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ABSTRACT

Occupational Choice and Matching in the Labor Market[*]

Integrating Roy with Becker, this paper studies occupational choice and matching in the labor market. Our model generates occupation earnings distributions which are right skewed, have firm fixed effects, and large changes in aggregate earnings inequality without significant changes in within firm inequality. The estimated model fits the earnings distribution both across and within firms in Brazil in 1999. It shows that the recent decrease in aggregate Brazilian earnings inequality is largely due to the increase in her educational attainment over the same years. A simulation of skilled biased technical change in the model also qualitatively fits the recent changes in earnings inequality in the United States.

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Despite differences in the distributions of employers by technologies and worker by skills across industries, regions and time, earnings distributions have some invariant characteristics:

1. In every market economy, there are many occupations. Most occupational earnings distributions are single peaked and right skewed.

2. Firm/establishment fixed effects continue to explain a significant fraction of the variance of log earnings after controlling for individual characteristics, industry and occupation fixed effects (E.g. Groshen (1991), Abowd et al. (1999)).

3. Recent changes of earnings inequality in many countries, either increasing or decreasing, are primarily due to changes in earnings inequality across and not within firms. E.g. Song et al. (2015) (United States); Benguria (2015) (Brazil); Faggio et al. (2010) (UK); Skans et al. (2009) (Sweden).

Characteristic 1 is well known. Figure 1, which is obtained from the US 2000 Census, plots the earnings distributions for different occupations selected by three different criteria: Occupations by sex ratios (men to women), size (measured by number of workers) and average earnings ranked at the 80th, 50th and 20th percentiles. All the earnings distributions have earnings which are weakly convex by percentile.

Since the distributions of the demand and supply of skills to an occupation will likely affect the sex composition, size and average earnings, the demand and supply distributions cannot be a first order determinant of convexity. Rather, there must be common mechanisms across occupations which generate convexity in the occupational earnings distributions. A single peaked right skewed distribution such as the log normal earnings distribution will approximate the earnings distributions on Figure 1.

Characteristic 2 was discovered shortly after economists started estimating earnings regressions. After controlling for individual characteristics, industry and occupational effects, firm/establishment fixed effects

explain a significant fraction of the residual variance of cross section log earnings (Groshen, 1991). Following Abowd et al. (1999), economists extended the analysis with panel data to estimate log earnings regressions with both worker and firm/establishment fixed effects. The explanatory power of the firm/establishment effects remain large. Some researchers (e.g. Card et al. (2013)), but not all, show that the correlation between individuals' and firms' fixed effects is quantitatively large. I.e. controlling for observables, including occupation, workers with high average earnings work primarily in firms with high average earnings. This potentially high correlation imply that there is positive assortative matching of co-workers by ability. The popular press also noticed this correlation:

> *The recruiting is not confined to the best engineers; sometimes it spills over to nontechnical employees too. Two of the chefs who prepared meals for Googlers, Alvin San and Rafael Monfort, have been hired away by Uber and Airbnb in the last 18 months. (NYT Aug 18, 2015)*

Characteristic 3 is a recent discovery. In recent decades, labor earnings inequality within many countries have changed significantly. For many countries, including the US, earnings inequality have risen. For other countries, as will be shown below for Brazil, it has fallen.[1] What about changes in across and within firm earnings inequality? In an important recent paper by Song et al. (2015), with Social Security Administration earnings data for more than 100 million workers per year, they showed that for 99.8% of the working population, there was no change in within firm earnings inequality from 1982 to 2012. Benguria (2015) showed that for male workers in the formal sector in Brazil, aggregate earnings inequality have fallen significantly from 1999 to 2013. To a first order, there was also no change in within firm earnings inequal-

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[1]Declining earnings inequalities were and are common in Latin America (Lustig et al. (2013)). For Spain, see Pijoan-Mas and Sánchez-Marcos (2010).

ity. We will describe the findings from both papers in more detail below.[2] Since aggregate changes in earnings inequality across countries recently occurred in both directions, how can there be so little change to within firms inequality?

In order to discuss earnings inequality within and across firms, workers in different firms have to differ in essential ways. In this paper, different firms produce different qualities of output. A firm can only produce higher quality output by hiring higher skill workers but not more workers of the same skill. E.g. we are assuming that a bakery produces high quality cakes by hiring a better baker and not more bakers.

To study occupation choice, we follow Roy (1951) and start with a bivariate distribution of workers' skills, each worker characterized by her cognitive and non-cognitive skills. There are two occupations: key role and support role. The occupational skill for each worker is an exogenous aggregation of the worker's cognitive and non-cognitive skills into occupational specific, one-dimensional indexes, which we label as key role and support role skills. Given their key role and support role skills, each worker decides which occupation to enter.

Workers work in firms/teams of two, one in a key role and one in a support role, producing according to a supermodular technology. Given occupational wages which depend on the skills of the workers, teams have to decide who to hire. With free entry of teams, the equilibrium revenue of each team is divided between its workers. As is well known from Beckers matching model, supermodularity of the team production function will lead to positive assortative matching by key role and support role skills for all teams. Due to frictionless occupational choice, the type space is segregated into two halves with equal mass, with full specialization within each. There is no long side of the market in equi-

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[2]In the face of large increases in aggregate earnings inequality, Faggio et al. (2010) also showed that there was little change in within firm inequality in the UK from 1984 to 1999. For Sweden where the wage setting institutions are significantly different than the US, Skans et al. (2009) concludes that "the trend in between-plant variance makes up the entire increase in wage dispersion over the period (i.e. 1985-2000)".

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As described, our model of the labor market integrates the Roy model of occupational choice with Becker's model of frictionless matching. Frictionless occupational choice mitigates within-firm inequality, while positive assortative assortative matching magnifies across-firm inequality. In order to obtain right skewed occupational earnings distributions, we will also assume that each team's production/revenue function of quality is convex in occupational skills. This assumption will lead to occupational earnings being convex in occupational skill.[3]

We use our framework to study the recent decline in earnings inequality in Brazil. The average years of educational attainment doubled in Brazil from 1999 to 2013. We ask whether this change in schooling can explain the observed decline in earnings inequality and also exhibit the three invariant features of the earnings distributions.

Using educational attainment as a proxy for cognitive skill, we first estimate the parameters of our model with the distributions of individual earnings and average earnings by firm in 1999. Then we simulate the earnings distributions predicted by our estimated model with the educational distribution in 2013. Our simulation replicates the Brazilian data where for both years, the occupational earnings distributions are single peaked and right skewed. Earnings inequality in both distributions are almost entirely due to the between-firm component. We also rationalize a significant decline in Brazilian earnings inequality between 1999 and 2013, as well as little change in within firm inequality between the two periods.[4]

What happens when there is skill biased technical change (SBTC)?

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[3]As Adam Smith pointed out long ago, workers are specialized by occupations. From a labor market point of view, what is the nature of wages which workers observe such that they will willingly specialize? If occupational wages are convex in occupational skills, workers will want to specialize in skills investments. Microfoundations for such specialization using indivisibilities and increasing returns are provided by Rosen (1983).

[4]Engbom and Moser (2016) argues that an increase in the minimum wage was also important.

5

When key role workers exogenously become more productive, more workers will want to become key role workers leading to a scarcity of support workers. Wages for support workers have to increase when workers from both occupations are necessary for production. PAM in the labor market also means that at the old wage gradient for support role workers, enhanced key role workers want to match with better qualified support role workers than before. This increased competition for better support teammates will lead to an increase in earnings inequality across support role workers. The two effects will mitigate an increase in within teams earnings inequality. Using our estimated model, we simulate the effect of SBTC. Compared with the 2013 benchmark, occupational choice and matching significantly mitigates the earnings inequality due to SBTC, particularly among the lesser skilled workers. Moreover, occupational choice reduces earnings inequality within firms.

Our model is highly stylized, ignoring some important features of labor markets. First, we do not consider variation in firm size. This does not allow us to discuss variation in the quantity of output across firms, an important concern of the standard model of SBTC. Second, we take the underlying skill (education) distribution as exogenous without considering why the Brazilian schooling has shifted. Third, we have a static model and we ignore search frictions in both occupational choice and matching. We leave these important concerns for further research.

This paper builds on previous work on frictionless occupation choice and matching in the labor market. We discuss our debt to this literature and present the empirical facts concerning earnings inequality in Section 1. Section 2 presents our model and Section 3 provides some characterizations. Section 4 shows the equivalence between our competitive model and a utilitarian social planner's linear programming problem. Section 5 provides our quantitative rationalization of the Brazilian experience. Our exploration of SBTC is in Section 6. Section 7 concludes.

1 Literature Review

1.1 Two Empirical Studies

This section reviews the above two papers which pertains to our work.[5] The two papers use the same empirical strategy. Song et al. (2015) analyzed data from the US Social Security Administration master file from 1982 to 2012. This data consists of the W2 forms filed annually by every employer for each employee to the US tax authority, the IRS. Their sample has between 66 million to 153 million workers per year, and between 0.8 million to 1 million firms per year. Individuals in firms with less than 10 workers were excluded. The earnings information per worker includes wages and salaries, bonuses, exercised stock options, the dollar value of vested restricted stock units and other sources of income. Wage earnings are top coded at the 99.999th percentile.

Benguria (2015) uses the Relação Anual de Informações (RIAS) data from 1999-2013. This data is filed annually by all registered employers on their employees. He analyzes a 10% random sample of the data set. The sample has over 5 million workers in 2013. The reported average montly earnings "are gross and include not only regular salary but also bonuses and other forms of compensation".

For any year t, let the log earnings of worker i and the mean of log earnings in firm j be w_t^{ij} and $\overline{w}_t^j$ respectively. For year t, we can decompose the variance of individual log earnings into a between firms variance of log earnings and a within firm variance of log earnings:

$$var(w_t^{ij}) = var(\overline{w}_t^j) + \sum_{j=1}^{J_t} P_t^j \times var(w_t^{ij}|i \in j)$$

$$J_t : \quad \text{number of firms in year } t$$

$$P_t^j : \quad j's \text{ share of employment in year } t$$

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[5]For the US, Barth et al. (2014) has similar results by establishments.

All US figures are from Song et al. (2015). Figure 2 shows the evolution of the variance decomposition for the US from 1980 to 2012. The top line is the evolution of the total variance over time. It has a significant upward trend which shows the well known increase in aggregate inequality in earnings in the US in recent decades. The middle line is the evolution of the variance of within firm earnings. Finally, the bottom line is the evolution of the between firm variance of earnings. Note that the variance of within firm earnings is larger than that of between firms earnings. So there are significant differences in earnings within firm. On the other hand, the slope of within firm earnings over time is significantly flatter than the slope for overall inequality. Rather, the slope of the between firm variance over time has the same slope as the slope for overall inequality.

All Brazil figures are from Benguria (2015). Figure 3 shows the evolution of the total variance of earnings for Brazil from 1999 to 2013. The slope of this (top) line is completely different from what happened in the US. The variance of aggregate earnings fell by 32% (21 log points) over the period 1999 to 2013.

Figure 4 shows the decomposition of the decline of the aggregate variance into between and across firms variation. Their panel A shows that most of the decline in aggregate variance is reflected in the decline in across firms variance. There is at best a modest decline in within firm variance. Unlike the US, there is more between firm inequality than within firm inequality.

For a finer decomposition within a year, the authors use another decomposition of earnings inequality for year t. Let W_{pt}^i be the mean of w_t^{ij} of all workers in the $p'th$ percentile in the earnings distribution in year t. Let $\overline{W}_{pt}^j$ be the mean of $\overline{w}_t^j$ for each worker in the $p'th$ percentile. Then:

$$W_{pt}^i = \overline{W}_{pt}^j + (W_{pt}^i - \overline{W}_{pt}^j)$$

W_{pt}^i is decomposed into a part which is the mean of the firms in which

these workers work in, $\overline{W}_{pt}^{j}$, and a residual, $(W_{pt}^{i} - \overline{W}_{pt}^{j})$, which is how these workers' mean log earnings deviate from their firms' mean. The change in earnings inequality by percentile from year t to year t' is:

$$W_{pt'}^{i} - W_{pt}^{i} = \overline{W}_{pt'}^{j} - \overline{W}_{pt}^{j} + (W_{pt'}^{i} - \overline{W}_{pt'}^{j}) - (W_{pt}^{i} - \overline{W}_{pt}^{j})$$

Figure 5 shows the changes in earnings inequality by percentile from 1982 to 2012 in the US. The diamond line represents the well known increase in overall inequality. The circle line, which is essentially on top of the diamond line, is the change in firm inequality by percentile. Since the difference between the two lines is the residual, the bottom line is the change in within firm inequality by percentile. What is remarkable is that there is, to a first order, no change in within firm inequality by percentile.[6] The change in overall inequality has received significant attention from the policy makers and researchers. It represents significant changes to how the US labor market evolved.

Figure 6 shows the changes in earnings inequality by percentile from 1999 to 2013 in Brazil. Earnings inequality fell significantly from 1999 to 2013. That is, earnings for the top percentiles did not increase as much as for the lower percentiles. On the other hand and similar to the US experience, there was essentially no change in within firm inequality except at the lowest percentiles.

Taken together, the above figures and the studies by Faggio et al. (2010) for the UK and Skans et al. (2009) for Sweden, showed that total inequality within a country can change significantly in a few decades and in different ways. In spite of sometimes large overall changes, there was essentially no change in within firm inequality by percentile. These effects are not well captured by existing models of the labor market.

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[6]Their paper showed that there was a large change in within firm inequality at the 99.8 percentile.

1.2 Theoretical Studies

Our model builds on classics in labor economics that evolved into the modern occupational choice and matching literatures. Adam Smith already emphasized specialization and the division of labor which are fundamental to what is discussed here. Ricardo first recognized that individuals may have more than one dimension of skills and argued that occupational choice should be based on comparative advantage. Roy (1951) sparked the analytic literature on occupational choice. French and Taber (2011) has a recent survey on Roy models.

Building on Katz and Murphy (1992), Acemoglu and Autor (2011) provides a survey of the standard model of SBTC and changes in earnings inequality. These models assume that firm output satisfies constant returns to scale in occupational skills. A worker who earns twice as much as an other worker in the same occupation has twice as much skill as the worker with lower earnings. I.e. skills are perfectly substitutable between workers in the same occupation. So labor market equilibrium is determined by an aggregate production function and the aggregate supply of skills to each occupation. By construction, the standard model is silent on earnings inequality between and within firms, and output quality differences across firms. We differ from them by ignoring variation in output across firms and focus only on quality differences.

Becker (1973, 1974) started the modern literature on frictionless matching by studying marriage matching. In his model, men and women are separately ordered by a one-dimensional index of ability. The marital output production function is supermodular (complementary) in spousal abilities. Under these two assumptions, Becker obtains positive assortative matching (PAM) in spousal abilities in equilibrium. Eeckhout and Kircher (2012) provides a state of the art summary and application to labor markets. PAM results in team fixed effects in earnings which are invariant to differences in technology and the distributions of workers' abilities. Most matching models of the labor market study one dimen-

sional matching. Lindenlaub (2016) is an exception. The optimal transport literature in mathematics also studies matching models. Galichon (2016) provides an accessible survey of this literature for economists.

Except for McCann and Trokhimtchouk (2010) and McCann et al. (2015), most previous work that combine occupational choice and matching, build on Kremer and Maskin (1996) in which workers differ by one dimension of heterogeneity. In one dimensional models, there is no comparative advantage in occupational choice. So both occupational choice and matching are based on absolute advantage which makes general characterization difficult to come by (E.g. Porzio (2015)). Garicano and Rossi-Hansberg (2004) and Lucas Jr (1978) provide elegant behaviorally motivated special cases. Erlinger et al. (2015) also studies a one dimensional occupational choice and matching model with investment and two labor market sectors. Geerolf (2016) showed that a Garicano model with occupational choice and matching generates an earnings distribution with a single peak and right skeweness. His techniques may also be useful in our context.

Our model has two dimensions of skills. For each worker, the two skills aggregate into two unidimensional occupational skills. There is PAM by occupational skills. With two skills, occupational choice is based on comparative advantage and PAM by occupational skills is due to absolute advantage. So occupation choice and matching coexist easily in contrast to the aforementioned one-dimensional models.

McCann and Trokhimtchouk (2010) has a general multidimensional skills model of occupational choice and matching. Our model is a special case of their framework and we owe our existence and uniqueness results to them. We differ from them by reducing our two dimensional skills problem into one dimensional occupational skill indices problem with which we provide sharper characterizations of equilibrium.

McCann et al. (2015) study a model of schooling investment, occupational choice and matching where workers differ by cognitive and communication skills. That model is richer in terms of behavior because

there is also investment, multi-sector considerations and heterogeneity in firm size. This paper builds on that work. Our environment here is simpler which leads to more transparent analytic results. McCann et al. (2015) provides a way to model variations in the quantities and qualities of firm output. Building on McCann et al. (2015), Melynk and Turner (2016) estimates a model of occupational choice, time use, matching in both labor and marriage market and where individuals differ by cognitive and communication skills.

Gola (2016) studies a two dimensional occupational choice problem similar to ours. He differs by studying the matching of firms to workers in each separate occupation. He derives comparative statics results for changes in the distribution of skills and/or the revenue functions which may be useful here.

As discussed earlier in footnote 3, our focus on quality differences across teams leads us to assume that the revenue function of a team is convex in occupational skills. Microfoundations for such specialization using indivisibilities and increasing returns are provided by Rosen (1983). Also see Yang and Borland (1991).

2 The Model

2.1 The Setup

Consider a labor market with a unit mass of workers. Each worker has two base skills (c, r), his or her cognitive skill and non-cognitive skill respectively. c and r are distributed according to the continuous bivariate density $b(c, r)$, such that $\bar{b} > b(c, r) > 0$ with positive domain $[\underline{c}, \bar{c}] \times [\underline{r}, \bar{r}]$. There is no atom in the density.

Production takes place in a team of two workers. One worker is a key role worker and the other is a support role worker. Consider a team with a key role worker with characteristics (c_1, r_1) and a support worker with

characteristics (c_2, r_2). The revenue they produce is:

$$\bar{R}(c_1, r_1; c_2, r_2) = R(k_1; s_2)$$

The cognitive skill of the key role worker, c_1, and her non-cognitive skill, r_1, interacts to form an index of key role skill, $k_1 = g_k(c_1, r_1)$. Analogously, the skill index of the support role worker is $s_2 = g_s(c_2, r_2)$. Consider a worker with base skills (c, r). We assume that $g_k(c, r)$ is not a monotone transform of $g_s(c, r)$ so that the two occupations rank at least some of the same workers differently.

We impose the following assumptions on the technology R:

Assumption 1 (Supermodularity). *R is strictly supermodular in k_1, s_2.*

As is well known since Becker and we will also show below, the supermodularity assumption will result in PAM by occupational skills.

Assumption 2 (Increasing Returns). *R is strictly increasing and convex in k_1, s_2.*

The convexity assumption is that quality is a convex function of occupational skills. This embodies two assumptions:

First, higher quality output/revenue is due to higher quality workers.

Second, convexity of skills is necessary if we want workers to specialize in their occupational skills investments. If the returns to skills are not convex, workers will diversify in the skills investment which is not what we see. As noted earlier, Rosen (1983), Yang and Borland (1991), and others have provided microfoundations for this convexity assumption. We will show below that the convexity of the revenue function in occupational skills will result in occupational wages which are convex in occupational skills. Both PAM and occupational wage convexity obtain without strong restrictions on the base skills distribution.

The density of (k, s) is a continuous function f, derived from b after a transform of variables, is also strictly bounded above and its domain

remains a rectangle $\Omega \equiv [\underline{k}, \bar{k}] \times [\underline{s}, \bar{s}]$. We assume that $R(k_1; s_2) > 0$ for all $(k, s) \in \Omega$.

Let $\pi(k)$ be the earnings of a key role worker with skill k. Let $w(s)$ be the earnings of a support worker with support role skill s. For the moment, assume that the earnings functions for both types of workers are increasing and convex in their occupational specific skills.

The following discusses occupational choice and matching given (π, w). We first show that the optimal occupational choice is characterized by complete segregation, with the (k, s) space partitioned into two equal halves by a separating line. Then we show that matching follows positive assortative matching (PAM). Therefore, the competitive equilibrium is characterized by a separating line and a matching line.

2.2 Occupational Choice

Workers choose the occupation which will maximize their net earnings. So a worker of type (k, s) will earn:

$$y(k, s) = \max[\pi(k), w(s)] \tag{1}$$

If $\pi(k) > w(s)$, the worker will be a key role worker. If $\pi(k) < w(s)$, the worker will be a support worker. If $\pi(k) = w(s)$, the worker will be indifferent between the two roles. Therefore, the type space Ω is partitioned into three sets $\Omega_k \equiv \{(k, s) \in \Omega | \pi(k) > w(s)\}$, $\Omega_s \equiv \{(k, s) \in \Omega | \pi(k) < w(s)\}$, and $\Omega_{ks} \equiv \{(k, s) \in \Omega | \pi(k) = w(s)\}$. Presuming that π, w are continuous, strictly increasing functions in their respective arguments (which we shall justify later), Ω_{ks} is an upward sloping line in Ω since for a higher k worker, he would be indifferent between the two roles only if his s is also higher. Accordingly, we define the separating function $\phi : [\underline{k}, \bar{k}] \to [\underline{s}, \bar{s}]$ such that

$$\phi(k) = \min\{w^{-1}(\pi(k)), \bar{s}\}$$

14

As such, if $\phi(k) < \bar{s}$, then workers with characteristics $(k, \phi(k))$ are indifferent between the two occupations:

$$w(\phi(k)) = \pi(k) \tag{2}$$

While if $\phi(k) = \bar{s}$, then a worker with skill k will always prefer the key role regardless of his s. In the discussion below and in the simulation, $\phi(k) < \bar{s}$ for all k. So we focus on (2).

The separating function is a central concept of the Roy model of occupational choice. We summarize the above discussion in Proposition 1.

Proposition 1 (Separating Function). *Consider a worker with characteristics (k, s). If $s > \phi(k)$, the worker will choose to be a support role worker. If $s < \phi(k)$, the worker will choose to be a key role worker. If $s = \phi(k)$, the worker will be indifferent between the two occupations. $\phi(k)$ is non-decreasing in k.*

Given $\phi(k)$, the cumulative distribution of key role workers from ability $\underline{k}$ to k is:

$$H(k) = \int_{\underline{k}}^{k} \int_{\underline{s}}^{\phi(u)} f(u,v) dv du \tag{3}$$

The cumulative distribution of support role workers from ability $\underline{s}$ to s is

$$G(s) = \int_{\underline{s}}^{s} \int_{\underline{k}}^{\phi^{-1}(v)} f(u,v) du dv \tag{4}$$

2.3 Matching

Since we are in a competitive environment and there is no cost of entry of firms/teams, a firm would hire a key role worker and a support role worker according to:

$$\max_{\tilde{k}, \tilde{s}} R(\tilde{k} : \tilde{s}) - \pi(\tilde{k}) - w(\tilde{s}) \tag{5}$$

15

The optimal choice of $(k; s)$ will satisfy:

$$R_k(k; s) = \pi'(k) \tag{6}$$

$$R_s(k; s) = w'(s) \tag{7}$$

Since the technology is strictly increasing in k and s, π, w are strictly increasing functions as we presumed.

We can invert either (6) or (7) to get the matching function, $s = \mu(k)$, where a key role worker of skill k will match with a support worker of type s. The matching function is a central concept in Becker's matching model and here as well.

Proposition 2 (Matching Function). $\mu' > 0$: *There is PAM between key role workers and support workers by occupational skills.*

Proof. The proof is by contradiction. Consider two key role workers, k_A and k_B, $k_A > k_B$ and two support workers s_A and s_B, $s_A > s_B$ such that $(k_A; s_A), (k_B; s_B)$ both satisfy the first-order conditions. Then we have PAM. Due to free entry of the entrepreneur,

$$R(k_A; s_A) + R(k_B; s_B) - \pi(k_A) - w(s_A) - \pi(k_B) - w(s_B) = 0 \tag{8}$$

Suppose that a non-PAM rearrangement $(k_A; s_B), (k_B; s_A)$ also satisfies the first-order condition. Then

$$R(k_A; s_B) + R(k_B; s_A) - \pi(k_A) - w(s_B) - \pi(k_B) - w(s_A) = 0 \tag{9}$$

Together, they imply that $[R(k_A; s_A) + R(k_B; s_A)] - [R(k_A; s_B) + R(k_B; s_A)] = 0$, which violates supermodularity of R. $\square$

Unlike one factor models of matching and occupational choice, there is no conflict between PAM and occupational choice. The reason for our lack of conflict is because in our two factor model of skills, occupational choice is due to comparative advantage and PAM is due to absolute advantage.

We are now ready to define an equilibrium for this labor market.

Definition 1. *An equilibrium consists of an earnings function for support workers, $w(r)$, an earnings function for key role workers, π, a separating function, ϕ, and a matching function, μ, such that:*

1. *All workers choose occupations which maximize their net earnings, i.e. solve equation (1).*

2. *A free-entry entrepreneur chooses key role workers and support role workers to maximize its net earnings (which is zero), i.e. solve equation (5).*

3. *The labor market clears. I.e. every worker of type (k,s) can find the job which maximizes his or her net earnings. Due to PAM, the labor market clearing condition can be written as:*

$$H(k) = G(\mu(k)), \forall k \tag{10}$$

Equation (10) says that for every k, the mass of key role workers up to skill k must be equal to the mass of support role workers up to skill $\mu(k)$.

3   Characterizations

First, we appeal to McCann et al. (2015) for the existence of competitive equilibrium.

Theorem 1 (Existence)**.** *An equilibrium, consisting of four unique functions, an earnings function for support workers, $w(s)$, an earnings function for key role workers, $\pi(k)$, a separating function, $\phi(k)$, and a matching function, $\mu(k)$, exists.*

Second,

Proposition 3 (Convex Earning Schudules). *$w(s)$ and $\pi(k)$ are convex in s and k respectively.*

Proof. From the optimal choices of key role workers, taking the second derivatives yields:

$$\pi''(k) = R_{kk}(k : \mu(k)) + R_{ks}(k : \mu(k))\mu'(k)$$

Convexity of the revenue function in occupational skills imply $R_{kk} > 0$ and supermodularity implies $R_{ks} > 0$. PAM implies $\mu' > 0$. So $\pi''(k) > 0$. By symmetry, $w''(s) > 0$.

$\square$

While sufficient, our empirical results below show that convexity of the revenue function is not necessary to obtain convexity of occupational wages.

Third, we discuss links between the separating and matching functions.

Proposition 4 (Identical Outside Options for the Worst Match). *$\phi(\underline{k}) = \mu(\underline{k})$, such that the worst match is a self-match. Since self-match splits the output evenly, the earnings inequality within this team is zero.*

Proof. If $\phi(\underline{k}) = \underline{s}$, the type $(\underline{k}; \underline{s})$ is the worst type among key role workers and support role workers in the equilibrium. Hence under PAM this type self-matches, equally splitting the output. Hence our proposition holds.

Now suppose not, where $\phi(\underline{k}) = s^* > \underline{s}$. Then $(\underline{k}, \underline{s}) \in \Omega_k$, such that this type works exclusively in the key role. Given that ϕ is strictly increasing, the lower support of the support role workers has support role skill s^*. Hence under PAM, $\mu(\underline{k}) = s^* = \phi(\underline{k})$.

$\square$

Proposition 5 (Within-Firm Inequality). *$\pi(k) - w(\mu(k)) > 0$ if and only if $\phi(k) > \mu(k)$.*

Proof. $\pi(k) = w(\phi(k))$. As w is strictly increasing, $w(\phi(k)) - w(\mu(k)) > 0$ if and only if $\phi(k) > \mu(k)$. □

$\phi(k) > \mu(k)$ implies that the worker type $(k, \mu(k))$ works exclusively in the key role. This means that although key role workers k will match with support role workers $\mu(k)$, the type $(k, \mu(k))$ will not self-match, i.e. there is specialization within the firm. This can only be the case when $\pi(k) > w(\mu(k))$, such that working in the key role is strictly better off for this type.

Proposition 5 implies that within-firm inequality depends on the wedge between the separating line and the matching line. Given that $\phi(\underline{k}) = \mu(\underline{k})$, we are interested in how ϕ and μ differ up along the ranks. We will now provide a link between the separating function, ϕ of occupational choice and the matching function μ. Differentiating the indifference condition (2) with respect to k,

$$w'(\phi(k))\phi'(k) = \pi'(k) \tag{11}$$

Substituting (6), (7) and $s = \mu(k)$ yields:

$$\phi'(k) = \frac{R_k(k; \mu(k))}{R_s(\mu^{-1}(\phi(k)); \phi(k))} \tag{12}$$

Equation (12) provides a restriction between the separating function and matching function that depends on the technology R. $\phi'(k)$ is the slope of the separating line, representing marginally how workers separate into key role and support role. The fraction on the right hand side is a ratio of marginal products: $R_k(k; \mu(k))$ is the marginal product of the indifferent type $(k, \phi(k))$ when he works as a key role worker; $R_s(\mu^{-1}(\phi(k)); \phi(k))$ is the marginal product of $(k, \phi(k))$ when he works as a support worker. The larger this fraction, optimally locally more types should be assigned to the key role.

19

4 Social Planner's Problem

Our model of frictionless occupational choice and matching is equivalent to a social planner determining occupational choices and matching for the population to maximize total revenue of the economy. In fact, the social planner's problem is a linear programming problem. McCann et al. (2015)'s proof of existence and uniqueness uses this equivalence. This section develops the Social Planner's problem in detail because a linear program is a much easier problem to numerically solve than looking for a fixed point of our competitive model. This is how we estimate the model and also produce simulation results.

Let $m : \mathbb{R}_+^2 \to \mathbb{R}_+$ be a density function such that $m(k_1; s_2)$ states the mass of team $(k_1; s_2)$, and let $\sigma_k, s_r : \mathcal{T} \to \mathbb{R}_+$ be density functions such that $\sigma_k(t), s_r(t)$ record the mass of agents of type t working in key role and support role respectively.

We have two accounting constraints:

$$\text{(Accounting Constraint for Key Role): } \int_{\tilde{s}} m(k; \tilde{s}) d\tilde{s} = \int_{\tilde{s} \in \mathbb{R}_+} \sigma_k(k, \tilde{s}) d\tilde{s}, \forall k \in \mathcal{K}$$

$$\text{(Accounting Constraint for Support Role): } \int_{\tilde{k}} m(\tilde{k} : s) d\tilde{k} = \int_{\tilde{k} \in \mathbb{R}_+} \sigma_k(\tilde{k}, s) d\tilde{k}, \forall s \in \mathcal{S}$$

The first accounting constraint states that the total mass of teams that involves key role workers k must be equal to the total mass of individuals whose key role skill is k, and that they select to work in the key role. The second accounting constraint is similarly defined.

We also have a resource constraint:

$$\sigma_k(t) + \sigma_r(t) = f(t) \ \forall t \in \mathcal{T}$$

Given $\{R, f\}$, the Social Planner allocates agents in teams to maximize social output, defined as the integral of team outputs. Let the space of $\{m, \sigma_k, \sigma_w\}$ under the resource and accounting constraints de-

20

fined above as Ω. The social planner's problem is:

$$S = \max_{\{\hat{m},\hat{s}_k,\hat{s}_w\}\in\Omega} \int_{\mathcal{T}} R(k:s)\hat{m}(k:s)dkds \tag{13}$$

The Lagrangian for the Social Planner's Problem is as follows:

$$
\begin{aligned}
\mathcal{L} = & \int_{\mathcal{T}} R(k;s)\hat{m}(k:s)dkds \\
& + \int_{\mathcal{K}} \left\{ \pi(k) \int_{\mathcal{S}} [\sigma_k(k,s) - \hat{m}(k;s)]ds \right\} dk \\
& + \int_{\mathcal{S}} \left\{ w(s) \int_{\mathcal{K}} [\sigma_s(k,s) - \hat{m}(k;s)]dk \right\} ds \\
& + \int_{\mathcal{T}} \psi(k,s)[f(k,s) - \hat{\sigma}_k(k,s) - \hat{\sigma}_s(k;s)]dkds \\
& + \int_{\mathcal{T}} \lambda_m(k,s)\hat{m}(k;s)dkds \\
& + \int_{\mathcal{T}} \lambda_{\sigma_k}(k,s)\hat{\sigma}_k(k,s)dkds + \int_{\mathcal{T}} \lambda_{\sigma_s}(k,s)\hat{\sigma}_s(k,s)dkds
\end{aligned} \tag{14}
$$

where, abusing notation, $\{\pi(k)\}_{k\in\mathbb{R}_+}$, $\{w(s)\}_{s\in\mathbb{R}_+}$ are the collection of Lagrangian multipliers for the accounting constraints; $\{\psi(k,s)\}_{(k,s)\in\mathbb{R}_+^2}$ is the collection of Lagrangian multipliers for the resource constraints, and

$$\{\lambda_m(k,s)\}_{(k,s)\in\mathbb{R}_+^2}, \{\lambda_{\sigma_k}(k,s)\}_{(k,s)\in\mathbb{R}_+^2}, \{\lambda_{\sigma_s}(k,s)\}_{(k,s)\in\mathbb{R}_+^2}$$

are the collection of Lagrangian multipliers for the non-negativity constraints.

The first-order condition with respect to $\hat{m}(k:s)$ is:

$$R(k:s) - [\pi(k) - w(s)] + \lambda_m(k,s) = 0 \tag{15}$$

If $m(k:s) > 0$, then $\lambda_m(k:s) = 0$ due to complementary slackness. This first-order condition is the same as that in the competitive equilibrium.

The first-order conditions with respect to $\sigma_k(k,s), \sigma_s(k,s)$ are:

$$\pi(k) - \psi(k,s) + \lambda_{\sigma_k}(k,s) = 0 \tag{16}$$

$$w(s) - \psi(k,s) + \lambda_{\sigma_s}(k,s) = 0 \tag{17}$$

For a type $(k,s) \in \mathcal{T}$ who works in both roles such that $\sigma_k(k,s) > 0, \sigma_s(k,s) > 0$, then $\lambda_{\sigma_k}(k,s) = \lambda_{\sigma_s}(k,s) = 0$. Hence:

$$\pi(k) = w(s) = \psi(k,s) \tag{18}$$

which is the occupational choice equation in the competitive equilibrium.

Note that $\psi(k,s) = \partial \mathcal{L}(.)/\partial f(k,s)$. Therefore, $\psi(k,s)$ is the social cost of employing an individual of the type $(k,s) \in \mathcal{T}$ at the margin. Also, $\psi(k,s) = \max\{\pi(k), w(s)\}$. Hence, the social marginal cost, due to occupational choice, is the maximum of the cost of hiring the individual as a key role worker and that as a support role worker.

5 Estimation and Simulations

5.1 Main Specification

This section estimates the model using 1999 Brazilian data. We also simulate the model to see how it matches the 2013 data.

The base skills distributions, c and r, are independent. c is the schooling by years in Brazil, taken from Benguria. Since we do not know what the non-cognitive skill distribution looks like, we treat it like an error distribution in our estimation. We assume that the non-cognitive skill r follows a time invariant symmetric truncated normal distribution (at 3 standard deviations) with the same support as the schooling distribution.

See Figure 7 for a plot of the density functions of base skills. The curve with dot markers shows the density of cognitive skill c in the year 1999, corresponding to the years of schooling. The curve with triangle

markers shows the density of cognitive skill in the year 2013. The two curves reveals that the schooling distribution shifts to the right from 1999 to 2013 in Brazil significantly; the average schooling almost doubled. The red curve in Figure 7 shows the density of non-cognitive skill r, which is assumed to be invariant across the two years 1999 and 2013.

The two occupations (key role and support role) have the following aggregators:

$$k_1 = c_1^{\beta_k} r_1^{1-\beta_k} \tag{19}$$

$$s_2 = c_2^{\beta_s} r_2^{1-\beta_s} \tag{20}$$

The simulation starts with a 50×50 square grid for (c,r). Because the aggregation is constant returns to scale with equal support of c and r, the grid for (k,s) is also a 50×50 square grid.

The production function in (k,s) is

$$R(k_1, s_2) = A k_1^{\alpha_k} s_2^{\alpha_s} \tag{21}$$

So the five parameters of the model consist of $\{A, \alpha_k, \alpha_s, \beta_k, \beta_s\}$. A is a scaling parameter; α_k, α_s control the (marginal) productivity of k and s; β_k, β_s control how cognitive skill c and non-cognitive skill r aggregate into role-specific skills k and s.

Note that the Cobb-Douglas form of the revenue function assumes supermodularity in (k_1, s_2) as the cross-deriative $R_{ks} = A \alpha_k \alpha_s k_1^{\alpha_k-1} s_2^{\alpha_s-1} > 0$. Whereas convexity of the revenue function is not assumed.

The key role and support role can be relabelled, such that if (α_k, β_k) and (α_s, β_s) are swapped, an equivalent model would result. To resolve this labelling issue, we impose $\beta_k > 0.5 > \beta_s$, such that the key role demands more cognitive skill than non-cognitive skill, and the support role demands more non-cognitive skill than cognitive skill.

The parameters of the model are estimated by fitting the model to the 1999 individual and between-firm inequality. The sum of squares devia-

tion, evaluated at each percentile between the simulated curves and the actual corresponding curves from Benguria (2015), is minimized.[7]

The main specification simulates the model using the following estimated parameters: $A = 2.2256$, $\alpha_k = 1.7397$, $\alpha_s = 0.9126$, $\beta_k = 0.9346$, $\beta_s = 0.3129$. Since $\alpha_k > 1 > \alpha_s$, the technology is strictly convex in k but not in s. Figure 8 plots the contour of occupational skills given the aggregation functions (19) and (20) for 1999 and 2013. The black lines that splits the contour maps in halves are the graphs of the separating function ϕ.

The graphs of simulated matching function μ and separating function ϕ are plotted in Figure 9 for 1999 and 2013. For both years, μ is strictly increasing in k, implying positive assortative matching. ϕ is strictly increasing in k as well. They are in line with our theoretical results. Across the two periods, μ shifted to the right less than ϕ. For any k, the average before-after difference in $\mu(k)$ is about 1.36 units of s (relative to a grid of 50 units). Whereas the average before-after difference in $\phi(k)$ is 2.51, which is larger.

Next we examine the simulated earnings functions π and w. Figure 10 shows their corresponding plots. In the first panel of Figure 10, the horizontal axis is skill level (k for key role, s for support role). Both $\pi(k)$ and $w(s)$ are strictly increasing and convex with respect to their arguments. Note that convexity of occupational wages obtain without the revenue function being strictly convex in both skills. In the second panel of Figure 10, we plot π, w by rank instead. Due to positive assortative matching, a key role worker and support role worker would match if and only if their respective ranks are equal. The second panel shows that a key role worker at any rank is earning more than his support role partner.

The earnings functions, being convex in their respective skills, do not

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[7]In the actual implementation, we transform the parameters as follows: A kept in levels; $\alpha_k = \exp(a), \alpha_s = \exp(a_s)$ where $a_k, a_s \in \mathbb{R}$; $\beta_k = 0.5 + 0.5\Phi(b_k)$ where $b_k \in \mathbb{R}$, $\beta_s = 0.5 + 0.5\Phi(b_s)$ where $b_s \in \mathbb{R}$. This transformation yields an objective function unconstrained in the parameters, while imposing $\beta_k > 0.5 > \beta_s$.

necessarily lead to convex earning distributions which also depend on the underlying distribution of skills. Hence we plot the earnings distributions in Figure 11. The upper panel of Figure 11 shows that for both key role and support role, the earnings distributions are skewed to the right. Aggregating across the two roles, the earnings distributions for 1999 and 2013 are both shown in the lower panel of Figure 11. The earnings distribution in 2013 is less skewed than that of 1999, corresponding to a decreased individual inequality.

The first panel of Figure 12 shows how well our estimated model fits the 1999 data. The deviations are mostly at the top and bottom percentiles. Quantitatively, the R-squared fits for individuals and firms are 0.8369 and 0.9343. For 2013, we hold the estimated parameters constant, only allowing the schooling distribution to shift to its 2013 values. This exercise tests whether this distributional shift alone can produce the observed changes in earnings inequality. The second panel of Figure 12 shows how well our estimated/simulated model predicts the 2013 data. Qualitatively, the simulated data replicates the slopes of individual and firm quantiles well, despite there is a misfit in levels. As is apparent from the figures, the R-squared fits for individuals and firms are 0.6194 and 0.2805 which are worse than the corresponding figures for 1999. This is not surprising because the parameters of the model were estimated to fit the 1999 figure. Once we allow the technology to have neutral change by restimating A alone to fit the 2013 data, the fit restores such that the corresponding figures become 0.7416 and 0.786.[8]

Given our goodness of fit results, it is not surprising that we can also replicate the changes in earnings inequality across individuals and firms between 1999 and 2013. The simulated changes in inequality is shown

---

[8]What is perhaps surprising is that the level of simulated earnings in 2013 exceed that of the actual. This implies that given the large shift of educational distribution and a fixed technology, Brazilian workers would have earned more on average in 2013. This phenomenon is probably due to the general equilibrium effects, such that the large increase in highly educated population deflates the value of high education. Since our objective is to discuss earnings inequality rather than its levels, we do not pursue this issue further.

in Figure 13 which largely resembles the actual data (Figure 6).

As discussed by Benguria and reviewed here, the Brazilian labor market changed significantly from 1999 to 2013. In particular, there was a marked decline in individual earnings inequality.

We have shown that a five parameter model of the Brazilian labor market can fit individual and across firm earnings inequality in 1999. The only observed heterogeneity of this model is the educational distribution. Based only on a shift in the educational distribution, we can, to a first order, replicate the changes in individual and across firms earnings inequality from 1999 to 2013. In spite of large changes in the distribution of skills and distribution of earnings, our model can also generate the lack of change in within firm earnings inequality observed between the two periods.

6 Skill biased technical change

The predominant explanation for the recent increase in the US is SBTC. See the survey by Autor and Acemoglu. SBTC is assumed to increase the marginal productivity of college educated workers relative to non-college educated workers. This divergence in productivity translates to divergence in earnings. In the standard model of SBTC, college enrollment is assumed to adjust slowly if at all to the change in earnings inequality over time.

Our model can be used to model the short and long run effect of SBTC. In the short run, there is no occupational choice even as SBTC change occurs. Key role workers and support workers cannot change occupations. They can change firms in response to SBTC. Since SBTC increases the marginal product of key role workers, at the old wage gradient for support workers, key role workers will want to hire higher skill support workers bidding up the earnings of support workers. This effect will mitigate the increase in earnings inequality in the short run. In the long-run, individuals can switch occupations. Thus we expect the

increase in earnings inequality due to SBTC will be even more muted.

To study how SBTC affects earnings inequality in our framework, we use the 2013 parameters as a benchmark. To model SBTC, we increase α_k, which raises the marginal rate of technical substitution $R_k/R_s = (\alpha_k/\alpha_s)(s/k)$, so that the key role becomes relatively more productive than before. We increase the value of α_k from 1.74 to 2 to make the effect apparent.

In the short-run, key role and support workers cannot change occupations in response to SBTC. The short run separating function is the same as before SBTC (Figure 14). Although earnings will change in response to SBTC, we know that there will be PAM in the new equilibrium. So short run equilibrium matching in teams is also the same as before. Since $R_k(k, \mu(k)) = \pi'(k)$ and $\pi(0) = 0$ after SBTC, the earnings schedule rotates upwards. All key role workers earn more than before, and their earnings diverge from before. Support role workers, whose earnings schedule is governed by $R_s(\mu^{-1}(s), s) = w'(s)$, will also increase but are less affected by SBTC. See the short run change in earnings inequality in Figure 15. The figure also plots the change in within firm inequality. Within firm inequality falls for low earnings firms which suggests that the increase in demand for higher skilled support workers benefited lower skill support workers relative to lower skill key role workers.

In the long-run, there is both occupational choice and matching. See the long run separating function in Figure 14. Some previously high skill support workers who also have high key role skills switch to the key role occupation. To maintain labor market equilibrium, previously low skill key role workers who also have low support role skills switch to support role occupations. For these previously key role workers to switch in spite of their increased productivity after SBTC, it must be the case that their support role wages increase significantly. I.e. for low productivity firms, within firm inequality must fall. See Figure 15. Although there is significant change in aggregate earnings inequality

due to SBTC, the increase in earnings inequality is smaller in the long run than in the short run. Also, there is minimal change in within firm inequality in the long run due to SBTC, consistent with the evidence in Song, et. al.. So our simulation of SBTC is able to qualitatively match the changes in both across and within firm inequality documented in Song, et. al..

The comparison between long run and short run effects of SBTC on earnings inequality show that occupational choice is central to mitigating the effects of SBTC on increasing earnings inequality.

7   Conclusion

This paper integrates Roy's model of occupational choice with Becker's model of matching in the labor market. It make three key modelling assumptions: (1) High earnings firms produce higher quality output with higher skill workers. (2) There is a bivariate distribution of occupational skills. (3) The firm revenue function is supermodular and convex in occupational skills. Our model generates earnings distributions which match the invariant characteristics discussed in the introduction without making strong parametric assumptions on the distributions of firm and worker characteristics.

The model is parameterized to quantitatively fit the aggregate and between firm earnings inequality in Brazil in 1999. Our simulation of the model for 2013 shows that the large increase in educational attainment between 1999 and 2013 was a first order factor in reducing aggregate Brazilian earnings inequality over that period.

SBTC in the model can also qualitatively rationalize the changes in the US earnings distribution discussed by Song et al. (2015).

Our model is highly stylized, ignoring some important features of labor markets. First, we do not consider variation in firm size. This does not allow us to discuss variation in the quantity of output across firms, an important concern of the standard model of SBTC. Second, we

take the underlying skill (education) distribution as exogenous without considering why the Brazilian schooling has shifted. Third, we have a static model and we ignore search frictions in both occupational choice and matching. We leave these important concerns for further research.

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A   Table and Figures
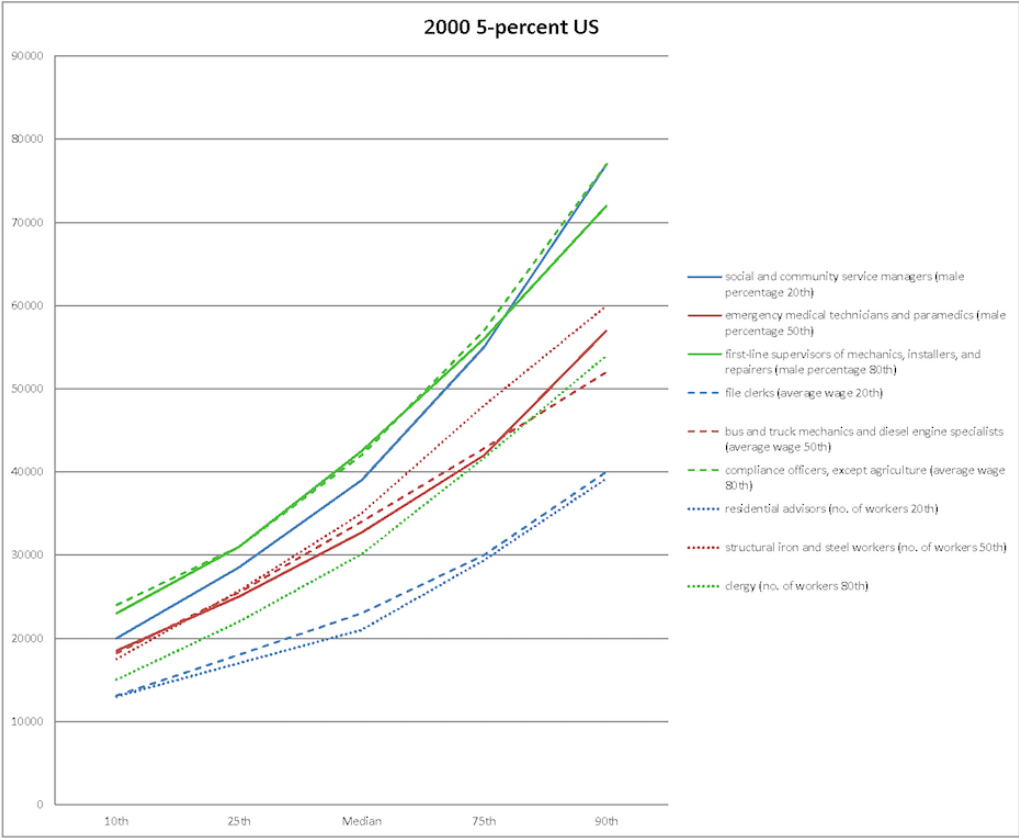
Figure 1: US 2000 Census
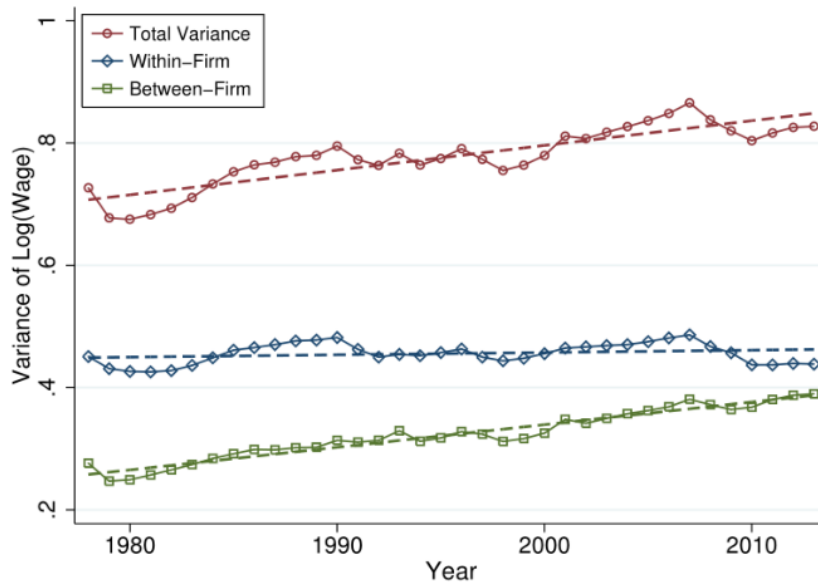
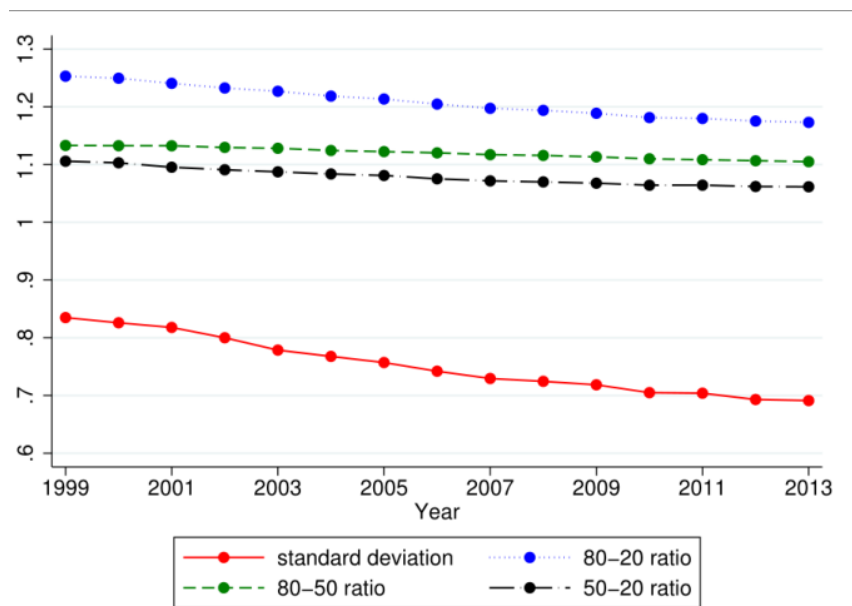Figure 2: U.S. Trends of Total, Between and Within Inequality

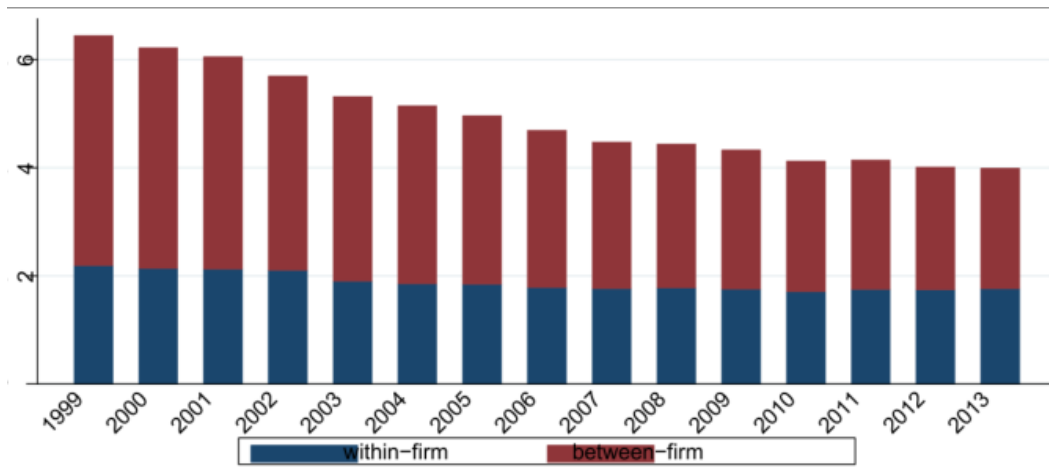Figure 3: Brazilian Trends of Total Inequality

34

Figure 4: Brazilian Between versus Within Firm Inequality

Note: Sample contains workers in firms with 20+ full-time equivalent employees.

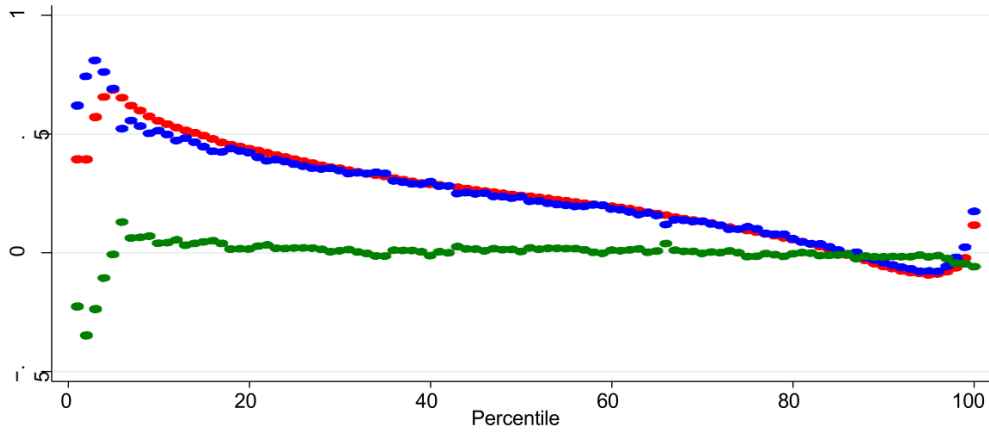Figure 5: Percentile Decomposition of US Wage Inequality

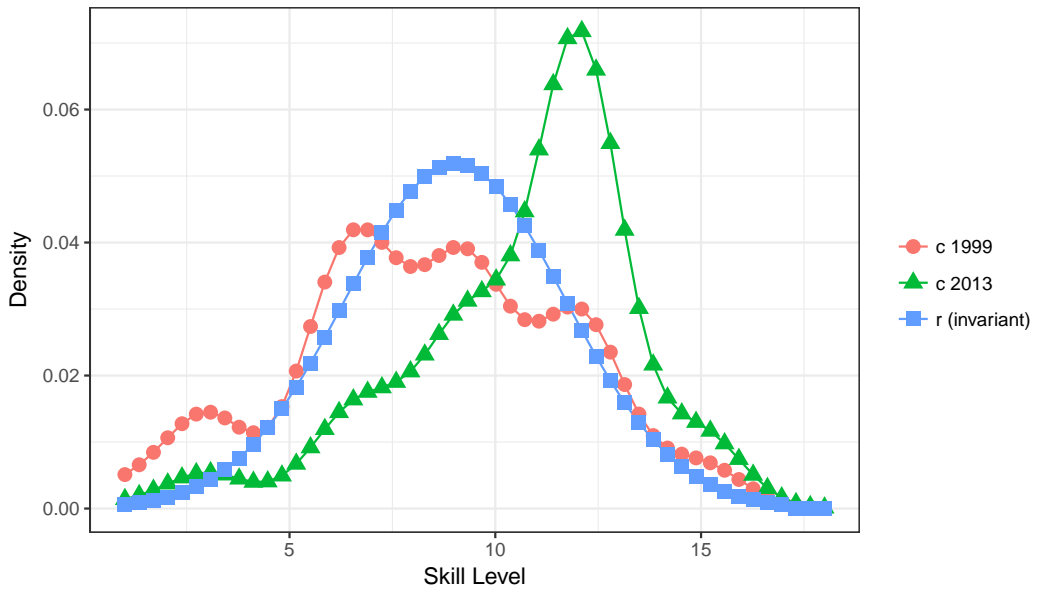Figure 6: Percentile Decomposition of Brazil Wage Inequality
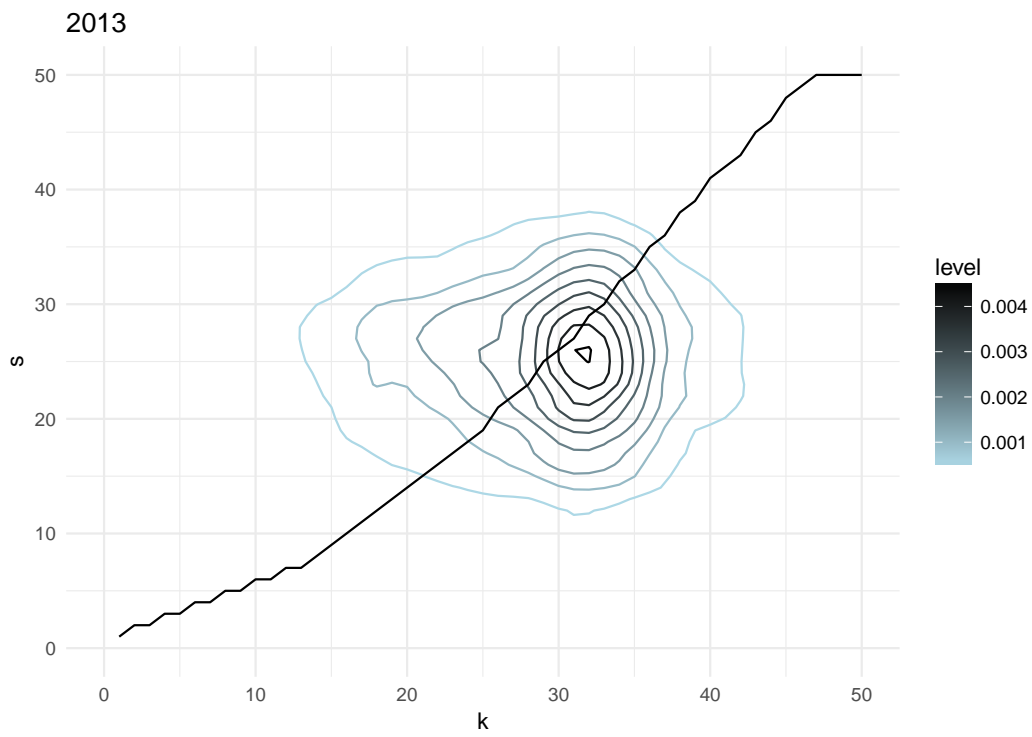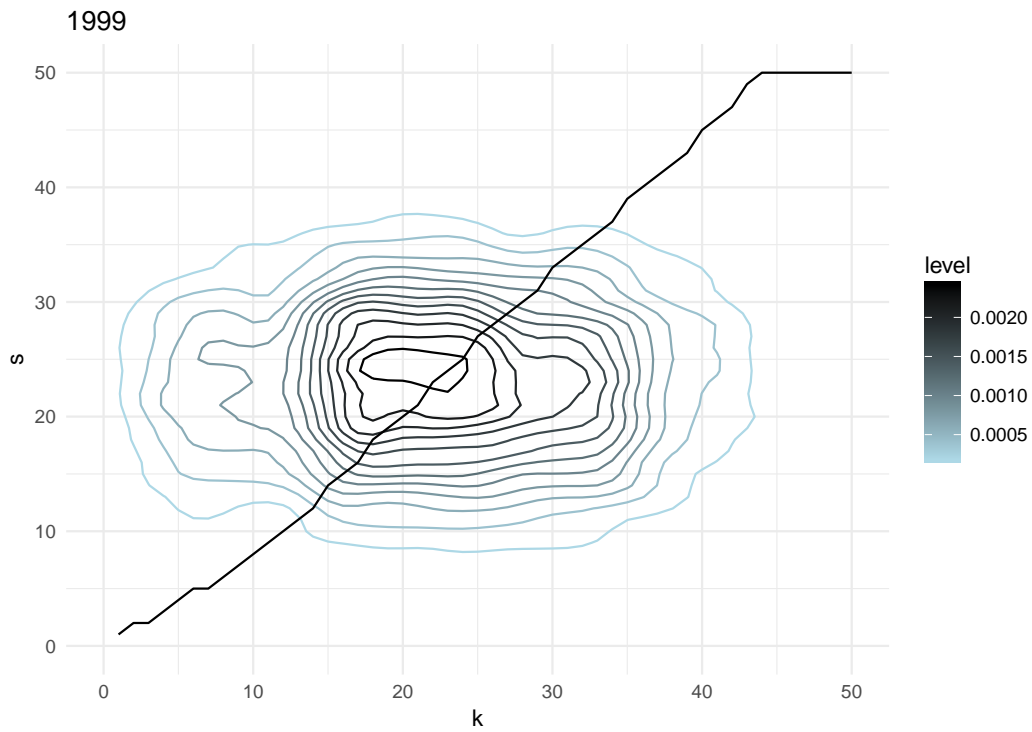

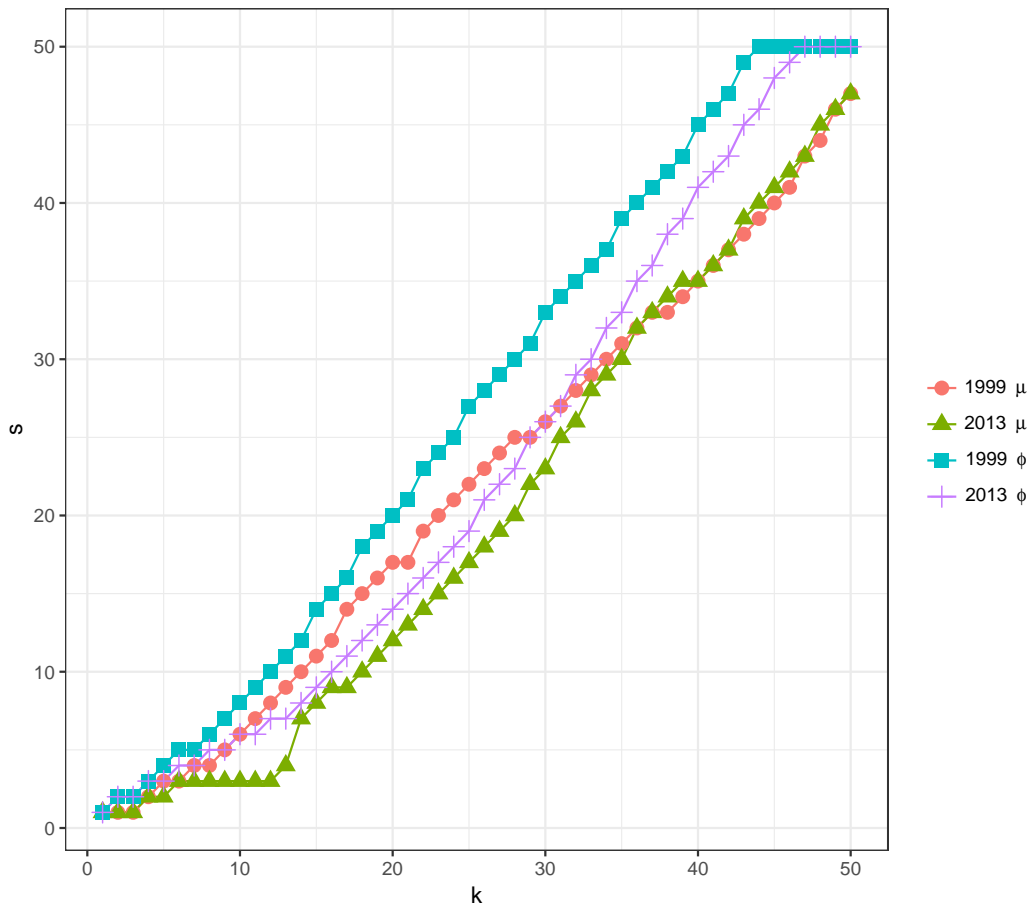Figure 7: Base Skill Distributions

Figure 8: Goodness of Fit Plots

Figure 9: Plots of Matching and Separating Functions in (k, s) space

Figure 10: Earnings Schedule

By Occupation

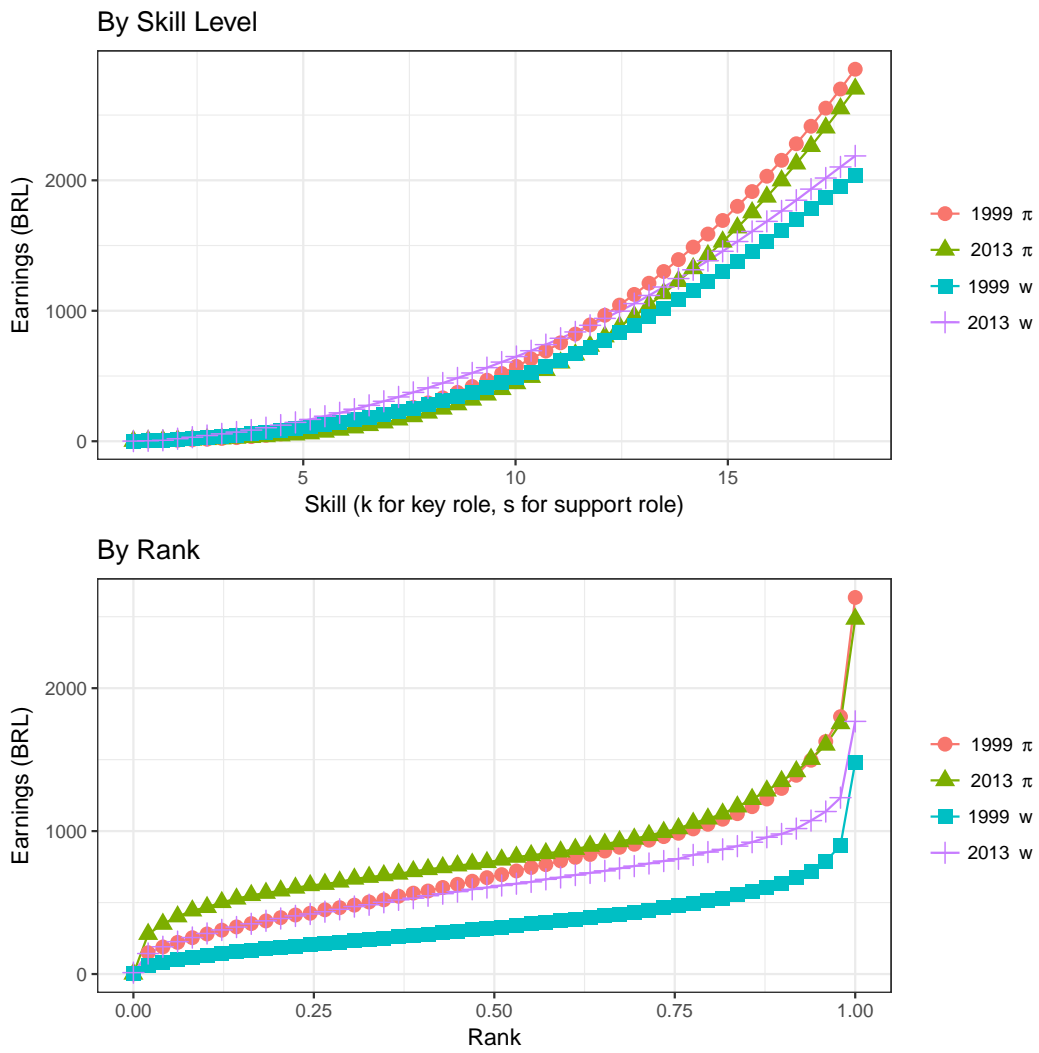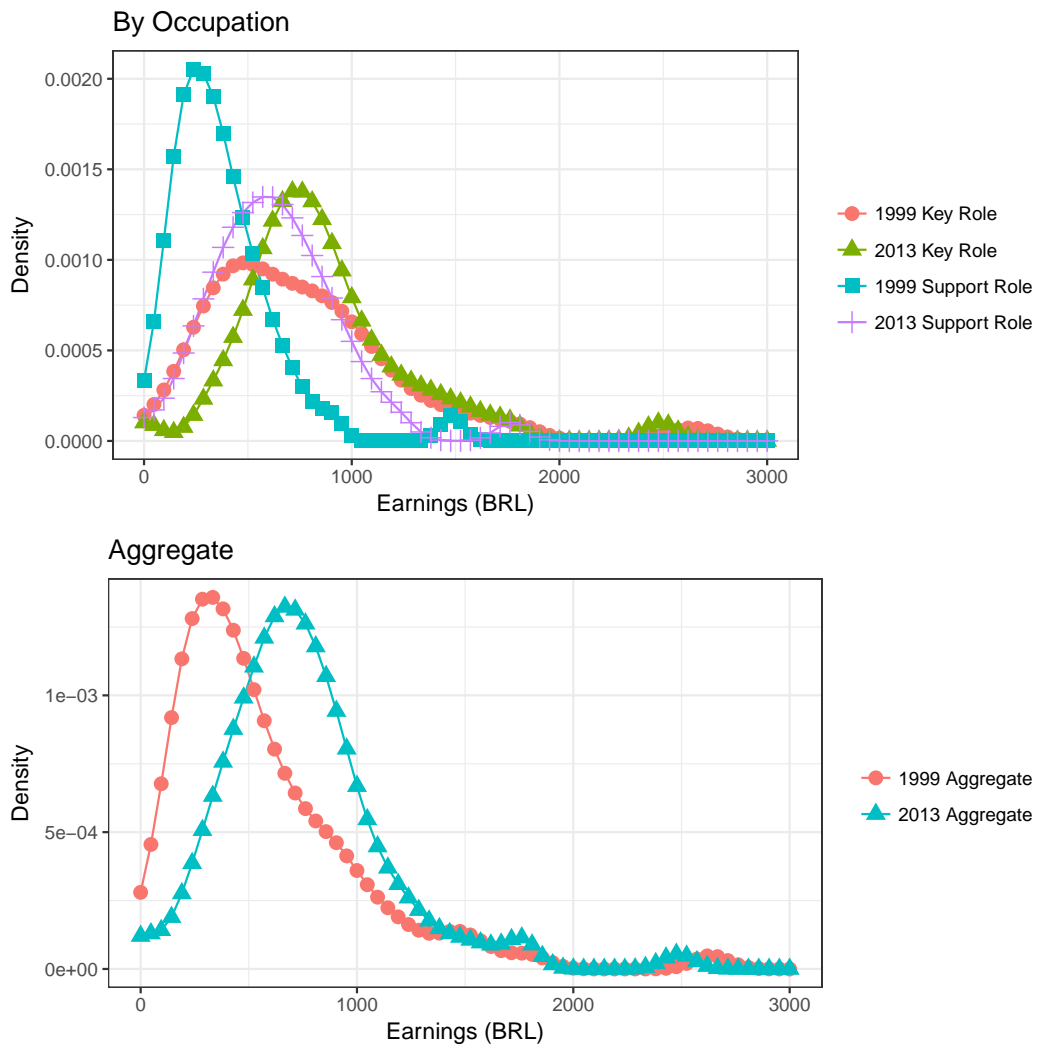Aggregate
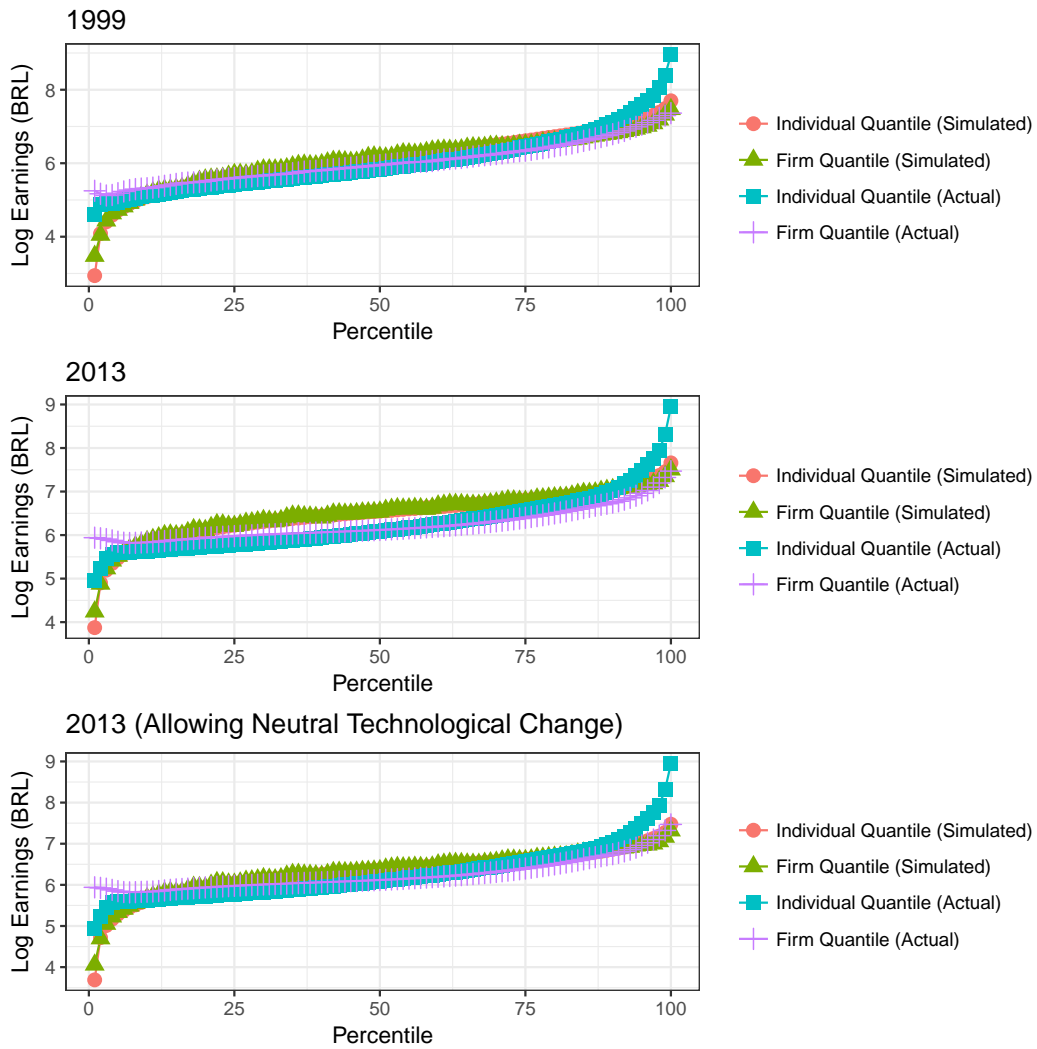
Figure 11: Earnings Distributions
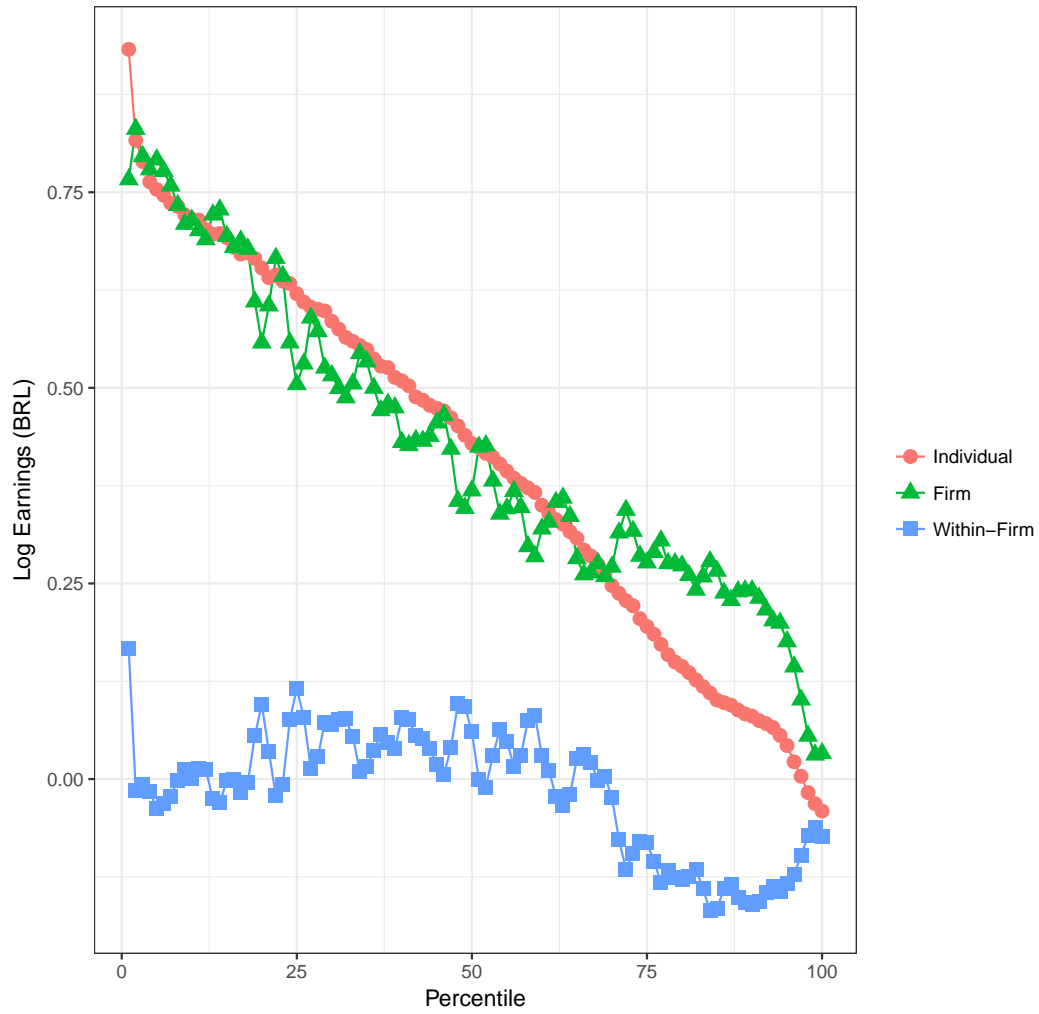
Figure 12: Goodness of Fit Plots

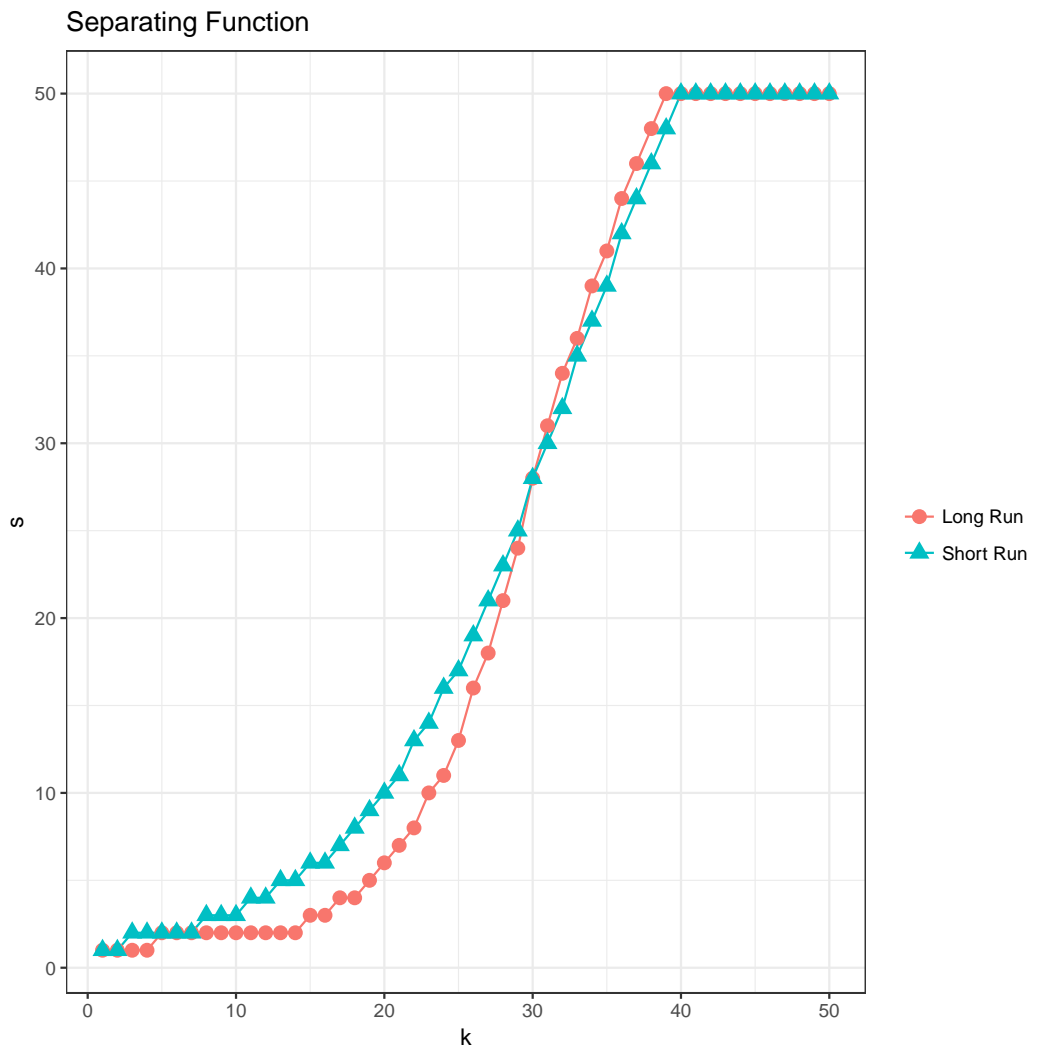Figure 13: Percentile Plot of Change (2013-1999)

Figure 14: Separating Function after SBTC
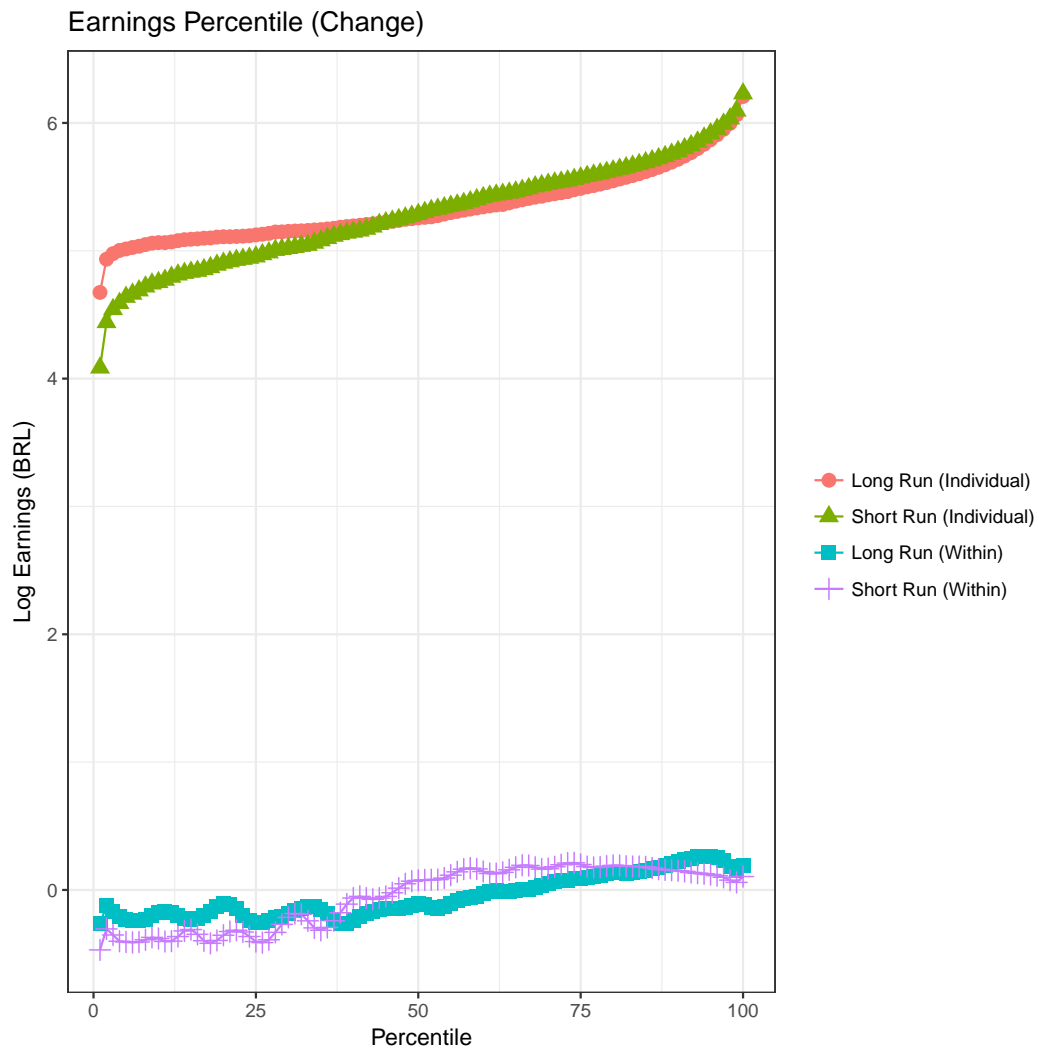
Figure 15: Earnings Percentiles after SBTC (Change)

Online Appendix for Occupational Choice and Matching in the Labor Market (Not for Publication)

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February 2, 2017

This online appendix reports the raw data input in our Brazilian calibration. Section 1 reports Brazilian inequality in levels, for the years 1999 and 2013 respectively, and its within-firm decompositions. Section 2 reports the education distributions in Brazil.

1 Brazil Inequality in Levels

Figure 1 and Figure 2 in this online appendix plot the inequality percentile decompositions for 1999 and 2013 respectively. For each figure, the red lines are individual percentiles, the blue lines are firm percentiles. The difference between red and green lines produces the green lines, which represent within-firm inequality. Taking time difference between Figure 1 and Figure 2 produces Figure 6: Percentile Decomposition of Brazil Wage Inequality in the main text.

Figure 1: Brazil Inequality 1999
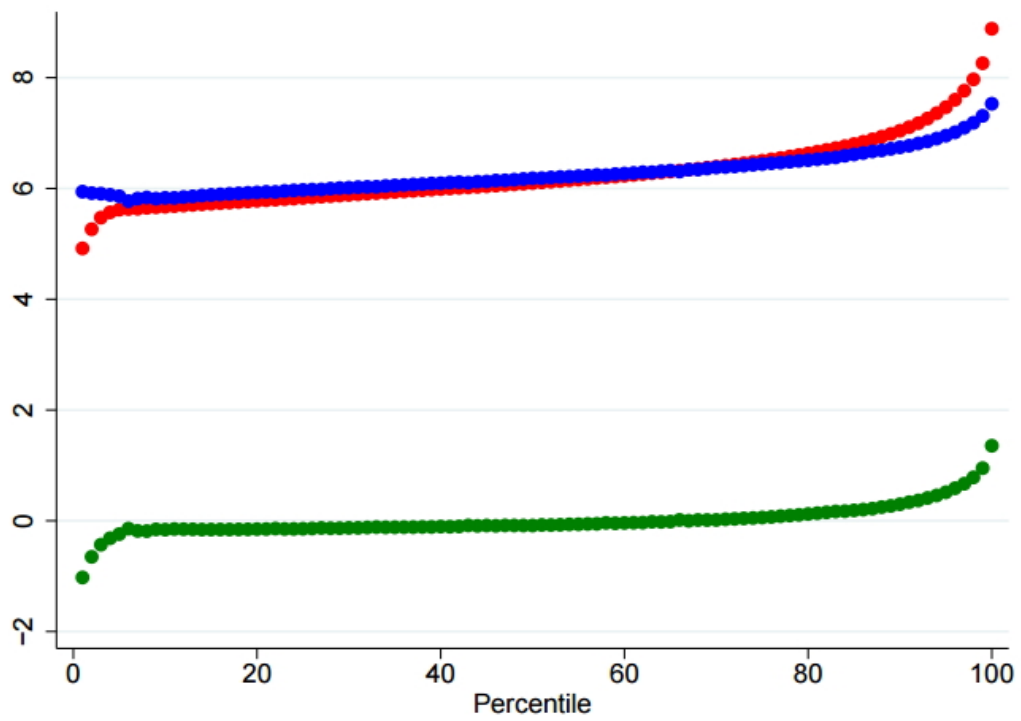
Figure 2: Brazil Inequality 2013

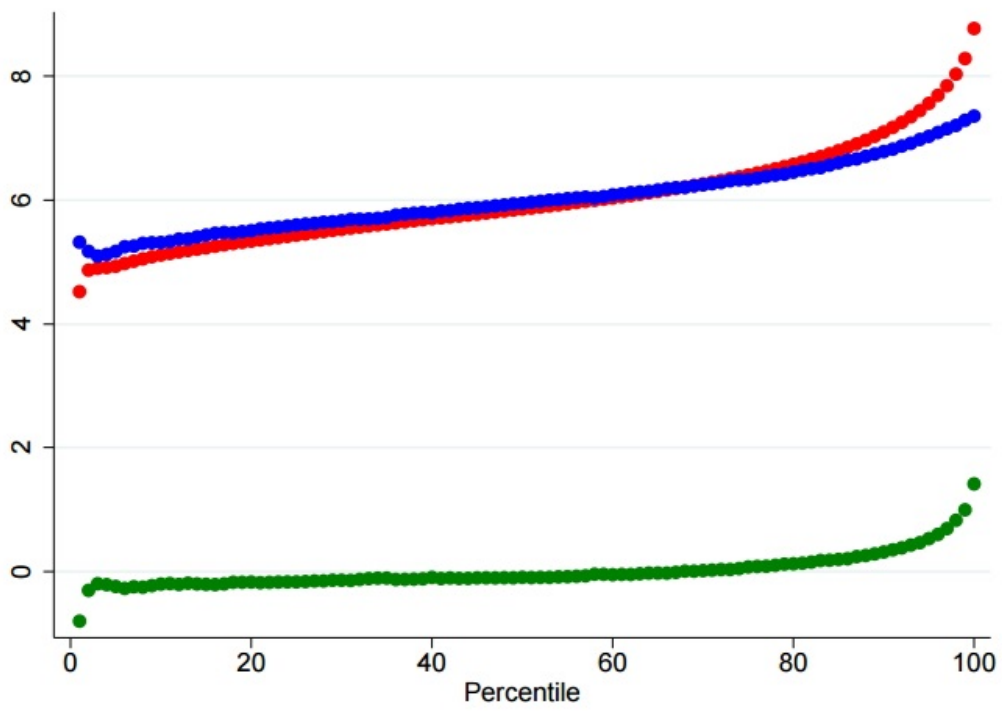We first extract raw data values from these graphs. Then we evaluate the individual and firm average wage at each percentile by interpolation (in log BRL). We print the resulting data table below.

Percentile	1999 Individual	2013 Individual	1999 Firm	2013 FIrm
1.00	4.59	4.96	5.25	5.89
2.00	4.88	5.23	5.17	5.87
3.00	4.94	5.43	5.12	5.85
4.00	4.91	5.55	5.14	5.83
5.00	4.93	5.60	5.18	5.80
6.00	4.98	5.60	5.23	5.78
7.00	5.02	5.61	5.27	5.77
8.00	5.05	5.61	5.29	5.77
9.00	5.08	5.62	5.31	5.77
10.00	5.11	5.63	5.32	5.78
11.00	5.13	5.64	5.34	5.78
12.00	5.15	5.65	5.36	5.79
13.00	5.18	5.66	5.38	5.81
14.00	5.20	5.67	5.41	5.82
15.00	5.23	5.68	5.44	5.83
16.00	5.25	5.69	5.46	5.84
17.00	5.27	5.70	5.47	5.85
18.00	5.29	5.71	5.48	5.86
19.00	5.30	5.72	5.49	5.87
20.00	5.32	5.73	5.51	5.88
21.00	5.34	5.74	5.53	5.88
22.00	5.36	5.75	5.55	5.89
23.00	5.38	5.76	5.57	5.90
24.00	5.39	5.77	5.58	5.91
25.00	5.41	5.78	5.60	5.92
26.00	5.43	5.79	5.61	5.93
27.00	5.45	5.80	5.63	5.93
28.00	5.46	5.81	5.64	5.94
29.00	5.48	5.82	5.65	5.95
30.00	5.50	5.83	5.67	5.96

Percentile	1999 Individual	2013 Individual	1999 Firm	2013 FIrm
31.00	5.51	5.83	5.68	5.97
32.00	5.52	5.84	5.69	5.97
33.00	5.53	5.85	5.70	5.98
34.00	5.55	5.86	5.71	5.99
35.00	5.57	5.87	5.73	6.00
36.00	5.59	5.88	5.75	6.01
37.00	5.61	5.89	5.77	6.01
38.00	5.62	5.90	5.79	6.02
39.00	5.64	5.91	5.80	6.03
40.00	5.65	5.92	5.81	6.04
41.00	5.66	5.93	5.82	6.05
42.00	5.68	5.93	5.84	6.05
43.00	5.70	5.94	5.85	6.05
44.00	5.71	5.95	5.87	6.06
45.00	5.72	5.96	5.88	6.07
46.00	5.74	5.97	5.89	6.08
47.00	5.75	5.98	5.91	6.09
48.00	5.77	5.99	5.92	6.10
49.00	5.78	6.00	5.94	6.11
50.00	5.80	6.02	5.95	6.12
51.00	5.82	6.03	5.97	6.13
52.00	5.83	6.04	5.98	6.14
53.00	5.85	6.04	6.00	6.15
54.00	5.87	6.05	6.01	6.15
55.00	5.88	6.07	6.02	6.16
56.00	5.89	6.08	6.03	6.17
57.00	5.90	6.09	6.04	6.18
58.00	5.92	6.10	6.04	6.19
59.00	5.94	6.11	6.06	6.19

Percentile	1999 Individual	2013 Individual	1999 Firm	2013 FIrm
60.00	5.96	6.12	6.08	6.21
61.00	5.98	6.13	6.10	6.22
62.00	6.00	6.15	6.12	6.23
63.00	6.01	6.17	6.13	6.24
64.00	6.03	6.20	6.14	6.24
65.00	6.05	6.24	6.16	6.25
66.00	6.08	6.28	6.18	6.26
67.00	6.12	6.33	6.20	6.27
68.00	6.18	6.38	6.22	6.28
69.00	6.24	6.43	6.23	6.30
70.00	6.30	6.47	6.25	6.31
71.00	6.36	6.50	6.27	6.32
72.00	6.40	6.53	6.29	6.33
73.00	6.44	6.54	6.31	6.35
74.00	6.46	6.56	6.33	6.36
75.00	6.48	6.58	6.34	6.38
76.00	6.51	6.60	6.36	6.39
77.00	6.53	6.62	6.39	6.40
78.00	6.56	6.64	6.41	6.42
79.00	6.60	6.66	6.43	6.43
80.00	6.63	6.68	6.46	6.45
81.00	6.66	6.71	6.49	6.46
82.00	6.69	6.73	6.51	6.48
83.00	6.73	6.76	6.54	6.49
84.00	6.77	6.79	6.57	6.52
85.00	6.82	6.82	6.61	6.55
86.00	6.87	6.85	6.64	6.57
87.00	6.92	6.89	6.68	6.60
88.00	6.98	6.93	6.71	6.62
89.00	7.05	6.98	6.75	6.64
90.00	7.12	7.03	6.79	6.67
91.00	7.19	7.10	6.83	6.70
92.00	7.28	7.18	6.88	6.74
93.00	7.37	7.26	6.93	6.78
94.00	7.48	7.36	6.99	6.83
95.00	7.60	7.48	7.05	6.88
96.00	7.71	7.60	7.11	6.94
97.00	7.85	7.73	7.17	7.02
98.00	8.06	7.96	7.23	7.12
99.00	8.38	8.36	7.30	7.24
100.00	8.99	8.89	7.36	7.39

2 Education Distribution

The other input in our Brazilian calibration is the education distribution. Felipe Benguria has provided the following table:

In 1999 educational attainment is divided in nine groups. They are based on the Brazilian educational system but are roughly the following.

1. Illiterate

2. Lower School - Incomplete

3. Lower School - Complete

4. Middle School - Incomplete

5. Middle School - Complete

6. High School - Incomplete

7. High School - Complete

8. College (or technical education) - Incomplete

9. College (or technical education) - Complete

CATEGORY	NUMBER 1999 (1000S)	SHARE 1999	NUMBER 2013 (1000S)	SHARE 2013
1	500.24	0.02	224.51	0.00
2	2150.89	0.10	1745.27	0.04
3	3224.57	0.15	1885.85	0.04
4	3885.97	0.18	3827.75	0.08
5	4179.40	0.19	6118.37	0.13
6	2124.18	0.10	4522.38	0.10
7	3968.14	0.18	22094.81	0.48
8	633.64	0.03	1623.57	0.04
9	1016.91	0.05	3681.21	0.08

We recode 1-9 into 1,3,6,7,9,10,12,13,15 years of schooling respectively. Our results does not depend on the precise definitions.