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ABSTRACT

Public-Sector Employment in an Equilibrium Search and Matching Model*

We extend the Diamond-Mortensen-Pissarides model of equilibrium unemployment to incorporate public-sector employment. We calibrate our model to Colombian data and analyze the effects of public-sector wage and employment policy on the unemployment rate, on the division of employment between the private and public sectors, and on the distributions of wages in the two sectors.

JEL Classification: J45, J64, D83

Keywords: public sector, search and matching, wages, unemployment

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1 Introduction

The public sector accounts for a substantial fraction of employment in both developed and developing economies. Algan et al. (2002) estimates that the public sector accounted for slightly less than 19% of total employment across 17 OECD countries in 2000, and Mizala et al. (2011) estimates that 13% of total urban employment over the period 1996-2007 across eleven Latin American countries was in the public sector.

In this paper, we incorporate a public-sector labor market into an extended version of the Diamond-Mortensen-Pissarides (DMP) search and matching model of equilibrium unemployment (Pissarides 2000). Our model is designed to address distributional questions. What types of workers tend to work in the public sector? What types tend to sort into the private sector? How do the size of the public sector and the hiring and wage-setting rules used in that sector affect the overall unemployment rate and the distributions of worker types and wages across the two sectors?

Our extension to the basic DMP model has three key ingredients. First, we assume an exogenous distribution, $Y \sim F(y)$, $\underline{y} \leq y \leq \overline{y}$, of human capital across workers.[1] This makes it possible to address questions about which types of workers tend to work in the two sectors. Second, we allow for *ex post* idiosyncratic match productivity. When a worker of type y meets a prospective employer with a vacancy, the worker draws a match-specific productivity, $X \sim G_s(x|y)$, $0 \leq x \leq \infty$, where the subscript $s \in \{p, g\}$ indicates whether the job in question is in the private or public (government) sector. To give content to our notion of human capital, we assume first-order stochastic dominance, i.e., $y' > y \Rightarrow G_s(x|y') < G_s(x|y)$. The higher is a worker's level of human capital, the more favorable is that worker's distribution of match-specific productivity, and this is the case in both sectors.[2] Finally, we take into account that the rules governing public-sector employment and wage determination are in general not the same as those used in the private sector. We assume that the public sector posts an exogenous measure of vacancies, v_g, and that a worker of type y who meets a public-sector vacancy and draws match-specific productivity x is offered the

[1] This assumption is also used in Albrecht, Navarro and Vroman (2009). That paper focused on the distribution of worker types across formal employment, informal employment and unemployment.

[2] This feature of our model is related to Dolado, Jansen and Jimeno (2007), who assume first-order stochastic dominance in conjunction with a two-point distribuion for y – "low-skill" and "high-skill" workers. Other papers achieve a similar effect by making a specific functional form assumption, typically that productivity is the product of worker type and an independent match-specific component.

job if and only if that productivity is no less than an exogenous threshold that varies with worker type, that is, if and only if $x \geq R_g(y)$. We also assume that a worker's wage in a public-sector job is determined by an exogenous rule, $w_g(x, y)$, and without loss of generality, we set $w_g(x, y) = 0$ for $x < R_g(y)$. Our combination of these three elements – *ex ante* worker heterogeneity, match-specific productivity with a first-order stochastic dominance assumption, and both private- and public-sector employment – is unique in the search and matching literature.

We calibrate our model using Colombian data and then use the calibrated model to simulate the effects of varying (i) the public-sector hiring and wage-setting rules and (ii) the level of public-sector vacancy creation. We also explore the effects of equalizing the level of job security in the two sectors. Colombia is an interesting case study because its public-sector wage premium is very large by international standards. Our baseline calibration indicates that most of this premium is attributable to different distributions of education across the two sectors. While more educated workers are more productive in either sector, we find that more highly educated workers sort into the government sector and this largely accounts for the wage premium. In general, our calibration and our numerical experiments suggest that to understand the differences between public- and private-sector wages and, more generally, to understand how the labor markets in the two sectors interact requires explicitly considering worker heterogeneity.

In terms of related literature, there are surprisingly few other papers that incorporate public-sector employment into an equilibrium search and matching framework. Two papers, namely, Burdett (2012) and Bradley, Postel-Vinay and Turon (2016), incorporate a public sector into the Burdett and Mortensen (1998) model of on-the-job search, while four papers, namely, Quadrini and Trigari (2007), Michaillat (2014), Gomes (2015) and Gomes (2016), use a DMP framework. Among these papers, only Gomes (2016) allows for worker heterogeneity, but his model differs from ours in several ways. Most importantly, he assumes that workers differ along two dimensions but only in a binary fashion - a worker is either of high or low ability and either has or doesn't have a college degree. As a result, his paper can only address the distributional questions that are the core of our paper in a limited way.[3] Our paper is also related to Albrecht, Navarro, and

[3]In addition to these six completed papers, Navarro and Tejada (work in progress) are applying an approach similar to the one developed in this paper to analyze how the minimum wage impacts the interaction between private- and public-sector labor markets in Chile, and Langot and Yassine (work in progress) are applying the approach used in Bradley et al. (2016) to analyze the three-way interaction among the informal sector and

Vroman (2009), which uses a DMP model with workers who are heterogeneous with respect to type y and has two sectors, a formal and an informal market. Our current paper adds match-specific productivity distributions that differ according to sector and while our private sector, like the formal sector in Albrecht et al. (2009), allows for search and matching frictions, the government sector is modeled quite differently and is not analogous to the informal market in Albrecht et al. (2009), which had differentiated productivity neither by match nor by worker type.

The rest of our paper is organized as follows. In the next section, we lay out our model and establish the existence of equilibrium. In Section 3, we discuss our calibration. Section 4 presents the results of our counterfactual experiments, and Section 5 concludes.

2 Model

We consider an equilibrium search model with worker and match-specific heterogeneity. Workers are heterogeneous with respect to human capital, $Y \sim F(y)$, and the productivity, x, of a match between an employer in sector $s \in \{p, g\}$ and a worker of type y is a draw from $G_s(x|y)$ with $G_s(x|y') < G_s(x|y)$ for $y' > y$. Only the unemployed search, and the rate at which they contact potential employers depends on overall labor market tightness, $\theta = (v_p + v_g)/u$, where v_p and v_g are the measures of private- and public-sector vacancies posted at any instant, and u is the fraction of the workforce that is unemployed. Search is random, so conditional on meeting a prospective employer, the probability that the job is in the private sector is $\phi = v_p/(v_p + v_g)$. Specifically, job seekers meet prospective employers at Poisson rate $m(\theta)$, and employers meet job seekers at rate $m(\theta)/\theta$. Not all meetings lead to matches. In the private sector, a match forms if and only if the realized value of match-specific productivity, x, is high enough so that the match is jointly worthwhile for the worker and firm. The threshold value of x depends in general on the worker's type, y. That is, a private-sector match forms if and only if $x \geq R_p(y)$, where $R_p(y)$ is a type-specific reservation productivity. In the public sector, a match forms if and only if $x \geq R_g(y)$. The key equilibrium objects are the reservation productivity schedule, $R_p(y)$, overall labor market tightness, θ, and the fraction, ϕ, of vacancy postings that are accounted for by the private sector. These objects are determined in equilibrium by (i) the condition that private-sector matches form if and only if doing so is in the joint interest of the worker and firm, (ii) a free-entry

two formal sector (private and public) labor markets using Egyptian and Jordanian data.

condition for private-sector vacancies, and (iii) steady-state conditions for worker flows into and out of unemployment, private-sector employment and public-sector employment.

Our assumptions reflect four important modeling choices. First, we are ruling out on-the-job search. Significant wage dispersion across *ex ante* identical workers can be generated in two ways in equilibrium search models, either by allowing for *ex post* match-specific differences in productivity or by assuming on-the-job search. We have opted for the assumption of match-specific heterogeneity since without panel data (only cross-sectional data are available for Colombia) we cannot track job-to-job transitions. We note that direct movements, i.e., movements without an intervening spell of unemployment, between jobs in the two sectors appear to be rare in Colombia. Using the method described in Robayo-Abril (2015), we estimate that on an annual basis, the probability of a direct transition from the public to the private sector is less than 0.04 and that the transition probability in the opposite direction is less than one quarter of one percent. Second, our assumption of random search means that all job seekers are active in both the private and public labor markets. An alternative approach would be to assume sector-specific search. We have opted for the random search specification both because it seems realistic to assume that unemployed workers are open to employment opportunities in both sectors and because in a model like ours with heterogeneous workers, a sector-specific search assumption would give the unrealistic prediction of perfect sorting. That is, all workers above some type y^* would search exclusively in one sector while all worker types below y^* would search exclusively in the other sector, and this, of course, is not what we see in the data. Third, our model restricts attention to formal-sector workers. We have chosen to leave the informal sector out of our model because the data indicate that the informal sector is not an important consideration for workers who have the possibility of public-sector employment. In our calibration, this consideration leads us to restrict our analysis to relatively highly educated males. Finally, we assume that $m(\theta)$ does not depend on y. In effect, we are ascribing differences in transition rates out of unemployment across worker types to differences in the fractions of job possibilities that are acceptable rather than to differences in the rate at which the different worker types hear about job possibilities.

2.1 Value Functions, Wages, Reservation Values

We start with the optimization problem for a worker of type y. Let $U(y)$, $N_p(x, y)$, and $N_g(x, y)$ be the values (expected discounted lifetime incomes) associated with unemployment and employment in, respectively, a private-sector job and a public-sector job with match-specific productivity x. The value of unemployment for a worker of type y is defined by

$$rU(y) = z(y) + \phi m(\theta) E \max[N_p(x,y) - U(y), 0] + (1-\phi) m(\theta) E \max[N_g(x,y) - U(y), 0] \quad (1)$$

This expression reflects the following assumptions. Time is continuous, and the worker lives forever, discounting the future at rate r. The worker of type y receives the type-specific flow value $z(y)$ while unemployed. Private-sector vacancies are met at rate $\phi m(\theta)$, and public-sector vacancies are met at rate $(1-\phi)m(\theta)$. When the worker meets a vacancy, a match-specific productivity is realized, and the worker realizes a capital gain, either $N_p(x,y) - U(y)$ or $N_g(x,y) - U(y)$, if the relevant difference is positive; zero otherwise.

Workers are assumed to be risk neutral and value jobs solely based on the wages paid and the rate at which the jobs break up. Job destruction is assumed to occur at exogenous Poisson rate $\delta_s(y)$, and we allow for the possibility that $\delta_p(y) \neq \delta_g(y)$. That is, we allow for the possibility that government jobs may be more or less secure than jobs in the private sector. There may, of course, be other non-wage benefits associated with employment in one sector versus the other, but we abstract from these. The two employment values for workers of type y with match-specific productivity x are defined by

$$rN_p(x,y) = w_p(x,y) + \delta_p(y)(U(y) - N_p(x,y)) \quad (2)$$

$$rN_g(x,y) = w_g(x,y) + \delta_g(y)(U(y) - N_g(x,y)). \quad (3)$$

The private-sector wage is determined by Nash bargaining with an exogenous worker share parameter, as described below, while the public-sector wage schedule is exogenous.

On the private-sector firm side, let $J(x,y)$ be the value (expected discounted profit) associated with a job filled by a worker of type y whose match-specific productivity is x, and let V be the value associated with posting a private-sector vacancy. These values are defined by

$$rJ(x,y) = x - w_p(x,y) + \delta_p(y)(V - J(x,y)) \quad (4)$$

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(x,y) - V, 0]. \quad (5)$$

The expectation in equation (5) is taken with respect to the joint distribution of (x,y) across the population of unemployed job seekers. A private-sector firm with a vacancy doesn't know what worker type it will meet next nor does it know what match-specific productivity this worker will draw. The firm does know, however, the distribution of worker types among the unemployed and the conditional distribution function $G_p(x|y)$.

We assume that the private-sector wage for a worker of type y with match-specific productivity x is determined via Nash bargaining with exogenous worker share parameter β. Imposing the free-entry condition for private-sector vacancy creation in advance, i.e., $V = 0$, the Nash bargaining solution implies

$$w_p(x, y) = \beta x + (1 - \beta) rU(y); \tag{6}$$

that is, the private-sector wage is a weighted average of the flow productivity of the match, x, and the flow value of the worker's outside option, $rU(y)$.

Substituting equation (6) into equation (2) and assuming, as will be the case in our calibration, that $w_g(x, y)$ is increasing in x for $x \geq R_g(y)$, it is clear that $N_p(x, y)$ and $N_g(x, y)$ are nondecreasing in x for any value of y. Accordingly, reservation productivities can be defined for the type-y worker. The private-sector reservation productivity for a type-y worker, $R_p(y)$, is defined by $N_p(R_p(y), y) = U(y)$. Using equations (2) and (6), $N_p(R_p(y), y) = U(y)$ implies $R_p(y) = rU(y)$. That is, at $x = R_p(y)$ the net surplus associated with the match equals zero. The public-sector reservation productivity for a type-y worker is simply $R_g(y)$. This is equivalent to assuming that, given the public-sector wage schedule, $N_g(R_g(y), y) \geq U(y)$. If $N_g(R_g(y), y) > U(y)$, there is rationing of public-sector jobs for type-y workers. If $N_g(R_g(y), y) = U(y)$, then $R_g(y) = rU(y) = R_p(y)$; that is, the public- and private-sector reservation productivities are equal for the type-y worker. Finally, we could in principle consider the case of $N_g(R_g(y), y) < U(y)$. In this case, however, matches would not form for $x \in [R_g(y), R_p(y))$ because workers would reject them. In this sense, it is without loss of generality to assume $N_g(R_g(y), y) \geq U(y)$.

To further characterize the private-sector reservation productivity, it is useful to rewrite our expression for $rU(y)$. Using equations (2) and (6) together with $rU(y) = R_p(y)$ gives

$$E \max[N_p(x, y) - U(y), 0] = \frac{\beta}{r + \delta_p(y)} \int_{R_p(y)}^{\infty} (x - R_p(y)) dG_p(x|y).$$

Similarly, using equation (3) gives

$$E \max[N_g(x, y) - U(y), 0] = \frac{1}{r + \delta_g(y)} \int_{R_g(y)}^{\infty} (w_g(x, y) - R_p(y)) dG_g(x|y).$$

Substituting into equation (1) then gives

$$R_p(y) = z(y) + \phi m(\theta)\frac{\beta}{r+\delta_p(y)}\int_{R_p(y)}^{\infty}(x - R_p(y))dG_p(x|y)$$
$$+(1-\phi)m(\theta)\frac{1}{r+\delta_g(y)}\int_{R_g(y)}^{\infty}(w_g(x,y) - R_p(y))dG_g(x|y). \quad (7)$$

Given overall labor market conditions, i.e., θ and ϕ, and the government's employment and wage-setting policy, equation (7) gives a unique solution for $R_p(y)$ since the RHS of equation (7) is positive at $R_p(y) = 0$ and the derivative of the RHS with respect to $R_p(y)$ is negative.[4]

2.2 Free-Entry and Steady State Conditions

The next step is to characterize optimal entry by private-sector firms. Imposing $V = 0$ and using equation (4), we have

$$J(x,y) = \frac{x - w_p(x,y)}{r+\delta_p(y)} = (1-\beta)\frac{x - R_p(y)}{r+\delta_p(y)}.$$

Letting $F_u(y)$ denote the distribution function of Y among the unemployed, the free-entry condition, i.e., equation (5) with $V = 0$, can be written as

$$c = \frac{m(\theta)}{\theta}\int_{\underline{y}}^{\overline{y}}(\frac{1-\beta}{r+\delta_p(y)})\int_{R_p(y)}^{\infty}(x - R_p(y))dG_p(x|y)dF_u(y). \quad (8)$$

The only unknown in equation (8) is the distribution function, $F_u(y)$. The distribution $F_u(y)$ is a contaminated in the sense that the distribution of types among the unemployed is affected by the different transition rates to and from unemployment by the various worker types and so differs from $F(y)$. Using Bayes Law, we can write

$$F_u(y) = \frac{u(y)F(y)}{u};$$

that is, the distribution of types among the unemployed, $F_u(y)$, can be written as the type-specific unemployment rate, $u(y)$, times the population

[4] If $z(y)$ is sufficiently negative, the RHS of equation (7) is negative. In this case, $R_p(y)$ is zero; i.e., the worker accepts any positive wage offer.

distribution function, $F(y)$, normalized by the overall unemployment rate,

$$u = \int_{\underline{y}}^{\overline{y}} u(y) dF(y).$$

To derive the type-specific unemployment rates, $u(y)$, let $n_p(y)$ and $n_g(y)$ be the fractions of time that a type-y worker spends in private-sector and public-sector employment, respectively. In steady state, the following two equations must hold:

$$\delta_p(y) n_p(y) = \phi m(\theta)(1 - G_p(R_p(y)|y))u(y) \tag{9}$$

$$\delta_g(y) n_g(y) = (1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))u(y). \tag{10}$$

The first condition equates the flow from private-sector employment to unemployment with the flow in the opposite direction, and the second condition equates the flow from public-sector employment to unemployment with its opposite flow. Using

$$u(y) + n_p(y) + n_g(y) = 1,$$

equations (9) and (10) imply

$$\begin{aligned} u(y) &= \frac{\delta_g(y)\delta_p(y)}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \\ n_p(y) &= \frac{\delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y))}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \\ n_g(y) &= \frac{\delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \end{aligned} \tag{11}$$

Substituting the expression for $u(y)$ into equation (8) completes the characterization of the private-sector free-entry condition.

The final unknown that needs to be characterized is ϕ, the fraction of vacancies that are posted by private-sector firms. To do this, note that since

$$v_p + v_g = \theta u,$$

$$\phi = v_p/(v_p + v_g)$$

implies

$$\phi = \frac{\theta u - v_g}{\theta u}. \tag{12}$$

This closes the model.

2.3 Equilibrium

Definition: A steady-state equilibrium is a function, $R_p(y)$, that satisfies equation (7) for all $y \in [\underline{y}, \overline{y}]$ together with scalars θ and ϕ that satisfy equations (8), (9), (10) and (12).

An equilibrium always exists. First, as noted above, for given values of θ and ϕ, the reservation productivity, $R_p(y)$, is uniquely determined. Second, given any value of ϕ, equation (8) has at least one solution for θ. The argument is standard. The RHS of equation (8) is continuous in θ, it approaches infinity as $\theta \to 0$, and it goes to zero as $\theta \to \infty$. Finally, once $R_p(y)$ and θ are determined as functions of ϕ, equation (12) has at least one solution in ϕ. (The complication, of course, is that u depends on ϕ.) Note that we do not claim uniqueness. In equation (8), $f_u(y)$ need not be monotonically decreasing in θ nor is it obvious that equation (12) has a unique solution. Uniqueness depends on the form of $F(y)$, $G_p(x|y)$, $G_g(x|y)$ and public-sector employment policy and needs to be investigated numerically.[5]

Given a parameter configuration and given an assumed public-sector wage and employment policy, once we know $\{R_p(y), \theta, \phi\}$, the model can be solved for the equilibrium distributions of wages, productivities and human capital across the two sectors. This can be done analytically. The model gives us the distribution of Y across the unemployed, namely, $F_u(y)$, and the conditional distributions, $G_p(x|y)$ and $G_g(x|y)$, are given exogenously. Then, using the reservation productivity rules, $R_p(y)$ and $R_g(y)$, together with the contact rates, $\phi m(\theta)$ and $(1-\phi)m(\theta)$, and the job destruction rates, $\delta_p(y)$ and $\delta_g(y)$, we can derive the joint distributions of (X, Y) across the two sectors. Finally, using the Nash bargaining rule for the private sector and the exogenous wage-setting rule, $w_g(x, y)$, for the public sector, we can derive the distributions of wages across the two sectors. Another approach is to find the equilibrium distributions by simulating the model. That is, we feed the assumed distribution of worker types into the model and use the distributions of wages across the two sectors that is generated by simulation. We solved for the equilibrium in our baseline calibration using both approaches and the results were essentially the same. We use the simulation approach for our calibration and counterfactuals since that method is computationally less demanding.

[5] The possibility of non-uniqueness of equilibrium is a common feature of DMP models with worker heterogeneity. See, e.g., Albrecht, Navarro and Vroman (2009) and Chéron, Hairault and Langot (2011).

3 Calibration

3.1 Data

To calibrate the model, we use data from the Colombian Household Survey (GEIH) from the second quarter of 2013. These surveys are repeated cross sections that are carried out by the Colombian Statistics Department (DANE) and are administered to a sample of employed and unemployed individuals in thirteen metropolitan areas.[6] We restrict our sample to male salaried full-time workers with more than 5 years of education, and we exclude the self-employed, domestic employees and unpaid family workers. The objective of these exclusions is to construct a sample that is primarily comprised of formal-sector workers. Our sample consists of 10,241 individuals, who represent 2.6 million people.

The data we work with are as follows. First, we know the number of years of education completed by each individual in the sample. We group workers into five educational categories indexed by $j = 1, ..., 5$, namely, (i) 6 to 11 years of completed education (completed or incomplete secondary), (ii) 12 to 15 years of education (incomplete tertiary), (iii) 16 years of education (completed tertiary), (iv) 17 years of education (post-graduate work, one-year specialization), and (v) 18 years of education or more (post-graduate, Masters or PhD). Second, we know whether each respondent is unemployed, employed in the private sector, or employed in the public sector; that is, we know the distribution of workers across the three labor market states of the model. Third, we observe wages for private and public employees. More precisely, we observe monthly earnings and weekly hours and use these to construct an hourly wage for each employed worker. Wages include tips and commissions. To reduce measurement error, we trim the top 1% and bottom 2% of wages in each educational class.[7] We convert Colombian pesos to 2011 US dollars using a purchasing power parity exchange rate. 2011 is the latest year the PPP rate is available.

Educational attainment, labor market state and wage all refer to the respondent's situation as of the survey date, so we are reasonably confident

[6] These areas are the following cities and their metropolitan areas: Bogotá, Cali, Medellín, Barranquilla, Bucaramanga, Manizales, Pasto, Pereira, Cucuta, Villavicencio, Ibague, Monteria and Cartagena.

[7] We trim more at the bottom of the observed wage distributions than at the top because we want to minimize the number of observed public-sector wages that fall below the legal minimum ($1.94 per hour in Colombia in 2011 dollars). In principle, no public-sector wages should be less than this value. When we trim the bottom 2% of wages in each educational category, 3.3% of observed public-sector wages fall below the legal minimum.

in these data. In addition, retrospective data are available on each respondent's labor market state in the previous year and on his elapsed duration in his current labor market state. Regarding previous labor market state, the data, especially the unemployment data, suffer from the standard time aggregation problem. A respondent who reports himself as unemployed as of the survey date and also reports that he was unemployed one year prior may have had an employment spell (or spells) in the intervening period. The unemployment duration data are also problematic. In particular, an individual who is currently employed reports how many months elapsed between the end of his previous job and the start of his current job, but we cannot tell whether he was unemployed or out of the labor force (a state not recorded in the data and not included in our model) in the intervening period. Accordingly, we primarily rely on the education, current labor market state and wage data in our calibration. We do, however, use data on average durations in private and public employment in our calibration procedure.

3.2 Stylized Facts

We emphasize the following broad facts about the Colombian labor market. First, the distribution of workers across the three labor market states varies strongly with educational attainment. As can be seen in Table 1, workers with some education beyond the tertiary level ($j = 4$ or 5) are less likely to be unemployed than are their less educated counterpart, and workers with a post-graduate degree are much more likely than are other workers to be employed in the public sector. Second, the level of public sector employment is quite low in Colombia, and the unemployment rate is quite high. As can be seen in Table 2, the public sector accounts for 7% of total employment ($150,411$ out of $2,155,156$),[8] which is quite low by developed and middle-income country standards and is considerably below the level of most Latin American countries.[9] Third, wages in the public sector are considerably higher than in the private sector. As shown in Table 2, the mean hourly wage in the public sector is \$7.84 as compared to \$4.50 in the private sector, a large public-sector premium.[10] Fourth, wages are more dispersed in the public sector than in the private sector – the standard deviation of hourly

[8] 8.7% of the employed workers in our sample have jobs in the public sector. When we use sample weights, this corresponds to 7% of the employed population.

[9] See Table 1 in Mizala et al. (2011). Note that the figures presented there include all urban workers.

[10] See Table 2 in Mizala et al. (2011) for Latin American wage gaps. See Borland and Gregory (1999) for a survey of results on the public-sector premium in developed countries.

wages is \$7.00 in the public versus \$5.50 in the private sector. This is in contrast to the typical developed and middle-income country pattern, which exhibits a tendency towards wage compression in the public sector. Finally, the duration data in Table 2 show that employment tends to last much longer in the public sector (44 quarters on average) than in the private sector (16 quarters on average).

Tables 1 and 2 go here

3.3 Wage Setting and Wage Gap Decomposition

3.3.1 Wage Setting

We begin with some notation and then describe our assumptions about government wage setting. We observe wages for all employed workers and denote them by $w_p^j(x)$ and $w_g^j(x)$. This notation reflects our assumption that wages depend on a match-specific productivity draw, x, in addition to the worker's type, i.e., his education level j, and whether he is employed in the private sector or in the government.

We assume the following wage-setting rule for public-sector employment:

$$w_g^j(x) = \psi^j + \gamma x + (1-\gamma) rU^j, \tag{13}$$

where ψ^j is a "pure public-sector premium" for type j workers, γ is the weight placed on productivity by the public sector, $1-\gamma$ is the weight placed on the worker's outside option (or, equivalently, on his qualifications). Recall that $R_p^j = rU^j$, i.e., the reservation productivity for private-sector employment for a type-j worker equals the flow value of unemployment for the worker. Private-sector wages are set by Nash bargaining with exogenous weight β as given above in equation (6). The private-sector wage for workers of type j is thus

$$w_p^j(x) = \beta x + (1-\beta) R_p^j. \tag{14}$$

Our assumed public-sector wage setting rule thus differs from the one used in the private sector in two ways. First, we allow for $\psi^j \geq 0$, and, second, we allow for the possibility that $\gamma \neq \beta$; that is, the wage-setting rules in the two sectors may differ in the relative weights placed on productivity versus education. In addition, conditional worker type, the distributions of match-specific productivity, x, may differ between the two sectors.

We also need to specify which workers the public sector is willing to hire. We do this by assuming that the public sector hires if and only if $x \geq w_g^j(x)$. That is, when an unemployed worker makes contact with a public-sector vacancy, that contact generates a match if and only if the worker's productivity is at least as great as the wage he would be paid in the match. This is in the spirit of a basic assumption of the DMP model in the private sector, namely, that a match forms if and only if it is in the joint interest of the worker and employer to do so. Let R_g^j be the reservation productivity for public-sector employment for a type-j worker. Setting $R_g^j = w_g^j(R_g^j)$ implies

$$R_g^j = \frac{\psi^j}{1-\gamma} + R_p^j.$$

The term $\dfrac{\psi^j}{1-\gamma}$ is interpreted as a public-sector rationing factor for type-j workers.[11]

Finally, public-sector employment policy is characterized by v_g, the measure of vacancies posted in the public sector. Rather than specifying the level of public-sector vacancy creation exogenously, we estimate v_g as a part of our calibration as described below.

3.3.2 Wage Gap Decomposition

The primary observation that motivates our calibration strategy is the fact that wages in the public sector are much higher than those in the private sector. The following Oaxaca-Blinder decomposition offers a first step towards understanding the gap in mean wages between the two sectors. Let η_s^j be the employment share of education group j in sector $s \in \{p, g\}$; similarly, let $\overline{w}_s^j$ be the average wage earned by worker type j who is employed in sector s. The difference in mean wages between the public and private sectors can be written as

$$\overline{w}_g - \overline{w}_p = \sum_{j=1}^{5} \eta_g^j \left(\overline{w}_g^j - \overline{w}_p^j\right) + \sum_{j=1}^{5} \left(\eta_g^j - \eta_p^j\right) \overline{w}_p^j. \tag{15}$$

The first term in this decomposition represents the part of the public-private wage gap accounted for by the difference between returns to public versus private employment within each educational class; the second term represents the part of the gap accounted for by the difference in the educational

[11] This discussion implicitly assumes that $\psi^j > 0$, which is what our calibration indicates.

composition of the workforce between the two sectors. As can be calculated from Table 3, differences in returns account for 15% of the wage gap while differences due to skill composition effects for 85%. This large composition effect is, of course, consistent with the pattern seen in Table 1.

Table 3 goes here

Our calibration allows us to go beyond this simple decomposition in two ways. First, given that skill composition accounts for much of the public-private wage gap, it is natural to ask what leads to the difference in worker educational attainment between the two sectors. Are the well educated more likely to flow into public-sector employment than are their less educated counterparts or are the composition effects driven by differences across education levels in the flows out of employment between the two sectors? Second, we are able to explore some of the factors underlying the different returns to public versus private employment. Specifically, the difference in returns can be due to several different factors. First, part of the difference may be due to a pure public-sector premium; that is, the public-sector wage may simply add a bonus to what an equally qualified worker employed in an equally productive job would earn in the private sector. Second, the two sectors may place different weights on productivity versus qualifications in their wage determination rules, i.e., γ and β may differ. Third, the distributions of match-specific productivity conditional on qualifications may not be the same in the two sectors. However, as is standard in models of this type, we face a fundamental identification problem, namely, that it is not possible to distinguish between the second and third explanation for different returns in the two sectors. If the returns to employment are higher in the public sector than in the private sector, it could be because productivity is higher in the public than in the private sector, e.g., because $G_g^j(x)$ first-order stochastically dominates $G_p^j(x)$, or it could be that the same level of productivity is more highly rewarded in the public sector, i.e., because $\gamma > \beta$. In our baseline calibration, we set $\beta = \gamma = 0.5$. In one of our counterfactual experiments, we explore the implications of choosing a different value for γ.

3.4 Calibration Strategy

Our calibration strategy consists of the following four steps:

Step 1: We begin by estimating reservation productivities for each worker type for private and public employment. A private-sector match with a worker of type j forms if and only if $x \geq R_p^j$, and a worker of this type with match-specific productivity R_p^j receives a wage of $w_p^j(R_p^j) = R_p^j$. Accordingly, we use the minimum observed private-sector wage (after trimming) among workers with j years of education to estimate R_p^j. Similarly, we use the minimum observed public-sector wage (after trimming) among workers with j years of education to estimate R_g^j. This procedure gives us estimates $\widehat{R}_p^j$ and $\widehat{R}_g^j$ for $j = 1, ...5$. Then, given γ, we back out estimates for the $\{\psi^j\}_{j=1}^5$.

Step 2: Once we have an estimate for R_p^j, we use the observed distribution of private-sector wages across workers with j years of education to estimate $G_p^j(x)$, that is, the distribution of private-sector productivity for workers with j years of education. Similarly, we use the observed distribution of public-sector wages for workers of type j, together with our estimates of R_g^j to estimate $G_g^j(x)$. To do this, we assume that $G_p^j(x)$ is a log-normal distribution function with parameters μ_p^j and σ_p^j; that is, we assume that the log of productivity in potential private-sector jobs across workers of type j is normally distributed with mean μ_p^j and standard deviation σ_p^j. Using equation (14) gives

$$\ln x = \ln \left(\frac{w_p^j - (1-\beta) R_p^j}{\beta} \right).$$

Given our assumed value for β, our estimate for R_p^j, and observed wages, we have a set of estimated values for the log productivities of workers of type j who are employed in private-sector jobs. We then use expressions for the mean and variance of a truncated ($\ln x \geq \ln R_p^j$) log-normal distribution to back out estimates of μ_p^j and σ_p^j. Given our assumed value for γ, we use an analogous procedure to estimate μ_g^j and σ_g^j, the parameters that characterize the log normal distribution of match-specific productivity across workers of type j in prospective public-sector jobs.

Step 3: Next, we estimate the parameters governing transitions from unemployment to private and public employment and vice versa. As we mentioned above, the duration information in our dataset is retrospective and subject to potential biases, e.g., time aggregation bias. Thus, we want to minimize the extent to which we use these data in our estimation procedure. Our assumption that workers contact private-sector vacancies at the same rate independent of type, that is, the assumption that $m(\theta)\phi$ does not vary with j and our similar assumption about the rate at which workers contact public-sector vacancies, helps us achieve this objective.

We proceed as follows. Workers of type j move from unemployment to private-sector employment at rate $m(\theta)\phi(1-G_p^j(R_p^j))$, and they flow in the opposite direction at rate δ_p^j; thus, in steady-state,

$$m(\theta)\phi(1-G_p^j(R_p^j))u^j = \delta_p^j n_p^j. \tag{16}$$

Similarly, the flow of type-j workers from unemployment to public-sector employment and vice versa satisfies

$$m(\theta)(1-\phi)(1-G_g^j(R_g^j))u^j = \delta_g^j n_g^j. \tag{17}$$

These steady-state equations hold for each worker type. Once we estimate $m(\theta)$ and ϕ, these equations give us estimates of the job destruction rates, $\{\delta_p^j\}_{j=1}^5$ and $\{\delta_g^j\}_{j=1}^5$.

To estimate $m(\theta)$ and ϕ, we use expressions for the average durations of private and public employment. The model assumes exponential durations; thus, for example, the expected duration of private-sector employment for a worker with j years of education is $1/\delta_p^j$. The expected duration of private-sector employment averaged across all worker types can therefore be written as

$$E[T_p] = \sum_{j=1}^{5} \eta_p^j \left(\frac{1}{\delta_p^j}\right),$$

Using equation (16),

$$E[T_p] = \sum_{j=1}^{5} \eta_p^j \left(\frac{n_p^j}{m(\theta)\phi(1-G_p^j(R_p^j))u^j}\right). \tag{18}$$

Similarly, the expected duration of public-sector employment across all worker types is

$$E[T_g] = \sum_{j=1}^{5} \eta_g^j \left(\frac{n_g^j}{m(\theta)(1-\phi)(1-G_g^j(R_g^j))u^j}\right). \tag{19}$$

The only "unknowns" on the right-hand sides of equations (18) and (19) are $m(\theta)$ and ϕ. Plugging in the sample counterparts for $E[T_p]$ and $E[T_g]$ together with our already-computed estimates of the various objects on the right-hand sides of equations (18) and (19) gives us estimates of $m(\theta)$ and ϕ.

Step 4: The fourth step in our calibration procedure ties up a number of loose ends. First, we back out an estimate for θ. To do this, we assume

Cobb-Douglas matching, namely,

$$m(\theta) = A\theta^{\alpha}.$$

Since reliable vacancy data are not available in Colombia, we set values for A and α. Specifically, we choose $\alpha = 0.5$, so the Hosios condition is satisfied, and then set $A = 0.25$. The latter choice is made to be consistent with the literature (e.g., Petrongolo and Pissarides 2001) and to produce a reasonable value of θ in the calibration. Given an estimate of $m(\theta)$ from the previous step, we then have an estimate for θ.

Next, we use our estimates of θ and ϕ together with equation (12) to set a value for v_g. We also use our estimates of θ and ϕ to back out an estimate of c, using the free-entry condition for private-sector vacancy creation. To do this, we need to fix a value for the discount rate, and we set $r = 0.0217$.[12] This is the final value that we set outside the model. Table 4 lists these values. Finally, the last parameters that we estimate are the type-specific flow values of unemployment, that is, z^j for $j = 1, ..., 5$. We do this using a discretized version of equation (1).

Table 4 goes here

4 Calibration Results

The results of our calibration are shown in Tables 5 and 6. Table 5 presents estimates of the parameters that are assumed to be the same for all worker types while Table 6 presents estimates of the parameters that we allow to vary with education level.

Tables 5 and 6 go here

We begin with the parameters that describe the public sector's employment and wage-setting rules. First, Table 5 indicates a steady-state level of public-sector vacancies of $\widehat{v}_g = 0.018$; that is, in steady state a bit less than two public-sector vacancies are posted per 100 workers in the labor force. Given the estimated value $\widehat{\phi}$ of 0.933, slightly less than 7% of posted vacancies are in the public sector. Second, Table 6 indicates a pure public-sector premium ($\widehat{\psi}^j > 0$) for workers at all levels of education, rising from a

[12] This is consistent with an annual real interest rate of 8.96%.

premium of a bit more than 5 cents per hour for the least educated workers to a bit less than 50 cents per hour for the most highly educated workers.

The wages that workers are paid in the private and public sectors depend not only on the wage-setting rules but also on how productive workers are in the two sectors. Table 6 presents the parameter estimates $\widehat{\mu}_s^j$ and $\widehat{\sigma}_s^j$ ($s = p, g$ and $j = 1, ..., 5$) that characterize the education-specific log normal distributions of match-specific productivity in the two sectors. These estimates are, of course, conditional on the assumption that $\beta = \gamma = 0.5$. Among workers with a college degree or less ($j = 1, 2$ and 3), we have $\widehat{\mu}_g^j > \widehat{\mu}_p^j$, while $\widehat{\mu}_g^j \approx \widehat{\mu}_p^j$ for $j = 4$ or 5, and, except among the least educated workers, $\widehat{\sigma}_p^j > \widehat{\sigma}_g^j$. Conditional on worker type j, the expected value of a match-specific productivity draw in sector s is $\exp\{\mu_s^j + \frac{(\sigma_s^j)^2}{2}\}$ and the corresponding variance is $\exp\{2\mu_s^j + (\sigma_s^j)^2\}\left(\exp\{(\sigma_s^j)^2\} - 1\right)$. Using our estimated log-normal parameters, for each worker type j, expected match-specific productivity is higher in the private sector than in the public sector, and, except among the less educated ($j = 1$ and 2) the variance of match-specific productivity is similarly higher for private-sector jobs. This reinforces the results of our Oaxaca-Blinder decomposition. Wages are higher on average in the public sector than in the private sector not because public-sector workers are more productive conditional on education but rather because more highly educated (and therefore more productive) workers are relatively more likely to be employed in the public sector. A similar conclusion holds for the dispersion in wages in the two sectors. The greater dispersion of productivity (and therefore wages) in the public sector is driven by composition effects. First, worker education is more dispersed in the public sector, and second, there are relatively many highly educated workers in the public sector. This second effect increases wage dispersion in the public sector because the variance of match-specific productivity is increasing in education irrespective of sector of employment.

Next we turn to the parameters that describe the frictions in the labor market. From Table 5, we have $\widehat{m(\theta)} = 0.314$. Since the unit of time is a quarter, this implies that, on average, it takes a bit less than a year for a worker to make a contact that can potentially lead to a job. Given our assumed values for A and α, that is, the parameters of the matching function, our implied estimate for labor market tightness is $\widehat{\theta} = 1.58$. That is, even though we estimate that this labor market is "tight" ($\widehat{\theta} > 1$), we find that it functions quite poorly, so it takes a long time on average for workers to find job possibilities. Not surprisingly, workers accept almost all job opportunities that arrive. The highest rejection rate of private-sector offers is among workers with a completed tertiary level of education ($j = 3$), and even these workers (jointly with their prospective employers) reject less than 2% of their prospective matches. Similarly, we estimate that employers take a long time to fill their open positions, a bit more than 5 quarters on average. As a result, there is no significant rationing of public-sector employment.

Estimates of the job destruction rates, i.e., $\widehat{\delta}_s^j$ for $s = p, g$ and $j = 1, ..., 5$ are shown in Table 6. Given the assumption that $m(\theta)$ and ϕ are the same across education levels, the fact that almost all contacts with an employer lead to a job means that the strong pattern of sorting into the two sectors by education level (as shown Table 3) is almost solely a matter of different job destruction rates by education level and sector. In the private sector, among workers with a tertiary education or less, we estimate that jobs last about 15 quarters on average while the expected duration of private-sector employment is considerably longer for more highly educated workers. In the public sector, we estimate that jobs for the least educated workers break up relatively quickly (twice as fast as the corresponding private-sector jobs) but for all other workers, jobs in the public sector last considerably longer than jobs in the private sector do. We also observe that estimated job destruction rates in the public sector fall very sharply with education. The bottom line is that public-sector jobs, except for those held by the least educated workers, last much longer than do the corresponding jobs in the private sector, and this difference in job stability between the two sectors is much greater for workers with higher levels of education. The ratios of the rates at which workers take jobs in the two sectors are essentially the same across the five levels of education, so the sorting of the more highly educated workers into the public sector, which in turn explains most of mean wage gap between the two sectors, is thus driven by the fact that jobs for the more highly educated are much more stable in the public than in the private sector. Explaining *why* the public-sector jobs held by highly educated workers are more secure than the corresponding private-sector jobs is beyond the scope of this paper.

Given our estimates of job accession and destruction rates by education level for the two sectors and of the sector-specific distributions of match-specific productivity for each worker type, we back out an estimate $\widehat{c} =$ \$2.91 of the vacancy-posting cost from the free-entry condition (equation 8). Relative to our estimates of mean productivities in the two sectors, this is not a particularly high figure; that is, private-sector employers are deterred from posting vacancies not primarily by a high per-period cost associated with that posting but rather by the fact that it takes more than five quarters on average to fill a vacancy. Finally, Table 6 shows our estimates of the type-specific flow values associated with unemployment, namely, the z^j's. As discussed in Hornstein, Krusell and Violante (2011), negative flow values are typically required to fit models of random search, at least those in which on-the-job search does not play a significant role. Examples can be seen in Table 7 of Eckstein and Wolpin (1995) and in Table II the survey paper by Bunzel et al. (2001).

As can be seen in Tables 7 and 8, the calibrated model does a good job of matching the data. The standard deviation of wages in the private sector as predicted by the model is a few cents below the standard deviation observed in the data and vice versa in the public sector, but otherwise the calibrated model fits the data at the aggregate level (Table 7) perfectly. Similarly, in Table 8, which shows employment shares and mean wages by worker type in the two sectors as predicted by the model and as observed in the data, the model predictions fit the data almost perfectly. Finally, Figure 1 compares the kernel densities of wages in the two sectors as predicted by the model to the corresponding kernels estimated from the data. Although the fit is not perfect, the wage distributions generated by the model do a reasonable job of matching what we see in the data.

Tables 7 and 8 plus Figure 1 go here

5 Counterfactual Experiments

We now turn to our counterfactual experiments. We explore four counterfactuals. In the first three, we change the parameters that characterize the public-sector wage and employment rules. Specifically, in succession we (i) eliminate the pure public-sector premium (we set $\psi^j = 0$ for all j), (ii) decrease the weight on match-specific productivity in the public-sector wage setting rule (we set $\gamma = 0.4$) and (iii) increase the measure of public-sector vacancies (we set $v_g = 0.02$) Then, since our baseline results are driven to a considerable extent by the difference in separation rates between the two sectors, we carry out a fourth counterfactual experiment in which we set the public-sector separation rates equal to the corresponding private-sector baseline values (we set δ_g^j equal to the baseline values of δ_p^j for all j). As expected, this last counterfactual shows the greatest effect.

In our baseline calibration, we take estimates of the reservation productivities directly from the wage data, and we infer the contact rate, the fraction of vacancies posted in the private sector, and the separation rates from the transition data. We then use a discretized version of the free-entry condition (equation 8) together with discretized versions of the recursions defining the reservation productivities (equation 7) to estimate the remaining parameters of the model. To carry out our counterfactual experiments, we essentially reverse this process. That is, after making the parameter changes indicated by the counterfactual (e.g., setting all of the $\psi^j = 0$) while holding all other parameters constant at the levels given in the baseline calibration, we use the equations of the model to solve for the implied values of the endogenous variables.

In our first counterfactual, we eliminate the pure public-sector premium. The results of this experiment are shown in Tables 9 and 10. The direct effect is to reduce the average public-sector wage paid to type-j workers by ψ^j; for example, before accounting for any spillover effects, we expect the average public-sector wage for the most highly educated workers to fall by 48 cents per hour. Taking a weighted average across the five worker types, i.e., computing $\sum_{j=1}^{5} \eta_g^j \psi^j$, gives an expected direct effect on the average public-sector wage of 22 cents per hour, a decrease that is small relative to the difference in mean wages between the two sectors ($7.84 − $4.50 = $3.34) that we observe in the data. The indirect, or equilibrium, effects on public-sector wages are also small. Eliminating the pure public-sector premium lowers the value of unemployment (equivalently, the private-sector reservation productivity) for all worker types, i.e., R_p^j falls for all j, and since $R_g^j = R_p^j + \dfrac{\psi^j}{1-\gamma}$, there is also an equilibrium effect on public-sector reservation productivities. This decrease in reservation productivities has two effects. First, wages fall in both sectors since wages are increasing in the value of worker outside options. Second, the fall in reservation productivities implies that some low-productivity matches form that would not otherwise have done so. The combined effect of the decrease in the R_p^j is a 1 cent decrease in the average private-sector wage and an additional 5 cent decrease in the average wage in the public sector.

Tables 9 and 10 go here

In our second counterfactual, we explore the implications of a second change to the public-sector wage-setting rule, namely, we reduce the weight on match-specific productivity in the public sector to $\gamma = 0.4$. That is, our counterfactual assumes that the public sector puts less weight on "performance" and more weight on "qualifications" than is the case in the private sector. As can be seen in Tables 11 and 12, even though this is not a particularly large decrease in γ, the effects on public-sector wages are substantial. The mean public-sector wage falls by \$1.33 (= \$7.84 − \$6.41) and the standard deviation of wages in the public sector falls by \$1.37 per hour. The latter effect occurs because the standard deviation of public-sector wages for workers of type j is simply γ times the standard deviation of match-specific productivity across workers of that type who are employed in public-sector jobs There are also equilibrium effects. The reduction in γ causes private-sector reservation productivities to fall, similar to the effect observed in our first counterfactual. The size of the fall in R_p^j depends on worker type. The private-sector reservation productivity for the least educated workers ($j = 1$) falls by 9 cents per hour while the fall is 69 cents for worker type $j = 5$. The overall effect on private-sector wages is still relatively small – on average, these wages fall by 9 cents per hour.

Tables 11 and 12 go here

The bottom line from our first two counterfactual experiments is that while changes in the public-sector wage-setting rule have obvious direct effects on public-sector wages, the associated spillover effects are relatively small. To get interesting equilibrium effects, we need to vary the parameters that determine the transitions that workers make across the three labor market states. We do this in our next two counterfactuals.

In our third counterfactual, we consider the effect of a small increase in public-sector vacancy postings, namely, we increase v_g to 0.02. The results of this counterfactual are shown in Tables 13 and 14. The small increase in v_g has a substantial direct effect: the fraction of workers employed in the public sector increases from 5.8% to 6.4%. Interestingly, however, this increase in public-sector vacancy postings leads to a slight *increase* in the unemployment rate, from 17.2% to 17.3%. The fall in private-sector vacancies more than offsets the increase in public-sector postings. In addition, the increase in v_g results in a relatively strong decrease (21 cents per hour) in the average wage in the public sector. The main driver of these initially somewhat counterintuitive results is the sorting of workers between the two sectors. Less educated workers get higher wages in the public sector; more highly educated workers prefer private-sector jobs. An increase in v_g helps the less educated, so their reservation productivities rise, and the wages they are paid, both in the private and in the public sector, increase. The opposite holds for the more highly educated. Since the less educated constitute the bulk of the work force (94% of the workers have a completed tertiary education or less) and since these workers are disproportionately employed in the private sector, the incentive to post private-sector vacancies takes a strong hit.

Tables 13 and 14 go here

Finally, in our fourth counterfactual, we consider the implication of a substantial parameter change. The large gap in mean wages that we observe between the public and private sectors is accounted for to a substantial extent by the fact that more educated workers are relatively more likely to work in the public sector, and this composition effect is in turn primarily due to differences in separation rates between the two sectors by worker type. Specifically, among the least educated workers ($j = 1$), private-sector jobs tend to last almost twice as long as public-sector jobs do, while for all other worker types, public-sector jobs tend to be more stable, and increasingly so as we move up the education distribution. What would happen if separation rates were the same in the two sectors? We address this question in our final counterfactual by setting the public-sector separation rates (the δ_g^j's) equal to the private-sector separation rates that we found in our baseline calibration. The results are shown in Tables 15 and 16. First, we consider the effects on worker sorting across labor market states. Changing the separation rates reduces average duration in public-sector jobs from 44 quarters to 16 quarters. Since v_g is fixed at its baseline value of 0.018, this mechanically decreases the measure of workers employed in the public sector from 5.8% to 5.3%. This decrease in public-sector employment is not, however, reflected in increased unemployment. Instead, private-sector employment increases from 77.0% to 78.3% of the workforce, and the unemployment rate falls from 17.2% to 16.5%. The strong increase in public-sector separation rates among the more highly educated workers means that more of these workers are available for private-sector employers to hire, and these employers respond by increasing their vacancy postings. Second, the change in separation rates has a strong effect on wages, especially in the public sector. The mean wage in the private sector increases by 31 cents (from \$4.49 to \$4.80) while the mean public-sector wage falls sharply by \$2.01 (from \$7.84 to \$5.83). That is, the change in separation rates eliminates almost two thirds of the public-private gap in mean wages. This strong result is driven by composition effects. In our baseline calibration, 30.9% of public-sector employment was accounted for by the least educated workers. After the parameter change, this share increases to 58%. Similarly, the most highly educated accounted for 19.1% of public-sector employment in our baseline calibration, but they account for only 5% in our counterfactual. These composition effects are sufficiently strong that the mean public-sector wage falls sharply even though the mean public-sector wage for each worker type increases. This last effect comes from the increase in reservation productivities across all worker types.

Tables 15 and 16 go here

The results of our counterfactual experiments are consistent with an approach that focuses on worker heterogeneity as a key to understanding the interaction between private- and public-sector labor markets. Wage differences between the public and private sector in Colombia appear to be driven primarily by productivity differences between the two sectors, and these productivity differences are in turn primarily driven by the different distributions of educational attainment across the workers in the two sectors. Although there is a (relatively small) pure public-sector wage premium in Colombia and while such pure premia may well exist in other countries, our approach suggests that it is of first-order importance to understand what lies behind the sorting of different worker types into the public versus the private sector. We focused on two potential explanations. First, more highly educated workers may reject private-sector jobs to wait for more attractive public-sector positions. This is more likely to happen when there is a pure public-sector premium that is increasing with worker qualifications. That is, in addition to the direct effect of adding a top-up to public-sector wages, a pure public-sector premium may attract more qualified workers to public-sector employment. This indirect effect is mostly absent in Colombia because job opportunities arrive too infrequently to allow workers to reject many private-sector jobs, but it is potentially important in other countries. Second, there may be differences in retention patterns for different worker types between the private and public sector. Jobs in the public sector are often viewed as "more secure" than private-sector jobs. In Colombia, this holds for jobs held by the more highly educated, but for the less educated, the opposite is the case. This is the primary reason that more highly educated workers are relatively more likely to be found in the public sector. To the extent that the underlying patterns that we observe in the Colombian data generalize to other countries, a better understanding of public-private wage differences and, more generally, how the public-sector and private-sector labor markets interact requires explicitly taking worker heterogeneity into account.

6 Conclusion

In this paper, we have developed a search-and-matching model to analyze the interaction between labor markets in the private and public sectors. The focus of our model is on distributional questions. What types of workers sort into the two sectors? How do the size of the public sector and the public sector's wage and employment policies affect the distribution of wages in the private sector and in the public sector? Given this focus, worker heterogeneity is a key element of our model. We calibrate our model using Colombian data. Colombia is an interesting case study because the wage differential between the public and private sectors there is very large. Our calibration and counterfactual experiments are motivated by a desire to differentiate among various potential explanations of this wage gap. Although there is a pure public-sector premium in Colombia, it is small relative to the differential that needs to be explained. Instead, the primary cause of the public-private wage differential in Colombia is that more highly educated workers, who tend to be more productive regardless of whether they are employed in the private or public sector, get differentially sorted into public-sector employment. A relatively minor aspect of this sorting is that there is rationing of public-sector jobs. More importantly, public-sector employment is unstable for the least educated workers but extremely stable for highly educated workers. Much more so than in the private sector, when a highly educated worker gets a public-sector job, he tends to keep that job for a very long time.

Public sector employment accounts for a significant fraction of employment in most economies, and the effect of public-sector labor market policy on overall labor market performance deserves more attention. The model and the calibration strategy developed in our paper can be applied more generally, and our focus on worker heterogeneity and the sorting of different worker types into the two sectors offers a useful complement to the existing literature.

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Table 1: Sample Composition				
Education			Employed	
	Total	Unemployed	Private	Public
j = 1	0.59	0.18	0.79	0.03
j = 2	0.25	0.17	0.76	0.07
j = 3	0.08	0.18	0.73	0.10
j = 4	0.03	0.12	0.76	0.12
j = 5	0.05	0.07	0.69	0.24
Total	1	0.17	0.77	0.06

Authors' calculations based on GEIH, 2nd quarter 2013, 13 Metropolitan Areas
Adjusted using sampling weights

Table 2: Descriptive Statistics for Employed Population			
	Employed	Private	Public
Mean Wage	4.74	4.50	7.84
SD Wage	5.68	5.50	7.00
Mean Duration	17.7	15.7	44.4

Sample Size	8276	7559	717
Population	2155156	2004745	150411

Authors' calculations based on GEIH, 2nd quarter 2013, 13 Metropolitan Areas
Adjusted using sampling weights
Wages in 2011 US$ per hour; duration in quarters

Table 3: Employment Shares and Mean Wages				
	Private		Public	
Education	η_p^j	$\bar{w}_p^j$	η_g^j	$\bar{w}_g^j$
j = 1	0.60	2.87	0.31	4.19
j = 2	0.25	3.82	0.30	5.17
j = 3	0.08	9.89	0.14	10.17
j = 4	0.03	11.84	0.06	10.26
j = 5	0.04	16.78	0.19	15.39
Total	1.00	4.50	1.00	7.84

Mean wages in 2011 US$ per hour. Adjusted using sampling weights

Table 4: Fixed Parameters – Based on Data or Previous Studies		
	Description	Value
Parameters		
r	Discount rate	0.022
β	Nash bargaining weight – private sector	0.5
γ	Productivity weight – public sector	0.5
A	Scale factor – contact function	0.25
α	Elasticity – contact function	0.5

Table 5: Estimated Parameters - Baseline		
Description		Value
m(Θ)	contact rate	0.314
ϕ	fraction private-sector vacancies	0.933
v_g	vacancies public sector	0.018
c	vacancy posting cost	2.914

Table 6: Estimated Parameters - Baseline										
Education	R_p^j	R_g^j	ψ^j	μ_p^j	μ_g^j	σ_p^j	σ_g^j	δ_p^j	δ_g^j	z^j
j = 1	0.57	0.70	0.06	1.56	1.94	0.41	0.44	0.067	0.125	-1.26
j = 2	1.03	1.24	0.10	1.73	2.10	0.56	0.46	0.068	0.054	-3.51
j = 3	2.16	2.93	0.38	2.51	2.71	0.83	0.53	0.070	0.039	-15.64
j = 4	2.31	3.31	0.50	2.87	2.77	0.62	0.39	0.045	0.021	-31.27
j = 5	2.48	3.44	0.48	3.10	3.04	0.82	0.72	0.031	0.006	-45.91

Table 7 – Calibration: Model vs Data		
	Model	Data
Unemployment rate (u)	0.172	0.172
Private-sector employment rate (n_p)	0.770	0.770
Public-sector employment rate (n_g)	0.058	0.058
Mean wage, private sector	4.49	4.50
Mean wage public sector	7.84	7.84
SD wage private sector	5.48	5.51
SD wage public sector	7.03	7.00
Mean duration private sector	15.72	15.72
Mean duration public sector	44.52	44.52
Wages in 2011 US$ per hour; durations in quarters		

Table 8 – Calibration: Model vs Data				
Private Sector				
	Employment Shares		Mean Wages	
	Model	Data	Model	Data
j = 1	0.60	0.60	2.87	2.87
j = 2	0.25	0.25	3.83	3.82
j = 3	0.08	0.08	9.87	9.89
j = 4	0.03	0.03	11.82	11.84
j = 5	0.04	0.04	16.80	16.78
Public Sector				
	Employment Shares		Mean Wages	
	Model	Data	Model	Data
j = 1	0.31	0.31	4.19	4.19
j = 2	0.30	0.30	5.17	5.17
j = 3	0.14	0.14	10.18	10.17
j = 4	0.06	0.06	10.27	10.26
j = 5	0.19	0.19	15.38	15.39
Wages in 2011 US$ per hour				

Table 9 – Counterfactual Experiment – $\psi = 0$		
	Baseline	Experiment
Unemployment rate (u)	0.172	0.172
Private-sector employment rate (n_p)	0.770	0.770
Public-sector employment rate (n_g)	0.058	0.058
Mean wage, private sector	4.49	4.48
Mean wage public sector	7.84	7.57
SD wage private sector	5.48	5.46
SD wage public sector	7.03	6.91
Mean duration private sector	15.7	15.7
Mean duration public sector	44.5	44.6

Table 10 - Counterfactual Experiment – $\psi = 0$						
Private Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.60	0.60	0.57	0.57	2.87	2.86
j = 2	0.25	0.25	1.03	1.01	3.83	3.82
j = 3	0.08	0.08	2.16	2.05	9.93	9.82
j = 4	0.03	0.03	2.31	2.11	11.85	11.72
j = 5	0.04	0.04	2.48	2.26	16.68	16.69
Public Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.31	0.31	0.70	0.57	4.19	4.12
j = 2	0.30	0.30	1.24	1.01	5.18	5.05
j = 3	0.14	0.14	2.93	2.05	10.19	9.74
j = 4	0.06	0.06	3.31	2.11	10.25	9.66
j = 5	0.19	0.19	3.44	2.26	15.41	14.80
Wages in 2011 US$ per hour						

Table 11 – Counterfactual Experiment – $\gamma = 0.4$		
	Baseline	Experiment
Unemployment rate (u)	0.172	0.172
Private-sector employment rate (n_p)	0.770	0.770
Public-sector employment rate (n_g)	0.058	0.058
Mean wage, private sector	4.49	4.40
Mean wage public sector	7.84	6.41
SD wage private sector	5.48	5.43
SD wage public sector	7.03	5.66
Mean duration private sector	15.7	15.7
Mean duration public sector	44.5	44.6
m(θ)	0.314	0.313
θ	1.578	1.572
ϕ	0.933	0.933
Wages in 2011 US$ per hour; durations in quarters		

Table 12 - Counterfactual Experiment – $\gamma = 0.4$						
Private Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.600	0.599	0.57	0.49	2.87	2.82
j = 2	0.250	0.250	1.03	0.84	3.83	3.73
j = 3	0.080	0.081	2.16	1.77	9.93	9.68
j = 4	0.030	0.030	2.31	1.71	11.85	11.52
j = 5	0.041	0.041	2.48	1.79	16.68	16.46
Public Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.309	0.310	0.70	0.61	4.19	3.43
j = 2	0.296	0.297	1.24	1.04	5.18	4.24
j = 3	0.142	0.140	2.93	2.53	10.19	8.42
j = 4	0.062	0.062	3.31	2.70	10.25	8.41
j = 5	0.191	0.192	3.44	2.75	15.41	12.48
Wages in 2011 US$ per hour						

Table 13 – Counterfactual Experiment – $v_g = 0.02$		
	Baseline	Experiment
Unemployment rate (u)	0.172	0.173
Private-sector employment rate (n_p)	0.770	0.763
Public-sector employment rate (n_g)	0.058	0.064
Mean wage, private sector	4.49	4.49
Mean wage public sector	7.84	7.81
SD wage private sector	5.48	5.43
SD wage public sector	7.03	6.98
Mean duration private sector	15.7	15.7
Mean duration public sector	44.5	44.5
m(θ)	0.314	0.313
θ	1.578	1.563
ϕ	0.933	0.926
Wages in 2011 US$ per hour; durations in quarters		

Table 14 - Counterfactual Experiment – $v_g = 0.02$						
Private Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.600	0.617	0.57	0.60	2.87	2.88
j = 2	0.250	0.245	1.03	1.09	3.83	3.86
j = 3	0.080	0.077	2.16	2.19	9.93	9.89
j = 4	0.030	0.028	2.31	2.30	11.85	11.79
j = 5	0.041	0.033	2.48	2.36	16.68	16.76
Public Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.309	0.311	0.698	0.726	4.191	4.206
j = 2	0.296	0.297	1.240	1.298	5.175	5.196
j = 3	0.142	0.141	2.926	2.948	10.186	10.195
j = 4	0.062	0.062	3.306	3.294	10.252	10.259
j = 5	0.191	0.188	3.441	3.318	15.410	15.313
Wages in 2011 US$ per hour						

Table 15 – Counterfactual Experiment – $\delta_g = \delta_p$		
	Baseline	Experiment
Unemployment rate (u)	0.172	0.165
Private-sector employment rate (n_p)	0.770	0.783
Public-sector employment rate (n_g)	0.058	0.053
Mean wage, private sector	4.49	4.80
Mean wage public sector	7.84	5.83
SD wage private sector	5.48	5.85
SD wage public sector	7.03	4.55
Mean duration private sector	15.7	15.9
Mean duration public sector	44.5	15.9
m(θ)	0.314	0.331
θ	1.578	1.751
ϕ	0.933	0.937
Wages in 2011 US$ per hour; durations in quarters		

Table 16 - Counterfactual Experiment – $\delta_g = \delta_p$						
Private Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.600	0.580	0.57	0.91	2.87	3.04
j = 2	0.250	0.250	1.03	1.07	3.83	3.84
j = 3	0.080	0.080	2.16	2.29	9.93	9.96
j = 4	0.030	0.030	2.31	2.69	11.85	12.02
j = 5	0.041	0.050	2.48	3.09	16.68	17.06
Public Sector						
	Employment Shares		Reservation Productivities		Mean Wages	
	Baseline	Experiment	Baseline	Experiment	Baseline	Experiment
j = 1	0.309	0.580	0.70	1.04	4.19	4.36
j = 2	0.296	0.250	1.24	1.27	5.18	5.19
j = 3	0.142	0.090	2.93	3.05	10.19	10.22
j = 4	0.062	0.030	3.31	3.62	10.25	10.47
j = 5	0.191	0.050	3.44	4.06	15.41	15.66
Wages in 2011 US$ per hour						

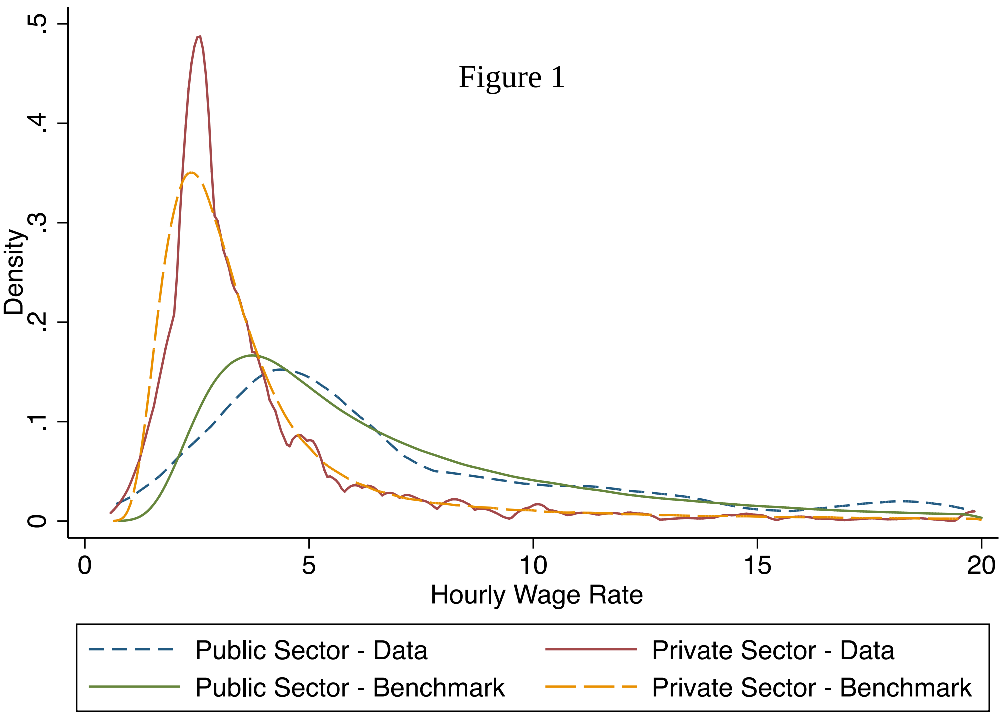
Figure 1
Density
0
.1
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0
5
10
15
20
Hourly Wage Rate
Public Sector - Data
Private Sector - Data
Public Sector - Benchmark
Private Sector - Benchmark