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ABSTRACT

Tournaments: There Is More Than Meets the Eye

By the well established tournament literature, incomplete information regarding the employees' productivity is essential for the rationalization of (efficiency-enhancing) tournaments. In this paper we propose an alternative rationalization of tournaments focusing on a fully informed principal whose objective is to maximize a weighted average of the profitability (productivity) of his team and of the promotion-seeking efforts of his employees. Our first main result clarifies the conditions under which the principal has an incentive to create a tournament that determines the promoted employee. We then examine the effect of the employees' productivity on their probability of promotion and on the extent of the resources wasted in the tournament. In particular, we specify the conditions that ensure that the most productive employee (the natural candidate for promotion) is less likely to be promoted and the conditions under which higher employee's productivity results in increased wasted promotion-seeking efforts.

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1. Introduction

The well established tournament literature demonstrates that it might be optimal for a principal to create a tournament when he does not have complete information regarding the employees' productivity. Workers compete in tournaments for promotion and the most productive one is promoted to the higher rung of the firm. Lazear and Rosen (1981), Rosen (1986), Gibbs (1989), Lazear (1996) and others, in papers based on a similar approach, investigated the incentives of prizes that enhance survival in sequential elimination tournaments that result in the selection of the most highly qualified contestant. Success is based on "survival of the fittest", which maintains "quality of play" as the game progresses. Their models identify the unique role of top-ranking prizes in maintaining performance incentives in career and other survival games and, in particular, in encouraging competitors to aspire to further heights, regardless of past achievements. The rationale of a tournament is thus based on its ability to induce the most productive worker to reveal himself via production.

In this paper we consider a fully informed principal whose employees are working at their highest capability. The principal's objective is to maximize a weighted average of two (endogenous) variables; the profitability (productivity) level of his team and the promotion efforts incurred by his employees. Our preliminary and main argument is that under such circumstances the principal may have an incentive to create a promotion tournament, despite the fact that he is completely informed about the productivity of his workers. The first result and its corollaries clarify the required conditions for the rationalization of a promotion tournament in our setting. The main idea behind the proposed tournament rationalization is that it induces the workers to invest effort in promotion-seeking activities that are aimed at and are appreciated by the principal.[1] One could interpret these activities as "influence costs" (see Tullock, 1967, Lockard and Tullock, 2001 and references therein and Milgrom

[1] The crucial point in the interpretation of the tournament in our setting is that the action of the employer is perceived as random by his employees. The perceived randomness could arise because the employees do not know the true preferences of the employer, or because there are other non-foreseeable factors that may impact on the employee's decision. According to this interpretation our assumption that the employer controls the CSF implies that he can transmit information or send signals that effectively determine the perception of the employees regarding the form of his random behavior.

and Robert, 1992), namely, the costs implied by attempts to influence others' decisions in a self-interested fashion, by attempts to counter such influence activities by others, and by the degradation of the quality of decisions because of such effective influence.

The firm often has a typical pyramidal structure: the number of employees decline at higher levels of the internal hierarchy and only one incumbent is situated at the top-level of the hierarchy. Internal tournaments that determine the promoted employees take place on each rung of the firm. A substantial literature has been concerned with the question how managers advance via competition through the ranks of the firm (see, for example, Beckmann 1978). A career path is typically the outcome of competition among peers with the objective of attaining higher rungs and, correspondingly, more remunerative positions during the life cycle. Successful contestants seek more prosperity and occupy themselves with winning further promotion at the expense of production. In the existing literature this is possible because of the ambiguities of measuring the individual’s contribution to output (see Radner 1993). In the current paper, it is shown that even under the extreme situation of no ambiguity regarding the workers' contribution to output, the management practice of holding a promotion tournament can be rationalized.

The analysis in our setting focuses on the effect of the employees' productivity on the probability of their promotion and on the extent of the resources wasted in the tournament. These resources can be viewed as another form of rent-seeking activities examined in the voluminous literature since Tullock’s (1967) seminal paper. In particular, we specify the conditions that ensure that the most productive employee in the team (the natural candidate for promotion) is less likely to be promoted and the conditions under which increased employee's productivity results in increased wasted promotion efforts. In the former case, since the employees differ in productivity (in production and in affecting promotion), the existing incentives in the firm, which are partly determined by the competitive market environment and partly by the principal, can result in senior management that is more likely to be dominated by low-productivity individuals. In the latter case, given the existing incentives in the firm, increased productivity may result in more intense competition in the promotion contest and, in turn, in increased wasteful promotion-seeking activities.

2. Promotion Tournaments: The Rationale

Suppose that two risk-neutral employees seek to maximize expected income over two periods. The firm of these employees has two hierarchical levels.[2] A principal/supervisor who has complete knowledge[3] of the workers' productivity must determine which of the workers to promote. In the first period both workers are on the first rung while in the second period only one of them is promoted to the second rung and the other one remains on the first rung.

The productivity of worker i is denoted by v_i $(i=1,2)$. v_i specifies the absolute productive efficiency per unit of time. To simplify, we assume that a worker's earnings are given by a linear function of his productivity, i.e. his added value to the firm.[4] The productivity of a worker on the second rung is denoted by p_i $(i=1,2)$. With no loss of generality, let worker 1 be more productive than worker 2: $v_1 > v_2$ and $p_1 > p_2$. The increase in a worker's productivity due to promotion is equal to $p_i - v_i$. The increase in productivity is assumed to be higher for the more productive worker, i.e., $p_1 - v_1 > p_2 - v_2$. Without this assumption, promotion of the less productive worker could be plausible because it results in increased profitability.[5]

If the principal wishes to maximize the total productivity of his team or its total profit, then, clearly, he would choose to promote the most productive worker. In this case the productivity/profit of the team in the two periods is:[6] [7]

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[2] Our results can be generalized to a larger number of rungs within a firm where the number of contestants (employees) in a tournament declines while climbing the rungs of the firm's ladder.

[3] The assumption that the principal has complete knowledge is made in order to accentuate our results.

[4] The results remain valid when the worker is assumed to earn some proportion of his contribution to the firm's profits and not his total contribution.

[5] The main results would not change by relaxing this assumption.

[6] To simplify the analysis, we assume that the discount factor is equal to 1.

$$v_1 + p_1 + 2v_2 \quad (1)$$

The fully informed principal's can be interested, however, in creating a contest between the two workers. In this promotion contest the winner is determined by a contest success function. This function transforms the promotion-seeking efforts and the productivities of the employees into their promotion probabilities. Each individual has an endowment of time, normalized to unity, which is allocated to productive and promotion-seeking activities. The promotion-seeking activities are targeted at the principal. A_i and L_i denote, respectively, the time allocated to these activities .That is,

$$A_i + L_i = 1. \quad (2)$$

In the first period the worker's contribution to the firm is equal to:[8]

$$I_i = v_i(1 - L_i) \quad (3)$$

Let Pr_1 be the probability that worker 1 wins the contest and, consequently, in the second period, has a productivity of $p_1(v_1)$ (henceforth p_1). For simplicity, we assume that if the worker loses the contest he works for one more period and does not compete during the second period. With probability $(1- \Pr_1)$ the worker loses the contest and therefore his productivity in the second period is equal to v_1 . The expected contribution of worker i to the firm's output is given by:

$$E(I_i) = v_i(1 - L_i) + v_i(1 - \Pr_i) + p_i \Pr_i = v_i(2 - L_i) + (p_i - v_i)\Pr_i \quad (4)$$

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[7] If the worker obtains δ , $0 < \delta < 1$, of his contribution and the firm obtains the remaining proportion $(1-\delta)$, then the firm's profits are equal to $(1-\delta)(v_1 + p_1 + 2v_2)$. Notice, however, that maximization of $(1-\delta)(w_1 + p_1 + 2w_2)$and $(w_1 + p_1 + 2w_2)$ are equivalent. We therefore disregard δ in our calculations.

[8] We disregard incentives of workers to sabotage each other (Lazear, 1989).

In our setting the principal's utility hinges on both profitability and the employees' promotion-seeking activities. The promotion seeking activities may benefit the principle because they can take the form of direct or indirect resources transferred to him or for example, various forms of activities that affect his status and self-esteem. [9] The principal therefore maximizes a weighted average of the profitability (productivity) level of his team and of the sum of the employees' promotion-seeking efforts. His objective function is thus given by:

$$S(.) = \alpha \left(E(I_1) + E(I_2)\right) + (1-\alpha)\left(v_1 L_1 + v_2 L_2\right) \tag{5}$$

where $\left(v_1 L_1 + v_2 L_2\right)$ is the value of the promotion-seeking activities.

If the principal does not create a contest and promotes the more productive worker, then the value of his objective function is $\alpha\left(v_1 + p_1 + 2v_2\right)$. The principal has an incentive to create a contest, if and only if [10]

$$\alpha \left(E(I_1) + E(I_2)\right) + (1-\alpha)\left(v_1 L_1 + v_2 L_2\right) > \alpha\left(v_1 + p_1 + 2v_2\right) \tag{6}$$

or,

$$\alpha \left(\Pr_1(p_1 - v_1) + \Pr_2(p_2 - v_2)\right) + (1-2\alpha)\left(v_1 L_1 + v_2 L_2\right) > \alpha\left(p_1 - v_1\right) \tag{7}$$

Inequality (6) therefore holds iff

$$\frac{(1-2\alpha)}{\alpha}\frac{\left(v_1 L_1 + v_2 L_2\right)}{\Pr_2} > (p_1 - v_1) - (p_2 - v_2) \tag{8}$$

[9] For a related analysis in the context of rent seeking and public policy, see Epstein and Nitzan (2002).

[10] For "optimal contest design" in the different context of research and labor tournaments that focus on the adverse selection problem associated with the selection of the most highly qualified contestants in auctions, see Fullerton and McAfee (1999) and the references therein

Note that $p_i - v_i$ is the increase in the team's productivity corresponding to the promotion of worker *i*. Whether this condition is satisfied or not hinges on the contest success function (CSF), on the parameters p_i and v_i, on the weights α and $1-\alpha$ assigned by the principal to his two utility components and, in turn, on the resulting equilibrium promotion-seeking activities of the contestants $v_1 L_1 + v_2 L_2$ and, therefore, on their contest winning probabilities $\Pr_1$ and $\Pr_2$. Since $p_1 - v_1 > p_2 - v_2$, given the CSF and the three parameters, the above condition is satisfied if the promotion-seeking activities of the workers are sufficiently large or the contest winning probability of worker 2 is sufficiently low. This simple condition has the following straightforward implications regarding the effect of the parameters:

(i) Inequality (8) requires that $\alpha < 0.5$. That is, a necessary condition for the existence of an effective incentive for a principal to create a tournament is that the weight he assigns to the productivity of the team is lower than the weight assigned to the contestants' promotion-seeking activities, $\alpha < 0.5$. More generally, equation (8) highlights which parameter values of $(1\text{-}\alpha)$ rationalize the tournament, viz., make the creation of the tournament the preferred option for the principal. A sufficiently low level of this parameter implies that a tournament is irrational. In such a case the principal would not bother to create a tournament. If $(1-\alpha)$ is sufficiently high, namely, the principal assigns a sufficiently high weight to the contestants' promotion-seeking activities, then it is sensible for him to create a tournament and act randomly in determining which of the workers to promote. A rational principal who only cares about the profitability of his team will never choose to create such a promotion tournament. If he cares just about extracting tangible rents for himself, that is, if $(1-\alpha)$=1, then creating a tournament is his preferred alternative.

(ii) If the tournament is symmetric in terms of the worker' productivities: $p_1 - v_1 = p_2 - v_2$ and $\alpha < 0.5$, then the principal always prefers to create a tournament rather than promote with certainty the (known) more productive worker. The reason for this is that when both workers are identical, $\Pr_1(p_1 - v_1) + \Pr_2(p_2 - v_2) = p - v$. This means that the principal always gains $\alpha(p - v)$, regardless of who wins the promotion contest. In such a case any tournament that generates positive promotion-seeking efforts is preferred to the 'no contest' alterative, provided that the weight assigned to the workers' promotion-seeking efforts is larger than the weight assigned to the profitability of the firm.

(iii) Preference of a tournament requires the existence of contest equilibrium. In the case of a pure strategy equilibrium, the following first and second order existence conditions, $\frac{\partial \Pr_i}{\partial(v_i L_i)} > 0$, $\frac{\partial \Pr_i}{\partial(v_j L_j)} < 0$ *and* $\frac{\partial^2 \Pr_i}{\partial(v_i L_i)^2} < 0$, are also required for the principal to prefer the seemingly ad hoc random behavior according to the CSF. Notice that these conditions also ensure that as the effect of promotion on a worker's productivity, $p_1 - v_1$, is increased, his promotion-seeking effort as well as his expected payoff are increased.[11]

(iv) The LHS of (6) can be rewritten as: $\beta_1(\Pr_1(p_1 - v_1) + \Pr_2(p_2 - v_2)) + \beta_2(v_1 L_1 + v_2 L_2)$, where $\beta_1 = \alpha$ and

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[11] To ensure that the equilibrium is a pure strategy Nash equilibrium and that it is unique, in addition to the assumed properties of the CSF, we have to add the following requirement: $\Pr_i(1 - \Pr_i)\frac{\partial^2 \Pr_i}{\partial(v_i L_i)\partial(v_j L_j)} + (2\Pr_i - 1)\frac{\partial \Pr_i}{\partial(v_i L_i)}\frac{\partial \Pr_i}{\partial(v_j L_j)} = 0$. This condition is satisfied if $\frac{\partial^2 \Pr_i}{\partial(v_j L_j)\partial(v_i L_i)} \gtreqless 0$ *iff* $\Pr_i \gtreqless 0.5$. This plausible assumption means that worker i has an advantage in terms of ability, if a change in *j*'s effort positively affects his marginal winning probability. In other words, a positive (negative) sign of the cross second-order partial derivative of $\Pr_i$, $\frac{\partial^2 \Pr_i}{\partial(v_j L_j)\partial(v_i L_i)}$, implies that *i* has an advantage (disadvantage) when *j*'s effort changes. Note that this assumption is satisfied by many contest success functions that have been studied in the contest literature.

$\beta_2 = 1-2\alpha$. To satisfy the inequality $\frac{(1-2\alpha)}{\alpha} > 1$, the weight β_1 assigned to the expected increase in the teams productivity $\Pr_1(p_1 - v_1) + \Pr_2(p_2 - v_2)$ must be smaller than the weight β_2 assigned to the promotion-seeking activities $v_1 L_1 + v_2 L_2$, that is, $\beta_1 < \beta_2$. In terms of the parameter α , this necessary condition becomes: $\alpha < 1/3$.

For $\alpha<1/3$, which satisfies the necessary condition for the rationalization of the tournament, let us consider the following simple contest success function: $\Pr_i(.) = \frac{L_i v_i}{L_i v_i + L_j v_j}$ $\forall i \neq j, \quad i, j = 1,2$. Under this CSF, in equilibrium, $L_i^* = \frac{1}{v_i} \frac{(p_j - v_j)(p_i - v_i)^2}{((p_i - v_i) + (p_j - v_j))^2}$ and $\Pr^*_i(.) = \frac{(p_i - v_i)}{(p_i - v_i) + (p_j - v_j)}$. Hence, in equilibrium, the ratio $\frac{(v_1 L^*_1 + v_2 L^*_2)}{\Pr^*_2}$ is equal to $p_1 - v_1$. Consequently, in this case $\alpha < \frac{1}{3}$, which implies that $\frac{(1-2\alpha)}{\alpha} > 1$ is also a sufficient condition for the rationalization of the promotion tournament (see inequality (8)). In other words, if the principal objective function is characterized by $\alpha < 1/3$, then he prefers a tournament based on the assumed contest success function to no tournament (certain promotion of the more productive employee).

3. Productivity and Promotion

Let us now examine the relationship between a worker's productivity and his probability of being promoted, i.e. of winning the tournament.

The objective of the workers is to maximize their expected income by determining the level of investment in promotion-seeking activities.[12] The equilibrium expected income is determined by the Nash equilibrium promotion-

[12] Recall that in our calculations δ is disregarded, see footnote 6 .

seeking efforts of the workers. The interior tournament equilibrium efforts L_1^* and L_2^* are characterized by the following equalities:

$$G_i = \frac{\partial E(I_i)}{\partial L_i} = -v_i + (p_i - v_i)\frac{\partial \Pr_i}{\partial (v_i L_i)} v_i = 0 \ , \ \forall \ i = 1,2 \tag{9}$$

The contest success function is assumed to satisfy the following plausible requirements (see (iv) in the preceding section): $\frac{\partial \Pr_i}{\partial (v_i L_i)} > 0$, $\frac{\partial \Pr_i}{\partial (v_j L_j)} < 0$ and $\left(\frac{\partial^2 \Pr_i}{\partial (v_i L_i)^2} < 0 \right)$. It can be verified that these conditions are sufficient for the existence of tournament equilibrium.

By the first order conditions ((9)), the marginal effect of a unit of investment in promotion-seeking activities on the probability of promotion is equal to:

$$\frac{\partial \Pr_i}{\partial (v_i L_i)} = \frac{1}{(p_i - v_i)} \quad , \forall \ i = 1,2 \tag{10}$$

and the equilibrium workers' investment in promotion-seeking activities satisfy the following equalities:

$$\frac{\partial L_i}{\partial v_i} = \frac{\frac{\partial G_i}{\partial L_j}\frac{\partial G_j}{\partial v_i} - \frac{\partial G_j}{\partial L_j}\frac{\partial G_i}{\partial v_i}}{\frac{\partial G_i}{\partial L_i}\frac{\partial G_j}{\partial L_j} - \frac{\partial G_j}{\partial L_i}\frac{\partial G_j}{\partial L_j}} \text{and} \ \frac{\partial L_i}{\partial v_j} = \frac{\frac{\partial G_i}{\partial L_j}\frac{\partial G_j}{\partial v_j} - \frac{\partial G_j}{\partial L_j}\frac{\partial G_i}{\partial v_j}}{\frac{\partial G_i}{\partial L_i}\frac{\partial G_j}{\partial L_j} - \frac{\partial G_j}{\partial L_i}\frac{\partial G_j}{\partial L_j}} \quad i,j\text{=1,2} \tag{11}$$

The ability of a contestant j to convert effort into probability of winning the tournament can be represented by the marginal effect of a change in his effort on his winning probability. By assumption, this marginal effect is declining with his own effort. A change in his effort also affects, however, the marginal winning probability of his rival i. The rival i has an advantage in terms of ability if a change in j's effort positively affects his marginal winning probability. In other words, a positive (negative) sign of the cross second-order partial derivative of $\Pr_i$, $\frac{\partial^2 \Pr_i}{\partial(L_i v_i)\partial(L_j v_j)}$, implies that i has an advantage (disadvantage) when j's effort changes. At some given combination of efforts $(L_i v_i, L_j v_j)$, the ratio between the effect of a change in j's effort on the marginal winning probability of i and the effect of a change in j's effort on his own ability, $\frac{\partial^2 \Pr_i}{\partial(L_i v_i)\partial(L_j v_j)} \Big/ \left(\frac{\partial^2 \Pr_j}{\partial(L_j v_j)^2}\right)$, is therefore a local measure of the asymmetry between the abilities of i and j. This asymmetry in the equilibrium ability to affect the outcome of the tournament can reflect the higher productivity of a worker, that is, his greater effectivity of turning effort into his probability of winning the tournament or the principal's preferences, that is, his bias in favor of one of the workers. This asymmetry together with two types of payoff-asymmetry that are presented below play a crucial rule in determining the comparative statics effects on which this section focuses.

The questions we would like to pose at this stage are what happens to the promotion-seeking efforts of the workers, to the total loss in the productivity of the team and to a worker's probability of promotion when one of the workers is replaced by a more productive one. Let us first calculate the effect of a change in v_i (a change in the productivity of a worker on the first rung) on the time allocated to promotion-seeking activities by him and by his rival, $\frac{\partial L_i}{\partial v_i}$ and $\frac{\partial L_j}{\partial v_i}$. Denote by η_i the elasticity of the increase in worker i's productivity (his added value to the firm) due to his promotion, $\eta_i = \frac{\partial(p_i - v_i)}{\partial v_i}\frac{v_i}{p_i - v_i}$. By assumption, η_i is positive, i.e., promotion positively affects the worker's productivity.

Let

$$D = (p_i - v_i)(p_j - v_j)v_j v_i \left(\frac{\partial^2 \Pr_j}{\partial(L_i v_i)^2}\frac{\partial^2 \Pr_i}{\partial(L_i v_i)^2} - \frac{\partial^2 \Pr_j}{\partial(L_j v_j)\partial(L_i v_i)}\frac{\partial^2 \Pr_i}{\partial(L_j v_j)\partial(L_i v_i)} \right) \quad (12)$$

Note that since $\Pr_i + \Pr_j = 1$, $\dfrac{\partial^2 \Pr_j}{\partial(L_j v_j)\partial(L_i v_i)} = -\dfrac{\partial^2 \Pr_i}{\partial(L_j v_j)\partial(L_i v_i)}$ and, therefore, *D>0*. By (9), (10) and (11),

$$\frac{\partial L_i^*}{\partial v_i} = -\frac{L_i}{v_i}\left(1+\frac{(p_j - v_j)v_j}{D\ L_i}\frac{\partial^2 \Pr_j}{\partial(L_j v_j)^2}\eta_i\right), \qquad i \neq j,\ \ i,j = 1,2 \tag{13}$$

Since $-\dfrac{\partial^2 \Pr_j}{\partial(L_j v_j)^2}>0$, we obtain that,

(v) Higher productivity of worker i increases his promotion-seeking effort, if the elasticity of the promotion effect on productivity is sufficiently large, specifically, if $\eta_i > \dfrac{1}{\dfrac{(p_j - v_j)v_j}{D\ L_i}\left(-\dfrac{\partial^2 \Pr_j}{\partial(L_j v_j)^2}\right)}$.

An increase in a worker's productivity raises his (alternative) cost of participation in the tournament. To induce the worker to invest in the tournament, his reward in case of winning must be sufficiently large. As worker *i*'s rival becomes more productive, $(p_j - v_j)$ *and* v_j are increased, the lower bound of the elasticity (ensuring that the worker increases his investment in the tournament) declines.

The effect of an increase in the productivity of worker *i* on the promotion-seeking effort of worker *j* is given by:

$$\frac{\partial L_j^*}{\partial v_i} = \frac{(p_j - v_j)}{D}\frac{\partial^2 \Pr_j}{\partial(L_j v_j)\partial(L_i v_i)}\eta_i\,, \qquad i \neq j,\ \ i,j = 1,2 \tag{14}$$

We therefore obtain that,

(vi) $\dfrac{\partial L_i^*}{\partial v_j}$ *is positive (negative) iff* $\dfrac{\partial^2 \Pr_j}{\partial(L_j v_j)\partial(L_i v_i)}$ *is positive (negative).*

In a symmetric contest where $\Pr_i\left(L_i v_i, L_j v_j\right) = 1 - \Pr_i\left(L_j v_j, L_i v_i\right)$ and $v_i > v_j$, $\frac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)} < 0$. If player j has a disadvantage (advantage) in terms of his equilibrium ability (marginal winning probability), that is, $\frac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)} < 0$ (>0), then an increase in worker i's productivity increases the disadvantage (advantage) of worker j and this induces him to reduce his investment in the tournament.

The probability that the more productive worker is promoted.

$$\frac{d \Pr_i}{d v_i} = \frac{\partial \Pr_i}{\partial v_i} + \frac{\partial \Pr_i}{\partial L_i} \frac{\partial L_i}{\partial v_i} + \frac{\partial \Pr_i}{\partial L_j} \frac{\partial L_j}{\partial v_i} \tag{15}$$

Using (9) through (15) we obtain that

$$\frac{d \Pr^*_i}{d v_i} = \eta_i \frac{v_j}{D}\left[\frac{\partial^2 \Pr_i}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)} - \frac{p_j - v_j}{p_i - v_i} \frac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)^2} \right] \tag{16}$$

We therefore get that,

(vii) For $\eta_i > 0$, $\frac{d \Pr^*_i}{d v_i} \begin{matrix} > \\ = \\ < \end{matrix} 0$ *iff* $\frac{\frac{\partial^2 \Pr_i}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)}}{\frac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)^2}} \begin{matrix} < \\ = \\ > \end{matrix} \frac{p_j - v_j}{p_i - v_i}$

This result establishes that the effect of productivity on worker i's probability of promotion hinges on the relationship between the local measure of asymmetry between the abilities of i and j, $\frac{\frac{\partial^2 \Pr_i}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)}}{\frac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)^2}}$ and the asymmetry between the contestants' promotion benefits, $\frac{p_j - v_j}{p_i - v_i}$. If $\frac{\partial^2 \Pr_i}{\partial(L_j v_j)\partial(L_i v_i)} > 0$, that is, if worker i has an advantage over worker j in terms of his equilibrium ability to affect the tournament outcome*,* since, by assumption, $\frac{\partial^2 \Pr_j}{\partial(L_j v_j)^2} < 0$, the local measure of the asymmetry between the abilities of *i* and *j* is negative, $\frac{\frac{\partial^2 \Pr_i}{\partial(L_j v_j)\partial(L_i v_i)}}{\frac{\partial^2 \Pr_j}{\partial(L_j v_j)^2}} < 0$. In such a case as worker i becomes more productive, his probability of winning the promotion is increased. However if worker *j* has an advantage over worker *i* in terms of his equilibrium ability to affect the outcome of the tournament*,* that is, if $\frac{\frac{\partial^2 \Pr_i}{\partial(L_j v_j)\partial(L_i v_i)}}{\frac{\partial^2 \Pr_j}{\partial(L_j v_j)^2}} > 0$, then it is not clear whether worker *i's* probability of promotion is increased. This depends on the symmetry between the contestants' promotion benefits, $\frac{p_j - v_j}{p_i - v_i}$. On the one hand, an increase in v_i decreases the advantage *j* has over *i*. On the other hand, however, worker i's alternative cost of participation in the tournament is increased. The overall effect on worker i's incentive to invest in the tournament is therefore ambiguous.

Finally, let us consider the effect of a change in the productivity of worker *i* on the productivity of the team, $v_i L_i^* + v_j L_j^*$. Using (9) through (14) we obtain that,

$$\frac{\partial\left(v_i L_i^* + v_j L_j^*\right)}{\partial v_i} = \frac{(p_j - v_j) v_j \eta_i}{D}\left[- \frac{\partial^2 \Pr_j}{\partial(L_j v_j)^2} - \frac{\partial^2 \Pr_i}{\partial(L_j v_j)\partial(L_i v_i)}\right] \qquad (17)$$

Hence,

(viii) For $\eta_i > 0$, $\dfrac{\partial\left(v_i L_i^* + v_j L_j^*\right)}{\partial v_i} \begin{array}{c} > \\ = \\ < \end{array} 0$ *iff* $\dfrac{\dfrac{\partial^2 \Pr_i^*}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)}}{-\dfrac{\partial^2 \Pr_j^*}{\partial\left(L_j v_j\right)^2}} \begin{array}{c} < \\ = \\ > \end{array} 1$

By this result, since, $\dfrac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)^2} < 0$, if worker *i* has a disadvantage in terms of his equilibrium ability to affect his probability of promotion, that is, if $\dfrac{\partial^2 \Pr_i}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)} < 0$, then an increase in worker *i's* productivity reduces his disadvantage and the total amount of resources invested in the tournament is increased. On the other hand, if worker *i* has an advantage in terms of his equilibrium ability to affect his probability of promotion, that is, if $\dfrac{\partial^2 \Pr_i}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)} > 0$, then an increase in worker *i's* productivity that further increases his advantage may reduce the total investment in the tournament. Whether the resources invested in the tournament are increased or decreased depends on whether the value of the local measure of the asymmetry between the abilities of *i* and *j,* $\dfrac{\dfrac{\partial^2 \Pr_i}{\partial\left(L_j v_j\right)\partial\left(L_i v_i\right)}}{\dfrac{\partial^2 \Pr_j}{\partial\left(L_j v_j\right)^2}}$ is larger or smaller than unity. In other words, this condition establishes that total effort is increased (decreased), if a change in *j*'s effort has a stronger (weaker) effect on the marginal promotion probability of worker *i* than on his own marginal probability of securing promotion. An alternative way of looking at this result is to consider the question: how does an increase in the variance of the productivity in the team (an increase in v_1 or a decrease in v_2) affects the total effort invested in the promotion-tournament. By (*viii*), an increase in the productivity variance increases the total effort invested in the contest if there is a sufficient asymmetry between the workers' abilities to affect the probability of promotion.

We conclude the comparative statics analysis with the following two special cases of zero and negative elasticities.:

a. When $\eta_i = 0$, the increase in productivity in both rungs are identical, that is, $\frac{\partial p_i}{\partial v_i} = 1$. In such a case if worker i becomes more productive, he lowers his investment in the tournament, $\frac{\partial L_i^*}{\partial v_i} < 0$. The reason for this result is that the cost of participation in the tournament becomes larger while the benefit from winning does not change. By (14), the rival, worker j, does not change his promotion-seeking effort, $\frac{\partial L_j^*}{\partial v_i} = 0$. The probability that worker i is promoted is not affected, $\frac{d \Pr^*{}_i}{d v_i} = 0$, and the total amount of resources invested in the tournament remains unchanged, $\frac{\partial \left(v_i L_i^* + v_j L_j^*\right)}{\partial v_i} = 0$. In this case the decline in L_i, corresponding to the increase in v_i is equal in absolute terms to the increase in v_i, which leaves $L_i\, v_i$ unchanged.

b. When $\frac{\partial p_i}{\partial v_i} = 0$, $\eta_i < 0$. In this situation, an increase in the productivity of worker i does not affect the productivity in the higher rung. In such a case worker i reduces his promotion-seeking effort. The alternative cost of participation in the tournament increases with an increase in v_i , while the benefit associated with promotion is reduced, $\frac{\partial (p_i - v_i)}{\partial v_i} = -1$. Consequently, $\frac{\partial L_i^*}{\partial v_i} < 0$. Worker j also reduces his effort, $\frac{\partial L_j^*}{\partial v_i} < 0$. If worker i has an advantage over worker j, then the probability that worker i wins the tournament declines and the total resources invested in the tournament are reduced. The situation may characterize state owned firms. In such firms wages in the higher rungs are often fixed (linked to the salaries of public officials), i.e., wages are independent of productivity. In such a situation we may well observe that the more productive workers have a lower probability of being promoted as their alternative cost of participation in the tournament increases with their productivity while the benefit from success, p_i-v_i, decrease with an increase in productivity.

4. Concluding Remarks

Tournaments are usually considered as efficiency-enhancing mechanisms that can successfully cope with the problem of identifying the more productive workers under incomplete information regarding individual skills. Incentives to perform are created by the increased benefit associated with the achievement of a higher rank. The effectivity of a tournament in eliciting information about productivity hinges on its appropriate design and on the production function. Our alternative rationalization of tournaments is based on the recognition that the benefit of principals often derives not only from the profit of the firm, but also from the promotion-seeking efforts of their employees. This alternative complementing rationalization of the common management practice of holding competition for promotion can be valid even when the principal has complete information on skills. The rationale, then, behind a tournament may not be the principal's lack of information but rather his mixed objectives. In other words, the existence of a tournament can be due to more than meets the eye of a reader of the vast tournament literature.

The effectivity of the proposed rationalization hinges on the weight assigned to the promotion-seeking efforts relative to the weight assigned to the profitability of the firm, on the effect of promotion on individual productivity and on the contest success function. Our first four results ((i)-(iv)) provide necessary and sufficient conditions in terms of these parameters for a promotion tournament to be preferred from the principal's viewpoint to certain promotion of the more productive employee. The remaining four comparative statics results ((v)-(viii)) focus on the effect of increased individual productivity on the individual workers' promotion-seeking efforts, on their probability of promotion and on the aggregate efforts invested by the workers in the promotion tournaments. These conditions clarify under what circumstances the less productive employees in the team are more likely to be promoted and under what conditions higher individual productivity results in increased wasted promotion-seeking efforts.

Usually, principals have incomplete information on the productivity of their workers and profit maximization is not their only concern. This implies that, in general, the existence of tournaments reflects both the principals' environmental constraint (the uncertainty regarding potential and realized productivity) and their mixed motives (the benefit derived from profits and from promotion-seeking efforts). One interesting question is how can one deduce from empirical data on tournaments to what extent they reflect the two possible rationalizations. Another challenging task is the design of an optimal tournament that takes into account the two possible rationalizations. The answer to these empirical and theoretical issues is beyond the scope of this paper and is left for future research.

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