1}} = f_7^t, \tag{D9}$$

$$h^{2t}_{y_{t-1}} = m_7^t, \tag{D10}$$

and the parameters A_t, B_t, A'_t, B'_t, C_t, C'_t, and D_t, can be expressed as:

$$A_t = \frac{f_2^t + f_4^t \log w_{1t}}{f_3^t w_{2t}}, \tag{D11}$$

$$B_t = \frac{m_1^t + m_4^t \log w_{2t}}{m_3^t w_{1t}}, \tag{D12}$$

$$C_t = f_9^t / f_3^t, \tag{D13}$$

$$D_t = m_9^t / m_3^t, \tag{D14}$$

$$A'_t = \frac{f_6^t + f_8^t \log w_{1t-1}}{f_7^t w_{2t-1}}, \tag{D15}$$

$$B'_t = \frac{m_5^t + m_8^t \log w_{2t-1}}{m_7^t w_{1t-1}}, \tag{D16}$$

$$C'_t = f_9^t / f_3^t. \tag{D17}$$

Finally, the partial derivatives of the sharing rule are given by Equations (57) to (63), and the cross derivatives that determine the necessary and sufficient conditions (Equations (31) to (51)), are all equalities between null terms, except for:

$$\varphi_{t,w_{1t}w_{2t}} = \varphi_{t,w_{2t}w_{1t}}, \tag{D18}$$

$$\varphi_{t,w_{1t-1}w_{2t-1}} = \varphi_{t,w_{2t-1}w_{1t-1}}, \tag{D19}$$

which are equivalent to:

$$\frac{f_9^t/f_3^t}{m_9^t/m_3^t - f_9^t/f_3^3} \frac{m_4^t}{m_3^t w_{1t} w_{2t}} = \frac{m_9^t/m_3^t}{m_9^t/m_3^t - f_9^t/f_3^3} \frac{f_4^t}{f_3^t w_{1t} w_{2t}}, \tag{D20}$$

and

$$\frac{K_1 m_8^t f_7^t/f_3^t}{m_7^t w_{1t-1} w_{2t-1}} = \frac{K_1 f_8^t f_7^t/f_3^t}{f_7^t w_{1t-1} w_{2t-1}}. \tag{D21}$$

Finally, these two equalities can be reduced to the following two restrictions: $f_9/m_9 = f_4/m_4$, and $f_8/m_8 = f_7/m_7$, respectively. □